The Projects Physics Course

Text and Handbook

Concepts of Motion





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UNIT Concepts of Motion



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This Text-Handbook is one of the many instructional materials developed for the Project Physics Course, including texts, laboratory experiments, films, and teacher guides. Development of the course has profited from the help of many colleagues listed at the front of the text units.

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Science is an adventure of the whole human race to learn to live in and perhaps to love the universe in which they are. To be a part of it is to understand, to understand oneself, to begin to feel that there is a capacity within man far beyond what he felt he had, of an infinite extension of human possibilities

I propose that science be taught at whatever level, from the lowest to the highest, in the humanistic way. It should be taught with a certain historical understanding, with a certain philosophical understanding, with a social understanding and a human understanding in the sense of the biography, the nature of the people who made this construction, the triumphs, the trials, the tribulations.

> I. I. RABI Nobel Laureate in Physics

Preface

Background The Project Physics Course is based on the ideas and research of a national curriculum development project that worked in three phases. First, the authors – a high school physics teacher, a university physicist, and a professor of science education – collaborated to lay out the main goals and topics of a new introductory physics course. They worked together from 1962 to 1964 with financial support from the Carnegie Corporation of New York, and the first version of the text was tried out in two schools with encouraging results.

These preliminary results led to the second phase of the Project when a series of major grants were obtained from the U.S. Office of Education and the National Science Foundation, starting in 1964. Invaluable additional financial support was also provided by the Ford Foundation, the Alfred P. Sloan Foundation, the Carnegie Corporation, and Harvard University. A large number of collaborators were brought together from all parts of the nation, and the group worked together for over four years under the title Harvard Project Physics. At the Project's center, located at Harvard University, Cambridge, Massachusetts, the staff and consultants included college and high school physics teachers, astronomers, chemists, historians and philosophers of science, science educators, psychologists, evaluation specialists, engineers, film makers, artists and graphic designers. The teachers serving as field consultants and the students in the trial classes were also of vital importance to the success of Harvard Project Physics. As each successive experimental version of the course was developed it was tried out in schools throughout the United States and Canada. The teachers and students in those schools reported their criticisms and suggestions to the staff in Cambridge. These reports became the basis for the next year's revision. The number of participating

teachers during this period grew from 2 in 1962-63 to over 100 in 1967-68. In that year over five thousand students participated in a large-scale formal research program to evaluate the results achieved with the course materials.

During 1968, the last of the experimental course materials was completed. With the culmination of course development and data gathering activities, the final phase of Harvard Project Physics got under way. During 1968-69 and 1969-70 the work of the Project concentrated on developing and conducting special training programs for teachers, disseminating information about the course to physics teachers, science department heads, school administrators and other interested persons, analyzing the large pool of final evaluation data and writing a complete report on the results, and trying to find out how the course might be reshaped to fit special audiences.

We wish it were possible to list in detail the contributions of each person who participated in some part of Harvard Project Physics. Unhappily it is not feasible, since more staff members worked on a variety of materials and had multiple responsibilities. Furthermore, every text chapter, experiment, piece of apparatus, film or other item in the experimental program benefitted from the contributions of a great many people. On the preceding pages is a partial list of contributors to Harvard Project Physics. There were, in fact, many other contributors too numerous to mention. These include school administrators in participating schools, directors and staff members of training institutes for teachers, teachers who tried the course after the evaluation year, and most of all the thousands of students who not only agreed to take the experimental version of the course, but who were also willing to appraise it critically and contribute their opinions and suggestions.

Aims. From the beginning Harvard Project Physics had three major goals in mind. These were to design a humanistically oriented physics course, to attract more students to the study of introductory physics, and to find out more about the factors that influence the learning of science in schools. The last of these involved extensive educational research, and has now been reported to the teaching profession in books and journals.

About ten years ago it became clear that a new physics course, having far wider appeal than the existing ones, was needed. Students who plan to go to college to study the humanities or social sciences, those already intent on scientific careers, and those who may not wish to go to college at all, can all benefit from a good introductory physics course. The challenge facing Harvard Project Physics was to design a humanistic course that would be useful and interesting to students with widely differing skills, backgrounds, and career plans. In practice, this meant designing a course that would have the following effect: 1. To help students increase their knowledge of the physical world by concentrating on ideas that characterize physics as a science at its best, rather than concentrating on isolated bits of information.

2. To help students see physics as the wonderfully many-sided human activity that it really is. This meant presenting the subject in historical and cultural perspective, and showing that the ideas of physics have a tradition as well as ways of evolutionary adaptation and change.

3. To increase the opportunity for each student to have immediately rewarding experiences in science even while gaining the knowledge and skill that will be useful in the long run.

4. To make it possible for teachers to adapt the course to the wide range of interests and abilities of their students.

5. To take into account the importance of the teacher in the educational process, and the vast spectrum of teaching situations that prevail.

How well did Harvard Project Physics meet the challenge? In a sense each student who takes this course must answer that question himself. It is a pleasure to report, however, that the largescale study of student achievement and student opinion in the participating schools throughout the United States and Canada showed gratifying results – ranging from the excellent scores on the College Entrance Examination Board achievement test in physics to the personal satisfaction of individual students. It is clear that the diverse array of individual students in the experimental groups responded well to the physics content, the humanistic emphasis of the course, and to its flexible multimedia course materials.

The Project Physics Course Today. Using the last of the experimental versions of the course developed by Harvard Project Physics in 1964-68 as a starting point, and taking into account the evaluation results from the try-outs, the three original collaborators set out to develop the version suitable for large-scale publication. We take particular pleasure in acknowledging the assistance of Dr. Andrew Ahlgren of Harvard University. Dr. Ahlgren was invaluable because of his skill as a physics teacher, his editorial talent, his versatility and energy, and above all, his commitment to the goals of Harvard Project Physics.

We would also especially like to thank Miss Joan Laws whose administrative skills, dependability, and thoughtfulness contributed so much to our work. The publisher, Holt, Rinehart and Winston, Inc. of New York, provided the coordination, editorial support, and general backing necessary to the large undertaking of preparing the final version of all components of the Project Physics Course, including texts, laboratory apparatus, films, etc. Damon, located in Needham, Massachusetts, worked closely with us to improve the engineering design of the laboratory apparatus and to see that it was properly integrated into the program. Since their last use in experimental form, all of the instructional materials have been more closely integrated and rewritten in final form. The course now consists of a large variety of coordinated learning materials of which this textbook is only one; in addition there are readers, handbooks, programmed instruction booklets, film loops, documentary films, transparencies, apparatus and various materials for teachers. With the aid of these materials and the guidance of your teacher, with your own interest and effort, you can look forward to a successful and worthwhile experience.

In the years ahead, the learning materials of the Project Physics Course will be revised as often as is necessary to remove remaining ambiguities, clarify instructions, and to continue to make the materials more interesting and relevant to students. We therefore urge all students and teachers who use this course to send to us (in care of Holt, Rinehart and Winston, Inc., 383 Madison Avenue, New York, New York 10017) any criticisms or suggestions they may have. And now—welcome to the study of physics!

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Physicist Enrico Fermi (1901–1954) at different stages of his career in Italy and America. Mrs. Laura Fermi is shown in the photograph at the top left of the page. UNIT

Concepts of Motion

CHAPTERS

- 1 The Language of Motion
- 2 Free Fall-Galileo Describes Motion
- 3 The Birth of Dynamics Newton Explains Motion
- 4 Understanding Motion

PROLOGUE It is January 1934, a dreary month in the city of Paris. A husband and wife, working in a university laboratory, are exposing a piece of ordinary aluminum to a stream of tiny charged bits of matter called alpha particles. Stated so simply, this certainly does not sound like a momentous event. But let us look more closely, for it is momentous indeed.

Never mind the technical details. Don't let them get in the way of the story. It all began as something of a family affair. The husband and wife are the French physicists Frédéric Joliot and Irène Curie. The alpha particles they are using in their experiment are shooting out of a piece of naturally radioactive metal, polonium, discovered 36 years before by Irène's parents, Pierre and Marie Curie, the famous discoverers of radium. What Frédéric and Irène have found is that when the aluminum is bombarded by alpha particles, the commonplace bit of material becomes radioactive for a while.

This is a surprise. Until this moment, nothing like this—a familiar, everyday substance becoming artificially radioactive—has ever been observed. But physicists in the laboratory cannot force new phenomena on nature, they can only show more clearly what nature is like. We know now that this sort of thing is a frequent occurrence. It happens, for example, in stars and in our atmosphere when it is bombarded by cosmic rays.

The news was exciting to scientists and traveled rapidly, though it made few, if any, newspaper headlines. Enrico Fermi, a young physicist on the staff of the University of Rome, became intrigued by the possibility of repeating the experiment of Frédéric and Irène – repeating it with one significant alteration. The story is told in the book *Atoms in the Family* by Enrico Fermi's wife, Laura. She writes:

... he decided he would try to produce artificial radioactivity with neutrons [instead of alpha particles]. Having no electric

charge, neutrons are neither attracted by electrons nor repelled by nuclei; their path inside matter is much longer than that of alpha particles; their speed and energy remain higher; their chances of hitting a nucleus with full impact are much greater.

Usually a physicist has some theory to guide him in setting up an experiment. This time, no good theory had yet been developed. Only through actual experiment could one tell whether or not neutrons would be good projectiles for triggering artificial radioactivity in the target nuclei. Therefore, Fermi, already an outstanding theoretical physicist at the age of 33, decided to design some experiments that could settle the issue. His first task was to obtain instruments suitable for detecting the particles emitted by radioactive materials. The best such laboratory instruments by far were Geiger counters, but in 1934 Geiger counters were still relatively new and not readily available. Therefore, Fermi built his own.

The counters were soon in operation detecting the radiation from radioactive materials. But Fermi also needed a source of neutrons. This he made by enclosing beryllium powder and the radioactive gas radon in a glass tube. Alpha particles from the radon, striking the beryllium, caused it to emit neutrons, which passed freely through the glass tube.

Now Enrico was ready for the first experiments. Being a man of method, he did not start by bombarding substances at random, but proceeded in order, starting from the lightest element, hydrogen, and following the periodic table of elements. Hydrogen gave no results; when he bombarded water with neutrons, nothing happened. He tried lithium next, but again without luck. He went on to beryllium, then to boron, to carbon, to nitrogen. None were activated. Enrico wavered, discouraged, and was on the point of giving up his researches, but his stubbornness made him refuse to yield. He would try one more element. That oxygen would not become radioactive he knew already, for his first bombardment had been on water. So he irradiated fluorine. Hurrah! He was rewarded. Fluorine was strongly activated, and so were other elements that came after fluorine in the periodic table.

This field of investigation appeared so fruitful that Enrico not only enlisted the help of Emilio Segré and of Edoardo Amaldi but felt justified in sending a cable to Rasetti [a colleague who had gone abroad], to inform him of the experiments and to advise him to come home at once. A short while later a chemist, Oscar D'Agostino, joined the group, and systematic investigation was carried on at a fast pace.

With the help of his colleagues, Fermi's work at the laboratory was pursued with high spirit, as Laura Fermi's account shows:

Follow the story rather than worrying about the techniques of the experiment.

... Irradiated substances were tested for radioactivity with Geiger counters. The radiation emitted by the neutron source would have disturbed the measurements had it reached the

All quotations in the Prologue are from Laura Fermi, Atoms in the Family: My Life With Enrico Fermi, University of Chicago Press, Chicago, 1954 (available as a paperback book in the Phoenix Books series). Fermi was one of the major physicists of the twentieth century.

Prologue

counters. Therefore, the room where substances were irradiated and the room with the counters were at the two ends of a long corridor.

Sometimes the radioactivity produced in an element was of short duration, and after less than a minute it could no longer be detected. Then haste was essential, and the time to cover the length of the corridor had to be reduced by swift running. Amaldi and Fermi prided themselves on being the fastest runners, and theirs was the task of speeding shortlived substances from one end of the corridor to the other. They always raced, and Enrico claims that he could run faster than Edoardo....

And then, on the morning of October 22, 1934, a fateful discovery was made. Two of Fermi's co-workers were irradiating a hollow cylinder of silver with neutrons from a source placed at the center of the cylinder, to make it artificially radioactive. They found that the amount of radioactivity induced in the silver depended on other objects that happened to be present in the room!

... The objects around the cylinder seemed to influence its activity. If the cylinder had been on a wooden table while being irradiated, its activity was greater than if it had been on a piece of metal.

By now the whole group's interest has been aroused, and everybody was participating in the work. They placed the neutron source outside the cylinder and interposed objects between them. A plate of lead made the activity increase slightly. Lead is a heavy substance. "Let's try a light one next," Fermi said, "for instance, paraffin." [The most plentiful element in paraffin is hydrogen.] The experiment with paraffin was performed on the morning of October 22.

They took a big block of paraffin, dug a cavity in it, put the neutron source inside the cavity, irradiated the silver cylinder, and brought it to a Geiger counter to measure its activity. The counter clicked madly. The halls of the physics building resounded with loud exclamations: "Fantastic! Incredible! Black Magic!" Parrafin increased the artificially induced radioactivity of silver up to one hundred times.

By the time Fermi came back from lunch, he had already formulated a theory to account for the strange action of paraffin.

Paraffin contains a great deal of hydrogen. Hydrogen nuclei are protons, particles having the same mass as neutrons. When the source is enclosed in a paraffin block, the neutrons hit the protons in the paraffin before reaching the silver nuclei. In the collision with a proton, a neutron loses part of its energy, in the same manner as a billiard ball is slowed down when it hits a ball of its same size [whereas it loses little speed if it is reflected off a much heavier ball, or a solid wall]. Before emerging from the paraffin, a neutron will have collided with many protons in succession, and its velocity will be greatly reduced. This *slow* neutron will have neutron source

silver cylinden



paraffin block

Because of Fermi's earlier experiments, they knew the water would not become artifically radioactive. However, they now reasoned that it would slow down neutrons and so allow silver to become more strongly radioactive.

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a much better chance of being captured by a silver nucleus than a fast one, much as a slow golf ball has a better chance of making a hole than one which zooms fast and may bypass it.

If Enrico's explanations were correct, any other substance containing a large proportion of hydrogen should have the same effect as paraffin. "Let's try and see what a considerable quantity of water does to the silver activity," Enrico said on the same afternoon.

There was no better place to find a "considerable quantity of water" than the goldfish fountain . . . in the garden behind the laboratory . . .

In that fountain the physicists had sailed certain small toy boats that had suddenly invaded the Italian market. Each little craft bore a tiny candle on its deck. When the candles were lighted, the boats sped and puffed on the water like real motor-boats. They were delightful. And the young men, who had never been able to resist the charm of a new toy, had spent much time watching them run in the fountain.

It was natural that, when in need of a considerable amount of water, Fermi and his friends should think of that fountain. On that afternoon of October 22, they rushed their source of neutrons and their silver cylinder to that fountain, and they placed both under water. The goldfish, I am sure, retained their calm and dignity, despite the neutron shower, more than did the crowd outside. The men's excitement was fed on the results of this experiment. It confirmed Fermi's theory. Water also increased the artificial radioactivity of silver by many times.

This discovery – that slowed-down neutrons can produce much stronger effects in the transmutation of certain atoms than can fast neutrons – turned out to be a crucial step toward further discoveries that, years later, led Fermi and others to the controlled production of atomic energy from uranium.

About this course: We will return to the study of nuclear physics later in the course. The reason for presenting a description of Fermi's discovery of slow neutrons here was *not* to instruct you now on the details of the nucleus, but to present a quick, almost impressionistic, view of scientists in action. Not every discovery in science is made in just the way Fermi and his colleagues made this one. Nevertheless, the episode does illustrate many of the major themes or characteristics of modern science – some of which are discussed below. Look for these themes as you read through this course; you will find them appearing over and over again in many varied situations.

Progress in science over the years is the result of the work of many people in many lands—whether working alone, in pairs or small groups, or in large research teams. No matter how different the individual way of working, no matter where he works, each scientist expects to share his ideas and results with other scientists who will try independently to confirm and add to his findings. As important as such cooperation is, the most essential ingredient of science is individual thought and creativity.



The same process by which neutrons were slowed down in the fountain is used in today's large nuclear reactors. An example is the "pool" research reactor pictured above.

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Prologue

Fermi and his associates showed stubborn perseverance in the face of discouraging results, imagination in the invention of theories and experiments, alertness to the appearance of unexpected results, resourcefulness in exploiting the material resources at hand, and joy in finding out something new and important. Traits we usually think of as being distinctly humane are of value in pursuing scientific work no less than elsewhere in life.

Scientists build on what has been found out and reported by other scientists in the past. Yet, every advance in science raises new scientific questions. The work of science is not to produce some day a finished book that can be regarded as closed once and for all, but to carry investigation and imagination on into fields whose importance and interest had not been realized before.

Some work in science depends upon painstaking observation and measurement, which can sometimes stimulate new ideas and sometimes reveals the need to change or even completely discard existing theories. Measurement itself, however, is usually guided by a theory. One does not gather data just for their own sake.

All these are characteristics of science as a whole and not of physics alone. This being a physics text, you may well wish to ask, "Yes, but just what *is* physics?" The question is fair enough, yet there is no simple answer. Physics can be thought of as an organized body of tested ideas about the physical world. Information about this world is accumulating ever more rapidly; the great achievement of physics has been to find a fairly small number of basic principles which help to organize and to make sense of certain parts of this flood of information. This course will deal with some, but not nearly all, of the ideas that together make up the content of physics. The purpose of this course is to provide you with the opportunity to become familiar with some of these ideas, to witness their birth and development, and to share in the pleasure that comes from using them to view the world in a new light.

Physics is more than just a body of laws and an accumulation of facts. Physics is what each physicist does in his own way: It is a continuing activity – a process of search that sometimes leads to discovery. Look in on different physicists at work and you will see differences in problems being studied, in apparatus being used, in individual style, and in much more. Fermi has provided us with one example, but as the course proceeds, we will encounter other, sometimes very different examples. By the end of this course, you will have dealt with many of the ideas and activities which together comprise physics. You will not just have learned about it—you will have actually done some physics.

Science gives us no final answers. But it has come upon wondrous things, and some of them may renew our childhood delight in the miracle that is within us and around us. Take, for example, so basic a thing as size . . . or time.

The Project Physics Course has made two documentary films that you might like to see. One is called *The World of Enrico Fermi* and includes the discovery described here. The other is entitled *People and Particles* and shows what it is like to be working now on a research problem in elementary particle physics.

Our place in space

Physics deals with those laws of the universe that apply everywhere – from the largest to the smallest.

	ORDER OF MAGNITUDE
istance to the furthest observed galaxy	10 ²⁶ meters
Distance to the nearest galaxy	10 ²²
Distance to the nearest star	1017
Distance to the sun	1011
Diameter of the earth	107
One mile	10 ³
Human height	10º
Finger breadth	10-2
Paper thickness	10-4
Large bacteria	10 ⁻⁵
Small virus	10 ⁻⁸
Diameter of atom	10-10
Diameter of nucleus	10-14

A globular star cluster



Atomic sites in tungsten



The estimated size of the universe now is of the order of 100 million, million, million, million times a man's height (man's height \times 10,000,000, 000,000,000,000,000). The smallest known constituent units of the universe are less in size than a hundreth of a millionth of a millionth of a man's height (man's height \times 0.000,000,000,000,01).

F

Our place in time

Physicists study phenomena in the extremes of time-space and the whole region between the longest and shortest. ORDER OF MAGNITUDE Age of universe 10¹⁷ seconds

Age of universe	
Precession of the earth's axis	10 ¹²
Human life span	10 ⁹
One year	107
One day	10 ⁵
Light from sun to earth	10 ³
Time between heartbeats	10 ⁰
One beat of fly's wings	10 ⁻³
Duration of strobe flash	10-5
Short laser pulse	10 ⁻⁹
Time for light to cross an atom	10-18
Shortest-lived subatomic particles	10-23

Particle tracks in a bubble chamber

Fossilized trilobites



The history of the universe has been traced back as far into the past as a hundred million times the length of a man's life (man's life \times 100,000, 000).



Events have been recorded that last only a few millionths of a millionth of a millionth of a man's heartbeat (man's heartbeat \times 0.000,000,000,000,000,000,000,001).

It is hard to resist the temptation to say more about these intriguing extremes; however, this is not where physics started. Physics started with the human-sized world—the world of horse-drawn chariots, of falling rain, and of flying arrows. It is with the physics of phenomena on this scale that we shall begin.

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	The motion of things A motion experiment that does not quite work A better experiment Leslie's "50" and the meaning of average speed Graphing motion and finding the slope Time out for a warning Instantaneous speed Acceleration – by comparison



CHAPTER ONE

The Language of Motion

1.1 The motion of things

The world is filled with things in motion: things as small as dust and as large as galaxies, all continually moving. Your textbook may seem to be lying quietly on the desk, but each of its atoms is incessantly vibrating. The "still" air around it consists of molecules tumbling chaotically, at various speeds, most of them moving as fast as rifle bullets. Light beams dart constantly through the room, covering the distance from wall to wall in about a hundred-millionth of a second, and making about ten million vibrations during that time. Even the whole earth, our majestic spaceship, is moving at about 18 miles per second around the sun.

There is a very old maxim: "To be ignorant of motion is to be ignorant of nature." Of course we cannot investigate all motions. So, from this swirling, whirling, vibrating world of ours let us choose just one moving object for our attention, something interesting and typical, but above all, something manageable. Then let us describe its motion.

But where shall we start? A machine, such as a rocket or a car? Though made and controlled by man, they or their parts move in fast and complicated ways. We really ought to start with something simpler and slower, something that our eyes can follow in detail. Then how about a bird in flight? Or a leaf falling from a tree?

Surely, in all of nature there is no motion more ordinary than that of a leaf fluttering down from a branch. Can we describe how it falls or explain why it falls? As we think about it we quickly realize that, while the motion may be "natural," it is very complicated. The leaf twists and turns, sails to the right and left,

Study for "Dynamism of a Cyclist" (1913) by Umberto Boccioni. Courtesy Yale University Art Gallery.





Section 1.2

back and forth, as it floats down. Even a motion as ordinary as this may turn out, on closer examination, to be more complicated than the motion of machines. And even if we could describe it in detail, what would we gain? No two leaves fall in quite the same way; therefore, each leaf would seem to require its own detailed description. Indeed, this individuality is typical of most events occurring spontaneously on earth.

And so we are faced with a dilemma. We want to describe motion, but the motions we encounter under ordinary circumstances appear too complex. What shall we do? The answer is that we should go, at least for a while, into the physics laboratory – because the laboratory is the place to separate the simple ingredients that make up all complex natural phenomena and to make those phenomena more easily visible to our limited human senses.

1.2 A motion experiment that does not quite work

A billiard ball, hit squarely in the center, speeds easily across a tabletop in a straight line. An even simpler motion (simpler because there is no rolling) can be obtained in *this* way: Take a disk of what is called "dry ice" (really frozen carbon dioxide), put it on a smooth floor, and give it a gentle push. It will move slowly and with very little friction, supported on its own vapor. We did this in front of a camera to get a photograph that would "freeze" the action for easier measurement later. While the dry ice disk was moving,





Time exposure of the disk in motion

Laboratory setup

Close-up of a dry ice disk



the shutter of the camera was kept open; the resulting timeexposure shows the path taken by the disk.

What can we learn about the disk's motion by examining the photographic record? Our question is easy enough to answer: as nearly as we can judge by placing a ruler on the photograph, the disk moved in a straight line. This is a very useful result, and we shall see later that it is really quite surprising. It shows how simplified the laboratory can be: the kinds of motion one ordinarily sees are almost never that simple. But did it move steadily, or did it slow down? From this photograph we cannot tell. Let us improve our experiment. Before we do so, however, we must be clear on just how we might expect to measure the speed.

Why not use something like an automobile speedometer? A speedometer is supposed to tell us directly the speed at which the car is moving at any time. Everyone knows how to read that most popular of all meters, even though few of us have a clear notion of how it works. Think of how speeds are expressed. We say, for example, that a car is moving at 60 *miles per hour*. This means that if the car continues to move with the same speed it had at the instant the speed reading was taken, the car would move a distance of 60 miles in a time interval of 1.0 hour. Or we could say that the car would move 1.0 mile in 1/60 of an hour, or 6.0 miles in 1/10 of an hour—or *any* distance and time intervals for which the ratio of distance to time is 60 miles per hour.

Unfortunately, an automobile speedometer cannot be hooked to a disk of dry ice, or to a bullet, or to many other objects whose speed we might wish to measure. (See SG 1.2.) However, there is a way to measure speeds in most cases that would interest us.

As a clue, remember what you would have to do if the speedometer in your car were broken and you still wanted to know your speed as you moved along a turnpike. You would do one of two things (the result is the same in either case): you would count the number of mile markers passed in one hour (or some fraction of it) and find the average speed by getting the ratio of miles and hours; or, you would determine the fraction of an hour it takes to go from one mile marker to the next (or to another marker a known number of miles away) and find again the average speed as a ratio of miles to hours.

Either method gives, of course, only the *average* speed for the interval during which speed is measured. That is not the same as the speed at any given instant, which a speedometer registers, but it is good enough for a start. After we get average speeds clear, we shall see a simple way of getting instantaneous speeds.

Therefore, to find the speed of an object, we measure the distance it moves and the time it takes to move that distance. Then we divide the distance by the time, and the speed comes out in miles per hour, or feet per second, or meters per second, depending upon the units used to measure distance and time. With this plan of attack, we return to the experiment with the dry ice disk. Our task now is to find the speed of the disk as it moves along its straight-line path. If we can do it for the disk, we can do it for many other objects as well.

The speed of an object, is, of course, how fast it moves from one place to another. A more formal way to say the same thing is: Speed is the time rate of change of position.

From time to time you will be referred to items in the *Study Guide*, a few pages found at the end of each chapter. Usually the letters SG plus a number will indicate this. See SG 1.1 on page 31 for more information on how to study for this course and on the use of the Study Guide. There will usually be one or more brief questions at the end of each section in a text chapter. Q1 below is the first. Use these to check on your own progress. Answer the questions *before* continuing to the next section. Check your answers to the end-of-section questions at the back of this book (page 197); whenever you find you did not get the correct answers, study through the section again. And of course, if anything is still unclear after you have tried to study it on your own or together with other students, then ask your teacher!

Q1 Why is it not possible to determine the speed of the dry ice puck in the time-exposure photograph on page 11?

1.3 A better experiment

To find speed, we need to be able to measure both distance and time. So let's repeat the experiment with the dry ice disk after first placing a meter stick (100 cm) on the table parallel to the expected path of the disk. This is the photograph we obtain:

We now have a way of measuring the distance traveled by the disk, but we still need a way to measure the time it takes the disk to travel a given distance.

This can be done in various ways but here is a fine trick that you can try in the laboratory. The camera shutter is again kept open and everything else is the same as before, except that the only source of light in the darkened room comes from a stroboscopic lamp. This lamp produces bright flashes of light at a frequency which can be set as we please. Since each pulse or flash of light lasts for only about 10 millionths of a second (10 microseconds), the moving disk appears in a series of separate, sharp exposures, rather than as a continuous blur. The photograph below was made by using such a stroboscopic lamp flashing 10 times a second, after the disk had been gently pushed as before.

 $\begin{array}{c} & & & & & & & & \\ & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & & \\$

Now we're getting somewhere. Our special setup enables us to record accurately a series of positions of the moving object. The meter stick helps us to measure the distance moved by the front edge of the disk between successive light flashes. The time interval between images is, of course, equal to the time interval between stroboscopic lamp flashes (which is 0.10 second in these photos).

We can now determine the speed of the disk at the beginning and end of its photographed path. The front edge of the first clear image of the disk at the left is 6 cm from the zero mark on the meter stick. The front edge of the second image from the left is at the 19-cm position. The distance traveled during that time was the difference between those two positions, or 13 cm. The corresponding time interval was 0.01 second. Therefore, the speed at the start must have been 13 cm/0.10 sec, or 130 cm/sec.

Turning now to the two images of the disk farthest to the right in the photograph, we find that the distance traveled during 0.10 sec was 13 cm. Thus the speed at the right end was 13 cm/0.10 sec, or 130 cm/sec.

The disk's motion was not measurably slower at the right end than at the left end. Its speed was 130 cm/sec near the beginning of the path – and 130 cm/sec near the end of the path. However, that does not yet prove that the speed was constant all the way. We might well suspect that it was, and you can easily check for yourself that this suspicion is justified. Since the *time* intervals between images are equal, the speeds will be equal if the *distance* intervals are equal to one another. Is the distance between images always 13 cm? Did the speed stay constant, as far as you can tell from the measurements?

When you think about this result, there is something really unusual in it. Cars, planes, and ships do not move in neat, straight lines with precisely constant speed even when they go under power. Yet this disk did it, coasting along on its own, without anything to keep it moving. You might well think it was just a rare event and it would not happen again. In any case, *you* should try it. The equipment you will use for this study of physics will include cameras, strobe lamps (or mechanical strobes, which work just as well), and low-friction disks of one sort or another. Repeat the experiment several times at different initial speeds, and then compare your results with those we found above.

You may have a serious reservation about the experiment. If you ask, "How do you know that the disk didn't slow down an amount too small to be detected by your measurements?" we can only answer that we don't know. All measurements involve some uncertainty which one can usually estimate. With a meter stick we can measure distances reliably to the nearest 0.1 cm. If we had been able to measure to the nearest 0.01 cm or 0.001 cm, we might have detected some slowing down. But if we again found no change in speed, you could still raise the same objection. There is no way out of this. We must simply admit that *no* physical measurements are ever infinitely precise. Thus it is wise to leave open to question

See the articles "Motion in Words" and "Representation of Movement" in Project Physics Reader 1.

Section 1.4

the results of any set of measurements and the findings based on them if increased precision could reveal other results.

Let us briefly review the results of our experiment. We devised a way to measure the successive positions of a moving dry ice disk at known time intervals. From this we calculated first the distance intervals and then the speed between selected positions. We soon discovered that (within the limits of accuracy of our measurement) the speed did not change. Objects that move in such a manner are said to have *uniform speed* or *constant speed*. We know now how to measure uniform speed. But, of course, actual motions are seldom uniform. What about the more usual case of *non*uniform speed? That is our next concern.

Q2 Suppose the circles below represent the successive positions of a moving object as photographed stroboscopically. Did the object move with uniform speed? How do you know?

0 0 0 0 0 0

Q3 Describe uniform speed without referring to dry ice pucks and strobe photography or to *any* particular object or technique of measurement.

1.4 Leslie's "50" and the meaning of average speed

Consider the situation at a swimming meet. At the end of each race, the name of the winner is announced – the swimmer with the shortest time; but since in a given race – say the 100-yard back-stroke – every swimmer goes the same distance, the swimmer with the shortest time is the one having the highest average speed while covering the measured distance. The ratio of the distance traveled to the elapsed time is the measure of average speed. This relationship is expressed in the following equation:

average speed = $\frac{\text{distance traveled}}{\text{elapsed time}}$

What information does a knowledge of the average speed give us? We shall answer this question by studying a real example.

Leslie is not the fastest girl freestyle swimmer in the world, but Olympic speed is not necessary for our purposes. One day after school, Leslie was timed while swimming two lengths of the Cambridge High School pool. The pool is 25.0 yards long, and it took her 56.1 seconds to swim the two lengths. Thus her average speed over the whole 50-yard distance was

$$\frac{50.0 \text{ yd}}{56.1 \text{ sec}} = 0.89 \text{ yd/sec}$$
, or nearly 2.7 ft/sec

Did Leslie swim the 50 yards at uniform (or constant) speed? If not, which length did she cover more quickly? What was her greatest speed? her least speed? How fast was she moving when she passed the 10-yard, or 18-yard or 45-yard mark? These are Some practice problems dealing with constant speed are given in Study Guide 1.3 (a, b, c, and d).



2.7 ft/sec is the equivalent of 1.8 miles per hour. No great speed! A sailfish can do over 40 mph. But man is a land animal. For short distances he can run better than 20 mph.

useful things to know when training for a meet. But so far we do not have a way to answer any of these questions. The value 0.89 yd/sec probably comes closer than any other *one* value to describing the whole event.

To compare Leslie's speed at different parts of the swim, we must observe the times and distances covered as we did in experimenting with the dry ice disk. That is why we arranged the event as shown on the photograph below.

Observers stationed at 5-yard intervals from the 0 mark along the length of the pool started their stopwatches when the starting signal was given. Each observer had two watches, one which he stopped as Leslie passed his mark going down the pool, and another which he stopped as she passed on her return trip. The data are tabulated in the margin.

_ <u></u>	t
0,0 yd	0.0 sec
5.0	2.5
10.0	5.5
15.0	11.0
20.0	16. O
25.0	22. O
30.0	26.5
35.0	32.0
40.0	39.5
45.0	47.5
50.0	56.1



From these data we can determine Leslie's average speed for the first 25 yards and for the last 25 yards separately.

Average speed for first 25 yards =	_ d	listance traveled
		elapsed time
Average speed for the <i>last</i> 25 yards	_	25.0 yards
	2	22.0 seconds
	= 1	1.10 yd/sec
		distance traveled
		elapsed time
		25.0 yards
	- 5	56.0 sec – 22.0 sec
		25.0 yd = 0.735 yd/sec
	- 3	34.0 sec = 0.735 yu/sec

It is now clear that Leslie did not swim with uniform speed. She swam the first length much faster (1.10 yd/sec) than the second length (0.74 yd/sec). Notice that the overall average speed (0.89 yd/sec) does not describe either lap very well. Here and elsewhere

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in our study of motion, the more we refine our measurements to look at detail, the more variation we find.

In a moment we shall continue our analysis of the data we have obtained for Leslie's swim—mostly because the concepts we are developing here, to discuss this everyday type of motion, will be needed later to discuss other motions, ranging from that of planets to that of atoms. First, we shall introduce some shorthand notation with which the definition of average speed can be simplified from

average speed =
$$\frac{\text{distance traveled}}{\text{elapsed time}}$$

to the more concise statement that says exactly the same thing:

$$v_{av} = \frac{\Delta d}{\Delta t}$$

In this equation v_{ar} is the symbol for the average speed, Δd is the symbol for change in position, and Δt is the symbol for an elapsed interval of time. The symbol Δ is the fourth letter in the Greek alphabet and is called delta. When Δ precedes another symbol, it means "the change in..." Thus, Δd does *not* mean " Δ multiplied by *d*" but rather "the change in *d*" or "the distance interval." Likewise, Δt stands for "the change in *t*" or "the time interval."

We can now go back to the data to see what we can learn about Leslie's average speed for each 5-yard interval, from beginning to end. This calculation is easily made, especially if we reorganize our data as in the table on page 19. The values of v_{ar} calculated at 5-yard intervals for the first lap are entered in the right-hand column. (The computations for the second lap are left for you to complete.)

Much more detail is emerging from the picture. Looking at the speed column, we see that Leslie's speed was at its greatest right near the beginning. Her racing jump into the water gave her extra speed at the beginning. In the middle of her first length she was swimming at a fairly steady rate, and she slowed down coming into the turn. Use your own figures to see what happened after the turn.

Although we have determined Leslie's speeds at various intervals along the path, we are still dealing with *average* speeds. The intervals are smaller – 5 yards rather than the entire 50 – but we do not know the details of what happened *within* any of the 5-yard intervals. Thus, Leslie's average speed between the 15- and 20-yard marks was 1.0 yd/sec, but we don't know yet how to compute her speed at the very instant and point when she was, say, 18 yards or 20 yards from the start. Even so, we feel that the average speed computed over the 5-yard interval between the 15- and 20-yard marks is probably a better estimate of her speed as she went through the 18-yard mark than is the average speed computed over the whole 50 yards, or over either 25-yard length. We shall come back to this problem of the determination of "speed at a particular instant and point" in Sec. 1.7. **Q5** If you have not already completed the table on page 19, do so now before going on to the next section.

1.5 Graphing motion and finding the slope

What can we learn about motion by graphing the data rather than just tabulating them? Let us find out by preparing a distanceversus-time graph, using the data from Leslie's 50-yard swim. As shown in the first graph on the next page, all we really know are the data points. Each point on the graph shows the time when Leslie was at a particular position along her path. In the second graph, dotted lines have been drawn to connect the points. We don't actually know what the values are between the data points-the straight-line connections are just a very simple way of suggesting what the overall graph might look like. In fact, the straight lines are not likely to be a very good approximation, because the resulting broken-line graph would indicate very abrupt changes. If we believe that Leslie changed speed only gradually, we can get a better approximation by drawing the smoothest curve possible through the data points. One experimenter's idea of a smooth curve is shown in the last graph.

Now let us "read" the graph. Notice that the line is steepest at the start. This means that there was a comparatively large change in position during the first seconds – in other words, she got off to a fast start! The steepness of the graph line is an indication of how fast she was moving. From 10 yards to 20 yards the line appears to be straight, becoming neither more nor less steep. This means that her speed in this stretch was constant. Reading the graph further, we see that she slowed down noticeably before she reached the 25-yard mark, but gained in speed right after the turn. The steepness decreases gradually from the 30-yard mark to the finish as Leslie was slowing down. There was no final spurt over the last 5 yards. (She could barely drag herself out of the pool after the trial.)

Looked at in this way, a graph provides us at a glance with a visual representation of motion. But this way of representing motion so far does not help us if we want to know actual values of her speed at various times. For this, we need a way of measuring the steepness of the graph line. Here we can turn to mathematics for help, as we often shall. There is an old method in geometry for solving just this problem. The steepness of a graph at any point is related to the change in the vertical direction (Δy) and the change in the horizontal direction (Δx) . By definition, the ratio of these two changes $(\Delta y/\Delta x)$ is the slope:

slope =
$$\frac{\Delta y}{\Delta x}$$

Slope is a widely-used mathematical concept, and can be used to indicate the steepness of a line in any graph. In a distance-time graph like the one for Leslie's swim, distance is usually plotted on





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the vertical axis (d replaces y) and time on the horizontal axis (t replaces x). Therefore, in such a graph, the slope of a straight line is given by

slope =
$$\frac{\Delta a}{\Delta t}$$

But this reminds us of the definition of average speed, $v_{av} = \Delta d/\Delta t$. Therefore, $v_{av} =$ slope! In other words, the slope of any straight-line part of a graph of distance versus time gives a measure of the average speed of the object during that interval. What we do when we measure slope on a graph is basically the same thing that highway engineers do when they specify the steepness of a road. They simply measure the rise in the road and divide that rise by the horizontal distance one must go in order to achieve the rise. The only difference between this and what we have done is that the Above are shown four ways of representing Leslie's swim: a table of data, a plot of the data points, broken straight-line segments that connect the points, and a smooth curve that connects the points.

If this concept is new to you or if you wish to review it, turn now to Study Guide 1.11 before continuing here. highway engineers are concerned with an actual physical slope: on a graph of their data the vertical axis and horizontal axis would both show distance. We, on the other hand, are using the *mathematical concept* of slope as a way of expressing distance measured against *time*.

We can get a numerical value quickly and directly for the slope of each straight-line segment in the graph on p. 19, so we will have the value of the average speed for each of the 5-yard intervals between data points. For example, we used our data table to calculate Leslie's average speed between the 5- and 10-yard markers as 1.4 yd/sec. She moved 5 yards on the vertical (distance) axis during a lapse of 3.5 seconds on the horizontal (time) axis. Therefore, the slope of the line segment connecting the 5-yard and 10-yard points is equal to 5 yards divided by 3.5 seconds, or 1.4 yd/sec.

The slope, as we have defined it here, is not exactly the same thing as the steepness of the line on the graph paper. If we had chosen a different scale for either the distance or time axis (making the graph, say, twice as tall or twice as wide), then the apparent steepness of the entire graph would be different. The slope, however, is measured by the ratio of the distance and time units – a Δd of 10 meters in a Δt of 5 seconds gives a ratio of 2 meters/second, no matter how much space is used for meters and seconds on the graph.

But the graph is more useful than just leading us back again to the values in the table. We can now ask questions that cannot be answered directly from the original data: What was Leslie's speed 10 seconds after the start? What was her speed as she crossed the 37-yard mark? Questions like these can be answered by finding the slope of a fairly straight portion of the graph line around the point of interest. Two examples are worked out on the



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graph at the bottom of page 20. For each example, Δt was chosen The 4-sec value for t is just for to be a 4-sec interval – from 2 sec before the point in question to 2 convenience; some other value sec after it: then the Δd for that Δt was measured.

The reasonableness of using the graph in this way can be checked by comparing the results with the values listed in the table on p. 19. For example, the speed near the 10-second mark is found from the graph to be about 3.0 vd/4.0 sec = 0.75 vd/sec. This is somewhat less than the value of 0.9 vd/sec given in the table for the average speed between 6 and 11 seconds; and that is just what we would expect, because the smooth-curve graph does become momentarily less steep around the 10-second point. If the smooth curve that was drawn really is a better description of Leslie's swimming than the broken line is, then we can get more information out of the graph than we put into it.

Q6 Turn back to p. 13 and draw a distance-time graph for the motion of the dry ice disk.

Q7 Which of the two graphs below has the greater slope?



Q8 Where was Leslie swimming most rapidly? Where was she swimming most slowly?

Q9 From the graph, find Leslie's speed at the 47-yard mark. From the table on p. 19, calculate her average speed over the last 5 yards. How do the two values compare?

1.6 Time out for a warning

Graphs are useful – but they can also be misleading. You must always be aware of the limitations of any graph you use. The only actual data in a graph are the plotted points. There is a limit to the precision with which the points can be plotted, and a limit to how precisely they can be read from the graph.

The placement of a line through a series of data points, as in the graph on page 19, depends on personal judgment and interpretation. The process of estimating values between data points is called *interpolation*. That is essentially what you are doing when you draw a line between data points. Even more risky than interpolation is *extrapolation*, where the graph line is

could have been used. Or, we could have chosen a value for Δd and then measured the corresponding Δt .

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The Language of Motion



19 minutes

17 minutes

27 minutes

The Language of Motion

These photographs show a stormy outburst of incandescent gas at the edge of the sun, a developing chive plant and a glacier. From these pictures and the time intervals given between pictures, you can determine the average speeds (1) of the growth of the solar flare with respect to the sun's surface (radius of sun is about 432,000 mi), (2) of the growth of one of the chive shoots with respect to the graph paper behind it (large squares are one inch), (3) of the moving glacier with respect to its "banks."



17 hours

33 hours





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extended to provide estimated points beyond the known data.

An example of a high-altitude balloon experiment carried out in Lexington, Massachusetts, nicely illustrates the danger of extrapolation. A cluster of gas-filled balloons carried cosmic ray detectors high above the earth's surface, and from time to time a measurement was made of the height of the cluster. The graph on the right shows the data for the first hour and a half. After the first 20 minutes the balloons seem to be rising in a cluster with unchanging speed. The average speed can be calculated from the slope: speed of ascent = $\Delta d/\Delta t$ = 27,000 ft/30 min = 900 ft/min. If we were asked how high the balloons would be at the very end of the experiment (500 min), we might be tempted to extrapolate, either by extending the graph or by computing from the speed. In either case we would obtain the result 500 min \times 900 ft/min = 450,000 ft, which is over 90 miles high! Would we be right? Turn to Study Guide 1.12 to see for yourself. (The point is that mathematical aids, including graphs, can be a splendid help, but only within the limits set by physical realities.)

Q10 What is the difference between extrapolation and interpolation?

Q11 Which estimate from the graph would you expect to be less accurate: Leslie's speed as she crossed the 30-yard mark, or her speed at the end of an additional lap?





2.0

1.7

1.7 Instantaneous speed

Now let us wrap up the chief lessons of this first chapter. In Sec. 1.5 we saw that distance-time graphs could be very helpful in describing motion. When we reached the end of the section, we were speaking of specific speeds at particular points along the path (like "the 14-yard mark") and at particular instants of time (like "the instant 10 seconds after the start"). You probably were bothered by this manner of talking, since at the same time we admitted that the only kind of speed we can actually measure is average speed. To find average speed we need a ratio of distance and time intervals. A particular point on the path, however, does not have any interval. Nevertheless, it makes sense to speak about the speed at a point. We will summarize what reasons there are for using "speed" in this way, and see how well we can get away with it.

You remember that our answer to the question (page 20), "How fast was Leslie swimming at time t = 10 sec?" was 0.85 yd/sec. That answer was obtained by finding the slope of a small portion of the curve encompassing the point P when t = 10 sec. That section of the curve has been reproduced in the margin here. Notice that the part of the curve we used appears to be nearly a straight line. As the table under the graph shows, the value of the slope for each interval changes very little as we decrease the time interval Δt . Now imagine that we closed in on the point where t = 10 sec



0.85

until the amount of curve remaining became vanishingly small. Could we not safely assume that the slope of that infinitesimal part of the curve would be the same as that on the straight line of which it seems to be a part? We think so. That is why we took the slope of the straight line from t = 8 sec to t = 12 sec, and *called* it the speed *at* the midpoint, the speed at t = 10 sec, or to use the correct term, "the instantaneous speed" at t = 10 sec.

In estimating a value for Leslie's instantaneous speed at a particular time, we actually measured the average speed over a 4.0-sec interval. We then made the conceptual leap that we have described. We decided that the *instantaneous* speed at a particular instant can be equated to an *average* speed $\Delta d/\Delta t$ provided: 1) that the particular instant is included in Δt , and 2) that the ratio $\Delta d/\Delta t$ is obtained for a small enough part of the curve, one which is nearly a straight-line segment, so that it does not change appreciably when we compute it over a still smaller time interval.

A second concrete example will help here. In the oldest known study of its kind, the French scientist de Montbeillard periodically recorded the height of his son during the period 1759-1777. A graph of height versus age for his son is shown in the margin.

From the graph, we can compute the average growth rate (v_{av}) over the entire 18-year interval or over any shorter time interval within that period. Suppose, however, we wanted to know how fast the boy was growing just as he reached his fifteenth birthday. The answer becomes evident if we enlarge the graph in the vicinity of the fifteenth year. (See the second graph.) His height at age 15 is indicated as point P, and the other letters designate



(m) + (m)



24
instants of time on either side of P. The boy's average growth rate over a two-year interval is given by the slope of the line segment **AB** in the enlarged figure in the margin. Over a one-year interval this average growth rate is given by the slope of CD. (See the third graph.) The slope of EF gives the average growth rate over six months, etc. The four lines, **AB**, **CD**, **EF**, **GH**, are not parallel to each other and so their slopes are different. However, the difference in slope gets smaller and smaller. It is large when we compare **AB** and **CD**, less if we compare **CD** and **EF**, less still between **EF** and **GH**. For intervals less than $\Delta t = 1$ yr, the lines appear to be more nearly parallel to each other and gradually to merge into the curve, becoming nearly indistinguishable from it. For very small intervals, you can find the slope by drawing a straight line *tangent* to this curve at **P**, placing a ruler at **P** (approximately parallel to line **GH**), and extending it on both sides as in Study Guide 1.11.

The values of the slopes of the straight-line segments in the middle and lower graphs have been computed for the corresponding time intervals and are tabulated at the right. We note that values of v_{av} calculated for shorter and shorter time intervals approach closer and closer to 6.0 cm/yr. In fact, for any time interval less than 2 months, the average speed v_{av} will be 6.0 cm/yr within the limits of accuracy of measuring height. Thus we can say that, on his fifteenth birthday, young de Montbeillard was growing at a rate of 6.0 cm/yr. At that instant in his life, t = 15.0 yr, this was his instantaneous growth rate (or if you will, the *instantaneous speed* of his head with respect to his feet!)

Average speed over a time interval Δt , we have said, is the ratio of distance traveled to elapsed time, or in symbols,

$$v_{av} = \frac{\Delta a}{\Delta t}$$

We now have added the definition of *instantaneous* speed at an instant as the final limiting value approached by the average speeds when we compute v_{av} for smaller and smaller time intervals including the instant t. In almost all physical situations, such a limiting value can be accurately and quickly estimated by the method described on the previous page.

