Laboratory guide of the “Variable Friction Force Virtual Experiment” – Part I

A) Introduction

We recorded the motion of a pierced coin sliding either on an inclined transparent acrylic panel, placed over a checkered paper, or directly along the checkered paper, which allows measuring the coordinates of its two-dimensional trajectory. From this video, we selected frames that allow following the coin displacement. The purpose of the experience is to provide data for the study of the motion of a body under the action of variable forces, in particular, to understand the relation between the magnitudes involved with motion.

This experiment is divided in two parts. In the first, the kinematical quantities and the friction force in several points of the trajectory of the coin will be determined. In the second part, we will use these data to elaborate a theoretical model that can predict the coin trajectory, that will be compared with the experimental one.

B) Analysis procedure

B1. As the recorded experimental arrangement is made of real things, the friction between the coin and the plane cannot be ignored. Watch the video on the “*Apresentação*” flap and just guess about the questions below. Take notes of your hypotheses and inquiries.

i. What is the shape of the trajectory of the coin?

ii. Is it possible to predict the trajectory of the coin? Which are the physical quantities you would have to know to predict the motion of the coin? Try to elaborate a reasoning based on Newton's laws and the empirical laws of contact friction to describe your observations.

B2. Watch the videos of the experimental cases with attention (they are in the “*Vídeos”* flap, from the page “*Filmes e Quadros*”). Check whether the movement happened according to your predictions.

B3. Observe the images of the case assigned to you, which should be similar to that on Fig. 1. The checkered paper has thick lines 1 cm apart, forming big squares, and little squares made by thin lines 0.2 cm apart. Notice that you do not need to use SI units, and, in this experiment, it is more natural to measure distance in centimetres. Anyway, register the adopted unit in your spreadsheet, since you may use other units in different parts of this analysis.

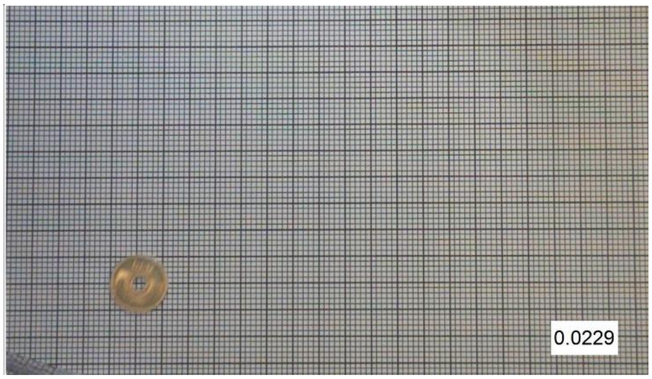

Figure 1. One of the images of the coin over the gridded panel in case A1.5. Time is coded in the box at right, $t_i = 0.0229$ s. See text for the other details.

Choose a *x*O*y* reference system to read the coordinates of the coin in each instant of time. It is better to choose the origin as the crossing point of two thick lines near the place where the motion of the coin begins.

Henceforward, x identifies the horizontal coordinate oriented to the right and y, a coordinate on the plane perpendicular to Ox, oriented upwards.

Open a spreadsheet in the computer, and prepare to insert the experimental data. Number the successive images by integer numbers i in sequence starting from 1. Make a table with the instants of time t_i and the respective positions $x(t_i)$ and $y(t_i)$ of the coin center for every image i. To facilitate the reading procedure, the mouse cursor has a circular shape with two hairs crossing: centre it on the coin and use its cross lines to determine the coordinates. Adopt half of the minor division that you can read on the gridded paper as the standard deviation of the position measurement (we suggest to adopt $\sigma_x = \sigma_y = 0{,}04\ cm$). Ignore the uncertainty in time.

B4. Plot the coin coordinates $x(t)$ and $y(t)$ in function of the time. Remember to include the uncertainty bars. Plot the trajectory, y vs. x, remembering that in this case the x and y scales must be the same.

