Rolling with slipping experiment
Virtual Laboratory Guide

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We recorded the motion of a ring rolling and slipping in front of a gridded panel, whose cells allow to take measurements of the angular and linear ring positions. From the angular and translational velocity graphs, the dynamics of the ring motion can be deduced.

Important: The experiment was recorded four times, and from each take we selected three sets of frames. You should analyse the set of frames assigned to your team by the professor. Each set of frames is identified by the number of the take (01 to 04) and a letter (A, B or C). Although you should acquire the data from just one set of frames, named "situação 01-A" or "situação 03-B", etc., it will be useful to inspect all the frames from the take.

1 Analysis procedure

1. Watch the video snippet and observe the peculiar motion of the ring, in particular:

    - The initial linear motion of the ring, its direction and its changes along the experiment.
    - The initial angular motion of the ring, in special its direction of rotation (clockwise or counter-clockwise), and the evolution of the angular velocity during the experiment.
    - If the ring velocity goes to zero sometime.

Before starting the quantitative analysis of the ring motion as directed below, watch once more the video snippet, and inspect all the frames from the take assigned to you. However, acquire data just for the set of frames assigned to you.

2. Fill a spreadsheet containing, for each frame i, the values of linear position, x_i, angular position, θ_i, and time, t_i, considering that the cells on the background gridded panel are squares of 2 cm side and the time code is in seconds. The procedure to obtain these values are detailed in section 3 below. The uncertainty in time is negligible, but adopt 1.0 cm and 0.04 rad for the uncertainties in the linear and angular positions, respectively; you can find more information about uncertainties at section 4 if you wish, but that is not required by now. **Important:**

   - Note that there is an instant when the translational movement ceases and the ring rotates apparently in the same position during some time. The fact that the position is the same does not mean that the data are identical, because the *times* are different; register the same position for the different time codes in your data table.
   - During all the data acquisition, keep the reference system fixed to the adopted origin and pointing to the adopted direction, even after the ring did reverse its motion.

3. Plot the graphs of x_i and θ_i in function of time and draw the respective uncertainty bars.

4. With the collected data, compute linear and angular average velocities for each time interval $[t_{i-1}; t_{i+1}]$. The formulas are

$$\overline{v}_{[t_{i-1};t_{i+1}]} = \frac{x_{(i+1)} - x_{(i-1)}}{t_{(i+1)} - t_{(i-1)}} \tag{1}$$

   and

$$\overline{\omega}_{[t_{i-1};t_{i+1}]} = \frac{\theta_{(i+1)} - \theta_{(i-1)}}{t_{(i+1)} - t_{(i-1)}} \tag{2}$$

   where x, θ and t are the positions read in item 2 above.

   As $[t_{i-1}; t_{i+1}]$ intervals are short, we will adopt

$$\overline{v}_{[t_{i-1};t_{i+1}]} \cong v(\bar{t}_i) \tag{3}$$

$$\overline{\omega}_{[t_{i-1};t_{i+1}]} \cong \omega(\bar{t}_i) \tag{4}$$

where $v(\bar{t}_i)$ and $\omega(\bar{t}_i)$ correspond to linear and angular instantaneous velocities, respectively, in the mean time $\bar{t}_i$ given by[1]

$$\bar{t}_i = \frac{t_{i+1} + t_{i-1}}{2} \tag{5}$$

Fill a spreadsheet with the obtained values ($\bar{t}_i$, $v(\bar{t}_i)$ and $\omega(\bar{t}_i)$), with their respective uncertainties. If you need help with the uncertainties propagation, see the text "Roteiro de Cálculo de Incertezas" (Uncertainties calculation roadmap, in Portuguese) in the flap "Guias".

5. Plot the calculated speeds with their respective uncertainties in function of time and *check whether your expectations of their evolution match what you have obtained*. Interpret the results on view of the dynamics of the motion and identify regions with different behaviours. From the kinematics, deduce the time intervals when the resultant force has different values. Evaluate and plot the linear and angular velocity tendency lines for each stage of the motion. Which physical magnitudes can be obtained from the slopes of the tendency lines?