From now on we will use the letter v without the subscript av to mean the instantaneous speed defined in this way. You may wonder why we have used the letter "v" instead of "s" for speed. The reason is that speed is closely related to velocity. We shall reserve the term "velocity" for the concept of *speed in a specified direction* (such as 50 mph to the north) and denote it by the symbol \vec{v} . When the direction is not specified and only the magnitude (50 mph) is of interest, we remove the arrow and just use the letter v, calling the magnitude of the velocity "speed." This crucial distinction between speed and velocity, and why velocity is more important in physics, will be discussed in more detail in later sections.

Q12 Define instantaneous speed, first in words and then in symbols.

Line between points	Δt	Δd	Growth rate $v_{av} = \frac{\Delta d}{\Delta t}$
AB	2 yr	19.0 cm	9.5 cm/year
CD	1 yr	8.0	8.0
DEF	6 mo	3.5	7.0
GH	4 mo	2.0	6.0
IJ	2 mo	1.0	6.0

SG 1.14

See SG 1.15, 1.16, and 1.17 for problems that check your understanding of the chapter up to this point.



1. Paris street scene, 1839. A daguerrotype made by Louis Daguerre himself.

2. American street scene, 1859



3. Boys on skateboards

Photography 1839 to the Present

1. Note the lone figure in the otherwise empty street. He was getting his shoes shined. The other pedestrians did not remain in one place long enough to have their images recorded. With exposure times several minutes long, the outlook for the possibility of portraiture was gloomy.

2. However, by 1859, due to improvements in photographic emulsions and lenses, it was not only possible to photograph a person at rest, but one could capture a bustling crowd of people, horses and carriages. Note the slight blur of the jaywalker's legs.

3. Today, one can "stop" action with an ordinary camera.

4. A new medium—the motion picture. In 1873 a group of California sportsmen called in the photographer Eadweard Muybridge to settle the question, "Does a galloping horse ever have all four feet off the ground at once?" Five years later he answered the question with these photos. The five pictures were taken with five cameras lined up along the track, each camera being triggered when the horse broke a string which tripped the shutter. The motion of the horse can be restructured by making a flip pad of the pictures.

With the perfection of flexible film, only one camera was needed to take many pictures in rapid succession. By 1895, there were motion picture parlors throughout the United States. Twenty-four frames each second were sufficient to give the viewer the illusion of motion.



4. Muybridge's series, 1878



5. A light can be flashed successfully at a controlled rate and a multiple exposure (similar to the strobe photos in this text) can be made. In this photo of a golfer, the light flashed 100 times each second.

6. It took another ninety years after the time the crowded street was photographed before a bullet in flight could be "stopped." This remarkable picture was made by Harold Edgerton of MIT, using a brilliant electric spark which lasted for about one millionth of a second.

7. An interesting offshoot of motion pictures is the high-speed motion picture. In the frames of the milk drop series shown below, 1000 pictures were taken each second (by Harold Edgerton). The film was whipped past the open camera shutter while the milk was illuminated with a flashing light (similar to the one used in photographing the golfer) synchronized with the film. When the film is projected at the rate of 24 frames each second, action which took place in 1 second is spread out over 42 seconds.

It is clear that the eye alone could not have seen the elegant details of this event. This is precisely why photography of various kinds is used in the laboratory. 5. Stroboscopic photo of golfer's swing. (See the article "The Dynamics of a Golf Club" in Project Physics *Reader 1.*)



6. Bullet cutting through a playing card.



7. Action shown in high-speed film of milk drop.

Q13 Explain the difference in meaning between average speed and instantaneous speed.

1.8 Acceleration – by comparison

You can tell from the photograph below of a rolling baseball that it was changing speed – accelerating. The increasing distances between the instantaneous images of the ball give you this information, but how can you tell how *much* acceleration the ball has?

To answer this question we have only one new thing to learn – the definition of acceleration. The definition itself is simple; our task is to learn how to *use* it in situations like the one above. For the time being, we will define acceleration as *rate of change of speed*. Later this definition will have to be modified somewhat when we encounter motion in which change in *direction* becomes an important additional factor. But for now, as long as we are dealing only with straight-line motion, we can equate the rate of change of speed with acceleration.

Some of the effects of acceleration are familiar to everyone. It is acceleration, not speed, that you notice when an elevator suddenly starts up or slows down. The flutter in one's stomach comes only during the speeding up and slowing down, not during most of the ride when the elevator is moving at a steady speed. Likewise, much of the excitement of the roller coaster and other rides at amusement parks is a result of their unexpected accelerations. Speed by itself does not cause these sensations. Otherwise they would occur during a smooth plane ride at 650 mph, or even just during the continuous motion of the earth around the sun at 65,000 mph.

Simply stated, speed is a relationship between two objects, one of which is taken to be the reference object while the other moves with respect to it. Some examples are the speed of the earth with respect to the stars, the speed of the swimmer with respect to the pool edge, the speed of the top of the growing boy's head with respect to his feet. . . In a perfectly smooth-riding train, we could tell that we were moving at a high speed only by seeing the scenery whizzing by. We would have just the same experience if the train were somehow fixed and the earth with rails, etc., were to whiz by in the other direction. And if we "lost the reference object" (by pulling down the shades, say) we might not know at all whether we were moving or not. In contrast, we "feel" accelerations and do not need to look out the train window to realize that the engineer has suddenly started the train or has slammed on the



Unless noted otherwise, "rate of change" will always mean "rate of change with respect to time."

Section 1.8

brakes. We might be pushed against the seat, or the luggage might fly from the rack.

All this suggests a profound physical difference between motion at constant speed and motion with acceleration. While it is best to learn about acceleration at first hand (in the laboratory and through the film loops), we can summarize the main ideas here. For the moment let us focus on the similarities between the concepts speed and acceleration; for motion in a straight line:

The rate of <i>change of position</i>	The rate of <i>change of speed</i>
is called <i>speed</i> .	is called <i>acceleration</i> .

This similarity of form will enable us to use what we have just learned about the concept of speed as a guide for making use of the concept of acceleration. For example, we have learned that the slope of the line of a *distance*-time graph is a measure of the instantaneous *speed*. The slope of a *speed*-time graph is a measure of the *instantaneous acceleration*.

This section concludes with a list of six statements about motion along a straight line. The list has two purposes: 1) to help you review some of the main ideas about speed presented in this chapter, and 2) to present the corresponding ideas about acceleration. For this reason, each statement about speed is immediately followed by a parallel statement about acceleration.

1. *Speed* is the rate of change of position. *Acceleration* is the rate of change of speed.

2. *Speed* is expressed in units of distance/time. *Acceleration* is expressed in units of speed/time.

3. Average speed over any time interval is the ratio of the change of position Δd and the time interval Δt :

$$v_{av} = \frac{\Delta c}{\Delta t}$$

Average acceleration over any time interval is the ratio of the change of speed Δv and the time interval Δt :

$$a_{av} = \frac{\Delta v}{\Delta t}$$

4. Instantaneous speed is the value approached by the average speed as Δt is made smaller and smaller. Instantaneous acceleration is the value approached by the average acceleration as Δt is made smaller and smaller.

5. On a distance-time graph, the instantaneous speed at any instant is the slope of the straight line tangent to the curve at the point of interest. On a speed-time graph, the instantaneous acceleration at any instant is the slope of the straight line tangent to the curve at the point of interest.

6. For the particular case of *constant speed*, the distance-time graph is a straight line; everywhere on it the instantaneous speed has the same value, equal to the average speed computed for the whole trip. For the particular case of *constant acceleration*, the speed-time graph is a straight line; everywhere on it the

For example, if an airplane changes its speed from 500 mph to 550 mph in 10 minutes, its average acceleration would be

$$\frac{\Delta v}{\Delta t} = \frac{550 \text{ mi/hr} - 500 \text{ mi/hr}}{10 \text{ min}}$$
$$= \frac{50 \text{ mi/hr}}{10 \text{ min}}$$
$$= \frac{5 \text{ mi/hr}}{\text{min}}$$

That is, its speed changed at a rate of 5 mph per minute. (If the speed was decreasing, the value of the acceleration would be negative.)



Constant speed and constant acceleration are often called "uniform" speed and "uniform" acceleration. In the rest of this course, we will use the terms interchangeably. SG 1.18 provides an opportunity to work with distance-time and speed-time graphs and to see their relationship to one another. Transparencies T3 and T4 may be helpful also.

SG 1.19 to 1.21 are review problems for this chapter. Some of these will test how thoroughly you grasp the language used for describing straight-line motion. instantaneous acceleration has the same value, equal to the average acceleration computed for the whole trip. When speed is constant, its value can be found from any corresponding Δd and Δt . When acceleration is constant, its value can be found from any corresponding Δv and Δt . (This is useful to remember because constant acceleration is the kind of motion we shall encounter most often in the following chapters.)

We now have most of the tools needed to get into some real physics problems. The first of these is the accelerated motion of bodies caused by gravitational attraction. It was by studying motion of falling objects that Galileo, in the early 1600's, was first able to shed light on the nature of accelerated motion. His work remains to this day a wonderful example of how scientific theory, mathematics, and actual measurements can be combined to develop physical concepts. More than that, Galileo's work was one of the early and most crucial battlegrounds of the scientific revolution. The specific ideas he introduced are even now fundamental to the science of mechanics, the study of bodies in motion.

Q14 What is the average acceleration of an airplane which goes from 0 to 60 mph in 5 seconds?

Q15 What is your average acceleration if, while walking, you change your speed from 4.0 miles per hour to 2.0 miles per hour in an interval of 15 minutes? Is your answer affected by how your change of speed is distributed over the 15 minutes?



1.1 This book is probably different in many ways from textbooks you have had in other courses. Therefore we feel it might help to make some suggestions about how to use it.

1. Do not write in this book unless your teacher gives you permission to do so. In many schools the books must be used again next year by other students. However, if you are fortunate enough to be in a situation in which the teacher can permit you to mark in the book, we encourage you to do so. You will note that there are wide margins. One of our reasons for leaving that much space is to enable you to record questions or statements as they occur to you when you are studying the material. Mark passages that you do not understand so that you can seek help from your teacher.

2. If you may not write in the textbook itself. try keeping a notebook keyed to the text chapters. In this study notebook jot down the kinds of remarks, questions and answers that you would otherwise write in the textbook as suggested above. Also, you ought to write down the questions raised in your mind by the other learning materials you will use, by the experiments you do, by demonstrations or other observations, and by discussions you may have with fellow students and others with whom you talk physics. Most students find such an informal notebook to be enormously useful when studying, or when seeking help from their teachers (or, for that matter, from advanced students, parents, scientists they may know, or anyone else whose understanding of physics they have confidence in).

3. You will find answers to all of the end-ofsection review questions on page 197. Always try to answer the questions yourself first and then check your answers. If your answer agrees with the one in the book, it is a good sign that you understand the main ideas in that section – although it is true that you can sometimes get the right answer for the wrong reason, and also that there may sometimes be other answers as good (or better than!) those given in the book.

4. There are many different kinds of items in the Study Guide at the end of each chapter. Brief answers to some of them are given on page 199. It is not intended that you should do every item. Sometimes we include material in the Study Guide which we think will especially interest only some students. Notice also that there are several kinds of problems. Some are intended to give practice in the use of a particular concept, while others are designed to help you bring together several related concepts. Still other problems are intended to challenge those students who particularly like to work with numbers.

5. This text is only one of the learning materials of the Project Physics course. The course includes several other materials such as film loops, programmed instruction booklets, and transparencies. Use those. Be sure to familiarize yourself also with the *Handbook*, which describes outside activities and laboratory experiments, and with the *Reader*, in which we

have collected interesting articles related to physics. Each of these learning aids makes its own contribution to an understanding of physics, and all are designed to be used together.

The Project Physics learning materials particularly appropriate for Chapter 1 include:

Experiments (in the Handbook) Naked Eye Astronomy Regularity and Time Variations in Data Measuring Uniform Motion

Activities (in the Handbook) Using the Electronic Stroboscope Making Frictionless Pucks

Reader Articles Motion in Words Representation of Motion Motion Dynamics of a Golf Club Bad Physics in Athletic Measurements

Transparencies Analyzing a Stroboscopic Photograph Stroboscopic Measurements Graphs of Various Motions Instantaneous Speed Instantaneous Rate of Change

In addition the following Project Physics materials can be used with Unit 1 in general:

Reader Articles The Value of Science Close Reasoning How to Solve It Four Pieces of Advice to Young People On Being the Right Size The Vision of Our Age Becoming a Physicist Chart of the Future

1.2 One type of automobile speedometer is a small electric generator driven by a flexible cable run off the drive shaft. The current produced increases with the rate at which the generator is turned by the drive shaft. The speedometer needle indicates the current. Until the speedometer is calibrated it cannot indicate actual speeds in

miles per hour. Try answering the questions below. If you have trouble you may want to try again after you have studied through Sec. 1.9.

- (a) How would you calibrate the speedometer in a car if the company had forgotten to do the job?
- (b) If you replaced the 24"-diameter rear wheels with 28"-diameter wheels, what would your actual speed be if the speedometer read 50 mph?
- (c) Would the speedometer read too high or too low if you loaded down the rear end of your car and had the tire pressure too low?
- (d) Does the operation of the speedometer itself affect the motion of the car?
- (e) How would you test to see if a bicycle speedometer affects the speed of a bike?
- (f) Can you invent a speedometer that has no effect on the motion of the vehicle that carried it?

OUT UN TION

1.3 Some practice problems:

	SITUATION	FIND
а	Speed uniform, dis- tance = 72 cm, time = 12 sec	Speed
b	Speed uniform at 45 miles per hour	Distance traveled in 20 minutes
с	Speed uniform at 36 ft/min	Time to move 9.0 feet
d	$d_1 = 0$ $t_1 = 0$ $d_2 = 15 \text{ cm } t_2 = 5.0 \text{ sec}$ $d_3 = 30 \text{ cm } t_3 = 10 \text{ sec}$	Speed and posi- tion at 8.0 sec
е	You drive 240 miles in 6.0 hr	Average speed
f	Same as e	Speed and position after 3.0 hr
g	Average speed is 76 cm/sec, computed over a distance of 418 cm	Time taken
h	Average speed is 44 m/sec, computed over time interval of 0.20 sec	Distance moved

1.4 A tsunami caused by an earthquake occurring near Alaska in 1946 consisted of several sea waves which were found to travel at the average speed of 490 mph. The first of the waves reached Hawaii 4 hrs and 34 min after the earthquake occurred. From these data, calculate how far the origin of the tsunami was from Hawaii.

1.5~ Light and radio waves travel through a vacuum in a straight line at a speed of very nearly $3\times10^8~m/sec.$

- (a) How long is a "light year" (the *distance* light travels in a year)?
- (b) The nearest star, Alpha Centauri, is 4.06×10^{16} m distant from us. If this star possesses planets on which highly intelligent beings live, how soon, at the earliest, could we expect to receive a reply after sending them a radio or light signal strong enough to be received there?

1.6 If you traveled one mile at a speed of 1000 miles per hour and another mile at a speed of 1 mile per hour, your average speed would *not* be 1000 mph + 1 mph/2 or 500.5 mph. What *would* be your average speed? (Hint: What is the total distance and total time?)

1.7 What is your average speed in each of these cases?

- (a) You run 100 m at a speed of 5.0 m/sec and then you walk 100 m at a speed of 1.0 m/sec.
- (b) You run for 100 sec at a speed of 5.0 m/sec and then you walk for 100 sec at a speed of 1.0 m/sec?

1.8 Design and describe experiments to enable you to make estimates of the average speeds for some of the following objects in motion.

- (a) A baseball thrown from outfield to home plate
- (b) The wind
- (c) A cloud
- (d) A raindrop
- (e) A hand moving back and forth as fast as possible
- (f) The tip of a swinging baseball bat
- (g) A person walking on level ground, upstairs, downstairs
- (h) A bird flying
- (i) An ant walking
- (j) A camera shutter opening and closing
- (k) An eye blinking
- (1) A whisker growing
- (m) The center of a vibrating guitar string

1.9 What problems arise when you attempt to measure the speed of light? Can you design an experiment to measure the speed of light?

1.10 Sometime, when you are a passenger in an automobile, compare the speed as read from the speedometer with the speed calculated from $\Delta d/\Delta t$. Explain any differences. Refer again to SG1.2. (For other activities see your Project Physics Handbook.)



1.11 Take a look at the graph of *y* versus *x* shown below:

Although in this particular graph the steepness of the line increases as x increases, the method presented below would also hold for a curve of any other shape. One way to indicate the steepness of the line at a point P is by means of its "slope." The numerical value of the slope at a point P is obtained by the following procedure (diagrammed above): At a very short distance along the line from point P to either side of it, mark 2 points, A and B. Choose these points so close to P that although they also lie on the curve, the line APB is a straight line as nearly as one can determine with a ruler. Measure Δy (the change in y) in going from A to B. In this example y = 0.6. Measure Δx (the corresponding change in x) in going from A to B. Δx here is 0.3. The slope of the segment AB is defined as the ratio of Δy to Δx of the short straight-line-segment APB. By definition, the slope of the curve at point P is taken to be equal to the slope of the straight-linesegment APB.

slope =
$$\frac{\Delta y}{\Delta x}$$

In this example,

slope =
$$\frac{\Delta y}{\Delta x} = \frac{0.6}{0.3} = 2$$

Q. What are the dimensions or units for the slope?

A. The dimensions are just those of y/x. For example, if y represents a distance in meters and x represents a time in seconds, then the units for slope will be meters per second (or m/sec).

Q. In practice, how close must A and B be to point P? (Close is not a very precise adjective. Baltimore is close to Washington if you are flying over both by jet. If you are walking, it is not close.)

A. Choose A and B near enough to point P so that a straight line drawn carefully to connect A and B also goes through point P.

Q. Suppose A and B are so close together that you cannot adequately read Δx or Δy from your graph. How would you try to calculate the slope?



A. Extend the straight line **AB** in both directions, as shown in the figure, as far as you wish, and compute its slope. What you are then doing is putting a *tangent line* to the curve at the chosen point between **A** and **B**. Notice that the small triangle is similar to the large triangle, and, therefore

$$\Delta y / \Delta x = \Delta Y / \Delta X$$

Problem:

- (a) Determine the slope of this graph of distance versus time (y in meters, t in seconds) at four different points or instants, namely when t = 1, 2, 3, and 4 seconds.
- (b) Find the instantaneous speed at these 4 points, and plot a graph of speeds vs. time.

1.12 (Answer to question in text, page 23.) Indeed the prediction based upon the first hour and a half would be vastly wrong. A prediction based on an extrapolation from the first $1\frac{1}{2}$ hour's observation neglects all the factors which limit the maximum height obtainable by such a cluster of balloons, such as the bursting of some of the balloons, the change in air pressure and density with height and many others. Actually, at the end of 500 minutes the cluster was not 450,000 feet high but had come down again, as the distance-time graph for the entire experiment shows. See top of next page. For another extrapolation problem, see SG 1.13.



1.13 World's 400-meter swimming records in minutes and seconds for men and women (numbers in parentheses are ages):

1926	4:57.0	Johnny Weissmuller (18)
	5:53.2	Gertrude Ederle (17)
1936	4:46.4	Syozo Makino (17)
	5:28.5	Helene Madison (18)
1946	4:46.4	(1936 record unbroken)
	5:00.1	R. Hveger (18)
1956	4:33.3	Hironoshin Furuhashi (23)
	4:47.2	Lorraine Crapp (18)
1966	4:11.1	Frank Weigand (23)
	4:38.0	Martha Randall (18)

By about how many meters would Martha Randall have beaten Johnny Weissmuller if they had raced each other? Could you predict the 1976 records for the 400-meter race by extrapolating the graphs of world's records vs. dates up to the year 1976?

1.14 How can we justify defining instantaneous speed as we have on p. 25? How can we be sure the definition is right?

1.15 Using the graph on p. 20 find the instantaneous speeds v at several points (0, 10, 20, 30, 40, and 50 sec, and near 0, or at other points of your choice) by finding the slopes of lines tangent to the curve at each of those points. Make a graph of v vs. t. Use your graph to describe her swim.

1.16 Turn back to p. 28. At the bottom of this page there is a multiple-exposure photograph of a baseball rolling to the right. The time interval between successive flashes was 0.20 sec. The distance between marks on the meter stick was 1 centimeter. You might tabulate your measurements of the ball's progress between flashes and construct a distance-time graph. From the distance-time graph, you can determine the instantaneous speed at several instants and

construct a speed-time graph. You can check your results by referring to the answer page at the end of this unit.

1.17 Careful analysis of a stroboscopic photograph of a moving object yielded information which was plotted on the graph below. By placing your ruler tangent to the curve at appropriate points estimate the following:



- (a) At what moment or interval was the speed greatest? What was the value of the speed at that time?
- (b) At what moment or in which interval was the speed least? What was it at that time?
- (c) What was the speed at time t = 5.0 sec?
- (d) What was the speed at time t = 0.5 sec?
- (e) How far did the object move from time t = 7.0 sec to t = 9.5 sec?

1.18 The data below show the instantaneous speed in a test run of a car starting from rest. Plot the speed-vs-time graph, then derive data from it and plot the acceleration-vs-time graph.

- (a) What is the speed at t = 2.5 sec?
 - (b) What is the maximum acceleration?

ime (sec)	Speed (m/sec)	Time (sec)	Speed (m/sec)
0.0	0.0	6.0	27.3
1.0	6.3	7.0	29.5
2.0	11.6	8.0	31.3
3.0	16.5	9.0	33.1
4.0	20.5	10.0	34.9
5.0	24.1		

1.19 The electron beam in a typical TV set sweeps out a complete picture in 1/30 sec and each picture is composed of 525 lines. If the width of the screen is 20 inches, what is the speed of that beam over the surface of the screen?

1.20 Suppose you must measure the instantaneous speed of a bullet as it leaves the barrel of a rifle. Explain how you might do this.

STUDY GUIDE 1

1.21 Discuss the motion of the cat in the following series of photographs, "Cat in trot changing to gallop." The numbers on each

photograph indicate the number of inches measured from the fixed line marked "0." The time interval between exposures is 0.030 sec.



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Portrait of Galileo by Ottavio Leoni, a contemporary of Galileo.

CHAPTER TWO

Free Fall – Galileo Describes Motion

2.1 The Aristotelian theory of motion

In this chapter we shall follow the development of an important piece of basic research: Galileo's study of freely falling bodies. While the physical problem of free fall is interesting in itself, our emphasis will be on the way Galileo, one of the first modern scientists, presented his argument. His view of the world, his way of thinking, his use of mathematics, and his reliance upon experimental tests set the style for modern science. These aspects of his work, therefore, are as important to us as the actual results of his investigation.

To understand the nature of Galileo's work and to appreciate its significance, we must first examine the previous system of physical thought that it eventually replaced. In medieval physical science, as Galileo learned it at the University of Pisa, a sharp distinction was thought to exist between the objects on the earth and those in the sky. All terrestrial matter, the matter within our physical reach, was believed to contain a mixture of four "elements" -Earth, Water, Air, and Fire. These elements were not thought of as identical with the natural materials for which they were named. Ordinary water, for example, was thought to be a mixture of all four elements, but mostly the element Water. Each of the four elements was thought to have a natural place in the terrestrial region. The highest place was allotted to Fire. Beneath Fire was Air, then Water, and finally, in the lowest position, Earth. Each was thought to seek its own place. Thus Fire, if displaced below its natural position, would tend to rise through Air. Similarly, Air would tend to rise through Water, whereas Earth would tend to fall through both Air and Water. The movement of any real object depended on its particular mixture of these four elements, and on where it was in relation to the natural places of these elements.

SG 2.1



A sketch of a medieval world-system.

A good deal of common-sense experience supports this naturalplace view. See SG 2.2

From quinta essentia, meaning fifth essence. In earlier Greek writings the term for it was aether (also written ether).

The painting entitled "School of Athens," was done by Raphael in the beginning of the sixteenth century. It reflects a central aspect of the Renaissance, the rebirth of interest in classical Greek culture. The central figures are Plato (on the left, pointing to the heavens) and Aristotle (pointing to the ground). When water boiled, for example, the element Water would be joined by the element Fire, whose higher natural place would cause the mixture to rise as steam. A stone, on the other hand, being composed primarily of the element Earth, would fall when released and would pass through Fire, Air, and Water until it came to rest on the ground, its natural place.

The medieval thinkers also believed that the stars, planets, and other celestial bodies differed in composition and behavior from objects on or near the earth. The celestial bodies were believed to contain none of the four ordinary elements, but instead to consist solely of a fifth element, the *quintessence*. The natural motion of objects composed of this element was neither rising nor falling, but endless revolution in circles around the center of the universe. That center was considered to be identical with the center of the earth. Heavenly bodies, although moving, were at all times in their natural places. Thus heavenly bodies were altogether different from terrestrial objects, which displayed natural motion only as they returned to their natural places from which they had been displaced.

This theory, so widely held in Galileo's time, had originated almost 2000 years before, in the fourth century B.C. We find it stated clearly in the writings of the Greek philosopher Aristotle. This physical science, built on order, class, place, and purpose, fits well many facts of everyday observation. It seemed particularly plausible in societies like those in which Aristotle and Galileo lived, where rank and order were dominant in human experience. Moreover, these conceptions of matter and motion were part of an allembracing universal scheme or "cosmology." In his cosmology Aristotle sought to relate ideas which are nowadays discussed separately under such headings as science, poetry, politics, ethics, and theology.

Not very much is known of Aristotle's physical appearance or life. It is thought that he was born in 384 B.C. in the Greek province of Macedonia. His father was the physician to the King of Macedonia, and so Aristotle's early childhood was spent in an environment of court life. He completed his education in Athens and later returned to Macedonia to become the private tutor to Alexander the Great. In 335 B.C., Aristotle came back to Athens and founded the Lyceum. a school and center of research.





After the decline of the ancient Greek civilization, the writings of Aristotle remained virtually unknown in Western Europe for 1500 years. They were rediscovered in the thirteenth century A.D. and were later incorporated into the works of Christian scholars and theologians. Aristotle became such a dominant influence in the late Middle Ages that he was referred to simply as "The Philosopher."

The works of Aristotle make up almost an encyclopedia of ancient Greek thought. Some of it was summarized from the work of others, but much of it seems to have been created by Aristotle himself. Today it is hard to believe that one man could have been so well informed on such different subjects as logic, philosophy, theology, physics, astronomy, biology, psychology, politics, and literature. Some scholars doubt that it was all the work of one man.

Unfortunately, Aristotle's physical theories had serious limitations. (This does not, of course, detract from his great achievements in other fields.) According to Aristotle, the fall of a heavy object toward the center of the earth is an example of "natural" motion. He evidently thought that any object, after release, quickly achieves some final speed of fall at which it continues to move to the end of its path. What factors determine the final speed of a falling object? It is a common observation that a rock falls faster than a leaf. Therefore, he reasoned, weight is a factor that governs the speed of fall. This fitted in well with his idea that the *cause* of weight was the presence of the element Earth, whose natural tendency was to the center of the earth. Thus a heavier object, having a greater content of Earth, has a greater tendency to fall to its natural place, and hence develops a greater speed in falling.

The same object falls more slowly in water than in air, so it seemed to Aristotle that the resistance of the medium must also be a factor. Other factors, such as the color or temperature of the falling object, could conceivably affect the rate of fall, but Aristotle decided that their influence could not be significant. He concluded that the rate of fall must increase in proportion to the weight of the object and decrease in proportion to the resisting force of the medium. The actual rate of fall in any particular case would be found by dividing the weight by the resistance.

Aristotle also discussed "violent" motion – that is, any motion of an object other than going freely toward its "natural place." Such motion, he argued, must always be caused by a *force*, and the speed of the motion will increase as the force increases. When the force is removed, the motion must stop. This theory agrees with our common experience, say in pushing a chair or a table across the floor. It doesn't work quite so well for objects thrown through the air, since such projectiles keep moving for a while even after we have stopped exerting a force on them. To account for this kind of motion, Aristotle proposed that the air itself somehow exerts a force that keeps the object moving.

Later scientists proposed some modifications in Aristotle's

Aristotle: rate of fall is proportional to weight divided by resistance.

SG 2.3

Section 2.1

theory of motion. For example, in the fifth century A.D. John Philoponus of Alexandria argued that the speed of an object in natural motion should be found by *subtracting* the resistance of the medium from the weight of the object, rather than dividing by the resistance. Philoponus claimed that his experimental work supported his theory, though he did not report the details; he simply said that he dropped two weights, one of which was twice as heavy as the other, and observed that the heavy one did not reach the ground in half the time taken by the light one.

There were still other difficulties with Aristotle's theory of motion. However, the realization that his teachings concerning motion had limitations did little to modify the importance given to them in the universities of France and Italy during the fifteenth and sixteenth centuries. Aristotle's theory of motion did, after all, fit much of ordinary experience in a general—if qualitative—way. Besides, the study of motion through space was of major interest to only a few scholars, just as it had been only a very small part of Aristotle's own work.

Two other influences stood in the way of radical changes in the theory of motion. First, Aristotle believed that mathematics was of little value in describing terrestrial phenomena. Second, he put great emphasis upon direct, qualitative observation as the basis for theorizing. Simple qualitative observation was very successful in Aristotle's biological studies. But as it turned out, real progress in physics began only when the value of mathematical prediction and detailed measurement was recognized.

A number of scholars in the fifteenth and sixteenth centuries had a part in this change to a new way of doing science. But of all these, Galileo was by far the most eminent and successful. He showed how to describe mathematically the motions of simple, ordinary objects – falling stones and balls rolling on an incline. This work not only paved the way for other men to describe and explain the motions of everything from pebbles to planets, it also began an intellectual revolution which led to what we now consider modern science.

Q1 Describe two ways in which, according to the Aristotelian view, terrestrial and celestial bodies differ from each other.

Q2 Which of these statements would be accepted in the fifteenth and sixteenth centuries by persons who believed in the Aristotelian system of thought?

- (a) Ideas of motion should fit in with poetry, politics, theology and other aspects of human thought and activity.
- (b) Heavy objects fall faster than light ones.
- (c) Except for motion toward their natural location, objects will not move unless acted on violently by a force.
- (d) Mathematics and precise measurement are especially important in developing a useful theory of motion.

John Philoponus: rate of fall is proportional to weight *minus* resistance.

SG 2.4

Qualitative refers to quality – the sort of thing that happens. Quantitative refers to quantity – the measurement or prediction of numerical values. This distinction will appear often in the course.



2.2 Galileo and his times

Galileo Galilei was born in Pisa in 1564 – the year of Michelangelo's death and Shakespeare's birth. Galileo was the son of a nobleman from Florence, and he acquired his father's active interest in poetry, music, and the classics. His scientific inventiveness also began to show itself early. For example, as a young medical student at the University of Pisa, he constructed a simple pendulum-type timing device for the accurate measurement of pulse rates.

Lured from medicine to physical science by reading Euclid and Archimedes, Galileo quickly became known for his unusual ability in science. At the age of 26, he was appointed Professor of Mathematics at Pisa. There he showed an independence of spirit unmellowed by tact or patience. Soon after his appointment, he began to challenge the opinions of his older colleagues, many of whom became his enemies. He left Pisa before his term was completed, apparently forced out by financial difficulties and by his enraged opponents. Later, at Padua in the Republic of Venice, he began his work in astronomy. His support of the sun-centered theory of the universe eventually brought him additional enemies, but it also brought him immortal fame. We shall deal with that part of his work in Unit 2.

Drawn back to his native province of Tuscany in 1610 by a generous offer of the Grand Duke, Galileo became Court Mathematician and Philosopher, a title which he chose himself. From then until his death at 78, despite illness, family troubles, occasional brushes with poverty, and quarrels with his enemies, he continued his research, teaching and writing.

2.3 Galileo's Two New Sciences

Galileo's early writings on mechanics (the study of the behavior of matter under the influence of forces) were in the tradition of the standard medieval theories of physics, although he was aware of some of the shortcomings of those theories. During his mature years his chief interest was in astronomy. However, when his important astronomical book, *Dialogue on the Two Great World Systems* (1632), was condemned by the Roman Catholic Inquisition and he was forbidden to teach the "new" astronomy, Galileo decided to concentrate again on mechanics. This work led to his book *Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion* (1638), usually referred to as *Two New Sciences*. This treatise signaled the beginning of the end, not only of the medieval theory of mechanics, but also of the entire Aristotelian cosmology which it supported.

Galileo was old, sick, and nearly blind at the time he wrote *Two New Sciences.* Yet, as in all his writings, his style is spritely





Italy about 1600



Title page of Dialogue on Two Great World Systems (1632).

Free Fall-Galileo Describes Motion

DISCORSI DIMOSTRAZIONI MATEMATICHE,

intorno à due nuoue scienze

Attenenti alla MECANICA & I MOVIMENTI LOCALI,

del Signor GALILEO GALILEI LINCEO, Filosofo e Matematico primario del Serenissimo Grand Duca di Tofcana.

Con una Appendice del centro di granità d'alcuni Solidi.



D

IN LEIDA. Appresso gli Elsevirii. M. D. C. XXXVIII.

Title page of Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion (1638).

SG 2.5

DEL GALILEO. 65 Farmo non fi farbbe il moto, la policion del Vacuo affolutamente preface non in relazione al moto, non vien deitrutta, mà per dire quel cho per aunentura potebber erifondere quegli antichi, acciò meglio fi forges, quanto concoldud al admonttrazione d'artittotel, mi parche fi potrebbe andar contro à gli affanti di quello, negandogli amendue. Equanto al primofi ograndemente dabito, che Aritto-tel non ferimentaffe mai quanto fa vera, che due pietre vun più arte dell'altra dicei volte lafine nel medefimo initante cader dava altezza, vo.gr. di conto bracia fuffer talmente differento auffe non bauere anaco fiefo dicei braccia for dimonte differento define no bauere anaco fiefo dicei braccia. Simp. Si vede pare dalle fue parale, cho ei mostra el buerlo fe-terenne thourem faita le forencetta. Sagt. Maio S. Simp. chen'hò faito la prona, vi afficuro, che vun palitiferio, che nefi cento, dagento, canzo più libbe, non maniciperò di von palmo filemente la ritto in sitera tale apula dan motori si voglamo filemente taritto in sitera dei la palla dan palitiferio, che nefi cento, adagento, canzo più libbe, non palitiferio si pare più von mezza, venendo anco dedt' altezza di turavo netica con sitera della palla dan paliti parto di von palmo folimente taritto in sitera di la palla dan succione thoure di site la conto in setta della palla dan paliti parto di von palmo folimente taritto in sitera da la palla dan paliti parto da site pie von mezza, venendo anco dall' altezza di turavo hevento.

moschesso, che ne pesi una mezza, venendo anco dall'alsezza di

dugento braccia. Salu. Mà finz'altre efferienze con brene, e concludente dimo-Salta. Mà/fm: altre efferienze con breue, e concludente dimo-litrazione posfiamo chiaramente prevare non effer vero, che vu mo-bile più grave fimmaus più velacemente d'un' altre omen grave, in-tendenda di mobili dell' itieffa materia i, chia fomma di quelli dei quali parta d'arilotele. Però ditemi S. Simp-fe voi ammettete, che di tiafcheduno corpo grane cadente fia vun da matura determinata velocità i, fiche l'accreftergliela, à diminuirgilelanon fi polfa fenon ton vigragi violenza, à opporgi quadeba impedimento. Simp. Non fi può dubtere, che l'itieffo mobile nell' itieffo mez-zo habbia vun flatnita, e da natura determinata velocità, la qua-lenn fi ce ji polfa accreftere fe nan com nuona impeto conferito.

le non fe gli polla accestere fe nan con nuova impeto conferito. gli, à diminni glicla falvo che con qualche impedimento the lori-audi.

Salu. Quando dunque noi hanessimo due mobili, le naturali velo-

A page from the original Italian edition of Two New Sciences, showing statements that are translated in this text. and delightful. He used the dialogue form to allow a lively conversation among three 'speakers': Simplicio, who competently represents the Aristotelian view; Salviati, who presents the new views of Galileo; and Sagredo, the uncommitted man of good will and open mind, eager to learn. Eventually, of course, Salviati leads his companions to Galileo's views. Let us listen to Galileo's three speakers as they discuss the problem of free fall:

Salviati: I greatly doubt that Aristotle ever tested by experiment whether it is true that two stones, one weighing ten times as much as the other, if allowed to fall at the same instant from a height of, say, 100 cubits, would so differ in speed that when the heavier had reached the ground, the other would not have fallen more than 10 cubits. [A "cubit" is equivalent to about 20 inches.]

Simplicio: His language would indicate that he had tried the experiment, because he says: We see the heavier: now the word see shows that he had made the experiment.

Sagredo: But, I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span [hand-breadth] ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits.

Here, perhaps, one might have expected to find a detailed report on an experiment done by Galileo or one of his colleagues. Instead, Galileo uses a "thought experiment" - an analysis of what would happen in an imaginary experiment – to cast grave doubt on Aristotle's theory of motion:

Salviati: But, even without further experiment, it is possible to prove clearly, by means of a short and conclusive argument, that a heavier body does not move more rapidly than a lighter one provided both bodies are of the same material and in short such as those mentioned by Aristotle. But tell me, Simplicio, whether you admit ' that each falling body acquires a definite speed fixed by nature, a velocity which cannot be increased or diminished except by the use of violence or resistance?

Simplicio: There can be no doubt but that one and the same body moving in a single medium has a fixed velocity which is determined by nature and which cannot be increased except by the addition of impetus or diminished except by some resistance which retards it.

Salviati: If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. Do you not agree with me in this opinion?

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Simplicio: You are unquestionably right.

Salviati: But if this is true, and if a large stone moves with a speed of, say, eight, while a smaller moves with a speed of four, then when they are united, the system will move with a speed less than eight; but the two stones when tied together make a stone larger than that which before moved with a speed of eight. Hence the heavier body moves with less speed than the lighter one; an effect which is contrary to your supposition. Thus you see how, from your assumption that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly.

Simplicio: I am all at sea. . . . This is, indeed, quite beyond my comprehension. . . .

Simplicio retreats in confusion when Salviati shows that the Aristotelian theory of fall is self-contradictory. But while Simplicio cannot refute Galileo's logic, his own eyes tell him that a heavy object *does* fall faster than a light object:

Simplicio: Your discussion is really admirable; yet I do not find it easy to believe that a birdshot falls as swiftly as a cannon ball.

Salviati: Why not say a grain of sand as rapidly as a grindstone? But, Simplicio, I trust you will not follow the example of many others who divert the discussion from its main intent and fasten upon some statement of mine that lacks a hairsbreadth of the truth, and under this hair hide the fault of another that is as big as a ship's cable. Aristotle says that "an iron ball of one hundred pounds falling from a height of 100 cubits reaches the ground before a one-pound ball has fallen a single cubit." I say that they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two fingerbreadths. . . . Now you would not hide behind these two fingers the 99 cubits of Aristotle, nor would you mention my small error and at the same time pass over in silence his very large one.

This is a clear statement of an important principle: even in careful observation of a common natural event, the observer's attention may be distracted by what is really a minor effect, with the result that he fails to see a much more significant regularity. Different bodies falling in air from the same height, it is true, do *not* reach the ground at exactly the same time. However, the important point is not that the times of arrival are *slightly different*, but that they are *very nearly the same*! Galileo regarded the failure of the bodies to arrive at exactly the same time as a minor effect which could be explained by a deeper understanding of motion in free fall. Galileo himself correctly attributed the observed results to differences in the effect of the resistance of the air on bodies of SG 2.6



A stroboscopic photograph of two freely falling balls of unequal weight. The balls were released simultaneously. The time interval between images is 1/30 sec. The phrase "free fall" as now used in physics generally refers to fall when the only force acting is gravity; that is, when air friction is negligible. different size and weight. A few years after Galileo's death, the invention of the vacuum pump allowed others to show that Galileo was right. Once the effect of air resistance was eliminated – for example, when a feather and a heavy gold coin were dropped from the same height at the same time inside an evacuated container – the different bodies fell at the same rate and struck the bottom of the container at the same instant. Long after Galileo, it became possible to formulate the laws of air resistance, so one could understand exactly why and by how much a light object falls behind a heavier one.

Learning what to ignore has been almost as important in the growth of science as learning what to take into account. In the case of falling bodies, Galileo's explanation depended on his being able to imagine how an object would fall if there were no air resistance. This may be easy for us who know of vacuum pumps, but in Galileo's time it was an explanation that was difficult to accept. For most people, as for Aristotle, common sense said that air resistance is always present in nature. Thus a feather and a coin could never fall at the same rate. Why should one talk about hypothetical motions in a vacuum, when a vacuum could not be shown to exist? Physics, said Aristotle and his followers, should deal with the world all around us that we can readily observe, not with some imaginary world which might never be found.

Aristotle's physics had dominated Europe since the thirteenth century, mainly because many intelligent scientists were convinced that it offered the most rational method for describing natural phenomena. To overthrow such a firmly established doctrine required much more than writing reasonable arguments, or simply dropping heavy and light objects from a tall building, as Galileo is often said to have done (but probably did not) at the Leaning Tower of Pisa. It demanded Galileo's unusual combination of mathematical talent, experimental skill, literary style, and tireless campaigning to discredit Aristotle's theories and to begin the era of modern physics.

A chief reason for Galileo's success was that he exposed the Aristotelian theory at its weakest point: he showed that physics can deal better with the world around us if we realize that the world of common observation is not the simple starting point the Aristotelians thought it to be. On the contrary, the world as we ordinarily observe it is usually quite complex. For example, in observing the fall of bodies you see the effects of both the law of fall and the law of resistance on objects moving through air. To understand what you see, you should start from a simple case (such as fall without resistance), even if this has to be "seen" only in your mind or by a mathematical model. Or you may turn to an experiment in the laboratory, where the usual conditions of observation can be changed. Only after you understand each of the different effects by itself should you go back to face the complexities of the ordinary case. Q3 If a nail and a toothpick are simultaneously dropped from the same height, they do not reach the ground at exactly the same instant. (Try it with these or similar objects.) How would Aristotelian theory explain this? What was Galileo's explanation?

2.4 Why study the motion of freely falling bodies?

In Galileo's attack on the Aristotelian cosmology, few details were actually new. However, his approach and his findings together provided the first coherent presentation of the science of motion. Galileo realized that, out of all the observable motions in nature, free-fall motion is the key to the understanding of all motions of all bodies. To decide *which* is the key phenomenon to study is the real gift of genius. But Galileo is also in many ways typical of scientists in general. His approach to the problem of motion makes a good "case" to be used in the following sections as an opportunity to discuss strategies of inquiry that are still used in science.

These are some of the reasons why we study in detail Galileo's attack on the problem of free fall. Galileo himself recognized another reason – that the study of motion which he proposed was only the starting phase of a mighty field of discovery:

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless, I have discovered some properties of it that are worth knowing that have not hitherto been either observed or demonstrated. Some superficial observations have been made, as for instance, that the natural motion of a heavy falling body is continuously accelerated; but to just what extent this acceleration occurs has not yet been announced....

Other facts, not few in number or less worth knowing I have succeeded in proving; and, what I consider more important, there have been opened up to this vast and most excellent science, of which my work is merely the beginning, ways and means by which other minds more acute than mine will explore its remote corners.

2.5 Galileo chooses a definition of uniform acceleration

Two New Sciences deals directly with the motion of freely falling bodies. In studying the following paragraphs from it, we must be alert to Galileo's overall plan. First, he discusses the mathematics of a possible, simple type of motion (which we now call uniform acceleration or constant acceleration). Then he proposes that heavy bodies actually fall in just that way. Next, on the basis of this proposal, he derives a prediction about balls rolling down an incline. Finally, he shows that experiments bear out these predictions. By Aristotelian cosmology is meant the whole interlocking set of ideas about the structure of the physical universe and the behavior of all the objects in it. This was briefly mentioned in Sec. 2.1. Other aspects of it will be presented in Unit 2.

In fact, more than mere "superficial observations" had been made long before Galileo set to work. For example, Nicolas Oresme and others at the University of Paris had by 1330 discovered the same distancetime relationship for falling bodies that Galileo was to announce in the *Two New Sciences*. Some of their reasoning is discussed in SG 2.7.

It will help you to have a plan clearly in mind as you progress through the rest of this chapter. As you study each succeeding section, ask yourself whether Galileo is

- -presenting a definition
- stating an assumption (or hypothesis)
- deducing predictions from his hypothesis
- experimentally testing the predictions

The first part of Galileo's presentation is a thorough discussion of motion with uniform speed, similar to our discussion in Chapter 1. That leads to the second part, where we find Salviati saying:

We pass now to . . . naturally accelerated motion, such as that generally experienced by heavy falling bodies.

... in the investigation of naturally accelerated motion we were led, by hand as it were, in following the habit and custom of nature herself, in all her various other processes, to employ only those means which are most common, simple and easy ...

When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal timeintervals, conceive additions of speed as taking place without complication....

Hence the definition of motion which we are about to discuss may be stated as follows:

A motion is said to be uniformly accelerated when, starting from rest, it acquires during equal timeintervals, equal increments of speed.

Sagredo: Although I can offer no rational objection to this or indeed to any other definition devised by any author whosoever, since all definitions are arbitrary, I may nevertheless without defense be allowed to doubt whether such a definition as the foregoing, established in an abstract manner, corresponds to and describes that kind of accelerated motion which we meet in nature in the case of freely falling bodies

Here Sagredo questions whether Galileo's arbitrary definition of acceleration actually corresponds to the way real objects fall. Is acceleration, as defined, really useful in describing their observed change of motion? Sagredo wonders about a further point, so far not raised by Galileo:

From these considerations perhaps we can obtain an answer to a question that has been argued by philosophers, namely, what is the *cause* of the acceleration of the natural motion of heavy bodies

But Salviati, the spokesman of Galileo, rejects the ancient tendency to investigate phenomena by looking first for their causes. It is premature, he declares, to ask about the cause of any motion until an accurate description of it exists:

This is sometimes known as the Rule of Parsimony: unless forced to do otherwise, assume the simplest possible hypothesis to explain natural events.

Rephrasing Galileo and using our symbols: for uniform speed v, the ratio $\Delta d/\Delta t$ is constant. Similarly, recall that for accelerated motion, as we saw in Chapter 1, we defined uniform acceleration as

$$a = \frac{\Delta v}{\Delta t} = \text{constant}$$

Other ways of expressing this relationship are discussed in SG 2.8 and 2.9.

Salviati: The present does not seem to be the proper time to investigate the cause of the acceleration of natural motion concerning which various opinions have been expressed by philosophers, some explaining it by attraction to the center, others by repulsion between the very small parts of the body, while still others attribute it to a certain stress in the surrounding medium which closes in behind the falling body and drives it from one of its positions to another. Now, all these fantasies, and others, too, ought to be examined; but it is not really worth while. At present it is the purpose of our Author merely to investigate and to demonstrate some of the properties of accelerated motion, whatever the cause of this acceleration may be.

Galileo has now introduced two distinct propositions: 1) "uniform" acceleration means equal speed increments Δv in equal time intervals Δt ; and 2) things actually fall that way. Let us first look more closely at Galileo's proposed definition.

Is this the only possible way of defining uniform acceleration? Not at all! Galileo says that at one time he thought a more useful definition would be to use the term uniform acceleration for motion in which speed increased in proportion to the distance traveled, Δd , rather than to the time Δt . Notice that both definitions met Galileo's requirement of simplicity. (In fact, both definitions had been discussed since early in the fourteenth century.) Furthermore, both definitions seem to match our common sense idea of acceleration about equally well. When we say that a body is "accelerating," we seem to imply "the farther it goes, the faster it goes," and also "the longer time it goes, the faster it goes." How should we choose between these two ways of putting it? Which definition will be more useful in the description of nature?

