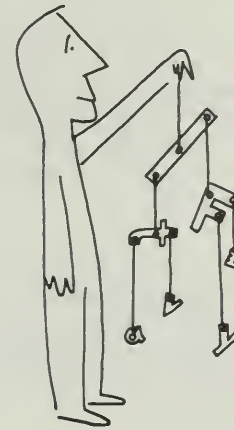
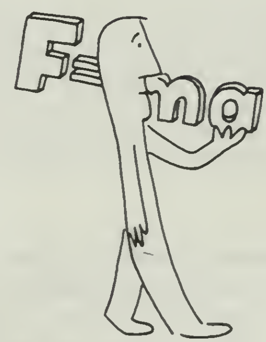
The Project Physics Course

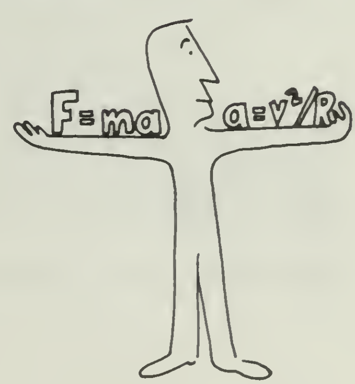
Equations 1 Simple Equations



Equations 2 Applications of Simple Equations



Equations 3 Combining Two Relationships



INTRODUCTION

You are about to use a programmed text. You should try to use this booklet where there are no distractions—a quiet classroom or a study area at home, for instance. Do not hesitate to seek help if you do not understand some problem. Programmed texts require your active participation and are designed to challenge you to some degree. Their sole purpose is to teach, not to quiz you.

This book is designed so that you can work through one program at a time. The first program, Equations 1, runs page by page across the top of each page. Equations 2 parallels it, running through the middle part of each page, and Equations 3 similarly across the bottom.

This publication is one of the many instructional materials developed for the Project Physics Course. These materials include Texts, Handbooks, Teacher Resource Books, Readers, Programmed Instruction Booklets, Film Loops, Transparencies, 16mm films and laboratory equipment. Development of the course has profited from the help of many colleagues listed in the text units.

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Equations 1 SIMPLE EQUATIONS

In physics and many other subjects it is useful to be able to handle simple algebraic equations. The purpose of this first program is to help you review the basic operations necessary to solve equations.

Equations 2 APPLICATIONS OF SIMPLE EQUATIONS

In this program you will gain practice in applying the basic algebraic operations you learned in Equations 1 to equations actually used in physics.

Equations 3 Combining Two Relationships

You saw in Equations 2 that relationships among physical quantities may be represented by equations and with the help of operations developed in Equations 1 you gained experience in solving these equations for some particular quantity or variable.

Often in physics we find a particular quantity in more than one relationship. For example, the quantity F occurs in each of the two equations $F = ma$ and $W = Fd$.

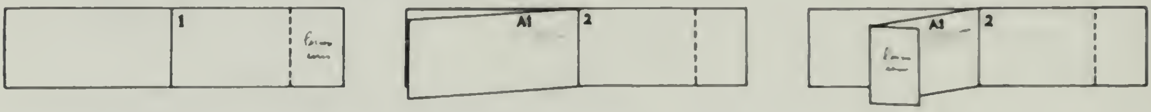
In this program we are going to see how two relationships having some quantity in common may be combined into a new relationship. For example, we can combine the two equations above and find an expression for W in terms of d, m, and a.

Often in this program more than one frame is used to solve a certain problem. We have followed the practice of repeating the problem in a box at the top of each of the frames concerned.

INSTRUCTIONS

1. *Frames:* Each frame contains a question. Answer the question by writing in the blank space next to the frame. Frames are numbered 1, 2, 3, . . .

2. *Answer Blocks:* To find an answer to a frame, turn the page. Answer blocks are numbered A1, A2, A3, . . . This booklet is designed so that you can compare your answer with the given answer by folding back the page, like this:



3. Always write your answer *before* you look at the given answer.

4. If you get the right answers to the sample questions, you do not have to complete the program.

INSTRUCTIONS: Same as for Equations 1, above.

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Introductory Frame A

Answer Space

Adding and subtracting. Study a, b, and c; then write down the results for d, e, and f.

a. $6b + 4b = 10b$
b. $4 - 10 = -6$
c. $p + 2q - 5p = 2q - 5p + p = 2q - 4p$
d. $6k + 3k =$
e. $4t - 10t =$
f. $-x + 2y + x =$

d. [ ]

e. [ ]

f. [ ]

1

Answer Space

In physics we often represent physical quantities by symbols and then represent relationships among physical quantities by equations.

For example, suppose a car which is moving with an initial speed v_i is given an acceleration of magnitude a and thus acquires a final speed of v_f in time t. The equation which describes the relationship among these four quantities is

$$v_f - v_i = at$$

(i) How many symbols representing physical quantities appear in the above equation?
(ii) List the symbols.

Turn page to begin Equations 3.

