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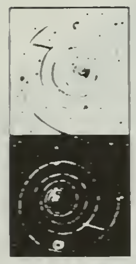

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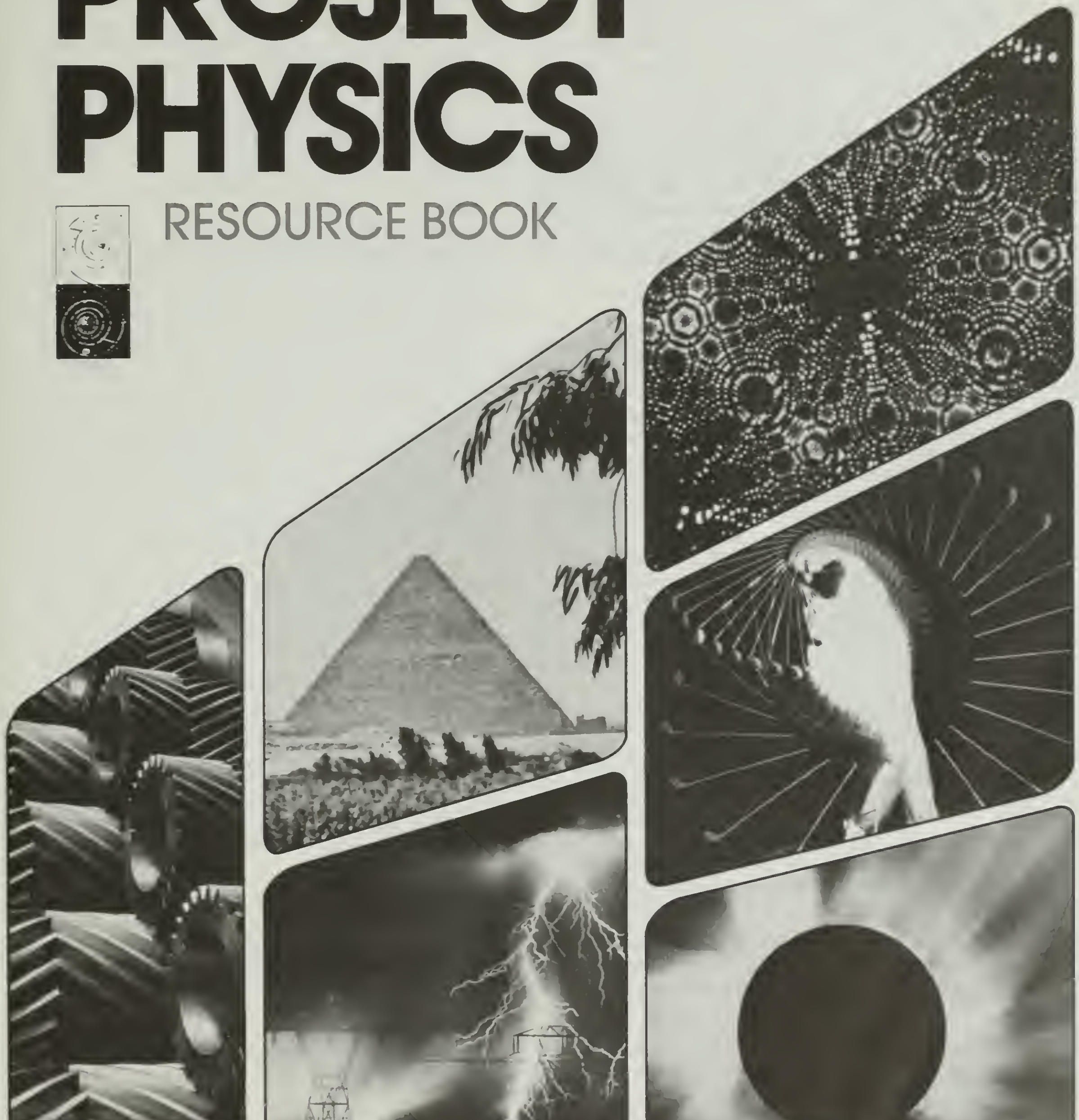

Directors of Harvard Project Physics

F. James Rutherford
Department of Science Education, New York University

Gerald Holton
Department of Physics, Harvard University

Fletcher G. Watson
Harvard Graduate School of Education

Editorial Development: William N. Moore, Roland Cormier, Lorraine Smith-Phelan
Editorial Processing: Margaret M. Byrne, Regina Chilcoat, Holly Massey
Art, Production, and Photo Resources: Vivian Fenster, Fred C. Pusterla, Robin M. Swenson
Annette Sessa, Beverly Silver, Anita Dickhuth, Dorina Virdo
Product Manager: Laura Zuckerman
Advisory Board: John Taggart, Maurice E. Fey, Norman Hughes, David J. Miller, John W. Griffiths
William L. Paul
Consultant: John Matejowsky
Researchers: Pamela Floch, Gerard LeVan

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Introduction

General Background

The Project Physics Course is based on the ideas and research results of the Harvard Project Physics curriculum development group. This national course improvement effort formally began in the spring of 1964. At that time Gerald Holton, James Rutherford, and Fletcher Watson of Harvard University received support from the United States Office of Education and the National Science Foundation, which enabled them to bring together professional people from all parts of the nation to work on the improvement of physics education.

Informally, the Project had started several years earlier, when Rutherford was a physics teacher and science department head in a public high school. Holton and Watson agreed to collaborate with him in testing the feasibility of designing a new physics course. With the story-line and aims in Gerald Holton's college text, *Introduction to Concepts and Theories in Physical Science*, as a general guide, preparation of a course outline and instructional materials was begun. In 1962, the founders obtained initial support from the Carnegie Corporation in New York, which allowed them to test their materials. The success of these tests, coupled with the increasing national awareness that something needed to be done about decreasing high school physics enrollments, led to the formation of Harvard Project Physics. The decision to expand to a national program was stimulated by a request from the National Science Foundation late in 1963.

The general purposes of Project Physics remained constant from the beginning, when three individuals worked without support, through the time of peak developmental activity involving hundreds of scientists, teachers, psychologists, artists, and other professional participants from throughout the United States and Canada, as well as thousands of students in trial classes. To some degree, the purposes reflected the fact that the directors of the Project were, respectively, a university physicist, a professor of science education, and an experienced high-school physics teacher. The chief purposes were:

1. *To design a humanistically oriented physics course.* Harvard Project Physics would show the science of physics in its proper light as a broadly based intellectual activity that has firm historical roots and that profoundly influences our whole culture.

2. *To develop a course that would attract a large number of high school students to the study of introductory physics.* Such a course must be meaningful not only to those who are already intent on a scientific career, but also to those who may not go on to college and to those who, while in college, will concentrate on the humanities or social sciences.

3. *To contribute to the knowledge of the factors that influence science learning.* In addition to its long-term value, this extensive educational research should supply information needed by teachers and administrators in deciding whether to introduce the course and, if so, in what way and for which students. The research results have been reported in professional journals and dissertations, and in the book *A Case Study in Curriculum Evaluation: Harvard Project Physics.*

Specific Goals of the Project Physics Course

The first two general aims of Harvard Project Physics (to develop a humanistically oriented physics course, and to help increase high school physics enrollments) can be restated in somewhat more specific terms. The Project Physics Course and course materials were designed to accomplish the following goals:

1. To help students to increase their knowledge of the physical world by concentrating on the ideas that characterize physics as a science at its best (for example, the conservation laws), rather than concentrating on isolated bits of information (such as the lens formula).

2. To help students see physics as the many-sided human activity that it really is. This means presenting the subject in historical and cultural perspective, and showing that the ideas of physics have not only a tradition but methods of adaptation and change.

3. To increase the opportunity for each student to have immediate rewarding experiences in science while gaining knowledge and skill that will be useful throughout life.

4. To make it possible for teachers to adapt the physics course to the wide range of interests and abilities among their students.

5. To recognize the importance of the teacher in the educational process, and the vast spectrum of teaching situations that prevail.

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Unit 6 / The Nucleus

Concepts of Motion

Organization of Instruction

THE MULTI-MEDIA SYSTEMS APPROACH

The Multi-media Systems approach is just one of many possible styles of classroom management. Here the teacher is a manipulator of environment and a tutor. The manipulation affords the control of the program by the teacher. At the same time, the students experience a measure of freedom in styles of learning. Much of the time the teacher tutors by answering and asking specific questions of small groups of individuals. The style is informal and nonauthoritative. However, on occasion the teacher makes a presentation to the entire class. For example, in the Chapter 1 daily plan the teacher presents graphs, velocity, and acceleration, on the 6th day. The students can request additional presentations on specific topics.

These are styles of teaching as good as this one. There are many different organizations of work within the framework of Multi-media Systems. However, this plan is offered so that a new teacher may see one organization of a program for a unit of *Project Physics*. Teachers are invited to modify this plan or invent their own style.

THE MULTI-MEDIA SCHEDULE

Day 1
Devote the time necessary for the opening of school. Take the class through a tour of six or seven media of instruction. Mention that the first reading assignment is not about physics but about what a physicist does and the materials available to learn physics.

Day 2
This day is used to explain the Multi-media System and to charge students with the responsibility of self-directed instruction.

Day 3
After the *Film Loop*, divide the class at random into small groups. Pass out three or four open-ended questions about the *Film Loop*. Be a listener.

Take a minute to comment on how to use the *Text* most effectively.

Day 4
Lab Stations: Uniform Motion

Students are to make qualitative observations of objects undergoing uniform motion. Students spend

8 to 10 minutes at each station. Brief instruction of what to look for at each station will be helpful.

1. balloon pucks on glass tray
2. pucks on plastic beads
3. *D2* (dynamics cart with accelerometer)
4. Polaroid photograph of tractor, blinky
5. *L8* and *L9* (*Film Loops*)
6. *T0* or *T1* (*Transparencies*)

Take a minute to describe the *Handbook*.

Day 5
Lab Stations: Accelerated Motion

1. *D4* (dynamics cart with accelerometer)
2. strobe photo of free fall
3. *L4* (a matter of relative motion)
4. *D3* (analysis of strobe photo)
5. *L9* (analysis of hurdle race)

Take a minute to mention your presentation tomorrow. Encourage the recording of data.

Day 6
Although the time is overdue for explanations, you have the students where you want them. Each has a head full of questions and a fist full of data. Various demonstrations, transparencies, and examples may be used to clarify the concepts and their measurements. Comment on *Study Guide* questions.

Day 7
Post answers. Let students who have many correct answers go on to other activities you have set up.

Design problem-solving group procedures carefully.

Allow individual problem solving. Drop in on all groups.

Take a minute to comment on the assignment with the first student evaluation in mind.

Day 8
Divide the class into small discussion groups. Have each group read and discuss the quoted dialogues in Galileo's *Two New Sciences*. Give some open-ended questions to each group.

Day 9
Explain *E1–5* "A Seventeenth Century Experiment" in detail. Concentrate on the structure of scientific thought, including definitions, assumptions, and Galileo's difficulty in testing his notion of acceleration.

Day 10
Students perform "A Seventeenth Century Experiment" (*E1–5*). Near the end of the period disturb each group with many questions about this experiment.

Day 11
Assign a few problems on free-fall acceleration and "A Seventeenth Century Experiment" to each group. You can work toward developing mathematical skills for the free-fall laboratory tomorrow.

Take a minute to mention to students that they should survey the Activities section of the *Handbook* with the idea of choosing their own activity tomorrow.

Day 12
Students can elect to do a detailed study on one of the following:

1. *L1* or *L2*
2. a_g by any of the methods in *E1–7*
3. any activity

Day 13
This lecture should touch upon the life and times of Galileo and also on the need to test theories by performing experiments.

Discuss: Is free fall the same for different masses? Is it the same at all positions in space?

Day 14
E1–1 "Naked-Eye Astronomy" requires the taking of data systematically over a period of weeks. Assign students and/or groups specific objects on which to gather data: sun, moon, specific stars, planets, etc. These observations will be very useful if they are carried out before Unit 2.

Day 15
Lab Stations: Vectors

1. *D7* (two ways to demonstrate addition of vectors)
2. *D8* (direction of acceleration and velocity)
3. *L3* (vector addition)
4. PSSC film, "Vectors"

Day 16
Explain vectors. Use *Film Loop 3*.

Day 17
Lab Stations: Force, Mass, and Acceleration

1. *D11* (inertia)
2. PSSC Exp. 21 (dependence of acceleration upon force and mass)
3. PSSC Exp. 20 (changes in velocity with a constant force)
4. *D12* (Newton's laws—air track)
5. *T8* (tractor-log problem)
6. *E1–8* (Newton's second law)

Day 18
Teacher-led discussion: Newton's Laws

Clarify points still unclear about Newton's three laws of motion. This should be the most crucial class so far this year.

Day 19
Small-group problem-solving session

Post answers. Let students who have many of the correct answers go on to other activities you have set up. Design problem-solving group procedure carefully. Allow individual problem solving. Drop in on all activities to help.

Day 20
Invite student leaders from different research groups to present observations about the sun, moon, and planets. Also encourage individual presentations

It is very important for the teacher to summarize the findings and to make clear what should be learned from this session. Suggest that students work on one more naked-eye observation of the heavens during the next five school days.

Day 21

Student Evaluation

This evaluation may be an examination. Or, the teacher can use more imaginative devices, such as laboratory reports, poetry, science fiction, additional problem sets, etc.

Day 22

Lab Stations: Complex Motion

1. *E1–10* (curves of trajectories)
2. *E1–11* (prediction of trajectories)
3. *E1–12* (centripetal force)
4. *E1–13* (centripetal force on a turntable)
5. *L6* (Galilean relativity)

Students are to use the apparatus at each station, making qualitative observations.

Day 23

Lab Stations

Same stations as Day 22 but students are to pick one experiment and do it quantitatively.

Day 24

Small-group problem solving

Have students discuss problems in small groups. Be sure an outstanding student is in each group. Circulate among groups.

Day 25

Students report to the rest of the class on results of experiments on Day 23. Urge that presentations be very short and clear to allow plenty of discussion time.

Day 26

Show the first 13 minutes of the film "Frames of Reference." Divide class at random into small groups, pass out three or four open-ended questions related to film, and use the rest of the period for discussion.

Day 27

Review projectile motion and uniform circular motion. Discuss satellite motion in detail.

Days 28–30

Evaluation

One method of evaluation is to review, test, and discuss the test. Take one day for each activity.

Another method is to evaluate each student individually during three days of conferences.

Unit 1 SAMPLE MULTI-MEDIA SYSTEMS APPROACH

Note: This is just one path of many that a teacher may take through Unit 1.

1 **Survey:** Multi-media Text: Introduction Handbook: Introduction	2 Introduce: Multi-media procedures	3 5-min film loop Small-group discussion Text: 1.1–1.4	4 Lab stations: Uniform motion Handbook: E1-2, E1-3, E1-4
5 Lab stations: Acceleration Text: 1.5–1.8	6 Teacher presentations: graphing velocity acceleration Selected Study Guide questions	7 Small-group problem solving Write up lab	8 Small group discussion of Sec. 2.3 Text: 2.1–2.4
9 Teacher presentation: 17th century Exp. Text: 2.5–2.10	10 Lab stations: 17th century Exp. Handbook: E1-5	11 Small-group problem solving Handbook: Survey Chapter 2	12 Lab. Stations: Free Fall Write up observations
13 Teacher presentation: Galileo & free fall Quiz Handbook: Survey E1-1	14 Organize E1-1: Naked-Eye Astronomy Observe sky Text: 3.1–3.4	15 Lab stations: Vectors	16 Teacher presentation: Vectors Observe sky Text: 3.5–3.9
17 Lab stations: force mass acceleration Observe sky Text: 3.10–3.11	18 Discussion: Newton's laws Observed sky Selected Study Guide questions	19 Small-group problem solving Observe sky Write up E1-1 & E1-8	20 Student presentation: Naked-Eye Astronomy Summary by teacher Review Ch. 3
21 Student evaluation Text: 4.1-4.3 Handbook: Survey Ch. 4	22 Lab stations: ← Text: 4.4–4.6	23 Complex motion → Write up lab	24 Student presentations Selected Study Guide questions
25 Small-group problem solving session Text: Reread 4.4	26 PSSC Film: Frames of Reference Small-group discussion of film Text: 4.7–4.8 Epilogue	27 Teacher presentation: Satellites Review Unit 1	28 Review or Individual student evaluation
29 Test or Individual student evaluation	30 Discuss test or Individual student evaluation		

Unit 1 SUGGESTED SCHEDULE BLOCKS AND TIMETABLE

Each block represents one day of classroom activity and implies approximately a 50-minute period. The words in each block indicate only the basic material under consideration or the main activity of the day. The suggested homework (listed above each block) refers mainly to the **Text** and **Handbook**, but is not meant to preclude the use of other learning resources.

CHAPTER 1 THE LANGUAGE OF MOTION

	Text: Introduction HB: Introduction		**Text 1.1–1.4**	**HB: Use E1-4 for Lab Analysis**
Open school Show multi-media samples	Introduce multi-media procedures	Any film loop on motion (5 min) Small-group discussion	Lab Stations: Uniform Motion (See day 4.)	Lab stations: Acceleration (See day 5.)

CHAPTER 2 FREE-FALL: GALILEO DESCRIBES MOTION

Text: 1.5–1.8	**Text: selected S G questions**	**Write up E 1-4**	**Text: 2.1–2.4**	**Text: 2.5–2.10**
Teacher presentation: Graphing, velocity, acceleration, instantaneous acceleration	Small-group problem solving or Special activity	Small-group discussion: Section 2.3	Teacher presentation E1-5: A Seventeenth Century Experiment	A Seventeenth Century Experiment (cont'd.)

CHAPTER 3 THE BIRTH OF DYNAMICS:

Handbook: E1-5 Write up lab	**Handbook: Survey Chapter 2**	**Write up lab**	**HB: E1-1 Survey Naked-Eye Astronomy**	**Observe sky Text: 3.1–3.4**
Small-group problem solving	Lab stations: Free Fall	Teacher presentation: Galileo Free Fall Quiz	Organize E1-1: Naked-Eye Astronomy	Lab stations: Vectors (See day 15.)