This is where experimentation becomes important. Galileo chose to define uniform acceleration as the motion in which the change of speed Δv is proportional to elapsed time Δt , and then demonstrate that this matches the behavior of real moving bodies, in laboratory situations as well as in ordinary, "un-arranged," experience. As you will see later, he made the right choice. But he was not able to prove his case by direct or obvious means, as you shall also see.

Q4 Describe uniform speed without referring to dry ice pucks and strobe photography or to *any* particular object or technique of measurement.

Q5 Express Galileo's definition of uniformly accelerated motion in words and in the form of an equation.

Q6 What two conditions did Galileo want his definition of uniform acceleration to meet?

2.6 Galileo cannot test his hypothesis directly

After Galileo defined uniform acceleration so that it would match the way he *believed* freely falling objects behaved, his next Here Salviati refers to the Aristotelian assumption that air propels an object moving through it (see Sec. 2.1). task was to devise a way of showing that the definition for uniform acceleration was useful for describing observed motions.

Suppose we drop a heavy object from several different heights say, from windows on different floors of a building. We want to check whether the final speed increases in proportion to the time it takes to fall—that is, whether $\Delta v \propto \Delta t$, or what amounts to the same thing, whether $\Delta v/\Delta t$ is constant. In each trial we must observe the time of fall and the speed just before the object strikes the ground. But there's the rub. Practically, even today, it would be very difficult to make a *direct measurement* of the speed reached by an object just before striking the ground. Furthermore, the entire time intervals of fall (less than 3 seconds even from the top of a 10-story building) are shorter than Galileo could have measured accurately with the clocks available to him. So a direct test of whether $\Delta v/\Delta t$ is constant was not possible for Galileo.

Q7 Which of these are valid reasons why Galileo could not test directly whether the final speed reached by a freely falling object is proportional to the time of fall?

- (a) His definition was wrong.
- (b) He could not measure the speed attained by an object just before it hit the ground.
- (c) There existed no instruments for measuring time.
- (d) He could not measure ordinary distances accurately enough.
- (e) Experimentation was not permitted in Italy.

2.7 Looking for logical consequences of Galileo's hypothesis

Galileo's inability to make *direct* measurements to test his hypothesis – that $\Delta v/\Delta t$ is constant in free fall – did not stop him. He turned to mathematics to derive from this hypothesis some other relationship that *could* be checked by measurement with equipment available to him. We shall see that in a few steps he came much closer to a relationship he could use to check his hypothesis.

Large distances of fall and large time intervals for fall are, of course, easier to measure than the small values of Δd and Δt that would be necessary to find the final speed just before the falling body hits. So Galileo tried to find, by reasoning, how total fall distance ought to increase with total fall time if objects did fall with uniform acceleration. You already know how to find total distance from total time for motion at constant *speed*. Now we will derive a new equation that relates total fall distance to total time of fall for motion at constant *acceleration*. In this we shall not be following Galileo's own derivation exactly, but the results will be the same. First, we recall the definition of average speed as the distance traversed Δd divided by the elapsed time Δt :

$$v_{av} = \frac{\Delta d}{\Delta t}$$

The symbol \propto means "directly proportional to."

SG 2.10

Section 2.7

This is a general definition and can be used to compute the average speed from measurement of Δd and Δt , no matter whether Δd and Δt are small or large. We can rewrite the equation as

$$\Delta d = v_{av} \times \Delta t$$

This equation, still being really a definition of v_{av} , is always true. For the special case of motion at a constant speed v, then $v_{av} = v$ and therefore, $\Delta d = v \times \Delta t$. When the value of v is known (as, for example, when a car is driven with a steady reading of 60 mph on the speedometer), this equation can be used to figure out how far (Δd) the car would go in any given time interval (Δt). But in uniformly accelerated motion the speed is continually *changing* – so what value can we use for v_{av} ?

The answer involves just a bit of algebra and some plausible assumptions. Galileo reasoned (as others had before) that for any quantity that changes uniformly, the average value is just halfway between the beginning value and the final value. For uniformly accelerated motion starting from rest (where $v_{initial} = 0$ and ending at a speed v_{final} , this rule tells us that the average speed is halfway between 0 and v_{final} – that is, $v_{ar} = \frac{1}{2} v_{final}$. If this reasoning is correct, it follows that

$$\Delta d = \frac{1}{2} v_{\text{final}} \times \Delta t$$

for uniformly accelerated motion starting from rest.

This relation could not be directly tested either, because the last equation still contains a speed factor. What we are trying to arrive at is an equation relating total distance and total time, without any need to measure speed.

Now we look at Galileo's definition of uniform acceleration: $a = \Delta v / \Delta t$. We can rewrite this relationship in the form $\Delta v = a \times \Delta t$. The value of Δv is just $v_{\text{final}} - v_{\text{initial}}$; and $v_{\text{initial}} = 0$ for motion that begins from rest. Therefore we can write

$$\Delta v = a imes \Delta t$$

 $v_{ ext{final}} - v_{ ext{initial}} = a imes \Delta t$
 $v_{ ext{final}} = a imes \Delta t$

Now we can substitute this expression for v_{final} into the equation for Δd above. Thus *if* the motion starts from rest, and *if* it is uniformly accelerated (and *if* the average rule is correct, as we have assumed) we can write

$$\Delta d = \frac{1}{2} v_{\text{final}} \times \Delta t$$
$$= \frac{1}{2} (a \times \Delta t) \times \Delta t$$

Or, regrouping terms,

$$\Delta d = \frac{1}{2}a(\Delta t)^2$$

This is the kind of relation Galileo was seeking – it relates total distance Δd to total time Δt , without involving any speed term.

Before finishing, though, we will simplify the symbols in the equation to make it easier to use. If we measure distance and time from the position and the instant that the motion starts ($d_{initial} = 0$)

More generally the average speed would be

$$V_{\rm av} = rac{V_{\rm initial} + V_{\rm final}}{2}$$

SG 2.11 and 2.12

and $t_{\text{initial}} = 0$), then the intervals Δd and Δt have the values given by d_{final} and t_{final} . The equation above can therefore be written more simply as

SG 2.13 and 2.14

$$l_{\text{final}} = \frac{1}{2}at^2_{\text{final}}$$

Remember that this is a very specialized equation—it gives the total distance fallen as a function of total time of fall but only if the motion starts from rest ($v_{initial} = 0$), if the acceleration is uniform (a = constant), and if time and distance are measured from the start ($t_{initial} = 0$ and $d_{initial} = 0$).

Galileo reached the same conclusion, though he did not use algebraic forms to express it. Since we are dealing only with the special situation in which acceleration *a* is constant, the quantity $\frac{1}{2}a$ is constant also, and we can cast the conclusion in the form of a proportion: in uniform acceleration from rest, the distance traveled is proportional to the square of the time elapsed, or

$d_{\rm final} \propto t^2_{\rm final}$

For example, if a uniformly accelerating car starting from rest moves 10 m in the first second, in *twice* the time it would move *four* times as far, or 40 m in the first two seconds. In the first 3 seconds it would move 9 times as far—or 90 m.

Another way to express this relation is to say that the ratio d_{final} to t^2_{final} has a constant value, that is,

$$\frac{d_{\text{final}}}{t^2_{\text{final}}} = \text{constant}$$

Thus a logical result of Galileo's original proposal for defining uniform acceleration can be expressed as follows: if an object accelerates uniformly from rest, the ratio d/t^2 should be constant. Conversely, any motion for which this ratio of d and t^2 is found to be constant for different distances and their corresponding times, we may well suppose to be a case of motion with *uniform acceleration* as defined by Galileo.

Of course, we still must test the hypothesis that freely falling bodies actually *do* exhibit just such motion. Recall that earlier we confessed we were unable to test directly whether $\Delta v/\Delta t$ has a constant value. Galileo showed that a logical consequence of a constant value of $\Delta v/\Delta t$ would be a constant ratio of d_{final} to t^2_{final} . The values for total time and distance of fall would be easier to measure than the values of short intervals Δd and Δt needed to find Δv . However, measuring the time of fall still remained a difficult task in Galileo's time. So, instead of a *direct* test of his hypothesis, Galileo went one step further and deduced an ingenious, indirect test.

Q8 Why was the equation $d = \frac{1}{2}at^2$ more promising for Galileo than $a = \Delta v / \Delta t$ in testing his hypothesis?

Q9 If you simply combined the two equations $\Delta d = v\Delta t$ and $\Delta v = a\Delta t$ it looks as if one might get the result $\Delta d = a\Delta t^2$. What is wrong with doing this?

SG 2.15

Because we will use the expression $d_{\text{final}}/t^2_{\text{final}}$ many times, it is simpler to write it as d/t^2 —it is understood that d and t mean total distance and time interval of motion, starting from rest.

2.8 Galileo turns to an indirect test

Realizing that a direct quantitative test with a rapidly and freely falling body would not be accurate, Galileo proposed to make the test on an object that was moving less rapidly. He proposed a new hypothesis: if a freely falling body has an acceleration that is constant, then a perfectly round ball rolling down a perfectly smooth inclined plane will also have a constant, though smaller, acceleration. Thus Galileo claimed that if d/t^2 is constant for a body falling freely from rest, this ratio will also be constant, although smaller, for a ball released from rest and rolling different distances down a straight inclined plane.

Here is how Salviati described Galileo's own experimental test in *Two New Sciences*:

A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken: on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse beat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter of the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the . . . channel along which we rolled the ball . . .

Note the careful description of the experimental apparatus. Today an experimenter would add to his verbal description any detailed drawings, schematic layouts or photographs needed to make it possible for other competent scientists to duplicate the experiment.

This picture painted in 1841 by G. Bezzuoli, attempts to reconstruct an experiment Galileo is alleged to have made during his time as lecturer at Pisa. Off to the left and right are men of ill will: the blase Prince Giovanni de Medici (Galileo had shown a dredging-machine invented by the prince to be unusable) and Galileo's scientific opponents. These were leading men of the universities; they are shown here bending over a book of Aristotle, where it is written in black and white that bodies of unequal weight fall with different speeds. Galileo, the tallest figure left of center in the picture, is surrounded by a group of students and followers.







Galileo's technique for measuring time is discussed in the next section.

SG 2.16



Spheres rolling down planes of increasingly steep inclination. At 90° the inclined plane situation matches free fall. (Actually, the ball will start slipping instead of rolling long before the angle has become that large.) Galileo has packed a great deal of information into these lines. He describes his procedures and apparatus clearly enough to allow other investigators to repeat the experiment for themselves if they wished. Also, he gives an indication that consistent measurements can be made, and he restates the two chief experimental results which he believes support his free-fall hypothesis. Let us examine the results carefully.

(a) First, he found that when a ball rolled down an incline at a fixed angle to the horizontal, the ratio of the distance covered to the square of the corresponding time was always the same. For example, if d_1 , d_2 , and d_3 represent distances measured from the same starting point on the inclined plane, and t_1 , t_2 , and t_3 the corresponding times taken to roll down these distances, then

$$\frac{d_1}{t_1^2} = \frac{d_2}{t_2^2} = \frac{d_3}{t_3^2}$$

In general, for each angle of incline, the value of d/t^2 was constant. Galileo did not present his experimental data in the full detail which has become the custom since. However, his experiment has been repeated by others, and they have obtained results which parallel his (see data in SG 2.16). This is an experiment which you can perform yourself with the help of one or two other students. (The photographs on the next page show students in the Project Physics course doing this experiment and also show some of their results.)

(b) Galileo's second experimental finding relates to what happens when the angle of inclination of the plane is changed. He found that whenever the angle changed, the ratio d/t^2 took on a new value, although for any one angle it remained constant regardless of distance of roll. Galileo confirmed this by repeating the experiment "a full hundred times" for each of many different angles. After finding that the ratio d/t^2 was constant for each angle of inclination for which measurements of t could be carried out conveniently, Galileo was willing to extrapolate. He concluded that the ratio d/t^2 is a constant even for larger angles, where the motion of the ball is too fast for accurate measurements of t to be made. Finally, Galileo reasoned that in the particular case when the angle of inclination became 90°, the ball would move straight down-and so becomes the case of a falling object. By his reasoning, d/t^2 would still be some constant in that extreme case (even though he couldn't say *what* the numerical value was.)

Because Galileo had deduced that a constant value of d/t^2 was characteristic of uniform acceleration, he could conclude at last that free fall was uniformly accelerated motion.

Q10 In testing his hypothesis that free fall motion is uniformly accelerated, Galileo made the unproved *assumption* that (check one or more):

⁽a) d/t^2 is constant.







- (b) the acceleration has the same value for all angles of inclination of the plane.
- (c) the results for small angles of inclination can be extrapolated to large angles.
- (d) the speed of the ball is constant as it rolls.
- (e) the acceleration of the rolling ball is constant if the acceleration in free fall is constant, though the value of the two constants is not the same.

Q11 Which of the following statements best summarizes the

work of Galileo on free fall when air friction is negligible? (Be prepared to defend your choice.) Galileo:

- (a) proved that all objects fall at exactly the same speed regardless of their weight.
- (b) proved that for any freely falling object the ratio d/t^2 is constant for any distance of fall.
- (c) proved that an object rolling down a smooth incline accelerates in the same way as (although more slowly than) the same object falling freely.
- (d) supported indirectly his assertion that the speed of an object, falling freely from rest is proportional to the elapsed time.
- (e) made it clear that until a vacuum could be produced, it would not be possible to settle the free-fall question once and for all.

2.9 Doubts about Galileo's procedure

This whole process of reasoning and experimentation looks long and involved on first reading, and some doubts may well arise concerning it. For example, was Galileo's measurement of time precise enough to establish the constancy of d/t^2 even for the case of a slowly rolling object? In his book, Galileo tries to reassure possible critics by providing a detailed description of his experimental arrangement (thereby inviting any skeptics to try it for themselves):

For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small cup during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the time intervals, and this with such accuracy that, although the operation was repeated many, many times, there was no appreciable discrepancy in the results.

The water clock described by Galileo was not invented by him. Indeed, there are references to water clocks in China as early as the

For problems that will check and extend your understanding of uniform acceleration. See SG 2.17 through 2.24.

Section 2.9

sixth century B.C., and they were probably used in Babylonia and India even earlier. In the early 16th century a good water clock was the most accurate of the world's instruments for measuring short time intervals. It remained so until shortly after Galileo's death, when the work of Christian Huygens and others led to practical pendulum clocks. When better clocks became available, Galileo's results on inclined-plane motion were confirmed.

Another reason for questioning Galileo's results is related to the great difference between free fall and rolling motion on a slight incline. Galileo does not report what angles he used in his experiment. However, as you may have found out from doing a similar experiment, the angles must be kept rather small. As the angle increases, the speed of the ball soon becomes so great that it is difficult to measure the times involved. The largest usable angle reported in a recent repetition of Galileo's experiment was only 6°. (See SG 2.15) It is not likely that Galileo worked with much larger angles. This means that the extrapolation to free fall (90° incline) is a large one, perhaps much too large for a cautious person – or for one not already convinced of Galileo's argument.

Still another reason for questioning Galileo's results is the observation that, as the angle of incline is increased, there comes a point where the ball starts to slide as well as roll. This change in behavior could mean that the motion is very different at large angles. Galileo does not discuss these cases. It is surprising that he apparently did not repeat the experiment with blocks which would slide, rather than roll, down a smooth incline. If he had, he would have found that for accelerated sliding motion the ratio d/t^2 is also a constant, although the constant has a different numerical value than for rolling at the same angle.

Q12 Which of the following statements could be regarded as major reasons for doubting the validity of Galileo's procedure?

- (a) His measurement of time was not sufficiently accurate.
- (b) He used too large an angle of inclination in his experiment.
- (c) It is not clear that his results apply when the ball can slide as well as roll.
- (d) In Galileo's experiment the ball was rolling, and therefore he could not extrapolate to the case of free fall where the ball did not roll.
- (e) d/t^2 was not constant for a sliding object.

2.10 Consequences of Galileo's work on motion

Galileo seems to have been well aware that one cannot get the correct numerical value for the acceleration of a body in free fall simply by extrapolating the results to increasingly large angles of inclination. He did not attempt to calculate a numerical value for the acceleration of freely falling bodies. But for his purposes it was enough that he could support the hypothesis that the acceleration is *constant* for any given body, whether rolling or falling. This is the



Early water clock

SG 2.25

We now know by measurement that the magnitude of the acceleration of gravity, symbol a_{u} , is about 9.8 m/sec per sec, or 32 ft/sec per sec, at the earth's surface. The Project Physics Handbook contains five different experiments for finding a value of a_{u} . (For many problems, the approximate value 10 m/sec/sec is satisfactory.)

SG 2.26

You can derive this equation. (See SG 2.27)

SG 2.28 and 2.29

first consequence of Galileo's work, one that has been fully borne out by all subsequent tests.

Second, if spheres of different weights are allowed to roll down an inclined plane set at a given angle, they turn out to have the same acceleration. We do not know how much experimental evidence Galileo himself had for this conclusion, but it is consistent with the observations for freely falling objects. It is consistent also with his "thought experiment" by which he argued that bodies of different weights fall at the same rate (aside from the comparatively small effects of air resistance). His results provided a decisive refutation of Aristotle's theory of motion.

Third, Galileo developed a mathematical theory of accelerated motion from which other predictions about motion could be derived. We will mention just one example here, which will turn out to be very useful in Unit 3. Recall that Galileo chose to define acceleration as the rate at which the speed changes with time. He then found by experiment that falling bodies actually do experience equal changes of speed in equal times, and not in equal distances as some had supposed. Still, the idea of something changing by equal amounts in equal distances has an appealing simplicity, too. One might ask if there isn't something that *does* change in that way during uniform acceleration. In fact, there is. It follows without any new assumptions that, during uniform acceleration from rest, the *square* of the speed changes by equal amounts in equal distances. There is a mathematical equation which expresses this result: If $v_{initial} = 0$, and a = constant, then

$v_{\text{final}}^2 = 2ad_{\text{final}}$

In words: if an object starts from rest and moves with uniform acceleration, then the square of its speed at any point is equal to twice the product of its acceleration and the distance it has moved. (We shall see the importance of this relation in Unit 3.)

These consequences of Galileo's work, important as they are to the development of physics, would scarcely have been enough to bring about a revolution in science by themselves. No sensible scholar in the seventeenth century would have given up his belief in the Aristotelian cosmology only because some of its predictions had been refuted in the case of falling (or rolling) bodies. But Galileo's work on free-fall motion helped to prepare the way for the development of a new kind of physics, and indeed a new cosmology, by planting the seeds of doubt about the crucial assumptions of Aristotelian science. For example, when it was recognized that all bodies fall with equal acceleration if air friction is negligibly small, then the whole Aristotelian explanation of falling motion (Section 2.1) broke down.

The most agitating scientific problem during Galileo's lifetime was not in mechanics but in astronomy. A central question in cosmology was whether the earth or the sun is the center of the universe. Galileo supported the view that the earth and other planets revolve around the sun, a view entirely contrary to

Section 2.10

Aristotelian cosmology. But to support such a view required a physical theory of why and how the earth itself moved. Galileo's work on free fall and other motions turned out to be just what was needed to begin to construct such a theory. His work did not have its full effect, however, until it had been combined with the investigations of forces and motion by the English scientist Isaac Newton. But as Newton acknowledged, Galileo was the pioneering pathfinder. (In the next chapter we will consider Newton's work on force and motion. In Chapter 8, after studying about motion in the heavens, we will return to Newton's laws and the revolution they began in science.)

Galileo's work on motion introduced a new and significant method of doing scientific research, a method as applicable today as when Galileo demonstrated it. The basis of this procedure is a cycle, repeated as often as necessary, entirely or in part, until a satisfactory theory has emerged: general observation \rightarrow hypothesis \rightarrow mathematical analysis or deduction from hypothesis \rightarrow experimental test of deduction \rightarrow modification of hypothesis in light of test, and so forth.

While the steps in the mathematics are often determined mainly by "cold logic," this is not so for the other parts of the process. A variety of paths of thought can lead to the hypothesis in the first place. A new hypothesis can come from an inspired hunch based on general knowledge of the experimental facts, or from a desire for mathematically simple statements, or from modifying a previous hypothesis that failed. Moreover, there are no general rules about exactly how well the experimental data must agree with the theoretical predictions. In some areas of science, a theory is expected to be accurate to better than one 1/1000th of one percent; in other areas, or at an early stage of any new work, one might be delighted to find a theory from which he could make predictions with an error of only 50 percent. Finally note that while experiment has an important place in this process, it is not at all the only or even the main element. On the contrary, experiments are worthwhile only in conjunction with the other steps in the process.

The general cycle of observation, hypothesis, deduction, test, modification, etc., so skillfully demonstrated by Galileo in the seventeenth century, commonly appears in the work of scientists today. Though there is no such thing as *the* scientific method, some form of this cycle is almost always present in scientific research. It is used not out of respect for Galileo as a towering figure in the history of science, but because it works so well so much of the time.

Galileo himself was aware of the value of both the results and the methods of his pioneering work. He concluded his treatment of accelerated motion by putting the following words into the mouths of the commentators in his book:

Salviati: . . . we may say the door is now opened, for the

SG 2.30

first time, to a new method fraught with numerous and wonderful results which in future years will command the attention of other minds.

Sagredo: I really believe that . . . the principles which are set forth in this little treatise will, when taken up by speculative minds, lead to another more remarkable result; and it is to be believed that it will be so on account of the nobility of the subject, which is superior to any other in nature.

During this long and laborious day, I have enjoyed these simple theorems more than their proofs, many of which, for their complete comprehension, would require more than an hour each; this study, if you will be good enough to leave the book in my hands, is one which I mean to take up at my leisure after we have read the remaining portion which deals with the motion of projectiles; and this if agreeable to you we shall take up tomorrow.

Salviati: I shall not fail to be with you.

Q13 Which one of the following was *not* a result of Galileo's work on motion?

- (a) The correct numerical value of the acceleration in free fall was obtained by extrapolating the results for larger and larger angles of inclination.
- (b) If an object starts from rest and moves with uniform acceleration *a* through a distance *d*, then the square of its speed will be proportional to *d*.
- (c) Bodies rolling on a smooth inclined plane are uniformly accelerated (according to Galileo's definition of acceleration).

Many details of physics, mathematics and history have appeared in this chapter. For a review of the most important ideas, see SG 2.31, 2.32, and 2.33.
2.1 Note that at the beginning of each chapter in this book there is a list of the section titles. This is a sort of road map you can refer to from time to time as you study the chapter. It is important, expecially in a chapter such as this one, to know how the part you are studying relates to what preceded it and to have some idea of where it is leading. For this same reason, you will find it very helpful at first to skim through the entire chapter, reading it rapidly and not stopping to puzzle out parts that you do not quickly understand. Then you should return to the beginning of the chapter and work your way through it carefully, section by section. Remember also to use the end-of-section questions to check your progress. The Project Physics learning materials particularly appropriate for Chapter 2 include:

Experiments

A Seventeenth-Century Experiment Twentieth Century Version of Galileo's Experiment Measuring the Acceleration Due to Gravity, a_g

Activities

When is Air Resistance Important? Measuring Your Reaction Time Falling Weights Extrapolation

Reader Article

On the Scientific Method

Film Loops

Acceleration Due to Gravity–Method I Acceleration Due to Gravity–Method II

Transparency Derivation of $d = v_i t + \frac{1}{2}at^2$

2.2 Aristotle's theory of motion seems to be supported to a great extent by common sense experience. For example, water bubbles up through earth at springs. When sufficient fire is added to water by heating it, the resulting mixture of elements (what we call steam) rises through the air. Can you think of other examples?

2.3 Drop sheets of paper with various degrees of "crumpling." Try to crumple a sheet of paper tight enough that it will fall at the same rate as a tennis ball. Can you explain the results with Aristotle's theory?

2.4 Compare Aristotle's hypothesis about falling rate (weight divided by resistance) with Philoponus' (weight minus resistance) for some extreme cases: a very heavy body with no resistance, a very light body with great resistance. Do the two hypotheses suggest very different results?

2.5 Consider Aristotle's statement "A given weight moves [falls] a given distance in a given time; a weight which is as great and more moves the same distances in less time, the times being in inverse proportion to the weights. For instance, if one weight is twice another, it will take half as long over a given movement." (*De Caelo*)

Indicate what Simplicio and Salviati each would predict for the falling motion in these cases:

- (a) A 2-pound rock falls from a cliff and, while dropping, breaks into two equal pieces.
- (b) A hundred-pound rock is dropped at the same time as one hundred 1-pound pieces of the same type of rock.
- (c) A hundred 1-pound pieces of rock, falling from a height, drop into a draw-string sack which closes, pulls loose and falls.

2.6 Tie two objects of greatly different weight (like a book and a pencil) together with a piece of string. Drop the combination with different orientations of objects. Watch the string. In a few sentences summarize your results.



2.7 A good deal of work preceded that of Galileo on the topic of motion. In the period 1280-1340, mathematicians at Merton College, Oxford, carefully considered different quantities that change with the passage of time. One result that had profound influence was a general theorem known as the "Merton Theorem" or "Mean Speed Rule."

This theorem might be restated in our language and applied to uniform acceleration as follows: the distance an object goes during some time while its speed is changing uniformly is the same distance it would go if it went at the average speed the whole time.

- (a) First show that the total distance traveled at a constant speed can be expressed as the area under the graph line on a speedtime graph. ("Area" must be found in speed units × time units.)
- (b) Assume that this area represents the total distance even when the speed is not constant. Draw a speed vs. time graph for uniformly increasing speed and shade in the area under the graph line.

(c) Prove the "Merton Rule" by showing that the area is equal to the area under a constant-speed line at the average speed.

2.8 According to Galileo, uniform acceleration means equal Δv 's in equal Δt 's. Which of the following are other ways of expressing the same idea?

- (a) Δv is proportional to Δt
- (b) $\Delta v / \Delta t = \text{constant}$
- (c) the speed-time graph is a straight line
- (d) v is proportional to t

2.9 In the *Two New Sciences* Galileo states, "... for so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity (namely 1:3:5:7...)...."

The area beneath the curve in a speed-time graph represents the distance traveled during some time interval. Using that idea, give a proof that the distances an object falls in successive equal time intervals will be in the ratios of the odd numbers.

2.10 Using whatever modern equipment you wish, describe how you could find an accurate value for the speed of a falling object just before striking the ground.

2.11 Show that the expression

$$v_{av} = \frac{v_{\text{initial}} + v_{\text{final}}}{2}$$

is equivalent to the "Merton Rule" discussed in SG 2.7.

2.12 For any quantity that *changes uniformly*, the average is the sum of the initial and final values divided by two. Try it out for any quantity you may choose – for example: what is the average age in a group of five people having individually the ages of 15, 16, 17, 18, and 19 years? What is your average earning power over five years if it grows steadily from \$5000 per year at the start to \$9000 per year at the end?

2.13 Several special assumptions have been made in arriving at the equation $d = \frac{1}{2}at^2$. What is the "unwritten text" behind it?

2.14 Lt. Col. John L. Stapp achieved a speed of 632 mph (284 m/sec) in an experimental rocket sled at the Holloman Air Base Development Center, Alamogordo, New Mexico, on March 19, 1954. Running on rails and propelled by nine rockets, the sled reached its top speed within 5 seconds. Stapp survived a maximum acceleration of 22 g's in slowing to rest during a time interval of $1\frac{1}{2}$ seconds (one g is an acceleration equal in magnitude to that due to gravity; 22 g's means $22 \times a_{\rm gc}$)

- (a) Find the average acceleration in reaching maximum speed.
- (b) How far did the sled travel before attaining maximum speed?

(c) Find the *average* acceleration while stopping.

2.15 Derive the expression $d/t^2 = constant$ from the expression $d = \frac{1}{2}at^2$.

2.16 Table 2.1 reports results from a recent repetition of Galileo's experiment in which the angle of inclination was 3.73° (*Science*, 133, 19-23, June 6, 1961). A water clock with a constant-level reservoir was used.

TABLE 2.1			
DISTANCE (ft)	TIME (measured in milliliters of water)	d/t²	
15	90	0.00185	
13	84	0.00183	
10	72	0.00192	
7	62	0.00182	
5	52	0.00185	
3	40	0.00187	
1	23.5	0.00182	

Do these data really support Galileo's assertion that d/t^2 is constant? Explain your conclusion.

2.17 Indicate whether the following statements are true or false when applied to the strobe photo below:

- (a) The speed of the ball is greater at the bottom than at the top.
- (b) This could be a freely falling object. (Make measurements on photograph.)
- (c) This could be a ball thrown straight upward.
- (d) If (b) is true, the speed increases with time because of the acceleration due to gravity.
- (e) If (c) is true, the speed decreases with time because of the effect of gravity; this effect could still be called acceleration due to gravity.



- 2.18 (a) Show by means of equations that Galileo's statement in SG 2.9 follows from $d/t^2 = \text{constant}$ for free fall from rest.
 - (b) The time interval between strobe flashes was 0.35 sec. Use this information to make a rough graph of d vs. t, also one of v vs. t, and find the acceleration of the ball.

2.19 The photograph in the figure below is of a ball thrown upward. The acceleration due to gravity increases the speed of the ball as it goes down from its highest point (like any free-falling object), if air friction is negligible. But the acceleration due to gravity, which does not change, acts also while the ball is still on its way up, and for that portion of the path causes the ball to slow down as it rises.

B

Stroboscopic photograph of a ball thrown into the air.

When there is both up and down motion, it will help to adopt a sign convention, an arbitrary but consistent set of rules, similar to designating the height of a place with respect to sea level. To identify distances measured *above* the point of initial release, give them *positive* values, for example, the distance at B or at D, measured from the release level, is about +60 cm and +37 cm, respectively. If measured *below* the release level, give them *negative* values; for example, E is at -23 cm. Also, assign a positive value to the speed of an object on its way up to the top (about +3 m/sec at A) and a negative value to a speed a body has on the way down after reaching the top (about -2 m/sec at D and -6 m/sec at E).

(a) Fill in the table with + and – signs.

AT POSITION	SIGN GIVEN TO VALUE OF	
	d	V
A B C D E		

- (b) Show that it follows from this convention and from the definition of $a = \Delta v / \Delta t$ that the value or sign given to the acceleration due to gravity is *negative*, and for both parts of the path.
- (c) What would the sign of acceleration due to gravity be in each case if we had chosen the + and - sign conventions just the other way, that is associating - with up, + with down?

2.20 Draw a set of points (as they would appear in a strobe photo) to show the successive positions of an object that by our convention in SG 2.19 had a positive acceleration, that is, "upward." Can you think of any way to produce such an event physically?

2.21 Memorizing equations will not save you from having to think your way through a problem. You must decide if, when and how to use equations. This means analyzing the problem to make certain you understand *what information is given* and *what is to be found*. Test yourself on the following problem. Assume that the acceleration due to gravity is nearly enough equal to 10 m/sec/sec.

Problem: A stone is dropped from rest from the top of a high cliff.

- (a) How far has it fallen after 1 second?
- (b) What is the stone's speed after 1 second of fall?
- (c) How far does the stone fall during the second second? (That is, from the end of the first second to the end of the second second.)

2.22 From the definition for *a*, show it follows directly that $v_{\text{final}} = v_{\text{initial}} + at$ for motion with constant acceleration. Using this relation, and the sign convention in SG 2.19, answer the questions below. (Assume $a_{\text{g}} = 10$ m/sec/sec.) An object is thrown straight upward with an initial speed of 20 m/sec.

(a) What is its speed after 1.0 sec?

(b) How far did it go in this first second?

- (c) How long did the object take to reach its maximum height?
- (d) How high is this maximum height?
- (e) When it descends, what is its final speed as it passes the throwing point?

If you have no trouble with this, you may wish to try problems SG 2.23 and 2.24.

2.23 A batter hits a pop fly that travels straight upwards. The ball leaves his bat with an initial speed of 40 m/sec. (Assume $a_g = 10 \text{ m/sec/sec}$)

- (a) What is the speed of the ball at the end of 2 seconds?
- (b) What is its speed at the end of 6 seconds?
- (c) When does the ball reach its highest point?
- (d) How high is this highest point?
- (e) What is the speed of the ball at the end of 10 seconds? (Graph this series of speeds.)
- (f) What is its speed just before it is caught by the catcher?

2.24 A ball starts up an inclined plane with a speed of 4 m/sec, and comes to a halt after 2 seconds.

- (a) What acceleration does the ball experience?
- (b) What is the average speed of the ball during this interval?
- (c) What is the ball's speed after 1 second?
- (d) How far up the slope will the ball travel?
- (e) What will be the speed of the ball 3
- seconds after starting up the slope? (f) What is the total time for a round trip to the top and back to the start?

2.25 As Director of Research in your class, you receive the following research proposals from physics students wishing to improve upon Galileo's free-fall experiment. Would you recommend support for any of them? If you reject a proposal, you should make it clear why you do so.

- (a) "Historians believe that Galileo never dropped objects from the Leaning Tower of Pisa. But such an experiment is more direct and more fun than inclined plane experiments, and of course, now that accurate stopwatches are available, it can be carried out much better than in Galileo's time. The experiment involves dropping, one by one, different size spheres made of copper, steel, and glass from the top of the Leaning Tower and finding how long it takes each one to reach the ground. Knowing d (the height of the tower) and time of fall t, I will substitute in the equation $d = \frac{1}{2}at^2$ to see if the acceleration *a* has the same value for each sphere."
- (b) "An iron shot will be dropped from the roof of a 4-story building. As the shot falls, it passes a window at each story. At each window there will be a student who starts his stopwatch upon hearing a signal that the shot has been released, and stops the watch as the shot passes his window. Also, each student records the speed of the

shot as it passes. From his own data, each student will compute the ratio v/t. I expect that all four students will obtain the same numerical value of the ratio."

(c) "Galileo's inclined planes dilute motion all right, but the trouble is that there is no reason to suppose that a ball rolling down a board is behaving like a ball falling straight downward. A better way to accomplish this is to use light, fluffy, cotton balls. These will not fall as rapidly as metal spheres, and therefore it would be possible to measure the time of the fall t for different distances. The ratio d/t^2 could be determined for different distances to see if it remained constant. The compactness of the cotton ball could then be changed to see if a different value was obtained for the ratio."

2.26 A student on the planet Arret in another solar system dropped an object in order to determine the acceleration due to gravity at that place. The following data are recorded (in local units):

TIME (in surgs)	DISTANCE (in welfs)	TIME (in surgs)	DISTANCE (in welfs)
0.0	0.0	2.2	10.41
0.5	0.54	2.4	12.39
1.0	2.15	2.6	14.54
1.5	4.84	2.8	16.86
2.0	8.60	3.0	19.33

- (a) What is the acceleration due to gravity on the planet Arret, expressed in welfs/surg²?
- (b) A visitor from Earth finds that one welf is equal to about 6.33 cm and that one surg is equivalent to 0.167 sec. What would this tell us about Arret?
- 2.27 (a) Derive the relation $v^2 = 2ad$ from the equations $d = \frac{1}{2}at^2$ and v = at. What special conditions must be satisfied for the relation to be true?
 - (b) Show that if a ball is thrown straight upward with an initial speed v it will rise to a height

$$h = \frac{v^2}{2a_g}$$

2.28 Sometimes it is helpful to have a special equation relating certain variables. For example, for constant acceleration a, the final speed v_t is related to initial speed v_i and distance traveled d by

$$v_i^2 = v_i^2 + 2ad$$

Try to derive this equation from some others you are familiar with.

2.29 Use a graph like the one sketched below, and the idea that the area under the graph line in a speed-time graph gives a value for the distance traveled, to derive the equation



2.30 List the steps by which Galileo progressed from his first definition of uniformly accelerated motion to his final confirmation that this definition is useful in describing the motion of a freely falling body. Identify each step as a hypothesis, deduction, observation, or computation, etc. What limitations and idealizations appear in the argument?

2.31 In these first two chapters we have been concerned with motion in a straight line. We have dealt with distance, time, speed and acceleration, and with the relationships among them. Surprisingly, most of the results of our discussion can be summarized in the three equations listed below.

$$v_{av} = \frac{\Delta d}{\Delta t}$$
 $a_{av} = \frac{\Delta v}{\Delta t}$ $d = \frac{1}{2}at$

The last of these equations can be applied only to those cases where the acceleration is constant. Because these three equations are so useful, they are worth remembering (together with the limitation on their use).

- (a) State each of the three equations in words.
- (b) Make up a *simple* problem to demonstrate the use of each equation. (For example: How long will it take a jet plane to travel 3200 miles if it averages 400 mi/hr?) Then work out the solution just to be sure the problem can be solved.
- (c) Derive the set of equations which apply *whether or not* the initial speed is zero.

2.32 Show to what extent the steps taken by Galileo on the problem of free fall, as described in Sections 2.5 through 2.8, follow the general cycle in the scientific process.

2.33 What is wrong with the following common statements? "The Aristotelians did not observe nature. They took their knowledge out of old books which were mostly wrong. Galileo showed it was wrong to trust authority in science. He did experiments and showed everyone directly that the old ideas on free fall motion were in error. He thereby started science, and also gave us the scientific method."

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CHAPTER THREE

The Birth of Dynamics – Newton Explains Motion

3.1 "Explanation" and the laws of motion

Kinematics is the study of *how* objects move, but not *why* they move. Galileo investigated many topics in kinematics with insight, ingenuity, and gusto. The most valuable part of that work dealt with special types of motion, such as free fall. In a clear and consistent way, he showed how to describe the motion of objects with the aid of mathematical ideas.

When Isaac Newton began his studies of motion in the second half of the seventeenth century, Galileo's earlier insistence that "the present does not seem to be the proper time to investigate the cause of the acceleration of natural motion" was no longer appropriate. Indeed, because Galileo had been so effective in describing motion, Newton could turn his attention to *dynamics*, the study of *why* an object moves the way it does – why it starts to move instead of remaining at rest, why it speeds up or moves on a curved path, and why it comes to a stop.

How does dynamics differ from kinematics? As we have seen in the two earlier chapters, kinematics deals with the description of motion. For example, in describing the motion of a stone dropped from a cliff, we can write an equation showing how the distance *d* through which the stone has dropped is related to the time *t* the stone has been falling. We can find the acceleration and the final speed attained during any chosen time interval. But when we have completed our description of the stone's motion, we are still not satisfied. Why, we might ask, does the stone accelerate rather than fall with a constant speed? Why does it accelerate uniformly as long as air friction is negligible? To answer these questions, we will have to add to our store of concepts those of *force* and *mass*; and in answering, we are doing *dynamics*. Dynamics goes beyond kinematics by taking into account the cause of the motion. SG 3.1

Some kinematics concepts: position, time, speed, acceleration. Some dynamics concepts: mass, force, momentum (Ch. 9), energy (Ch. 10). In our study of kinematics in Chapters 1 and 2, we encountered four situations: an object may:

(a) remain at rest; (b) move uniformly in a straight line; (c) speed up during straight-line motion; (d) slow down during straight-line motion.

Because the last two situations are examples of acceleration, the list could really be reduced to:

(a) rest; (b) uniform motion; and (c) acceleration.

Rest, uniform motion, and acceleration are therefore the phenomena we shall try to explain. But the word "explain" must be used with care. To the physicist, an event is "explained" when he can demonstrate that the event is a logical consequence of a law he has reason to believe is true. In other words, a physicist with faith in a general law "explains" an observation by showing that it is consistent with the law. In a sense, the physicist's job is to show that the infinite number of separate, different-looking occurrences all around and within us are merely different manifestations or consequences of some general rules which describe the way the world operates. The reason this approach to "explanation" works is still quite remarkable: the number of general rules or "laws" of physics is astonishingly *small*. In this chapter we shall learn three such laws. Taken together with the mathematical schemes of Chapters 1 and 2 for describing motion, they will suffice for our understanding of practically all motions that we can readily observe. And in Unit 2 we shall have to add just one more law (the law of universal gravitation), to explain the motions of stars, planets, comets, and satellites. In fact, throughout physics one sees again and again that nature has a marvelous simplicity.

To explain rest, uniform motion, and acceleration of any object, we must be able to answer such questions as these: Why does a vase placed on a table remain stationary? If a dry-ice disk resting on a smooth, level surface is given a brief push, why does it move with uniform speed in a straight line rather than slow down noticeably or curve to the right or left? Answers to these (and almost all other) specific questions about motion are contained either directly or indirectly in the three general "Laws of Motion" formulated by Isaac Newton. These laws appear in his famous book, *Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy*, 1687), usually referred to simply as *The Principia*. They are among the most basic laws in physics to this day.

We shall examine Newton's three laws of motion one by one. If your Latin is fairly good, try to translate them from the original. A modernized version of Newton's text of these laws, in English, is reproduced in the margin at the left.

Before we look at Newton's contribution, it will be instructive to find out how other scientists of Newton's time, or earlier, might have answered questions about motion. One reason for doing this now is that many people who have not studied physics still show intuitively a bit of the pre-Newtonian viewpoint! Let us look at what we must overcome.

In Chapter 4 we will take up motion also along curved paths.

Newton's First Law: Every object continues in its state of rest or of uniform motion in a straight line unless acted upon by an unbalanced force.

Newton's Second Law: The acceleration of an object is directly proportional to, and in the same direction as, the unbalanced force acting on it, and inversely proportional to the mass of the object.

Newton's Third Law: To every action there is always opposed an equal reaction; or, mutual actions of two bodies upon each other are always equal and in opposite directions.



Q1 A baseball is thrown straight upward. Which of these questions about the baseball's motion are kinematic and which dynamic?

- (a) How high will the ball go before coming to a stop and starting downward?
- (b) How long will it take to reach that highest point?
- (c) What would be the effect of throwing it upward twice as hard?
- (d) Which takes longer, the trip up or the trip down?
- (e) Why does the acceleration remain the same whether the ball is moving up or down?

3.2 The Aristotelian explanation of motion

The idea of force played a central role in the dynamics of Aristotle, twenty centuries before Newton. You will recall from Chapter 2 that in Aristotle's physics there were two types of motion – "natural" motion and "violent" motion. For example, a falling stone was thought to be in "natural" motion (towards its natural place), but a stone being steadily lifted was thought to be in "violent" motion (away from its natural place). To maintain this uniform violent motion, a force had to be continuously applied. Anyone lifting a large stone is very much aware of this as he strains to hoist the stone higher.

The Aristotelian ideas were consistent with many commonsense observations. But there were also difficulties. Take a specific example—an arrow shot into the air. It cannot be in violent motion without a mover, or something pushing on it. Aristotelian physics

Keeping an object in motion at uniform speed.

SG 3.2

required that the arrow be continually propelled by a force; if the propelling force were removed, the arrow should immediately stop its flight and fall directly to the ground in "natural" motion.

But of course the arrow does not fall to the ground as soon as it loses direct contact with the bowstring. What then is the force that propels the arrow? Here, the Aristotelians offered an ingenious suggestion; the motion of the arrow through the air was maintained by the air itself! A commotion is set up in the air by the initial movement of the arrow. That is; as the arrow starts to move, the air is pushed aside; the rush of air to fill the space being vacated by the arrow maintains it in its flight.

More sophisticated ideas to explain motion were developed before the mid-seventeenth century. But in every case, a force was thought to be necessary to sustain uniform motion. The explanation of uniform motion depended on finding the force, and that was not always easy. There were also other problems. For example, a falling acorn or stone does not move with uniform speed – it accelerates. How is acceleration explained? Some Aristotelians thought the speeding up of a falling object was associated with its approaching arrival at its natural place, the earth. In other words, a falling object was thought to be like the tired horse that starts to gallop as it approaches the barn. Others claimed that when an object falls, the weight of the air above it increases while the column of air below it decreases, thus offering less resistance to its fall.

When a falling object finally reaches the ground, as close to the center of the earth as it can get, it stops. And there, in its "natural place," it remains. Rest, being regarded as the natural state of objects on earth, required no further explanation. The three phenomena-rest, uniform motion, and acceleration-could thus be explained in a more or less plausible fashion by an Aristotelian. Now, let us examine the Newtonian explanation of the same phenomena. The key to this approach is a clearer understanding of the concept of force.

Q2 According to Aristotle, what is necessary to maintain uniform motion?

Q3 Give an Aristotelian explanation of a dry-ice puck's uniform motion across a table top.

3.3 Forces in equilibrium

Our common-sense idea of force is closely linked with our own muscular activity. We know that a sustained effort is required to lift and support a heavy stone. When we push a lawn mower, row a boat, split a log, or knead bread dough, our muscles let us know we are applying a force to some object. Force and motion and muscular activity are naturally associated in our minds. In fact, when we think of changing the shape of an object, or moving it or changing its motion, we naturally think of the muscular sensation

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of applying a force to the object. We shall see that many-but not all-of the everyday common-sense ideas about force are useful in physics.

We know intuitively that forces can make things move, but they can also hold things still. The cable supporting the main span of the Golden Gate Bridge is under the influence of mighty forces, yet it remains at rest. Apparently, more is required to start motion than just the application of forces.

Of course, this is not surprising. We have all seen children quarrelling over a toy. If each child pulls determinedly in his own direction, the toy may go nowhere. On the other hand, the tide of battle may shift if one of the children suddenly makes an extra effort, or if two children cooperate and pull side by side against the third.



Likewise, in the tug-of-war between the two teams shown above, large forces were exerted on each side, but the rope remained at rest: one may say the forces balanced, or they "cancelled." A physicist would say that the rope was *in equilibrium* when the sum of the forces on each side of it were equally large and acting in opposite directions. Equally well, he might say the *net force* is zero. Thus a body in equilibrium would not start to move until a new, "unbalanced" force was added which destroyed the equilibrium.

In all these examples, both the magnitude of the forces and their directions are important. The effect of a force depends on the direction in which it is applied. We can represent this directional nature of forces in a sketch by using arrows: The direction the arrow points represents the direction in which the force acts; the length of the arrow represents how large the force is (for example, a 10-lb force is shown by an arrow twice as long as a 5-lb force).

Now we discover a surprising result. If we know separately each of the forces applied to any object at rest, we can predict whether it will remain at rest. It is as simple as this: The object acted on by forces will be in equilibrium under these forces and will remain at rest only if the arrows representing the forces all "add up to zero."

How does one "add up" arrows? By a simple graphical trick. Take the tug-of-war as an example. Let us call the force exerted by the team pulling to the right \vec{F}_{1} . (The little arrow over the F





There are several ways of expressing the idea of unbalanced force: net force, resultant force, total force, vector sum of forces. All mean the same thing.

indicates that we are dealing with a quantity for which direction is important.) The force of the second team is then called \vec{F}_2 . Figure (a) in the margin shows the two arrows corresponding to the two forces, each applied to the central part of the rope, but in opposite directions. Let us assume that these forces, \vec{F}_1 and \vec{F}_2 , were accurately and separately measured, for example, by letting each team in turn pull on a spring balance as hard as it can. The arrows for \vec{F}_1 and \vec{F}_2 are carefully drawn to a chosen scale, such as 1'' = 1000 lb, so that 750 lb of force in either direction would be represented by an arrow of 3/4'' length. Next, in Figure (b), we take the arrows \vec{F}_1 and \vec{F}_2 and draw them again in the correct direction and to the chosen scale, but this time we put them "head to tail." Thus \vec{F}_1 might be drawn first, and then \vec{F}_2 is drawn with the tail of \vec{F}_2 starting from the head of \vec{F}_1 . (Since they would of course overlap in this example, we have drawn them a little apart in Figure (b) to show them both more clearly.) The trick is this: If the head end of the second arrow falls exactly on the tail end of the first, then we know that the effects of \vec{F}_1 and \vec{F}_2 balance each other. The two forces, acting in opposite directions and equally large, add up to zero. If they did not, the excess of one force over the other would be the *net force* and the rope would accelerate instead of being at rest.

To be sure, this was an obvious case, but the graphical technique turns out to work also for cases that are not simple. For example, apply the same procedure to the toy, or to a boat that is to be secured by means of three ropes attached to different moorings.



We are defining equilibrium without worrying about whether the object will rotate. For example: The sum of the forces on the plank in the diagram below is zero, but it is obvious that the plank will rotate.