Answer A

d. | $9k$ |

e. | $-6t$ |

f. | $2y$ |

A1

(i) four

(ii) v_f

v_i

a

t

Introductory Frame B

Multiplying and dividing. Study a, b, and c; then write the results for d, e, and f.

a. $(-1)(+3) = -3$

b. $(-5)(-a) = +5a = 5a$

c. $\frac{6a}{-6} = -a$ (Note: $\frac{6a}{-6} = \frac{-6a}{6} = -\frac{6a}{6} = -a$)

d. $(-1)(-3b) =$

e. $(4)(-2p) =$

f. $\frac{-2a}{-a} =$

Answer Space

d.

e.

f.

2

First operation: If we add the same quantity to both sides of an equation, both sides will still be equal to each other.

Example:

$$a - b = 4c$$
$$a - b + b = 4c + b$$
$$a = 4c + b$$

Note that by adding b to both sides, we get a alone on one side of the equation, that is, we have solved the equation for a.

Solve the following equation for v_f in the same way:

$$v_f - v_i = at$$

1

If $m = 2p$ and $p = 3$, what is the value of m?

Answer B

d. $3b$

e. $-8p$

f. $+2$

A2

$$v_f - v_i = at$$

$$v_f - v_i + v_i = at + v_i$$

$$\boxed{v_f = at + v_i}$$

A1

$$m = 2p$$

$$m = (2)(3)$$

$$\boxed{m = 6}$$

Introductory Frame C

Operations with parentheses. Study a, b, c, and d; then write results for e, f, and g.

a. $3(a + 2c) = 3a + 6c$
b. $3(a - 2c) = 3a - 6c$
c. $-3(a - 2c) = -3a + 6c$
d. $-(p + q) = -p - q$
e. $5(k - 2m) =$
f. $-(t - 5) =$
g. $-4(2a + b) =$

Answer Space

e.

f.

g.

3

Second operation: If we subtract the same quantity from both sides of an equation, both sides will still be equal to each other.

Example:

$$b + 4 = a$$
$$b + 4 - 4 = a - 4$$
$$b = a - 4$$

Note that by subtracting 4 from both sides, we get *b* alone on one side of the equation, that is, we have solved the equation for *b*.

Solve the following equation for v_i:

$$v_i + at = v_f$$

2

If $v_1 = 4v_2$ and $v_2 = 2$, what is the number which v_1 represents?

Answer C

e. $5k - 10m$

f. $-t + 5$

g. $-8a - 4b$

A3

$$v_i + at = v_f$$

$$v_i + at - at = v_f - at$$

$$v_i = v_f - at$$

A2

$$v_1 = 4v_2$$

but $v_2 = 2$

so $v_1 = (4)(2)$

$$v_1 = 8$$

Introductory Frame D

More dividing! Study a, b, and c; then write results for d, e, and f.

a. $\frac{6a - 9b}{3} = 2a - 3b$

b. $\frac{6a + 9b}{-3} = -2a - 3b$

c. $\frac{6a - 9b}{-2} = -3a + \frac{9b}{2}$

d. $\frac{4a + 2b}{2} =$

e. $\frac{4a - 2b}{-2} =$

f. $\frac{3a + b}{-3} =$

Answer Space

d.

e.

f.

4

Third operation: If we divide both sides of an equation by the same quantity, both sides will still be equal to each other.

Example:

$$c = 4a$$
$$\frac{c}{4} = \frac{4a}{4}$$
$$\frac{c}{4} = a$$

Note that by dividing both sides by 4, we have solved for a.

If we return to the physics relationships we have been using and if the initial speed v_i is equal to 0, we have this relationship between v_f, a, and t:

$$v_f = at$$

Solve for v_f as in the example above.

3

If $v_1 = 4v_2$ and $v_2 = 4k$, what is the value of v_1 in terms of k?

Answer D

d. $2a + b$

e. $-2a + b$

f. $-a - \frac{b}{3}$

A4

$$v_f = at$$

$$\frac{v_f}{t} = \frac{at}{t}$$

$$\frac{v_f}{t} = a$$

or $a = \frac{v_f}{t}$

A3

$$v_1 = 4v_2$$

but $v_2 = 4k$

so $v_1 = 16k$

1

The equal sign (=) in an equation means that the symbols on one side represent the same quantity as the symbols on the other side. For example,

$2 + 4 = 3 + 3$ $\qquad$ $(3)(4) = (2)(6)$

$a + 4 = b + 3$ $\qquad$ $3m = 2n$

2, 3, 4, and 6 are called *numerals*. a, b, m, and n are called *variables* and may be replaced by *numerals*.

In the equation $(4)(3) = b + 2$, what numeral will replace b so that both sides of the equal sign represent the same quantity?

5

Fourth operation: If we multiply both sides of an equation by the same quantity, both sides will still be equal to each other.

Example:

$$\frac{a}{4} = b$$

$$(4)\frac{(a)}{4} = (4)(b)$$

$$a = 4b$$

Note by this operation we have solved for a.
Solve the following equation for v_i in the same manner:

$$a = \frac{v_f}{f}$$

4

In the previous frame we began with two equations $v_1 = 4v_2$ and $v_2 = 4k$, and we obtained a new equation $v_1 = 16k$.

Note that the new equation does not contain v_2. We have *eliminated* the quantity v_2 which is common to both of these equations.

How was v_2 eliminated? By taking the value of v_2 given in the second equation and substituting it for v_2 in the first equation.