NEWTON EXPLAINS MOTION

Observe sky	**Observe sky Text: 3.5–3.9**	**Observe sky Text: 3.10–3.11**	**Observe sky Text: selected SG questions**	**Observe sky Write up E1-1 and E1-8**
Teacher presentation: Vectors Film Loop 3	Lab stations: Newton's 2nd Law	Teacher-led discussion: Newton's Laws	Small-group problem solving	Student presentations: Naked-Eye Astronomy

CHAPTER 4 UNDERSTANDING MOTION

Review Ch. 3	**Text: 4.1–4.3 HB: Survey Ch. 4**	**Observe sky**	**Observe sky Text: 4.4–4.8**	**Observe sky Write up conclusions**
Student evaluation	Lab stations: Projectile Motion	Teacher discusses projectile motion	Lab stations: Circular Motion	Student presentations: Circular Motion, Naked-Eye Astronomy

Write up labs Text: Reread 4.4	**Text: selected S G questions**	**Unit 1 Epilogue**	**Review Unit 1**	
PSSC Film: Frames of Reference Small-group discussion	Teacher-led discussion: Uniform Circular Motion S G Questions	Review or Individual student evaluation	Test or Individual student evaluation	Discuss Test or Individual student evaluation

CHAPTER 1 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities
1.1 The motion of things				E 1-1 Naked-eye astronomy E 1-2 Regularity and time E 1-3 Variations in data D 1 Recognizing simple motion	R 7 Motion in words R 8 The Representation of Movement	1.1
1.2 A motion experiment that does not quite work	3				T 0 Using stroboscopic photographs	Electronic stroboscope Making frictionless pucks 1.2
1.3 A better experiment	4 5	2	9	D 2 Uniform motion using accelerometer and dynamics cart	L 8 Analysis of a hurdle race. I	1.3
1.4 Leslie's "50" and the meaning of average speed		6 7 8	17 19		L 9 Analysis of a hurdle race. II R 13 The Dynamics of a Golf Club	1.4
1.5 Graphing motion and finding the slope	11	15		E 1-4 Measuring uniform motion	T 2 Graphs of various motions	1.5
1.6 Time out for a warning	12	13				1.6
1.7 Instantaneous speed	14	10 17	20	D 3 Instantaneous speed	T 3 Instantaneous speed T 4 Instantaneous rate of change R 8 Motion	1.7
1.8 Acceleration—by comparison		21	18	D 4 Uniform acceleration using liquid accelerometer	T 0 Using stroboscopic photographs T 3 Graphs of various motions F 1 Straight-line kinematics	1.8
					R 1 The Value of Science R 14 Bad Physics and Athletic Measurements	

CHAPTER 2 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
2.1 The Aristotelian theory of motion	1 2 3	4		D 5 Comparative fall rates of light and heavy objects		When is air resistance important?	2.1
2.2 Galileo and his times							2.2
2.3 Galileo's "Two New Sciences"	6	5		D 6 Coin and feather			2.3
2.4 Why study the motion of freely falling bodies?	8		9		R 6 Haldane, "On Being the Right Size"		2.4
2.5 Galileo chooses a definition of uniform acceleration							2.5
2.6 Galileo cannot test his hypothesis directly		10					2.6
2.7 Looking for logical consequences of Galileo's hypothesis	15	7 12 13	11 14		T 6 Derivation of $d = v_i t + \frac{1}{2} at^2$	Measuring your reaction time	2.7
2.8 Galileo turns to an indirect test	16 17 20	18 19 21 32	22 23 24	E 1-5 A seventeenth century experiment or E 1-6 A twentieth century version of Galileo's experiment		Falling weights	2.8
2.9 Doubts about Galileo's procedure	25			E 1-7 Measuring acceleration of gravity a_g	L 1 Accleration due to gravity Method I L 2 Acceleration due to gravity Method II		2.9
2.10 Consequences of Galileo's work on motion	33	26 30 32	27 28 29 31		T 6 Derivation of $d = v_i t + \frac{1}{2} at^2$		2.10

CHAPTER 3 RESOURCE CHART

Text	Study Guide E	M	H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
3.1 "Explanation" and the laws of motion	1						3.1
3.2 The Aristotelian explanation of motion	2						3.2
3.3 Forces in equilibrium			3				3.3
3.4 About vectors	4	8		D 7 Addition of vectors D 8 Direction of $\vec{v}$ and $\vec{a}$ D 9 Direction of $\vec{v}$ and $\vec{a}$ (air track) D 10 Noncommutative rotation	L 3 Vector Addition I—Velocity of a boat R 10 Introducing Vectors		3.4
3.5 Newton's first law of motion				D 11 Newton's first law	L 4 A matter of relative motion (qualitative) R 12 Newton's Laws of Dynamics F 2 PSSC-Inertia	Newton's first law	3.5
3.6 The significance of the first law	5 6 9	 10	7				3.6
3.7 Newton's second law of motion	11 15 16 19	12 13 14 17	18 20 23 21 22	E 1-8 Newton's second law D 12 Newton's laws (air track) D 13 Effects of friction on $\vec{a}$		Newton's second law	3.7
3.8 Mass, weight, and free fall	26 27 28 29	25		E 1-9 Mass and weight D 14 Demonstrations with rockets D 15 Making an inertial balance		Accelerometers	3.8
3.9 Newton's third law of motion	31 33	30 32		D 16 Action–reaction (rope). I D 17 Action–reaction (rope). II D 18 Reaction force of a wall D 19 Newton's third law	T 8 The tractor-log problem		3.9
3.10 Using Newton's laws of motion	34			D 20 Action–reaction (car) D 21 Action–reaction (nail) D 22 Action–reaction (jumping)			3.10
3.11 Nature's basic forces					R 15 The Scientific Revolution R 16 How the Scientific Revolution of the 17th Century Affected Other Branches of Thought		3.11

CHAPTER 4 RESOURCE CHART

Text	Study Guide E	M	H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
4.1 A trip to the moon	2						4.1
4.2 Projectile motion	3	4			T 9 Projectile motion R 11 Galileo's Discussion of Projectile Motion	Speed of a stream of water Projectile motion demonstrations Photographing projectile motion	4.2
4.3 What is the path of a projectile?	7		5 6	E 1-10 Curves of trajectories	T 9 Projectile motion T 10 Path of a projectile F 3 PSSC—Free fall and projectile motion		4.3
4.4 Moving frames of reference	9 10	8		D 23 Frames of reference D 24 Inertial versus noninertial reference frames	L 4 A matter of relative motion L 5 Galilean relativity 1 (ball dropped from ship) L 6 Galilean relativity 2 (object dropped from aircraft) L 7 Galilean relativity 3 (projectile fired vertically) F 4 PSSC—Frames of reference		4.4
4.5 Circular motion	11			D 25 Uniform circular motion E 1-11 Prediction of trajectories E 1-12 Centripetal force		Motion in a rotating reference frame	4.5
4.6 Centripetal acceleration and centripetal force	13 14 18	15 17	12 16	E 1-13 Centripetal force on a turntable	R 13 The Dynamics of a Golf Club	Penny and coat hanger	4.6
4.7 The motion of earth satellites		21 24	19 20 22 23	D 26 Simple harmonic motion D 27 Simple harmonic motion (airtrack)	T 11 Centripetal acceleration—graphical F 5 PSSC—Vector Kinematics		4.7
4.8 What about other motions?	25 26 27				L 8 Analysis of a hurdle race. I L 9 Analysis of a hurdle race. II R 19 The Vision of our Age R 21 Chart of the Future		4.8

Background and Development

OVERVIEW OF UNIT 1

How do things move? *Why* do things move? The principal task of Unit 1 is to provide answers to these questions. A secondary task is to provide insight into the way scientists go about their work.

The first question, "How do things move?" is answered gradually, starting with a very simple motion and proceeding to more complex motions. One of the main reasons for starting the course with kinematics is that it provides an immediate opportunity for student laboratory activity. Furthermore, these activities, though easily carried out, are both interesting and significant. The students usually like kinematics experiments and gain confidence in their ability to do physics.

Most of Chapter 1 is spent developing the intellectual tools to describe straight-line motion. The key concepts are average speed and instantaneous speed. The chapter concludes by making an analogy between the change in position with time (speed) and the change in speed with time (an example of acceleration).

Chapter 2 extends the description of motion to accelerating objects, specifically an object in free fall. We follow Galileo through his own analysis as he seeks to confirm that the speed of a freely falling object is proportional to the elapsed time of this fall. By using Galileo as an example, Chapter 2 also serves to provide the student with some understanding of the scientist as a person working within a social milieu.

The second question, "Why do things move?" is the fundamental question of dynamics. Newton provided the answer to this question with his three laws of motion. These three laws are developed in Chapter 3. Vector concepts are introduced and are used throughout the remainder of the unit.

The final chapter in Unit 1 brings together the concepts learned in the first three chapters and applies them to projectile motion and uniform circular motion. The chief example used to develop these ideas is that of a journey from the surface of the earth to the surface of the moon.

Chapter 1 begins by citing an old maxim: "To be ignorant of motion is to be ignorant of nature." Indeed, kinematics and dynamics are to physics what grammar is to language or what scales are to music. The techniques and ideas learned in Unit 1 are used throughout the course.

CHAPTER 1 / THE LANGUAGE OF MOTION

1.1 | The Motion of Things

The most significant case of motion in the development of science is not mentioned in this section, although it forms the main topic of Unit 2. The first scientific problem facing humanity dealt with motion in the heavens. From earliest times, people questioned the nature and causes of the motions of the various astronomical bodies. Galileo's study of the motion of objects at the earth's surface (terrestrial motion) led to an understanding of the motions of the inaccessible heavenly bodies.

Thus, an understanding of the basic concepts of motion as formulated in seventeenth century physics is taken up at the beginning of this course. These concepts are still useful in explaining and understanding much of the physical world that surrounds us. Moreover, the concepts have historical importance.

Do not devote much class time to the justification of starting the course with the study of motion. The students will not yet know enough physics to know what alternatives there are. It is more crucial to get the course going quickly, raise interesting questions, and encourage student participation. (Some teachers, after having taught the course for one year, prefer to start with Chapters 5 and 6 and part of Chapter 7 to establish motivation for study of motion.)

Motion goes on about us all the time. It may be complex and confusing, or it may have regularities that make it simpler to understand and classify. To help students begin to think about the motions around them, ask them to classify several motions into those that are regular and those that are irregular. Examples might be a pendulum, a sewing machine, a leaf blowing on a tree, and a bird in flight. Some motions may contain both regular and irregular features.

Students will certainly recognize on an intuitive level that while events such as falling leaves or flying birds may be commonplace, they are not necessarily simple. As a first approximation, the motion of one object is more complicated than that of another if it is undergoing more erratic changes of direction and/or speed. In the long run, however, the distinction depends upon experience and our ability to find functional relationships with which to describe events or statements. Generally, when we undertake investigations in a relatively new field, we look around for what appear to be simple, straightforward examples of the phenomenon being investigated. The simplicity may later on turn out to be deceptive, but at least we have made a start. We can correct ourselves later.

The Greeks took uniform circular motion, rather than uniform straight-line motion as the simplest.

Both their physics and their metaphysics helped to direct attention to circular motion as primary. It is pointless to debate which is *really* simpler—but progress in physics was greatly helped by adoption of the Galilean view in the early and middle part of the seventeenth century.

1.2 | A Motion Experiment that Does Not Quite Work

Section 1.1 ends with a suggestion that we can learn from experiment. Section 1.3 suggests an experimental means for establishing regular time intervals and measuring distance as a function of time, which leads to the definition of *speed.*

Section 1.2 is a bridge to help the students bring intuitive feelings about motion and speed into an experimental environment.

1.3 | A Better Experiment

The main burden of teaching the student how to interpret and use strobe photographs rests on the laboratory and audio-visual aids under the direction of the teacher. The treatment in the *Text* is not sufficient by itself.

Quickly get the students started on an analysis of motion and on laboratory work associated with it. Don't start the course with protracted philosophical discussions about the role of experiment or the nature of simplicity. After they understand more physics, there is time enough to come back to some of these questions.

Experiments are done so that events can be manipulated and made "simple." Furthermore, they can be reproduced and done over and over again while measurements and observations are made.

It is in the laboratory that students should learn about the role of laboratory in science. Laboratory experiments, which sometimes may seem very artificial and almost trivial, do lead to understandings that better explain the complex and interesting events seen in the world outside of a lab. If, however, we begin with the study of complex motions, such as falling leaves, we may never find the regularities for which we search.

If you use *E3-2,* Method B Stroboscopic Photographs (Unit 3, page 104 in the *Handbook*), develop the idea of "freezing" motion. However, this need not be done rigorously; the idea that regular motion viewed at regular intervals can cause the motion to seem to stop or move slowly will be enough.

1.4 | Leslie's Swim and the Meaning of Average Speed

This section introduces the crucial definitions of *average speed* and *interval* and applies them quantitatively to a kind of real motion that may previously have interested the student.

The preceding section hinted rather casually at the profound and essential notion that all measurements are approximations. This statement is incomplete, of course, until it specifies what measurements are approximations *to.* In this section, there is a clear example of one measurement (the overall average speed) that is an approximation to each of several other measurements (the average speeds over the intervals). Likewise, each of these average speeds is presented as an approximation to "what really happened" at every moment of the motion. This may well be the student's first exposure to the idea of an experimentally unreachable concept that can nevertheless be approached, one step at a time, as nearly as one wishes, until one's measuring instruments are no longer good enough to improve the picture further.

1.5 | Graphing Motion and Finding the Slope

This section presents no more than a bare outline of constructing and interpreting graphs related to speed. For a good many students this will be extremely elementary, and for them this section may be enough.

In addition to helping students to use graphs correctly, an effort should be made to get them into the habit of trying to interpret them physically. For example, you would like students to be able to look at a graph and to describe in words the physical behavior illustrated.

PSSC Lab I–4 is an excellent exercise in graphing that can be done either under classroom supervision or as a homework assignment.

Below are some general rules that should be observed in plotting a graph. This list is far from complete and is used as a minimum standard. The following four ideas should be stressed to your students.

1. Proper graph format. Each graph prepared should include a title, experiment name or number, and student name and date, presented in block form near the top of the graph.

Each axis should be labeled clearly with the quantity plotted and the unit of measure used. The scale values should also be clearly given. All of this information should be easily readable from the lower righthand corner of the graph sheet without rotating the page.

2. Size. All graph presentations should be large enough to show clearly the behavior of the quantities being plotted. The number of points included in the graph should also affect the choice of size. Poor choices include numerous points shown on too small a graph as well as very few points presented on too large a graph.

3. Scales. The choice of scale on any graph is arbitrary but should be made to maximize clarity. The range of values to be plotted should determine the placement of the origin and the maximum scale value for each axis. Whenever possible the scale should be chosen that allows decimal multiples (such as 100, 10, 1, or 0.1) of the units being graphed to be located easily along the axis.

4. Plotting techniques. Experimental points

should be plotted with small, sharp dots. To avoid "losing" a data point, a small circle should be drawn around each point.

The uncertainty in the values on the coordinate axes can be indicated by the size of the circle used, or perhaps by the length of horizontal and vertical bars drawn through the point as a cross.

Seldom will data points fall on a totally smooth curve. Whenever there is some reason to believe that whatever is being graphed actually does change smoothly, a smooth curve should be drawn as close as possible to the data points plotted. Encourage students to consider what is implied in a broken-line graph connecting data points in contrast to the implications of the smooth curve.

When two or more curves are plotted on the same graph, students should use either different colors for each curve, or dotted and broken lines. In either of these cases, a key or legend should appear on the graph defining the use of each line.

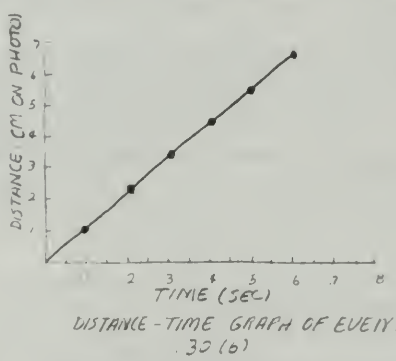


Fig. 1 From the graph, we can see that at time 0.25 sec the distance traversed is about 2.8 cm.

1.6 | Time Out for a Warning

The process of estimating values between data points is called *interpolation.*

The process of predicting values that extend beyond the range of data points is called *extrapolation.*

A discussion of the uncertainties involved in interpolation and extrapolation might be warranted. Stress the fact that *interpolation* is usually more reliable than extrapolation. Both are risky and should be undertaken with care, and the values should not be ascribed greater certainty than is warranted.

The danger of extrapolation can be illustrated with a rubber band and a set of weights. Suspend the rubber band and load it successively with heavier and heavier weights, recording and graphing the amount of extension for each weight. After suspending 500 g ask the students to predict by extrapolation the extension of the rubber band when the 1000-g weight is suspended. The actual extension will turn out to be much less than the extrapolated value because the elastic characteristics of the rubber band change. This will appeal to your students, and it makes the point that not all graphs are linear.

Note: The particular selection of weights will depend on the size of the rubber band available.

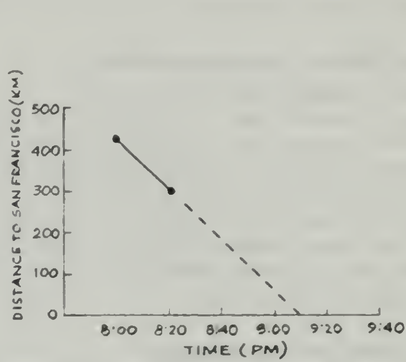


Fig. 2 From the graph we can see that the plane will be about 96 km from San Francisco at 9:00 P.M. if it does not change speed and direction.

1.7 | Instantaneous Speed

The concepts of *instantaneous speed* and *limit* are introduced here, although the latter concept is not given its customary name. The now-familiar notion of average speed is associated with its graphical representation as the slope of a chord joining the endpoints of an interval on a distance–time curve. The student should already know about tangents from geometry, and this section should help to identify the slope of a tangent as a graphical representation of an instantaneous speed.

It may be puzzling to some students (and even disturbing to others) to learn that their common-sense notion of instantaneous speed corresponds to nothing that can be specified exactly by the "exact" science of physics. They may even resist the idea and thus miss the conceptual leap involved. Instantaneous speed is a conceptual "invention." It is justified in physics by its usefulness in describing and explaining motion and its consistency with other physics concepts. The point is not that there really is or isn't such a thing, but that the *idea* is fruitful.

Although the idea of speed is introduced with a car speedometer, a car speedometer does not give instanteous speed any more accurately than this method. A speedometer also averages over a time interval. (Note the lag of a speedometer in registering as you begin with large acceleration.)

1.8 | Acceleration by Comparison

The scalar definitions of average and instantaneous acceleration are presented by analogy with the speed definitions. The common-sense basis of Galilean relativity also appears, but only casually.

Since Galilean relativity will play a major role in later chapters, it would be well to pause here and make a point of the fact that there is a real qualitative difference between speed and acceleration that the equations do not show (at least at this level). Every student's experience with carnival rides and automobiles is wide enought to recall incidents that will make the distinction real, if each is stimulated to search his or her memory. If students come out of this section convinced that "you can't tell when you're moving uniformly, but you can tell when you're accelerating," they will be better equipped to tackle the physics of Galileo and Newton.

Beginning physics students are sometimes confused by the units for accelerations, m/sec^2. It may be helpful to show that the units come from the definition of acceleration, $\Delta v/\Delta t$, and that change in velocity per unit time, $\frac{\text{m/sec}}{\text{sec}}$, is written just for convenience as m/sec^2. If your students can't get comfortable with the "square time," stick with the more obvious expression: m/sec/sec. Conventions of notation are the least important aspects of physics you can teach, and ought not to be purchased at the cost of understanding the ideas.

CHAPTER 2 / FREE FALL: GALILEO DESCRIBES MOTION

Many discussions of Galileo and his study of mechanics are quite critical of Aristotle. It is, perhaps, as unfair to condemn Aristotle for not accepting what the vacuum pump would prove as it would be unfair to criticize Galileo for not discovering radio astronomy.

It should be pointed out that the physics inherited by Galileo is really a very different and advanced kind of physics compared to the original work of Aristotle.

A frequently overlooked contribution to Aristotelian physics was made by the Arabs. After the decline of the Alexandrian period of Greek science (about 200 A.D.), the knowledge of the Greeks was not lost to the West. During the so-called Dark Ages in Europe, there was great activity in the Arab world. From the eighth century through the twelfth, considerable scientific and scholarly work was done by the Muslims. Working in Damascus, Baghdad, Cairo, and ultimately in several centers in Spain, the Muslims modified the work of Aristotle and other Greeks in many ways. Furthermore, the Muslims were influenced by the studies of the Persians, Hindus, Chinese, and others in the East, and by certain Christians from the West.

Muslim science flourished in Toledo, Cordoba, and other Spanish cities. As these cities were gradually reconquered by the Christians during the eleventh to fifteenth centuries, Muslim and ancient Greek knowledge filtered into Europe.