Consider a situation where \vec{F}_1 is a force of 34 lb, \vec{F}_2 is 26 lb, \vec{F}_3 is 28 lb, each in the direction shown. (The scale for the magnitude of the forces here is 0.1 cm = 1 lb of force.) Is the boat in equilibrium under the forces? Yes, if the forces add up to zero. Let's see. With rule and protractor the arrows are drawn to scale and in exactly the right direction. Then, adding \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 head to tail, we see that the head of the last arrow falls on the tail of the first. Yes, the forces cancel; they add up to zero; the net force is zero. Therefore

the object is in equilibrium. This method tells us when an object is in equilibrium, no matter how many different forces are acting on it.

We can now summarize our understanding of the state of rest as follows: if an object remains at rest, the sum of all forces acting on it must be zero. We regard rest as an example of the condition of equilibrium, the state in which all forces on the object are balanced.

An interesting case of equilibrium, very different from the disputed toy or rope, is part of the "free fall" of a sky-diver. In fact his fall is "free" only at the beginning. The force of air friction increases with speed, and soon the upward frictional force on the sky-diver is great enough to balance the force of gravity downward. Under those circumstances he falls with constant speed, much like a badminton bird or falling leaf. The sensation is not of falling but, except for the wind, the same as lying on a soft bed. During part of a dive from an airplane you can be as much in equilibrium as lying in bed! In both cases the net force acting on you is zero.

Q4 A vase is standing at rest on a table. What forces would you say are acting on the vase? Show how each force acts (to some scale) by means of an arrow. Can you show that the sum of the forces is zero?

Q5 In which of these cases are the forces balanced?



3.4 About vectors

Graphical construction with arrows really works. With it we can predict whether the forces balance and will leave the object in equilibrium or whether any net force is left over, causing the object to accelerate. Why can we use arrows in this way? The reason involves the precise mathematical definitions of displacement and of force, but you can demonstrate for yourself the reasonableness of the addition rule by trying a variety of experiments. For example, you could attach three spring scales to a ring and have some friends pull on the scales with forces that just balance, leaving the ring at rest. While they are pulling, you read the magnitudes of the forces on the scales and mark the directions of the pulls. You can then make a graphical construction with arrows representing the forces and see whether they add to zero. Many different experiments of this kind ought all to show a net force of zero. SG 3.3





It is not obvious that forces should behave like arrows. But arrows drawn on paper happen to be useful for calculating how forces add. (If they were not, we simply would look for other symbols that do work.) Forces belong in a class of concepts called *vector quantities*, or just *vectors* for short. Some characteristics of vectors are well represented by arrows. In particular, vector quantities have *magnitude* which we can represent by the length of an arrow drawn to scale. They have *direction* which can be shown by the direction of an arrow. By experiment, we find that they can be *added* in such a way that the total effect of two or more, called the *vector resultant*, can be represented by the headto-tail addition of arrows.

In the example of the tug-of-war we talked about the effect of equally large, opposing forces. If two forces act in the *same* direction, the resultant force is found in essentially the same way, as shown below.





If two forces act at some angle to each other, the same type of construction is still useful. For example, if two forces of equal magnitude, one directed due east and the other directed due north, are applied to an object at rest but free to move, the object will accelerate in the northeast direction, the direction of the resultant force. The magnitude of the acceleration will be proportional to the magnitude of the resultant force which is shown by the length of the arrow representing the resultant.



The same adding procedure is used if the forces are of any magnitude and act at any angles to each other. For example, if one force were directed due east and a somewhat larger force were directed northeast, the resultant vector sum could be found as shown below.





You can equally well use a graphical construction called the "parallelogram method." It looks different from the "head-to-tail" method, but is really exactly the same. In the parallelogram construction, the vectors to be added are represented by arrows joined tail-to-tail instead of head-to-tail, and the resultant is obtained by completing the diagonal of the parallelogram.

To summarize, we can now define a vector quantity. It is a quantity which has both direction and magnitude and which can be added by the graphical construction of the head-to-tail representation of arrows, or by the equivalent parallelogram method. (It also has other properties which you will study if you take further physics courses.) By this definition, many important physical concepts are vectors - for example, displacement, velocity, and acceleration. Some other physical concepts, including volume, distance, and speed, do not require specification of direction, and so are not vector quantities; these are called scalar quantities. When you add 10 liters of water to 10 liters of water, the result is always 20 liters, and direction has nothing to do with the result. Similarly, the term speed has no directional meaning; it is the magnitude of the velocity vector, as given by the length of the arrow, without regard to its direction. By contrast, when you add two forces of 10 lb each, the resultant force may be anywhere between zero and 20 lb, *depending* on the directions of the two individual forces.

We shall soon have to correct an oversimplification we had to make in Sec. 1.8, where we defined acceleration as the rate of change of speed. That was only partly correct, because it was incomplete. We shall also want to consider changes in the *direction* of motion as well. The more useful definition of acceleration is the rate of change of *velocity*, where velocity is a vector having both magnitude and direction. In symbols,

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

where $\Delta \vec{v}$ is the change in velocity. Velocity can change in two ways: by changing its magnitude (speed), and by changing its direction. In other words, an object is accelerating when it speeds up, or slows down, or changes direction. We shall explore this definition more fully in later sections.

Q7 List three properties of vector quantities.

Q8 How does the new definition of acceleration given above differ from the one used in Chapter 1?

3.5 Newton's first law of motion

Were you surprised when you first watched a dry-ice disk or some other nearly frictionless device? Remember how smoothly it glides along after just the slightest nudge? How it shows no sign of slowing down or speeding up? Although our intuition and everyday experience tell us that some force is constantly needed to keep an object moving, the disk fails to live up to our Aristotelian expectations. It is always surprising to see this for the first time.

Yet the disk is behaving quite naturally. If the forces of friction were absent, a gentle, momentary push would make tables and Any vector quantity is indicated by a letter with an arrow over it; for example, \vec{F} , \vec{a} , or \vec{v} .



We shall use vectors frequently. To learn more about them you can use the Project Physics Programmed Instruction booklets on vectors. See also *Reader 1* article "Introduction to Vectors."

chairs take off and glide across the floor just like a dry-ice disk. Newton's first law directly challenges the Aristotelian notion of what is "natural." It declares that the state of rest and the state of uniform, unaccelerated motion in a straight line are equally natural. Only the existence of some force, friction for example, keeps a moving object from moving *forever*! Newton's first law of motion can be stated as follows in modern terminology:

Every object continues in its state of rest or of uniform rectilinear motion unless acted upon by an unbalanced force. Conversely, if an object is at rest or in uniform rectilinear motion, the unbalanced force acting upon it must be zero.

In order to understand the motion of an object, we must take into account all the forces acting on it. If *all* forces (including friction) are in balance, the body will be moving at constant \vec{v} .

Although Newton was the first to express this idea as a general law, Galileo had made similar statements fifty years before. Of course, neither Galileo nor Newton had dry-ice disks, and so they were unable to observe motion in which friction had been reduced so significantly. Instead, Galileo devised a thought experiment in which he imagined the friction to be zero.

This thought experiment was based on an actual observation. If a pendulum bob on the end of a string is pulled back and released from rest, it will swing through an arc and rise to very nearly its starting height. Indeed, as Galileo showed, the pendulum bob will rise almost to its starting level even if a peg is used to change the path.



It was from this observation that Galileo generated his thought experiment. He predicted that a ball released from a height on a frictionless ramp, would roll up to the same height on a similar facing ramp, regardless of the actual path length. For example, in the diagram at the top of the next page, as the ramp on the right is changed from position (a) to (b) and then to (c), the ball must roll further in each case to reach its original height. It slows down more gradually as the angle of the incline decreases. If the second ramp is exactly *level* as shown in (d), the ball can *never* reach its original height. Therefore, Galileo believed, the ball on this frictionless surface would roll on in a straight line and at an

Because constant velocity means both constant speed and constant direction, we can write Newton's first law more concisely:

 $\vec{v} = \text{constant}$ if and only if $\vec{F}_{\text{net}} = 0$

This statement includes the condition of rest, since rest is a special case of unchanging velocity – the case where $\vec{v} = 0$.

SG 3.5





unchanged speed forever. This could be taken to be the same as Newton's first law, and some historians of science do give credit to Galileo for having come up with the law first. Other historians, however, point out that, for Galileo, rolling on forever meant staying at a constant height above the earth—not moving in a straight line through space.

This tendency of objects to maintain their state of rest or of uniform motion is sometimes called "the principle of inertia." Newton's first law is therefore sometimes referred to as the "law of inertia." Inertia is a property of all objects. Material bodies have, so to speak, a stubborn streak so far as their state of motion is concerned. Once in motion, they continue to move with unchanging velocity (unchanging speed and direction) unless compelled by some externally applied force to do otherwise. If at rest, they remain at rest. This is why seat belts are so helpful when the car stops very suddenly, and also why a car may not follow an icy road around a turn, but travel a straighter path into a field or fence. The greater the inertia of an object, the greater its resistance to a change in its state of motion, and hence the greater is the force needed to produce a desired change in the state of its motion. This is why it is more difficult to start a train or a ship and to bring it up to speed than it is to keep it going once it is moving at the desired speed. (In the absence of friction, it would keep moving without any applied force at all.) But for the same reason it is difficult to bring it to a stop, and passengers and cargo keep going forward if the vehicle is suddenly braked.

Newton's first law tells us that if we see an object moving with a constant speed in a straight line, we know at once that the forces acting on it must be balanced, that is, it is in equilibrium. In Sec. 3.4 we established that an object at *rest* is in equilibrium. Does this mean that in Newtonian physics the state of rest and the state of uniform motion are equivalent? It does indeed. When we know that a body is in equilibrium, we know only that $\vec{v} = \text{constant}$. Whether the value of this constant is zero or not depends in any case on which body is chosen as reference for measuring the magnitude of \vec{v} . We can decide whether to say that it is at rest or that it is moving with constant \vec{v} larger than zero only by reference to some other body.

Take, for example, a tug-of-war. Suppose two teams were sitting on the deck of a barge that was drifting with uniform velocity down a lazy river. Two observers—one on the same barge and one on the shore—would each give a report on the incident as viewed from his Inside the laboratory there is no detectable difference between a straight (horizontal) line and a constant height above the earth. But on a larger scale, Galileo's eternal rolling would become motion in a circle around the earth. Newton made clear what is really important: that in the absence of the earth's gravitational pull or other external forces, the ball's undisturbed path would extend straight out into space.



Galileo's idea of a straight line.



Newton's idea of a straight line.

own frame of reference. The observer on the barge would observe that the forces on the rope were balanced and would report that it was at rest. The observer on the shore would report that the forces on the rope were balanced and that it was in uniform motion. Which observer is right? They are both right; Newton's first law of motion applies to both observations. Whether a body is at rest or in uniform motion depends on which reference frame is used to observe the event. In both cases the forces on the object involved are balanced.

Q9 What is the net force on the body in each of the four cases sketched in the margin of the opposite page?

Q10 What may have been a difference between Newton's concept of inertia and Galileo's?

3.6 The significance of the first law

You may have found Galileo's thought experiment convincing. But think how you might try to verify the law of inertia experimentally. You could start an object moving (perhaps a dryice disk) in a situation in which you believe there is no unbalanced force acting on it. Then you could observe whether or not the object continued to move uniformly in a straight line, as the first law claims it should.

The experiment is not as simple as it sounds; in fact, Newton's laws involve some profound philosophical content (see SG 3.7); but we can see the significance of Newton's first law even without going into all these subtleties. For convenience let us list the important insights the first law provides.

- 1. It presents the idea of inertia as a basic property of all material objects. Inertia is the tendency of an object to maintain its state of rest or uniform motion.
- 2. It points up the equivalence of a state of rest for an object and a state of uniform motion in a straight line. Both states indicate that the net force is zero.
- 3. It raises the whole issue of frame of reference. An object stationary for one observer might be in motion for another observer; therefore, if the ideas of rest or uniform motion are to have any significance, a frame of reference must be specified from which the observations of events are to be made.
- 4. It purports to be a universal law. It emphasizes that a single scheme can deal with motion anywhere in the universe. For the first time no distinction is made between terrestrial and celestial domains. The same law applies to objects on earth as well as on the moon and the planets and the stars. And it applies to balls, dry-ice pucks, magnets, atomic nuclei, electrons everything!

Of course, the idea of inertia does not explain why bodies resist change in their state of motion. It is simply a term that helps us to talk about this basic, experimentally observed fact of nature. (See SG 3.6 and 3.7.)

The correct reference frame to use in our physics turns out to be any reference frame that is at rest or in uniform rectilinear motion with respect to the stars. The rotating earth is, therefore, strictly speaking not allowable as a Newtonian reference frame; but for most purposes the earth rotates so little during an experiment that the rotation can be neglected. (See SG 3.8.)

5. The first law describes the behavior of objects when no unbalanced force acts on them. Thus, it sets the stage for the question: precisely what happens when an unbalanced force *does* act on an object?

3.7 Newton's second law of motion

In Section 3.1 it was stated that a theory of dynamics must account for rest, uniform motion, and acceleration. So far we have met two of our three objectives: the explanation of rest and of uniform motion. In terms of the first law, the states of rest and uniform motion are equivalent; they are different ways of describing the state of equilibrium—that state in which no unbalanced force acts on an object.

The last section concluded with a list of insights provided by the first law. You noticed that there was no quantitative relationship established between force and inertia. Newton's second law of motion enables us to reach our third objective – the explanation of acceleration – and also provides a quantitative expression, an equation for the relationship between force and inertia. We shall study separately the way in which force and inertia enter into the second law. Later in this section we will look more closely at how force and inertia are measured. But first we will take some time to be sure that Newton's statement is clear. First we consider the situation in which different forces act on the same object, and then the situation in which the same force acts on different objects.

Force and Acceleration. To emphasize the force aspect, Newton's second law can be stated as follows:

The net, unbalanced force acting on an object is directly proportional to, and in the same direction as, the acceleration of the object.

More briefly, this can be written as: "acceleration is proportional to net force." If we let \vec{F}_{net} stand for net force and \vec{a} stand for acceleration, we can write this relationship precisely as:

$$\vec{a} \propto \vec{F}_{ne}$$

Both \vec{a} and \vec{F}_{net} are vectors; the statement that they are proportional includes the understanding that they also point in the same direction.

To say that one quantity is proportional to another is to make a precise mathematical statement. Here it means that if a given net force (\vec{F}_{net}) causes an object to move with a certain acceleration (\vec{a}) , then a new force equal to twice the previous force $(2\vec{F}_{net})$ will cause the same object to have a new acceleration equal to twice the earlier acceleration (or $2\vec{a}$); three times the net force will cause three times the acceleration; and so on. Using symbols, this principle can be expressed by a statement like the following:

SG 3.9 SG 3.10



Apple falling-negligible friction



Feather falling at nearly constant speed



Kite held suspended in the wind



Man running against the wind

If a force \vec{F}_{net} will cause

 \vec{a} , then a force equal to $2\vec{F}_{net}$ will cause $2\vec{a}$ $3\vec{F}_{net}$ will cause $3\vec{a}$ $\frac{1}{2}\vec{F}_{net}$ will cause $\frac{1}{2}\vec{a}$ $5.2\vec{F}_{net}$ will cause $5.2\vec{a}$

and so on.

One can readily imagine a rough experiment to test the validity of the law-more easily as a thought experiment than as a real one. Take a nearly frictionless dry-ice puck on a flat table, attach a spring balance, and pull with a steady force so that it accelerates continuously. The pull registered by the balance will be the net force since it is the only unbalanced force acting. Measure the forces and the corresponding accelerations in various tries, then compare the values of \vec{F}_{net} and \vec{a} . We shall look into this method in detail in the next section.

Mass and Acceleration. Now we can consider the inertia aspect of the second law, the effect of the same net force acting on different objects. In discussing the first law, we said inertia is the resistance an object exhibits to any change in its velocity. We know from experience and observation that some objects have greater inertia than others. For instance, if you were to throw a baseball and then put a shot with your full effort, you know that the baseball would be accelerated more and hence would reach a greater speed than the shot. Thus, the acceleration given a body depends as much on the body as it does on the force applied to it. The concept of the amount of inertia a body has is expressed by the term mass.

Mass is a familiar word, but it becomes useful in physics only after it is disentangled from some aspects of its common sense meaning. For example, mass is often used as a synonym for weight. But although mass and weight are closely related, they are not at all the same thing. Weight is a force, the force with which gravity is acting on an object; mass, on the other hand, is a measure of an object's resistance to acceleration. It is true that on or near the surface of the earth, objects that are hard to accelerate are also heavy, and we will return to this relationship in Sec. 3.8.

If you supply the same force to several different objects, their

resulting acceleration of each object is inversely proportional to its mass. Using the symbol *m* for mass (a scalar quantity), and the symbol *a* for the magnitude of the vector acceleration \vec{a} , we can write "*a* is inversely proportional to *m*," or what is mathematically

accelerations will not be the same. Newton claimed that the

the same, "*a* is proportional to $\frac{1}{m}$," or

SG 3.11

What does it mean to say that mass is a scalar quantity?

This means that if a certain force makes a given object have a certain acceleration, then the same force will cause an object having twice the mass to have one-half the acceleration, an object

 $a \propto \frac{1}{m}$

having three times the mass to have one-third the acceleration, an object of one-fifth the mass to have five times the acceleration, and so on. This is why, for example, a truck takes much longer to reach the same cruising speed when it is full than when it is nearly empty. Using symbols, we can express this as follows:

> If a given force \vec{F}_{net} is applied, and an object of mass m experiences a, then an object of mass 2m will experience $\frac{1}{2}a$, of mass 3m will experience $\frac{1}{3}a$, of mass $\frac{1}{5}m$ will experience 5a, of mass 2.5m will experience 0.4a, and so on.

This can be demonstrated by experiment. Can you suggest how it might be done?

The roles played by force and mass in Newton's second law can be combined in a single statement:

The acceleration of an object is directly proportional to, and in the same direction as, the unbalanced force acting on it, and inversely proportional to the mass of the object.

The ideas expressed in this long statement can be summarized by the equation

$$\vec{F} = \frac{\vec{F}_{\text{net}}}{m}$$
 SG 3.13

We can regard this equation as one possible way of expressing Newton's second law of motion. The same relation may of course be equally well written in the form

a

$$\vec{F}_{net} = m\vec{a}$$

In either form, this is probably the most fundamental single equation in all of Newtonian mechanics. Like the first law, the second has an incredible range of application: It holds no matter whether the force is mechanical or electric or magnetic, whether the mass is that of a star or a nuclear particle, whether the acceleration is large or small. We can use the law in the easiest problems and the most sophisticated ones. By measuring the acceleration which an unknown force gives a body of *known mass*, we can compute a numerical value for the force from the equation $\vec{F}_{net} = m\vec{a}$. Or, by measuring the acceleration that a *known force* gives a body of unknown mass, we can compute a numerical value for the force from the equation $\vec{F}_{net} = m\vec{a}$. Clearly we must be able to *measure* two of the three quantities in order to be able to *compute* the other.

Units of mass and force. Even before we can make such measurements, however, we must establish units for mass and force that are consistent with the units for acceleration (which have already been defined in terms of standards of length and time – for example, meters per second per second). SG 3.12

One way to do this is to choose some convenient object, perhaps a piece of corrosion-free metal, as the universal standard of mass, just as a meter is a universal standard of length. We can arbitrarily *assign* to this object a mass of one unit. Once this unit has been selected we can proceed to develop a measure of force.

Although we are free to choose any object as a standard of mass, ideally it should be exceedingly stable, easily reproducible, and of reasonably convenient magnitude. Such a standard object has, in fact, been agreed on by the scientific community. By international agreement, the primary standard of mass is a cylinder of platinum-iridium alloy, kept near Paris at the International Bureau of Weights and Measures. The mass of this platinum cylinder is *defined* as exactly 1 *kilogram* (abbreviated 1 kg). Accurately made copies of this international primary standard of mass are kept in the various standards laboratories throughout the world. Further copies have been made from these for distribution to manufacturers and research laboratories.



1 kg corresponds to the mass of about 1 liter of water, or about 2.2. lb (more precisely 2.205 lb). The 1/1000th part of 1 kg is 1 gram (1 g).

SG 3.14

The standard kilogram and meter at the U.S. Bureau of Standards.

SG 3.15, 3.16

SG 3.17, 3.18

In this equation we use only the magnitudes – the direction is not part of the definition of the unit of force.

Now we can go on to answer the question of how much "push" or "pull" should be regarded as one unit of force. We *define* 1 unit of force as a force which, when acting alone, causes an object that has a mass of 1 kilogram to accelerate at the rate of exactly 1 meter/second/per second.

Imagine an experiment in which the standard 1-kg object is pulled with a spring balance in a horizontal direction across a level, frictionless surface. The pull is regulated to make the 1-kg object accelerate at exactly 1 m/sec². The required force will by definition be one unit in magnitude:

Thus, $1 \text{ kg} \cdot \text{m/sec}^2$ of force is that quantity of force which causes a mass of 1 kg to accelerate 1 m/sec².

The unit kg·m/sec² has been given a shorter name, the *newton* (abbreviated as N). The newton is therefore a derived unit, defined in terms of a particular relationship between the *m*eter, the *k*ilogram, and the second. Thus the newton is part of the "mks" system of units, which is used almost universally in modern scientific work.

The "hidden text" in Newton's second law involves both definitions and experimental facts. There are several possible ways of analyzing it: if you choose to define some part, you must prove others experimentally—or vice-versa. Textbooks do not all agree on how best to present the relation of definition and experiment in Newton's second law, and Newton himself may have not thought it through entirely. However, as a system of ideas (whichever way it is analyzed), it was powerful in leading to many discoveries in physics.

Newton did not "discover" the concepts of force and mass. But he did recognize that these concepts were basic to an understanding of motion. He clarified these concepts, and found a way to express them in numerical values, and so made a science of dynamics possible.

Q11 Which three fundamental units of distance, mass and time are used to define the unit of force?

Q12 A net force of 10 N gives an object a constant acceleration of 4 m/sec^2 . What is the mass of the object?

Q13 True or false? Newton's second law holds only when frictional forces are absent.

Q14 A 2-kg object, shoved across the floor with a speed of 10 m/sec, slides to rest in 5 sec. What was the magnitude of the force producing this acceleration?

Q15 Complete the table in the margin which lists some accelerations resulting from applying equal forces to objects of different mass.

3.8 Mass, weight, and free fall

The idea of force has been generalized in physics to include much more than muscular pushes and pulls. Whenever we observe an acceleration, we infer that there is a force acting. Forces need not be "mechanical" or exerted by contact only; they can be due to gravitational, electric, magnetic, or other actions. Newton's laws are valid for all of them.

The force of gravity acts without direct contact between objects that are separated not only by a few feet of air, as is the case with the earth and a falling stone, but also across empty space such as separates the earth from an artificial satellite in orbit.

We shall use the symbol \vec{F}_{g} for gravitational force. The

The units of acceleration "m/sec per second" can be written as "m/sec/sec" or "m/sec". The sec² means that division by time units occurs twice, not something like "square time."

SG 3.19, 3.20, 3.21, 3.22, 3.23.

MASS	ACCELERATION
m	30 m/sec ²
2 <i>m</i>	15 m/sec ²
3 <i>m</i>	
1/5 <i>m</i>	
0.5 <i>m</i>	
45 <i>m</i>	
	3 m/sec ²
	75 m/sec ²

magnitude of the gravitational pull \vec{F}_g is, roughly speaking, the same anywhere on the surface of the earth for a particular object. When we wish to be very precise, we must take into account the facts that the earth is not exactly spherical, and that there are irregularities in the composition of the earth's crust. These factors cause slight differences – up to 1/2% – in the gravitational force on the same object at different places. An object having a constant mass of 1 kg will experience a gravitational force of 9.812 newtons in London, but only 9.796 newtons in Denver, Colorado. Geologists make use of these variations in locating oil and other mineral deposits.

The term weight is often used in everyday conversation as if it meant the same thing as bulk or mass. In physics, we define the weight of an object as the gravitational force acting on the body. Weight is a vector quantity, as are all forces. Your weight is the downward force our planet exerts on you whether you stand or sit, fly or fall, orbit the earth in a space vehicle or merely stand on a scale to "weigh" yourself.

Think for a moment what a scale does. The spring in it compresses until it exerts on you an upward force sufficient to hold you up. So what the scale registers is really the force with which it pushes up on your feet. When you and the scale stand still and are not accelerating, the scale must be pushing up on your feet with a force equal in magnitude to your weight. That is why you are in equilibrium – the sum of the forces on you is zero.

Now imagine for a moment a ridiculous but instructive thought experiment: as you stand on the scale, the floor (which, sagging slightly, has been pushing up on the scale) suddenly gives way, and you and the scale are dropping into a deep well in free fall. At every instant, your fall speed and the scale's fall speed will be equal. since you started falling together and fall with the same acceleration. Your feet would now touch the scale only barely (if at all), and if you looked at the dial you would see that the scale registers zero. This does not mean you have lost your weight – that could only happen if the earth suddenly disappeared, or if you were suddenly removed to far, interstellar space. No, \vec{F}_g still acts on you as before, accelerating you downward, but since the scale is accelerating *with* you, you are no longer pushing down on it – nor is it pushing up on you.

You can get a fairly good idea of the difference between the properties of weight and mass by holding a big book: First, just lay the book on your hand; you feel the weight of the book acting down. Next, grasp the book and shake it back and forth sideways. You still feel the weight downwards, but you also feel how hard the book is to accelerate back and forth—its mass. You could make your sensation of the book's weight disappear by hanging the book on a string, but the sensation of its inertia as you shake it remains the same. This is only a crude demonstration, and it isn't clear that the shaking sensation doesn't still depend on the pull of the earth. More elaborate experiments would show, however, that weight can

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Is the boy weightless? Explain. (See SG 3.24.)

change without changing mass. Thus when an astronaut on the moon's surface uses a big camera, he finds it much easier to hold – its *weight* is only 1/6 of its weight on earth. But its *mass* or inertia is not less, and it is as hard to swing around suddenly into a new position as it is on earth.

We can now understand the results of Galileo's experiment on falling objects in a more profound way. Galileo's discussion of falling objects showed that any given object (at a given locality) falls with uniform acceleration, $\vec{a_g}$. What is responsible for its uniform acceleration? A constant net force—in this case of free fall, just \vec{F}_g . Now Newton's second law expresses the relationship between this force and the resulting acceleration. Applying the equation $\vec{F}_{net} = m\vec{a}$ to this case, where $\vec{F}_{net} = \vec{F}_g$ and $\vec{a} = \vec{a_g}$, we can write

$$\vec{F}_{q} = m \vec{a}_{q}$$

We can, of course, rewrite this equation as

$$\vec{a}_g = \frac{\vec{F}_g}{m}$$

We conclude from Newton's second law that the reason why the acceleration of a body in free fall is constant is that for an object of given mass m the gravitational force \vec{F}_g over normal distances of fall is nearly constant.

Galileo, however, did more than claim that every object falls with constant acceleration: he found that all objects fall with the same uniform acceleration, which we now know has the value of about 9.8 m/sec at the earth's surface. Regardless of the mass m or weight \vec{F}_{g} , all bodies in free fall (in the same locality) have the same acceleration \vec{a}_g . Is this consistent with the relation $\vec{a}_g = \vec{F}_g/m$? It is consistent only if for every object \vec{F}_{q} is directly proportional to mass m: that is, if m is doubled, \vec{F}_{q} must double; if m is tripled, \vec{F}_{a} must triple. This is a significant result indeed. Weight and mass are entirely different concepts. Weight is the gravitational force on an object (hence weight is a vector). Mass is a measure of the resistance of an object to change in its motion, a measure of inertia (hence mass is a scalar). Yet the fact that different objects fall freely with the same acceleration means that the magnitudes of these two quite different quantities are proportional in any given locality.

Q16 An astronaut is orbiting the earth in a space vehicle. The acceleration due to gravity at that distance is half its value on the surface of the earth. Which of the following are true?

- (a) His weight is zero.
- (b) His mass is zero.
- (c) His weight is half its original value.
- (d) His mass is half its original value.
- (e) His weight remains the same.
- (f) His mass remains the same.

Consider SG 3.14 again.

SG 3.25, 3.26, 3.27, 3.28

Q17 A boy jumps from a table top. When he is halfway between the table top and the floor, which of the statements in Q16 are true?

3.9 Newton's third law of motion

In his first law, Newton described the behavior of objects when they are in a state of equilibrium; that is, when the net force acting on them is zero. His second law explained how their motion changes when the net force is not zero. Newton's third law added a new and surprising insight about forces.

Consider this problem: In a 100-meter dash, an athlete will go from rest to nearly his top speed in less than a second. We could measure his mass before he makes the dash, and we could use high-speed photography to measure his initial acceleration. With his mass and acceleration known, we could use F = ma to find the force acting on him during the initial acceleration. But where does the force come from? It must have something to do with the runner himself. Is it possible for him to exert a force on himself as a whole? Can he lift himself by his own bootstraps?

Newton's third law of motion helps us to understand just such puzzling situations. First, let us see what the third law claims. In Newton's words:

To every action there is always opposed an equal reaction: or, mutual actions of two bodies upon each other are always equal and directed to contrary parts.

This is a word-for-word translation from the *Principia*. It is generally agreed, however, that in Newton's statement the expression force on one object may be substituted for the word action, and the expression equally large force on another object for the words equal reaction. Read it over with this change.

The most startling idea to come out of this statement is that forces always exist in mirror-twin pairs, and on two different objects. Indeed, the idea of a single force unaccompanied by another force acting somewhere else is without any meaning whatsoever. On this point Newton wrote: "Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone." This suggests that forces always arise as a result of interactions between objects: object A pushes or pulls on B, while at the same time object B pushes or pulls with precisely equal amount on A. These paired pulls and pushes are always equal in magnitude, opposite in direction, and on two different objects.

Applying this idea to the athlete, we now see that his act of pushing his feet on the earth (one may call it here the action) is accompanied by a push of the earth on him (one can call it the reaction)—and the latter is what propels him forward. In this and all other cases it really makes no difference which we call the action

He is, to be sure, pushing against the ground-but that is a force acting on the ground.

SG 3.29

and which the reaction, because they occur at exactly the same time. The action does not "cause" the reaction — if the earth could not "push back" on his feet, the athlete could not push on the earth in the first place, but would slide around — as on slippery ice. Action and reaction coexist. You can't have one without the other. And most important, the two forces are not acting on the same body. In a way, they are like debt and credit: one is impossible without the other; they are equally large but of opposite sign; and they happen to two different objects.

Any body A that affects body B must itself be affected by B- equally and oppositely. We can use the efficient shorthand of algebra to express the idea that whenever bodies A and B interact:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

This is the equivalent of Newton's explanatory statement: Whenever two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction.

A host of everyday observations illustrate Newton's third law: A boat is propelled by the water that pushes forward on the oar while the oar pushes back on the water. A car is set in motion by the push of the ground on the tires as they push back on the ground; when friction is not sufficient, the tires cannot start the car forward. While accelerating a bullet forward, a rifle experiences a recoil kick. A balloon jumps forward while the air spurts out the opposite direction. Many such effects are not easily observed; for example, when an apple falls, pulled down by its weight, the earth accelerates upward, pulled up by the attraction to the apple.

Now note what the third law does not say-this, too, is important. The third law speaks of forces, not of the effects these forces produce. Thus in the last example, the earth accelerates upward as the apple falls down; the forces on each are equally large, but the accelerations produced by the forces are quite different; owing to the enormous mass of the earth, the earth's upward acceleration is insensibly small. The third law also does not describe how the push or pull is applied, whether by contact or by magnetic action or by electrical action. Nor does the law require that the force be either an attraction or repulsion. The third law really does not depend on any particular kind of force. It applies equally to resting objects and to moving objects, to accelerating objects as well as to objects in uniform motion. It applies whether or not there is friction present. Indeed, the universality of the third law makes it extremely valuable throughout physics.



force on earth



The force on the moon due to the earth is equal and opposite to the force on the earth due to the moon.



In the collision between the ball and the club, the force the ball exerts on the club is equal and opposite to the force the club exerts on the ball. Both the club and the ball get deformed by the forces acting on them.



Q18 According to Newton's third law, what are the four general characteristics of forces?

Q19 Identify the forces that act according to Newton's third law when a horse accelerates; when a swimmer moves at constant speed.

Q20 A piece of fishing line breaks if the force exerted on it is greater than 500 N. Will the line break if two people at opposite ends of the line pull on it, each with a force of 300 N?

Q21 State Newton's three laws of motion as clearly as you can in your own words.

3.10 Using Newton's laws of motion

We have discussed each of Newton's three laws of motion in some detail. The first law emphasizes the modern point of view in the study of motion: What requires explanation is not motion itself, but *change* of motion. The first law stresses that one must account for why an object speeds up or slows down or changes direction. The second law asserts that the rate of change of velocity of an object is related to both the mass of the object and the net force applied to it. In fact, the very meanings of force and mass are shown by the second law to be closely related to each other. The third law is a statement of a force relationship between interacting objects.

Despite their individual importance, Newton's three laws are most powerful when they are used together. So successful was the mechanics based on Newton's laws that until the late nineteenth century it seemed that all of creation must be understood as "matter in motion." Let us examine a specific example that illustrates the use of these laws.

Example 1

On September 12, 1966, a dramatic experiment based on Newton's second law was carried out high over the earth. In this experiment, the mass of an orbiting Agena rocket case was determined by accelerating it with a push from a Gemini spacecraft. After the Gemini spacecraft made contact with the Agena rocket case, the aft thrusters on the Gemini, calibrated to give an average thrusting force of 890 N, were fired for 7.0 sec. The change in velocity of the spacecraft and rocket case was found to be 0.93 m/sec. The mass of the Gemini spacecraft was known to be about 3400 kg. The question to be answered was: What is the mass of the Agena?

(Actually, the mass of the Agena had already been measured independently. The purpose of the experiment was to develop a technique to find the unknown mass of a foreign satellite in orbit.)

SG 3.30, 3.31, 3.32

In this case, a known force of magnitude 890 N was acting on two objects in contact, with a total mass of m_{total} , where

$$m_{\text{total}} = m_{\text{Gemini}} + m_{\text{Agena}}$$

= 3400 kg + m_{Agena}

The magnitude of the average acceleration produced by the thrust is found as follows:

 $a = \frac{\Delta v}{\Delta t}$ $= \frac{0.93 \text{ m/sec}}{7.0 \text{ sec}}$ $= 0.13 \text{ m/sec}^2$

Newton's second law gives us the relation

or

$$F = m_{\text{total}} \times a$$

Solving for m_{Agena} gives

$$m_{\text{Agena}} = \frac{F}{a} - 3400 \text{ kg} = \frac{890 \text{ N}}{0.13 \text{ m/sec}} - 3400 \text{ kg}$$
$$= 6900 \text{ kg} - 3400 \text{ kg}$$
$$= 3500 \text{ kg}$$

 $= (m_{Agena} + 3400 \text{ kg}) \times a$

The actual mass of the Agena, as previously determined, was about 3660 kg. The technique of finding the mass by nudging the Agena while in orbit therefore gave a result that was accurate to within 5% – well within the margin of error expected in making this measurement.







Example 2

Imagine taking a ride on an elevator: (A) At first it is at rest on the ground floor; (B) it accelerates upward uniformly at 1m/sec/sec for a few seconds; then (C) continues to go up at a constant speed of 5m/sec.



If a 100-kg man (whose weight would therefore be about 1000 newtons) is standing in the elevator, with what force is the elevator floor pushing up on him during (A), (B), and (C)?

Parts (A) and (C) are dynamically the same: Since he is not accelerating, the net force on him must be zero. So the floor must be pushing up on him just as hard as gravity is pulling him down. The gravitational force on him, his weight, is 1000 N. So the floor must be exerting an upward force of 1000 newtons.

Part (B): Since the man is accelerating upward, there must be a net force upward on him; the unbalanced force is

$$F_{net} = ma_{up}$$

= 100 kg × 2 m/sec/sec
= 200 N

So the floor must be pushing up on him with a force 200 N greater than what is required just to balance his weight; therefore, the total force upward on him is 1200 N.

3.11 Nature's basic forces

Our study of Newton's laws of motion has increased our understanding of objects at rest, moving uniformly, and accelerating. However, we have accomplished much more in the process. Newton's first law alerted us to the importance of frames

SG 3.33 is an elaboration of a similar example. For a difficult worked-out example see SG 3.34.

of reference. A critical analysis of the relationship between descriptions of the same event seen from different frames of reference was in fact the necessary first step toward the theory of relativity.

Newton's second law shows the fundamental importance of the concept of force. In fact, it presents us with a mandate: when you observe acceleration, find the force! This is how we were first directed to the gravitational force as an explanation of Galileo's kinematics: For all objects, at a given place, $\vec{a_g}$ is constant for all objects; since $\vec{a_g} = \vec{F_g}/m$ by Newton's second law, we must conclude that the magnitude of $\vec{F_g}$ is always proportional to m.

But this is only a halfway solution. Now we want to know why F_g is proportional to m for all bodies at a given place and how F_g changes for a given body as it is moved to places more distant from the earth. Is there a law connecting F_g , m, and distance – a "force law"? As Unit 2 will show, there is indeed. Knowing that force law, we shall be able to claim to understand all gravitational interactions among objects.

Gravitational attraction is not the only basic force by which objects interact. However, it is satisfying to realize that there appear to be very few such basic forces. In fact, physicists now believe that everything we observe in nature is the consequence of just four basic interactions. In terms of our present understanding, all the events of nature – subnuclear and nuclear, atomic and molecular, terrestrial and solar, galactic and extragalactic – are the manifestations of one or more of these few types of forces.

There is, of course, nothing sacred about the number four. New discoveries or theoretical insights might increase or reduce the number. For example, two (or more) of the basic interactions might some day be seen as consequences of something even more basic.

The first of the interactions is the gravitational force, which becomes important only on a relatively large scale, that is, when tremendous numbers of atoms of matter are involved. Between individual atoms, gravitational force is so weak so to be insignificant, but it is this weak force that literally holds the parts of the universe together. The second interaction involves electric and magnetic processes and is most important on the atomic and molecular scale. It is electromagnetic force that holds together objects in the range between the atom and the earth.

We know the force laws governing gravitational and electromagnetic interactions; therefore these interactions are fairly well "understood." The situation changes completely when we consider the two remaining basic interactions. They are the subject of vigorous research today. The third interaction (the so-called "strong" interaction) somehow holds the particles of the nucleus together. The fourth interaction (the so-called "weak" interaction) governs certain reactions among subnuclear particles.

We do, of course, have other *names* for forces, but each of these belongs to one of the basic types. One of the most common is the "frictional" force; it is thought to be an electrical interaction – that is,

Refer to K. Ford's: The World of Elementary Particles for brief discussion of four forces.

Einstein spent most of the latter half of his life seeking a theory that would express gravitational and electromagnetic effects in a unified way. A satisfactory "unified field theory" is still being sought. the atoms on the surfaces of the objects sliding or rubbing against each other interact electrically.

We shall be encountering these ideas again. We shall deal with the gravitational force in Unit 2, the electrical and magnetic forces in Units 4 and 5, and the forces between nuclear particles in Unit 6. In all these cases, an object subjected to the force will behave in accordance with Newton's laws of motion.

The knowledge that there are so few basic interactions is both surprising and encouraging. It is surprising because at first glance the events all around us seem so varied and complex. It is encouraging because our elusive goal—an understanding of the events of nature—looks more attainable.

"The Starry Night," by Vincent Van Gogh.

The intuitive feeling that all of nature's phenomena are interlinked on a grand scale is shared by scientists as well as artists.



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3.1 The Project Physics learning materials particularly appropriate for Chapter 3 include the following:

Experiments

Newton's Second Law Mass and Weight

Activities

Checker Snapping Beaker and Hammer Pulls and Jerks Experiencing Newton's Second Law Make One of These Accelerometers

Reader Articles

Introduction to Vectors Newton's Laws of Dynamics The Scientific Revolution How the Scientific Revolution of the 17th Century Affected Other Branches of Thought

Film Loops

Vector Addition - Velocity of a Boat

3.2 The Aristotelian explanation of motion should not be dismissed lightly. Great intellects of the Renaissance period, such as Leonardo da Vinci, who among other things designed devices for launching projectiles, did not challenge such explanations. One reason for the longevity of these ideas is that they are so closely aligned with our common sense ideas.

In what ways do your common sense notions of motion agree with the Aristotelian ones?

3.3 Three ants are struggling with a crumb. One ant pulls toward the east with a force of 8 units. Another pulls toward the north with a force of 6 units, and the third pulls in a direction 30° south of west with a force of 12 units.

- (a) Using the "head-to-tail" construction of arrows, find whether the forces balance, or whether there is a net (unbalanced) force on the crumb.
- (b) If there is a net force, you can find its direction and magnitude by measuring the line drawn from the tail of the first arrow to the head of the last arrow. What is its magnitude and direction?

3.4 Show why the parallelogram method of adding arrows is geometrically equivalent to the head-to-tail method.

3.5 There are many familiar situations in which the net force on a body is zero, and yet the body moves with a constant velocity. One example of such "dynamic equilibrium" is an automobile traveling at constant speed on a straight road: the force the road exerts on the tires is just balanced by the force of air friction. If the gas pedal is depressed further, the tires will push against the tires harder and the road will push against the tires harder; so the car will accelerate forward – until the air friction builds up enough to balance the greater drive force. Give another example of a body moving with constant velocity under balanced forces. Specify the source of each force on the body and, as in the automobile example, explain how these forces could be changed to affect the body's motion.

- 3.6 (a) You exert a force on a box, but it does not move. How would you explain this? How might an Aristotelian explain it?
 - (b) Suppose now that you exert a greater force and the box moves. Explain this from your (Newtonian) point of view and from an Aristotelian point of view.
 - (c) You stop pushing on the box and it quickly comes to rest. Explain this from both the Newtonian and the Aristotelian points of view.

3.7 There are at least two drawbacks to an experimental test of Newton's law of inertia.

- (a) How can you really be sure that there is no unbalanced force acting on the object, even if you see that the object moves uniformly in a straight line? We can answer that we are sure because the object does continue to move uniformly in a straight line. But this answer is merely a restatement of the first law, which we wanted to prove by experiment. Surely we cannot use the first law to verify the first law! But we are not really caught in a circular argument. Practically, we can expect to find forces on an object only when other objects are in contact with it, or somewhere near it. The influences may be of unfamiliar kinds, and we may have to stretch what we mean by "near"; but whenever a force is detected we look for the source of the influence. If all known influences on an object were balanced, and yet it didn't move uniformly, we would suspect an unknown influence and track it down-and we would find it. At least, that's how it has always turned out so far. As a practical example, consider the demonstration involving low friction pucks on a level surface. Without using Newton's first law, how could you be sure the surface was level?
- (b) What is meant by a straight line?
- 3.8 (a) Assume that the floor of a laboratory could be made perfectly horizontal and perfectly smooth. A dry ice puck is placed on the floor and given a small push. Predict the way in which the puck would move. How would this motion differ if the whole laboratory were moving uniformly during the experiment? How would it differ if the whole laboratory were accelerating along a straight line? If the puck were seen to move in a curved path along the floor, how would you explain this?
 - (b) A man gently starts a dry ice puck in motion while both are on a rotating

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platform. What will he report to be the motion he observes as the platform keeps rotating? How will he explain what he sees if he believes he can use Newton's first law to understand observations made in a rotating reference frame? Will he be right or wrong?

3.9 In terms of Newton's first law, explain:

- (a) Why people in a moving car lurch forward when the car suddenly slows down;
- (b) What happens to the passengers of a car that makes a sharp, quick turn;
- (c) When a coin is put on a phonograph turntable and the motor started, does the coin fly off when the turntable reaches a certain speed? Why doesn't it fly off sooner?

3.10 A balloon-like object stands before you, unmoving, suspended in mid-air. What can you say about the forces that may be acting on it? Suddenly it moves off in a curved path. Give two different explanations. How can you test which is right?

3.11 In an actual experiment on applying the same force to different masses, how would you know it was the "same force"?

3.12 Several proportionalities can be combined into an equation only if care is taken about the units in which the factors are expressed. When we wrote $\Delta d = v \times \Delta t$ in Chapter 1, we chose *meters* as units for *d*, *seconds* as units for *t*, and then made sure that the equation came out right by using meters/second as units for *v*. In other words, we let the equation *define* the unit for *v*. If we had already chosen some *other* units for *v*, say miles per hour, then we would have had to write instead something like

$\Delta d = k \times v \Delta t$

where k is a constant factor that matches up the units of d, t, and v.

What value would k have if d were measured in miles, t in seconds, and v in miles per hour?

Writing $\vec{a} = \vec{F}_{net}/m$ before we have defined units of F and m is not the very best mathematical procedure. To be perfectly correct in expressing Newton's law, we would have had to write:

$$\vec{a} = k \times \frac{\vec{F}_{\text{net}}}{m}$$

where k is a constant factor that would match up whatever units we choose for a, F, and m. In fact, we will take the easiest way out and let the equation define the units of F in terms of the units we choose for a and m, so the equation comes out right without using k. (Or if you prefer to say it that way, we choose units so that k = 1.)

3.13 A body is being accelerated by an unbalanced force. If the magnitude of the net force is doubled and the mass of the body is reduced to one-third of the original value, what will be the ratio of the second acceleration to the first?

3.14 What does a laboratory balance measure – mass or weight? What about a spring balance? (Hint: consider what would happen to readings on each if they were on the moon instead of the earth.) You might want to consider this question again after reading Sec. 3.8.

 $_{3.15}$ Describe as a thought experiment how you could calibrate a spring balance in force units. If you actually tried to do the experiments, what practical difficulties would you expect?

3.16 "Hooke's law" says that the force exerted by a stretched or compressed spring is directly proportional to the amount of the compression or extension. As Robert Hooke put it in announcing his discovery:

... the power of any spring is in the same proportion with the tension thereof: that is, if one power stretch or bend it one space, two will bend it two, three will bend it three, and so forward. Now as the theory is very short, so the way of trying it is very easie.



If Hooke says it's "easie," then it might well be so. You can probably think immediately of how to test this law using springs and weights. (a) Try designing such an experiment; then after checking with your instructor, carry it out. What limitations do you find to Hooke's law? (b) How could you use Hooke's law to simplify the calibration procedure asked for in SG 3.15?

3.17 Refer to the discussion in SG 3.12. Show that k = 1 when we define a newton as we do on p. 83.

3.18 When units for different terms in a relation are defined completely independently from one another, the numerical value of the constant must be found experimentally. (Later in this course you will see how finding the value of k in certain relations was very important in the development of physics.) Say, for example, that we had decided to measure force in "tugs," defining a tug as the force required to stretch a standard rubber band one inch. How could we go about finding k? 3.19 Complete this table:

				RESULTING
NET	FORCE	N	IASS	ACCELERATION
а	1.0 N	1.0	kg	1.0 m/sec ²
b	24.0	2.0		12.0
С		3.0		8.0
d		74.0		0.2
е		0.006	66	130.0
f	72.0			8.0
g	3.6			12.0
h	1.3			6.4
i	30.0	10.0		
j	0.5	0.20		
k	120.0	48.0		



3.20 A rocket-sled has a mass of 4440 kg and is propelled by a solid-propellent rocket motor of 890,000-N thrust which burns for 3.9 seconds.

- (a) What is the sled's average acceleration and maximum speed?
- (b) This sled has a maximum acceleration of $30 g (= 30 a_g)$. How can that be, considering the data given?
- (c) If the sled travels a distance of 1530 m while attaining a top speed of 860 m/sec (how did it attain *that* high a speed?), what is its average acceleration?

