“Variable Friction Force” Experiment Script - Part II

A) Introduction

In the first part of the experiment, we determined the physical quantities related to the movement of the coin, including the resistive force along its path on the inclined plane. It should now be clear that the trajectory is not a parabola, because the net force on the coin varies.

In this second part, we will elaborate an equation of motion, find its solution and explain the observed trajectory. Tests of theoretical hypotheses about the magnitude and direction of the resistive force will allow us to investigate whether it can be interpreted as the kinetic friction force, according to Amonton's laws. The results obtained will provide the basis for a theoretical model for the equations of motion, with which we will calculate the *x* and *y* positions as functions of time. Finally, fitting the model parameters to the experimental trajectory and comparing the calculated and observed trajectories will reveal whether the model is adequate.

B) Analysis procedure (Continuation)

Amontons' laws on dry contact friction, applied to a moving body, determine the vector characteristics of the friction force, $\vec{f}_{at}$: its direction is aligned with the velocity vector $\vec{v}$, but in the opposite direction to the displacement, and its magnitude is constant. So:

$$\vec{f}_{at} = \mu_c mg \cos\theta \left(-\frac{\vec{v}}{v}\right) \tag{10}$$

where μ_c is the coefficient of kinetic friction and $N = \left|\vec{N}\right| = \left|\vec{P}\right| \cos\theta = mg\cos\theta$ is the magnitude of the normal force and θ the angle of inclination of the surface on which the coin slides with respect to a horizontal plane.

In order to verify that these laws are applicable, the resistive force $\vec{f}$ will be subjected to a statistical hypothesis test against the $\vec{f}_{at}$ of equation (10). Item **B15** describes a way to investigate whether the magnitude of the resistive force is compatible with a constant value. Corroborating the opposition between resistive force and velocity vectors requires an elaborate calculation, described in item **B16**.

B15. ***Test of hypothesis I, on the resistive force module.*** Retrieve the resistive force magnitude graph f as a function of time, constructed in Part I, and draw a line parallel to the time axis at the mean value of this force. Check that the distribution of points around the mean value is consistent with a resistive force of constant magnitude, as per eq. (10); the appendix to this guide suggests a methodology for performing this comparison. Once confirmed that the measured values for the resistive force are compatible with a constant value approximately equal to its average value, $\bar{f}$, use this average value and the known values of m, g and θ to determine a first estimation $\mu_e = \bar{f}/(mg\cos\theta)$ for the coefficient of friction.

B16. ***Test of hypothesis II, on the direction of the resistive force***. The components of velocity and resistive force on the coin were calculated in Part I. Figure 1 shows one of the images, where the velocity and friction force vectors $\vec{f}_{at}$ are represented, and the sketch next to this image shows the angle between them, of π radians according to eq. (10). We will investigate whether the values found for the resistive force $\vec{f}$ are consistent with this theoretical expectation. In order to compare the relative orientation between the velocity and the resistive force measured along the trajectory – and to verify if this force can be described as that of kinetic friction – it is necessary to measure the angle between the vectors $\vec{v}$ and $\vec{f}$ using a uniform criterion.

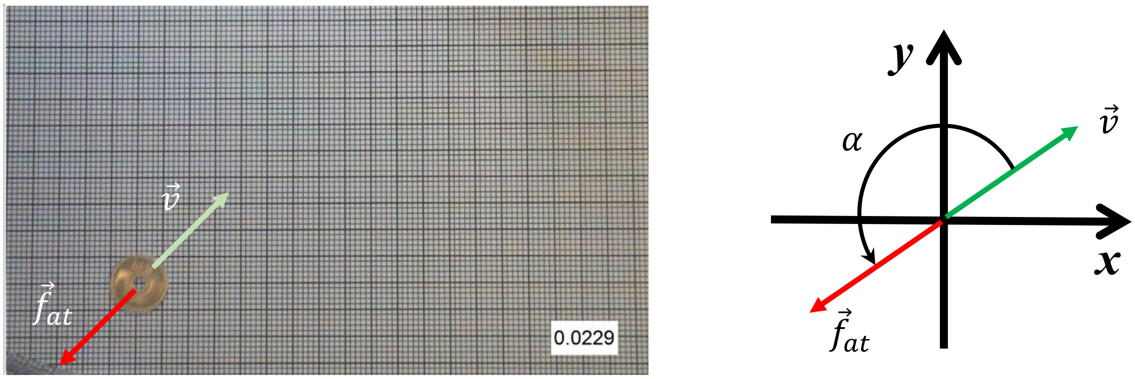

Figure 1. On the left, an image extracted from the video, where the representations of the velocity vectors and expected friction force are drawn. The sketch on the right indicates the angle $\alpha = \pi$ counterclockwise using velocity as the angular coordinate origin. The experimental measurements of α (between $\vec{v}$ and $\vec{f}$) may fluctuate up or down, given random variations.