6. From the velocities graphs, try to identify the instant when the ring begins the rolling without slipping motion. Think about the procedure you can use to determine this instant.

7. Try to find answers for the questions below, that will be asked in your homework. Notice, however, that you do not need to get them right, since this content was not yet explored in the lecture classes.

   i. Does the ring move to the left or to the right at the start? Does it change direction along the experiment?

   ii. Does the ring rotates clockwise or counter-clockwise at the start? Does it change the direction of rotation during the motion?

   iii. When or where the ring has zero speed? Search an explanation when and why does it happen.

[1] What we are doing, in fact, is to evaluate numerically the speed as the derivative of the position with respect to time. For more information, see the text "Como calcular a derivada de uma função numericamente" (How to compute numerically the derivative of a function) in the flap "Guias" of the virtual experiment site.

iv. Identify any forces acting on the ring. Make two diagrams of the ring, one for the forward motion and the other for the backward motion, and represent the forces acting on it, as well as the linear and angular motion directions in both situations.

v. Find the sets of points where the ring is moving with a constant translational acceleration and interpret the slope of the tendency lines for each region. Do the same for the angular motion.

vi. Try to develop mathematical expressions to describe the translation and rotation speeds in function of time. Pay attention to the time ranges in which the expressions are valid.

2 Homework

Every team should hand in a report. We stress that you will get good marks if you accomplished the tasks and use *your own words* in the writing. The report should reflect your thinkings about the subject. By now, interpretation accuracy is not being graded.

The report must consider the following items:

I. **Introduction**: Describe briefly the experiment, the arrangement used and its objectives. Provide the answers to questions i and ii in the last topic of the preceding section, using *your own words*.

II. **Data analysis**: Identify the set of frames assigned to your team, and give the corresponding tables and graphs with:

- the measured positions (x_i and θ_i along with t_i) and
- the calculated velocities ($\overline{v}_i$ and $\overline{\omega}_i$ for all $\overline{t}_i$), with the respective tendency lines.

III. **Discussion**: Provide the answers of items iii to vi in the last topic of the preceding section — *your findings in your own words.*

IV. **Conclusion**: State what you have discovered and understood about the rolling with slipping motion. If you find connections with the main objectives presented in the Introduction, show them. Explain, if possible, why this experiment was useful (or not) to your study and if it gave a hint on the underlying physical laws. If you have an idea for a different analysis procedure for this experiment or even for another experiment on this subject, explain it.

3 Data acquisition

Before you start taking data, it is interesting to scan all frames of the take assigned to you, in order to get a feeling about the motion, which is easier when seeing a sequence of images taken with smaller time intervals. After that, restrict yourself to the assigned set of frames, and measure first the linear positions and then the angular positions.

3.1 Reading linear position

- Observe in figure 1 the presence of two strips attached to the ring and intersecting perpendicularly at its center. Use the intersection point, P, to track the ring translational motion.

- To begin with the measurement procedure you should choose, in the background gridded panel, the origin of the reference system; see figure 1 for an example. The back grid provides the measurement unities: blue squares have sides of 10 cm and red squares, 2 cm. It is worth remarking that the reference system origin O for the linear positions must remain fixed throughout the experiment.

- The consecutive linear positions are obtained measuring the distance from P in each frame to the adopted origin, as in the example of figure 1.

3.2 Reading angular position

Any fixed point on the ring can, in principle, be used to follow its rotation, and the quadrant where the angle is measured is arbitrary. Since it is easier to follow the motion of one line than one point, we suggest to refer the angular position to a half of one of the strips — the *guide radius*. Also, we have found that the measurement of the position of the guide radius in any quadrant but the first is prone to error. We suggest, therefore, to measure the angle formed between the Px' axis and the strip that is seen on the first quadrant — the