Before the time of Galileo, Aristotelian science had been blended with Christian philosophy, particularly by Thomas Aquinas. There were, however, various criticisms and interpretations made during the later Middle Ages at Oxford, Paris, Padua, and other centers of intellectual activity. You should be aware of these points, not necessarily to bring them up in class for discussion, but to avoid overemphasizing the conflict between Aristotle and Galileo, which would thereby seem to imply that nothing happened during the 2,000 years separating these two great men.

For a more complete (but by no means extensive) treatment of Aristotle's physics the student may be referred to any of the following:

C. B. Boyer, "Aristotle's Physics," *Scientific American*, May 1950.

M. R. Cohen and I. E. Drabkin, *A Sourcebook in Greek Science*, New York: McGraw-Hill, 1948. See pp. 200–203 on natural and unnatural motions, and especially pp. 207–212 on falling bodies.

Alexandre Koyre, "Galileo," pp. 147–175, in Philip P. Weiner and Aaron Noland (editors), *Roots of Scientific Thought*, New York: Basic Books, 1957. Pp. 153–158 clearly describe the Aristotelian theory of motion.

There are other papers in the Weiner and Noland anthology that will help you. We recommend the book for your library and for the school library.

O. L. O'Leary, *How Greek Science Passed to the Arabs*, London: Routledge and Keegan Paul, 1948.

S. F. Mason, *Main Currents of Scientific Thought*, New York: Henry Schuman, 1953.

The influence of China, India, and the craft tradition in medieval Europe, as well as the influence of the Arabic world, is outlined in Chapters 7 through 11 (pp. 53–98).

A. C. Crombie, *Medieval and Early Modern Science*, Garden City: Doubleday-Anchor, 1959. Volume I deals with the fifth through the thirteenth centuries; Volume II treats the thirteenth to the seventeenth centuries.

2.1 | The Aristotelian Theory of Motion

The Aristotelian scheme is a complex and highly successful one. For approximately 2,000 years this scheme dominated intelligent thinking.

Aristotle was, perhaps, the first to realize that an explanation of the universe must be based on careful descriptions and classifications of what is in it. He was primarily an encyclopedist, and his writings were authoritative accounts of what was known at the time in such widely diverse fields as logic, mechanics, physics, astronomy, meteorology, botany, zoology, psychology, ethics, economics, geography, politics, metaphysics, music, literature, and mathematics. He was among the first to understand and to discuss such things as the principle of the lever, the concept of the center of gravity, and the concept of density.

Aristotle's notion that the motion of an object moving with constant speed requires a force proportional to the speed is not true for an object falling in a vacuum. It is true, however, for an object moving in a viscous medium, and most terrestrial motion is in the air, a viscous medium. Remember that vacuum pumps were not invented until nearly 2,000 years after Aristotle.

Students certainly should not be required to learn the details of the Aristotelian or medieval physics of motion. There are, however, some general points that might well be emphasized. These are:

1. The ideas on motion appear as logical parts of a larger theory about the nature and structure of the universe. In a sense, the "grand structure" existed first and the various aspects of it could be learned by deduction from this grand structure. This contrasts with the modern approach in which individual topics and disciplines are studied and only gradually merge into a larger, more comprehensive structure.

2. The rules governing the motion of bodies on or near the earth were different from the rules governing the motion of nonterrestrial objects. Thus it was the nature of objects near the earth to be stationary once they reached their "natural place." There was no conflict in saying at the same time that the natural behavior of stars and planets was to move continuously in circles.

3. The Aristotelian scheme was essentially qualitative and nonexperimental.

The main Aristotelian ideas about motion survived for a long time for many reasons. One of the reasons was that they did not seem to violate "common sense." Even today the instinctive physics of most people is probably Aristotelian. For example, recall how hard it is to convince students that a 10-kg mass and a 1-kg mass will fall at essentially the same speed. Recall also the difficulties of teaching Newton's first and third laws, not to mention special relativity theory or quantum mechanics.

2.2 | Galileo and His Times

The roots of Galileo's thinking extend far back to the Greek tradition. He was able to apply the traditions of Pythagoras and Plato to a new context and to give them new vitality. Galileo contributed greatly to shaping the new science, but he did not do it alone, and, indeed, he never entirely escaped from the past.

A thumbnail biography of Galileo cannot do justice to his colorful life and career. Students who would like to know more about Galileo should be referred to one of the following:

Laura Fermi and G. Bernadini, *Galileo and the Scientific Revolution*, New York: Basic Books, Inc., 1961. Short and readable.

I. B. Cohen, "Galileo," *Scientific American*, August 1949.

E. J. Greene, *One Hundred Great Scientists*, Leander, Texas: Washington Press, 1964.

The time-line chart on page 42 is one of a series of similar charts that appear throughout the *Text*. These charts are to help students place the individual and the events into the larger context of history. Most students know something about Shakespeare and Mary Stuart and Galileo, but frequently they are not aware that these people were contemporaries. We hope that the students will gain more perspective by being able roughly to relate Galileo to his contemporaries: to Elizabeth I, Kepler, and to the founding of Jamestown, the first American settlement.

Under no circumstances should students be required to memorize the names or dates appearing on these charts. The names and events here represent only a sampling. Students might wish to add additional names to the chart.

2.3 | Galileo's *Two New* Sciences

The mention of the Inquisition and Galileo's confinement may stimulate students to raise questions about this whole affair. Most of the controversy had to do with the concept of the solar system, that is, with Galileo's astronomy. More about this will be encountered in Unit 2.

Since the focus in this chapter is on one aspect of Galileo's study of motion at the earth's surface, it may be well to defer the more dramatic aspects of Galileo's career until Unit 2. However, for students who might like to prepare themselves for the issue, you can recommend:

Georgio de Santillana, *The Crime of Galileo*, Chicago: The University of Chicago Press, 1955.

F. Sherwood Taylor, *Galileo and the Freedom of Thought*, London: C. A. Watts, 1938.

The dialogue of Sagredo, Simplicio, and Salviati is a discussion of a new book on mechanics by an unnamed author who is a friend of theirs. The "eminent academician" who wrote the book is, of course, Galileo, whose views are presented through Salviati.

Several copies of the Dover paperback edition of the Crew and de Salvio translation of *Two New Sciences* should be on hand for students who wish to locate these quotations and follow them in greater length.

A technique used in present-day experiments can be mentioned in discussing the argument between Simplicio and Salviati. It is easier to determine the *difference* between the outcome of two events when they are simultaneous. For example, it is easy to tell which runner has won a 200-m race, even when one leads the other by only 1 m. It would be more difficult to make this determination by timing two runners in separate races.

2.4 | Why Study the Motion of Freely Falling Bodies?

This brief section emphasizes that our main interest is in studying the approach used by Galileo.

The quotations from *Two New Sciences* show that Galileo himself realized that his work was of significance and that it would lead to a new science of physics.

2.5 | Galileo Chooses a Definition of Uniform Acceleration

This section and the three sections that follow it deal with Galileo's free-fall experiment. There is some danger that the student will get lost before reaching the end. For this reason, the opening paragraph of Sec. 2.5 summarizes the overall plan of attack. Point out, especially to students who are not accustomed to involved derivations or proofs, that it is important to consider the plan of attack before beginning to study such a logical argument.

The summary (and the marginal commentary accompanying it) should prove useful, yet is is potentially misleading. This, like all summaries, makes the events sound more organized and systematic than they were. Galileo is, after all, giving an *ex post facto* description of work that he did over a period of years. Furthermore, he embedded that description in a controversial document that had wider aims than merely to present some research findings. Take care, therefore, to see that the student does not accept these activities as a model for all scientific endeavor.

2.6 | Galileo Cannot Test His Hypothesis Directly

Frequently a direct test of a particular hypothesis cannot be made. One or more of the quantities involved cannot be measured accurately because the means for making the measurement has not yet been established.

Suppose that, in an attempt to test directly the hypothesis that v/t is a constant, Galileo had permission to use a 20-story building 65 m high. Suppose that he put marks on the building at 1, 4, 9, 16, 25, 36, 49, and 64 m from the top. An object dropped from the top of the building should pass these marks at equal time intervals of a little less than 0.5 sec.

But observing that the marks are passed at equal time intervals is not really a direct test that v/t is constant. For this, he would have to determine the instantaneous speed as the object passed each mark and the distance used for these speed measurements should be very small. Suppose instead that he considered a rather large distance, 1 m. The time to cover the first space interval, from 0.5 to 1.5 m, would be about 0.23 sec. If we assume that Galileo could measure to within 0.1 sec, his probable error for the first distance would be about half the quantity he was trying to measure. The time elapsed while the object moved through the last interval, from 63.5 to 64.5, would be less than 0.03 sec. To measure this time interval to 5% accuracy would require a clock that one could read to about 0.001 sec, at least 10 and probably 100 times better than anything Galileo had available. No wonder he resorted to an indirect test.

Now there are many methods available to us, such as stroboscopic pictures or electrically driven timers, which allow us to test directly whether or not v/t is constant for a freely falling body. But all of these methods depend on our ability to measure small time intervals (0.001 sec) with precision. Such methods were simply not available to Galileo.

2.7 | Looking for Logical Consequences of Galileo's Hypothesis

Part of the reason for the scientific breakthrough begun in the sixteenth and seventeenth centuries was use of a mathematical approach to the study of motion.

The *Text* derives $d = at^2/2$ but you should repeat the derivation carefully in class. The main point of this section is not to teach the derivation; it is to emphasize the value of mathematics in science. Simple algebra allows us to arrive at a relationship that is self-evident at the beginning. While the final equation contains no new information, it presents the information in a different and useful way. For instance, it allows us to make predictions concerning distances traveled by accelerating bodies that are not evident in the parent equations.

The word *constant* is used in several ways in physics. In the context of this section, constant means: in uniform acceleration, the numerical value of the ratio d/t^2 is the same (is constant) for each and every interval for which distance and time measurements are made, provided other parameters are fixed. The numerical value of that constant ratio will depend on the value of acceleration in a particular case.

2.8 | Galileo Turns to an Indirect Test

This section contains what is probably the largest conceptual leap in Galileo's argument. He assumed that the inclined plane was primarily a device for diluting free fall without changing its fundamental nature, and he was able to proceed to experimental tests of his hypothesis. If the students can be made to see that this is a reasonable though not necessarily true assumption, the inclined-plane experiment should not be difficult.

Encourage all students to carry out this experiment. With reasonable effort the students will find that for any distance along the incline, the ratio d/t^2 will be constant for a given angle of incline. For practical reasons the experiment is limited to relatively small angles.

In summary, the main purpose of this section is to make the association between the inclined plane experiment and the overall problem of free fall. An understanding of the actual experiment itself should come from the laboratory.

2.9 | Doubts about Galileo's Procedure

Healthy skepticism is one of the characteristics of scientists. Students from the beginning should be encouraged to be critical of scientific claims and experiments. In a textbook it is difficult not to sound authoritarian from time to time. A section such as this, which lists several reasons for questioning Galileo's results, is intended to counteract that tendency.

2.10 | Consequences of Galileo's Work on Motion

The purpose of this section is to show that Galileo's work on motion had consequences far beyond the particular issues at hand. His contributions to the advancement of science were both substantive and methodological.

CHAPTER 3 / THE BIRTH OF DYNAMICS: NEWTON EXPLAINS MOTION

3.1 | "Explanation" and the Laws of Motion

The purpose here is to place the kinds of motion we studied in kinematics into perspective. These are motions that Newton's laws will explain.

Dynamics is introduced by contrasting it to kinematics. The distinction between these was mentioned in Chapter 2 when Salviati (Galileo) said that the time to talk about causes of motion was after accurate descriptions existed.

One of the preoccupations of science is to provide systematic explanations of observable phenomena. Newtonian mechanics (Newton's laws of motion, the law of universal gravitation, and various force functions) represent one such explanatory system. As the students progress through this chapter, studying the three laws individually, they should be reminded from time to time of this overall theme of explanation.

3.2 | The Aristotelian Explanation of Motion

Aristotelian ideas concerning motion should be presented for better appreciation of the Newtonian development. Contrast between the two will and should be made. Aristotelian and so-called "common-sense" observations should be connected.

3.3 | Forces in Equilibrium

Develop the ideas of unbalanced force and equilibrium for the condition of "rest." This will set the stage for the equilibrium condition of constant velocity with no unbalanced forces to be considered, in Sec. 3.5.

3.4 | About Vectors

The concept of a vector is developed briefly in this section. The *Text* states, but does not show, that acceleration can be treated as a vector. Actually the section will not stand alone as a way of teaching vectors. It is necessary that all students understand what vector quantities are and why they are important. They should also be able to do vector addition and subtraction graphically.

3.5 | Newton's First Law of Motion

The significance of the first law of motion cannot be overstated. Most physics textbooks point out that the first law is really a special case of the second one, since by the second law the acceleration is zero if the force is zero. While this is true, it misses the point.

The law of inertia is fundamental to modern mechanics, for it states what is to be the starting place of the entire theory of motion. The first law makes it perfectly clear from the beginning what is to be basic, what requires further explanation and what does not. In so doing, the law of inertia dramatically exposes the difference between the Newtonian system and the Aristotelian system.

The main points to be emphasized about the first law of motion are these:

1. Fundamentally, the law is a definition. It states the convention to be followed in studying forces. Forces are not to be considered as the causes of motion but rather as whatever creates acceleration.

2. The law of inertia cannot be proven by observation or experiment. One reason is that the ordinary method for deciding whether or not there are unbalanced forces operating is to observe whether or not there is acceleration.

Many teachers demonstrate the *plausibility* of the first law by showing that, as the retarding friction on a moving body is reduced, the object appears to behave more and more in accordance with the first law. The teacher should be extremely careful not to pass off a classroom demonstration as proof of the first law: Definitions cannot be proven

3.6 | The Significance of the First Law

This persents one of the first opportunities in the course to delve into deep philosophical issues. Students are usually fascinated by the first law (and somewhat skeptical about it) and are more than willing to discuss several of the issues listed in this section, especially frame of reference and universality.

3.7 | Newton's Second Law of Motion

Force and mass are very difficult concepts to master. This section postpones a definition of those terms and avoids consideration of the empirical content of the second law. The section does not explain the equation

$$\vec{F}_{net} = m\vec{a}$$

The student should understand that for a single object, a is proportional to F. For different objects acted on by a constant force, a is inversely proportional to m. We want the student to realize that if the second law is true, then certain mathematical relationships must exist between force and acceleration and between mass and acceleration.

The newton is the only force unit mentioned in this chapter. Perhaps the student should know that there are other force units depending upon the system of units being employed. However, there is little to be gained from comparing them or being able to convert from one to the other.

Perhaps it is worth noting that implicit in the equation $F = ma$ is a proportionality constant that does not appear because it was set equal to 1 ($F = kma$ and $k = 1$). An alternative approach would have been to define a standard unit of force as well as standard units of mass and acceleration, and then do experiments measuring force, mass, and acceleration from which the value of the constant could be computed. Examples of this approach include G in the universal gravitation equation, and the spring constant in Hooke's law equations.

The first law of motion is mathematically a necessary consequence of the second law, while the reverse is not true. One can conceive that the second law of motion might have been very much more difficult to formulate.

The first law does not really take on meaning and is not at all useful in the real world of physics until certain additional operational definitions have been given for such terms as rectilinear motion, equal time intervals and constant speed. Also, a frame of reference must be established for all the measurements.

The main point to be emphasized is that, while the first law provides a general explanation of an event, the second law provides a quantitative and therefore more useful explanation. For example, when we can say that an object slows down because there is a retarding force of 4.0 N acting on it, we know a great deal more than when we merely say that it slows down because there is a retarding force.

3.8 | Mass, Weight, and Free Fall

Why do all objects in free fall at a given location fall with the same acceleration a_g? The answer is in the proportionality between weight F_g and mass m. It is imperative, therefore, that students understand the distinction between weight and mass. The only thing that really makes this difficult is that they are accustomed to using the terms interchangeably and usually think of weight as a measure of mass. (There is some dispute among physics educators about how weight ought to be defined. Here we are using weight as synonymous with gravitational force.) Perhaps the relationship between mass and weight is easier to understand if the second law equation is put in the form

$$a_g = \frac{F_g}{m}$$

Then it is clear that the force is proportional to the mass: no matter what the value of the mass, the acceleration will remain constant.

Why in a given location do objects fall with uniform rather than nonuniform acceleration? That the acceleration is constant is an experimental fact described by the second law.

3.9 | Newton's Third Law of Motion

The purpose of this section is to enable the students to understand what the third law says.

Once students grasp the notion that forces always appear or disappear in pairs due to the interaction of real objects and that the two forces act on different objects, then the rest is not difficult. However, these ideas are contrary to everyday experience and are not easily accepted. The tremendous inertia of the earth, the ever-present forces of friction, and the imperceptible distortion of rigid objects (like floors and walls) all help conceal action–reaction.

The third law allows one to examine a small part of a complex chain of events. Students find it hard to believe that the earth can exert a force on a runner. One way of demonstrating the value of the third law at this point is to ask students to invent an explanation for the acceleration of a runner that is *not* like the third law in form.

If forces are equal and opposite, how can an object accelerate? The point to be emphasized in going over this example is that the opposite forces act on different objects. In discussing the forces between two objects in a system, the third law is needed. It describes the location and magnitude of various force pairs. On the other hand, when one becomes interested in the motion of a particular object, then one must ask about the net unbalanced force acting on that object and apply the second law to determine its acceleration. Distinguishing which law to use is not easy and students should be furnished with other examples.

3.10 | Using Newton's Laws of Motion

In many real situations the laws of motion are used together. The first and third law help keep the qualitative situation clear and the second law permits a quantitative analysis. Two examples of the usefulness of the three laws in dealing with real physical situations are presented in this section. The main purpose is to demonstrate the application of the laws of motion and *not* to make all students highly skilled at numerical problem solving.

3.11 | Nature's Basic Forces

Treat this section as a reading assignment. It generalizes and extends the laws of motion and helps introduce Chapter 4. The four basic interactions in nature are mentioned to reduce the complexity the world so far seems to present and to point to what lies ahead in the study of physics.

4.1 | A Trip to the Moon

What is simple and what is complex is not altogether easy to decide. Certainly one's purposes for making such distinctions will have something to do with the criteria. In general, there are three criteria being used in this unit. The first two are uniformity and symmetry. If the parameters that describe the motion are uniform or constant in value, or if the path of motion is symmetrical, the motion is regarded as "simple."

The third criterion has to do with dimensionality. Motion becomes more complex as you go from one or two to three dimensions. By this standard, projectile motion and uniform circular motion can be considered as being more complex than rectilinear motion and less complex than, say, helical motion.

4.2 | Projectile Motion

Not much is gained by emphasizing the definition of projectile motion. Students should understand that a projectile is an object moving through space without the aid of any self-contained motive power.

The historical significance of the problem of projectile motion does not receive as much emphasis as it could in this section or in the chapter as a whole. Many historians of science feel that it was one of the key issues in the whole controversy over the nature of motion. Aristotle's theory was least able to explain projectile motion.

The concept of independent horizontal and vertical motion may be difficult for students to accept because it conflicts with their common-sense notion that horizontal speed affects the rate of fall. The students should carry through the analysis. Demonstrate the apparatus that projects one sphere and drops another at the same instant. (See *Handbook* page 28.) Also, students should make their own measurements, or at least see measurements made on photographs or transparencies similar to the one on *Text* page 105.

Two major and quite separate points need to be made:

1. It is an undeniable experimental fact that a short-range projectile launched horizontally will reach the ground at the same time as a similar object dropped at the same instant from the same height. This fact, that the gravitational acceleration of a projectile is exactly the same as the gravitational acceleration of an object falling freely from rest, comes from observation, not from deduction from first principles.