First, verify that the dot product $\vec{f} \cdot \vec{v} < 0$ for all times, meaning that $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ and that the relative orientation is similar to what is shown on the left in Figure 1. In this range, the angle α between the velocity vectors and the resistive force, as suggested in the sketch to the right of Figure 1, can be calculated for all instants using the cross product between the velocity and resistive force vectors[1]:

$$\alpha = \text{arcsen}\left(\frac{v_x f_y - v_y f_x}{v\, f}\right) \tag{11}$$

where v_x, v_y and f_x, f_y are the projections of velocity and resistive force in the directions Ox and Oy, respectively, and

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} \tag{3}$$

as defined in Part I of this guide and will be used throughout this document. Since $\sin\theta$ has the same value for many angles θ (the correspondence between sine and angle is not 1 to 1), the sine function can be inverted only restricting the argument to a domain, making a *branch cut*, as explained in the *Guide Appendix*, which details how to work with the arcsine function normally provided in computer systems.

Build the graph of angle α as a function of time and include a line parallel to the time axis in the expected value $\alpha = \pi$ rad. Check that the points on the graph are distributed around π rad, in accordance with the expected behavior of the kinetic friction force.

B17. ***Building a model.*** If the tests of hypothesis of items **B15** and **B16** corroborate that the measured resistive force is compatible with the kinetic friction force of eq. (10), it is promising to build a theoretical model based on these laws and Newton's laws and predict the trajectory of the coin. Returning to eqs. (4), (7), and (8), we observe that the resultant forces in the directions Ox and Oy are, respectively, the horizontal component of the friction force and the sum of the components of the weight and friction forces:

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[1] In eq. (11), we use the projection with sign of $\vec{v} \times \vec{f}$ on an axis perpendicular to the plane of motion and relate it to the angle α shown in Fig. 1. Calculating the angle between the vectors, which is in the range $[0;\pi]$, would make it difficult to verify that velocity and resistive force are opposites, because it would place the hypothetical value of the angle at the extreme of that range. See the Appendix for details.

$$\vec{F}_R = f_{at_x}\hat{\imath} + \left(f_{at_y} + P_y\right)\hat{\jmath} \tag{12}$$

Equation (10) provides an expression that accounts for the direction and intensity of the kinetic friction force. Substituting it in eq. (12),

$$\vec{F}_R = \left(-\frac{\mu_c mg\cos\theta\, v_x}{v}\right)\hat{\imath} + \left(-\frac{\mu_c mg\cos\theta\, v_y}{v} - mg\operatorname{sen}\theta\right)\hat{\jmath} \tag{13}$$

Applying Newton's 2nd Law,

$$\vec{F}_R = m\vec{a} = m(a_x\hat{\imath} + a_y\hat{\jmath}) \tag{14}$$

Finally, the equations of motion are:

$$\begin{cases} a_x = \dfrac{dv_x}{dt} = -\dfrac{\mu_c g\cos\theta\, v_x}{v} \\ a_y = \dfrac{dv_y}{dt} = -\dfrac{\mu_c g\cos\theta\, v_y}{v} - g\operatorname{sen}\theta \end{cases} \tag{15}$$

The temporal function of velocity of the coin is the solution of these coupled and nonlinear differential equations, which can be solved numerically.

B18. *Computing the position over time*. The time interval between successive images is small enough to assume that the velocity is approximately constant and equal to the velocity at the beginning of the interval, but large enough to make the velocity during the following interval different; this last speed can be calculated from the previous one with eq. (15). See the guide *Integração Numérica*, in the flap *Guias Auxiliares* on the MExI page for more details.