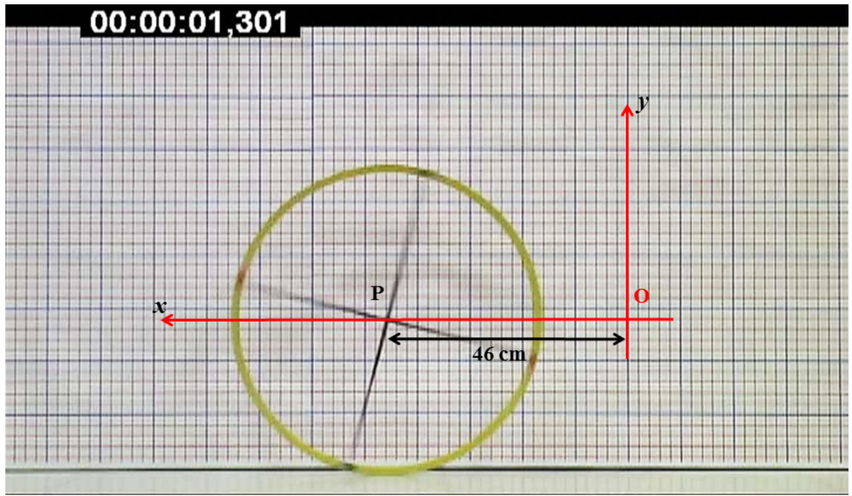

Figure 1: Distances must be measured from the origin O to the center of the ring, P, in this frame 23 red squares = 46 cm. The time is 1,301 s.

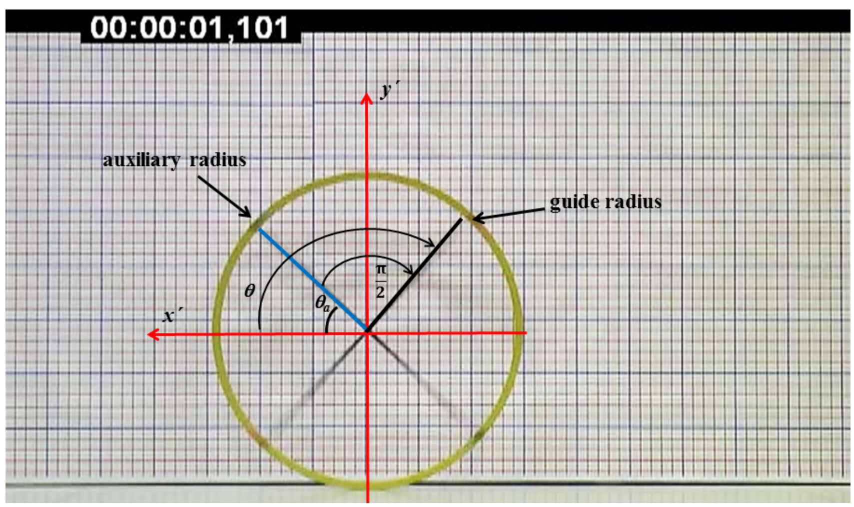

Figure 2: The angular position of the guide radius in the second quadrant can be obtained from the angular position of the auxiliary radius by adding $\frac{\pi}{2}$ to the value measured in the first quadrant.

auxiliary radius — and add the angle it forms with the guide radius, always a multiple of $\pi/2$, see Figure 2.

Below, we list the tasks needed to obtain the angular position in function of time.

- The angle θ_a between the auxiliary radius and the axes Px' can be obtained from the orthogonal projections of the auxiliary radius using the cells on the gridded back panel, see figure 3, and is given by

$$\tan\theta_a = \frac{y'_a}{x'_a}$$

Figure 3 highlights the relationship between θ_a and the number of cells in each x'-y' projection of the auxiliary radius.