Let us consider two other equations: $F = ma$ and $a = \frac{v^2}{R}$. What quantity is common to both of these equations?

A1

10

since the left side (3) (4) is equal to 12, and the right side will equal 12 if *b* is replaced by 10.

A5

$$a = \frac{v_f}{t}$$

$$(a)(t) = \frac{v_f}{t}(t)$$

$$at = v_f$$

or $\boxed{v_f = at}$

A4

a

2

We can write a new equation by adding the same quantity to both sides of a given equation. For example,

if $a = b$

then $a + 2 = b + 2$

and, in general, $a + k = b + k$

Change the equation

$$a - 4b = c$$

into a new equation by adding $6b$ to both sides.

6

We can apply any or all of these operations to manipulate symbols of an equation to express a given relationship in a more useful form.

For example: the equation $F = ma$ describes a relationship among the amount of net force F applied to an object, the mass m of the object, and the amount of acceleration a acquired by the object. We can show how this acceleration depends upon the mass and the force by solving the equation $F = ma$ for a.

Divide both sides by m, $\frac{F}{m} = \frac{ma}{m}$

$$\frac{F}{m} = a$$

or $a = \frac{F}{m}$

Now solve $F = ma$ for m in a similar way.

5

In the two equations, $F = ma$ and $a = \frac{v^2}{R}$, we may substitute the value of a in the second equation for a in the first equation, and by doing so get a new expression for F.

What is this new expression for F?

A3

$$x - y = c + d$$

$$x - y + y = c + d + y$$

but $-y + y = 0$

so $\boxed{x = c + d + y}$

A7

$$v = \frac{2\pi R}{T}$$

$$vT = \left(\frac{2\pi R}{T}\right)(T)$$

$$\boxed{vT = 2\pi R}$$

A6

$\boxed{a}$

This is the variable which is common to the two original equations.

4

Note that in answer frame A 3 by adding y to both sides of the equation we get x alone on the left hand side. Later we shall see an advantage in getting a variable by itself on one side of an equation.

What would you add to both sides of the equation

$$k - 20 = 6$$

so that k will be alone on the left hand side?

8

Now solve the new equation $vT = 2\pi R$ for T by dividing both sides by v.

7

Final speed v_f, amount of acceleration a, and time t are three characteristics of the motion of an object. For an object starting from rest, these are related in two equations:

$$v_f = at \quad \text{and} \quad d = \frac{1}{2} v_f t$$

Combine these two relationships into a new equation by eliminating v_f. This will give us an expression for d in terms of a and t.

A4

Add 20 to both sides

Thus $k - 20 = 6$

$$k - 20 + 20 = 6 + 20$$

$$k = 26$$

A8

$$vT = 2\pi R$$

$$\frac{vT}{v} = \frac{2\pi R}{v}$$

$$T = \frac{2\pi R}{v}$$

A7

Substitute the value of v_f in the first equation (that is, at) for v_f in the second equation

$$d = \tfrac{1}{2}(at)\,(t)$$

or $$d = \tfrac{1}{2}at^2$$

5

Subtracting the same quantity from both sides of an equation also gives a new equation. For example,

if $a = b$

then $a - 4 = b - 4$

and, in general, $a - c = b - c$

Make a new equation by subtracting 2 from both sides of the equation

$$x + 2 = y$$

9

Another equation which describes circular motion relates speed v, radius of path R, and acceleration toward the center of revolution a_c is

$$a_c = \frac{v^2}{R}$$

Solve this equation for the variable R. You can follow the same steps used in the previous two frames.

8

Given the two equations $a = \frac{v^2}{R}$ and $v = \frac{2\pi R}{T}$, find a in terms of R, T, and π.

A5

$$x + 2 = y$$
$$x + 2 - 2 = y - 2$$
$$\boxed{x = y - 2}$$

Note that by subtracting 2 from both sides we got x by itself on the left hand side of the equation.

A9

$$a_c = \frac{v^2}{R}$$

Multiply both sides by R:

$$a_c R = \frac{v^2 R}{R}$$

or $a_c R = v^2$

Divide both sides by a_c:

$$\frac{a_c R}{a_c} = \frac{v^2}{a_c}$$

$$\boxed{R = \frac{v^2}{a_c}}$$

A8

$$a = \frac{v^2}{R} = v^2 \left(\frac{1}{R}\right)$$

but $v = \frac{2\pi R}{T}$

so $a = \left(\frac{2\pi R}{T}\right)^2 \left(\frac{1}{R}\right)$

$$a = \frac{4\pi^2 R^2}{T^2 R}$$

or $\boxed{a = \frac{4\pi^2 R}{T^2}}$

6

We can make a new equation by multiplying both sides of an equation by the same quantity. Here are three examples to study.

(i) If $a = b$, then $5a = 5b$

(ii) If $p = (q + r)$, then $2p = 2(q + r)$
or, $2p = 2q + 2r$

(iii) If $\frac{m}{2} = 4$, then $(2)\left(\frac{m}{2}\right) = (2)(4)$

or, $m = 8$

Make a new equation by multiplying both sides of the equation

$$\tfrac{1}{3}a = b + 2$$

by 3.

10

Suppose we wish to solve the equation

$$a_c = \frac{v^2}{R}$$

for v.