3.21 If you have "dynamics carts" available, here is one way of doing an experiment to demonstrate the inverse proportionality between acceleration and mass:

- (a) Add load blocks to one or the other of two carts until the carts balance when placed on opposite platforms of a laboratory balance. Balance a third cart with one of the first pair. Each cart now has the same mass *m*. (State two main assumptions involved here.)
- (b) Accelerate one cart on a level surface, using a rubber band; that is, pull the cart with the rubber band, keeping the rubber band stretched a known constant amount so that it supplies a constant force. Any other method can be used that will assure you that, within reason, the same force is

being applied each time. Record the position of the cart at equal time intervals by means of stroboscopic photography.

- (c) Repeat the last step in all details, but use two carts hooked together. Repeat again using all three carts hooked together. In all three cases it is crucial that the applied force be essentially the same.
- (d) Determine the value of acceleration for masses of m (1 cart), 2m (2 carts), and 3m (3 carts).
- (e) Prepare a graph of a vs. m, of a vs. 1/m, and of 1/a vs. m. Comment on your results.

3.22 Describe in detail the steps you would take in an idealized experiment to determine the unknown mass m of a certain object (in kilograms) if you were given nothing but a frictionless horizontal plane, a 1-kg standard, an uncalibrated spring balance, a meter stick, and a stopwatch.

3.23 A block is dragged with constant velocity along a rough horizontal table top by means of a spring balance horizontally attached to the block. The balance shows a reading of 0.40 N at this and any other constant velocity. This means that the retarding frictional force between block and table is 0.40 N, and is not dependent on speed.

Now the block is pulled harder and given a constant acceleration of 0.85 m/sec²; the balance is found to read 2.1 N. Compute the mass of the block.

3.24 We have claimed that any body in free fall is "weightless" because any weight-measuring device falling with it would read zero. This is not an entirely satisfactory explanation, because you feel a definite sensation during free fall that is exactly the same sensation you would feel if you were truly without weight – say deep in space far from any star or planet. (The sensation you feel on jumping off a roof or a diving board, or when someone pulls a chair out from under you.) Can you explain why your insides react in the same way to lack of weight and to free fall?

3.25 Explain the statement that while the mass of an object is the same everywhere, its weight may vary from place to place.

- 3.26 (a) A replica of the standard kilogram is constructed in Paris and then sent to the National Bureau of Standards near Washington. D.C. Assuming that this secondary standard is not damaged in transit, what is
 - (i) its mass in Washington?
 - (ii) its weight in Paris and in Washington?
 (In Paris, a_g = 9.81 m/sec²; in Washington, a_g = 9.80 m/sec².)
 - (b) What is the change in your own weight as you go from Paris to Washington?

- 3.27 (a) Find your mass in kg, and your weight in newtons.
 - (b) How much force is needed to accelerate you 1 m/sec²? How many kilograms can you lift? How many newtons of force must you exert to do this?

3.28 Why is it often said that astronauts in orbit are weightless?

3.29 When a runner pushes on the earth with the sole of his shoe, the earth pushes with an equal and opposite force on the sole of the shoe. This latter force has an accelerating effect on the runner, but what does the force acting on the earth do to the earth? From Newton's second law we would conclude that such an unbalanced force would accelerate the earth. The mass of the earth is very great, however, so the acceleration caused by the runner is very small. A reasonable value for the average acceleration of a runner when he starts is 5 m/sec/sec, and a reasonable value for his mass would be 60 kg. The mass of the earth is approximately 60×10^{23} kg.

- (a) What acceleration of the earth would the runner cause?
- (b) If the acceleration lasts for 2 seconds, what speed will the runner have reached?
- (c) What speed will the earth have reached?

3.30 In terms of Newton's third law, assess the following statements:

- (a) You are standing perfectly still on the ground; therefore you and the earth exert equal and opposite forces on each other.
- (b) The reason that a propeller airplane cannot fly above the atmosphere is that there is no air to push one way while the plane goes the other.
- (c) Object A rests on object B. The mass of object A is 100 times as great as that of object B, but even so, the force A exerts on B is no greater than the force of B on A.

3.31 Consider a tractor pulling a heavy log in a straight line. On the basis of Newton's third law, one might argue that the log pulls back on the tractor just as strongly as the tractor pulls the log. But why, then, does the tractor move? (Make a large drawing of the tractor, rope, log, and earth, and enter the forces.)



3.32 Consider the system consisting of a 1.0-kg ball and the earth. The ball is dropped from a short distance above the ground and falls freely. Assuming that the mass of the earth is approximately 6.0×10^{24} kg,

- (a) make a vector diagram illustrating the important forces acting on each member of the system.
- (b) calculate the acceleration of the earth in this interaction.
- (c) find the ratio of the magnitude of the ball's acceleration to that of the earth's acceleration (a_b/a_e) .
- (d) make a vector diagram as in (a) but showing the situation when the ball has come to rest after hitting the ground.
- 3.33 (a) A 75-kg man stands in an elevator. What force does the floor exert on him when the elevator
 - (i) starts moving upward with an acceleration of 1.5 m/sec²?
 - (ii) moves upward with a constant speed of 2.0 m/sec?
 - (iii) starts accelerating downward at 1.5 m/sec^2 ?
 - (b) If the man were standing on a bathroom (spring) scale during his ride, what readings would the scale have under conditions (i), (ii), and (iii) above?
 - (c) It is sometimes said that the "apparent weight" changes when the elevator accelerates. What could this mean? Does the weight really change?

3.34 Useful hints for solving problems about the motion of an object and the forces acting on it.

- (a) make a light sketch of the physical situation.
- (b) in heavy line, indicate the limits of the particular object you are interested in, and draw all the forces acting on that object. (For each force acting on it, it will be exerting an opposite force on something else – but we don't care about those.)
- (c) find the vector sum of all these forces, for example, by graphical construction.
- (d) using Newton's second law, set this sum, \vec{F}_{net} , equal to $m\vec{a}$.
- (e) solve the equation for the unknown quantity.
- (f) put in the numerical values you know and calculate the answer.

Example:

A ketchup bottle whose mass is 1.0 kg rests on a table. If the friction force between the table and the bottle is a constant 6 newtons, what horizontal pull is required to accelerate the bottle from rest to a speed of 6 m/sec in 2 sec?

First, sketch the situation:


Second, draw in arrows to represent all the forces acting on the object of interest. There will be the horizontal pull \vec{F}_p , the friction \vec{F}_r , the gravitational pull \vec{F}_q (the bottle's weight), and the upward force \vec{F}_T exerted by the table. (There is, of course, also a force acting down on the table, but we don't care about that – we're interested only in the forces acting on the bottle.)



Next, draw the arrows alone. In this sketch all the forces can be considered to be acting on the center of mass of the object.



Because the bottle is not accelerating up or down, we know there is no net force up or down – so \vec{F}_{τ} must just balance \vec{F}_{p} . So the net force acting on the bottle is just the vector sum of \vec{F}_{p} and \vec{F}_{f} . Using the usual tip-to-tail addition:

As the last arrow diagram shows, the horizontal pull must be greater than the force required for acceleration by an amount equal to the friction. We already know \vec{F}_{f} . We can find \vec{F}_{net} from Newton's second law if we know the mass and acceleration of the bottle, since $\vec{F}_{net} = m\vec{a}$. The net force required to accelerate the case is found from Newton's second law:

$$\vec{F}_{net} = m\vec{a}$$

The mass m is given as 1.0 kg. The acceleration involved in going from rest to 6.0 m/sec in 2 seconds is

$$a = \frac{\Delta v}{\Delta t}$$
$$= \frac{6.0 \text{ m/s}}{2 \text{ sec}}$$

= 3.0 m/sec/sec

So the net force required is

3

$$F_{\rm net} = 1.0 \text{ kg} \times 3.0 \text{ m/sec/sec}$$

sec

= 3.0 kg m/sec/sec

= 3.0 newtons

If we consider toward the right to be the positive direction, F_{net} is 3.0 newtons and F_i , which is directed to the left, is -3.0 newtons.

$$\vec{F}_{net} = \vec{F}_p + \vec{F}_f$$

.0N = $\vec{F}_p + (-3.0N)$
 $\vec{F}_p = 3.0N + 3.0N$
 $\vec{F}_p = 6.0N$

If you prefer not to use + and - signs, you can work directly from your final diagram and use only the magnitudes of the forces:



from which the magnitude of F_p is obviously 6.0N.



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"... the greater the velocity ... with which [a stone] is projected, the farther it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it." – Newton's System of the World

CHAPTER FOUR

Understanding Motion

4.1 A trip to the moon

Imagine a Saturn rocket taking off from its launching pad at Cape Kennedy. It climbs above the earth, passing through the atmosphere and beyond. Successive stages of the rocket shut off leaving finally a capsule hurtling through the near-vacuum of space toward its destination 240,000 miles away. Approximately 65 hours after take-off, the capsule circles the moon and descends to its target – the center of the lunar crater Copernicus.

The complexity of such a voyage is enormous. To direct and guide the flight, a great number and variety of factors must be taken into account. The atmospheric drag in the early part of the flight depends upon the rocket's speed and altitude. The engine thrust changes with time. The gravitational pulls of the sun, the earth, and the moon change as the capsule changes its position relative to them. The rocket's mass is changing. Moreover, it is launched from a spinning earth, which in turn is circling the sun, and the target – the moon – is moving around the earth at a speed of about 2,300 miles per hour.

Yet, as for almost any complex motion, the flight can be broken down into small portions, each of which is relatively simple to describe. What we have learned in earlier chapters will be useful in this task.

In simplified form, the earth-moon trip can be divided into these eight parts:

- Part 1. The rocket accelerates vertically upward from the surface of the earth. The force acting on the rocket is not really constant, and the mass of the rocket decreases as the propellent escapes. The value of the acceleration at any instant can be computed using Newton's second law; it is given by the ratio of net force (thrust minus weight) at that instant to the mass at that instant.
- Part 2. The rocket, still accelerating, follows a curved path as it is "injected" into an orbit about the earth.



In his science-fiction novels of more than a hundred years ago, the French author Jules Verne (1828-1905) launched three spacemen to the moon by means of a gigantic charge fixed in a steel pipe deep in the earth.







- Part 3. In an orbit 115 miles above the earth's surface, the capsule moves in a nearly circular arc at a constant speed of 17,380 miles/hr.
- Part 4. The rocket engines are fired again, increasing the capsule's speed so that it follows a much less curved path into space. (The minimum speed necessary to escape the earth completely is 24,670 miles/hr.)
- Part 5. In the flight between earth and moon, only occasional bursts from the capsule's rockets are required to keep it precisely on course. Between these correction thrusts, the capsule moves under the influence of the gravitational forces of earth, moon, and sun; we know from Newton's first law that the capsule would move with constant velocity if it were not for these forces.
- Part 6. On nearing the moon, the rocket engines are fired again to give the capsule the correct velocity to "inject" into a circular orbit around the moon.
- Part 7. The capsule is moving with a constant speed of about 1 mile/sec in a nearly circular path 50 miles above the moon's surface.
- Part 8. After its rockets are fired in the direction of motion to reduce the speed, the capsule accelerates downward as it falls toward the surface of the moon. It follows an arcing path before it lands in the crater Copernicus. (Just before impact, the rocket engines fire a final time to reduce speed of fall and prevent a hard landing.)







SG 4.2

Motion along a straight line (as in Parts 1 and 5) is easy enough to describe. But let us analyze in greater detail other parts of this trip: moving on a circular arc, as in Parts 3 and 7, and projectile motion, as in Part 8, are two important cases.

How shall we go about making this analysis? Following the example of Galileo and Newton, we can try to learn about the behavior or moving objects beyond our reach, even on the moon or in the farthest parts of the universe, by studying the motion of objects near at hand. If we believe that physics is the same everywhere, then the path of a lunar capsule moving as in Part 8 can be understood by studying a marble rolling off the edge of a table or a bullet fired from a horizontal rifle.

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Section 4.2

4.2 Projectile motion

Consider this experiment: a rifle is mounted on a tower with its barrel parallel to the ground; the ground over which the bullet will travel is level for a very great distance. At the instant a bullet leaves the rifle, an identical bullet is dropped from the height of the barrel of the rifle. The second bullet has no horizontal motion relative to the ground; it goes only straight down. Which bullet will reach the ground first?

We do not need to know anything about the speed of the bullet or the height of the tower in order to answer this question. Consider first the motion of the second bullet, the one that is dropped. As a freely falling object, it accelerates toward the ground with constant acceleration. As it falls, the time t and the corresponding downward displacement y are related by



$$y = \frac{1}{2}a_g t^2$$

where a_g is the acceleration due to gravity at that location.

Now consider the bullet that is fired horizontally from the rifle. When the gun is fired, the bullet is driven by the force of expanding gases and accelerates very rapidly until it reaches the muzzle of the rifle. On reaching the muzzle these gases escape and no longer push the bullet. At that moment, however, the bullet has a large horizontal speed, v_x . The air will slow the bullet slightly, but we shall ignore that fact and imagine an ideal case with no air friction. As long as air friction is ignored, there is no force acting on the projectile in the horizontal direction. Therefore, we expect



the horizontal speed will remain constant. From the instant the bullet leaves the muzzle, we would expect its horizontal motion to be described by the equation

$$x = v_x t$$

So much for the forward part of the motion. There is, however, another part that becomes more and more important as t increases. From the moment the bullet leaves the gun, it falls toward the earth while it moves forward, like any other unsupported body. Can we use the same equation to describe its fall that we used to describe the fall of the dropped bullets? And how will falling affect the bullet's horizontal motion? These doubts raise a more



fundamental question that goes beyond just the behavior of the bullets; namely, is the vertical motion of an object affected by its horizontal motion? Or vice versa?

To answer these questions, we can carry out a real experiment similar to our thought experiment. We can use a special laboratory device designed to fire a ball in a horizontal direction at the moment that a second ball is released to fall freely from the same height. We set up our apparatus so that both balls are the same height above a level floor. The balls are released and, although the motions of the balls may be too rapid for us to follow with the eye, we will hear that they reach the floor at the same time. This result suggests that the vertical motion of the projected ball is unaffected by its horizontal velocity.

In the margin is a stroboscopic photograph of this experiment. Equally spaced horizontal lines aid our examination. Look first at the ball on the left, which was released without any horizontal motion. You see that it accelerates because it moves greater distances between successive flashes. Careful measurement of the photograph shows that the acceleration is constant, within the uncertainty of our measurements.

Now compare the vertical positions of the second ball, fired to the right, with the vertical positions of the ball which is falling freely. The horizontal lines show that the distances of fall are the same for corresponding time intervals. The two balls obey the same law for motion in a vertical direction. That is, at every instant they both have the same constant acceleration a_g , the same downward velocity and the same vertical displacement. The experiment therefore supports the idea that the vertical motion is the same whether or not the ball has a horizontal motion also. The horizontal motion does not affect the vertical motion.

We can also use the strobe photo to see if the vertical motion of the projectile affects its horizontal velocity, by measuring the *horizontal* distance between successive images. We find that the horizontal distances are practically equal. Since the time intervals between images are equal, we can conclude that the horizontal velocity v_x is constant. So we can conclude that the vertical motion doesn't affect the horizontal motion.

The experiment shows that the vertical and horizontal components of the motion are independent of each other. This experiment can be repeated from different heights, and with different horizontal velocities, but the results lead to the same conclusion.

The independence of motions at right angles has important consequences. For example, it is easy to predict the displacement and the velocity of a projectile at any time during its flight. We need merely to consider the horizontal and vertical aspects of the motion *separately*, and then add the results – vectorially. We can predict the magnitude of the components of displacement (x and y) and of the components of velocity (v_x and v_y) at any instant by application of the appropriate equations. For the horizontal



The two balls in this strobscopic photograph were released simultaneously. The one on the left was simply dropped from rest position; the one on the right was given an initial velocity in the horizontal direction.

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component of motion, the equations are

and

and for the vertical component of motion,

and

$$u = \frac{1}{2}a_a t^2$$

 $x = v_{r}t$

 $v_v = a_a t$



Q1 If a body falls from rest with acceleration a_g , with what acceleration will it fall if it has an initial horizontal speed v_x ?

4.3 What is the path of a projectile?

It is easy to see that a thrown object, such as a rock, follows a curved path, but it is not so easy to see just what kind of curve it traces. For example, arcs of circles, ellipses, parabolas, hyperbolas, and cycloids (to name only a few geometric figures) all provide likely-looking curved paths.

Better knowledge about the path of a projectile was gained when mathematics was applied to the problem. This was done by deriving the equation that expresses the shape of the path. Only a few steps are involved. First let us list equations we already know for a projectile launched horizontally:

and

$$x = v_v t$$
$$y = \frac{1}{2}a_v t^2$$

We would know the shape of the trajectory if we had an equation that gave the value of y for each value of x. We can find the fall distance y for any horizontal distance x by combining these two equations in a way that eliminates the time variable. Solving the equation $x = v_x t$ for t we get

$$t = \frac{x}{v_x}$$

Because t means the same in both equations, we can substitute x/v_r for t in the equation for y:

$$y = \frac{1}{2}a_g t^2$$

and thus

$$y = \frac{1}{2}a_g \left(\frac{x}{v_x}\right)^2$$

In this last equation there are two variables of interest, x and y, and three constant quantities: the number $\frac{1}{2}$, the uniform acceleration of free fall a_g , and the horizontal speed v_x which we



VX



Specialized equations such as these need not be memorized.

take to be constant for any one flight from launching to the end. Bringing these constants together between one set of parentheses, we can write the equation as

$$y = \left(\frac{a_g}{2v_x^2}\right) x^2$$

or, letting k stand for constant $(a_g/2v_x^2)$

 $y = kx^2$

This equation shows a fairly simple relationship between x and y for the trajectory. We can translate it as: the distance a projectile falls away from a straight path is proportional to the square of the distance it moves sideways. For example, when the projectile goes twice as far horizontally from the launching point, it drops vertically four times as far.

The mathematical curve represented by this relationship between x and y is called a *parabola*. Galileo deduced the parabolic shape of trajectories by an argument similar to the one we used. (Even projectiles not launched horizontally – as in the photographs on p. 103 and 123 – have parabolic trajectories.) With this discovery, the study of projectile motion became much simpler. because the geometric properties of the parabola had been established centuries earlier by Greek mathematicians.

Here we find a clue to one of the important strategies in modern science. When we express the features of a phenomenon quantitatively and cast the relations between them into equation form, we can use the rules of mathematics to manipulate the equations, and so open the way to unexpected insights.

Galileo insisted that "the proper language of nature is *mathematics*," and that an understanding of natural phenomena is aided by translating our qualitative experiences into quantitative terms. If, for example, we find that trajectories have a parabolic shape, we can apply all we know about the mathematics of parabolas to describe - and predict - trajectories. Physicists have often drawn on the previously developed parts of pure mathematics to express (or to extend) their conceptions of natural phenomena. Sometimes, as in the case of Newton's inventing calculus, they have to develop new parts of mathematics. The physical scientist often tries to use methods from another branch of science, in addition to mathematics, to find a solution for his particular problem. For example, just as Galileo used the alreadyknown mathematics of parabolas to deal with actual projectile motions, so the modern sound engineer solves problems in acoustics using ideas and mathematical techniques developed independently by electrical engineers. Whatever the methods of science may be, many ideas and concepts can often be extended from one specialty to another, with fruitful results.

We can now apply our theory of projectile motion to the case mentioned earlier, the free motion of a space capsule toward the



Drawing of a parabolic trajectory from Galileo's *Two New Sciences*.

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"Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it." (Galileo, cited in Discoveries and Opinions of Galileo, translated by Stillman Drake. Anchor Books, pp. 237-238.)

moon's surface. Let us assume that the orbit is a low one, so that the acceleration due to gravity is almost constant between the orbit and the surface. If the rocket engines are fired forward, in the direction of motion, the capsule's speed will be reduced and it will begin to fall closer to the surface. After firing, the reduced horizontal speed remains constant, so the capsule falls toward the surface on a parabolic path. Spaceflight engineers apply ideas like these to land a space capsule on a desired moon target. (See SG 4.23).

Q2 Which of the conditions below must hold in order for the relationship $y = kx^2$ to describe the path of a projectile?

- (a) a_g is a constant
- (b) a_g depends on t
- (c) a_g is straight down
- (d) v_x depends on t
- (e) air friction is negligible

4.4 Moving frames of reference

Galileo's work on projectiles leads to thinking about reference frames. As you will see in Unit 2, Galileo ardently supported the idea that the preferred reference frame for discussing motions in our planetary system is one fixed to the sun, not the earth. From that point of view, the earth both revolves around the sun and rotates on its own axis. For many scientists of Galileo's time, this idea was impossible to accept, and they thought they could prove their case. If the earth rotated, they said, a stone dropped from a tower would not land directly at its base. For if the earth rotates once a day, the tower would move on for hundreds of feet for every second the stone is falling; hence, the stone would be left behind while falling through the air and consequently would land far behind the base of the tower. But this is not what happens. As near as one can tell, the stone lands directly under where it was released. Therefore, many of Galileo's critics believed that the tower and the earth could not be considered to be in motion.

To answer these arguments, Galileo showed the same observation can support his view that, during the time of fall, the tower and the ground supporting it were moving forward together with the same uniform velocity. While the stone was being held at the top of the tower, it had the same horizontal velocity as the tower. Releasing the stone allows it to gain vertical speed, but by the principle of independence of v_x and v_y discussed in Section 4.3, this does not diminish any horizontal speed it had initially on being released. In other words, the falling stone behaves like any other projectile: the horizontal and vertical components of its motion are independent of each other. Since the stone and tower continue to have the same v_x throughout, the stone will not be left behind as it falls. Therefore, no matter what the speed of the earth, the stone



The critics of Galileo claimed that if the earth moved, a dropped stone would be left behind and land beyond the foot of the tower.





will land at the foot of the tower. The fact that falling stones are not left behind is not a proof that the earth is standing still.

Similarly, Galileo said, an object released from a crow's nest at the top of a ship's perpendicular mast will land at the foot of the mast, whether the ship is standing still in the harbor or moving with constant velocity through quiet water. This was actually tested by experiment in 1642 (and is also the subject of three Project Physics film loops). We know this to be the case from everyday observation: when you drop or throw a book in a bus or train or plane that is moving with constant velocity, you will see it moving just as it would if the vehicle were standing still. Or again, if an object is projected vertically *upward* from inside an open car that is moving at constant velocity, it will fall back into the car. A person in the car will see the same thing happen whether the car has been continuously moving at constant velocity or has been standing still.

From these and other observations has come a valuable generalization: If there is any one laboratory in which Newton's laws hold, then these laws will hold equally well in any other lab (or "reference frame") that moves at constant velocity with respect to the first. This generalization is called the *Galilean relativity principle*. It holds true for all "classical" mechanical phenomena – that is, phenomena involving a tremendous range of relative velocities, up to millions of miles per hour.

If the laws of mechanics are found to be the same for all reference frames moving with constant velocity with respect to each other, then there is no way to find the *speed* of one's own reference frame from any mechanical experiment done *in* the reference frame, nor can one pick out any one reference frame as the "true" frame – the one that is, say, at absolute rest. Thus there can be no such thing as the "absolute" velocity of a body – all measured velocities are only *relative*.

What about observations of phenomena outside of one's own frame of reference? Certainly some outside phenomena can appear differently to observers in different reference frames – for example, the velocity of an airplane will have a different value when seen from the earth and from a moving ship. But other measurables such as mass, acceleration, and time interval will have the *same* values when a phenomenon is observed from different reference frames that move with constant velocity with respect to one another. Moreover, certain *relationships* among such measurements will be found to be the same for these different reference frames. Newton's laws of motion are examples of such "invariant" relationships, and so are all the laws of mechanics that follow from them.

Notice that the relativity principle, even in this restricted classical form, does not say "everything is relative." On the contrary, it asks us to look for relationships that do *not* change when reference systems are changed.

At high speeds, air drag will affect the results considerably. The situation is still indistinguishable from a car at rest—but in a high wind!

When relative speeds become a noticeable fraction of the speed of light (almost a billion mph), some deviations from this simple relativity principle begin to appear. We will consider some of them in Unit 5. Q3 If the laws of mechanics are found to be the same in two reference frames, what must be true of their motions?

4.5 Circular motion

A projectile launched horizontally from a tall tower strikes the earth at a point determined by the speed of the projectile, the height of the tower, and the acceleration due to the force of gravity. As the projectile's launch speed is increased, it strikes the earth at points farther and farther from the tower's base, and we would have to take into account that the earth is not flat but curved. If we suppose the launch speed to be increased even more, the projectile would strike the earth at points even farther from the tower, till at last it would rush around the earth in a nearly circular orbit. At this orbiting speed, the fall of the projectile away from the forward, straight line motion is matched by the curvature of the surface, and it stays at a constant distance above the surface.

What horizontal launch speed is required to put an object into a circular orbit about the earth or the moon? We shall be able to answer this question quite easily after we have learned about circular motion.

The simplest kind of circular motion is *uniform* circular motion, that is, motion in a circle at constant speed. If you are in a car or train that goes around a perfectly circular track so that at every instant the speedometer reading is forty miles per hour, you are executing uniform circular motion. But this is not the case if the track is any shape other than circular, or if your speed changes at any point.

How can we find out if an object in circular motion is moving at constant speed? The answer is to apply the same test we used in deciding whether or not an object traveling in a straight line does so with constant speed: we measure the instantaneous speed at many different moments and see whether the values are the same. If the speed is constant, we can describe the circular motion of the object by means of two numbers: the radius R of the circle and the speed v along the path. For regularly repeated circular motion, we can use a quantity more easily measured than speed: either the time required by an object to make one complete revolution, or the number of revolutions the object completes in a unit of time. The time required for an object to complete one revolution in a circular path is called the period of the motion. The period is usually denoted by the capital letter T. The number of revolutions completed by the same object in a unit time interval is called the *frequency* of the motion. Frequency will be denoted by the letter *f*.

As an example, we will use these terms to describe a car moving with uniform speed on a circular track. Let us suppose the car takes 20 seconds to make one lap around the track. Thus, T = 20 seconds. Alternatively, we might say that the car makes 3 laps in a minute.

In discussing circular motion it is useful to keep clearly in mind a distinction between revolution and rotation. We define these terms differently: revolution is the act of traveling along a circular or elliptical path; rotation is the act of spinning rather than traveling. A point on the rim of a phonograph turntable travels a long way; it is revolving about the axis of the turntable. But the turntable as a unit does not move from place to place: it merely rotates. In some situations both processes occur simultaneously; for example, the earth rotates about its own axis, while it also revolves (in a nearly circular path) around the sun.

Thus f = 3 revolutions per minute, or f = 1/20 revolution per second. The relationship between frequency and period (*when the same time unit is used*) is f = 1/T. If the period of the car is 20 sec/rev, then the frequency is

$$\frac{1}{20 \frac{\sec}{\mathrm{rev}}} = \frac{1}{20} \frac{\mathrm{rev}}{\mathrm{sec}}$$

All units are a matter of convenience. Radius may be expressed in terms of centimeters, kilometers, miles, or any other distance unit. Period may be expressed in seconds, minutes, years, or any other time unit. Correspondingly, the frequency may be expressed as "per second," "per minute," or "per year." The most widely used units of radius, period, and frequency in scientific work are meter, second, and per second.

 Table 4.1 Comparison of the frequency and period for various kinds of circular motion. Note the differences between units.

PHENOMENA	PERIO	D	FREQUENCY
Electron in circular accelerat	or 10 ⁻⁶	sec	10 ⁶ per sec
Ultra-centrifuge	0.00033	sec	3000 per sec
Hoover Dam turbine	.33	sec	3 per sec
Rotation of earth	24	hours	0.0007 per min
Moon around the earth	27	days	0.0015 per hour
Earth about the sun	365	days	0.0027 per day

If an object is in uniform circular motion, and if we know the frequency of revolution f and the radius R of the path, we can compute the speed v of the object without difficulty. The distance traveled in one revolution is simply the perimeter of the circular path, that is, $2\pi R$. The time for one revolution is by definition the period T. Since for uniform motion it is always true that

speed =
$$\frac{\text{distance traveled}}{\text{time elapsed}}$$

by substitution we get

$$v = \frac{2\pi R}{T}$$

To express this equation for circular motion in terms of the frequency f, we rewrite it as

$$v = 2\pi R \times \frac{1}{T}$$

now, since by definition

$$f = \frac{1}{T}$$

we can write

$$v = 2\pi R \times f$$

If the body is in *uniform* circular motion, the speed computed

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The term "revolutions" is not assigned any units because it is a pure number, a count. There is no need for a standard as there is for distance, mass, and time. So, the unit for frequency is usually given without "rev." This looks strange, but one gets used to it—and it is not very important, because it is merely a matter of terminology, not a fact of physics. with the aid of this equation is both its instantaneous speed and its average speed. If the motion is not uniform, the formula gives only the *average* speed; the *instantaneous* speed for any point on the circle can be determined if we find $\Delta d/\Delta t$ from measurements of very small segments of the path.

Let us now see how the last equation can be used. We can, for example, calculate the speed of the tip of a helicopter rotor blade in its motion around the central shaft. On one model, the main rotor has a diameter of 7.50 m and a frequency of 480 revolutions/minute under standard conditions. Thus f = 480 per minute = 8.00 per second and R = 3.75 m, and

 $v = 2\pi R f$ v = 2 (3.14)(3.75)(8.00) meters/second v = 189 m/sec

or about 420 miles/hr.

Q4 If a phonograph turntable is running at 45 revolutions per minute,

(a) What is its period (in minutes)?

(b) What is its period (in seconds)?

(c) What is its frequency in cycles per second?

Q5 What is the period of the minute hand of an ordinary clock? If the hand is 3.0 cm long, what is the linear speed of the tip of the minute hand?

Q6 The terms frequency and period can also be used for any other periodic, repetitive phenomenon. For example, if your heart beats 80 times per minute, what are the frequency and period for your pulse?

4.6 Centripetal acceleration and centripetal force

Let us assume that a stone on a string is moving with uniform circular motion, for example in a horizontal plane as the stone is whirled overhead. The speed of the stone is constant. The velocity, however, is always changing. Velocity is a vector quantity, which includes both speed and direction. Up to this point we have dealt with accelerations in which only the speed was changing. In uniform circular motion the speed of the revolving object remains the same, while the direction of motion changes continually. The figure shows the whirling stone at three successive moments in its revolution. At any instant, the direction of the velocity vector is tangent to the curving path. Notice that its speed, represented by the length of the velocity arrow, does not vary; but its direction changes from moment to moment. Since acceleration is defined as a change in velocity, the stone is in fact accelerating.

But to produce an acceleration a net force is needed. In the case of the whirling stone, a force is exerted on the stone by the string, and if we neglect the weight of the stone or air resistance, that







 $\vec{a_c}$ and $\vec{F_c}$ are parallel, but \vec{v} is perpendicular to $\vec{a_c}$ and $\vec{F_c}$. Note that usually one should *not* draw different kinds of vector quantities on the same drawing.

Understanding Motion

The adjective centripetal means literally "moving, or directed, toward the center."

In uniform circular motion, the Instantaneous velocity and the centripetal force at any instant of time are perpendicular, one being along the tangent, the other along the radius. So instantaneous velocity and the acceleration are also always at right angles. will be the net force. If the string were suddenly cut, the stone would go flying off on a tangent with the velocity it had at the instant the string was cut—on a tangent to the circular path. As long as the string holds, the stone is forced into a circular path.

The direction of this force acting on the stone is along the string. Thus the force vector is always pointing toward the center of rotation. This kind of force – always directed toward the center of rotation – is called *centripetal* force.

From Newton's second law we know that force and acceleration are in the same direction, so the acceleration vector is also directed toward the center. We shall call this acceleration *centripetal* acceleration, and give it the symbol $\vec{a_c}$. Any object moving along a circular path has a centripetal acceleration.



We know now the direction of centripetal acceleration. What is its magnitude? An expression for a_c can be derived from the definition of acceleration $\vec{a_c} = \Delta v / \Delta t$. The details of such a derivation are given on the next page. The result shows that $\vec{a_c}$ depends on \vec{v} and R, and in fact the magnitude of a_c is given by

This is about 4/10 of a_0 , and could be called an acceleration of "0.4q."

 $a_c = \frac{v^2}{R}$

Let us verify this relationship with a numerical example. If, as sketched in the diagram, a car goes around a circular curve of radius R = 100 m at a uniform speed of v = 20 m/sec, what is its centripetal acceleration a_c toward the center of curvature? By the equation derived on the gray page:

$$a_c = \frac{v^2}{R}$$
$$= \frac{\left(20 \frac{m}{sec}\right)^2}{100 m}$$
$$= \frac{400}{100} \frac{\frac{m^2}{sec^2}}{m}$$
$$= 4.0 \frac{m}{sec^2}$$

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Derivation of the equation $a_c = \frac{v^2}{R}$

Assume the stone is moving uniformly in a circle of radius R. We can find what the relationship between a_c , v, and R is by treating a small part of the circular path as the combination of a tangential motion and an acceleration toward the center. To follow the circular path, the stone must accelerate toward the center through a distance h in the same time that it would move through a tangential distance d. The stone, with speed v, would travel a tangential distance d given by $d = v\Delta t$. In the same time Δt , the stone, with acceleration a_c , would travel toward the center through a distance hgiven by $h = \frac{1}{2}a_c\Delta t^2$. (We can use this last equation because at t = 0, the stone's velocity toward the center is zero.)

We can now apply the Pythagorean Theorem to the triangle in the figure at the right.

$$R^{2} + d^{2} = (R + h)^{2}$$

= $R^{2} + 2Rh + h^{2}$

When we subtract R^2 from each side of the equation we are left with

$$d^2 = 2Rh + h^2$$

We can simplify this expression by making an approximation: since h is very small compared to R, h^2 will be very small compared to Rh. If we choose Δt to be vanishingly small (as we must to get the instantaneous acceleration), h^2 will become vanishingly small compared to Rh; so we shall neglect h^2 and write

$$d^2 = 2Rh$$

Also, we know $d = v\Delta t$ and $h = \frac{1}{2}a_c\Delta t^2$, so we can substitute for d^2 and for h accordingly. Thus

 $a_c = \frac{v^2}{R}$

 $\begin{aligned} (v\Delta t)^2 &= 2R \cdot \frac{1}{2}a_c(\Delta t)^2 \\ v^2(\Delta t)^2 &= Ra_c(\Delta t)^2 \\ v^2 &= Ra_c \end{aligned}$

or

The approximation becomes better and better as Δt becomes smaller and smaller. In other words, v^2/R is the magnitude of the *instantaneous* centripetal acceleration for a body moving on a circular arc of radius R. For uniform circular motion, v^2/R is the magnitude of the centripetal acceleration at every point of the path. (Of course it does not have to be a stone on a string. It can be a point particle on the rim of a rotating wheel, or a house on the rotating earth, or a coin sitting on a rotating phonograph disk, or a car in a curve on the road.)





Does this make sense? We can check the result by going back to the basic vector definition of acceleration: $\vec{a}_{av} = \Delta \vec{v} / \Delta t$. We will make a scale drawing of the car's velocity vector at two instants a short time Δt apart, measure the change in velocity $\Delta \vec{v}$ between them, and divide the magnitude of $\Delta \vec{v}$ by Δt to get \vec{a}_{av} over the interval.

Consider a time interval of $\Delta t = 1$ second. Since the car is moving at 20 m/sec, its position will change 20 m during Δt . Two positions P and P', separated by 20 m, are marked in diagram B.

Now draw arrows representing velocity vectors. If we choose a scale of 1 cm = 10 m/sec, the velocity vector for the car will be represented by an arrow 2 cm long. These are drawn at P and P' in diagram C.

If we put these two arrows together tail to tail as in diagram D, it is easy to see what the change in the velocity vector has been during Δt . Notice that if $\Delta \vec{v}$ were drawn halfway between P and P', it would point directly toward the center of the curve; so the average acceleration between P and P' is indeed directed centripetally. Measurement of the $\Delta \vec{v}$ arrow in the diagram shows that it has a magnitude of 0.40 cm; so it represents a velocity change of 4.0 m/sec. This change occurred during $\Delta t = 1$ second, so the rate of change is 4.0 m/sec/sec – the same value we found using the relation $a_c = v^2/R!$

The best way of showing that $a_c = v^2/R$ is entirely consistent with the mechanics we have developed in Unit 1 is to do some experiments to measure the centripetal force required to keep an object moving in a circle. If, for example, the mass of the car were 1000 kg, there would have to be a centripetal force acting on the car:

 $F_c = m \times a_c$ $= 1000 \text{ kg} \times 4.0 \frac{\text{m}}{\text{sec}^2}$

 $= 4000 \text{ kg} \frac{\text{m}}{\text{sec}^2} = 4000 \text{ N}$ (or about 1800 pounds).

This force would be directed toward the center of curvature of the road – that is, it would always be sideways to the direction the car is moving. This force is exerted on the tires by the road. If the road is wet or icy, and can not exert a force of 4000 N sideways on
SG 4.13 the tires, the centripetal acceleration will be less than 4.0 m/sec – so
SG 4.14 the car will follow a less curved path as sketched in the margin on the next page. In situations where the car's path is less curved than the road, we would say the car "left the road" – although it might be just as appropriate to say the road left the car.

The sideways force exerted on tires by a road is not easy to measure. But in Project Physics *Handbook 1* there are a number of ways suggested for you to check experimentally whether $F_c = ma_c$ or $F_c = m v^2/R$.

For uniform motion in repeated cycles, it is often easier to measure the frequency f or period T than it is to measure v directly.

Section 4.7

We can substitute the relations $v = 2\pi R f$ or $v = 2\pi R/T$ into the equation equation for a_c to get alternative and equivalent ways of calculating a_c :

$c = \frac{v^2}{R}$	$a_c = \frac{v^2}{R}$
$=\frac{(2\pi Rf)^2}{R}$	$=\frac{\left(rac{2\pi R}{T} ight)^2}{}$
$=\frac{4\pi^2 R^2 f^2}{R}$	R $4\pi^2 R^2$
$=4\pi^2 R f^2$	$=\frac{T^2}{R}$
	$=rac{4\pi^2 R}{T^2}$

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Q7 In which of the following cases can a body have an acceleration?

- (a) moves with constant speed
- (b) moves in a circle with constant radius
- (c) moves with constant velocity

а

Q8 In what direction would a piece from a rapidly spinning fly-wheel go if it suddenly shattered?

Q9 If a car of mass m going at speed v enters a curve of radius R, what is the force required to keep the car curving with the road?

Q10 If a rock of mass m is being whirled overhead at 1 revolution/second on a string of length R, what is the force which the string must be exerting?

4.7 The motion of earth satellites

Nature and technology provide many examples of the type of motion where an object is in uniform circular motion. The wheel has been a main characteristic of our civilization, first as it appeared on crude carts and then later as an essential part of complex machines. The historical importance of rotary motion in the development of modern technology has been described by the historian V. Gordon Childe:

Rotating machines for performing repetitive operations, driven by water, by thermal power, or by electrical energy, were the most decisive factors of the industrial revolution, and, from the first steamship till the invention of the jet plane, it is the application of rotary motion to transport that has revolutionized communications. The use of rotary machines, as of any other human tools, has been cumulative and progressive. The inventors of the eighteenth and nineteenth centuries were merely extending the applications of rotary motion that had been devised in previous generations, reaching back thousands of years into the prehistoric past. . . . [*The History of Technology*.]



Chariot. Alberto Giacometti, 1950.

As you will see in Unit 2, there is another rotational motion that has also been one of the central concerns of man throughout recorded history: the orbiting of planets around the sun and of the moon around the earth.

Since the kinematics and dynamics for any uniform circular motion are the same, we can apply what you have learned so far to the motion of artificial earth satellites in circular (or nearly circular) paths. As an illustration, we will select the satellite Alouette I, Canada's first satellite, which was launched into a nearly circular orbit on September 29, 1962.

Tracking stations located in many places around the world maintain a record of any satellite's position in the sky. From the position data, the satellite's distance above the earth at any time and its period of revolution are found. By means of such tracking, we know that Alouette I moves at an average height of 630 miles above sea level, and takes 105.4 minutes to complete one revolution.

We can now quickly calculate the orbital speed and the centripetal acceleration of Alouette I. The relationship $v = 2\pi R/T$ allows us to find the speed of any object moving uniformly in a circle if we know its period T and its distance R from the center of its path (in this case, the center of the earth). Adding 630 miles to the earth's radius of 3963 miles, we get R = 4594 miles, and

$$v = \frac{2\pi R}{T}$$
$$= \frac{2\pi \times 4593 \text{ min}}{105.4 \text{ min}}$$
$$= \frac{28,860 \text{ min}}{105.4 \text{ min}}$$
$$= 274 \text{ mi/min}$$

or roughly 16,400 mi/hr.

To calculate the centripetal acceleration of Alouette I, we can use this value of v along with the relationship $a_c = v^2/R$. Thus

 $a_c = \frac{v^2}{R}$ $=\frac{(274 \text{ mi/min})^2}{4,594 \text{ mi}}$ $= 16.3 \text{ mi}/\text{min}^2$

which is equivalent to 7.3 m/sec². (To get the same result, we could just as well have used the values of R and T directly in the relationship $a_c = 4\pi^2 R/T^2$.)

What is the origin of the force that gives rise to this acceleration? Although we will not make a good case for it until Chapter 8, you surely know already that it is due to the earth's attraction. Evidently the centripetal acceleration a_c of the satellite is just the gravitational acceleration a_g at that height, which has a value 25% less than a_g very near the earth's surface.

SG 4.19

SG 4.20 SG 4.21



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Section 4.7

Earlier we asked the question, "What speed is required for an object to stay in a circular orbit about the earth?" You can answer this question now for an orbit 630 miles above the surface of the earth. To get a general answer, you need to know how the acceleration due to gravity changes with distance. In Chapter 8 we will come back to the problem of injection speeds for orbits.

The same kind of analysis applies to an orbit around the moon. For example, on the first manned orbit of the moon (Apollo 8, in 1968), the mission control group wanted to put the capsule into a circular orbit 70 miles above the lunar surface. They believed that the acceleration due to the moon's gravity at that height would be $a_g = 1.43$ m/sec². What direction and speed would they give the capsule to "inject" it into lunar orbit?

The direction problem is fairly easy—to stay at a constant height above the surface, the capsule would have to be moving horizontally at the instant the orbit correction was completed. So injection would have to occur just when the capsule was moving on a tangent, 70 miles up, as shown in the sketch in the margin. What speed (relative to the moon, of course) would the capsule have to be given? The circular orbit has a radius 70 miles greater than the radius of the moon, which is 1080 miles; so R = 1080 mi + 70 mi= 1150 mi; this is equal to $1.85 \times 10^6 \text{ meters}$. The centripetal acceleration is just the acceleration caused by gravity, which was supposed to be 1.43 m/sec^2 , so



$$a_c = a_g$$

$$\frac{v^2}{R} = a_g$$

$$v^2 = Ra_g$$

$$v = \sqrt{Ra_g}$$

$$= \sqrt{(1.85 \times 10^6 \text{ m}) \times 1.43 \frac{\text{m}}{\text{sec}^2}}$$

$$= \sqrt{2.65 \text{ m} \times 10^6 \frac{\text{m}}{\text{sec}^2}}$$

$$= 1.63 \times 10^3 \frac{\text{m}}{\text{sec}}$$

The necessary speed for an orbit at 70 miles above the surface is therefore 1630 m/sec (about 3600 mi/hr). Knowing the capsule's speed, ground control could calculate the necessary speed changes to reach 1630 m/sec. Knowing the thrust force of the engines and the mass of the capsule, they could calculate the time of thrust required to make this speed change.

Q11 What information was necessary to calculate the speed for an orbit 70 miles above the moon's surface?

SG 4.22 SG 4.23 SG 4.24

NAME	LAUNCH DATE	WEIGHT (Ib)	PERIOD (min)	Perigee-Apogee	REMARKS (Including purpose)
Sputnik 1 1957 (USSR)	Oct. 4, 1957	184	96.2	142-588	First earth satellite. Internal temperature, pressure inside satellite.
Explorer 7 1958 (USA)	Jan. 31, 1958	30.8	114.8	224-1573	Cosmic rays, micrometeorites, internal and shell temperatures, discovery of first Van Allen belts.
Lunik 3 1959 (USSR)	Oct. 4, 1959	959	22,300	30,000-291,000	Transmitted photographs of far side o moon.
Vostok 1 1961 (USSR)	Apr. 12, 1961	10,416	89.34	109-188	First manned orbital flight (Major Yuri Gagarin; one orbit)
Midas 3 1961 (USA)	July 12, 1961	3,500	161.5	2,129-2,153	Almost circular orbit.
Telestar 1 1962 (USA)	July 10, 1962	170	157.8	593-3,503	Successful transmission across the Atlantic: telephony, phototelegraphy, and television.
Alouette 1 1962 (USA- Canada)	Sept. 29, 1962	319	105.4	620-640	Joint project between NASA and Canadian Defense Research Board; measurement in ionosphere.
Luna 4 1963-08 (USSR)	Apr. 2, 1963	3,135	42,000	56,000-435,000	Passed 5,300 miles from moon; very large orbit.
Vostok 6 1963-23 (USSR)	June 16, 1963	''about 5 tons''	88.34	106-134	First orbital flight by a woman; (Valentina Terishkive; 48 orbits)
Syncom 2 1963-31 (USA)	July 26, 1963	86	1,460.4	22,187-22,192	Successfully placed in near- Synchronous orbit (stays above same spot on earth).

Table 4.2 Some information on selected artificial satellites.

4.8 What about other motions?



So far we have described straight-line motion, projectile motion, and uniform circular motion. In all these cases we considered only examples where the acceleration was constant—at least in magnitude if not in direction—or very nearly constant. There is another basic kind of motion that is equally common and important in physics, where the acceleration is always changing. A common example of this type of motion is that seen in playground swings, or in vibrating guitar strings. Such back and forth motion, or *oscillation*, about a center position occurs when there is a force always directed toward the center position. When a guitar string is pulled aside, for example, a force arises which tends to restore the string to its undisturbed center position. If it is pulled to the other side, a similar restoring force arises in the opposite direction.

A very common type of such motion is one for which the restoring force is proportional, or nearly proportional, to how far the object is displaced. This is true for the guitar string, if the displacements are not too large; pulling the string aside 2 mm will produce twice the restoring force that pulling it aside 1 mm will. Oscillation with a restoring force proportional to the displacement

Section 4.8

is called *simple harmonic motion*. The mathematics for describing simple harmonic motion is relatively simple, and many phenomena, from pendulum motion to the vibration of atoms, have aspects that are very close to simple harmonic motion. Consequently, the analysis of simple harmonic motion is used very widely in physics. The Project Physics *Handbook 1* describes a variety of activities you can do to become familiar with oscillations and their description.

Either simply or in combination, the dynamics discussed in this chapter will cover most of the motions that will interest us, and is a good start toward understanding apparently very complicated motions, whether those of water ripples on a pond, a person running, the swaying of a tall building or bridge in the wind, a small particle zig-zagging through still air, an amoeba seen under a microscope, or a high-speed nuclear particle moving in the field of a magnet. The methods we have developed in this and the preceding chapters give us means for dealing with any kind of motion whatsoever, on earth or anywhere in the universe.

When we considered the forces needed to produce motion, Newton's laws supplied us with the answers. Later, when we shall discuss other motions ranging from the elliptical motion of planets to the hyperbolic motion of an alpha particle passing near a nucleus, we shall continue to find in Newton's laws the tool for inferring the magnitude and direction of the forces acting in each case.

Conversely, if we know the magnitude and direction of the forces acting on an object, we can determine what its change in motion will be. If in addition we know also the present position, velocity and mass of an object, we can reconstruct how it moved in the past, and we can predict how it will move in the future under these forces. Thus Newton's laws provide a comprehensive view of forces and motion. It is not surprising that Newtonian mechanics became a model for many other sciences: here seemed to be a method for understanding all motions, no matter how mysterious they previously may have appeared to be. SG 4.25 SG 4.26







EPILOGUE The purpose of this Unit was to deal with the fundamental concepts of motion. We decided to start by analyzing particularly simple kinds of motion in the expectation that they are indeed the "ABC's" of physics. These ideas would allow us to turn our attention back to some of the more complex features of the world. To what extent were these expectations fulfilled?