If Δt is the interval between successive images, then

$$t_{n+1} = t_n + \Delta t \tag{16}$$

From the positions and speeds in $t = t_n$, the speed in t_{n+1} can be calculated using a_x and a_y from eq. (15):

$$\begin{cases} v_{x(n+1)} = v_{x(n)} + \left(-\dfrac{\mu_k g\cos\theta\, v_{x(n)}}{\sqrt{{v_{x(n)}}^2 + {v_{y(n)}}^2}}\right)\Delta t \\ v_{y(n+1)} = v_{y(n)} + \left(-\dfrac{\mu_k g\cos\theta\, v_{y(n)}}{\sqrt{{v_{x(n)}}^2 + {v_{y(n)}}^2}} - g\operatorname{sen}\theta\right)\Delta t \end{cases} \tag{17}$$

the positions *x* and *y* in t_{n+1} can be calculated as:

$$\begin{cases} x_{n+1} = x_n + v_{x(n)}\Delta t \\ y_{n+1} = y_n + v_{y(n)}\Delta t \end{cases} \tag{18}$$

Note that in (17) we have expanded the modulus of velocity v into its components, since these are the quantities calculated in the iterative process of solving the equation of motion.

Start the process with the first velocity and position values obtained in the worksheet. For the coefficient of friction μ_c, start with the first estimation μ_e, determined in the item **B15**. Then, these new velocities and positions calculated with eqs. (17) and (18) must be introduced into the *right* side of these equations to compute the velocity and position at the subsequent instant; this *iterative* process must be repeated for all instants t_i successively until the last moment of interest.

B19. *Fitting the model parameters*. Compare the positions calculated by numerical integration with those read from the images. To facilitate the comparison, superimpose the experimental data (already plotted in item **B4** of Part I) and the calculated values (item **B17**) on the same graph. If the trajectories are not close, change slightly (within one or two standard deviations) *one* of the parameters μ_c, x_0, y_0, v_{0x} or v_{0y} and recalculate the worksheet (changing the value in the parameter cell and pressing *Enter* should work if you correctly implemented the equations in the worksheet). Start looking for the best values of v_{0x} or v_{0y}, then try to adjust μ_c and change x_0 and y_0 only after understanding the effect of changes in the other parameters. Normally, a few trials in a few minutes lead to a set of parameter values that provides a trajectory compatible with the experimental one, although likely the fit will not be perfect.

C) Report preparation procedure

Write a report for an audience that knows neither the experiment nor the analysis procedures but has knowledge on physics. Describe what was done, formulate the conclusion and explain how it was reached. Try to be clear, objective and synthetic; use your own words. Each group must deliver a single report, with the sections listed below.

C6. *Identification*: list the names of group members and identify the analyzed image set.

C7. *Introduction*: explain the objectives of the experiment and how they were investigated.

C8. *Experimental Description*: In your own words, briefly describe the experimental arrangement, mentioning the components and their characteristics.

C9. *Results Obtained*: present the numerical results of items **B15** and **B16**, in the form of tables and graphs. Check that you have expressed the values of the quantities in appropriate units and with the appropriate number of significant figures, as well as that you have inserted uncertainty bars in all graphs.

C10. *Data analysis:* Summarize the reasons why the graphs in the previous item did or did not validate the tested hypotheses I and II, and how the statistical interpretation of the data contributed to the conclusion. Present in the same system of axes the graphs of the measured and calculated trajectories in the first attempt (with the initial estimates from item **B18**), informing the initial values of μ_c, x_0, y_0, v_{0x} and v_{0y} used in this calculation. Then, plot the graphs of the measured and calculated trajectories on the same axis system with the parameters adjusted according to item **B19**, and inform the best values found for μ_c, x_0, y_0, v_{0x} and v_{0y}. **Do not** include raw data or data extracted from the initial analysis of the set of images, as these have already been presented in the synthesis of Part I.

C11. *Discussion.* Based on the tests performed in items **B15** and **B16**, justify how and why you considered the hypotheses about the resistive force to be valid. Point out any discrepancies between the measured and calculated trajectories, presented in item **C10**, and suggest possible causes for the observed differences. Comment if the adjusted values of μ_c, x_0, y_0, v_{0x} and v_{0y} are compatible with the measured values and, if you find discrepancies, suggest reasons for this. Explain how the trajectory of the coin would change if the following parameters were changed:

a. the inclination angle of the plane;
b. the coin flip angle;
c. the coefficient of friction between the coin and the surface of the plane;
d. the magnitude of the coin initial velocity;
e. the coin mass;
f. the local acceleration of gravity, g.

C12. *Conclusion:* Go back to the introduction, pay attention to the objective of the experiment and comment on whether it was fully, partially or not achieved. *Suggestion*: go back to items **C5** (Part I) and **C11** (Part II) and comment on how the theoretical hypotheses about Amonton's laws were tested and how the application of a theoretical model to predict the trajectory allows for a deeper study of the mechanics of a body subjected to a variable force.