First solve for v^2 by performing the necessary operation.

9

Let us look at a situation which requires an additional step.

If $v = at$ and $F = ma$, find an expression for F in terms of m, v, and t. In other words, combine the equations to eliminate a. To do this we can

(i) solve one equation for a, and
(ii) substitute this value for a in the remaining equation.

To solve the first equation $v = at$ for a, what operation would you perform?

A6

$\frac{1}{3}a = b + 2$

$\frac{3}{3}a = 3(b + 2)$

Note that $\frac{3}{3} = 1$

and $3(b + 2) = 3b + 6$

so $\boxed{a = 3b + 6}$

A10

$$a_c = \frac{v^2}{R}$$

Multiply both sides by R:

$$a_c R = \frac{v^2 R}{R}$$

$$a_c R = v^2$$

or $\boxed{v^2 = a_c R}$

A9

Divide both sides by t.

7

Make $c - b = 3$

into a new equation by multiplying both sides by -1.

11

If $v^2 = a_c R$,

find an expression for v, that is, solve this equation for v.

10

OUR PROBLEM:	If $v = at$ and $F = ma$, find F in terms of m, v, and t.

When we divide both sides of $v = at$ by t, we get

$$\frac{v}{t} = \frac{at}{t}$$

$$\frac{v}{t} = a$$

$$\text{or} \quad a = \frac{v}{t}$$

Now substitute this value for a in the second equation, $F = ma$.

A7

$$c - b = 3$$
$$-1(c - b) = (-1)(3)$$

Note that $(-1)(c) = -c$

and $(-1)(-b) = +b$

so $-c + b = -3$

or $\boxed{b - c = -3}$

A11

$$v^2 = a_c R$$

Take the square root of both sides:

$$\sqrt{v^2} = \sqrt{a_c R}$$

$$\boxed{v = \sqrt{a_c R}}$$

If you are not familiar with square roots, ask your teacher for help.

A10

$$F = ma$$

but $a = \frac{v}{t}$

so $F = m\left(\frac{v}{t}\right)$

$$\boxed{F = \frac{mv}{t}}$$

8

Dividing both sides of an equation by the same quantity gives a new equation.

Make a new equation by dividing both sides of the equation

$$3b = 12$$

by 3.

12

In the equation

$$2d = at^2$$

d represents the distance an object moves from rest in time *t* when given a constant acceleration *a*.

To solve this equation for *t*, solve first for t^2 and for *t*.

11

In the study of electricity you will become familiar with power *P*, voltage *V*, current *I*, and resistance *R*.

Suppose we are given the relationships represented by the equations $P = VI$ and $I = \frac{V}{R}$ and we wish to find *P* in terms of *I* and *R*.

What two steps would you perform?

A8

$3b = 12$

$\frac{3b}{3} = \frac{12}{3}$

$\boxed{b = 4}$

A12

$2d = at^2$

Divide both sides by a:

$\frac{2d}{a} = \frac{at^2}{a}$

$\frac{2d}{a} = t^2$

or $t^2 = \frac{2d}{a}$

Take the square root of both sides

$\boxed{t = \sqrt{\frac{2d}{a}}}$

A11

(1) Solve $I = \frac{V}{R}$ for V.

(2) Substitute this value for V in $P = VI$.

(Alternatively, it is possible to solve $P = VI$ for V and to substitute this value in $I = \frac{V}{R}$. This would not be as direct a method, however.)

9

Make a new equation by dividing both sides of the equation

$$2b = c - 4a$$

by 2.

13

Consider again the equation:

$$v_f = v_i + at$$

If we wish to find an expression for t, we must get at alone on the right hand side of the equation. How can we do this?

12

OUR PROBLEM:	If $P = VI$ and $I = \frac{V}{R}$ find P in terms of I and R.

Perform the two steps listed in answer A11.

A9

$$2b = c - 4a$$

$$\frac{2b}{2} = \frac{c - 4a}{2}$$

$$b = \frac{c - 4a}{2}$$

or $b = \frac{c}{2} - 2a$

A13

Subtract v_i from both sides of the equation.

A12

If $I = \frac{V}{R}$

then $IR = V$

or $V = IR$

Substitute this value for V in $P = VI$:

thus $P = (IR)(I)$

or $P = I^2R$

10

The two sides of an equation will still be equal to each other if we do any of the following:

(i) *add* the same quantity to both sides
(ii) *subtract* the same quantity from both sides
(iii) *multiply* both sides by the same quantity
(iv) *divide* both sides by the same quantity.

These four operations which you have learned will be used many times in this program. Use one of these operations to change the equation

$$c + b = 3$$

so that c is the only symbol on one side of the equation; we call this "solving the equation for c."

14

Subtracting v_i from both sides of the equation

$$v_f = v_i + at$$

gives us:

$$v_f - v_i = v_i + at - v_i$$

$$v_f - v_i = at$$

or $$at = v_f - v_i$$

Now solve this new equation for *t*.

13

We are going to use the following equations to introduce one further step in combining two relationships:

If $5v + 2t = k,$

and $v + t = m,$

find an expression for v in terms of *k* and *m*.

First of all, what quantity should we eliminate?