2. This experimental fact can be explained or rationalized by assuming that the observed motion of a projectile is the vector sum of two other motions that are completely independent of each other: uniform horizontal motion and accelerated vertical motion.

Students who are interested in projectile motion and who understand some trigonometry ought to be encouraged to analyze the general case of projectile motion; the case in which the projectile is launched at any angle. Once they have derived what they believe to be general equations, they should show that the equations used in this and the next section can be deduced from the general equations.

4.3 | What Is the Path of a Projectile?

The purpose of this section is to establish and demonstrate the power of mathematics in science and to justify the need for continued scholarly work in pure mathematics. In order to do this effectively, it is important that prior to the derivation of the equation of the parabola, two other points be made.

1. There is no *a priori* reason to favor one curve over another. In fact, there is no reason even to suppose that the trajectory of a projectile will always have the same mathematical shape.

2. The question cannot be settled simply by observing the paths of projectiles with the unaided eye. For one thing, the angle of observation and problems of perspective make observation difficult. Secondly, many mathematical curves look very much alike and can be distinguished only by analysis. Finally, many objects that are thrown do not follow a parabolic path because of the large and changing air resistance they encounter along the flight path.

The difficulties in determining the shape of the projectile's trajectories can be easily demonstrated by throwing objects inside the classroom, or, preferably, out on the playing field. See *Project Physics Transparency T10* "Path of a Projectile."

The support that experiment gives to the purely mathematical combination of motions can provide the student with evidence that mathematical manipulation of symbols that express known principles can lead to new relations among the symbols—relations that are also true.

With a particularly able class, the teacher might wish to develop the relationship between the range of a projectile and its velocity and angle of fire. This problem could also be assigned as a project for better students who are familiar with trigonometry. They should try to derive the general projectile equation:

$$d_y = \tan\theta \, d_x + \tfrac{1}{2} a_g \left(\frac{d_x^2}{v_h^2 \cos 2\theta} \right)$$

See "Projectile Motion" in *Foundations of Modern Physical Science* by G. Holton and D. Roller (Addison-Wesley, 1958).

4.4 | Moving Frames of Reference

There are two related points to be made in this section. First, there is the Galilean relativity principle, which is merely a formal statement or generalization of the observable fact that mechanical experiments give the same results no matter what the (constant) velocity of the laboratory may be. The second point is that the *laws of motion* are the same for all reference frames moving uniformly with respect to each other.

Make certain, however, that students realize that the appearance of any motion they see *does* depend on the relative motion of the viewer. "The Perception of Motion," by H. Wallach, in *Scientific American*, July 1959, may be of interest. It concerns the fact that people view relative motion as if it were absolute.

4.5 | Circular Motion

This is an introduction to the terminology of circular motion and does not deserve great emphasis in class. Although it is not taken up in the *Text*, it is probably worthwhile to demonstrate the difficulty of deciding whether or not an object is in circular motion when observing it from a frame of reference that is moving with respect to the object.

For some interesting results of being located on the earth in a noninertial frame of reference, see the article by J. McDonald, "The Coriolis Effect," in *Scientific American*, May, 1952.

4.6 | Centripetal Acceleration and Centripetal Force

The difficulty in this section is to show that the acceleration of an object moving uniformly in a circle is truly centripetal. The *Text* gives only a plausibility argument; and, while this will convince some students, there may well be skeptics. In most classes it may be worthwhile for you to go through the derivation on the chalk-board, or use *Transparency 11*.

Material is included to provide the students with practice in thinking in terms of vectors. It also provides an opportunity to review and compare the three kinds of motion considered: rectilinear, projectile, and circular.

This might also be an appropriate time to suggest that circular motion is really a special case of projectile motion. This can be done in one of two ways. One way is to compare the vector relationship of an object moving with uniform circular motion with the vector relationship of a projectile at the top of its trajectory. The second way is to approach it through the use of a diagram such as that used by Newton (the figure on page 112).

The relationship $a = v^2/R$ is used to carry out an arithmetic solution to the same uniform circular motion problem solved graphically in the text. This should provide some of the brighter or more mathematically inclined students with an opportunity to check it out for themselves. By no means should all students be held responsible for studying the derivation.

If a sample problem is worked in class, an interesting example might be to find the acceleration of a point on the earth's equator due to the rotation of the earth ($R = 6{,}400$ km, $t = 24$ hr $= 8.64 \times 10^4$ sec). The result of this calculation can be compared with the value of the acceleration of gravity. The question can then be asked: "What would happen if the earth were rotating at a speed such that the centripetal acceleration were equal to the acceleration of gravity?" This idea will be taken up again in this chapter.

4.7 | The Motion of Earth Satellites

There is no new physics in this section. What the student has learned about circular motion up to this point is almost entirely theoretical or, at least, deals with cases, such as a blinky on a turntable, about which most students care very little.

The satellite Alouette is used because it has a nearly circular orbit and because it has some historical significance.

The trouble with such a list as the one in Table 4.2 is that it becomes obsolete almost as fast as it is printed. An effort was made to select satellites of continuing interest. Perhaps some students should be assigned the task of finding the additional entries needed to bring the list up to date. See *Sky and Telescope* magazine for frequent articles on satellites.

The important questions that will really show whether the student has learned what has been covered up to this point are "Why does the satellite not fall back to earth?" and "Why does a satellite not fly off into outer space?"

After most of the students in the class have been able to answer these questions successfully in terms of the kinematics of circular motion, you may impress them with the progress they have made by asking them to respond to those questions as the Aristotelians probably would have.

The section ends by suggesting that speed, distance above the earth, and period of rotation are not independent variables. This is probably *not* the time to take up questions about how satellites get from one orbit to another, and what effect this has on their speed. However, it may not be possible to avoid the issue altogether, especially if some dramatic event has recently happened.

Although satellite orbits (in particular, planetary orbits) will be taken up in greater detail in Unit 2, the statement is made in this section that at a particular height a satellite must have a certain velocity in order to maintain a circular orbit. The question "Why have all of the satellite launchings to date been in an easterly direction?" is useful. The answer involves having the students think about vector addition of velocities in terms of a frame of reference related to the center of the earth rather than the more familiar frame of reference of the individual on the surface of the earth.

4.8 | What About Other Motions?

This section should be treated merely as a reading assignment. Its only purpose is to remind the student that there are many interesting kinds of motion that we have not dealt with.

Brief Descriptions of Learning Materials

SUMMARY LIST OF UNIT 1 MATERIALS

Experiments

E1-1 Naked-Eye Astronomy
E1-2 Regularity and Time
E1-3 Variations in Data
E1-4 Measuring Uniform Motion
E1-5 A Seventeenth Century Experiment
E1-6 Twentieth Century Version of Galileo's Experiment
E1-7 Measuring the Acceleration of Gravity a_g
E1-8 Newton's Second Law
E1-9 Mass and Weight
E1-10 Curves of Trajectories
E1-11 Prediction of Trajectories
E1-12 Centripetal Force
E1-13 Centripetal Force on a Turntable

Demonstrations

D1 Recognizing simple motions
D2 Uniform motion, using accelerometer and dynamics cart
D3 Instantaneous speed
D4 Uniform acceleration, using liquid accelerometer
D5 Comparative fall rates of light and heavy objects
D6 Coin and feather
D7 Two ways to demonstrate the addition of vectors
D8 Direction of acceleration and velocity
D9 Direction of acceleration and velocity: an air-track demonstration
D10 Noncommutative rotations
D11 Newton's first law
D12 Newton's law experiment (air track)
D13 Effect of friction on acceleration
D14 Demonstrations with rockets
D15 Making an inertial balance
D16 Action–reaction forces in pulling a rope. I
D17 Action–reaction forces in pulling a rope. II
D18 Reaction force of a wall
D19 Newton's third law
D20 Action–reaction forces between car and road
D21 Action–reaction forces in hammering a nail
D22 Action–reaction forces in jumping upwards
D23 Frames of reference
D24 Inertial versus noninertial reference frames
D25 Uniform circular motion
D26 Simple harmonic motion
D27 Simple harmonic motion: air track

Film Loops

L1 Acceleration Due to Gravity. I
L2 Acceleration Due to Gravity. II
L3 Vector Addition: Velocity of a Boat
L4 A Matter of Relative Motion
L5 Galilean Relativity: Ball Dropped from Mast of Ship
L6 Galilean Relativity: Object Dropped from Aircraft
L7 Galilean Relativity: Projectile Fired Vertically
L8 Analysis of a Hurdle Race. I
L9 Analysis of a Hurdle Race. II

Reader Articles

R1 *The Value of Science* by Richard P. Feynman
R2 *Close Reasoning* by Fred Hoyle
R3 *On Scientific Method* by P. W. Bridgman
R4 *How to Solve It* by G. Polya
R5 *Four Pieces of Advice to Young People* by Warren W. Weaver
R6 *On Being the Right Size* by J. B. S. Haldane
R7 *Motion in Words* by J. B. Gerhart and R. H. Nussbaum
R8 *Motion* by R. P. Feynman, R. B. Leighton, and M. Sands
R9 *The Representation of Movement* by Gyorgy Kepes
R10 *Introducing Vectors* by Banesh Hoffmann
R11 *Galileo's Discussion of Projectile Motion* by G. Holton and D. H. D. Roller
R12 *Newton's Laws of Dynamics* by R. P. Feynman, R. B. Leighton, and M. Sands
R13 *The Dynamics of a Golf Club* by C. L. Stong
R14 *Bad Physics in Athletic Measurements* by P. Kirkpatrick
R15 *The Scientific Revolution* by Herbert Butterfield
R16 *How the Scientific Revolution of the Seventeenth Century Affected other Branches of Thought* by Basil Willey
R17 *Report on Tait's Lecture on Force, at British Association, 1876* by James Clerk Maxwell
R18 *Fun in Space* by Lee A. DuBridge
R19 *The Vision of Our Age* by J. Bronowski
R20 *Becoming a Physicist*, by Anne Roe
R21 *Chart of the Future*, by Arthur C. Clarke

Sound Films (16 mm)

People and Particles
Electron Synchrotron
The World of Enrico Fermi

F1 Straight-Line Kinematics
F2 Inertia (PSSL)
F3 Free Fall and Projectile Motion (PSSC)
F4 Frames of Reference (PSSC)
F5 Vector Kinematics (PSSC)

Transparencies

T0 Using Stroboscopic Photographs
T1 Stroboscopic Measurements
T2 Graphs of Various Motions
T3 Instantaneous Speed
T4 Instantaneous Rate of Change
T6 Derivation of $d = v_1 t + \frac{1}{2} at^2$
T8 Tractor-Log Problem
T9 Projectile Motion
T10 Path of a Projectile
T11 Centripetal Acceleration: Graphical Treatment

FILM LOOPS

Quantitative measurements can be made with *Film Loops* marked (Lab), but these loops can also be used qualitatively.

L1 ACCELERATION DUE TO GRAVITY. I
Slow-motion photography in one continuous sequence allows measurement of average speed of a falling bowling ball during two 30-cm intervals separated by 1.5 m. (Lab)

L2 ACCELERATION DUE TO GRAVITY. II
Slow-motion photography allows measurement of average speed of a falling bowling ball as it passes through four 20-cm intervals spaced 1 m apart. (Lab)

L3 VECTOR ADDITION: VELOCITY OF A BOAT
A motorboat is viewed from above as it moves upstream, downstream, across stream, and at an angle upstream. Vector triangles can be drawn for the various velocities. (Lab)

L4 A MATTER OF RELATIVE MOTION
A collision between two equally massive carts is viewed from various stationary and moving frames of reference.

L5 GALILEAN RELATIVITY: BALL DROPPED FROM MAST OF SHIP
A realization of the experiment digested in Galileo's *Dialogue on the Two Great World Systems;* the ball lands at the base of the mast of the moving ship.

L6 GALILEAN RELATIVITY: OBJECT DROPPED FROM AIRCRAFT
A flare is dropped from an aircraft that is flying horizontally. The parabolic path of the flare is shown, and freeze frames are provided for measurement of the position at ten equally spaced intervals. (Lab)

L7 GALILEAN RELATIVITY: PROJECTILE FIRED VERTICALLY
A flare is fired vertically from a Ski-doo that moves along a snow-covered path. Events are shown in which the Ski-doo's speed remains constant and in which the speed changes after firing.

L8 ANALYSIS OF A HURDLE RACE. I
Slow-motion photography allows measurement of speed variations during a hurdle race. (Lab)

L9 ANALYSIS OF A HURDLE RACE. II
A continuation of the preceding loop. (Lab)

Note: A fuller discussion of each *Film Loop* and suggestions for its use will be found in the section of this *Resource Book* entitled "Film Loop Notes."

SOUND FILMS (16 mm)

FILM NOTES ON *PEOPLE AND PARTICLES*

In planning this course and preparing all the necessary texts, laboratory equipment, film loops, teacher guides, and so forth, we felt that we should also make a film of what it is like to be working on a real physics problem at the research frontier. We did not want to film a set-up interview or a prepared lecture; we wanted to show people who are working in science.

Of course, we could choose only one out of the great variety of physics research problems of interest today. Our preference was to film a group of moderate size to show the variety of people involved in experimental work. Also, we had to select a problem that was not too difficult to understand, since the first showing of the film will be near the beginning of the course.

We decided to focus the camera on a group of Harvard University students and professors struggling with their work at the Cambridge Electron Accelerator (CEA). For over 2 years everything was filmed as it happened. (With regret we report that the Cambridge Electron Accelerator has been dismantled. As the cost of building and operating even more powerful machines increased, money was not available to continue operating many older machines, so they were dismantled.)

The film traces the work of the participants in the experiment as they design, construct, and assemble the equipment to a point where the physicists are prepared to take actual data. The experiment itself took several more months. As this course is particularly concerned with the human element in the making of science, it is appropriate that the film should not emphasize advanced physical theory. Instead, it concentrates more on the style of work in a lab, on the men and women who are working together, and on some of the joys and pains of doing original scientific work. It raises a number of themes, from the international character of science to the fact that work on this scale requires a great range of skilled people, including shop machinists, scientists, engineers, secretaries, and so forth.

Most people have no way of being directly involved in any scientific work and cannot look over the shoulders of scientists. If they could, even through such a film, there might be fewer strange and false notions about work in a laboratory; false notions of exaggerated glamor, just as much as of dark doings. We hope our film shows that work on a real research problem, whether in physics, in other sciences, or in any field, can be a truly human enterprise.

An extensive guide to the themes and physics of the film has been prepared, which can be studied in connection with a second showing of the film late in the course. Refer to the separate document entitled *People and Particles*. Furthermore, the 15-minute film *Electron Synchrotron* gives additional technical information about the Cambridge Electron Accelerator. Even before the first showing, however, the following brief notes will be helpful.

The term *pair production* refers to the production of a pair of elementary particles, an electron and an anti-electron (a positron). Under the right conditions such a pair of particles may be produced when a very energetic packet of light, a photon (which may also be considered an elementary particle), passes near a massive object, such as an atomic nucleus. The electron beam from the Cambridge Electron Accelerator is used to produce the particle pairs by a several-step process. The motion of the particles is detected with spark chambers in which electric sparks jump along the paths of the particles.

The Strouch-Walker group (named for the two physicists who head the group, Professors Karl Strouch and James Walker) is using the electron–positron pairs to look into a newly suspected flaw in one of the most solidly established theories of physics. They are trying to establish whether the present theory of the influence of electrical charges on one another is correct. Some physicists think the present theory is bound to show its limitations when it becomes possible to experiment with charges that are extremely near each other.

This procedure has often occurred in science. By testing the limits of a theory, by looking into the contradictions between the experimental result and the theoretical predictions, the scientists are led to new theoretical structures. Of the several themes that run through the film, the relation of research to education, the international character of science, communication among scientists, and the relation of science to technology, the last is particularly evident on the first showing. Pure physics of the last 100 years or so has made possible the development of such practical devices as the oscilloscope, the electronic computer, the scintillation detector, the high-voltage generator, and the electron accelerator itself. These devices are being put to use in this experiment to produce more advances in pure physics; without these technological devices the performance of this experiment would be impossible. Conversely, without the development of physics itself, these devices might never have been invented. It is a never-ending interplay.

Most of the pieces of equipment seen in the film are technological devices that are not based on the physical laws being tested. Such equipment is often referred to as "hardware." It is the construction of the hardware that is so costly, and yet it would be quite impossible to do experiments like this without it.

In a sense, the film does not and cannot show new physics "being done." It shows construction of new equipment, on the basis of known laws. From the operation of this equipment the "new physics" will be fashioned in the minds of the experimenters.

FILM NOTES ON *ELECTRON SYNCHROTRON*

Electron Synchrotron is a 15-minute film showing one of the world's great accelerators, the 6-billion-electron-volt Cambridge Electron Accelerator (CEA). This accelerator in Cambridge, Massachusetts, was operated jointly by the Massachusetts Institute of Technology and Harvard University.

Dr. William A. Shurcliff, Senior Research Associate at the CEA, is the narrator. Using simple diagrams, he explains the principle of operation of the accelerator. Then he conducts a "guided tour" of the accelerator itself, showing and explaining how electrons are injected into a 60-m-diameter orbit, how they are accelerated almost to the speed of light while making 10,000 turns around the orbit, and how they are then directed into a 90-m-long Experimental Hall where they are used in high-energy physics experiments.

The enormous size of the accelerator and its clean, functional design are dramatically evident; and the physical principles of its operation are indicated.

The film ends with some glimpses of the type of mammoth experimental equipment that is necessary for analyzing the new kinds of fundamental particles that are created when the 6-billion-volt electrons collide with a small target.

The movie is designed for high school or college students, and may be used at the beginning of or during their first course in physics. No prior understanding of accelerators or of physics is assumed. The movie *Electron Synchrotron* was designed as a companion to the 28-minute black-and-white movie *People and Particles*, also prepared by *Project Physics*. The latter film concentrates on one actual experiment performed by a team of physicists at the CEA, but does not go into the details of operation of the accelerator. Thus, the two films complement one another. Together, they give the viewer a direct feel for how particles are accelerated to high energy and how they are then used in explorations of the fundamental particles of nature.

Further data on the operation of the CEA are to be found in the Film Guide to the documentary film *People and Particles*.

The following questions and answers will serve as a discussion guide:

1. Why accelerate the electron?

   To give it great energy. An electron accelerated to almost the speed of light has an energy concentration exceeding anything known to us 30 years ago.

2. Why choose the electron to accelerate?

   It is the easiest kind of particle to accelerate since it has an electric charge and extremely little mass. We don't yet know how to accelerate neutral particles, and to accelerate a much more massive particle, such as the proton, requires far more time and effort.

3. Is the great concentration of energy resulting from particle acceleration of any special use?

   A particle having a concentration of several billion electron-volts of energy can break up any existing atomic nucleus. More importantly, it can transform any of the smallest parts of the nucleus (the so-called fundamental particles) into particles of new and even more puzzling types. By producing and analyzing such new particles, high-energy physicists are beginning to find out what matter is and does on the smallest scale known to us. Also, by means of this device physicists are beginning to understand these subnuclear particles in terms of charge, spin, and angular momentum.

4. In an electron synchrotron, what keeps the electrons traveling in a curved path?

   A long series of magnets arranged accurately in a circle supply vertical magnetic fields that do this with no friction at all. The magnets neither speed up nor slow down the electrons. They control only the shape of the path, which is a circle in this accelerator.

5. What makes the electrons speed up: that is, what accelerates them?

   Intense electric fields are fluctuating (oscillating); and the timing of the oscillations is controlled accurately so that, as the electrons come by, the electric forces are in a forward direction. The electric fields are produced at certain special locations along the circular orbit, in copper chambers called radio frequency cavities, or rf cavities for short.