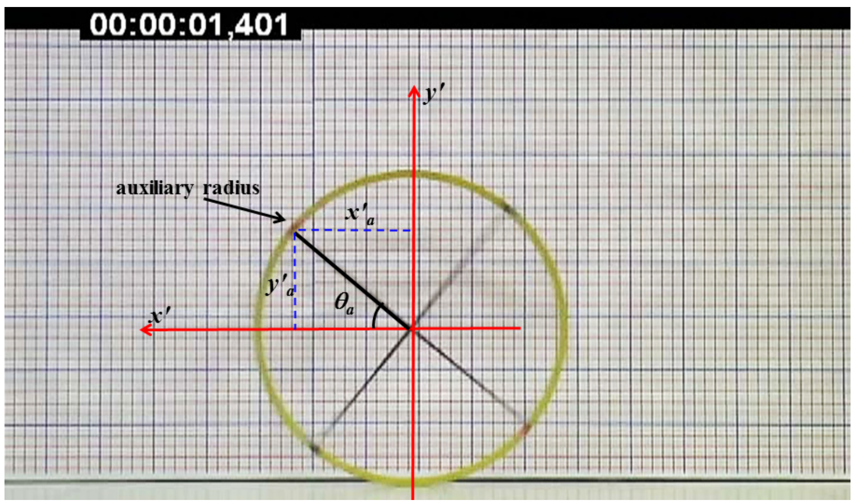

Figure 3: x'_a and y'_a auxiliary radius projections on x' and y' axes, respectively.

- Prepare a spreadsheet to list the raw data and deduce the angles. Reserve columns for t, x'_a and y'_a, since for each frame you will count and register the number of cells of x' and y' auxiliary radius projections, besides the time code.

- The angles will increase with time because the ring rotates always in the same direction, but when the measurement is done in a single quadrant, you must correct for the angle between the auxiliary and guide radii, as illustrated by figures 2 and 4. When q designates the quadrant occupied by the guide ray, this angle is

$$\Delta\theta_q = (q-1)\frac{\pi}{2}$$

Also, the number of turns completed, n, must be taken into account, and generates another angular displacement

$$\Delta\theta_n = 2\pi\, n$$

Therefore, add two more columns to the spreadsheet to accommodate q and n.

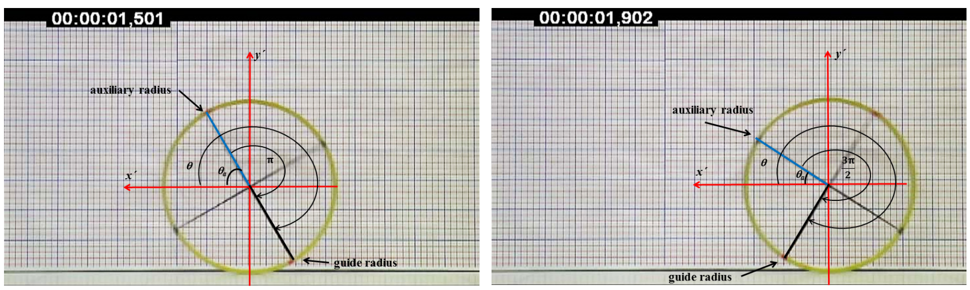

Figure 4: Corrections for measurements in the first quadrant when the guide-ray is in the third quadrant (left) and fourth quadrant (right).

- In the last column of the spreadsheet, evaluate the angle according to

$$\theta = \arctan\frac{y'_a}{x'_a} + (q-1)\frac{\pi}{2} + 2\pi\, n$$

where q is the number of the quadrant where the guide radius is, and n is the number of completed turns. Table 1 shows an example of how to tabulate the data.

You should verify that all the angular positions are in ascending order (as in Table 1).

Table 1: Spreadsheet model and data organization. The symbols defining the columns are explained in the text.

t (s)	x'_a	y'_a	q	n	θ (rd)
0.067	13	7			0.49
0.1	9.5	10.5	1	0	0.84
0.133	5.5	14			1.2
0.167	14	1			1.64
0.2	13	6			2.00
0.234	9.5	10	2	0	2.30
0.267	5.5	13			2.74
0.3	0.5	14			3.11
0.334	14	5			3.48
0.367	10.5	9	3	0	3.85
0.4	7	12			4.18
0.434	2.5	14			4.54
0.467	13.5	2.5			4.90
0.5	12.5	6			5.16
0.534	10	10	4	0	5.50
0.567	6.5	12			5.79
0.601	2.5	13.5			6.10
0.634	13.5	2			6.43
0.667	12.5	5.5			6.70
0.701	10.5	9	1	1	6.99
0.734	8	11.5			7.25
0.767	4.5	13			7.52
0.801	0.5	14			7.82
0.843	13.5	3.5			8.11
0.868	11.5	7			8.40
0.901	10	9	2	1	8.59
0.934	7.5	12			8.87
0.968	4	13			9.13
1.001	1	14			9.35
1.034	13.5	2			9.57
1.068	12.5	5	3	1	9.81
1.101	11.5	7			9.97
1.134	9.5	10			10.24