A10

To get c alone on the left side of the equation, subtract b from both sides.

$$c + b = 3$$
$$c + b - b = 3 - b$$
$$c = 3 - b$$

You have thus solved the equation $c + b = 3$ for c.

A14

$$at = v_f - v_i$$

Divide both sides by a:

$$\frac{at}{a} = \frac{v_f - v_i}{a}$$

$$t = \frac{v_f - v_i}{a}$$

A13

t

11

Change the equation

$$v_1 + v_2 = 6h$$

so that v_2 is the only symbol on one side of the equation; that is, solve the equation for v_2.

15

Here is another equation similar to the one which we just solved:

$$v_f^2 = v_i^2 + 2ad$$

Use similar steps to solve the equation for *d*.

14

OUR PROBLEM:	If $5v + 2t = k$ and $v + t = m$ find an expression for *v* in terms of *k* and *m*.

To eliminate *t* we first solve one equation for *t*. Which equation should we select so that solving for *t* will require the lest number of steps?

A11

$$v_1 + v_2 = 6h$$

$$v_1 + v_2 - v_1 = 6h - v_1$$

$$\boxed{v_2 = 6h - v_1}$$

A15

$$v_f^2 = v_i^2 + 2ad$$

Subtract v_i^2 from both sides:

$$v_f^2 - v_i^2 = v_i^2 + 2ad$$

$$v_f^2 - v_i^2 = 2ad$$

Divide both sides by $2a$:

$$\frac{v_f^2 - v_i^2}{2a} = \frac{2ad}{2a}$$

$$\frac{v_f^2 - v_i^2}{2a} = d$$

or $$\boxed{d = \frac{v_f^2 - v_i^2}{2a}}$$

A14

The second equation,
$v + t = m$

To solve for t we subtract v from both sides.
To solve for t in the first equation, $5v + 2t = k$, we must first subtract $5v$ from both sides and then divide both sides by 2.

12

To solve the equation

$$x - y = 3$$

for y, we may first change the equation so that $-y$ appears alone on the left side. How can we do this?

16

If $$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

write an equation to show how T_2 depends on the other variables.

We want to write an equation in the form T_2 = _____. To get T_2 on top, begin by multiplying both sides of the equation T_2.

15

OUR PROBLEM:	If $5v + 2t = k$ and $v + t = m$, find an expression for v in terms of k and m.

Solve the second equation for t.

A12

Subtract x from both sides.

A16

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$\frac{P_1V_1T_2}{n_1T_1} = \frac{P_2V_2T_2}{n_2T_2}$$

$$\boxed{\frac{P_1V_1T_2}{n_1T_1} = \frac{P_2V_2}{n_2}}$$

A15

Subtract v from both sides:

$v + f = m$

$\boxed{f = m - v}$

13

When x is subtracted from both sides of the equation

$$x - y = 3$$

we get $\quad x - y - x = 3 - x$

or $\quad x - x - y = 3 - x$

so $\quad -y = 3 - x$

Now solve this new equation for y by multiplying both sides by (-1).

17

In A16, note that T_2 is multiplied by P_1V_1 and divided by n_1T_1. To solve for T_2 we must multiply both sides of the equation by n_1 and T_1 (or by n_1T_1) and then divide both sides by P_1 and V_1 (or by P_1V_1).

First multiply by n_1T_1.

16

OUR PROBLEM:	If $5v + 2t = k$ and $v + t = m$, find an expression for v in terms of k and m.

Substitute this value for t in the first equation.

A13

$$-y = 3 - x$$
$$(-1)(-y) = (-1)(3 - x)$$
$$y = -3 + x$$

or $\boxed{y = x - 3}$

A17

$$\frac{P_1V_1T_2}{n_1T_1}(n_1T_1) = \frac{P_2V_2}{n_2}(n_1T_1)$$

$$\boxed{T_2P_1V_1 = \frac{P_2V_2n_1T_1}{n_2}}$$

A16

$$5v + 2t = k$$

but $t = m - v$

so $\boxed{5v + 2(m - v) = k}$

14

Use the two steps described in frames 12 and 13 to solve the equation

$$s_2 - s_1 = 3$$

for s_1.

18

Now divide both sides of the equation in A17 by $P_1 V_1$ to give an expression for T_2.

17

OUR PROBLEM:	If $5v + 2t = k$ and $v + t = m$, find an expression for v in terms of k and m.

Finally solve the equation in answer frame A16 for v.

A14

$$s_2 - s_1 = 3$$

Subtract s_2 from both sides.

$$s_2 - s_1 - s_2 = 3 - s_2$$

$$-s_1 = 3 - s_2$$

Multiply both sides by (−1).

$$(-1)(-s_1) = (-1)(3 - s_2)$$

$$s_1 = -3 + s_2$$

or $\boxed{s_1 = s_2 - 3}$

A18

$$\frac{P_1V_1T_2}{P_1V_1} = \frac{P_2V_2n_1T_1}{(P_1V_1)n_2}$$

$$\boxed{T_2 = \frac{P_2V_2n_1T_1}{P_1V_1n_2}}$$

A17

$$5v + 2(m - v) = k$$

Remove porenthesis:

$$5v + 2m - 2v = k$$

or $3v + 2m = k$

Subtract $2m$ from both sides

$$3v = k - 2m$$

Divide both sides by 3·

$$\boxed{v = \frac{k - 2m}{3}}$$

15

Dividing both sides of the equation

$$3b = 3a - c$$

by 3 will get b alone on the left hand side of the equation.