6. Why bother to make the electrons travel in a circular orbit? Why not let them travel straight ahead, along a straight line?

   If each electron travels in a circular orbit and travels around this orbit several thousand times, each of the rf cavities pushes it forward several thousand times. But if the electrons always traveled straight ahead, they would pass through each rf cavity only once and each rf cavity would give only one push. To get enough acceleration, you would have to provide thousands of rf cavities, instead of only 16, as at the CEA. Thousands of rf cavities would cost many millions of dollars, and providing power to thousands of rf cavities would also be very expensive. (Note: The electron accelerator at Stanford University in California, which uses straight-ahead acceleration and thousands of rf cavities, costs $100,000,000, eight times the cost of the CEA. It has, of course, advantages of its own, too. For example, the flow of electrons is continuous and strong instead of in pulses and relatively weak.)

7. Is there any limit on how fast the electrons can go? Can they be accelerated to speeds exceeding the speed of light?

   According to Einstein's principle of special relativity, no object can travel as fast as light does in a vacuum. Electrons in electron accelerators reach speeds almost but not quite equal to the speed of light. Electrons accelerated at the CEA reach a speed of 0.999,999,99 c, where c stands for the speed of light (about 3×10^8 m/sec).

8. As the CEA synchrotron is so huge (72 m in diameter) and so complicated, does it not use very complicated physical principles?

   No. The principles are exactly the same simple ones that can be demonstrated in a laboratory with crude, inexpensive apparatus. The people who designed the CEA accelerator made magnificent use of simply physics.

9. How about the experiments in which the high-energy electrons strike a target made of hydrogen atoms and produce new kinds of particles? Are the results exactly predictable here too?

   No. This is just the point! Physicists do not yet know all the basic principles of particles. They don't know yet how many kinds of particles there are. They don't know yet how many different families of particles there are. They don't know yet how many kinds of properties the particles can display. They don't know yet how many kinds of forces there are. These are among the most exciting current problems in science, and many physicists are working on them today.

Further data on the operation of the CEA are to be found in the Film Guide to the documentary film "People and Particles."

FILM NOTES ON *THE WORLD OF ENRICO FERMI*

The film "The World of Enrico Fermi" is a documentary film, primarily for pedagogical purposes, produced by Harvard Project Physics with a grant from the Ford Foundation. Its main aim is to give the viewer an appreciation of the life and contribution of a nearly contemporary physicist, one who was widely honored and loved, and whose work helped to transform not only physics and the style of doing science, but even the course of history itself.

The film serves several purposes. Viewers meet a number of the foremost scientists in documentary film footage or stills. They see some of the equipment Fermi used and glimpse the way Fermi made teams work so well. They see the locations at which Fermi was active (including footage of laboratories in Rome, Columbia, Chicago, and Los Alamos). But in following this work from the discovery of slow-neutron-induced radioactivity to the nuclear reactor and the A-bomb, they also find the kinds of questions raised that are much on the minds of students and the public today. These questions concern the relations between physics, technology, and social concerns.

(Note: In discussions with students, it is essential to keep certain distinctions in mind. Enrico Fermi and his group contributed to the *basic* physics of nuclear reactions between 1934 and 1938. Then, in his work on reactors and the nuclear bomb during the war years, Fermi became concerned primarily with applied and developmental research rather than with the pursuit of pure physics.)

In focusing on a major historical person, this film provides an opportunity for comparisons and contrasts with the other major documentary film of *Project Physics, People and Particles*, which should be shown in class for the first time preferably not too long before or after the Fermi film showing. *People and Particles* is trying to capture what goes on in a typical basic research laboratory in which a group of younger and older scientists collaborate on a problem in pure physics. The Fermi film makes another "cut" through the growth of modern physics, and presents a coherent life history of one physicist. We have carefully selected a person who was at all times directly accessible, and not one who was completely out of reach of ordinary appreciation as were Newton and Einstein. Fermi had many students, some of whom are shown in this film, and of whom a remarkably large number have become distinguished physicists themselves. Also, Fermi was always closely associated with other prominent physicists of his own generation, many of whom are shown in this film, too. In this way the Fermi film can be regarded as a kind of guided tour that introduces some of the most important physicists of today and of the recent past.* They are all identified in documentary footage and photographs or in interviews that were conducted specifically for the film. In a sense, the film in its own right is itself a contribution to the history of science.

In its final version, the 46-minute-long film divides into two halves; the two parts should be shown together if possible but can be shown separately. The first half brings the story up to about 1938, when Fermi obtained his Noble Prize and fled to America. The second half includes the war years and after.

In order to make the film useful to the early stages of an introductory physics course, and to wider audiences, the amount of detailed physics instruction is kept to a minimum. The ideas necessary for understanding slow neutron fission are the only ones from Fermi's voluminous and important work that are treated at any length. If the film is used late in an introductory physics course (either for the first or for the second time), the discussion of nuclear physics can be elaborated in class by referring to the significant portions in Unit 6, *The Nucleus*. The unit develops a theme first introduced in the Prologue of Unit 1.

The film provides opportunity for a good discussion among students. Among some of the points that invite treatment are the following:

- the simplicity and directness of Fermi's methods;
- the ability of his group to do outstanding work, evidently without sacrificing the kind of fun that students rarely associate with the image of the physicist;
- Fermi's own remarkable ability to differentiate between what was merely fascinating and what was truly interesting and relevant in physics—the mark of genius;
- the fact that his career coincided with that of the arrival of the Nuclear Age, and contributed to it;
- that the possibility of an A-bomb effort by the Nazis during World War II necessitated a relationship between basic research scientists and the U.S. military, a relationship that many of the scientists wished to modify or discontinue immediately after the war;
- Enrico Fermi's own isolation from political battles, and the fact that not until the last years of his life did he see the need for scientists to make clear their views to policy makers;
- the fact that the nuclear reactor, initially a by-product of the program to make the A-bomb in war time, now has become one of the chief hopes for cheap energy;
- the great variety that exists in the personal characteristis of physicists, within this country and among different countries;
- the maturing of America during the past 40–50

*The list of persons shown includes: Fermi, Allison, Amaldi, Anderson, Bohr, Chamberlain, Chandrasekhar, Chew, Compton, Marie Curie, Ehrenfest, Einstein, Goudsmit, Heisenberg, Lawrence, Meitner, Michelson, Millikan, Morrison, Oppenheimer, Rabi, Rasetti, Seaborg, Segrè, Szilard, Urey, Yang.

years in top-level research in most branches of physics, and the contributions of immigrant scientists to this process;

• the great fondness with which some of the most eminent persons look back upon their associations with Fermi, and particularly Fermi as teacher;

• his ability to excel both in theoretical work and in experiments.

After the film, the interested student will enjoy reading further. We suggest the paperback biography, *Atoms in the Family,* written by his wife, Laura Fermi.

FILM SOURCES

Combined film catalog and order forms are available on request from:

U.S. Department of Energy
Office of Public Affairs
C-460
Washington, DC 20545

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION FILMS

Requests for the free loan of NASA films should be addressed to the library assigned responsibility for your area, as indicated by list below. An up-to-date catalog of films available from NASA can be obtained free on request from NASA Headquarters, Audio-Visual Branch, Code FAM, Washington, DC 20545.

WHO MAY BORROW FILMS FROM NASA

Residents of the United States and Canada, who are bona fide representatives of educational, civic, industrial, professional, youth activity, and government organizations are invited to borrow films from the NASA Film Library that services their area. There is no film rental charge, but the requestor must pay return postage and insurance costs. In view of the wear and tear that results from repeated projection, films are loaned for group showing and not for screenings before individuals or in homes. Because custody of the films involves both legal and financial responsibility, films cannot be loaned to minors.

To expedite shipment of film, requestor should give name and address of person and organization, specifying showing date and alternate date. It is also advisable to indicate a substitute film.

Unless otherwise noted, television stations may order films for unsponsored public service or sustaining telecasts.

PSSC FILM SOURCES

PSSC films are available from Modern Talking Picture Service. To order prints or for additional information, contact the appropriate film rental office listed below.

412 West Peachtree Street N.W.
Atlanta, Georgia 30308

230 Boylston Street
Chestnut Hill, Massachusetts 02167

1687 Elmhurst Road
Elk Grove Village, Illinois 60007

1145 North McCadden Place
Los Angeles, California 90038

315 Springfield Avenue
Summit, New Jersey 07901

2323 New Hyde Park Road
New Hyde Park, New York 11040

ADDITIONAL FILM SOURCES FROM WHICH PHYSICS FILMS ARE AVAILABLE

Physics films may be obtained on a rental basis from the distributors listed below. It is advisable to review films you may select before class use.

Contemporary Films, Inc.
McGraw-Hill Book Company
Princeton Road
Heightstown, New Jersey 08520
Attention: Film Rental

Encyclopedia Britannica Films
180 East Post Road
White Plains, New York 10601

General Dynamics
Convair Division M.Z. 251–30
P.O. Box 80847
San Diego, California 92138

General Electric Company
Educational Films
705 Corporation Park
Scotia, New York 12302

International Film Bureau, Inc.
332 South Michigan Avenue
Chicago, Illinois 60604

National Film Board of Canada
1251 6th Avenue
New York, New York 10020

NET Film Service
Audio-visual Center, Indiana University
Bloomington, Indiana, 47401

Shell Film Library
1433 Sadlier Circle, West Drive
Indianapolis, Indiana 46329

U.S. Navy, Commanding Officer
Naval Education & Support Center
Atlantic Naval Station, Building Z 86
Norfolk, Virginia 23571

Western Electric Company
Motion Picture Bureau
195 Broadway, Room 1626
New York, New York 10007

T0 USING STROBOSCOPIC PHOTOGRAPHS
This transparency provides an opportunity to analyze the motion of a golf club during the full swing.

T1 STROBOSCOPIC MEASUREMENTS
Stroboscopic facsimiles of uniform speed and uniform acceleration are shown. Measurements may be taken directly, data recorded on tables and graphs plotted on grids.

T2 GRAPHS OF VARIOUS MOTION
Multiple examples of distance–time, speed–time and acceleration–time graphs. Useful for slope concept, area-under-the-curve concept, and review.

T3 INSTANTANEOUS SPEED
Stroboscopic facsimilie of body-on-spring oscillation, data table, and grid. Find approximate instantaneous speed by approaching the limit and graphical estimation.

T4 INSTANTANEOUS RATE OF CHANGE
Determines v_{av} from enlarged portion of a distance–time curve as time intervals are decreased. Shows slopes of chords approach the slope of the tangent as the slope of the tangent approaches zero.

T6 DERIVATION OF $d = v_1 t + \frac{1}{2} at^2$
Colored overlays illustrate graphical procedures using area-under-the-curve technique. Space is provided for teacher-directed derivation.

T8 TRACTOR-LOG PROBLEM
Classic third law horse-and-wagon paradox is updated with this tractor-log version.

T9 PROJECTILE MOTION
Stroboscopic facsimile of objects projected horizontally and falling freely are analyzed graphically. Space is provided for derivation of equation of the trajectory.

T10 PATH OF A PROJECTILE
This demonstration transparency suggests that students approximate portion of circles, hyperbolas, parabolas, and ellipses by throwing objects. Leads to determination of actual path of projectile. Use with *T9*.

T11 CENTRIPETAL ACCELERATION—GRAPHICAL TREATMENT
Stroboscopic facsimiles allow derivation of v^2R and graphical measurement of a_c.

Demonstration Notes

D1 RECOGNIZING SIMPLE MOTIONS
As an introduction to the section, perform the events listed below. Ask the students to select the event which would be the best starting point for a study of motion. Ask them to give reasons for their selection.

(a) roll a football
(b) roll a marble
(c) drop a sheet of paper
(d) bounce a ball
(e) swing an object around in a circle

D2 UNIFORM MOTION, USING ACCELEROMETER AND DYNAMICS CART
Tape a large liquid-surface accelerometer to a dynamics cart and show the students that the water surface is horizontal when the cart moves with uniform motion. Do the same with the cork-in-bottle accelerometer (see *Handbook*), stressing the fact that the cork remains vertical when the motion is uniform. Contrast uniform motion with the rest condition.

D3a INSTANTANEOUS SPEED (Using Strobe Photos of Body on Spring)
In this demonstration-activity the class analyzes a complex motion (that of a body on a spring) that is definitely nonuniform. Simple equipment is used to develop step by step the quite sophisticated concept of instantaneous speed, introduced in Sec. 1.7 of the *Text*. A stroboscopic record is made of one-half oscillation of the body-spring assembly and this record is used to estimate the instantaneous speed of the mass at one point of the observed oscillation.

Equipment
Body-and-spring assembly hung so as to oscillate freely, with a light source taped to the oscillating body and a sliding pointer arranged so as to indicate the point of interest. (See Fig. 1.)

Polaroid camera
Motor strobe and disk
or xenon strobe
Overhead projector for projection of print. You may find *Transparency T3* useful for recording and analyzing the data.

Procedure
The body-spring assembly is shown to the class extended, released, and allowed to oscillate briefly. A problem is posed orally to the students, "How fast was the body moving?" Stated in the terminology of Sec. 1.7, this question is "What is the instantaneous speed v of the body?" The students will recognize that the body had different speeds

at different instants and that they can't even begin to answer your question until you make it more specific. Now choose a point fairly near (but not at) the end of the oscillation and ask for the speed at that point. Attach the pointer to mark the point of interest. If we could tie a speedometer to the body, we could watch and record its reading as it passes the pointer. Since we cannot do this, we have to estimate the value of v from distance and time measurements. Two possible approaches are suggested in the *Text*.

Method 1 Measure the average speed $v_{av} = \Delta d/\Delta t$ over some interval that includes the point of interest. Begin with long time intervals and then progressively shorten the interval until there is no longer any trend in the values of v_{av} as the interval is reduced still further. This final value of v_{av} is, within the experimental uncertainty, equal to the value of the instantaneous speed v. (Note that as the distances and time intervals measured become smaller, the percentage uncertainty in v_{av} increases. Therefore, for small enough Δt, the calculated values of v_{av} will have random variations due to experimental uncertainties in Δd and Δt.)

Method 2 Make a graph of displacement against time. Draw a tangent to the curve through the chosen point P and compute its slope, which is approximately the instantaneous speed at P. This is a straightforward exercise in graphical analysis of a complex straight-line motion. Since the drawing of tangents to curves is not a very precise operation, many students will get more satisfying results from Method 1. Method 1 also has the advantage of emphasizing the concept of approaching a limit.

Fig. 1

Method 1 Approaching the limit

Set up the body-spring assembly and camera as shown in Fig. 1. The best strobe rate to use will depend on the characteristics of your spring and the mass of the body used. Try a rate of 30/sec (6-slot disk, 300-rpm motor). You will want at least 15 or 20 intervals to measure.

Alternatively, if you have a polished steel ball that can be attached to the spring, a xenon strobe light gives good results.

With the apparatus aligned and the lights out, extend the spring by pulling straight down on the body. Open the camera shutter just before releasing the body, and then close the shutter again just as the body reaches its highest point and starts down again (to avoid the confusion of overlapping traces).

Hints on photography and techniques for making the information on a single photograph quickly available to the whole class are discussed in the notes on photography in the *Equipment Notes* section of this *Resource Book*.

Calculate $v_{av} = \Delta d/\Delta t$ for several asymmetric intervals containing point P and having one endpoint in common (see Fig. 2). Start with a Δt of 20 or so time intervals and work down to two intervals. Have some of your faster students repeat the process for a different interval. Others might try symmetric intervals (see Fig. 2).

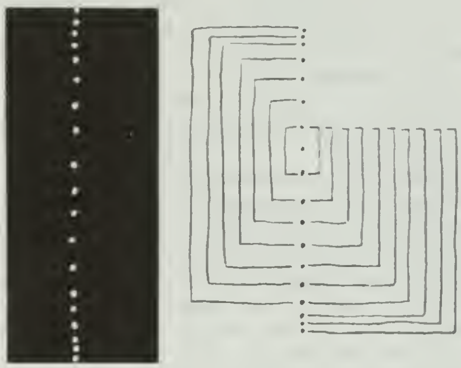


Fig. 2 A facsimile of a typical strobe photograph of the motion of a mass on a spring, showing two possible ways of choosing a set of decreasing time intervals.

Sample results are shown in Table 1, as measured from a stroboscopic photo like that shown in Fig. 2. Precision of this order is obtained by using the 0.1 mm scale and magnifier. (One millimeter in Fig. 2 represents 1 cm in real space.)

TABLE 1
Asymmetric Intervals

t (in intervals)	d (mm on photo)
0	0.0
1	0.5
2	1.6
3	3.1
4	4.8
5	7.0
6	9.5
7	12.2
8	15.0
9	17.8
10	20.7
11	23.6
12	26.3
13	28.9
14	31.1
15	33.0
16	34.5
17	35.6
18	36.5

TABLE 2
Symmetric Intervals

Δt one interval is 1/30 sec	Δd mm	$v_{av} = \frac{\Delta d}{\Delta t}$ mm/1/30 sec (on photo)	v_{av} cm/sec (real space)
18 intervals	36.5	2.03	60.9
16 intervals	35.1	2.19	65.7
14 intervals	32.9	2.35	70.5
12 intervals	29.9	2.49	74.7
10 intervals	26.3	2.63	78.9
8 intervals	21.9	2.74	82.2
6 intervals	16.8	2.80	84.0
4 intervals	11.4	2.85	85.5
2 intervals	5.7	2.85	85.5

We see in this example that the value of v_{av} does not change as Δt is decreased below six intervals. This value of v_{av} is equal, within the precision of this experiment, to the value of the instantaneous speed v at the point P at the center of each of the intervals tabulated above.

Have students graph the results tabulated in Table 2: average speed versus size of time interval (Fig. 3). Ask students:

"What would we find if we could make measurements over even smaller time intervals? Is there a point on the curve whose value represents the instantaneous speed?" You may be able to suggest that it is reasonable (since the body doesn't suddenly speed up or slow down at P) that v is the point where the curve would cut the v_{av} axis, and that in this case (the case of a limiting process) it is legitimate to extend the curve (extrapolate) to that axis. The important idea of extrapolation must be introduced with some care and a variety of examples.

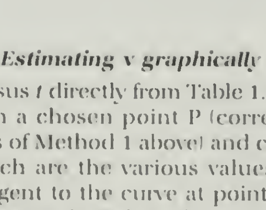


Fig. 3

Method 2 Estimating v graphically

Graph d versus t directly from Table 1. Draw chords centered on a chosen point P (corresponding to the intervals of Method 1 above) and compute their slopes, which are the various values of v_{av}. Construct a tangent to the curve at point P, and compute its slope as the value of v at P. The slope of the tangent at any point gives the value of v at that point.

Estimate the slope of the tangent at each of the data points, and plot a graph of v versus t. Repeat the process, estimating the slopes of the v–t curve at the data points, and plotting acceleration versus time. (See Fig. 4.)

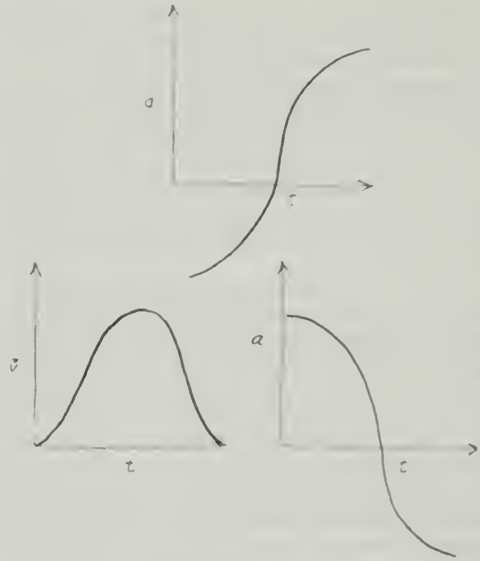


Fig. 4 Typical plots of d, v, and a against t. The lowest point of the body's motion is taken as $d = 0$, $t = 0$.