We did find that a relatively few basic concepts allowed us to gain a considerable understanding of motion. First of all, we found that useful descriptions of the motion of objects can be given using the concepts of distance, displacement, time, speed, velocity, and acceleration. If to these we add force and mass and the relationships expressed in Newton's three laws of motion, it becomes possible to account for observed motion in an effective way. The surprising thing is that these concepts of motion, which were developed in extraordinarily restricted circumstances, can in fact be so widely applied. For example, our work in the laboratory centered around the use of sliding dry ice pucks and steel balls rolling down inclined planes. These are not objects found moving around ordinarily in the everyday "natural" world. Even so, we found that the ideas obtained from those specialized experiments could lead us to an understanding of objects falling near the earth's surface, of projectiles, and of objects moving in circular paths. We started by analyzing the motion of a disk of dry ice moving across a smooth surface and ended up analyzing the motion of a space capsule as it circles the moon and descends to its surface.

Thus, we have made substantial progress in analyzing complex motions. On the other hand, we cannot be satisfied that we have here all the intellectual tools needed to understand all of the phenomena that interest us. In Unit 3 we shall add to our stock of fundamental concepts a few additional ones, particularly those of momentum, work, and energy. They will help us when we turn our attention away from interactions involving a relatively few objects of easily discernible size, and to interactions involving countless numbers of submicroscopic objects-molecules and atoms.

In this Unit we have dealt primarily with concepts that owe their greatest debts to Galileo, Newton, and their followers. If space had permitted, we should also have included the contributions of René Descartes and the Dutch scientist Christian Huyghens. The mathematician and philosopher, A. N. Whitehead has summarized the role of these four men and the significance of the concepts we have been dealing with in the following words:

This subject of the formation of the three laws of motion and of the law of gravitation [which we shall take up in Unit 2] deserves critical attention. The whole development of thought occupied exactly two generations. It commenced with Galileo and ended with Newton's *Principia*; and Newton was born in the year that Galileo died. Also the lives of Descartes and Huyghens fall within the period occupied by these great terminal figures. The issue of the combined labours of these four men has some right to be

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Epilogue

considered as the greatest single intellectual success which mankind has achieved. (Science and the Modern World)

The laws of motion Whitehead speaks of, the subject of this Unit, were important most of all because they suddenly allowed a new understanding of celestial motion. For at least twenty centuries man had been trying to reduce the complex motions of the stars, sun, moon, and planets to an orderly system. The genius of Galileo and Newton was in studying the nature of motion of objects as it occurs on earth, and then to assume the same laws would apply to objects in the heavens beyond man's reach.

Unit 2 is an account of the immense success of this idea. We shall trace the line of thought, starting with the formulation of the problem of planetary motion by the ancient Greeks, through the work of Copernicus, Tycho Brahe, Kepler, and Galileo to provide a planetary model and the laws for planetary motion, and finally to Newton's magnificent synthesis of terrestrial and celestial physics in his Law of Universal Gravitation.









4.1 The Project Physics learning materials particularly appropriate for Chapter 4 include the following: Experiments

Curves of Trajectories Prediction of Trajectories Centripetal Force Centripetal Force on a Turntable

Activities

Projectile Motion Demonstration Speed of a Stream of Water Photographing a Waterdrop Parabola Ballistic Cart Projectiles Motion in a Rotating Reference Frame Penny and Coat Hanger Measuring Unknown Frequencies

Reader Articles

Galileo's Discussion of Projectile Motion Newton's Laws of Dynamics Rigid Body Fun in Space

Film Loops

A Matter of Relative Motion Galilean Relativity – Ball Dropped from Mast of Ship Galilean Relativity – Object Dropped from Aircraft Galilean Relativity – Projectile Fired vertically Analysis of Hurdle Race I Analysis of Hurdle Race II

4.2 The thrust developed by a Saturn Apollo rocket is 7,370,000 newtons (approximately 1,650,000 lbs.) and its mass is 540,000 kg. What is the acceleration of the vehicle relative to the earth's surface at lift off? How long would it take for the vehicle to rise 50 meters?

The acceleration of the vehicle increases greatly with time (it is 47 m/sec² at first stage burnout) even though the thrust force does not increase appreciably. Explain why the acceleration increases.

4.3 A hunter points his gun barrel directly at a monkey in a distant palm tree. Will the bullet follow the line of sight along the barrel? If the animal, startled by the flash, drops out of the branches at the very instant of firing, will it then be hit by the bullet? Explain.

4.1 The displacement \vec{d} of an object is a vector giving the straightline distance from the beginning to the end of an actual path; \vec{d} can be thought of as made up of a horizontal (x) and a vertical (y) component of displacement; that is, $\vec{d} = \vec{x} + \vec{y}$ (added vectorially).

In a trajectory, x, y, and the total displacement d can be thought of as the magnitudes of the sides of right triangles. So can v_x , v_y and the magnitude of the velocity v.

(a) Find an expression for d in terms of x and y.

- (b) Find an expression for v in terms of v_x and v_y
- (c) Rewrite the expression for d and v in terms of v_x , a_g , and t.



4.5 If you like algebra, try this general proof.

If a body is launched with speed v at some angle other than 0°, it will initially have both a horizontal speed v_x and a vertical speed v_y . The equation for its horizontal displacement is $x = v_x t$, as before. But the equation for its vertical displacement has an additional term: $y = v_y t + \frac{1}{2}a_y t^2$. Show that the trajectory is still parabolic in shape.

4.6 A lunch pail is accidently kicked off a steel beam on a skyscraper under construction. Suppose the initial horizontal speed v_x is 1.0 m/sec. Where is the pail (displacement), and what is its speed and direction (velocity) 0.5 sec after launching?

4.7 In Galileo's drawing on page 104, the distances *bc*, *cd*, *de*, etc. are equal. What is the relationship among the distances *bo*, *og*, *gl*, and *ln*?

4.8 You are inside a van that is moving with a constant velocity. You drop a ball.

- (a) What would be the ball's path relative to the van?
- (b) Sketch its path relative to a person driving past the van at a high uniform speed.
- (c) Sketch its path relative to a person standing on the road.

You are inside a moving van that is accelerating uniformly in a straight line. When the van is traveling at 10mph (and still accelerating) you drop a ball from near the roof of the van onto the floor.

- (d) What would be the ball's path relative to the van?
- (e) Sketch its path relative to a person driving past the van at a high uniform speed.
- (f) Sketch its path relative to a person standing on the road.

4.9 Two persons watch the same object move. One says it accelerates straight downward, but the other claims it falls along a curved path. Describe conditions under which each would be reporting correctly what he sees.

4.10 An airplane has a gun that fires bullets straight ahead at the speed of 600 mph when tested on the ground while the plane is stationary.

The plane takes off and flies due east at 600 mph. Which of the following describes what the pilot of the plane will see? In defending your answers, refer to the Galilean relativity principle:

- (a) When fired directly ahead the bullets move eastward at a speed of 1200 mph.
- (b) When fired in the opposite direction, the bullets dropped vertically downward.
- (c) If fired vertically downward, the bullets move eastward at 600 mph. while they fall.

Specify the frames of reference from which (a). (b), and (c) are the correct observations.

4.11 Many commercial record turntables are designed to rotate at frequencies of 16 2/3 rpm (called transcription speed). 33 1/3 rpm (long playing). 45 rpm (pop singles). and 78 rpm (old fashioned). What is the period corresponding to each of these frequencies?

4.12 Two blinkies are resting on a rotating turntable and are photographed in a setup as shown in the figure below. The outer blinky has a frequency of 9.4 flashes/sec and is located 15.0 cm from the center. For the inner blinky, the values are 9.1 flashes/sec and 10.6 cm.



- (a) What is the period of the turntable?
- (b) What is the frequency of rotation of the turntable? Is this a standard phonograph speed?
- (c) What is the speed of the turntable at the position of the outer blinky?
- (d) What is the speed of the turntable at the position of the inner blinky?
- (e) What is the speed of the turntable at the very center?
- (f) What is the angular speed of each blinky-that is, the rate of rotation measured in degrees/sec? Are they equal?
- (g) What is the centripetal acceleration experienced by the inner blinky?
- (h) What is the centripetal acceleration experienced by the outer blinky?
- (i) If the turntable went faster and faster, which would leave the turntable first, and why?

4.13 Passengers on the right side of the car in a left turn have the sensation of being "thrown

against the door." Explain what actually happens to the passengers in terms of force and acceleration.

4.14 The tires of the turning car in the example on page 112 were being pushed sideways by the road with a total force of 1800 lb. Of course the tires would be pushing on the road with a total force of 1800 lb also. (a) What happens if the road is covered with loose sand or gravel? (b) How would softer (lower pressure) tires help? (c) How would banking the road (that is, tilting the surface toward the center of the curve) help? (Hint: consider the extreme case of banking in the bob-sled photo on p. 110.)

4.15 Using a full sheet of paper, make and complete a table like the one below.

NAME OF CONCEPT	SYMBOL	DEFINITION	EXAMPLE
		Length of a	
		path between	
		any two points,	
		as measured	
		along the path.	Straight ling
			distance and
			direction from
			Detroit to
			Chicago.
	V		
Instan-			
taneous			
speed			
			An airplane
			flying west
			at 400 mph at
			constant
		Time rate of	altitude.
		change of	
		velocity	
		Velocity.	
	a_g		
Centripetal			
acceler-			
ation			
			The drive shaft
			of some
			turne 600 rem
			in low gear
		The time it	in low year.
		takes to make	
		one complete	
		revolution.	



4.16 Our sun is located at a point in our galaxy about 30,000 light years (1 light year = 9.46×10^{12} km) from the galactic center. It is thought to be revolving around the center at a linear speed of approximately 250 km/sec.

- (a) What is the sun's centripetal acceleration with respect to the center of the galaxy?
- (b) The sun's mass can be taken to be 1.98 \times 10³⁰ kg; what centripetal force is required to keep the sun moving in a circular orbit about the galactic center?
- (c) Compare the centripetal force in (b) with that necessary to keep the earth in orbit about the sun. (The earth's mass is 5.98×10^{24} kg and its average distance from the sun is 1.495×10^8 km.)

4.17 The hammer thrower in the photograph below is exerting a large centripetal force to keep the hammer moving fast in a circle, and applies it to the hammer through a connecting wire. The mass of the 16-pound hammer is 7.27 kg. (a) Estimate the radius of the circle and the period, and calculate a rough value for the amount of force required just to keep it moving in a circle. (b) What other components are there to the total force he exerts on the hammer?



- 4.18 Contrast rectilinear motion, projectile motion, and uniform circular motion by
 - (a) defining each
 - (b) giving examples.
 - (c) describing the relation between velocity and acceleration in each case.

- 4.19 These questions are asked with reference to Table 4.2 on page 116.
 - (a) Which satellite has the most nearly circular orbit?
 - (b) Which satellite has the most eccentric orbit? How did you arrive at your answer?
 - (c) Which has the longest period?
 - (d) How does the position of Syncom 2 relative to a point on earth change over one day?

4.20 If the earth had no atmosphere, what would be the period of a satellite skimming just above the earth's surface? What would its speed be? 4.21 Explain why it is impossible to have an earth satellite orbit the earth in 80 minutes. Does this mean that it is impossible for any object to go around the earth in less than 80 minutes? 4.22 What was the period of the "70 mi" Apollo 8 lunar orbit?

4.23 Knowing a_g near the moon's surface, and the orbital speed in an orbit near the moon's surface, we can now work an example of Part 8 of the earth-moon trip described in Sec 4.1. The Apollo 8 capsule was orbiting about 100 kilometers above the surface. The value of a_g near the moon's surface is about 1.5 m/sec².

If the capsule's rocket engines are fired in the direction of its motion, it will slow down. Consider the situation in which the rockets fire long enough to reduce the capsule's horizontal speed to 100 m/sec^2 .

- (a) About how long will the fall to the moon's surface take?
- (b) About how far will it have moved horizontally during the fall?
- (c) About how far in advance of the landing target might the "braking" maneuver be performed?

4.24 Assume that a capsule is approaching the moon along the right trajectory, so that it will be moving tangent to the desired orbit. Given the speed v_o necessary for orbit and the current speed v, how long should the engine with thrust F fire to give the capsule of mass m the right speed? 4.25 The intention of the first four chapters has been to describe "simple" motions and to progress to the description of more "complex" motions. Put each of the following examples under the heading "simplest motion." "more complex." or "very complex." Be prepared to say why you place any one example as you did and state any assumptions you made.

- (a) helicopter shown on p. 109
- (b) "human cannon ball" in flight
- (c) car going from 40 mph to a complete stop
- (d) tree growing
- (e) child riding a Ferris wheel
- (f) rock dropped 3 mi.
- (g) person standing on a moving escalator
- (h) climber ascending Mt. Everest
- (i) person walking
- (j) leaf falling from a tree

4.26 Write a short essay on the physics involved in the motions shown in one of the four pictures on the opposite page, using the ideas on motion from Unit 1.

STUDY GUIDE 4



Acknowledgments

Prologue

Pp. 1-4 Fermi, Laura, *Atoms in the Family*, U. of Chicago Press, pp. 83-100 not inclusive. *Chapter Two*

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P. 7 (left) Museum of Comparative Zoology, Harvard University; (right) Brookhaven National Laboratory. Chapter 1

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The Projects Physics Course

Handbook

Concepts of Motion



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This *Handbook* is your guide to observations, experiments, activities, and explorations, far and wide, in the realms of physics.



Prepare for challenging work, fun and some surprises. One of the best ways to learn physics is by *doing* physics, in the laboratory and out. Do not rely on reading alone. Also, this *Handbook* is different from laboratory manuals you may have worked with before. Far more projects are described here than you alone can possibly do, so you will need to pick and choose. Although only a few of the experiments and activities will be assigned, do any additional ones that interest you. Also, if an activity occurs to you that is not described here, discuss with your teacher the possibility of doing it. Some of the most interesting science you will experience in this course will be the result of the activities which you choose to pursue beyond the regular assignments of the school laboratory.

This *Handbook* contains a section corresponding to each chapter of the *Text*. Usually each section is divided further in the following way:

The *Experiments* contain full instructions for the investigations you will be doing with your class.

The Activities contain many suggestions for construction projects, demonstrations, and other activities you can do by yourself.

The *Film Loop* notes give instructions for the use of the variety of film loops that have been specially prepared for the course.

In each section, do as many of these things as you can. With each, you will gain a better grasp of the physical principles and relationships involved.



Keeping Records

Your records of observations made in the laboratory or at home can be kept in many ways. Your teacher will show you how to write up your records of observations. But regardless of the procedure followed, the key question for deciding what kind of record you need is this: "Do I have a clear enough record so that I could pick up my lab notebook a few months from now and explain to myself or others what I did?"

Here are some general rules to be followed in every laboratory exercise. Your records should be neatly written without being fussy. You should organize all numerical readings in tables, if possible, as in the sample lab write up on pages 130 and 131. You should always identify the units (centimeters, kilograms, seconds, etc.) for each set of data you record. Also, identify the equipment you are using, so that you can find it again later if you need to recheck your work.

In general, it is better to record more rather than less data. Even details that may seem to have little bearing on the experiment you are doing—such as the temperature and whether it varied during the observations, and the time when the data were taken—may turn out to be information that has a bearing on your analysis of the results.

If you have some reason to suspect that a particular datum may be less reliable than

other data—perhaps you had to make the reading very hurriedly, or a line on a photograph was very faint—make a note of the fact. But don't erase a reading. When you think an entry in your notes is in error, draw a single line through it—don't scratch it out completely or erase it. You may find it was significant after all.

There is no "wrong" result in an experiment, although results may be in considerable error. If your observations and measurements were carefully made, then your result will be correct. What ever happens in nature, including the laboratory, cannot be "wrong." It may have nothing to do with your investigation. Or it may be mixed up with so many other events you did not expect that your report is not useful. Therefore, you must think carefully about the interpretation of your results.

Finally, the cardinal rule in a laboratory is to choose in favor of "getting your hands dirty" instead of "dry-labbing." In 380 B.C., the Greek scientist, Archytas, summed this up pretty well:

In subjects of which one has no knowledge, one must obtain knowledge either by learning from someone else, or by discovering it for oneself. That which is learnt, therefore, comes from another and by outside help; that which is discovered comes by one's own efforts and independently. To discover without seeking is difficult and rare, but if one seeks, it is frequent and easy; if, however, one does not know how to seek, discovery is impossible.

SEPT. 1969 EXPERIMENT. This experiment is to see how a rubber band stretches under the influence of forces rubber band I put different masses on the YOUR OWN SKETCH WILL ALWAYS end of the rubber band and HELP YOUR recorded the position of the MEMORY top of the hook that holds Greight the weight Room Temperature 26°C Position of the top of rubber band 36.3 cm INCLUDE ANY DATA YOU mass. Force Ploof Bottom Extension (n) (cm) (cm) THINK MAY BE RELEVANT ALWAYS SHOW UNITS OF TABULATED QUANTITIES (weights from 0. set, no enor) 44.0±.1 0 0 10 45.1 .098 1.1±.2 ESTIMATE THE ERROR OF EVERY QUANTIFY YOU ,196 45.8 20 1.8 MEASURE 30 .294 46.8 2.8 49.6 5.6 50 .490 second 20g 60 57.5 .588 7.5 weight is misser 70 .686 53.7 9.7 INCLUDE COMMENTS ON YOUR DATA 80 ,784 56.1 12.1 KEEP DATA 60.6 100 ,980 16.6 TABLES IN NEAT. 80 .784 56,2 12.2 recheck.

On these two pages is shown an example of a student's lab notebook report. The table is used to record both observed quantities (mass, scale position) and calculated quantities (force, extension of rubber band). The graph shows at a glance how the extension of the rubber band changes as the force acting on it is increased.

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Using the Polaroid Camera

You will find the Polaroid camera is a very useful device for recording many of your laboratory observations. Section 1.3 of your textbook shows how the camera is used to study moving objects. In the experiments and activities described in this *Handbook*, many suggestions are made for photographing moving objects, both with an electronic stroboscope (a rapidly flashing xenon light) and with a mechanical disk stroboscope (a slotted disk rotating in front of the camera lens). The setup of the rotating disk stroboscope with a Polaroid camera is shown below.



In the opposite column is a check list of operations to help you use the modified Polaroid Land camera model 210. For other models, your teacher will provide instructions. Check list of operations for Polaroid Land camera model 002

1. Make sure that there is film in the camera. If no white tab shows in front of the door marked "4" you must put in new film.

2. Fasten camera to tripod or disk strobe base. If you are using the disk strobe technique, fix the clip-on slit in front of the lens.

3. Check film (speed) selector. Set to suggested position (75 for disk strobe or blinky; 3000 for xenon strobe).

4. If you are taking a "bulb" exposure, cover the electric eye.

5. Check distance from lens to plane of object to be photographed. Adjust focus if necessary. Work at the distance that gives an image just one-tenth the size of the object, if possible. This distance is about 120 cm.

6. Look through viewer to be sure that whatever part of the event you are interested in will be recorded. (At a distance of 120 cm the field of view is just under 100 cm long.)

7. Make sure the shutter is cocked (by depressing the number 3 button).

8. Run through the experiment a couple of times without taking a photograph, to accustom yourself to the timing needed to photograph the event.

9. Take the picture: keep the cable release depressed only as long as necessary to record the event itself. Don't keep the shutter open longer than necessary.

10. Pull the white tab all the way out of the camera. Don't block the door (marked "4" on the camera).

11. Pull the large yellow tab straight out—all the way out of the camera. Begin timing development.

12. Wait 10 to 15 seconds (for 3000-speed black-and-white film).

13. Ten to 15 seconds after removing film from the camera, strip the white print from the negative.

14. Take measurements immediately. (The magnifier may be helpful.)

15. After initial measurements have been taken, coat your picture with the preservative supplied with each pack of film. Let this dry thoroughly, label it on the back for identification and mount the picture in your (or a partner's) lab report.

16. The negative can be used, too. Wash it carefully with a wet sponge, and coat with preservative.

17. Recock the shutter so it will be set for next use.

18. Always be careful when moving around the camera that you do not inadvertently kick the tripod.

19. Always keep the electric eye covered when the camera is not in use. Otherwise the batteries inside the camera will run down quickly.
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The Physics Readers

Your teacher probably will not often assign reading in the *Project Physics Reader*, but you are encouraged to look through it for articles of interest to you. In the Unit 1 *Reader* most students enjoy the chapter from Fred Hoyle's science fiction novel, *The Black Cloud*. This chapter, "Close Reasoning," is fictional, but nevertheless accurately reflects the real excitement of scientists at work on a new and important problem.

Since different people have very different interests, nobody can tell you which articles you will most enjoy. Those with interests in art or the humanities will probably like Gyorgy Kepes' article "Representation of Movement." If you are interested in history and in the role science plays in historical development, you are urged to read the Butterfield and Willey articles.

The *Reader* provides several alternative treatments of mechanics which either supplement or go beyond the Unit 1 *Text*. Thus Sawyer gives a discussion of the concept of speed different from that used in the *Text*. Clifford's approach is interesting because it uses geometry rather than algebra in explaining fundamental ideas. For those seeking a deeper understanding of mechanics, we particularly recommend the article from the *Feynman Lectures on Physics*. For articles that deal with applications of physics, you can turn to Strong on "The Dynamics of the Golf Club," Kirkpatrick on "Bad Physics in Athletic Measurements," and DuBridge on "Fun in Space."

Practice the art of browsing! Don't decide from the titles alone whether you are interested, but read portions of articles here and there, and you may well discover something new and interesting.



EXPERIMENTS

EXPERIMENT 1 NAKED EYE ASTRONOMY

The purpose of this first experiment is to familiarize you with the continually changing appearance of the sky. By watching the heavenly bodies closely day and night over a period of time, you will begin to understand what is going on up there and gain the experience you will need in working with Unit 2, *Motion in the Heavens*.

Do you know how the sun and the stars, the moon and the planets, appear to move through the sky? Do you know how to tell a planet from a star? Do you know when you can expect to see the moon during the day? How do the sun and planets move in relation to the stars?

The Babylonians and Egyptians knew the answers to these questions over 5000 years ago. They found them simply by watching the everchanging sky. Thus, astronomy began with simple observations of the sort you can make with your unaided eye.

You know that the earth appears to be at rest while the sun, stars, moon, and planets are seen to move in various paths through the sky. Our problem, as it was for the Babylonians, is to describe what these paths are and how they change from day to day, from week to week, and from season to season.

Some of these changes occur very slowly. In fact, this is why you may not have noticed them. You will need to watch the motions in the sky carefully, measuring them against fixed points of reference that you establish. You will need to keep a record of your observations for at least four to six weeks.

Choosing References

To locate objects in the sky accurately, you first need some fixed lines or planes to which your measurements can be referred, just as a map maker uses lines of latitude and longitude to locate places on the earth.

For example, you can establish a northsouth line along the ground for your first reference. Then with a protractor held horizontally, you can measure the position of an object in the sky around the horizon from this north-south line. The angle of an object around the horizon from a north-south line is called the object's *azimuth*. Azimuths are measured from the north point (0°) through east (90°) to south (180°) and west (270°) and around to north again (360° or 0°).

To measure the height of an object in the sky, you can measure the angle between the object and a horizontal plane, such as the horizon, for your second reference. This plane can be used even when the true horizon is hidden by trees or other obstructions. The angle between the horizontal plane and the line to an object in the sky is called the *altitude* of the object.



Establishing References

You can establish your north-south line in several different ways. The easiest is to use a compass to establish magnetic north but this may not be the same as true north. A magnetic compass responds to the total magnetic effect of all parts of the earth, and in most localities the compass does not point true north. The angle between magnetic north and true north is called *the angle of magnetic declination*. At some places the magnetic declination is zero, and the compass points toward true north.

At places east of the line where the declination is zero, the compass points west of true north; at places west of the line, the compass points east of true north. You can find the angle of declination and its rate of change per year for your area from the map below.

At night you can use the North Star (Polaris) to establish the north-south line. Polaris is the one fairly bright star in the sky that moves least from hour to hour or with the seasons. It is almost due north of an observer anywhere in the Northern Hemisphere.

To locate Polaris, first find the "Big Dipper" which on a September evening is low in the sky and a little west of north. (See the star map, Fig. 1–1 page 136.) The two stars forming the end of the dipper opposite the handle are known as the "pointers," because they point to the North Star. A line passing through them and extended upward passes very close to a bright star, the last star in the handle of the "Little Dipper." This bright star is the Pole Star, Polaris. On September 15 at 8:30 P.M. these constellations are arranged about as shown in the diagram below. B.C. by John Hart



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This chart of the stars will help you locate some of the bright stars and the constellations. To use the map, face north and turn the chart until today's date is at the top. Then move the map up nearly over your head. The stars will be in these positions at 8 P.M. For each hour *earlier* than 8 P.M., rotate the chart 15 degrees (one sector) clockwise. For each hour *later* than 8 P.M., rotate the chart counter-clockwise. If you are observing the sky outdoors with the map, cover the glass of a flashlight with fairly transparent red paper to look at the map. This will prevent your eyes from losing their adaptation to the dark when you look at the map.



Imagine a line from Polaris straight down to the horizon. The point where this line meets the horizon is nearly due north of you.

Now that you have established a northsouth line, either with a compass or from the North Star, note its position with respect to fixed landmarks, so that you can use it day or night.

You can establish the second reference, the plane of the horizon, and measure the altitude of objects in the sky from the horizon, with an astrolabe, a simple instrument you can obtain easily or make yourself, very similar to those used by ancient viewers of the heavens. Use the astrolabe in your hand or on a flat table mounted on a tripod or on a permanent post. A simple hand astrolabe you can make is described in the Unit 2 *Handbook*, in the experiment dealing with the size of the earth.

Sight along the surface of the flat table to be sure it is horizontal, in line with the horizon in all directions. If there are obstructions on your horizon, a carpenter's level turned in all directions on the table will tell you when the table is level.

Turn the base of the astrolabe on the table



until the north-south line on the base points along your north-south line. Or you can obtain the north-south line by sighting on Polaris through the astrolabe tube. Sight through the tube of the astrolabe at objects in the sky you wish to locate and obtain their altitude above the horizon in degrees from the protractor on the astrolabe. With some astrolabes, you can also obtain the azimuth of the objects from the base of the astrolabe.

To follow the position of the sun with the astrolabe, slip a large piece of cardboard with a hole in the middle over the sky-pointing end of the tube. (Caution: Never look directly at the sun. It can cause permanent eye damage!) Standing beside the astrolabe, hold a small piece of white paper in the shadow of the large cardboard, several inches from the sighting end of the tube. Move the tube about until the bright image of the sun appears through the tube on the paper. Then read the altitude of the sun from the astrolabe, and the sun's azimuth, if your instrument permits.



Observations

Now that you know how to establish your references for locating objects in the sky, here are suggestions for observations you can make on the sun, the moon, the stars, and the planets. Choose at least one of these objects to observe. Record the date and time of all your observa-

138 Experiment 1

tions. Later compare notes with classmates who concentrated on other objects.

A. Sun

CAUTION: NEVER look directly at the sun; it can cause permanent eye damage. Do not depend on sun glasses or fogged photographic film for protection. It is safest to make sun observations on shadows.

1. Observe the direction in which the sun sets. Always make your observation from the same observing position. If you don't have an unobstructed view of the horizon, note where the sun disappears behind the buildings or trees in the evening.

2. Observe the time the sun sets or disappears below your horizon.

3. Try to make these observations about once a week. The first time, draw a simple sketch on the horizon and the position of the setting sun.

4. Repeat the observation a week later. Note if the position or time of sunset has changed. Note if they change during a month. Try to continue these observations for at least two months.

5. If you are up at sunrise, you can record the time and position of the sun's rising. (Check the weather forecast the night before to be reasonably sure that the sky will be clear.)

6. Determine how the length of the day, from sunrise to sunset, changes during a week; during a month; or for the entire year. You might like to check your own observations of the times of sunrise and sunset against the times as they are often reported in newspapers. Also if the weather does not permit you to observe the sun, the newspaper reports may help you to complete your observations.

7. During a single day, observe the sun's azimuth at various times. Keep a record—of the azimuth and the time of observation. Determine whether the azimuth changes at a constant rate during the day, or whether the sun's apparent motion is more rapid at some times than at others. Find how fast the sun moves in degrees per hour. See if you can make a graph of the speed of the sun's change in azimuth. Similarly, find out how the sun's angular altitude changes during the day, and at what time its altitude is greatest. Compare a graph of the speed of the sun's change in altitude with a graph of its speed of change in azimuth. 8. Over a period of several months—or even an entire year—observe the altitude of the sun at noon—or some other convenient hour. (Don't worry if you miss some observations.) Determine the date on which the noon altitude of the sun is a minimum. On what date would the sun's altitude be a maximum?



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B. Moon

1. Observe and record the altitude and azimuth of the moon and draw its shape on successive evenings at the same hour. Carry your observations through at least one cycle of phases, or shapes, of the moon, recording in your data the dates of any nights that you missed.

For at least one night each week, make a sketch showing the appearance of the moon and another "overhead" sketch of the relative positions of the earth, moon, and sun. If the sun is below the horizon when you observe the moon, you will have to estimate the sun's position.

2. Locate the moon against the background of

the stars, and plot its position and phase on a sky map supplied by your teacher.

3. Find the full moon's maximum altitude. Find how this compares with the sun's maximum altitude on the same day. Determine how the moon's maximum altitude varies from month to month.

4. There may be a total eclipse of the moon this year. Consult Table 1 on page 140, or the *Celestial Calendar and Handbook*, for the dates of lunar eclipses. Observe one if you possibly can.

C. Stars

1. On the first evening of star observation, locate some bright stars that will be easy to find on successive nights. Later you will identify some of these groups with constellations that are named on the star map in Fig. 1–1, which shows the constellations around the North Star, or on another star map furnished by your teacher. Record how much the stars have changed their positions compared to your horizon after an hour; after two hours.

2. Take a time exposure photograph of several minutes of the night sky to show the motion of the stars. Try to work well away from bright street lights and on a moonless night. Include some of the horizon in the picture for reference. Prop up your camera so it won't move during the time exposures of an hour or more. Use a *small* camera lens opening (large fnumber) to reduce fogging of your film by stray light.

3. Viewing at the same time each night, find



This multiple exposure picture of the moon was taken with a Polaroid Land camera by Rick Pearce, a twelfthgrader in Wheat Ridge, Colorado. The time intervals between successive exposures were 15 min, 30 min, 30 min, and 30 min. Each exposure was for 30 sec using 2000-speed film. Which way was the moon moving in the sky?



A time exposure photograph of Ursa Major (The Big Dipper) taken with a Polaroid Land camera on an autumn evening in Cambridge, Massachusetts.

whether the positions of the star groups are constant in the sky from month to month. Find if any new constellations appear after one month; after 3 or 6 months. Over the same periods, find out if some constellations are no longer visible. Determine in what direction and how much the positions of the stars shift per week and per month.

D. Planets and meteors

1. The planets are located within a rather narrow band across the sky (called the ecliptic) along which the sun and the moon also move. For details on the location of planets, consult Table 1 on page 140, or the *Celestial Calendar and Handbook*, or the magazine *Sky and Telescope*. Identify a planet and record its position in the sky relative to the stars at two-week intervals for several months.

2. On almost any clear, moonless night, go outdoors away from bright lights and scan as much of the sky as you can see for meteors. Probably you will glimpse a number of fairly bright streaks of meteors in an hour's time. Note how many meteors you see. Try to locate on a star map like Fig. 1–1 where you see them in the sky.

Look for meteor showers each year around November 5 and November 16, beginning around midnight. Dates of other meteor showers are given in Table 2 on page 141. Remember that bright moonlight will interfere with meteor observation.

Additional sky observations you may wish to make are described in the Unit 2 *Handbook*.

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TABLE 1 A GUIDE FOR PLANET AND ECLIPSE OBSERVATIONS

Check your local newspaper for eclipse times and extent of eclipse in your locality. Mercury Venus Mars Jupiter Saturn Lunar Solar Visible for about one Visible for several months Very bright for Especially Especially Eclipses Eclipses week around stated around stated time. one month on bright for sevbright for two time each side of en months bemonths on yond stated each side of given time. Mercury and Venus are best viewed the hour Observable for time. given time. before dawn when indicated as a.m. and the Visible for 13 16 months surhour after sunset when indicated as p.m. rounding given months. time mid Feb.: a.m. Mar. 7: total 1 late Apr.: p.m. late May: early Dec .: in Fla., par-9 early June: a.m. early Nov .: p.m. overhead overhead Feb. 21 tial in east-7 mid Aug.: p.m. mid Dec.: a.m. at at Aug. 17 ern and 0 late Sept .: a.m. midnight midnight southern early Dec.: p.m. U. S. mid Jan.: a.m. 1 late Mar.: p.m. early Sept .: late June: late Dec .: 9 mid May: a.m. overhead overhead overhead Feb. 10 7 late July: p.m. at at at 1 mid Sept.: a.m. midnight midnight midnight late Nov .: p.m. early Jan.: a.m. 1 mid Mar.: p.m. late July: 9 early May: a.m. overhead July 10 7 mid July: p.m. mid May: p.m. at Jan. 30 partial in 2 late Aug.: a.m. early Aug.: a.m. midnight July 26 northern U.S. early Nov .: p.m. mid Dec.: a.m. late Feb.: p.m. 1 late Apr.: a.m. late Nov.: early Sept .: early Jan.: 9 late June : p.m. late Dec.: p.m. overhead overhead overhead Dec. 10 7 early Aug.: a.m. at at at 3 mid Oct.: p.m. midnight midnight midnight early Dec.: a.m. mid Feb.: p.m. 1 late Mar.: a.m. mid Oct.: late Jan.: 9 carly June: p.m. early Mar.: a.m. overhead overhead June 4 7 mid July: a.m. at Nov. 29 at 4 late Sept .: p.m. midnight midnight early Nov.: a.m. late Jan.: p.m. 1 early Mar.: a.m. early Nov .: early Feb .: 9 mid May: p.m. mid-late July: p.m. overhead overhead May 25 7 early July: a.m. early Oct.: a.m. at at Nov. 18 5 mid Sept.: p.m. midnight midnight late Oct.: a.m. mid Jan.: p.m. 1 late Feb.: a.m. late Jan.: early Dec .: late Feb.: 9 early May: p.m. overhead overhead overhead 7 mid June: a.m. at at at 6 late Aug.: p.m. midnight midnight midnight early Oct.: a.m. mid Dec.: p.m. early Feb.: a.m. 1 early Apr.: p.m. early Mar.: p.m. 9 late May: a.m. mid Apr.: a.m. Apr. 4 7 mid Aug.: p.m.

7 late Sept.: a.m.

TABLE 2 FAVORABILITY OF OBSERVING METEOR SHOWERS THE BEST TIME FOR VIEWING METEOR SHOWERS IS BETWEEN MIDNIGHT AND 6 A.M., IN PARTICULAR DURING THE HOUR DIRECTLY PRECEDING DAWN.

Quadrantids Jan. 3-5 Virgo	Lyrids Apr. 19-23 Lyra	Perseids July 27-Aug. 17 Perseus	Orionids Oct. 15-25 Orion	Leonids Nov. 14-18 Leo	Geminids Dec. 9-14 Gemini
Rises in the east around 2 a.m., upper eastern sky at 5 a.m.	Rises in the east around 10 p.m., western sky at 5 a.m.	Rises in the east at 10 p.m., towards the west at 5 a.m.	Rises in the east at midnight, di- rectly overhead at 5 a.m.	Rises in the east at 2 a.m., upper eastern sky at 5 a.m.	Rises in the east at 8 p.m., towards the far west at 5 a.m.
Good	Good	Through early August, after Aug. 10	Good	Good	Good 1 9 6 8
Poor	Good	Aug. 3-17	After Oct. 20	Good	Good 1 9 6 9
Good	Poor	July 27-Aug.11	Oct. 18-25	Poor	Poor 1 9 7 0
Good	Good	July 27-Aug. 2 Aug. 7-17	Good	Good	Good 1 9 7 1
Good	Good	Aug. 2-17	Oct. 15-20	Nov. 14-16	Good 1 9 7 2
Good	Apr. 21-23	July 27-Aug. 9	Good	Good	Poor 1 9 7 3
Poor	Good	Aug. 7-17	Good	Good	Good 1 9 7 4
Good	Good	Good	Oct. 21-25	Poor	Dec. 9-12 1 9 7 5
Good	Good	July 27-Aug. 5 Aug. 12-17	Good	Good	Good 1 9 7 6
Poor	Good	Aug. 3-17	Oct. 15-21	Good	Good 1 9 7

142 Experiment 2

EXPERIMENT 2 REGULARITY AND TIME

You will often encounter regularity in your study of science. Many natural events occur regularly—that is, over and over again at equal time intervals. But if you had no clock, how would you decide how regularly an event recurs? In fact, how can you decide how regular a clock is?

The first part of this exercise is intended merely to show you the regularity of a few natural events. In the second part, you will try to measure the regularity of an event against a standard and to decide what is really meant by the word "regularity."

Part A

You work with a partner in this part. Find several recurring events that you can time in the laboratory. You might use such events as a dripping faucet, a human pulse, or the beat of recorded music. Choose one of these events as a "standard event." All the others are to be compared to the standard by means of the strip chart recorder.

One lab partner marks each "tick" of the standard on one side of the strip chart recorder tape while the other lab partner marks each "tick" of the event being tested. After a long run has been taken, inspect the tape to see how the regularities of the two events compare. Run for about 300 ticks of the standard. For each 50 ticks of the standard, find on the tape the number of ticks of the other phenomenon, estimating to $\frac{1}{10}$ of a tick. Record your results in a table something like this:

STANDARD EVENT	TEST EVENT
First 50 ticks	ticks
Second 50 ticks	ticks
Third 50 ticks	ticks
Fourth 50 ticks	ticks

The test event's frequency is almost certain to be different from test to test. The difference could be a real difference in regularity, or it could come from your error in measuring.

Q1 If you think that the difference is larger than you would expect from human error, then which of the two events is *not* regular?

Part B

In this part of the lab, you will compare the regularity of some devices specifically designed to be regular. The standard here will be the time recording provided by the telephone company or Western Union. To measure two peri-





Experiment 2 143

ods of time, you will have to make three calls to the time station, for example, 7 P.M., 7 A.M., and 7 P.M. again. Agreement should be reached in class the day before on who will check wall clocks, who will check wristwatches, and so on. Watch your clock and wait for the recording to announce the exact hour. Tabulate your results something like this:

TIME STATION

Time	Period
"7 P.M. exactly"	12:00:00 h
"7 P.M. exactly"	12:00:00 h

ELECTRIC WALL CLOCK

7:	:	}	 :	:
7:	:	j)		
7:	:	}	 :	:

In Part I, you found that to test regularity you need a standard that is consistent, varying as little as possible. The standard is understood, *by definition*, to be regular.

Q2 What is the standard against which the time station signal is compared? Call to find out what this standard is. Try to find the final standard that is used to define regularity—the time standard against which all other recurring events are tested. How can we be sure of the regularity of *this* standard?

B.C By Johnny Hart







144 Experiment 3

EXPERIMENT 3 VARIATIONS IN DATA

If you count the number of chairs or people in an ordinary sized room, you will probably get exactly the right answer. But if you measure the length of this page with a ruler, your answer will have a small margin of uncertainty. That is, numbers read from measuring instruments do not give the exact measurements in the sense that one or two is exact when you count objects. Every measurement is to some extent uncertain.

Moreover, if your lab partner measures the length of this page, he will probably get a different answer from yours. Does this mean that the length of the page has changed? Hardly! Then can you possibly find the length of the page without any uncertainty in your measurement? This lab exercise is intended to show you why the answer is "no."

Various stations have been set up around the room, and at each one you are to make some measurement. Record each measurement in a table like the one shown here. When you have completed the series, write your measurements on the board along with those of your classmates. Some interesting patterns should emerge if your measurements have not been influenced by anyone else. Therefore, do not talk about your results or how you got them until everyone has finished.

TYPE OF		
MEASUREMENT	REMARKS	MEASUREMENT

Chapter 1 The Language of Motion

EXPERIMENT 4 MEASURING UNIFORM MOTION

If you roll a ball along a level floor or table, eventually it stops. Wasn't it slowing down all the time, from the moment you gave it a push? Can you think of any things that have uniform motion in which their speed remains constant and unchanging? Could the dry-ice disk pictured in Sec. 1–3 of the *Text* really be in uniform motion, even if the disk is called "frictionless"? Would the disk just move on forever? Doesn't everything eventually come to a stop?



In this experiment you check the answers to these questions for yourself. You observe very simple motion, like that pictured below, and make a photo record of it, or work with similar photos. You measure the speed of the object as precisely as you can and record your data in tables and draw graphs from these data. From the graphs you can decide whether the motion was uniform or not.

Your decision may be harder to make than you would expect, since your experimental measurements can never be exact. There are likely to be ups and downs in your final results. Your problem will be to decide whether the ups and downs are due partly to real changes in speed or due entirely to uncertainty in your measurements.

If the speed of your object turns out to be constant, does this mean that you have produced an example of uniform motion? Do you think it is possible to do so?

Doing the Experiment

Various setups for the experiment are shown on pages 145 and 146. It takes two people to photograph a disk sliding on a table, or a glider





Fig. 1–1. Stroboscopic photograph of a moving CO₂ disk.

on an air track, or a steadily flashing light (called a blinky) mounted on a small box which is pushed by a toy tractor. Your teacher will explain how to work with the set up you are using. Excellent photographs can be made of any of them.

If you do not use a camera at all, or if you work alone, then you may measure a transparency or a movie film projected on the chalk board or a large piece of paper.

Or you may simply work from a previously prepared photograph such as Fig. 1–1, above. If there is time, you might try several of these methods.

One setup uses for the moving object a disk made of metal or plastic. A few plastic beads sprinkled on a smooth, dust-free table top (or a sheet of glass) provide a surface for the disk to slide with almost no friction. Make sure the surface is quite level, too, so that the disk will not start to move once it is at rest.

Set up the Polaroid camera and the stroboscope equipment according to your teacher's instructions. Instructions for operating the Polaroid model 210, and a diagram for mounting this camera with a rotating disk stroboscope is shown on page 132. A ruler need not be included in your photograph as in the photograph above. Instead, you can use a magnifier with a scale that is more accurate than a ruler for measuring the photograph.

Either your teacher or a few trials will give you an idea of the camera settings and of the speed at which to launch the disk, so that the images of your disk are clear and wellspaced in the photograph. One student launches the disk while his companion operates the camera. A "dry run" or two without taking a picture will probably be needed for practice before you get a good picture. A good picture is one in which there are at least five sharp and clear images of your disk far enough apart for easy measuring on the photograph.



Fig. 1-2. Estimating to tenths of a scale division.

Making Measurements

Whatever method you have used, your next step is to measure the spaces between successive images of your moving object. For this, use a ruler with millimeter divisions and estimate the distances to the nearest tenth of a millimeter, as shown in Fig. 1–2 above. If you use a magnifier with a scale, rather than a ruler, you may be able to estimate these quite precisely. List each measurement in a table like Table 1.

Since the intervals of time between one image and the next are equal, you can use that interval as a unit of time for analyzing the event. If the speed is constant, the distances of travel would turn out to be all the same, and the motion would be uniform.

Q1 How would you recognize motion that is not uniform?

Q2 Why is it unnecessary for you to know the time interval in seconds?

TABLE 1			
TIME INTERVAL	DISTANCE TRAVELED IN EACH TIME INTERVAL		
1st	0.48 cm		
2nd	0.48		
3rd	0.48		
4th	0.48		
5th	0.48		
6th	0.48		

Table 1 has data that indicate uniform motion. Since the object traveled 0.48 cm during each time interval, the speed is 0.48 cm per unit time.

It is more likely that your measurements go up and down as in Table 2, particularly if you measure with a ruler.

TABLE 2				
	DISTANCE TRAVELED IN			
lst	0.48 cm			
2nd	0.46			
3rd	0.49			
4th	0.50			
5th	0.47			
6th	0.48			

Q3 Is the speed constant in this case? Since the distances are *not* all the same, you might well say, "No, it isn't." Or perhaps you looked again at a couple of the more extreme data in Table 2, such as 0.46 and 0.50 cm, checked these measurements, and found them doubtful. Then you might say, "The ups and downs are because it is difficult to measure to 0.01 cm with the ruler. The speed really *is* constant as nearly as I can tell." Which statement is right?

Look carefully at the divisions or marks on your ruler. Can you read your ruler accurately to the nearest 0.01 cm? If you are like most people, you read it to the nearest mark of 0.1 cm (the nearest whole millimeter) and *estimate* the next digit between the marks for the nearest tenth of a millimeter (0.01 cm), as illustrated in Fig. 1-2 at the left.

In the same way, whenever you read the divisions of any measuring device you should

read accurately to the nearest division or mark and then estimate the next digit in the measurement. Then probably your measurement, including your estimate of a digit between divisions, is not more than half of a division in error. It is not likely, for example, that in Fig. 1–2 on page 146 you would read more than half a millimeter away from where the edge being measured comes between the divisions. In this case, in which the divisions on the ruler are millimeters, you are at most no more than 0.5 mm (0.05 cm) in error.

Suppose you assume that the motion really is uniform and that the slight differences between distance measurements are due only to the uncertainty in reading the ruler. What is then the best estimate of the constant distance the object traveled between flashes?

Usually, to find the "best" value of distance you must average the values. The average for Table 2 is 0.48 cm, but the 8 is an uncertain measurement.

If the motion recorded in Table 2 really is uniform, the measurement of the distance traveled in each time interval is 0.48 cm plus or minus 0.05 cm, written as 0.48 ± 0.05 cm. The ±0.05 is called the *uncertainty* of your measurement. The uncertainty for a single measurement is commonly taken to be half a scale division. With many measurements, this uncertainty may be less, but you can use it to be on the safe side.

Now you can return to the big question: Is the speed constant or not? Because the numbers go up and down you might suppose that the speed is constantly *changing*. Notice though that in Table 2 the changes of data above and below the average value of 0.48 cm are always smaller than the uncertainty, 0.05 cm. Therefore, the ups and down *may* all be due to the difficulty in reading the ruler to better than 0.05 cm—and the speed may, in fact, be constant.

Our conclusion from the data given here is that the speed is constant to within the uncertainty of measurement, which is 0.05 cm per unit time. If the speed goes up or down by less than this amount, we simply cannot reliably detect it with our ruler.





150 Experiment 4

Study your own data in the same way. Q4 Do they lead you to the same conclusion? If your data vary as in Table 2, can you think of anything in your setup that could have been making the speed actually change? Even if you used a magnifier with a scale, do you still come to the same conclusion?

Measuring More Precisely

A more precise measuring instrument than a ruler or magnifier with a scale might show that the speed in our example was *not* constant. For example, if we used a measuring microscope whose divisions are 0.001 cm apart to measure the same picture again more precisely, we might arrive at the data in Table 3. Such precise measurement reduces the uncertainty greatly from ± 0.05 cm to ± 0.0005 cm.

	TABLE 3
TIME INTERVAL	DISTANCE TRAVELED IN EACH TIME INTERVAL
1st	0.4826 cm
2nd	0.4593
3rd	0.4911
4th	0.5032
5th	0.4684
6th	0.4779

Q5 Is the speed constant when we measure to such high precision as this?

The average of these numbers is 0.4804, and they are all presumably correct within half a division which is 0.0005 cm. Thus our best estimate of the true value is 0.4804 \pm 0.0005 cm.

Drawing a Graph

If you have read Sec. 1.5 in the *Text*, you have seen how speed data can be graphed. Your data provide an easy example to use in drawing a graph.