Solve the equation for b by performing this operation.

19

Returning again to the equation

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}.$$

Use the steps we used in the last three frames to solve this equation for T_1.

18

Let us review the steps we have been studying in combining two relationships to obtains the value of a particular variable.

(i) Examine the two equations to see which quantity should be eliminated—the quantity we are *not* interested in for the moment.

(ii) Select one of these equations and solve for this quantity.

(iii) Substitute the value you obtained for this quantity into the other equation.

(iv) Solve the new equation for the variable whose value is desired.

Check these steps carefully and look back at previous frames if necessary to see where we applied each step.

A15

$$3b = 3a - c$$

$$\frac{3b}{3} = \frac{3a - c}{3}$$

$$b = \frac{3a - c}{3}$$

or $$b = a - \frac{c}{3}$$

A19

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

Multiply both sides by T_1:

$$\frac{P_1V_1}{n_1} = \frac{P_2V_2T_1}{n_2T_2}$$

Multiply both sides by n_2T_2:

$$\frac{P_1V_1n_2T_2}{n_1} = P_2V_2T_1$$

Divide both sides P_2V_2:

$$\frac{P_1V_1n_2T_2}{P_2V_2n_1} = T_1$$

or $$T_1 = \frac{P_1V_1n_2T_2}{P_2V_2n_1}$$

16

It is not necessary of course that the symbol being solved for always appear alone on the *left* side of the equation; the right side will do as well. For example, an alternative way to solve the equation

$$s_2 - s_1 = 3$$

for s_1 would be to add s_1 to both sides:

$$s_2 - s_1 + s_1 = 3 + s_1$$
$$s_2 = 3 + s_1$$

and then subtract 3 from both sides:

$$s_2 - 3 = 3 + s_1 - 3$$
$$s_2 - 3 = s_1$$

Solve the following equation for x in a similar way:

$$6 - x = y$$

20

Examine carefully the equation

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2},$$

and then write down the two operations which you would perform to solve for V_2.

19

Consider these two equations describing uniform acceleration from rest:

$$v = at \quad \text{and} \quad d = \tfrac{1}{2}at^2$$

(i) If we desire to find a in terms of v and d, what quantity would you eliminate?

(ii) Which equation would you select to solve for this quantity most easily?

(iii) Solve this equation for the quantity to be eliminated.

(i) ____

(ii) ____

(iii) ____

A16

$$6 - x = y$$

$$6 - x + x = y + x$$

$$6 = y + x$$

$$6 - y = y + x - y$$

$$\boxed{6 - y = x}$$

A20

(i) Multiply both sides by $n_2 T_2$

(ii) Divide both sides by P_2

A19

(i) $\boxed{t}$

(ii) $\boxed{v = at}$

(iii) Divide both sides by a.

$$\frac{v}{a} = t$$

or $\boxed{t = \frac{v}{a}}$

17

Here is another similar equation:

$$5k = 5t + 15s$$

Solve this equation for k.

21

Perform the two operations listed in A20 to solve the equation

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

for V_2.

20

OUR PROBLEM:	If $v = at$ and $d = \frac{1}{2}at^2$, find a in terms of v and d.

Substitute the value we have found for t into the second equation, and do any necessary simplification.

A17

$$5k = 5t + 15s$$

$$\frac{5k}{5} = \frac{5t + 15s}{5}$$

$$\boxed{k = t + 3s}$$

A21

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$\frac{P_1V_1n_2T_2}{n_1T_1} = P_2V_2$$

$$\frac{P_1V_1n_2T_2}{n_1P_2T_1} = V_2$$

$$\boxed{V_2 = \frac{P_1V_1n_2T_2}{P_2n_1T_1}}$$

A20

$$d = \frac{1}{2}at^2$$

but $t = \frac{v}{a}$

so $d = \frac{1}{2}(a)\left(\frac{v}{a}\right)^2$

$$d = \frac{1}{2}(a)\ \frac{v^2}{a^2}$$

and $\boxed{d = \frac{v^2}{2a}}$

18

We can solve the equation

$$\frac{b}{5} = 2a + c$$

for b if we multiply both sides of this equation by 5.

Solve this equation for b.

22

Solve $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ for P_1.

21

OUR PROBLEM:	If $v = at$ and $d = \frac{1}{2}at^2$ find a in terms of v and d.

Finally solve the new equation in the answer block A20. for a.

A18

$$\frac{b}{5} = 2a + c$$

$$\frac{5b}{5} = 5(2a + c)$$

$$\boxed{b = 10a + 5c}$$

A22

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

Multiply both sides by n_1T_1:

$$P_1V_1 = \frac{n_1T_1P_2V_2}{n_2T_2}$$

Divide both sides by V_1:

$$\boxed{P_1 = \frac{n_1T_1P_2V_2}{n_2T_2V_1}}$$

A21

$$d = \frac{v^2}{2a}$$

Multiply both sides by a:

$$ad = \frac{v^2}{2}$$

Divide both sides by d:

$$\boxed{a = \frac{v^2}{2d}}$$

19.