Ask students, "At what point was the mass moving fastest? Was it at one of the data points? How accurately do you know where and when the maximum speed occurred?" Ask similar questions about the time (and position) of zero acceleration.

D3b INSTANTANEOUS SPEED (Using Strobe Photos of Pendulum Swing)

As an alternative to the body-and-swing demonstration, a pendulum swing can be analyzed. In *E1–4, Measuring Uniform Motion*, students measured the average speed of a Uniform Motion Device (UMD) over long and short time intervals and probably concluded that the Uniform Motion Device was moving with nearly constant speed. In this demonstration-experiment the class looks at and analyzes a more complex motion (the swing of a pendulum), which is definitely nonuniform. Simple equipment is used to develop step by step the quite sophisticated concept of instantaneous speed introduced in Sec. 1.7 of the *Text*. A value for the instantaneous speed of the pendulum bob at the bottom (center) of its swing is estimated experimentally.

Equipment

- Pendulum about 50 cm long, hung from rigid support
- Polaroid camera
- Motor strobe disk and light source taped to pendulum bob,
  - *or* ac blinky
  - *or* xenon strobe
- Overhead projector, for projection of print
- Flexible scale, for measuring projection of print

Procedure

A pendulum is shown to the class, drawn back, released, and allowed to describe a full arc. A problem is posed for the students: "How fast was the pendulum bob moving at the very bottom of its swing?" Stated in the terminology of Sec. 1.7, this question is: "What is the instantaneous speed v of the bob at the lowest point P?" This is the reading we might get from a speedometer at the moment of passing through the bottom position if we could possibly attach one to the pendulum. Since we cannot do this, we have to estimate the value of v from distance and time measurements. Two possible approaches are suggested in the *Text*.

Method 1 Measure the average speed, $v_{av} = \frac{\Delta d}{\Delta t}$, over some interval centered on the point P. The pendulum clearly moves more slowly the farther away it is from the bottom point. Therefore, the longer the interval over which v_{av} is measured the lower v_{av} will be. All values of v_{av} will be less than the speed right <u>at</u> the bottom. To get an estimate of the instantaneous speed we must progressively shorten the time interval until there is no <u>trend</u> in the values of v_{av} as the time interval is further reduced. This value of v_{av} is, within the experimental uncertainty, equal to the value of the instantaneous speed v at P. (Note that as the distances and time intervals measured become smaller, the percentage uncertainty in v_{av} increases. Therefore, for small enough Δt, the calculated values of v_{av} will have random variations due to experimental uncertainties in Δd and Δt.

Method 2 Make a graph of displacement against time, and draw a tangent to the curve through the points for the highest observed velocities. The slope of the tangent is approximately the instantaneous speed at P.

The drawing of tangents to curves is not a very precise operation. For this reason, and because it emphasizes the idea of the approach towards a limit, Method 1 is recommended.

Method 1 Approaching the limit

There are several alternative experimental procedures here. The one described first is the simplest experimentally.

Set up the pendulum, light source, and camera as shown in Fig. 5. Use a strobe rate of 60/sec (12-slot disk, 300-rpm motor). Alternatively, you may use the ac blinky, with some added mass, as the pendulum. (In this case, of course, you do not need the strobe disk in front of the camera.) It is important to set up a marker to indicate the bottom point of the swing, and to have a rigid stop so that the bob can be drawn back to the same position for each release. Obviously the instantaneous speed at the bottom point depends on the amplitude of the swing. Be careful not to pull down on the string prior to release; the stretch will disturb the motion of the bob. Photograph as much of the swing as possible. Be sure to close the camera shutter before the bob begins the return swing to avoid the confusion of overlapping traces.

Hints on photography and techniques for making the information on a single photograph quickly available to the whole class are discussed in the *Equipment Notes* on photography.

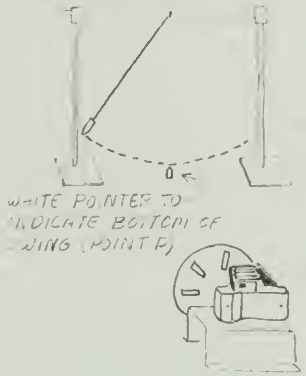


Fig. 5

Calculate $v_{av} = \frac{\Delta d}{\Delta t}$ for several intervals centered on the bottommost point or the bottommost interval, depending on the particular photograph (Fig. 6). Start with a Δt of between 30 and 40 time intervals and work down to 2 or 3 intervals. The distance intervals Δd are measured *along the arc*, which requires use of a flexible scale.

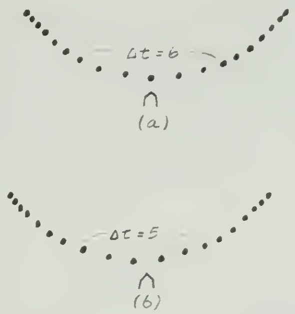


Fig. 6 (a) Trace is symmetrical about bottommost point. (b) Trace is symmetrical about bottommost interval.

Sample results are shown in Table 3. (One mm on the photograph represents 1 cm in real space.)

TABLE 3

Δt one interval is 1/60 sec	Δd mm	$v_{av} = \frac{\Delta d}{\Delta t}$ (mm/1/60 sec) on photograph	v_{av} cm/sec real space
26 intervals	84.0	3.23	194
22 intervals	76.0	3.46	208
18 intervals	66.0	3.66	220
14 intervals	53.5	3.82	229
10 intervals	39.5	3.95	237
6 intervals	24.0	4.0	240
2 intervals	8.0	4.0	240

In this example, the value of v_{av} does not change as Δt is decreased below 6 intervals. This value of v_{av} is, within the precision of this experiment, equal to the value of the instantaneous speed v at the point P.

Method 2 Alternative procedure

A slight variation perhaps emphasizes more clearly that what we are doing here is measuring v_{av} over successively shorter time intervals. A series of strobe photographs taken, each one at a higher stobe rate (smaller time interval between images) than the previous one. This is most conveniently done with the light source and disk strobe method by progressively opening up more slots. In this method one measures v_{av} over the lowest interval only on each trace. At the lowest strobe rate(s) it may be impossible to find an interval that is adequately centered on the bottom point. The *change* between the value of v_{av} over the longest Δt and its value over the shortest Δt will be less in this method, because the range of time intervals over which the measurement is made is less.

A calibrated xenon strobe and steel-ball pendulum bob could be used for this method. Yet another possibility is to feed the ac blinky with various known frequencies from a (calibrated) audio oscillator via an amplifier and transformer. (Remember that the neon lamp does not glow below about 70 V peak voltage.)

Possible extensions

1. In Method 2 above, plot d against t, draw chords centered on P to find various values of v_{av}, and draw the tangent at P to find the value of v at P. The slope of the tangent at any point gives the value of v at that point.

2. Plot a graph of the results obtained above average; that is, speed against time interval (Fig. 7).

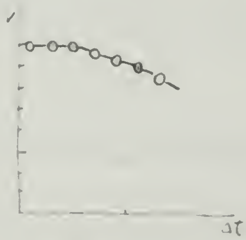


Fig. 7

Ask students: "What would we find if we could make measurements over even smaller time intervals?" Can they find a point on the curve whose value represents the instantaneous speed? You may be able to suggest that it is reasonable (since the bob doesn't suddenly speed up or slow down at P) that v is the point where the curve would cut the v_{av} axis, and that in this case it is legitimate to extend the curve (extrapolate) to that axis. The important idea of extrapolation must be introduced with some care and a variety of examples.

3. There are many other measurements that can be made with this simple experimental setup. It could be instructive for students to make graphs of d against t, v_{av} (measured over one interval) against t, and, if possible, of acceleration, a, against t. (Measure t and d from the point P, Fig. 8.)

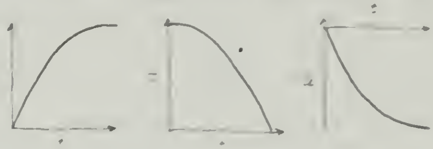


Fig. 8 Plots of s, v_{av}, a against t. The bottommost point of the pendulum swing is taken as $s = 0$, $t = 0$.

4. The concept of instantaneous speed will come up again in Unit 3. The kinetic energy of a body at a given instant depends upon its speed at that instant. The interchange of potential and kinetic energy in a pendulum will be referred to specifically. It is worthwile to make photographs that encompass both the topmost point, the bottom point P, and some scale to give absolute measures of distance. The photographs should be kept for use later in Unit 3.

Questions for discussion

1. Could one ever measure v experimentally? How?

2. A car speedometer appears to measure instantaneous speed. Does any student know how it works? How is it calibrated? (This is done at constant speed, that is, by measuring Δt for a known Δd while the speedometer reading is unchanging. So really all that we know is that the speedometer tells us instantaneous speed for the special case of uniform motion, that is when $v = v_{av}$ at every point.)

D4 UNIFORM ACCELERATION, USING LIQUID ACCELEROMETER

This demonstration allows you to show that when a cart moves with constant acceleration a, the surface of the liquid is a straight line tilted in the direction of the acceleration.

Give the cart a uniform acceleration by suspending an object over a pulley, as in Fig. 9.

It is best to use objects whose masses range from 100 to 400 g. It is important to keep the string as long as possible, so that you use the entire length of the table. By changing the mass of the suspended object, you can vary the acceleration of the cart. Notice that the slope of the liquid increases with greater acceleration. The slope is thus a measure of the acceleration. It can be shown that $\tan \theta = a/g$ so that $a = g \tan \theta$. You will find detailed comments on quantitative work with the liquid-surface accelerometer in the *Equipment Notes* for Unit 1. The fan cart, Fig. 35, can be used as a nongravitational source of uniform acceleration. A small accelerator may be mounted on the fan cart or on another cart pulled by the fan cart.

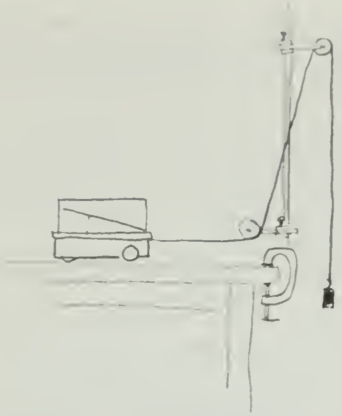


Fig. 9 Arrangement to demonstrate uniform acceleration.

D5 COMPARATIVE FALL RATES OF LIGHT AND HEAVY OBJECTS

Drop several pairs of objects, such as a marble and a lead shot, simultaneously from the same height. Decide whether the theory of Aristotle or that of Galileo agrees best with the observations. Account for any discrepancies.

On a large book place several objects, such as a small piece of paper, a marble, and a paper clip. Drop the book. Do the objects fall at the same rate and stay on the book?

D6 COIN AND FEATHER

If the equipment is available, do the coin and feather experiment. Failures are usually due to a poor vacuum pump or to a defective seal on the apparatus. Check ahead of time to see how long it takes to evacuate the apparatus sufficiently to show that the coin and feather fall together.

D7 TWO WAYS TO DEMONSTRATE THE ADDITION OF VECTORS

Method 1

Apparatus:

- 50 cm × 50 cm board
- Two dynamics carts
- Two Uniform Motion Devices (UMD)
- Two sheets of clear plastic (Kodak Safety 3, for overhead transparencies)
- Three marking pens of different colors
- Clamps, stands, etc., to support pens
- Stopwatch
- Three people to operate Uniform Motion Devices (UMD) and stopwatch

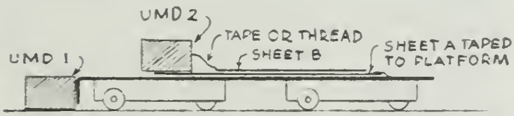


Fig. 10 The rollowing platform and the arrangement of the plastic sheets.

Fasten the two carts underneath the board to form a rolling platform, as shown in Fig. 10. Hook up UMD 1 to push the platform along the table. Attach one plastic Sheet A to one corner of the platform, as in Fig. 11.

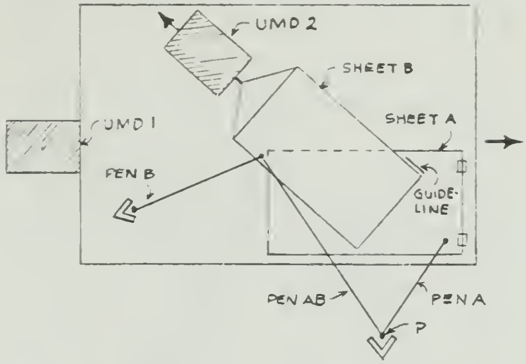


Fig. 11 The apparatus as seen from above.

Attach plastic Sheet B to UMD 2 as shown, so that the UMD tows the sheet along smoothly behind it. Adjust the tow rope so that v_b (the velocity of UMD 2) is parallel to the long edges of Sheet B.

Choose a direction for v_b, and aim UMD 2 in that direction, laying Sheet B across Sheet A as shown. Draw a guide line on Sheet A, using the edge of Sheet B as a ruler. This is your only record of the direction of v_b.

Attach one marker (Pen A) by means of a ringstand and stiff wires so that it makes a line on Sheet A as the platform rolls along. From the length and direction of this line, you will be able to figure out the magnitude and direction of v_a, assuming that v_a is constant.

Attach Pen B to the rolling platform. It makes a line on Sheet B that indicates the motion of UMD 2 relative to the platform.

Pen AB also marks on Sheet B, but it is fastened to the stationary ringstand on the table. The motion of Sheet B with respect to the table is made up of the two simple motions added together vectorially. From the line that Pen AB makes, you can deduce the vector $(v_a + v_b)$.

Adjust Pen B and Pen AB so that they begin at the same point P on Sheet B.

With the pens in place, set the Uniform Motion Devices in motion at the same time. (This will take a little practice.) Shut them off, again simultaneously, when the longest line that has been drawn is 10–15 cm long. Use the stopwatch to time the motion.

You now have three lines of different lengths, colors, and directions. If you make certain assumptions, you can treat these lines as direct representations of v_a, v_b, and $(v_a + v_b)$. Add an arrowhead to each line to indicate the actual direction of the velocity that it represents. Remove both plastic sheets from the apparatus and slide Sheet B over Sheet A until the head of v_a is at point P. Be sure to keep the edge of Sheet B parallel to the guideline.

Ask the students if $(v_a + v_b)$ seems to be the vector sum of v_a and v_b, using the parallelogram rule. Convince them that if these velocities have added as vectors, the three vectors should form a triangle. Is this the case (see Fig. 12)?

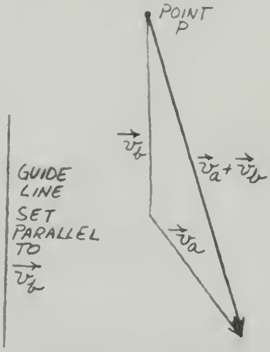


Fig. 12 Try the same procedure for a few other directions of v_b (v_a and v_b parallel, opposite, at right angles, etc).

Method 2

Apparatus:

- dc blinky, set to about 1 flash per second
- The same rolling platform as in Method 1, painted black
- Two Uniform Motion Devices
- Polaroid camera mounted on tripod (for 3000 speed film, the lens setting is about EV 16)
- Bench stand and pointer to indicate the starting point of the blinky
- Three people to operate Uniform Motion Devices and camera

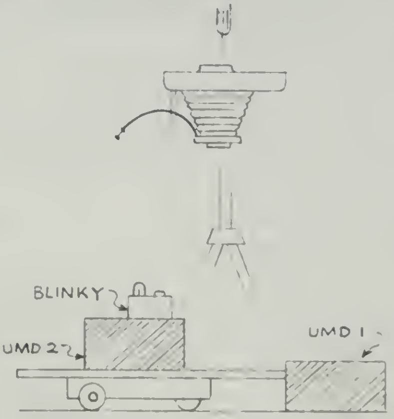


Fig. 13 Arrangement of apparatus when v_1 and v_2 are parallel.

Set up the rolling platform, pushed by UMD 1, as in Method 1. The velocity of the platform as it moves past the camera is again v_a. Place UMD 2 on the platform: its velocity with respect to the platform is v_b. Point the camera downward, so that it takes a picture of the apparatus from directly overhead. Mount the blinky on UMD 2 and position the pointer so that you will be able to put the blinky back in its original position after taking the first picture.

Turn out the lights, open the camera shutter and set UMD 1 in motion. Let it tow the platform across a good part of the camera's field of view. Ask the students which velocity is obtained from the strobe record of this motion.

Replace the platform so that everything is the same as it was before the last step. Repeat the process without advancing the film (when you are through, you will have a triple exposure) but this time have only UMD 2 in motion. Which velocity can you calculate from this strobe record?

Return to the starting point again, and take a third picture (on the same film) of the motion of the blinky, this time with both Uniform Motion Devcies moving.

Develop the print and calculate the three velocities (speeds and directions) $\vec{v}_a$, $\vec{v}_b$ and $(\vec{v}_a + \vec{v}_b)$. Skip the first interval in each strobe record: It takes the Uniform Motion Devices a little time to get up to speed. Draw arrows representing the three velocities, and check as in Method 1 to see if the parallelogram law of vector addition holds for the motion you have observed. Do the three vectors form a triangle? Should they?

Again, try the cases where $\vec{v}_a$ and $\vec{v}_b$ are parallel, at right angles, and at several other angles of your choosing.

D8 DIRECTION OF ACCELERATION AND VELOCITY

Using the same arrangement as *D4*, demonstrate that acceleration and velocity can have different directions. Hang an object of 100- or 200-g mass over the pulley and give the cart a push to the left so that it goes nearly to the end of the table before it stops and reverses direction. You should try to give a short, smooth push so that the liquid reaches its steady state quickly.

Once the water has reached its steady state, the surface is a straight line whose slope does not change, even when the velocity reverses direction. The explanation, of course, is that the acceleration is constant and independent of the velocity. Only the weight of the object over the pulley determines the acceleration of the cart.

D9 DIRECTION OF ACCELERATION AND VELOCITY—AN AIR-TRACK DEMONSTRATION

Mount the small accelerometer on an air-track cart. When the track is horizontal and the cart is at rest or moving with uniform speed, the surface of the liquid is also horizontal. Only when a horizontal force causes the cart to accelerate (for example when the cart starts or stops or collides with something else) is the slope of the surface not horizontal.

Next, place the track at a slight incline. When the cart slides freely on the air track, the surface is parallel to the track. These interesting facts are explained in the *Equipment Notes.*

Again, you can show that velocity and acceleration can have different directions. Give the cart a push up the incline. If friction is negligible, the slope of the liquid remains the same while the cart slows down, reverses direction, and moves down the incline. If frictional forces are increased by adding mass to the cart, the slope will decrease when the cart begins to move downhill.

D10 NONCOMMUTATIVE ROTATIONS

One of the points frequently made about vector addition is that it is commutative; that is, the order of addition does not affect the sum. Students are frequently convinced from their experience with arithmetic that this is true of all operations. It is useful to be able to show them an example of an operation that is not commutative.

If a closed book is placed on the desk in front of a student, rotated 90° about an axis along the spine of the book, and then rotated another 90° about an axis parallel to the near edge of the desk, the final orientation of the book is different than if the opposite order of the two operations is followed.