4 Uncertainties

4.1 Uncertainty in position measurements

The uncertainty in linear position was estimated considering the gridded background. The small cells are 2 cm wide, an the crossing of the strips is often blurred, therefore the minimum readable unit is somewhere between 1 and 2 cm. Assigning a confidence level of 95 % to this reading, a reasonable value for the standard deviation would be 0.5 to 1.0 cm. When we analysed all frames of all takes, we have found that the higher of these values represent better the standard deviation. We suggest, therefore, using 1.0 cm for the linear position uncertainty, that represents an overall average.

4.2 Angular position uncertainty estimate

The angular position uncertainty can be estimated considering the gridded background panel. As the numbers of cells in x and y directions are used to compute the angular position $\left(\theta = \arctan \frac{y'_a}{x'_a}\right)$, the uncertainty in θ can be determined by the usual propagation formula:

$$\sigma_\theta^2 = \left[\frac{\partial \theta}{\partial y}\sigma_{y'_a}\right]^2 + \left[\frac{\partial \theta}{\partial x}\sigma_{x'_a}\right]^2$$

From this expression, we find, after some algebra

$$\sigma_\theta^2 = \left[\frac{x'_a}{{x'_a}^2 + {y'_a}^2}\right]^2 \sigma_{x'_a}^2 + \left[\frac{y'_a}{{x'_a}^2 + {y'_a}^2}\right]^2 \sigma_{y'_a}^2$$

Adopting $\sigma_{y'_a} = \sigma_{x'_a}$ and noting that ${x'_a}^2 + {y'_a}^2 = R^2$, the expression simplifies to

$$\sigma_\theta = \frac{\sigma_{x'_a}}{R} \cong 0.04 \text{ rd} \tag{6}$$

where we adopted $\sigma_{x'_a} = 1.0$ and rounded the result to one significant figure.

4.3 Uncertainty in the slope of a tendency line

Spreadsheets evaluate the tendency line parameters using the least-squares method assuming that all data have the same weight, which is a good assumption in this experiment. However, the determination of the uncertainty in the slope of the tendency line requires some calculations in the framework of the least squares method. Here we show a shortcut if you don't want to go through these calculations. When all coordinate values have the same standard deviation, σ_{speed}, the abscissas are equally spaced, and the number of data points, N, is reasonably big, a good estimate of the standard deviation of the slope is given by

$$\sigma_c = \frac{\sigma_{speed}\sqrt{12}}{T\sqrt{N}}, \tag{7}$$

where T is the time interval sampled, i.e., the difference between the maximum and minimum values of the abscissa values.

5 Ring mass, size and rotational inertia

In table 2, you will find the parameters of the ring used in the experiment and are important to explain its mechanical behaviour.

Table 2: Size and inertia parameters of the ring used in the experiment.

Parameter	Value
Internal radius (R_i)	(27.1 ± 0.1) cm
External radius (R)	(28.6 ± 0.1) cm
Moment of Inertia — CM axis	$(288 \pm 5) \times 10^3$ g cm^2
Mass	(381.3 ± 0.5) g

The radii were obtained with a measuring tape and the mass with a balance. The moment of inertia was calculated from the ring dimensions and shape, neglecting small holes for the rays and the air valve in the bicycle hoop we have used. It was calculated as the sum of rings with different masses and radii, filling all the range $R_i < r < R$ and the uncertainty evaluated as 1/6 of the extreme values - all mass located in the internal or the external radius.