We wish to solve the equation

$$3(a + b) = c$$

for b.

To solve for a quantity within parentheses, we can first remove the parentheses by performing the indicated operation.

Remove the parentheses in the equation

$$3(a + b) = c$$

by multiplying out $(a + b)$ by 3.

23

An expression for kinetic energy E_k in terms of mass (m) and speed (v) is

$$E_k = \frac{1}{2}mv^2$$

Solve this equation for v.

22

Rewrite this relationship as an expression for v instead of a.

That is, solve $a = \frac{v^2}{2d}$ for v.

A19

$3(a + b) = c$

$$\boxed{3a + 3b = c}$$

A23

$$E_k = \frac{1}{2}mv^2$$

Multiply both sides by 2:

$$2E_k = mv^2$$

Divide both sides by *m*:

$$\frac{2E_k}{m} = v^2$$

Take the square root of both sides

$$\sqrt{\frac{2E_k}{m}} = v$$

or $\boxed{v = \sqrt{\frac{2E_k}{m}}}$

A22

$$a = \frac{v^2}{2d}$$

Multiply both sides by 2*d*:

$$2ad = v^2 \quad \text{or} \quad v^2 = 2ad$$

Take the square root of both sides:

$$\boxed{v = \sqrt{2ad}}$$

20

Now we wish to solve the equation

$$3a + 3b = c$$

for b. First get $3b$ by itself on the left hand side.

24

Our last equation is one which you will study in connection with Newton's Law of Gravitation:

$$\vec{F} = \frac{Gm_1m_2}{R^2}$$

Solve this equation for R.

23

We could use the same two equations

$$v = at \quad \text{and} \quad d = \tfrac{1}{2}at^2$$

to find d in terms of v and t.

Remember to (i) decide which quantity should be eliminated;
(ii) select one equation to solve for this quantity;
(iii) substitute this value into the other equation;
and (iv) solve, if necessary, to get the required variable alone.

Now find d in terms of v and t.

A20

$$3a + 3b = c$$

Subtract $3a$ from both sides.

$$3a + 3b - 3a = c - 3a$$

$$\boxed{3b = c - 3a}$$

A24

$$\vec{F} = \frac{Gm_1m_2}{R^2}$$

Multiply both sides by R^2:

$$\vec{F}R^2 = Gm_1m_2$$

Divide both sides by $\vec{F}$:

$$R^2 = \frac{Gm_1m_2}{\vec{F}}$$

Take the square root of both sides:

$$\boxed{R = \sqrt{\frac{Gm_1m_2}{\vec{F}}}}$$

A23

Eliminate a by solving the equation $v = at$ for a

$$a = \frac{v}{t}$$

and substituting its value in the equation $d = \frac{1}{2}at^2$:

$$d = \frac{1}{2}\left(\frac{v}{t}\right)(t^2)$$

$$\boxed{d = \frac{1}{2}vt}$$

21

Now divide the equation

$$3b = c - 3a$$

by 3 to solve for *b*.

This is the end of Equations 2. The last program in this series, Equations 3 Combining Two Relationships, starts at the front of the book.

24

Here is a sequence of three equations

$$W = Fd \text{ - - - - - - - - - - - } (1)$$

$$F = ma \text{ - - - - - - - - - - - } (2)$$

$$d = \frac{v^2}{2a} \text{ - - - - - - - - - - - } (3)$$

relating to work *W*, force *F*, distance *d*, acceleration *a*, and speed *v* (for motion beginning from rest).

We want to find the amount of work *W* required to get a body of mass *m* moving at speed *v*. That is, we want to find *W* in terms of *m* and *v*. First, combine the equations (1) and (2) to eliminate *F*, a term we are not presently interested in.

A21

$$3b = c - 3a$$

$$\frac{3b}{3} = \frac{c - 3a}{3}$$

$$b = \frac{c - 3a}{3}$$

or $$b = \frac{c}{3} - a$$

A24

Substitute the value of F in equation (2) into equation (1):

$$W = Fd$$

but $F = ma$

so $W = (ma)(d)$

or $W = mad$

22

Here is another equation with a quantity in parentheses:

$$5(x - 3) = 5$$

Solve for x following the three steps used in the last three frames.

25

OUR PROBLEM:	$W = Fd$ - - - - - - - - - - (1)
	$F = ma$ - - - - - - - - - - (2)
	$d = \frac{v^2}{2a}$ - - - - - - - - - - (3)
	Find W in terms of m and v.

Combine the new equation $W = mad$ with equation (3) above and solve for W in terms of m and v by eliminating d.

A22

$$5(x - 3) = 5$$

Remove parentheses by multiplying $(x - 3)$ by 5.

$$5x - 15 = 5$$

Get $5x$ alone on one side of the equation by adding 15 to both both sides.

$$5x - 15 + 15 = 5 + 15$$

$$5x = 20$$

Divide both sides by 5.

$$\frac{5x}{5} = \frac{20}{5}$$

$$\boxed{x = 4}$$

A25

$$W = mad$$

$$d = \frac{v^2}{2a}$$

so $W = ma\left(\frac{v^2}{2a}\right)$

$$W = \frac{mav^2}{2a}$$

or $\boxed{W = \frac{mv^2}{2}}$

23

Suppose you are asked to solve the equation

$$3a + 4b = a + b$$

for b. Note that the quantity b appears on both sides of the equation.