D11 NEWTON'S FIRST LAW

There is an aesthetic appreciation in science for simple statements that describe very complex phenomena. $E = mc^2$ is an example of such a statement. Newton's first two laws and the equation $F = ma$, which follows from them, are early examples. For the teacher, these simple statements often create difficulties because the students fail to realize their importance. There is a tendency to feel that what is not complex and filled with mathematical symbols cannot be very important. Nothing could be further from the truth. Many people contributed to the eventual development of the three laws, and Newton's own work was a perplexing amalgam of intuition, definition, and experiment. While one cannot say precisely how Newton came to his conclusions, he was deeply familiar with the related phenomena. Therefore, we suspect that the students' introduction to the laws of motion should avoid the didactic and favor direct experience and an intuitive approach.

The demonstration described below may seem trivial, but firsthand experience with very low-friction motion is valuable for understanding Newtonian physics. While this is listed as a demonstration, it should be conducted as an informal experiment. This has always been a very enjoyable experiment for the students, who frequently mention it as their favorite.

Equipment

- Several pucks with balloons or plastic beads
- Puck table
- Large rubber band
- Air track (optional)

Procedure

Student lab groups are given single pucks without the balloons or plastic beads, with the instructions that they are to play with them for several minutes so as to be able to describe how the pucks move. Then a brief discussion is held to establish what happens to the pucks' motions under various circumstances, for example, just resting on the table, being pushed briefly, being pushed steadily, when the table is tilted, etc. Friction may be mentioned; perhaps someone will suggest what the motion would be like without friction.

Immediately demonstrate the low-friction capability of the pucks, and supply students with balloons and/or plastic beads for another short period of investigation. (Half the class could use balloons and half plastic beads to make the results more general.) The instructions are, as before, to be able to describe the motion of the pucks. Fences made from the large rubber bands are excellent as reflectors because they allow long runs.

The leveling of the surface may be a problem, especially with the balloon pucks. The concluding discussion might become heated on the how-do-you-know-when-there's-no-force paradox, but that's fine. If students argue about these things, they are aware of the issue.

The disk magnets or air track can be used for a further extension of frictionless motion.

D12 NEWTON'S-LAW EXPERIMENT (AIR TRACK)

With the calibrated accelerometer you can perform experiments to define forces in terms of the accelerations of objects whose masses are known. The accelerometer would enable you to determine the accelerations directly. See *Equipment Notes* for information on how to use the liquid-surface accelerometer quantitatively.

D13 EFFECT OF FRICTION ON ACCELERATION

Demonstration 12 works only if friction is negligible. Since the direction of the frictional force F_{frict} is always opposite to the velocity, you can show the effect of friction on acceleration by attaching tape with adhesive on both sides to the wheels of the cart.

When the cart moves to the right, the horizontal forces acting on it are illustrated, as in Fig. 14. The acceleration is then

$$a = \frac{T - F_{\text{frict}}}{M}$$

where M is the mass of the cart plus the accelerometer. When the cart moves to the left, however, the forces act as in Fig. 15. The acceleration is now

$$a = \frac{T + F_{\text{frict}}}{M}$$

The tension T is simply the weight of the object hanging over the pulley and is independent of the velocity.

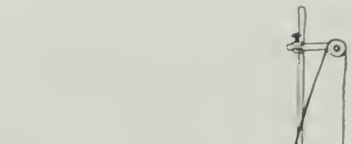


Fig. 14 Force diagram when cart is moving to right.

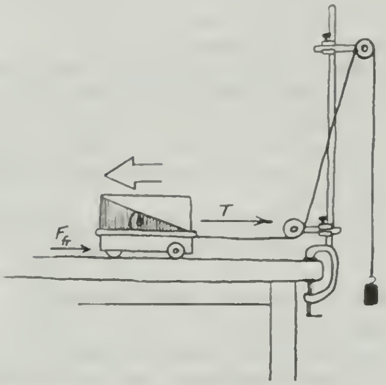


Fig. 15 Force diagram when cart is moving to left. The change in θ is exaggerated.

Since the acceleration is less when the cart moves to the right than when it moves to the left, the slope of the water when the cart moves to the right will also be less. This difference in slopes is slight, but noticeable.

D14 DEMONSTRATIONS WITH ROCKETS

Introduction

The demonstration experiments suggested here can accomplish two things. They are exciting, which makes them ideal as motivating experiments at the beginning of the course. Rockets and space flight are matters of great public interest today, and experiments like these could do much to arouse interest (and perhaps increase enrollment) in a physics course. The experiements can also be used to teach quite a lot of physics: free fall, force, impulse, conservation of energy, application of trigonometry, etc.

We cannot stress too strongly the need for strict supervision by the teacher at all times. Get permission and support from local officials and school administrators before starting model rocketry.

Small solid-fuel rocket engines, lightweight rockets, and a considerable body of supplementary information can be purchased from Estes Industries, Inc., Box 227, Penrose, Colorado 81240. Their catalog is available on request from the address given. We have tested the "Scout," the "Corporal," and the "V-2." Assembly for these models ranges from 1 to 2 hours and could be done by students.

When used with some care under strict supervision of the teacher, these rockets are probably considerably safer than a good number of other experiments that are performed in the classroom. However, students should not be permitted to take home rockets from the school's supply or to use the school's rockets during school hours without careful supervision. Although quantitative experiments of real precision are probably mathematically too involved, students can learn much from a series of demonstrations that permit some student participation.

Rocket engines come in a variety of sizes with maximum thrusts of either 6N or 39N and thrust durations from 1.7 sec to 2.0 sec. In addition, a special-purpose engine [B.8–O P] for use in static tests is available.

Experiments with rockets in free flight

If a large, open space is accessible to the class, a number of experiments can be performed with free-flight rockets. For example, one may use successively more powerful engines in several otherwise identical rockets. Another set of experiments would make use of rockets of identical exterior design but of different mass: in fact, one might make one of the rockets so heavy that it will not lift off. We all get a thrill from firing the small rocket and seeing it rise rapidly. Students should stand at known distances, at least 30 m from the launching pad, each with a simple altimeter, consisting of a protractor with a small plumbline and a viewing tube, made, for example, from a large soda straw (Fig. 16).

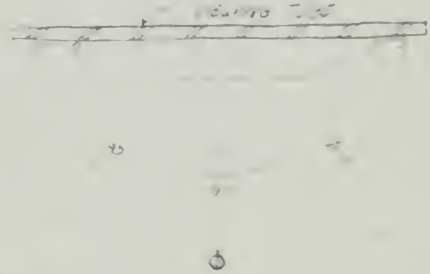


Fig. 16

Each student should try to measure the angle of elevation of the rocket at the same moment, preferably when the rocket has reached its maximum height. The teacher can call out the time for this measurement. Using simple trigonometry, students can calculate the height of the rocket. If there is little wind and the rocket rises vertically, they can calculate the height knowing the distance and elevation angle. Each student will find a value for

H. A comparison of the results will provide an opportunity to discuss errors of measurement.

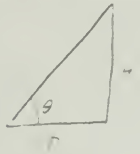


Fig. 17

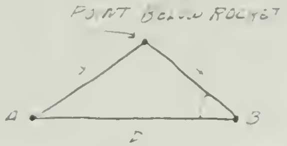


Fig. 18

In most cases, the rocket will not rise vertically. The computations will become fairly involved unless students can measure both the angle of elevation at points of maximum *H* and the angle through which they must turn from a fixed line when measuring *H*. For example, if two students, A and B, stand at a fixed distance *D*, and each has to turn from the line connecting their position by angles of θ and ϕ, respectively, we can then at once find the point above which the rocket was at its highest point and determine the distances x and y. Knowing x and y, each student can calculate H (see Fig. 18). Write to Estes Industries for copies of their Technical Report TR3 "Altitude Tracking," which gives detailed instructions. This exercise and excursion into trigonometry, although not directly part of a physics course, is useful in showing the need for mathematics as a tool.

When firing rockets, all possible safety precautions should be followed. Estes Industries will supply an outline (Attachment #3) of how to handle the rockets and what methods to employ to prevent accidents. In fact, the safety code as supplied by Estes has an educational value in showing students how to handle potentially dangerous situations.

Experiments with test stands

The design of a simple test stand for rocket engines requires knowledge of fundamental physics principles. Basically, one wants to measure as accurately as possible the force (thrust) a rocket exerts as a function of time. Since the burning times of these rockets are short (from a minimum of 1.7 sec to a maximum of 2.0 sec), one needs to use a recording device. In order to measure thrust correctly, the apparatus should be truly static, that is, there should be as little motion as possible while the engine fires. If a spring is used to provide the balancing force, precautions must be taken to avoid oscillations; in fact the damping should be critical and furthermore should be velocity-dependent so that the recording pen will always return to the same zero position.

Test stands can be designed in a variety of ways. Two designs have been tested.

A. The first test stand consists of an engine holder (Figs. 19 and 20), made from a rocket-body tube (Estes Cat. #651–BT–40, 0.765" I.D., 0.028" wall thickness) connected to an aluminum rod R that is free to move in two bearing blocks B.

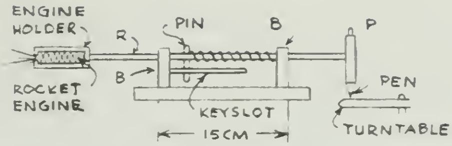


Fig. 19

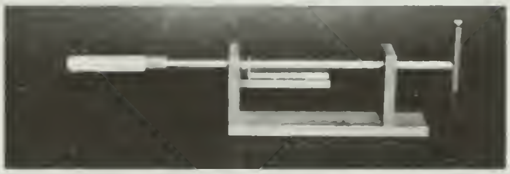


Fig. 20

Attached to the far end of the rod R is a tube P into which a marking pen or some similar easy and light writing device can be inserted. The rod R carries a pin that serves two purposes: It compresses a spring as the engine is fired and it prevents the rod from turning about its axis by riding in a key slot attached to one of the bearing blocks. The spring constant should be chosen so that a steady (static) force of 20 N will give a compression of approximately 7.5 cm. Friction in the bearings may just provide the necessary damping force; otherwise, one can add some damping by pressing a cloth strip against the rod. The test stand is set up radially near a turntable so that when no force is applied to the spring, the pen will leave a circular trace near the edge of a circular sheet of paper attached to the rotating turntable. When the rocket engine is fired, the pen is pushed toward the center of the turntable and plots a graph that can be analyzed for a measure of the force applied to the spring. With the turntable rotating at 33 rpm, a "firing" of an Estes A8–0 (P) rocket engine will leave a polar coordinate record that covers almost a complete revolution, indicating that the force was applied for approximately 1/33 min or just under 2 sec (Fig. 21). If a linear chart drive is available that will move the paper at a high enough speed to spread out the graph over a reasonable distance (at least 25 cm/sec), you can substitute this for the turntable. However, there is merit in using a polar graph, if only to show students a different method of recording and analyzing data.

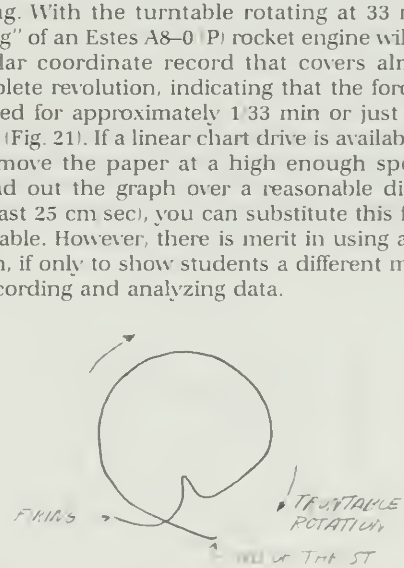


Fig. 21

To translate the curve drawn by the recording device during the firing into a force-versus-time plot, one needs to calibrate the test stand. This can be done by applying known forces; for instance, weights applied via a pulley to the spring, while the turntable is moved by hand through sections for each applied force. A calibration curve, relating displacement to (static) force can then be drawn.

Note that the spring used was *non*linear. The reason for this is that the initial large force acts for a short time only, and thus the impulse due to this force is fairly small. To measure accurately the much smaller sustained force, a spring is needed which will give reasonably large deflections for the small force acting over the longest part of the firing. Again, there is additional educational benefit to be derived from the fact that another illusion is shattered for most students (and many teachers); namely, that springs by nature are linear and that Hooke's law can be applied without thought.

It might be worthwhile to point out that there is another problem invovled in this analysis; namely, that the force applied by the rocket engine is an "impulsive" force, acting for a short time only, whereas the calibration of the test stand is done statically.

Fig. 22

Students would benefit from transferring the polar-coordinate graph to a Cartesian-coordinate graph. They can then compute the total impulse of the engine ($\int Fdt$) by finding the area under the curve.

Fig. 23

If this impulse is assumed to occur in a short time, compared with the total flight, a first approximation gives $\int Fdt = mv_{final} - mv_{initial}$, when m is the mass of the rocket plus engine and v is the speed of the rocket after the impulse has been applied. If we neglect all external forces except gravity, we can find the maximum height to which it would rise from simple kinematic considerations ($v_1^2 - v_g^2 = 2gh$). The actual height to which the rocket will rise is much less than the computed one.

B. A second type of test stand (Figs. 24 and 25) can be assembled easily in most schools from odds and ends. It involves a 37.5-cm wooden ruler in which a vertical shaft has been placed at the 30-cm mark. The ruler can turn freely about this shaft in a horizontal plane. (Inexpensive steel shafts with bearings are available from radio supply houses, for example, Allied #44Z094, panel-bearing assembly with 7.5-cm shaft. At the 35-cm mark a rocket motor holder is fastened securely by gluing it with a good contact cement, then tying it with string (Fig. 26). Finally, paint the string and motor holder with glue, coil dope, shellac, or some other material that will bind to the ruler, string, and rocket motor holder.

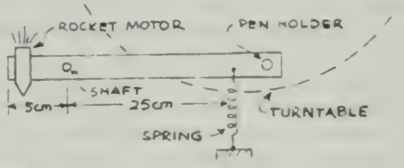


Fig. 24

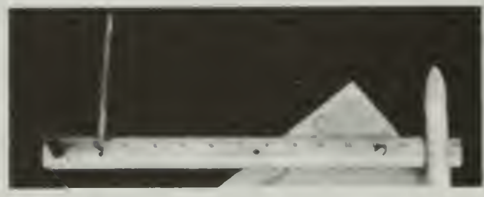


Fig. 25

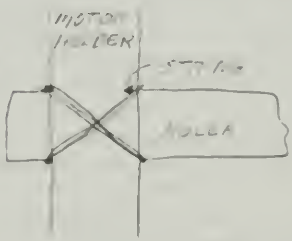


Fig. 26

At the 5-cm mark a spring is fastened that will extend not more than 7.5-cm when a force of 4 N is applied to it. A nonlinear spring would have the same advantages explained earlier

Note: There are various ways to make nonlinear springs. In this particular case one could, for example, have two springs attached (Fig. 27), such that for small forces Spring 1 will stretch, but Spring 2 will not be under any tension. As Spring 1 stretches, eventually the string that connects Spring 2 to the ruler will become tight and the force constant of the combination will become the sum of the force constants of both springs.

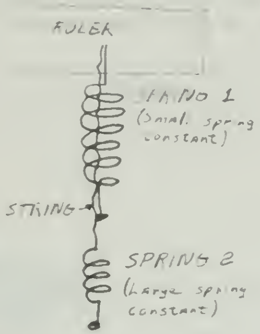


Fig. 27

A second method to obtain a nonlinear spring uses one single spring and a thin string loosely tied between some of the coils of the spring (Fig. 28). As the spring is stretched, all coils will open up at first, until the string becomes taut. From then on, only those coils can extend that are outside the tied-down section of the spring. It is easy to adjust the relative spring constants simply by shifting the position of the string, holding back more or fewer of the coils.

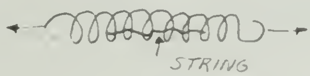


Fig. 28

A light tube which can hold a marker pen is attached to the far end of the ruler, near the 2.5-cm mark. We again use a turntable so that the pen can trace a graph of its excursion as a function of time on a paper disk fastened to the turntable.

The reason for using the unequal lever arms in this design is to have the rocket engine move through as small a distance as feasible, thus approaching a true static test, and also to have the moving parts of the device be as light as feasible while still giving a reasonably large trace on the graph paper.

Figure 29 shows the result of a firing using a linear spring and no damping force. A number (at least four) of oscillations following the initial excursion of the pen can be seen.

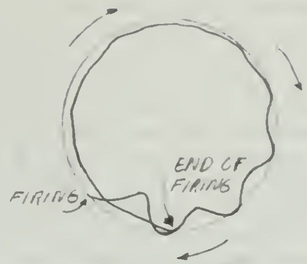


Fig. 29

Damping can be applied in a variety of ways and will provide a very interesting exercise in applied physics. The most obvious way to decrease oscillations is to apply a frictional force. A bottle brush held perpendicular to the ruler near the 7.5-cm mark and pushed against the flat side of the ruler (Fig. 30) will help dampen out the vibrations (Fig. 31), but the damping is not critical. In addition the friction will introduce sizeable shifts in the zero position. Ideally the damping force should be velocity-dependent. We have tested a viscous device, consisting of a metal vane being pushed through oil and find that it is also noncritical (Fig. 32) but does not have a zero correction. Another method would be to use a metal plate moving in a strong magnetic field (eddy-brake).

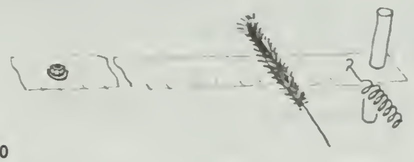


Fig. 30

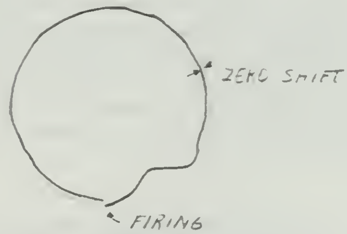


Fig. 31

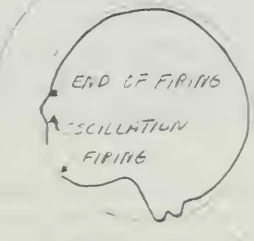


Fig. 32

This part of the project is completely open-ended. Students can undertake a systematic experimental study of damping forces and begin to appreciate the problems of the scientist or the engineer. They will also begin to realize that through systematic study of a problem one will slowly be able to approach better and better solutions.

This demonstration can teach a good deal about free fall, propelled flight, the operational meaning of force, momentum, conservation of energy, the use of trigonometry, experimental uncertainty, and the scattering of data; but it can also be justified as a motivating experiment that is interesting and exciting. Rockets and space flight today hold a unique position in the public eye. It seems reasonable to make use of this interest in attempting to attract students to the physics course. There is no question that the news of such firings in a course will spread rapidly through a school. As a consequence, students who otherwise might not have found out about the excitement and challenges of physics may become interested.

D15 MAKING AN INERTIAL BALANCE

An inertial balance may be an aid to help the students distinguish between mass and weight. One end of a hacksaw blade is clamped to a bench so that it can vibrate in a horizontal plane. Various masses are attached to it, but their weight is supported by suspending the masses from a string. The hacksaw blade is pulled to one side and then released so that it swings.

D16 ACTION–REACTION FORCES IN PULLING A ROPE. I

Attach a heavy spring balance to a wall and find two students whose maximum pull is about the same. Then place the spring balance between the two students and have them pull against each other with their maximum force. The balance will read the same in each case. This should help bring home the point that a "pushed or pulled" object, such as a wall, will exert an opposing force whenever a force is applied to it.

D17 ACTION–REACTION FORCES IN PULLING A ROPE. II

Place a student on each of two carts and pass a rope between them. First have one student pull alone, then the other, and finally both. Start the carts from the same position each time and note the place where they meet. Ask the class whether an observer, watching the carts alone, could tell which student was actively pulling in each case.

D18 REACTION FORCE OF A WALL

When you lean on a wall does it exert a force on you? Stand on a cart or roller skates and lean against the wall.