Just as in the example on *Text* page 19, lay off time intervals along the horizontal axis of the graph. Your units are probably not seconds; they are "blinks" if you used a stroboscope or simply "arbitrary time units" which mean here the equal time intervals between positions of the moving object.

Then lay off the *total* distances traveled along the vertical axis. The beginning of each scale is in the lower left-hand corner of the graph.

Choose the spacing of your scale division so that your data will, if possible, spread across most of the graph paper.

The data of Table 2 on page 147 are plotted as an example on the graph of the sample write up of Experiment 4 on pages 148 and 149. Q6 In what way does the graph on page 149 show uniform motion? Does your own graph show uniform motion too?

If the motion in your experiment was not uniform, review Sec. 1.9 of the *Text*. Then from your graph find the average speed of your object over the whole trip.

Q7 Is the average speed for the whole trip the same as the average of the speeds between successive measurements?

Additional Questions

Q8 Could you use the same methods you used in this experiment to measure the speed of a bicycle? a car? a person running? (Assume they are moving uniformly.)

Q9 The divisions on the speedometer scale of many cars are 5 mi/hr in size. You can estimate the reading to the nearest 1 mi/hr.

(a) What is the uncertainty in a speed measurement by such a speedometer?(b) Could you measure reliably speed changes as small as 2 mi/hr? 1 mi/hr?0.5 mi/hr? 0.3 mi/hr?

ACTIVITIES

USING THE ELECTRONIC STROBOSCOPE

Examine some moving objects illuminated by an electronic stroboscope. Put a piece of tape on a fan blade or mark it with chalk and watch the pattern as you turn the fan on and off. How can you tell when there is exactly one flash of light for each rotation of the fan blade?

Observe a stream of water from a faucet, objects tossed into the air, or the needle of a running sewing machine. If you can darken the room completely, try catching a thrown ball lighted only by a stroboscope. How many flashes do you need during the flight of the ball to be able to catch it reliably?

MAKING FRICTIONLESS PUCKS

Method 1. Use a flat piece of dry ice on a very smooth surface, like glass or Formica. When you push the piece of dry ice (frozen carbon dioxide), it moves in a frictionless manner because as the carbon dioxide changes to a vapor it creates a layer of CO_2 gas between the solid and the glass. (CAUTION: Don't touch dry ice with your bare hands; it can give you a severe frost bite!)

Method 2. Make a balloon puck if your lab does not have a supply. First cut a 4-inch diameter disk of 1-inch-thick Masonite. Drill a $\frac{1}{2}$ " diameter hole part way through the center of the disk so it will hold a rubber stopper. Then drill a $\frac{1}{32}$ " diameter hole on the same center the rest of the way through the disk. Drill a $\frac{1}{16}$ " hole through the center of a stopper in the hole in the masonite disk. Place the disk on glass or Formica.

Method 3. Make a pressure pump puck. Make a disk as described in Method 2. Instead of using a balloon, attach a piece of flexible tubing, attached at the other end to the exhaust of a vacuum pump as shown in the diagram. Run the tubing over an overhead support so



it does not interfere with the motion of the puck.

Method 4. Drill a $\frac{1}{32''}$ hole in the bottom of a smooth-bottomed cylindrical can, such as one for a typewriter ribbon. Break up dry ice (DON'T touch it with bare hands) and place the pieces inside the can. Seal the can with tape, and place it on a very smooth surface.



Chapter **2** Free Fall—Galileo Describes Motion



Accelerated motion goes on all around you every day. You experience many accelerations yourself, although not always as exciting as those shown in the photographs. What accelerations have you experienced today?

When you get up from a chair, or start to



walk from a standstill, hundreds of sensations are gathered from all over your body in your brain, and you are aware of these normal accelerations. Taking off in a jet or riding on an express elevator, you experience much sharper accelerations. Often this feeling is in the pit of your stomach. These are very complex motions.

Note how stripped down and simple the accelerations are in the following experiments, film loops, activities. As you do these, you will learn to measure accelerations in a variety of ways, both old and new, and become more familiar with the fundamentals of acceleration.

If you do either of the first two experiments of this chapter, that is, numbers 5 and 6, you will try to find, as Galileo did, whether d/t^2 is a constant for motion down an inclined plane. The remaining experiments are measurements of the value of the acceleration due to gravity which is represented by the symbol a_g .

EXPERIMENT 5 A SEVENTEENTH-CENTURY EXPERIMENT

This experiment is similar to the one discussed by Galileo in the *Two New Sciences*. It will give you firsthand experience in working with tools similar to those of a seventeenth-century scientist. You will make quantitative measurements of the motion of a ball rolling down an incline, as described by Galileo.

From these measurements you should be able to decide for yourself whether Galileo's definition of acceleration was appropriate or not. Then you should be able to tell whether it was Aristotle or Galileo who was correct about his thinking concerning the acceleration of objects of different sizes.

Reasoning Behind the Experiment

You have read in Sec. 2.6 of the *Text* how Galileo expressed his belief that the speed of freefalling objects increases in proportion to the time of fall—in other words, that they accelerate uniformly. But since free fall was much too rapid to measure, he assumed that the speed of a ball rolling down an incline increased in the same way as an object in free fall did, only more slowly. But even a ball rolling down a low incline still moved too fast to measure the speed for different parts of the descent accurately. So Galileo worked out the relationship $d \propto t^2$ (or $d/t^2 = \text{constant}$), an expression in which speed differences have been replaced by the *total time* t and *total* distance d rolled by the ball. Both these quantities can be measured.

Be sure to study *Text* Sec. 2.7 in which the derivation of this relationship is described. If Galileo's original assumptions were true, this relationship would hold for both freely falling objects and rolling balls. Since total distance and total time are not difficult to measure, seventeenth-century scientists now had a secondary hypothesis they could test by experiment. And so have you. Sec. 2.8 of the *Text* discusses much of this.

Apparatus

The apparatus that you will use is shown in Fig. 2–1 below. It is similar to that described by Galileo.

You will let a ball roll various distances down a channel about six feet long and time the motion with a water clock.

You use a water clock to time this experi-



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ment because that was the best timing device available in Galileo's time. The way your own clock works is very simple. Since the volume of water is proportional to the time of flow, you can measure time in milliliters of water. Start and stop the flow with your fingers over the upper end of the tube inside the funnel. Whenever you refill the clock, let a little water run through the tube to clear out the bubbles.

Compare your water clock with a stop watch when the clock is full and when it is nearly empty to determine how accurate it is. Q1 Does the clock's timing change? If so, by how much?

It is almost impossible to release the ball with your fingers without giving it a slight push or pull. Therefore, dam the ball up, with a ruler or pencil, and release it by quickly moving this dam away from it down the inclined plane. The end of the run is best marked by the sound of the ball hitting the stopping block.

Brief Comment on Recording Data

A good example of a way to record your data appears on page 154. We should emphasize again the need for neat, orderly work. Orderly work looks better and is more pleasing to you and everyone else. It may also save you from extra work and confusion. If you have an organized table of data, you can easily record and find your data. This will leave you free to think about your experiment or calculations rather than having to worry about which of two numbers on a scrap of paper is the one you want, or whether you made a certain measurement or not. A few minutes' preparation before you start work will often save you an hour or two of checking in books and with friends.

Operating Suggestions

You should measure times of descent for several different distances, keeping the inclination of the plane constant and using the same ball. Repeat each descent about four times, and average your results. Best results are found for very small angles of inclination (the top of the channel raised less than 30 cm). At greater inclinations, the ball tends to slide as well as to roll.

From Data to Calculations

Galileo's definition of uniform acceleration (*Text*, page 49) was "equal increases in speed in equal times." Galileo showed that if an object actually moved in this way, the total distance of travel should be directly proportional to the square of the total time of fall, or $d \propto t^2$.

Q2 Show how this follows from Galileo's definition. (See Sec. 2.7 in the Text if you cannot answer this.)

If two quantities are proportional, a graph of one plotted against the other will be a straight line. Thus, making a graph is a good way to check whether two quantities are proportional. Make a graph of d plotted against t^2 .

Q3 Does your graph support the hypothesis? How accurate is the water clock you have been using to time this experiment?

If you have not already done so, check your water clock against a stopwatch or, better yet, repeat several trials of your experiment using a stopwatch for timing.

Q4 How many seconds is one milliliter of time for your water clock? Can the inaccuracy of your water clock explain the conclusion you arrived at in Q2 above?

Going Further

1. In Sec. 2.7 of the *Text* you learned that $a = 2d/t^2$. Use this relation to calculate the actual acceleration of the ball in one of your runs.

2. If you have time, go on to see whether Galileo or Aristotle was right about the acceleration of objects of various sizes. Measure d/t^2 for several different sizes of balls, all rolling the same distance down a plane of the same inclination.

Q5 Does the acceleration depend on the size of the ball? In what way does your answer refute or support Aristotle's ideas on falling bodies.

Q6 Galileo claimed his results were accurate to $\frac{1}{10}$ of a pulse beat. Do you believe his results were that accurate? Did you do that well? How could you improve the design of the water clock to increase its accuracy?

EXPERIMENT 6 TWENTIETH-CENTURY VERSION OF GALILEO'S EXPERIMENT

Galileo's seventeenth-century experiment had its limitations, as you read in the Text, Sec. 2.9. The measurement of time with a water clock was imprecise and the extrapolation from acceleration at a small angle of inclination to that at a verticle angle (90°) was extreme.

With more modern equipment you can verify Galileo's conclusions; further, you can get an actual value for acceleration in free fall (near the earth's surface). But remember that the idea behind the improved experiment is still Galileo's. More precise measurements do not always lead to more significant conclusions.

Determine a_g as carefully as you can. This is a fundamental measured value in modern science. It is used in many ways—from the determination of the shape of the earth and the location of oil fields deep in the earth's crust to the calculation of the orbits of earth satellites and spacecrafts in today's important space research programs.

Apparatus and Procedure

For an inclined plane use the air track. For timing the air track glider use a stopwatch instead of the water clock. Otherwise the procedure is the same as that used in Experiment 5. As you go to higher inclinations you should stop the glider by hand before it is damaged by hitting the stopping block.

Instead of a stopwatch, you may wish to use the Polaroid camera to make a strobe photo of the glider as it descends. A piece of white tape on the glider will show up well in the photograph. Or you can attach a small light source to the glider. You can use a magnifier with a scale attached to measure the glider's motion recorded on the photograph. Here the values of d will be millimeters on the photograph and t will be measured in an arbitrary unit, the "blink" of the stroboscope, or the "slot" of the strobe disk.

Plot your data as before on a graph of d vs. t^2 . Compare your plotted lines with graphs of the preceding cruder seventeenth-century experiment, if they are available. Explain the differences between them.

Q1 Is d/t^2 constant for an air track glider?

Q2 What is the significance of your answer to the question above?

As further challenge, if time permits, try to predict the value of a_{g} , which the glider approaches as the air track becomes vertical. To do this, of course, you must express d and t in familiar units such as meters or feet, and seconds. The accepted value of a_g is 9.8 m/sec² or 32 ft/sec² near the earth's surface.

Q3 What is the percentage error in your calculated value? That is, what percent is your error of the accepted value?

Percentage error

 $=\frac{\text{accepted value} - \text{calculated value}}{\times 100}$

so that if your value of a_g is 30 ft/sec² your percentage error

 $=\frac{32 \text{ ft/sec}^2 - 30 \text{ ft/sec}^2}{32 \text{ ft/sec}^2} \times 100$ $=\frac{2}{32} \times 100 = 6\%$

а

Notice that you *cannot* carry this 6% out to 6.25% because you only know the 2 in the fraction $\frac{2}{32}$ to one digit. Hence, you can only know one digit in the answer, 6%. A calculated value like this is said to have one significant digit. You cannot know the second digit in the answer until you know the digit following the 2. To be significant, this digit would require a third digit in the calculated values of 30 and 32.

Q4 What are some of the sources of your error?

EXPERIMENT 7 MEASURING THE ACCELERATION OF GRAVITY a_g

Aristotle's idea that falling bodies on earth are seeking out their natural places sounds strange to us today. After all, we know the answer: It's gravity that makes things fall.

But just what is gravity? Newton tried to give operational meaning to the idea of gravity by seeking out the laws according to which it acts. Bodies near the earth fall toward it with a certain acceleration due to the gravitational "attraction" of the earth. But how can the earth make a body at a distance fall toward it? How is the gravitational force transmitted? Has the acceleration due to gravity always remained the same? These and many other questions about gravity have yet to be answered satisfactorily.

Whether you do one or several parts of this experiment, you will become more familiar with the effects of gravity—you find the acceleration of bodies in free fall yourself and you will learn more about gravity in later chapters.

Part A: a_g by Direct Fall*

In this experiment you measure the acceleration of a falling object. Since the distance and hence the speed of fall is too small for air resistance to become important, and since other sources of friction are very small, the acceleration of the falling weight is very nearly a_g .

Doing the Experiment

The falling object is an ordinary laboratory hooked weight of at least 200 g mass. (The drag on the paper strip has too great an effect on the fall of lighter weights.) The weight is suspended from about a meter of paper tape as shown in the photograph. Reinforce the tape by doubling a strip of masking tape over one end and punch a hole in the reinforcement one centimeter from the end. With careful handling, this can support at least a kilogram weight.



When the suspended weight is allowed to fall, a vibrating tuning fork will mark equal time intervals on the tape pulled down after the weight.

The tuning fork must have a frequency between about 100 vibrations/sec and about 400 vibrations/sec. In order to mark the tape, the fork must have a tiny felt cone (cut from a marking pen tip) glued to the side of one of its prongs close to the end. Such a small mass affects the fork frequency by much less than 1 vibration/sec. Saturate this felt tip with a drop or two of marking pen ink, set the fork in vibration, and hold the tip very gently against the tape. The falling tape is most conveniently guided in its fall by two thumbtacks in the edge of the table. The easiest procedure is to have an assistant hold the weighted tape straight up until you have touched the vibrating tip against it and said "Go." After a few practice runs, you will become expert enough to mark several feet of tape with a wavy line as the tape is accelerated past the stationary vibrating fork.

Instead of using the inked cone, you may press a corner of the vibrating tuning fork

^{*}Adapted from R. F. Brinckerhoff and D. S. Taft, *Modern Laboratory Experiments in Physics*, by permission of Science Electronics, Inc., Nashua, New Hampshire.

gently against a 1-inch square of carbon paper which the thumbtacks hold ink surface inwards over the falling tape. With some practice, this method can be made to yield a series of dots on the tape without seriously retarding its fall.

Analyzing Your Tapes

Label with an A one of the first wave crests (or dots) that is clearly formed near the beginning of the pattern. Count 10 intervals between wave crests (or dots), and mark the end of the tenth space with a **B**. Continue marking every tenth crest with a letter throughout the length of the record, which ought to be at least 40 waves long.

At **A**, the tape already had a speed of v_0 . From this point to **B**, the tape moved in a time t, a distance we shall call d_1 . The distance d_1 is described by the equation of free fall:

$$d_1 = v_0 t + \frac{a_g t^2}{2}$$

In covering the distance from **A** to **C**, the tape took a time exactly twice as long, 2t, and fell a distance d_2 described (on substituting 2t for t and simplifying) by the equation:

$$d_2 = 2\upsilon_0 t + \frac{4a_g t^2}{2}$$

In the same way the distances **AB**, **AE**, etc., are described by the equations:

$$d_3 = 3v_0t + \frac{9a_gt^2}{2}$$
$$d_4 = 4v_0t + \frac{16a_gt^2}{2}$$

and so on.

All of these distances are measured from A, the arbitrary starting point. To find the distances fallen in each 10-crest interval, you must subtract each equation from the one before it, getting:

$$\mathbf{AB} = v_0 t + \frac{a_g t^2}{2}$$

$$BC = v_0 t + \frac{3a_y t^2}{2}$$

$$CD = v_0 t + \frac{5a_y t^2}{2}$$
and
$$DE = v_0 t + \frac{7a_y t^2}{2}$$

2

From these equations you can see that the weight falls farther during each time interval. Moreover, when you subtract each of these distances, **AB**, **BC**, **CD**, . . . from the subsequent distance, you find that the *increase* in distance fallen is a constant. That is, each difference **BC** – **AB** = **CD** – **BC** = **DE** – **CD** = $a_g t^2$. This quantity is the increase in the distance fallen in each successive 10-wave interval and hence is an acceleration. Our formula shows that a body falls with a constant acceleration.

From your measurements of AB, AC, AD, etc., make a column of AB, BC, CD, ED, etc., and in the next column record the resulting values of a_gt^2 . The values of a_gt^2 should all be equal (within the accuracy of your measurements). Why? Make all your measurements as precisely as you can with the equipment you are using.

Find the average of all your values of $a_g t^2$, the acceleration in centimeters/(10-wave interval)². You want to find the acceleration in cm/sec². If you call the frequency of the tuning fork *n* per second, then the length of the time interval *t* is 10/n seconds. Replacing *t* of 10 waves by 10/n seconds gives you the acceleration, a_g in cm/sec².

The ideal value of a_g is close to 9.8 m/sec², but a small force of friction impeding a falling object is sufficient to reduce the observed value by several percent.

Q1 What errors would be introduced by using a tuning fork whose vibrations are slower than about 100 vibrations per second? higher than about 400 vibrations per second?

Part B: a_g from a Pendulum

You can easily measure the acceleration due to gravity by timing the swinging of a pendulum.

Of course the pendulum is not falling straight down, but the time it takes for a round-trip swing still depends on a_g . The time T it takes for a round-trip swing is

$$T = 2\pi \sqrt{\frac{l}{a_g}}$$

In this formula l is the length of the pendulum. If you measure l with a ruler and T with a clock, you should be able to solve for a_g .

You may learn in a later physics course how to derive the formula. Scientists often use formulas they have not derived themselves, as long as they are confident of their validity.

Making the Measurements

The formula is derived for a pendulum with all the mass concentrated in the weight at the bottom, called the bob. Hence the best pendulum to use is one whose bob is a metal sphere hung by a fine thread. In this case you can be sure that almost all the mass is in the bob. The pendulum's length, l, is the distance from the point of suspension to the *center* of the bob.

Your suspension thread can have any convenient length. Measure l as accurately as possible, either in feet or meters.

Set the pendulum swinging with *small* swings. The formula doesn't work well for large swings, as you can test for yourself later.

Time at least 20 complete round trips, preferably more. By timing many round trips instead of just one you make the error in starting and stopping the clock a smaller fraction of the total time being measured. (When you divide by 20 to get the time for a single round trip, the error in the calculated value for one will be only $\frac{1}{20}$ as large as if you had measured only one.)

Divide the total time by the number of swings to find the time *T* of one swing.

Repeat the measurement at least once as a check.

Finally, substitute your measured quantities into the formula and solve it for a_g .

If you measured *l* in meters, the accepted value of a_g is 9.80 meters/sec².

If you measured *l* in feet, the accepted value of a_g is 32.1 ft/sec².

Finding Errors

You probably did not get the accepted value. Find your *percentage error* by dividing your error by the accepted value and multiplying by 100:

Percentage error

$$= \frac{\text{accepted value} - \text{your value}}{\text{accepted value}} \times 100$$

$$= \frac{\text{your error}}{\text{accepted value}} \times 100$$

With care, your value of a_g should agree within about 1%.

Which of your measurements do you think was the least accurate?

If you believe it was your measurement of length and you think you might be off by as much as 0.5 cm, change your value of l by 0.5 cm and calculate once more the value of a_g . Has a_g changed enough to account for your error? (If a_g went up and your value of a_g was already too high, then you should have altered your measured l in the opposite direction. Try again!)

If your possible error in measuring is not enough to explain your difference in a_g try changing your *total* time by a few tenths of a second—a possible error in timing. Then you must recalculate T and hence a_g .

If neither of these attempts work (nor both taken together in the appropriate direction) then you almost certainly have made an error in arithmetic or in reading your measuring instruments. It is most unlikely that a_g in your school differs from the accepted value by more than one unit in the third digit.

Q2 How does the length of the pendulum affect your value of T? of a_q ?

Q3 How long is a pendulum for which T = 2 seconds? This is a useful timekeeper.

Part C: a_g with Slow-Motion Photography (Film Loop)

With a high speed movie camera you could photograph an object falling along the edge of a vertical measuring stick. Then you could determine a_g by projecting the film at standard speed and measuring the time for the object to fall specified distance intervals.

A somewhat similar method is used in *Film Loops 1* and 2. Detailed directions are given for their use in the Film Loop notes on pages 164–165.

Part D: a_g from Falling Water **Drops**

You can measure the acceleration due to gravity a_g simply with drops of water falling on a pie plate.

Put the pie plate or a metal dish or tray on the floor and set up a glass tube with a stopcock, valve, or spigot so that drops of water from the valve will fall at least a meter to the plate. Support the plate on three or four pencils to hear each drop distinctly, like a drum beat.

Adjust the valve carefully until one drop



strikes the plate at the same instant the next drop from the valve begins to fall. You can do this most easily by *watching* the drops on the valve while listening for the drops hitting the plate. When you have exactly set the valve, the time it takes a drop to fall to the plate is equal to the time interval between one drop and the next.

With the drip rate adjusted, now find the time interval t between drops. For greater accuracy, you may want to count the number of drops that fall in half a minute or a minute, or to time the number of seconds for 50 to 100 drops to fall.

Your results are likely to be more accurate if you run a number of trials, adjusting drip rate each time, and average your counts of drops or seconds. The average of several trials should be closer to actual drip rate, drop count, and time intervals than one trial would be.

Now you have all the data you need. You know the time *t* it takes a drop to fall a distance *d* from rest. From these you can calculate a_g , since you know that $d = \frac{1}{2}a_g t^2$ for objects falling from rest. Rewrite this relationship in the form $a_g = \ldots$

Q4 When you have calculated a_g by this method, what is your percentage error? How does this compare with your percentage error by any other methods you have used? What do you think led to your error? Could it be leaking connections, allowing more water to escape sometimes? How would this affect your answer?

Distance of fall lessened by a puddle forming in the plate: How would this change your results?

Less pressure of water in the tube after a period of dripping: Would this increase or decrease the rate of dripping? Do you get the same counts when you refill the tube after each trial?

Would the starting and stopping of your counting against the watch or clock affect your answer? What other things may have shown up in your error?

Can you adapt this method of measuring the acceleration of gravity so that you can do it at home? Would it work in the kitchen sink? or if the water fell a greater distance, such as down a stairwell?

Part E: a_g with Falling Ball and Turntable

You can measure a_g with a record-player turntable, a ring stand and clamp, carbon paper, two balls with holes in them, and thin thread.

Ball X and ball Y are draped across the prongs of the clamp. Line up the balls along a radius of the turntable, and make the lower ball hang just above the paper.

With the table turning, the thread is burned and each ball, as it hits the carbon paper, will leave a mark on the paper under it.

Measure the vertical distance between the balls and the angular distance between the marks. With these measurements and the speed of the turntable, determine the freefall time. Calculate your percentage error and suggest its probable source.



Part F: a_g with Strobe Photography

Photographing a falling light source with the Polaroid Land camera provides a record that can be graphed and analyzed to give an average value of a_g . The 12-slot strobe disk gives a very accurate 60 slots per second. (Or, a neon bulb can be connected to the ac line outlet in such a way that it will flash a precise 60 times per second, as determined by the line frequency. Your teacher has a description of the approximate circuit for doing this.)



ACTIVITIES

WHEN IS AIR RESISTANCE IMPORTANT?

By taking strobe photos of various falling objects, you can find when air resistance begins to play an important role. You can find the actual value of the terminal speed for less dense objects such as a Ping-Pong or styrofoam ball by dropping them from greater and greater heights until the measured speeds do not change with further increases in height. (A Ping-Pong ball achieves terminal speed within 2 m.) Similarly, ball bearings and marbles can be dropped in containers of liquid shampoo or cooking oil to determine factors affecting terminal speed in a liquid as shown in the adjoining photograph.





A magnet is a handy aid in raising the steel ball to the top of the container.

MEASURING YOUR REACTION TIME

Your knowledge of physics can help you calculate your reaction time. Have someone hold the top of a wooden ruler while you space your thumb and forefinger around the bottom (zero) end of the ruler. As soon as the other person releases the ruler, you catch it. You can compute your reaction time from the relation

$$d = \frac{1}{2}a_g t^2$$

by solving for t. Compare your reaction time with that of other people, both older and younger than yourself. Also try it under different conditions-lighting, state of fatigue, distracting noise, etc. Time can be saved by computing dfor $\frac{1}{10}$ sec or shorter intervals, and then taping reaction-time marks on the ruler.

A challenge is to try this with a one-dollar bill, telling the other person that he can have the dollar if he can catch it.

FALLING WEIGHTS

This demonstration shows that the time it takes a body to fall is proportional to the square root of the vertical distance $(d \propto t^2)$. Suspend a string, down a stairwell or out of a window, on which metal weights are attached at the following heights above the ground: 3", 1', 2'3", 4', 6'3", 9', 12'3", 16'. Place a metal tray or ashcan cover under the string and then drop or cut the string at the point of suspension. The weights will strike the tray at equal intervals of time—about $\frac{1}{8}$ second.

Compare this result with that obtained using a string on which the weights are suspended at equal distance intervals.

EXTRAPOLATION

Many arguments regarding private and public policies depend on how people choose to extrapolate from data they have gathered. From magazines, make a report on the problems of extrapolating in various cases. For example:

- 1. The population explosion
- 2. The number of students in your high school ten years from now
- 3. The number of people who will die in traffic accidents over next holiday weekend
- 4. The number of lung cancer cases that will occur next year among cigarette smokers
- 5. How many gallons of punch you should order for your school's Junior prom

To become more proficient in making statistics support your pet theory-and more cautious about common mistakes-read How to Lie with Statistics by Darrell Huff, published by W. W. Norton and Company.

FILM LOOP 1 ACCELERATION DUE TO GRAVITY – I

A bowling ball in free fall was filmed in real time and in slow motion. Using the slowmotion sequence, you can measure the acceleration of the ball due to gravity. This film was exposed at 3900 frames/sec and is projected at about 18 frames/sec; therefore, the slow-motion factor is 3900/18, or about 217. However, your projector may not run at exactly 18 frames/sec. To calibrate your projector, time the length of the entire film which contains 3331 frames. (Use the yellow circle as the zero frame.)

To find the acceleration of the falling body using the definition

$$acceleration = \frac{change in speed}{time interval}$$

you need to know the instantaneous speed at two different times. You cannot directly measure instantaneous speed from the film, but you can determine the average speed during small intervals. Suppose the speed increases steadily, as it does for freely falling bodies. During the first half of any time interval, the instantaneous speed is less than the average speed; during the second half of the interval, the speed is greater than average. Therefore, for uniformly accelerated motion, the average speed v_{av} for the interval is the same as the instantaneous speed at the mid-time of the interval.

If you find the instantaneous speed at the midtimes of each of two intervals, you can calculate the acceleration a from

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

where v_1 and v_2 are the average speeds during

the two intervals, and where t_1 and t_2 are the midtimes of these intervals.

Two intervals 0.5 meter in length are shown in the film. The ball falls 1 meter before reaching the first marked interval, so it has some initial speed when it crosses the first line. Using a watch with a sweep second hand, time the ball's motion and record the times at which the ball crosses each of the four lines. You can make measurements using either the bottom edge of the ball or the top edge. With this information, you can determine the time (in apparent seconds) between the midtimes of the two intervals and the time required for the ball to move through each $\frac{1}{2}$ -meter interval. Repeat these measurements at least once and then find the average times. Use the slowmotion factor to convert these times to real seconds; then, calculate the two values of v_{av} . Finally, calculate the acceleration a.

This film was made in Montreal, Canada, where the acceleration due to gravity, rounded off to $\pm 1\%$, is 9.8 m/sec². Try to decide from the internal consistency of your data (the repeatability of your time measurements) how precisely you should write your result.



FILM LOOP 2 ACCELERATION DUE TO GRAVITY –II

A bowling ball in free fall was filmed in slow motion. The film was exposed at 3415 frames/ sec and it is projected at about 18 frames/ sec. You can calibrate your projector by timing the length of the entire film, 3753 frames. (Use the yellow circle as a reference mark.)

If the ball starts from rest and steadily acquires a speed v after falling through a distance d, the change in speed Δv is v - 0, or v, and the average speed is $v_{av} = \frac{0+v}{2} = \frac{1}{2}v$. The time required to fall this distance is given by

$$\Delta t = \frac{d}{v_{av}} = \frac{d}{\frac{1}{2}v} = \frac{2d}{v}$$

The acceleration a is given by

$$a = \frac{\text{change of speed}}{\text{time interval}} = \frac{\Delta v}{\Delta t} = \frac{v}{2d/v} = \frac{v^2}{2d}$$

Thus, if you know the instantaneous speed v of the falling body at a distance d below the starting point, you can find the acceleration. Of course you cannot directly measure the

instantaneous speed but only average speed over the interval. For a small interval, however, you can make the approximation that the average speed is the instantaneous speed at the midpoint of the interval. (The average speed is the instantaneous speed at the mid*time*, not the mid*point*; but the error is small if you use a short enough interval.)

In the film, small intervals of 20 cm are centered on positions 1m, 2m, 3m, and 4m below the starting point. Determine four average speeds by timing the ball's motion across the 20 cm intervals. Repeat the measurements several times and average out errors of measurement. Convert your measured times into real times using the slow-motion factor. Compute the speeds, in m/sec, and then compute the value of $v^2/2d$ for each value of d.

Make a table of calculated values of a, in order of increasing values of d. Is there any evidence for a systematic trend in the values? Would you expect any such trend? State the results by giving an average value of the acceleration and an estimate of the possible error. This error estimate is a matter of judgment based on the consistency of your four measured values of the acceleration.

B.C. by John Hart



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Chapter **3** The Birth of Dynamics—Newton Explains Motion

EXPERIMENT 8 NEWTON'S SECOND LAW

Newton's second law of motion is one of the most important and useful laws of physics. Review *Text* Sec. 3.7 on Newton's second law to make sure you are familiar with it.

Newton's second law is part of a much larger body of theory than can be proved by any simple set of laboratory experiments. Our experiment on the second law has two purposes.

First, because the law *is* so important, it is useful to get a feeling for the behavior of objects in terms of force (F), mass (m), and acceleration (a). You do this in the first part of the experiment.

Second, the experiment permits you to consider the uncertainties of your measurements. This is the purpose of the latter part of the experiment.

You will apply different forces to carts of different masses and measure the acceleration.



Fig. 3–1 How the Apparatus Works

You are about to find the mass of a loaded cart on which you then exert a measurable force. From Newton's second law you can predict the resulting acceleration of the loaded cart.

Arrange the apparatus as shown in Fig. 3-1. A spring scale is firmly taped to a dynamics cart. The cart, carrying a blinky, is pulled along by a cord attached to the hook of the spring scale. The scale therefore measures the force exerted on the cart.

The cord runs over a pulley at the edge of the lab table and from its end hangs a weight.



(Fig. 3-2.) The hanging weight can be changed so as to produce various tensions in the cord and hence various accelerating forces on the cart.

Now You Are Ready to Go

Measure the total mass of the cart, the blinky, the spring scale, and any other weights you may want to include with it to vary the mass. This is the mass m being accelerated.

Release the cart and allow it to accelerate. Repeat the motion several times while watching the spring-scale pointer. You may notice that the pointer has a range of positions. The midpoint of this range is a fairly good measurement of the average force F_{av} producing the acceleration.

Record F_{av} in newtons.

Our faith in Newton's law is such that we assume the acceleration is the same and is constant every time this particular F_{av} acts on the mass m.

Use Newton's law to predict what the average acceleration a_{av} was during the run.

Then find a directly to see how accurate your prediction was.

To measure the average acceleration a_{av} take a Polaroid photograph through a rotating disk stroboscope of a light source mounted on the cart. As alternatives you might use a liquid surface accelerometer described in detail on page 170, or a blinky. Analyze your results just as in the experiments on uniform and accelerated motion 4, 5, and 6 to find a_{av} .

This time, however, you must know the distance traveled in meters and the time interval in seconds, not just in blinks, flashes or other arbitrary time units.

Q1 Does F_{av} (as measured) equal ma_{av} (as computed from measured values)?

You may wish to observe the following effects without actually making numerical measurements.

1. Keep the mass of the cart constant and observe how various forces affect the acceleration.

2. Keep the force constant and observe how various masses of the cart affect the acceleration.

Q2 Do your observations support Newton's second law? Explain.

Experimental Errors

It is unlikely that your values of F_{av} and ma_{av} were equal.

Does this mean that you have done a poor job of taking data? Not necessarily. As you think about it, you will see that there are at least two other possible reasons for the inequality. One may be that you have not yet measured everything necessary in order to get an accurate value for each of your three quantities.

In particular, the force used in the calculation ought to be the net, or resultant, force on the cart—not just the towing force that you measured. Friction force also acts on your cart, opposing the accelerating force. You can measure it by reading the spring scale as you tow the cart by hand at *constant speed*. Do it several times and take an average, F_f . Since F_f acts in a direction opposite to the towing force F_T ,

$$F_{net} = F_T - F_{f_1}$$

If F_f is too small to measure, then $F_{net} = F_T$, which is simply the towing force that you wrote as F_{av} in the beginning of the experiment.

Another reason for the inequality of F_{av} and m_{av} may be that your value for each of these quantities is based on *measurements* and every measurement is uncertain to some extent.

You need to estimate the uncertainty of each of your measurements.

Uncertainty in average force F_{av} Your uncertainty in the measurement of F_{av} is the amount by which your reading of your spring scale varied above and below the average force, F_{av} . Thus if your scale reading ranged from 1.0 to 1.4N the average is 1.2N, and the range of uncertainty is 0.2N. The value of F_{av} would be reported as 1.2 ± 0.2 N.

Q3 What is *your* value of F_{av} and its uncertainty?

Uncertainty in mass m Your uncertainty in m is roughly half the smallest scale reading of

the balance with which you measured it. The mass consisted of a cart, a blinky, and a spring scale (and possibly an additional mass). If the smallest scale reading is 0.1 kg, your record of the mass of each of these in kilograms might be as follows:

$$m \text{ cart} = 0.90 \pm 0.05 \text{ kg}$$

 $m \text{ blinky} = 0.30 \pm 0.05 \text{ kg}$
 $m \text{ scale} = 0.10 \pm 0.05 \text{ kg}$

The total mass being accelerated is the sum of these masses. The uncertainty in the total mass is the sum of the three uncertainties. Thus, in our example, $m = 1.30 \pm 0.15$ kg. Q4 What is *your* value of *m* and its uncertainty?

Uncertainty in average acceleration a_{av} Finally, consider a_{av} . You found this by measuring $\Delta d/\Delta t$ for each of the intervals between the points on your blinky photograph.



Fig. 3-3

Suppose the points in Fig. 3-3 represent images of a light source photographed through a single slot—giving 5 images per second. Calculate $\Delta d/\Delta t$ for several intervals.

If you assume the time between blinks to have been measured very accurately, the uncertainty in each value of $\Delta d/\Delta t$ is due primarily to the fact that the photographic images are a bit fuzzy. Suppose that the uncertainty in locating the distance between the centers of the dots is 0.1 cm as shown in the first column of the Table below.

Average speeds	Average accelerations
$\Delta d_1/\Delta t = 2.5 \pm 0.1 \text{ cm/sec}$	$\Delta y_1/\Delta t = 0.9 \pm 0.2 \text{ cm/sec}^2$
$\Delta d_2/\Delta t = 3.4 \pm 0.1 \text{ cm/sec}$	$\Delta y_2/\Delta t = 0.6 \pm 0.2 \text{ cm/sec}^2$
$\Delta d_3/\Delta t = 4.0 \pm 0.1 \text{ cm/sec}$	$\Delta y_3/\Delta t = 0.8 \pm 0.2 \text{ cm/sec}^2$
$\Delta d_4/\Delta t = 4.8 \pm 0.1 \text{ cm/sec}$	Average = $0.8 \pm 0.2 \text{ cm/sec}^2$

When you take the differences between successive values of the speeds, $\Delta d/\Delta t$, you get the accelerations, $\Delta v/\Delta t$, which are recorded in the second column. When a difference in two measurements is involved, you

find the uncertainty of the differences (in this case, $\Delta v/\Delta t$) by *adding* the uncertainties of the two measurements. This results in an uncertainty in acceleration of $(\pm 0.1) + (\pm 0.1)$ or ± 0.2 cm/sec² as recorded in the table.

Q5 What is *your* value of a_{av} and its uncertainty?

Comparing Your Results

You now have values of F_{av} , m and a_{av} , their uncertainties, and you consider the uncertainty of ma_{av} . When you have a value for the uncertainty of this product of two quantities, you will then compare the value of ma_{av} with the value of F_{av} and draw your final conclusions. For convenience, we have dropped the "av" from the symbols in the equations in the following discussion. When two quantities are multiplied, the percentage uncertainty in the product never exceeds the sum of the percentage uncertainties in each of the factors. In our example, $m \times a = 1.30 \text{ kg} \times 0.8$ $cm/\sec^2 = 1.04$ newtons. The uncertainty in $a (0.8 \pm 0.2 \text{ cm/sec}^2)$ is 25% (since 0.2 is 25%) of 0.8). The uncertainty in m is 11%. Thus the uncertainty in ma is 25% + 11% = 36% and we can write our product as ma = 1.04 N + 0.36%which is, to two significant figures,

 $ma = 1.04 \pm 0.36$ N

(The error is so large here that it really isn't appropriate to use the two decimal places; we could round off to 1.0 ± 0.4 N.) In our example we found from direct measurement that $F_{net} = 1.2 \pm 0.2$ N. Are these the same quantity?

Although 1.0 does not equal 1.2, the range of 1.0 ± 0.4 overlaps the range of 1.2 ± 0.2 , so we can say that "the two numbers agree within the range of uncertainty of measurement."

An example of lack of agreement would be 1.0 ± 0.2 and 1.4 ± 0.1 . These are presumably not the same quantity since there is no overlap of expected uncertainties.

In a similar way, work out your own values of F_{net} and ma_{ar} .

Q6 Do your own values agree within the range of uncertainty of your measurement?

Q7 Is the relationship $F_{net} = ma_{av}$ consistent with your observations?
EXPERIMENT 9 MASS AND WEIGHT

You know from your own experience that an object that is pulled strongly toward the earth (like, say, an automobile) is difficult to accelerate by pushing. In other words, objects with great weight also have great inertia. But is there some simple, exact relationship between the masses of objects and the gravitational forces acting on them? For example, if one object has twice the mass of another, does it also weigh twice as much?

Measuring Mass

The masses of two objects can be compared by observing the accelerations they each experience when acted on by the same force. Accelerating an object in one direction with a constant force for long enough to take measurements is often not practical in the laboratory. Fortunately there is an easier way. If you rig up a puck and springs between two rigid supports as shown in the diagram, you



can attach objects to the puck and have the springs accelerate the object back and forth. The greater the mass of the object, the less the magnitude of acceleration will be, and the longer it will take to oscillate back and forth.

To "calibrate" your oscillator, first time the oscillations. The time required for 5 complete round trips is a convenient measure. Tape pucks on top of the first one, and time the period for each new mass. (The units of mass are not essential here, for we will be interested only in the ratio of masses.) Then plot a graph of mass against the oscillation period, drawing a smooth curve through your experimental plot points. Do not leave the pucks stuck together.

Q1 Does there seem to be a simple relationship between mass and period? Could you write an algebraic expression for the relationship?

Weight

magnet.

To compare the gravitational forces on two objects, they can be hung on a spring scale. In this investigation the units on the scale are not important, because we are interested only in the ratio of the weights.

Comparing Mass and Weight

Use the puck and spring oscillator and calibration graph to find the masses of two objects (say, a dry cell and a stapler). Find the gravitational pulls on these two objects by hanging each from a spring scale.

Q2 How does the ratio of the gravitational forces compare to the ratio of the masses? Q3 Describe a similar experiment that would compare the masses of two iron objects to the magnetic forces exerted on them by a large

You probably will not be surprised to find that, to within your uncertainty of measurement, the ratio of gravitational forces is the same as the ratio of masses. Is this really worth doing an experiment to find out, or is the answer obvious to begin with? Newton didn't think it was obvious. He did a series of very precise experiments using many different substances to find out whether gravitational force was always proportional to inertial mass. To the limits of his precision, he found the proportionality to hold exactly. (Newton's results have been confirmed to a precision of $\pm 0.00000001\%$, and extended to gravitational attraction to bodies other than the earth).

Newton could offer no explanation from his physics as to why the attraction of the earth for an object should increase in exact proportion to the object's inertia. No other forces bear such a simple relation to inertia, and this remained a complete puzzle for two centuries until Einstein related inertia and gravitation theoretically. (See "Outside and Inside the Elevator" in the Unit 5 *Reader*.) Even before Einstein, Ernst Mach made the ingenious suggestion that inertia is not the property of an object by itself, but is the result of the gravitational forces exerted on an object by everything else in the universe.

ACTIVITIES

CHECKER SNAPPING

Stack several checkers. Put another checker on the table and snap it into the stack. On the basis of Newton's first law, can you explain what happened?

BEAKER AND HAMMER

One teacher suggests placing a glass beaker half full of water on top of a pile of three wooden blocks. Three quick back-and-forth swipes (NOT FOUR!) of a hammer leave the beaker sitting on the table.

PULLS AND JERKS



Hang a weight (such as a heavy wooden block that just barely supports it, and tie another identical string below the weight. A slow, steady pull on the string below the weight breaks the string *above* the weight. A quick jerk breaks it *below* the weight. *Why*?

EXPERIENCING NEWTON'S SECOND LAW

One way for you to get the feel of Newton's second law is actually to pull an object with a constant force. Load a cart with a mass of several kilograms. Attach one end of a long rubber band to the cart and, pulling on the other end, move along at such a speed that the rubber band is maintained at a constant length —say 70 cm. Holding a meter stick above the band with its 0-cm end in your hand will help you to keep the length constant.

The acceleration will be very apparent to the person applying the force. Vary the mass on the cart and the number of rubber bands (in parallel) to investigate the relationship between F, m, and a.

MAKE ONE OF THESE ACCELEROMETERS

An accelerometer is a device that measures acceleration. Actually, anything that has mass could be used for an accelerometer. Because you have mass, you were acting as an accelerometer the last time you lurched forward in the seat of your car as the brakes were applied. With a knowledge of Newton's laws and certain information about you, anybody who measured how far you leaned forward and how tense your muscles were would get a good idea of the magnitude and direction of the acceleration that you were undergoing. But it would be complicated.

Here are two accelerometers of a much simpler kind. With a little practice, you can learn to read accelerations from them directly, without making any difficult calculations.

A. The Liquid-Surface Accelerometer

This device is a hollow, flat plastic container



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partly filled with a colored liquid. When it is not being accelerated, the liquid surface is horizontal, as shown by the dotted line in Fig. 3-4. But when it is accelerated toward the left (as shown) with a uniform acceleration a, the surface becomes tilted, with the level of the liquid rising a distance h above its normal position at one end of the accelerometer and falling the same distance at the other end. The greater the acceleration, the more steeply the surface of the liquid is slanted. This means that the slope of the surface is a measure of the magnitude of the acceleration a.



The length of the accelerometer is 2l, as shown in Fig. 3-4 above. So the slope of the surface may be found by

slope =
$$\frac{\text{vertical distance}}{\text{horizontal distance}}$$

= $\frac{2h}{2l}$
= $\frac{h}{l}$

Theory gives you a very simple relation-

*SIGH



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ship between this slope and the acceleration *a*:

slope
$$=\frac{h}{l}=\frac{a}{a_g}$$

Notice what this equation tells you. It says that if the instrument is accelerating in the direction shown with just a_g (one common way to say this is that it has a "one-*G* acceleration"), the acceleration of gravity, then the slope of the surface is just 1; that is, h = l and the surface makes a 45° angle with its normal, horizontal direction. If it is accelerating with $\frac{1}{2}$ a_g , then the slope will be $\frac{1}{2}$; that is $h = \frac{1}{2} l$. In the same way, if $h = \frac{1}{4} l$, then $a = \frac{1}{4} a_g$, and so on with any acceleration you care to measure.

To measure h, stick a piece of centimeter tape on the front surface of the accelerometer as shown in Fig. 3–5 below. Then stick a piece of white paper or tape to the back of the instrument to make it easier to read the level of the liquid. Solving the equation above for a gives

$$a = a_g \times \frac{h}{l}$$





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This shows that if you place a scale 10 scale units away from the center you can read accelarations directly in $\frac{1}{10}$ th's of "G's." Since a_g is very close to 9.8m/sec² at the earth's surface if you place the scale 9.8 scale units from the center you can read accelerations directly in m/sec². For example, if you stick a centimeter tape just 9.8 cm from the center of the liquid surface, one cm on the scale is equivalent to an acceleration of one m/sec².

Calibration of the Accelerometer

You do not have to trust blindly the theory mentioned above. You can test it for yourself. Does the accelerometer really measure accelerations directly in m/sec²? Stroboscopic methods give you an independent check on the correctness of the theoretical prediction.

Set the accelerometer on a dynamics cart and arrange strings, pulleys, and masses as you did in Experiment 9 to give the cart a uniform acceleration on a long tabletop. Don't forget to put a block of wood at the end of the cart's path to stop it. Make sure that the accelerometer is fastened firmly enough so that it will not fly off the cart when it stops suddenly. Make the string as long as you can, so that you use the entire length of the table.

Give the cart a wide range of accelerations by hanging different weights from the string. Use a stroboscope to record each motion. To measure the accelerations from your strobe records, plot t^2 against d, as you did in Experiment 5. (What relationship did Galileo discover between d/t^2 and the acceleration?) Or use the method of analysis you need in Experiment 9.

Compare your stroboscopic measurements with the readings on the accelerometer during each motion. It takes some cleverness to read the accelerometer accurately, particularly near the end of a high-acceleration run. One way is to have several students along the table observe the reading as the cart goes by; use the average of their reports. If you are using a xenon strobe, of course, the readings on the accelerometer will be visible in the photograph; this is probably the most accurate method.

Plot the accelerometer readings against

the stroboscopically measured accelerations. This graph is called a "calibration curve." If the two methods agree perfectly, the graph will be a straight line through the origin at a 45° angle to each axis. If your curve turns out to have some other shape, you can use it to convert "accelerometer readings" to "accelerations"—if you are willing to assume that your strobe measurements are more accurate than the accelerometer. (If you are not willing, what can you do?)

B. Automobile Accelerometer-I

With a liquid-surface accelerometer mounted on the front-back line of a car, you can measure the magnitude of acceleration along its path. Here is a modification of the liquid-surface design that you can build for yourself. Bend a small glass tube (about 30 cm long) into a U-shape, as shown in Fig. 3–6 below.



Calibration is easiest if you make the long horizontal section of the tube just 10 cm long; then each 5 mm on a vertical arm represents an acceleration of $\frac{1}{10}$ g = (about) 1 m/sec², by the same reasoning as before. The two vertical arms should be at least three-fourths as long as the horizontal arm (to avoid splashing out the liquid during a quick stop). Attach a scale to one of the vertical arms, as shown. Holding the long arm horizontal, pour colored water into the tube until the water level in the arm comes up to the zero mark. How can you be sure the long arm is horizontal?

To mount your accelerometer in a car, fasten the tube with staples (carefully) to a piece of plywood or cardboard a little bigger than the U-tube. To reduce the hazard from broken glass while you do this, cover all but the scale (and the arm by it) with cloth or cardboard, but leave both ends open. It is essential that the accelerometer be horizontal if its readings are to be accurate. When you are measuring acceleration in a car, be sure the road is level. Otherwise, you will be reading the tilt of the car as well as its acceleration. When a car accelerates—in any direction—it tends to tilt on the suspension. This will introduce error in the accelerometer readings. Can you think of a way to avoid this kind of error?