To solve for a quantity which appears more than once in an equation, begin by changing the equation so that this quantity appears only on one side.

Change

$$3a + 4b = a + b$$

so that b appears only on the left side; that is, subtract b from both sides.

26

One final problem. For an object revolving around a central point, the amount of centripetal force F_c and centripetal acceleration a_c, the mass m of the object, its speed v, the radius of revolution R and the time of one revolution T, are related by these equations:

$$F_c = ma_c \qquad vT = 2\pi R \qquad a_c = \frac{v^2}{R}$$

Find an expression for F_c in terms of m, T and R.

A23

$$3a + 4b = a + b$$

$$3a + 4b - b = a + b - b$$

$$\boxed{3a + 3b = a}$$

A26

$$\boxed{F_c = \frac{4\pi^2 mR}{T^2}}$$

If you got this answer, you can feel confident of your ability to handle simple equations. If you missed this problem, review frames 24 and 25, and then try again. Should you still have trouble, ask your teacher for help.

24

Now solve

$$3a + 3b = a$$

for *b*, that is, get *b* alone on one side of the equation.

You have now reviewed simple equations and the combining of two relationships. You should have little difficulty in understanding the development of many relationships found in your study of physics.

You may want to review these programs at some time later. Just take some blank pages and place them over your former answers and record your answers again and compare with the answer blocks.

A24

$$3a + 3b = a$$

Subtract $3a$ from both sides.

$$3a + 3b - 3a = a - 3a$$

$$3b = -2a$$

Then divide both sides by 3.

$$\frac{3b}{3} = \frac{-2a}{3}$$

$$\boxed{b = -\frac{2a}{3}}$$

25

Here is an equation with q on both sides of the equation:

$$2(p - q) = 3h + q$$

We want to solve this equation for q. First remove the parentheses and get q on one side of the equation.

A25

$$2(p - q) = 3h + q$$

Remove parentheses.

$$2p - 2q = 3h + q$$

Subtract q from both sides:

$$\boxed{2p - 3q = 3h}$$

26

In the new equation in A25 on the opposite page, solve for q by getting $-3q$ by itself on the left hand side and then dividing both sides by -3.

A26

$$2p - 3q = 3h$$

$$2p - 3q - 2p = 3h - 2p$$

$$-3q = 3h - 2p$$

$$\frac{-3q}{-3} = \frac{3h - 2p}{-3}$$

$$q = \frac{3h - 2p}{-3}$$

or

$$q = -h + \frac{2}{3}p$$

or

$$\boxed{q = \frac{2}{3}p - h}$$

27

Following the steps described in frames 25 and 26, solve the following equation for s:

$$3(r - 2s) = r + 3s$$

Note: As was pointed out before, equations can be solved by isolating the symbol being solved for on the right hand side of the equation. Thus, an alternative way to solve for s after removing brackets from

$$3(r - 2s) = r + 3s$$

would be to (i) add 6s to both sides, or (ii) subtract r from both sides, and then (iii) divide by 9.

$$3r - 6s = r + 3s$$

(i) $3r = r + 9s$

(ii) $2r = 9s$

(iii) $\frac{2r}{9} = s$

A27

$$3(r - 2s) = r + 3s$$

Remove brackets.

$$3r - 6s = r + 3s$$

Subtract 3s from both sides.

$$3r - 6s - 3s = r + 3s - 3s$$

$$3r - 9s = r$$

Subtract 3r from both sides.

$$3r - 9s - 3r = r - 3r$$

$$-9s = -2r$$

Then divide both sides by −9.

$$\frac{-9s}{-9} = \frac{-2r}{-9}$$

$s = \frac{2r}{9}$	See note at left.

28

We shall conclude this program with a few simple equations to solve.

Solve $6p - 2t = s$

for p.

A28

$$6p - 2t = s$$

Add $2t$ to both sides.

$$6p = s + 2t$$

Divide both sides by 6.

$$p = \frac{s + 2t}{6}$$

or

$$p = \frac{s}{6} + \frac{t}{3}$$

29

Solve

$$m - \frac{1}{2}n = 8$$

for n.

A29

$m - \frac{1}{2}n = 8$

Subtract *m* from both sides.

$-\frac{1}{2}n = 8 - m$

Multiply both sides by −1.

$\frac{1}{2}n = -8 + m$

or $\frac{1}{2}n = m - 8$

Multiply both sides by 2.

$n = 2(m - 8)$

or $n = 2m - 16$

30

Solve

$$2(3a + b) = a + 7b$$

for a.

A 30

$$2(3a + b) = a + 7b$$
$$6a + 2b = a + 7b$$
$$6a - a + 2b = 7b$$
$$5a + 2b = 7b$$
$$5a = 7b - 2b$$
$$5a = 5b$$
$$\boxed{a = b}$$

You have now completed Equations 1 and are able to handle the main algebraic operations. You can practice this skill in the context of physics equations by going through the program Equations 2. It begins at the front of this book just below this program.

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