B5. In the spreadsheet built on item **B3**, compute the components of the mean velocity of the coin, $\bar{v}$**,** in the time interval $t_{i-1} \leq t \leq t_{i+1}$, given by:

$$\bar{v}_x(t_{i-1} \leq t \leq t_{i+1}) = \frac{x(t_{i+1}) - x(t_{i-1})}{t_{i+1} - t_{i-1}} \tag{1}$$

$$\bar{v}_y(t_{i-1} \leq t \leq t_{i+1}) = \frac{y(t_{i+1}) - y(t_{i-1})}{t_{i+1} - t_{i-1}} \tag{2}$$

where x and y are the position coordinates of the coin, t the instant of time and i, the number of the image. The notation $x(t_{i+1})$ in the equation means the horizontal position of the coin in the instant of time t_{i+1}.

B6. The mean velocity given by eqs. (1-2) is approximately equal to the instantaneous velocity in the mean time $\bar{t}_i$, which turns to be the time when the image between t_{i+1} and t_{i-1} was taken, since the time interval between successive images is always the same. In equation form,

$$\bar{t}_i = \frac{t_{i-1} + t_{i+1}}{2} = t_i \tag{3}$$

For this reason, we can adopt the approximation:

$$\bar{v}_x(t_{i-1} \leq t \leq t_{i+1}) \cong v_x(\bar{t}_i) = v_x(t_i) \tag{4}$$

$$\bar{v}_y(t_{i-1} \leq t \leq t_{i+1}) \cong v_y(\bar{t}_i) = v_y(t_i) \tag{5}$$

where $v_x(t_i)$ and $v_y(t_i)$ correspond to the instantaneous linear velocity projections in instant t_i. This approximation is very good, because the interval $t_{i-1} \leq t \leq t_{i+1}$ is very small. Finally, notice that it is not possible to compute the velocity for the first and last images, as can be observed in the Table 1.

B7. Determine the velocity magnitude of the coin in each instant, with:

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} \tag{6}$$

B8. Plot the projections and the magnitude of the velocity in function of the time, $v_x(t)$, $v_y(t)$ and $v(t)$, respectively. Remember to draw the uncertainty bars.

B9. Determine the F_x and F_y projections of the resultant force $\overrightarrow{F_R}$:

$$\overrightarrow{F_R} = F_x\hat{\imath} + F_y\hat{\jmath} \tag{7}$$

To do so, compute the mean acceleration, with expressions similar to eqs. (1 – 5), and use Newton's 2nd Law to obtain the O*x* and O*y* projections of the resultant force, just by multiplication of the acceleration by the mass of the coin, given in the table "*Massa da Moeda*", which shows an image of the mass measurement on the flap "*Filmes e Quadros"*.

B10. Compute the magnitude of the resultant force in each instant, using:

$$\left|\vec{F}_R\right| = \sqrt{F_x^2 + F_y^2} \tag{8}$$

B11. Build the graphs of the O*y* projection of the resultant force, $F_y(t)$, and of the intensity of the resultant force on the coin, $\left|\vec{F}_R\right|(t)$. Remember to draw the uncertainty bars.

B12. Find the angle formed by the plane with the horizontal, θ, in the table "*Inclinação do Plano"* on the flap "*Filmes e Quadros*", that links to the images of the measurements made with a protractor. Pay attention that different protractors were used, which adopt different references in the measurement (some angles were measured with respect to the vertical, others, to the horizontal). In the calculations below, it is assumed that θ is the angle formed between the inclined plane and the horizontal.

B13. Consider the resultant force:

$$\overrightarrow{F_R} = \overrightarrow{f_k} + \vec{N} + \vec{W} \tag{9}$$

where $\overrightarrow{f_k}$, $\vec{N}$ and $\vec{W}$ are the friction, normal and weight forces, respectively. Once obtained their respective projections on O*x* and O*y* axes, determine the projections of the friction force, given by:

$$f_{kx} = F_x \tag{10}$$

$$f_{ky} \quad = F_y + mg\sin\theta \tag{11}$$

where m is the mass of the coin and g the intensity of the gravitational acceleration (use $g = 9{,}786$ m/s^2).