C. Automobile Accelerometer-II

An accelerometer that is more directly related to F = ma can be made from a 1-kg cart and a spring scale marked in newtons. The spring scale is attached between a wood frame and the cart as in the sketch below. If the frame is kept level, the acceleration of the system can



be read directly from the spring scale, since one newton of force on the 1-kg mass indicates an acceleration of one m/sec². (Instead of a cart, any 1-kg object can be used on a layer of low-friction plastic beads.)

D. Damped-Pendulum Accelerometer

One advantage of liquid-surface accelerometers is that it is easy to put a scale on them and read accelerations directly from the instrument. They have a drawback, though; they give only the component of acceleration that is parallel to their horizontal side. If you accelerate one at right angles to its axis, it doesn't register any acceleration at all. And if you don't know the direction of the acceleration, you have to use trial-and-error methods to find it with the accelerometers we have discussed up to this point. A damped-pendulum accelerometer, on the other hand, indicates the direction of any horizontal acceleration; it also gives the magnitude, although less directly than the previous instruments do.

Hang a small metal pendulum bob by a short string fastened to the middle of the lid of a one-quart mason jar as shown on the left hand side of the sketch at the bottom of the page. Fill the jar with water and screw the lid on tight. For any position of the pendulum, the angle that it makes with the vertical depends upon your position. What would you see, for example, if the bottle were accelerating straight toward you? Away from you? Along a table with you standing at the side? (Careful: this last question is trickier than it looks.

To make a fascinating variation on the damped-pendulum accelerometer, simply replace the pendulum bob with a cork and turn the bottle upside down as shown on the right hand side of the sketch at the bottom of the page. If you have punched a hole in the bottle lid to fasten the string, you can prevent leakage with the use of sealing wax, parafin, or tape.

This accelerometer will do just the opposite from what you would expect. The explanation of this odd behavior is a little beyond the scope of this course: it is thoroughly explained in *The Physics Teacher*, vol. 2, no. 4 (April 1964) page 176.



FILM LOOP

FILM LOOP 3 VECTOR ADDITION— VELOCITY OF A BOAT

A motorboat was photographed from a bridge in this film. The boat heads upstream, then downstream, then directly across stream, and at an angle across the stream. The operator of the boat tried to keep the throttle at a constant setting to maintain a steady speed relative to the water. The task before you is to find out if he succeeded.



This photograph was taken from one bank of the stream. It shows the motorboat heading across the stream and the camera filming this loop fixed on the scaffolding on the bridge.

First project the film on graph paper and mark the lines along which the boat's image moves. You may need to use the reference crosses on the markers. Then measure speeds by timing the motion through some predetermined number of squares. Repeat each measurement several times, and use the average times to calculate speeds. Express all speeds in the same unit, such as "squares per second" (or "squares per cm" where cm refers to measured separations between marks on the moving paper of a dragstrip recorder). Why is there no need to convert the speeds to meters per second? Why is it a good idea to use a large distance between the timing marks on the graph paper?





The head-to-tail method of adding vectors. For a review of vector addition see Project Physics Programmed instruction Booklet entitled Vectors *II*.

The head-to-tail method of adding vectors is illustrated in Fig. 3-7. Since velocity is a vector with both magnitude and direction, you can study vector addition by using velocity vectors. An easy way of keeping track of the velocity vectors is by using subscripts:

 \vec{v}_{BE} velocity of boat relative to earth

 $\overrightarrow{v}_{\scriptscriptstyle BW}$ velocity of boat relative to water

 \overrightarrow{v}_{wE} velocity of water relative to earth

Then
$$\overrightarrow{v}_{BE} = \overrightarrow{v}_{BW} + \overrightarrow{v}_{WE}$$

For each heading of the boat, a vector diagram can be drawn by laying off the velocities to scale. A suggested procedure is to record data (direction and speed) for each of the five scenes in the film, and then draw the vector diagram for each.

Scene 1: Two blocks of wood are dropped overboard. Time the blocks. Find the speed of the river, the magnitude of v_{WE} .

Scene 2: The boat heads upstream. Measure \vec{v}_{BE} , then find \vec{v}_{BW} using a vector diagram similar to Fig. 3-8.



Fig. 3-8

Scene 3: The boat heads downstream. Measure \vec{v}_{BE} , then find \vec{v}_{BW} using a vector diagram.



Fig. 3–9

Scene 4: The boat heads across stream and drifts downstream. Measure the speed of the boat and the direction of its path to find \vec{v}_{BE} . Also measure the *direction* of \vec{v}_{BW} , the direction the boat points. One way to record data is to use a set of axes with the 0° - 180° axis passing through the markers anchored in the river. A diagram, such as Fig. 3-9, will help you record and analyze your measurements. (Note that the numbers in the diagram are deliberately not correct.) Your vector diagram should be something like Fig. 3-10.



Scene 5: The boat heads upstream at an angle, but moves directly across stream. Again find a value for \vec{v}_{BW} .

Checking your work: (a) How well do the four values of the magnitude of \vec{v}_{BW} agree with each other? Can you suggest reasons for any discrepancies? (b) From scene 4, you can *calculate* the heading of the boat. How well does this angle agree with the *observed* boat heading? (c) In scene 5, you determine a direction for \vec{v}_{BW} . Does this angle agree with the observed boat heading?

Chapter **4** Understanding Motion

EXPERIMENT 10 CURVES OF TRAJECTORIES

Imagine you are a ski-jumper. You lean forward at the top of the slide, grasp the railing on each side, and yank yourself out into the track. You streak down the trestle, crouch and give a mighty leap at the takeoff lip, and soar up and out, looking down at tiny fields far below. The hill flashes into view and you thump on its steep incline, bobbing to absorb the impact.

This exciting experience involves a more complex set of forces and motions than you can deal with in the laboratory at one time. Let's concentrate therefore on just one aspect: your flight through the air. What kind of a path, or trajectory, would your flight follow?

At the moment of projection into the air a skier has a certain velocity (that is, a certain speed in a given direction), and throughout his flight he must experience the downward acceleration due to gravity. These are circumstances that we can duplicate in the laboratory. To be sure, the flight path of an actual skijumper is probably affected by other factors, such as air, velocity and friction; but we now know that it usually pays to begin experiments with a simplified approximation that allows us to study the effects of a few factors at a time. Thus, in this experiment you will launch a steel ball from a ramp into the air and try to determine the path it follows.

How to Use the Equipment

If you are assembling the equipment for this experiment for the first time, follow the manufacturer's instructions.

The apparatus being used by the students in the photograph on page 177 consists of two ramps down which you can roll a steel ball. Adjust one of the ramps (perhaps with the help of a level) so that the ball leaves it horizontally.

Tape a piece of squared graph paper to the plotting board with its left-hand edge behind the end of the launching ramp.



To find a path that extends all across the graph paper, release the ball from various points up the ramp until you find one from which the ball falls close to the bottom righthand corner of the plotting board. Mark the point of release on the ramp and release the ball each time from this point.

Attach a piece of carbon paper to the impact board, with the carbon side facing the ramp. Then tape a piece of thin onionskin paper over the carbon paper.

Now when you put the impact board in its way, the ball hits it and leaves a mark that you can see through the onionskin paper, automatically recording the point of impact between ball and board. (Make sure that the impact board doesn't move when the ball hits it; steady the board with your hand if necessary.) Transfer the point to the plotting board by making a mark on it just next to the point on the impact board.

Do not hold the ball in your fingers to release it—it is impossible to let go of it in the same way every time. Instead, dam it up with



a ruler held at a mark on the ramp and release the ball by moving the ruler quickly away from it down the ramp.

Try releasing the ball several times (always from the same point) for the same setting of the impact board. Do all the impact points exactly coincide?

Repeat this for several positions of the impact board to record a number of points on the ball's path. Move the board *equal distances* every time and always release the ball from the same spot on the ramp. Continue until the ball does not hit the impact board any longer.

Now remove the impact board, release the ball once more, and watch carefully to see that the ball moves along the points marked on the plotting board.

The curve traced out by your plotted points represents the *trajectory* of the ball. By observing the path the ball follows, you have completed the first phase of the experiment.

If you have time, you will find it worth while to go further and explore some of the properties of your trajectory.

Analyzing Your Data

To help you analyze the trajectory, draw a horizontal line on the paper at the level of the

end of the launching ramp. Then remove the paper from the plotting board and draw a smooth continuous curve through the points as shown in the figure at the bottom of the page.

You already know that a moving object on which there is no net force acting will move at constant speed. There is no appreciable horizontal force acting on the ball during its fall, so we can make an *assumption* that its horizontal progress is at a constant speed. Then equally spaced lines will indicate equal time intervals.

Draw vertical lines through the points on your graph. Make the first line coincide with the end of the launching ramp. *Because of your plotting procedure* these lines should be equally spaced. If the horizontal speed of the ball is uniform, these vertical lines are drawn through positions of the ball separated by equal time intervals.

Now consider the vertical distances fallen in each time interval. Measure down from your horizontal line the vertical fall to each of your



plotted points. Record your measurements in a column. Alongside them record the corresponding horizontal distances measured from the first vertical line. A sample of results as recorded in a student notebook is shown on the right.

Q1 What would a graph look like on which you plot horizontal distance against time?

Earlier in your work with accelerated motion you learned how to recognize uniform acceleration (see Secs. 2.5–2.8 in the *Text* and Experiment 5). Use the data you have just collected to decide whether the vertical motion of the ball was uniformly accelerated motion.

Q2 What do you find?

Q3 Do the horizontal and the vertical motions affect each other in any way?

Q4 Write an equation that describes the horizontal motion in terms of horizontal speed v, the horizontal distance, Δx , and the time of travel, Δt .

Q5 What is the equation that describes the vertical motion in terms of the distance fallen vertically, Δy , the vertical acceleration, a_y , and the time of travel, Δt ?



Try These Yourself

There are many other things you can do with this apparatus. Some of them are suggested by the following questions.

Q6 What do you expect would happen if you repeated the experiment with a glass marble of the same size instead of a steel ball?

Q7 What will happen if you next try to repeat

the experiment starting the ball from a different point on the ramp?

Q8 What do you expect if you use a smaller or larger ball starting always from the same reference point on the ramp?

Q9 Plot the trajectory that results when you use a ramp that launches the ball at an angle to the horizontal. In what way is this curve similar to your first trajectory?

EXPERIMENT 11 PREDICTION OF TRAJECTORIES

You can predict the landing point of a ball launched horizontally from a tabletop at any speed. If you know the speed, v_0 , of the ball as it leaves the table, the height of the table above the floor and a_g , you can then use the equation for projectile motion to predict where on the floor the ball will land.

You know an equation for horizontal motion:

$$\Delta x = v \, \Delta t$$

and you know an equation for free-fall from rest:

$$\Delta y = \frac{1}{2}a_g \ (\Delta t)^2$$

The time interval is difficult to measure. Besides, in talking about the *shape* of the path, all we really need to know is how Δy relates to Δx . Since, as you found in the previous experiment, these two equations still work when an object is moving horizontally and falling at the same time, we can combine them to get an equation relating Δy and Δx , without Δt appearing at all. We can rewrite the equation for horizontal motion as:



 $\Delta t = \frac{\Delta x}{T}$

Then we can substitute this expression for t into the equation for fall:

$$\Delta y = \frac{1}{2}a_g \, \frac{(\Delta x)^2}{v^2}$$

Thus the equation we have derived should describe how Δy changes with Δx —that is, it should give us the shape of the trajectory. If we want to know how far out from the edge of the table the ball will land (Δx) , we can calculate if from the height of the table (Δy) , a_{q} , and the ball's speed v along the table.

Doing the Experiment

Find v by measuring with a stopwatch the time t that the ball takes to roll a distance d along the tabletop. (See Fig. 4–1 below.) Be sure to have the ball caught as it comes off the end of the table. Repeat the measurement a few times, always releasing the ball from the same place on the ramp, and take the average value of v.

Measure Δy and then use equation for Δy to calculate Δx . Place a target, a paper cup, perhaps, on the floor at your predicted landing spot as shown below. How confident are you of your prediction? Since it is based on *measurement*, some uncertainty is involved. Mark an area around the spot to indicate your uncertainty.



Now release the ball once more. This time, let it roll off the table and land, hopefully, on the target as shown in the figure above. If the ball actually does fall within the range of values of x you have estimated, then you have supported the assumption on which your calculation was based, that vertical and horizontal motion are not affected by each other.

Q1 How could you determine the range of a ball launched horizontally by a slingshot?

Q2 Assume you can throw a baseball 40 meters on the earth's surface. How far could you throw that same ball on the surface of the moon, where the acceleration of gravity is one-sixth what it is at the surface of the earth? Q3 Will the assumptions made in the equations $\Delta x = v\Delta t$ and $\Delta y = \frac{1}{2}ag(\Delta t)^2$ hold for a Ping-Pong ball? If the table were 1000 meters above the floor, could you still use these equations? Why or why not?



The path taken by a cannon ball according to a drawing by Ufano (1621). He shows that the same horizontal distance can be obtained by two different firing angles. Gunners had previously found this by experience. What angles give the maximum range?

EXPERIMENT 12 CENTRIPETAL FORCE

The motion of an earth satellite and of a weight swung around your head on the end of a string are described by the same laws of motion. Both are accelerating toward the center of their orbit due to the action of an unbalanced force.

In the following experiment you can discover for yourself how this centripetal force depends on the mass of the satellite and on its speed and distance from the center.

How the Apparatus Works

Your "satellite" is one or more rubber stoppers. When you hold the apparatus in both hands, as shown in the photo above, and swing the stopper around your head, you can measure the centripetal force on it with a spring scale at the base of the stick. The scale should read in newtons or else its readings should be converted to newtons.

You can change the length of the string so as to vary the radius R of the circular orbit, and you can tie on more stoppers to vary the satellite mass m.

The best way to set the frequency f is to swing the apparatus in time with some periodic sound from a metronome or an earphone attachment to a blinky. You keep the rate constant by adjusting the swinging until you see the stopper cross the same point in the room at every tick.

Hold the stick vertically and have as little motion at the top as possible, since this would change the radius. Because the stretch of the spring scale also alters the radius, it is helpful to have a marker (knot or piece of tape) on the string. You can move the spring scale up or down slightly to keep the marker in the same place.



Doing the Experiment

The object of the experiment is to find out how the force F read on the spring scale varies with m, with f, and with R.

You should only change *one* of these three quantities at a time so that you can investigate the effect of each quantity independently of the others. It's easiest to either double or triple m, f, and R (or halve them, and so on, if you started with large values).

Two or three different values should be enough in each case. Make a table and clearly record your numbers in it.

Q1 How do changes in m affect F when R and f are kept constant? Write a formula that states this relationship.

Q2 How do changes in f affect F when m and R are kept constant? Write a formula to express this too.

Q3 What is the effect of R on F?

Q4 Can you put m, f, and R all together in a single formula for centripetal force, R?

How does your formula compare with the expression derived in Sec. 4.7 of the *Text*.

EXPERIMENT 13 CENTRIPETAL FORCE ON A TURNTABLE

You may have had the experience of spinning around on an amusement park contraption known as the Whirling Platter. The riders seat themselves at various places on a large flat polished wooden turntable about 40 feet in diameter. The turntable gradually rotates faster and faster until everyone (except for the person at the center of the table) has slid off. The people at the edge are the first to go. Why do the people slide off?

Unfortunately you probably do not have a Whirling Platter in your classroom, but you do have a Masonite disk that fits on a turntable. The object of this experiment is to predict the maximum radius at which an object can be placed on the rotating turntable without sliding off.

If you do this under a variety of conditions, you will see for yourself how forces act in circular motion.

Before you begin, be sure you have studied Sec. 4.6 in your *Text* where you learned that the centripetal force needed to hold a rider in a circular path is given by $F = mv^2/R$.

Studying Centripetal Force

For these experiments it is more convenient to write the formula $F = mv^2/R$ in terms of the frequency *f*. This is because *f* can be measured more easily than *v*. We can rewrite the formula as follows:

- $v = \frac{\text{distance traveled}}{\text{in one revolution}} \times \frac{\text{number of revolu-inor for the sec}}{\text{tions per sec}}$
 - $=2\pi R \times f$

Substituting this expression for v in the formula gives:

$$F = \frac{m \times (2\pi Rf)}{R}$$
$$= \frac{4\pi^2 m R^2 f^2}{R}$$
$$= 4\pi^2 m R f^2$$

You can measure all the quantities in this equation.



Friction on a Rotating Disk

For objects on a rotating disk, the centripetal force is provided by friction. On a frictionless disk there could be no such centripetal force. As you can see from the equation we have just derived, the centripetal acceleration is proportional to R and to f^2 . Since the frequency f is the same for any object moving around with a turntable, the centripetal acceleration is directly proportional to R, the distance from the center. The further an object is from the center of the turntable, therefore, the greater the centripetal force must be to keep it in a circular path.

You can measure the maximum force F_{max} that friction can provide on the object, measure the mass of the object, and then calculate the maximum distance from the center R_{max} that the object can be without sliding off. Solving the centripetal force equation for R gives

$$R_{max} = \frac{F_{max}}{4\pi^2 m f^2}$$

Use a spring scale to measure the force needed to make some object (of mass m from 0.2 to 1.0 kg) start to slide across the motionless

disk. This will be a measure of the maximum friction force that the disk can exert on the object.

Then make a chalk mark on the turntable and time it (say, for 100 sec)—or accept the marked value of rpm—and calculate the frequency in rev/sec.

Make your predictions of R_{max} for turntable frequencies of 33 revolutions per minute (rpm), 45 rpm, and 78 rpm.

Then try it!

Q1 How great is the percentage difference

between prediction and experiment for each turntable frequency? Is this reasonable agreement?

Q2 What effect would decreasing the mass have on the predicted value of R? Careful! Decreasing the mass has an effect on F_{\parallel} also. Check your answer by doing an experiment.

Q3 What is the smallest radius in which you can turn a car if you are moving 60 miles an hour and the friction force between tires and road is one-third the weight of the car? (Careful! Remember that weight is equal to $a_n \times m$.)



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ACTIVITIES

PROJECTILE MOTION DEMONSTRATION

Here is a simple way to demonstrate projectile motion. Place one coin near the edge of a table. Place an identical coin on the table and snap it with your finger so that it flies off the table, just ticking the first coin enough that it falls almost straight down from the edge of the table. The fact that you hear only a single ring as both coins hit shows that both coins took the same time to fall to the floor from the table. Incidentally, do the coins *have* to be identical? Try different ones.

SPEED OF A STREAM OF WATER

You can use the principles of projectile motion to calculate the speed of a stream of water issuing from a horizontal nozzle. Measure the vertical distance Δy from the nozzle to the ground, and the horizontal distance Δx from the nozzle to where the water hits the ground.

Use the equation relating Δx and Δy that was derived in Experiment 11, solving it for v:



so

and

 $v^2 = \frac{1}{2}a_g \frac{(\Delta \chi)^2}{y}$

$$v = \Delta x \sqrt{\frac{a_g}{2\Delta y}}$$

The quantities on the right can all be measured and used to compute v.



PHOTOGRAPHING A WATERDROP PARABOLA

Using an electronic strobe light, a doorbell timer, and water from a faucet, you can photograph a water drop parabola. The principle of independence of vertical and horizontal motions will be clearly evident in your picture.

Remove the wooden block from the timer. Fit an "eye dropper" barrel in one end of some tubing and fit the other end of the tubing onto a water faucet. (Instead of the timer you can use a doorbell without the bell.) Place the tube through which the water runs under the clapper so that the tube is given a steady series of sharp taps. This has the effect of breaking the stream of water into separate, equally spaced drops (see photo on previous page).

To get more striking power, run the vibrator from a variable transformer (Variac) connected to the 110 volt a.c., gradually increasing the Variac from zero just to the place where the striker vibrates against the tubing. Adjust the water flow through the tube and eye dropper nozzle. By viewing the drops with the xenon strobe set at the same frequency as the timer, a parabola of motionless drops is seen. A spot-light and disk strobe can be used instead of the electronic strobe light, but it is more difficult to match the frequencies of vibrator and strobe. The best photos are made by lighting the parabola from the side (that is, putting the light source in the plane of the parabola). The photo above was made in that



way. With front lighting, the shadow of the parabola can be projected onto graph paper for more precise measurement.

Some heating of the doorbell coil results, so the striker should not be run continuously for long periods of time.

BALLISTIC CART PROJECTILES

Fire a projectile straight up from a cart or toy locomotive as shown in the photo below that is rolling across the floor with nearly uniform velocity. You can use a commercial device called a ballistic cart or make one yourself. A spring-loaded piston fires a steel ball when you pull a string attached to a trigger pin. Use the electronic strobe to photograph the path of the ball.



Of course projectile trajectories can be photographed of any object thrown into the air using the electronic strobe and Polaroid Land camera. By fastening the camera (securely!) to a pair of carts, you can photograph the action from a moving frame of reference.

MOTION IN A ROTATING REFERENCE FRAME

Here are three ways you can show how a moving object would appear in a rotating reference frame.

Method I Attach a piece of paper to a phonograph turntable. Draw a line across the paper as a turntable is turning (see Fig. 4–2 below), using as a guide a meter stick supported on books at either side of the turntable. The line should be drawn at a constant speed.







Method II Place a Polaroid camera on the turntable on the floor and let a tractor run along the edge of a table, with a flashlight bulb on a pencil taped to the tractor so that it sticks out over the edge of the table.



Method III How would an elliptical path appear if you were to view it from a rotating reference system? You can find out by placing a Polaroid camera on a turntable on the floor, with the camera aimed upwards. (See Fig. 4–3 below.) For a pendulum, hang a flashlight bulb and an AA dry cell. Make the pendulum long enough so that the light is about 4 feet from the camera lens.



Fig. 4-3

With the lights out, give the pendulum a swing so that it swings in an elliptical path. Hold the shutter open while the turntable makes one revolution. You can get an indication of how fast the pendulum moves at different points in its swing by using a motor strobe in front of the camera, or by hanging a blinky.

PENNY AND COAT HANGER

Bend a coat hanger into the shape shown in the sketch below in this right-hand column. Bend the end of the hook slightly with a pair of pliers so that it points to where the finger supports the hanger. File the end of the hook flat. Balance a penny on the hook. Move your finger back and forth so that the hanger (and balanced penny) starts swinging like a pendulum. Some practice will enable you to swing the hanger in a vertical circle, or around your head and still keep the penny on the hook. The centripetal force provided by the hanger keeps the penny from flying off on a straight-line path. Some people have done this demonstration successfully with a pile of as many as five pennies at once.



MEASURING UNKNOWN FREQUENCIES

Use a calibrated electronic stroboscope or a hand-stroboscope and stopwatch to measure the frequencies of various motions. Look for such examples as an electric fan, a doorbell clapper, and a banjo string.

On page 108 of the *Text* you will find tables of frequencies of rotating objects. Notice the enormous range of frequencies listed, from the electron in the hydrogen atom to the rotation of our Milky Way galaxy.

FILM LOOPS

FILM LOOP 4 A MATTER OF RELATIVE MOTION

Two carts of equal mass collide in this film. Three sequences labeled Event A, Event B, and Event C are shown. Stop the projector after *each* event and describe these events in words, as they appear to you. View the loop now, before reading further.

Even though Events A, B, and C are visibly different to the observer, in each the carts interact similarly. The laws of motion apply for each case. Thus, these events could be the *same* event observed from different reference frames. They *are* closely similar events photographed from different frames of reference, as you see after the initial sequence of the film.

The three events are photographed by a camera on a cart which is on a second ramp parallel to the one on which the colliding carts move. The camera is your frame of reference, your coordinate system. This frame of reference may or may not be in motion with respect to the ramp. As photographed, the three events appear to be quite different. Do such concepts as position and velocity have a meaning independently of a frame of reference, or do they take on a precise meaning only when a frame of reference is specified? Are these three events really similar events, viewed from different frames of reference?

You might think that the question of which cart is in motion is resolved by sequences at



the end of the film in which an experimenter, Franklin Miller of Kenyon College, stands near the ramp to provide a reference object. Other visual clues may already have provided this information. The events may appear different when this reference object is present. But is this *fixed* frame of reference any more fundamental than one of the moving frames of reference? fixed relative to what? Or is there a "completely" fixed frame of reference?

If you have studied the concept of momentum, you can also consider each of these three events from the standpoint of momentum conservation. Does the total momentum depend on the frame of reference? Does it seem reasonable to assume that the carts would have the same *mass* in all the frames of reference used in the film?

B.C. by John Hart







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FILM LOOP 5 GALILEAN RELATIVITY— BALL DROPPED FROM MAST OF SHIP

This film is a partial actualization of an experiment described by Sagredo in Galileo's *Two New Sciences:*

If it be true that the impetus with which the ship moves remains indelibly impressed in the stone after it is let fall from the mast; and if it be further true that this motion brings no impediment or retardment to the motion directly downwards natural to the stone, then there ought to ensue an effect of a very wondrous nature. Suppose a ship stands still, and the time of the falling of a stone from the mast's round top to the deck is two beats of the pulse. Then afterwards have the ship under sail and let the same stone depart from the same place. According to what has been premised, it shall take up the time of two pulses in its fall, in which time the ship will have gone, say, twenty yards. The true motion of the stone will then be a transverse line (i.e., a curved line in the vertical plane), considerably longer than the first straight and perpendicular line, the height of the mast, and yet nevertheless the stone will have passed it in the same time. Increase the ship's velocity as much as you will, the falling stone shall describe its transverse lines still longer and longer and yet shall pass them all in those selfsame two pulses.

In the film a ball is dropped three times:

Scene 1: The ball is dropped from the mast. As in Galileo's discussion, the ball continues to move horizontally with the boat's velocity, and also it falls vertically relative to the mast.

Scene 2: The ball is tipped off a stationary support as the boat goes by. It has no forward velocity, and it falls vertically relative to the water surface.

Scene 3: The ball is picked up and held briefly before being released.



The ship and earth are frames of reference in constant relative motion. Each of the three events can be described as viewed in either frame of reference. The laws of motion apply for all six descriptions. The fact that the laws of motion work for *both* frames of reference, one moving at constant velocity with respect to the other, is what is meant by "Galilean relativity." (The positions and velocities are relative to the frame of reference, but the *laws of motion* are not. A "relativity" principle also states what is *not* relative.)

Scene 1 can be described from the boat frame as follows: "A ball, initially at rest, is released. It accelerates downward at 9.8 m/ sec² and strikes a point directly beneath the starting point." Scene 1 described differently from the earth frame is: "A ball is projected horizontally toward the left; its path is a parabola and it strikes a point below and to the left of the starting point."

To test your understanding of Galilean relativity, you should describe the following: Scene 2 from the boat frame; Scene 2 in earth frame; Scene 3 from the boat frame; Scene 3 from the earth frame.

FILM LOOP 6 GALILEAN RELATIVITY— OBJECT DROPPED FROM AIRCRAFT

A Cessna 150 aircraft 23 feet long is moving about 100 ft/sec at an altitude of about 200 feet. The action is filmed from the ground as a flare is dropped from the aircraft. Scene 1 shows part of the flare's motion; Scene 2, shot from a greater distance, shows several flares dropping into a lake; Scene 3 shows the vertical motion viewed head-on. Certain frames of the film are "frozen" to allow measurements. The time interval between freeze frames is always the same.



Seen from the earth's frame of reference, the motion is that of a projectile whose original velocity is the plane's velocity. If gravity is the only force acting on the flare, its motion should be a parabola. (Can you check this?) Relative to the airplane, the motion is that of a body falling freely from rest. In the frame of reference of the plane, the motion is vertically downward.

The plane is flying approximately at uniform speed in a straight line, but its path is not necessarily a horizontal line. The flare starts with the plane's velocity, in both magnitude and in direction. Since it also falls freely under the action of gravity, you expect the flare's downward displacement below the plane to be $d = \frac{1}{2}at^2$. But the trouble is that you cannot be sure that the first freeze frame occurs at the very instant the flare is dropped. However, there is a way of getting around this difficulty. Suppose a time B has elapsed between the release of the flare and the first freeze frame. This time must be added to each of the freeze frame times (conveniently measured from the first freeze frame) and so you would have

$$d = \frac{1}{2}a(t+B)^2$$

To see if the flare follows an equation such as this, take the square root of each side:

$$\sqrt{d} = (\text{constant}) (t + B)$$

Now if we plot \sqrt{d} against *t*, we expect a straight line. Moreover, if B = 0, this straight line will also pass through the origin.

Suggested Measurements

(a) **Vertical motion**. Project Scene 1 on paper. At each freeze frame, when the motion on the screen is stopped briefly, mark the positions of the flare and of the aircraft cockpit. Measure the displacement *d* of the flare below the plane. Use any convenient units. The times can be taken as integers, $t = 0, 1, 2, \ldots$, designating successive freeze frames. Plot \sqrt{d} versus *t*. Is the graph a straight line? What would be the effect of air resistance, and how would this show up in your graph? Can you detect any signs of this? Does the graph pass through the origin?

(b) Analyze Scene 2 in the same way.

(c) **Horizontal motion.** Use another piece of graph paper with time (in intervals) plotted horizontally and displacements (in squares) plotted vertically. Using measurements from your record of the flare's path, make a graph of the two motions in Scene 2. What are the effects of air resistance in the horizontal motion? the vertical motion? Explain your findings between the effect of air friction on the horizontal and vertical motions.

(d) Acceleration due to gravity. The "constant" in your equation, d = (constant) (t + B), is $\frac{1}{2}a$; this is the slope of the straight-line graph obtained in part (a). The square of the slope gives $\frac{1}{2}a$ so the acceleration is twice the square of the slope. In this way you can obtain the acceleration in squares/(interval)². To convert your acceleration into ft/sec^2 or m/ sec^2 , you can estimate the size of a "square" from the fact that the length of the plane is 23 ft (7 m). The time interval in seconds between freeze frames can be found from the slow-motion factor.

FILM LOOP 7 GALILEAN RELATIVITY— PROJECTILE FIRED VERTICALLY

A rocket tube is mounted on bearings that leave the tube free to turn in any direction. When the tube is hauled along the snowcovered surface of a frozen lake by a "ski-doo," the bearings allow the tube to remain pointing vertically upward in spite of some roughness of path. Equally spaced lamps along the path allow you to judge whether the ski-doo has constant velocity or whether it is accelerating. A preliminary run shows the entire scene; the setting is in the Laurentian Mountains in the Province of Quebec at dusk.

Four scenes are photographed. In each case a rocket flare is fired vertically upward. With care you can trace a record of the trajectories.

Scene 1: The ski-doo is stationary relative to the earth. How does the flare move?

Scene 2: The ski-doo moves at uniform velocity relative to the earth. Describe the motion of



the flare relative to the earth; describe the motion of the flare relative to the ski-doo.

Scenes 3 and 4: The ski-doo's speed changes after the shot is fired. In each case describe the motion of the ski-doo and describe the flare's motion relative to the earth and relative to the ski-doo. In which cases are the motions a parabola?

How do the events shown in this film illustrate the principle of Galilean relativity? In which frames of reference does the rocket flare behave the way you would expect it to behave in all four scenes knowing that the force is constant, and assuming Newton's laws of motion? In which systems do Newton's laws fail to predict the correct motion in some of the scenes?

FILM LOOP 8 ANALYSIS OF A HURDLE RACE-I

The initial scenes in this film show a regulation hurdle race, with 1-meter-high hurdles spaced 9 meters apart. (Judging from the number of hurdles knocked over, the competitors were of something less than Olympic caliber!) Next, a runner, Frank White, a 75kg student at McGill University, is shown in medium slow-motion (slow-motion factor 3) during a 50-meter run. His time was 8.1 seconds. Finally, the beginning of the run is shown in extreme slow motion (slow-motion factor of 80). "Analysis of a Hurdle Race II" has two more extreme slow-motion sequences.

To study the runner's motion, measure the average speed for each of the 1-meter intervals in the slow-motion scene. A "dragstrip" chart recorder is particularly convenient for recording the data on a single viewing of the loop. Whatever method you use for measuring time, the small but significant variations in speed will be lost in experimental uncertainty unless you work very carefully. Repeat each measurement several times.

The extreme slow-motion sequence shows the runner from 0 m to 6 m. The seat of the runner's white shorts might serve as a reference mark. (What are other reference points on the runner that could be used? Are all ref-



erence points equally useful?) Measure the time to cover each of the distances, 0-1, 1-2, 2-3, 3-4, 4-5, and 5-6 m. Repeat the measurements several times, viewing the film over again, and average your results for each interval. Your accuracy might be improved by forming a grand average that combines your average with others in the class. (Should you use *all* the measurements in the class?) Calculate the average speed for each interval, and plot a graph of speed versus displacement. Draw a smooth graph through the points. Discuss any interesting features of the graph.

You might assume that the runner's legs push between the time when a foot is directly beneath his hip and the time when that foot is off the ground. Is there any relationship between your graph of speed and the way the runner's feet push on the track?

The initial acceleration of the runner can be estimated from the time to move from the starting point to the 1-meter mark. You can use a watch with a sweep second hand. Calculate the average acceleration, in m/sec², during this initial interval. How does this forward acceleration compare with the magnitude of the acceleration of a falling body? How much force was required to give the runner this acceleration? What was the origin of this force?

FILM LOOP 9 ANALYSIS OF A HURDLE RACE-II

This film loop, which is a continuation of "An-

alysis of a Hurdle Race I," shows two scenes of a hurdle race which was photographed at a slow-motion factor of 80.

In Scene 1 the hurdler moves from 20 m to 26 m, clearing a hurdle at 23 m. (See photograph.) In Scene 2 the runner moves from 40 m to 50 m, clearing a hurdle at 41 m and sprinting to the finish line at 50 m. Plot graphs of these motions, and discuss any interesting features. The seat of the runner's pants furnishes a convenient reference point for measurements. (See the film-notes about the "Analysis of a Hurdle Race I" for further details.)

No measurement is entirely precise; measurement error is always present, and it cannot be ignored. Thus it may be difficult to tell if the small changes in the runner's speed are significant, or are only the result of measurement uncertainties. You are in the best tradition of experimental science when you pay close attention to errors.

It is often useful to display the experimental uncertainty graphically, along with the measured or computed values.

For example, say that the dragstrip timer was used to make three different measurements of the time required for the first meter of the run: 13.7 units, 12.9 units, and 13.5 units, which give an average time of 13.28



STADENG 7

units. (If you wish to convert the dragstrip units to seconds, it will be easier to wait until the graph has been plotted using just units, and then add a seconds scale to the graph.) The lowest and highest values are about 0.4 units on either side of the average, so we could report the time as 13.3 ± 0.4 units. The uncertainty 0.4 is about 3% of 13.3, therefore the percentage uncertainty in the time is 3%. If we assume that the distance was exactly one meter, so that all the uncertainty is in the time, then the percentage uncertainty in the speed will be the same as for the time-3%. The slow-motion speed is 100 cm/13.3 time units, which equals 7.53 cm/unit. Since 3% of 7.53 is 0.23, the speed can be reported as 7.53 + 0.23 cm/unit. In graphing this speed value, you plot a point at 7.53 and draw an "error bar" extending 0.23 above and below the point. Now estimate the limit of error for a typical point on your graph and add error bars showing the range to each plotted point.

Your graph for this experiment may well look like some commonly obtained in scientific research. For example, in the figure at the right a research team has plotted its experimental data; they published their results in spite of



the considerable scattering of plotted points and even though some of the plotted points have errors as large as 5%.

How would you represent the uncertainty in measuring *distance*, if there were significant errors here also?

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Answers to End of Section Question

Chapter 1

Q1 We have no way of knowing the lengths of time involved in going the observed distances.

Q2 No; the time between stroboscope flashes is constant and the distance intervals shown are not equal.

Q3 An object has a uniform speed if it travels equal distances in equal time intervals; or, if the *distance traveled* = constant, regardless of the particular distances and times chosen.

Q4 Average speed is equal to the distance travelled divided by the elapsed time while going that distance.

Q5	Δt	$\Delta d/\Delta t$	
	(5.0)	(1.0)	
	(6.0)	(0.8)	(entries in brackets are those
	(4.5)	1.1	already given in the text)
	(5.5)	0.9	
	7.5	0.67	
	8.0	0.62	
	8.6	0.58	

Q6 Hint: to determine location of left edge of puck relative to readings on the meter stick, line up a straight edge with the edge of puck and *both* marks on meter stick corresponding to a given reading.



(cm)	t(sec)
0	0
13	1
26	2
39	3
52	.0
65	.5
78	.6
92	.7

d

Q7 The one on the left has the larger slope mathematically; it corresponds to 100 miles/hr whereas the one on the right corresponds to 50 miles/hr.

Q8 Most rapidly at the beginning when the slope is steepest; most slowly toward the end where the slope is most shallow.

 $\frac{\Delta d}{\Delta t} = \frac{3 \text{ yds}}{8.6 \text{ sec}} = 0.6 \text{ yd/sec from the table}$

Q10 Interpolation means estimating values *between* data points; extrapolation means estimating values *beyond* data points.

Q11 An estimate for an additional lap (extrapolation).

Q12 Instantaneous speed means the limit approached by the average speed as the time interval involved gets smaller and smaller.

 $v = limit \frac{\Delta d}{\Delta t}$ as Δt approaches zero.

Q13 Instantaneous speed is just a special case of average speed in which the ratio $\Delta d/\Delta t$ does not change as Δt is made smaller and smaller. However, $\Delta d/\Delta t$ always gives average speed no matter how large or how small Δt is.

Q14
$$a_{av} = \frac{\text{final speed} - \text{initial speed}}{\text{time elapsed}} = \frac{60 - 0 \text{ mph}}{5 \text{ sec}}$$

= 12 mph/sec

Q15
$$\frac{2 \text{ mph} - 4 \text{ mph}}{1/4 \text{ hr}} = -8 \text{ mph/hr, or } -0.13 \text{ mph/min.}$$

No, not since average is specified.

Chapter 2

Q1 Composition: terrestrial objects are composed of combinations of earth, water, air and fire; celestial objects of nothing but a unique fifth element. Motion: terrestrial objects seek their natural positions of rest depending on their relative contents of earth (heaviest), water, air and fire (lightest); celestial objects moved endlessly in circles.

Q2 (a), (b), and (c)

Q3 Aristotle: the nail is heavier than the toothpick so it falls faster.

Galileo: air resistance slows down the toothpick more than the nail.

Q4 See Q3 of Chapter 1 p. 15

Q5 An object is uniformly accelerated if its speed increases by equal amounts during equal time intervals. $\Delta v / \Delta t = constant$

Q6 The definition should (1) be mathematically simple and (2) correspond to actual free fall motion.Q7 (b)

Q8 Distances are relatively easy to measure as compared with speeds; measuring short time intervals remained a problem, however.

Q9 The expression d = v t can only be used if v is constant. The second equation refers to accelerated motion in which v is *not* constant. Therefore the two equations cannot be applied to the same event.

Q10 (c) and (e)

Q11 (d)

Q12 (a), (c) and (d)

Chapter 3

Q1 kinematic – (a), (b), (d) dynamic – (c), (e)

Q2 A continuously applied force

Q3 The air pushed aside by the puck moves around to fill the space left behind the puck as it moves along and so provides the propelling force needed.Q4 The force of gravity downward and an upward

force of equal size exerted by the table.

The sum of the forces must be zero because the vase is not accelerating.

Q5 The first three.

Q6 No, in many cases equilibrium involves frictional forces which depend on the fact that the object is in motion.

Q7 Vector quantities (1) have magnitude and direction

(2) can be represented graphically by arrows

(3) can be combined to form a single resultant vector by using either the head to tail or the parallelogram method. (Note: only vectors of the same kind are combined in this way; that is, we add force vectors to force vectors, *not* force vectors to velocity vectors, for example.)

Q8 Direction is now taken into account. (we must now consider a change of direction to be as valid a case of acceleration as speeding up or slowing down.)

Q9 W downward, 0,0,0

Q10 Galileo's "straight line forever" motion may have meant at a constant height above the earth whereas Newton's meant moving in a straight line through empty space.

Q11 Meter, Kilogram and Second

Q12 m =
$$\frac{F}{a} = \frac{10 N}{4m/sec^2} = 2.5 \text{ kg}$$

Q13 False; (frictional forces must be taken into account in determining the actual net force exerted.)

Q14 Acceleration
$$= \frac{0 - 10 \text{ m/sec}}{5 \text{ sec}} = -2 \text{ m/sec}^2$$

Force = ma = 2 kg \times (-2 m/sec²) = -4 Newtons 198 (the minus sign arises because the force and the acceleration are opposite in direction to the original motion. Since the question asks only for the magnitude of the force it may be disregarded.)
Q15 10 m/sec²
10 m/sec²
0.67 m/sec^r
10 m
0.4 m
Q16 (c) and (f)
Q17 (e) and (f)
Q18 (1) appear in pairs
(2) are equal in magnitude
(3) opposite in direction

(4) act on two different objects

Q19 The horse pushes against the earth, the earth pushes against the horse causing the horse to accelerate forward. (The earth accelerates also but can you measure it?) The swimmer pushes backward against the water; the water, according to the third law, pushes forward against the swimmer; however, there is also a backward frictional force of drag exerted by the water on the swimmer. The two forces acting *on* the swimmer add up to zero, since he is not accelerating.

Q20 No, the force "pulling the string apart" is still only 300 N; the 500 N would have to be exerted at both ends to break the line.

Q21 See text p. 68

Chapter 4

Q1 The same acceleration a_g ; its initial horizontal speed has no effect on its vertical accelerated motion.

Q2 (a), (c) and (e)

Q3 They must be moving with a uniform speed relative to each other.

- Q4 (a) T = $1/f = 1/45 = 2.2 \times 10^{-2}$ minutes
- (b) 2.2×10^{-2} minutes \times 60 seconds/minute = 1.32 sec.
- (c) f = 45 rpm \times 1/60 minutes/sec = 0.75 rps
- Q5 T = 1 hour = 60 minutes

$$v = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 3}{60} = .31$$
 cm/minute

Q6
$$f = 80$$
 vibrations/minute = 1.3 vib/sec
T = 1/f = 1/1.3 = .75 sec

Q7 (a) and (b)

Q8 Along a tangent to the wheel at the point where the piece broke loose.

$$\frac{mv^2}{R}$$

Q10 4π²mR

Q11 The value of the gravitational acceleration and the radius of the moon (to which 70 miles is added to determine R).

Brief Answers to Study Guide

Chapter 1 1.1 Information 1.2 (a) discussion (b) 58.3 mph (c) discussion (d) discussion (e) discussion 1.3 (a) 6 cm/sec (b) 15 mi. (c) 0.25 min. (d) 3 cm/sec 24 cm (e) 30 mi/hr (f) 30 mi/hr? 120 mi? (g) 5.5 sec (h) 8.8 m 1.4 22×10^{3} mi 1.5 (a) 9.5 × 1015 m (b) 2.7 × 108 sec or 8.5 years 1.6 1.988 mph or 2 mph 1.7 (a) 1.7 m/sec (b) 3.0 m/sec 1.8 discussion 1.9 discussion .10 discussion 1.11 (a) 0.5, 1.0, 1.5, and 2.0 (b) graph 1.12 Answer 1.13 25.6 meters; 4:00 for men, 4:30 for women 1.14 discussion 1.15 graph 1.16 graphs d vs t: d = 0.9,22,39,5,60.5,86cm (approx) at intervals of 0.2 sec v vs t: v = 45,65,87.5,105,127 cm/sec (approx) at intervals of 0.2 sec 1.17 (a) Between 1 and 4.5 sec; 1.3 m/sec (b) 0.13 m/sec (c) 0.75 m/sec (d) 1.0 m/sec (e) 0.4 m (approx) 1.18 (a) 14.1 m/sec (b) 6.3 m/sec² 1.19 315,000 in/sec 1.20 discussion 1.21 discussion Chapter 2 2.1 Information 2.2 discussion 2.3 discussion 2.4 discussion 2.5 discussion 2.6 discussion 2.7 proof 2.8 (a), (b), (c) 2.9 discussion 2.10 discussion 2.11 proof 2.12 17 years \$7000 2.13 discussion 2.14 (a) 57 m/sec² (b) 710 m (c) -190 m/sec2 2.15 proof 2.16 discussion 2.17 (a) true (b) true (based on measurements of 6 lower positions) (c) true (d) true (e) true 2.18 proof

2.19 (a) Position d



(b) proof (c) discussion 2.20 discussion 2.21 (a) 5.0 m (b) 10 m/sec (c) 15 m 2.22 (a) 10 m/sec (b) 15 m (c) 2 sec (d) 20 m (e) -20 m/sec 2.23 (a) 20 m/sec (b) -20 m/sec (c) 4 sec (d) 80 m (e) 0 mi/sec (f) -40 m/sec 2.24 (a) -2 m/sec² (b) 2 m/sec (c) 2 m/sec (d) 4 m (e) -2 m/sec (f) 4 sec 2.25 discussion 2.26 (a) 4.3 welfs/surg² (b) 9.8 m/sec² 2.27 proof 2.28 proof 2.29 proof 2.30 discussion 2.31 discussion discussion 2.32 2.33 discussion **Chapter 3** 3.1 information 3.2 discussion 3.3 (a) construction (b) 2.4 units, West 3.4 proof 3.5 discussion 3.6 discussion 3.7 discussion 3.8 discussion 3.9 discussion 3.10 discussion 3.11 discussion 3.12 2.8 × 10-4 hr/sec 3.13 6/1 3.14 discussion 3.15 discussion 3.16 discussion 3.17 proof 3.18 discussion 3.19 (c) 24N (d) 14.8N (e) 0.86N (f) 9.0 Kg (g) 0.30 Kg (h) 0.20 Kg (i) 3 m/sec2 (j) 2.5 m/sec2 (k) 2.50 m/sec2 3.20 (a) 2.0 × 10² m/sec² 7.8 × 10² m/sec (b) discussion (c) 2.4×10^2 m/sec² 3.21 discussion

3.22 discussion 3.23 2.0 Kg 3.24 discussion 3.25 discussion 3.26 (a) 1 Kg, 9.81N in Paris, 9.80N in Washington (b) individual calculation 3.27 individual calculation 3.28 discussion 3.29 (a) -5 × 10⁻²³ m/sec² (b) 10 m/sec (c) 1×10^{-22} m/sec 3.30 discussion 3.31 discussion **3.32** (a) diagram (b) 1.7×10^{-24} m/sec² (c) $\frac{6 \times 10^{24}}{10}$ (d) diagram 3.33 (a) 862N, 750N, 638N (b) The same as in (a) for scale calibrated in Newtons (c) discussion 3.34 hints for solving motion problems Chapter 4 4.1 Information 4.2 13.6 m/sec²; 2.71 sec; mass decreases 4.3 discussion 4.4 derivation 4.5 proof 4.6 1.3 m; at an angle of 67° below the horizontal; 5.1 m/sec, 78° below the horizontal 4.7 discussion 4.8 discussion 4.9 discussion 4.10 discussion 4.11 6.0×10^{-2} min, 3.0×10^{-2} min, 1.3×10^{-2} min 4.12 (a) 1.9 sec (b) 32 rpm (c) 50 cm/sec (d) 35 cm/sec (e) 0 (f) 190°/sec, yes (g) 120 cm/sec2 (h) 160 cm/sec² (i) discussion 4.13 discussion 4.14 discussion 4.15 table completion 4.16 (a) $2.2 \times 10^{-10} \text{ m/sec}^2$ (b) $4 \times$ 10²⁰ N (c) approximately 1/100 4.17 approximately 103 N 4.18 discussion 4.19 (a) Syncom 2 (b) Lunik 3 (c) Luna 4 (d) dosen't change 4.20 5.1 × 103 sec or 85 min 7.9×10^3 m/sec 4.21 discussion 4.22 7.1 × 103 sec or 120 min 4.23 (a) 3.6×10^2 sec (b) 36 Km (c) discussion

- 4.24 $t = (m/F)(V_0 V)$
- 4.25 discussion
- 4.26 essay



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