Note that, in Equation (11), the sign of $mg\sin\theta$ comes from the orientation chosen for the coordinate axis (paragraph **B3)**; if you have adopted other orientation, you will obtain a different sign. In the uncertainty propagation process, you can neglect the uncertainties in m, g and θ.

B14. Determine the friction force magnitude:

$$f_k = \left|\vec{f}_k\right| = \sqrt{{f_{kx}}^2 + {f_{ky}}^2} \tag{12}$$

B15. Plot the magnitude of the friction force in function of the time, $f_k(t) = \left|\vec{f}_k(\bar{t})\right|$. Remember to draw the uncertainty bars.

Until now you should have built a table similar to this one:

Table 1. Model of the table built for the analysis of the launching of the coin. There is no need to include another column for $\bar{t}_i$, because $\bar{t}_i = t_i$ for every i.

	t (s)	**x** (cm)	**y** (cm)	$\mathbf{v}_x$ (cm/s)	$\mathbf{v}_y$ (cm/s)	**v** (cm/s)	$\mathbf{a}_x$ (cm/s²)	$\mathbf{a}_y$ (cm/s²)	$\mathbf{F}_x$ (g.cm/s²)	$\mathbf{F}_y$ (g.cm/s²)	$\mathbf{F}_R$ (g.cm/s²)	$\mathbf{f}_{at,x}$ (g.cm/s²)	$\mathbf{f}_{at,y}$ (g.cm/s²)	$\mathbf{f}_{at}$ (g.cm/s²)
	###	###	###											
	###	###	###	###	###	###								
	###	###	###	###	###	###	###	###	###	###	###	###	###	###
	###	###	###	###	###	###	###	###	###	###	###	###	###	###
	###	###	###	###	###	###	###	###	###	###	###	###	###	###
	###	###	###	###	###	###								
	###	###	###											
standard deviation	0	0,04	0,04	###	###	###	###	###	###	###	###	###	###	###

C) Procedure to elaborate the brief report

In this first stage, you must deliver only a short description of your experimental results, that will be checked, corrected and returned to you, in order to ensure a good basis for the final report. In the flap "*Apresentação"* of the experiment page, there is a template of the document for this synopsis with the required sections.

C1. *Identification*: list the names of the group members (or write your name, if you worked alone) and the code corresponding to the case analysed.

C2. *Initial Expectations:* make a brief description of the experiment and register predictions and expectations found after thinking about the experiment, as suggested in paragraph **B1**.

C3. *Obtained Data:* include the plots made as asked in paragraph B4 as well as their interpretation. Check that you identified the axes with the correct units, with the appropriate number of significant digits, and included the uncertainty bars.

C4. *Data Analysis:* present the table with your measured data and calculations; it should be similar to that shown in **Table 1** above. Include the graphs from items **B8**, **B11** and **B15,** and describe them (say if the quantity varies monotonically or there is a maximum/minimum clearly visible; if the quantity is null or changes abruptly at some time; if the experimental values fluctuate much with respect its mean value, etc). Verify the units of the quantities and check whether they are represented with the suitable number of decimal places. Check that you plotted also the uncertainty bars.

C5. *Discussion:* explain why and how – by now, it is not worth to tell if you found the corrected answers according the theoretical formalism. Use your own words and ideas, rescuing, when necessary, comments of paragraph **C2**.

i. Describe the trajectory of the coin over the inclined plane. Comment if your prevision on item **B1** was correct, but note that this comment is not going to be evaluated – it was the enticing question, and we wonder if it has motivated you.
ii. List the quantities that influence the trajectory of the coin.
iii. List the quantities that, if modified, change the trajectory shape. What would be the change in each case?
iv. List the forces that act in the coin all along the motion. Sketch the free-body diagram and explain, with your own words, how the friction force vector evolves along the track.

v. Looking at the forces you listed in item **iv** above, which of them remains constant? Which of them vary? Remember that force is a vector.