D19 NEWTON'S THIRD LAW

The following simple demonstrations dramatically illustrate Newton's third law. Their simplicity, moreover, gives some indication of the elegance and profundity of this remarkable law.

To show that forces exist in pairs on different objects, and that the paired forces act in opposite directions, set up a linear equal-mass explosion between two dynamics carts. Propel the carts apart with a steel hoop, magnets, streams of water, or any other forces you can think of. See Fig. 33 for some suggestions. Stress that this concept of force-opposite-force is valid for all types of forces.

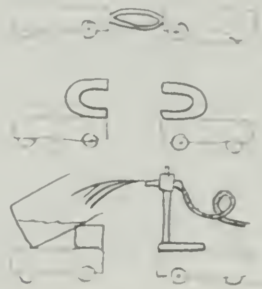


Fig. 33

The experiment on conservation of momentum, *E3–1*, gives detailed instructions about the explosion using the steel hoop. You can take a strobe photograph of the explosion, and show that if the carts have equal masses, they move apart at equal speeds. If the carts have equal speeds, the accelerations they received during the explosion were equal in magnitude. Since the carts have equal masses and since the duration of the interaction is the same for each cart, Newton's second law implies that they experienced equal forces during the explosion.

A more direct method to show that the forces are equal in magnitude is to modify the demonstration by propelling the two dynamics carts with large magnetron magnets. Move the magnets back about 3 cm on the carts. Place a pencil or dowel in the hole at the front of each cart and loop an 8-cm rubber band around the pencils. When you release the carts, they will separate, stretch the rubber band, oscillate, and finally come to rest.

When the carts are at rest, the forces acting on each cart are those shown in Fig. 34. The tension in a rubber band is uniform, so $T = T'$. Since each cart is at rest, then $T = F$ and $T' = F'$. Thus $F = F'$, and the magnetic forces on the carts are equal. Note that in this demonstration the carts can have different masses.

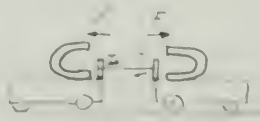


Fig. 34

Another exciting way to illustrate Newton's third law is to mount a sail on the fan cart that was used to illustrate uniform acceleration *D4*, and let the propeller blow against the sail. Since the sail bends forward, clearly there is a force on it. But the cart does not move because when the propeller pushes against the air, the air exerts a reaction force against the propeller. Thus, the net force on the glider is zero. (If the sail does not catch all the air from the propeller, the cart may move slightly.) If you remove the sail, the only force on the glider is the reaction force exerted by the air on the propeller. This force causes the glider to move backwards.

The fan cart rigged for uniform acceleration is sketched in Fig. 35. The placement of the sail to show action and reaction is sketched in Fig. 36.

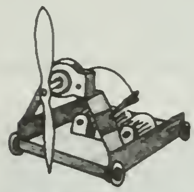


Fig. 35

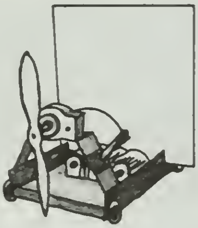


Fig. 36

D20 ACTION–REACTION FORCES BETWEEN CAR AND ROAD

Demonstrate the coupling of forces between a car and the road. Obtain a motorized toy car. Place a piece of cardboard on top of some plastic beads or an upside-down skate-wheel cart. Then place the wound-up car on the cardboard roadway. The opposing forces will cause the roadway to move backward when the car moves forward.

D21 ACTION–REACTION FORCES IN HAMMERING A NAIL

Hammer a nail into a plank while the plank is first on a bench, then on a soft pillow. The force exerted on the nail depends not only on the hammer but also on the opposing force of the plank.

D22 ACTION–REACTION FORCES IN JUMPING UPWARD

When you jump off the floor, does the floor push harder on you in order to cause the upward acceleration? Jump up from a bathroom scale and watch the scale.

D23 FRAMES OF REFERENCE

The following demonstration illustrates the idea that different motions can appear the same when observed from different reference frames.

One familiar example is the situation of two trains in a station on parallel tracks. An observer in one train cannot tell which train is moving, or whether both trains are moving, unless he or she watches the station.

In the following demonstration, a camera photographs a blinky, with either the camera or the blinky moving at constant velocity. From the photograph, one cannot tell which object was moving. The photos in the two cases are identical, unless part of the laboratory also appears in them.

This idea that an observer's view of a motion will depend on one's frame of reference will be a major theme in Unit 2. To an observer on the earth, the sun seems to move daily around the earth. But the same apparent motions would be seen if the sun were stationary and the earth rotated on an axis. The impossibility of distinguishing between the two motions caused much intellectual controversy in the sixteenth and seventeenth centuries.

Equipment

Polaroid camera, cable release, and tripod. With 3000-speed film, use the EV 15 setting.
Two dynamics carts
Two Uniform Motion Devices (UMD)
dc blinky
Black screen
Turntable

Straight-line motion

Mount the blinky on one cart and the camera on the other. Use the UMD to push the carts. It may be necessary to increase the mass of the cart with the blinky, so that both carts are driven at the same speed. Arrange the apparatus as shown in Fig. 37.

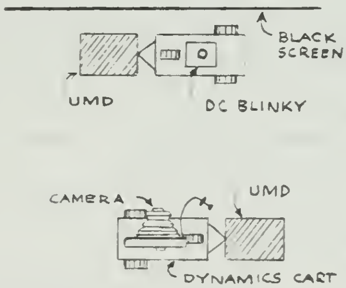


Fig. 37 Apparatus for linear motion.

Take two photographs, one with the blinky moving and the camera stationary, and the other with the cart moving and the blinky stationary. Use the cable release and be careful not to jar the camera when you open the shutter.

Circular motion

Mount the camera on the tripod and attach the blinky to a turntable. Aim the camera straight down. Figure 38 shows this arrangement.

Fig. 38 Apparatus for circular motion.

Take a time exposure with the camera at rest and the blinky moving one revolution in a circle. If you do not use the turntable, move the blinky by hand around a circle drawn faintly on the background. Then take a second print, with the blinky at rest and the camera moved steadily by hand about the axes of the tripod. Try to have the camera move at the same rotational speed as the blinky moved in the first photo.

Extension

Observers in the train cannot tell which train is moving if there is a relative velocity between the trains. If there were a relative acceleration, however, they could tell which train was accelerating. They could detect the acceleration of the train, for example, with a liquid-surface accelerometer. If the acceleration were great enough, they would also feel themselves being pushed back or thrown forward. An object cannot accelerate unless a force acts on it.

Strictly speaking, our observers could not be sure they were accelerating. According to Einstein's principle of equivalence, the effects of a uniform acceleration $\vec{a}$ are indistinguishable from those of a uniform gravitational field $-\vec{a}$. In the train, however, the observers can be reasonably confident that the accelerometer detects an acceleration, not some bizarre gravitational field.

D24 INERTIAL VERSUS NONINERTIAL REFERENCE FRAMES

Have a student toss a ball straight upwards and catch it again while walking at a constant speed. Ask for descriptions of the path of the ball as seen by the ball tosser and by a seated student. How do the accelerations compare as measured by the walker and by the seated student? (They are the same.) How would the path appear if the ball tosser had stood still and the student had moved sideways with the original speed of the walker? (It would appear to be the same as before to both viewers.)

Now toss the ball as you accelerate, walking faster and faster, and again as you slow down. Also toss the ball as you walk in a circle. Show that, in these cases, the two frames of reference give two different accelerations.

You might want to discuss this idea again in Chapter 4, where the idea is developed that acceleration is caused by an unbalanced force. An accelerated frame of reference requires apparent (or fictitious) forces to explain accelerations that are not present when viewed from a fixed frame of reference.

D25 UNIFORM CIRCULAR MOTION

To demonstrate the acceleration in uniform circular motion, place the accelerometer along the diameter of a phonograph turntable. When the turntable rotates, the liquid surface is parabolic. Figure 39 shows this situation. The acceleration increases with the distance from the center and is always directed inward. By changing the speed of the turntable, you can show that the acceleration is greater for higher speeds of rotation. This is also discussed in the *Equipment Notes* on the liquid-surface accelerometer, page 73.

Fig. 39 Accelerometer on rotating turntable. The surface of the liquid is parabolic.

D26 SIMPLE HARMONIC MOTION

Harmonic motion can be demonstrated as an example of a more complex motion. To show that harmonic motion can be discussed in terms of circular motion, set an object such as a peg on a phonograph turntable, moving in uniform circular motion. Then illuminate this motion from the side and project its shadow onto a screen so that all that can be seen is a back-and-forth motion. Harmonic motion can be developed further, but it is probably enough just to give several examples of objects that have this motion, such as a vibrating tuning fork, a pendulum, and an object suspended on a spring.

D27 SIMPLE HARMONIC MOTION: AIR TRACK

By attaching a long rubber band or string to each end of the cart and pulling back and forth, you can make the cart move in approximately simple harmonic motion. The class can see qualitatively that the acceleration is directed opposite to the velocity and is at maximum when the cart is farthest away from the equilibrium position.

Experiment Notes

E1–1 NAKED-EYE ASTRONOMY

Useful equipment:

SC–1 Constellation chart
Star and satellite pathfinder
Celestial calendar

The subject of motion in the heavens is not taken up until Unit 2. However, it is advisable to have students carefully observe the sky in advance of studying that unit. This is because the motions of the heavenly bodies appear to be very slow. The activity is unusual in that it continues over several weeks. However, the time required for each observation can be quite short. Start early in the year.

There are no substitutes for the students' own experiences in making astronomical observations for themselves. For some students this may well be the first time that their attention has been guided to the beauty of the night sky. At least they will come to appreciate the skill and patience of early astronomers working with the same sort of primitive instruments. Some students may be excited enough to continue their observations beyond the outlines suggested.

Suggest that each student or pair concentrate on only one of the observations. Later they can share their observations and make comparisons. No student should feel compelled to attempt all the observations, although anyone may do so.

Conditions will vary greatly, from areas where useful observation is nearly impossible, as in smoggy cities, to places where the sky is ideally clear. Even in good areas, there will be bad nights.

A planetarium visit can be used as a supplement to, or, in poor viewing areas as a substitute for, personal observation. Contact the nearest planetarium and explain briefly your need for a special program. Most planetarium directors will be willing to put on a special show for your class that emphasizes the celestial motions important in Unit 2. A suggested program is given at the end of these notes.

A. Sun

Warn your students never to look directly at the sun since this can cause permanent eye damage. They should make all their observations of the sun by indirect methods.

The sun's azimuth, its direction measured from north through east for 360°, changes continually. In your location it is not likely to be at its highest point in the sky at 12 o'clock noon. One reason is that you may be on daylight saving time, in which case noon is about 1 P.M. But even on standard time you may not be located in the center of your time zone. Places near the center of each time zone are given in Table 1. If you are east of the center for your time zone, the sun will cross your local meridian 4 minutes earlier for each 1° eastward. Similarly, if you are west of the center of your time zone, the sun will transit 4 minutes later for each 1° of longitude westward.

TABLE 1
Some Places Near the Centers of Time Zones

Zone	Mid-Longitude	Places Near Mid-Longitude
Eastern	75°W	Philadelphia
Central	90°W	Memphis, St. Louis, New Orleans
Mountain	105°W	Denver
Pacific	120°W	Lake Tahoe

Even if you were exactly on the central meridian for your time zone, only rarely would noon occur at 12 o'clock. Each day the sun moves east among the stars, but not at a constant rate because the earth's orbit is elliptical rather than circular. Your students will understand this when Kepler's second law is discussed in Chapter 7. Even a uniform motion of the sun along the ecliptic would result in uneven days because the sun's annual path also has a north–south component. So, our clocks run on a fictitious average day (Mean Solar Time) based on the length of a year. Actually the sun gains and loses on Mean Solar Time. The difference is called the Equation of Time and may amount to over 16 minutes.

B. Moon

The moon appears to move eastward among the stars approximately 360° per month. By plotting the position and shape of the moon on the constellation chart, the students may be able to confirm how the moon's phase depends on its position relative to the sun. (The sun's position at 10-day intervals may be given along the ecliptic on the chart.)

Students could use the astrolabe described in the *Handbook* to measure the altitude and azimuth of the moon. During a winter night, the full moon reaches a higher altitude than the sun did at noon. During a summer night, the full moon reaches a lower altitude at night than did the sun at noon.

The new moon is close to the sun, full moon is 180° from the sun, and quarter moon is 90° from the sun. Note that the new moon to first quarter moon can only be seen in the late afternoon and evening while the third quarter to new moon can only be seen in the morning. (Yes, the moon can often be seen while the sun is up.)

C. Stars

The "Star and Satellite Pathfinder" shows which stars are above the horizon at latitude 40°N at a particular date and time. The Constellation Chart shows the stars in a band 60°N and S around the celestial equator. This includes all the stars high in the sky at middle latitudes. The curved line across the middle of the chart is the ecliptic. The sun's path throughout the year and its position at 10-day intervals is marked on the ecliptic.

Relative to the sun, the stars move about 30° westward per month. Different stars appear in the sky as the seasons change. For example, Orion is prominent in winter but is not seen in summer when the sun is in that part of the sky. See the "Star and Satellite Pathfinder" for information on what stars are visible month by month.

D. Planets

Because the sun, moon, and planets stay in the same narrow band around the sky, we can conclude that they all move in nearly the same plane; that is, the planetary system is essentially "flat."

In "normal" motion planets move eastward among the stars; in retrograde motion they move westward. Consult the Celestial Calendar to find out when the different planets are in retrograde motion. Post the month's Celestial Calendar with eclipses, conjunctions, and similar data marked on it.

Supplementary note on coordinate systems

Although coordinate systems for locating objects in the sky are not an important aspect of this study, teachers may wish an explanation of the various systems used.

Coordinates on the Earth: The latitude–longitude system is used to locate objects on the earth's surface. The equator of the earth is established as a great circle along the earth's surface halfway between the north and south poles and perpendicular to the earth's polar axis. Meridians are a set of great circles passing through the poles and are perpendicular to the equator. The local meridian (your north–south line) establishes your east–west location. The meridian passing through Greenwich, England, is called the prime meridian and has an assigned longitude of 0°. Places *west* of the prime meridian up to halfway around the earth (to the International Date Line) have longitudes *west*. Places *east* from Greenwich up to the International Date Line have longitudes *east*. Maximum longitudes are therefore 180°E and 180°W.

Latitudes are angular distances measured north or south from the equator to the poles, a total distance of 90°. Thus, the latitude of a place is the angular distance between the place and the equator as one might see it from the earth's center.

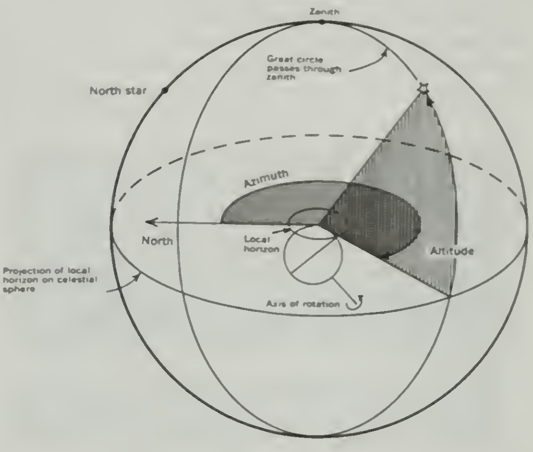
Coordinates in the Sky: One convenient way to establish the position of a star or other heavenly object is to use the *altitude–azimuth system*. The coordinates in this system are:

> Altitude: the angle of the object above the observer's local horizon.
>
> Azimuth: the direction around the horizontal plane measured eastward from true north.

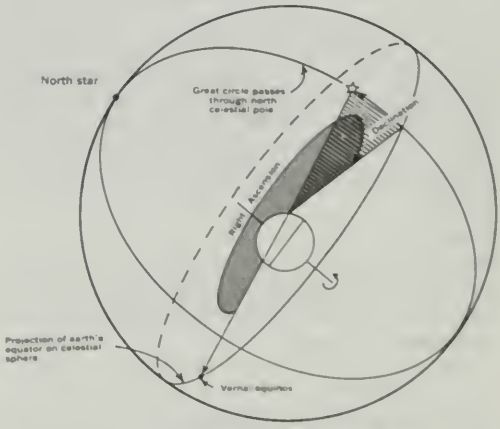
Such a system is local. No two observers (even a few miles apart) have at the same moment the same coordinates for the same star. Also, as the earth turns, a star's position on this system constantly changes.

For this reason, astronomers long ago devised a coordinate system attached to the so-called celestial sphere. This is sometimes referred to as the equatorial system and the elements measured are *right ascension* and *declination*. Imagine that we extended the earth's axis to the celestial sphere. Also, extend the plane of the equator until it intersects the celestial sphere. Great circles passing through the North Celestial Pole and crossing the celestial equator at right angles are called *hour circles*. These are similar to meridians on the earth's surface.

COORDINATES IN THE SKY
Altitude Azimuth System



Equatorial System



Rotate paper 35° to visualize the equatorial plane

The hour circle that passes through the vernal equinox is the reference circle from which *right ascension* is measured. The right ascension of a star is the angle measured eastward along the celestial equator from the vernal equinox to the hour circle passing through the body. The angle is measured in hours. Since it takes 24 hours for the celestial sphere to rotate through 360°, 1 hour is equivalent to 15°.

Declination establishes the distance of a star along an hour circle north or south of the celestial equator. Declinations are like latitudes on the earth's surface. A star having a declination of 40°N passes overhead at places having a latitude of 40°N.

Stars remain very nearly fixed with respect to their coordinates in the right ascension–declination system.

SUGGESTED INTRODUCTORY PLANETARIUM PROGRAM

This is an outline of the major phenomena that would be most useful and appropriate to *Project Physics* students. A program used at the Morrison Planetarium in San Francisco is used as a model. Other planetarium programs have been outlined in this *Resource Book.*

1. The current night sky
   (a) Set sky at predawn and at 9:00 P.M.
   (b) Locate Polaris
   (c) Point out a *few* constellations
      (1) Ursa Major
      (2) Cassiopeia
      (3) Cygnus (including the binary Albireo)
      (4) Sagittarius (on the ecliptic)
   (d) Show off the planets
   (e) Display sun and moon against starry field
2. Motions in the heavens
   (a) Circumpolar stars for 24 hours
   (b) Sun for
      (1) 12 hours demonstrating westward motion
      (2) 1 month demonstrating eastward motion
      (3) 6 months demonstrating north–south motion
   (c) Moon for
      (1) 6 hours
      (2) 1 month
   (d) Planets emphasizing
      (1) retrograde motion
      (2) maximum angle of elongation for Mercury or Venus
3. Celestial coordinate systems
   (a) Altitude and azimuth
   (b) Right ascension and declination
   (c) Constellations used to locate planets

E1–2 REGULARITY AND TIME

Equipment needed:

Dragstrip (chart recorder)
Blinky
Pendulum
Metronome

The first part of this experiment is designed to show the regularity of a few natural events. Students compare a variety of recurrent phenomena with a "standard clock," such as the blinky or metronome. The recurrent phenomena might include another blinky, pendulum, object on a spring, dripping burette, the human pulse, or tape-recorded crickets.

The mention of "time" should be avoided in this part, because the students' notion of absolute time will confuse the problem of regularity. However, do not force the issue. The point is to investigate first the regularity found naturally in the world and then move on to contrived measurement standards.

Caution the students not to bear too heavily on the recording tape with their pens, because the increased drag might affect their results. When students have completed the measurements from the dragstrip recorder on their own tapes, the information can be pooled on a master graph. It might look like the Fig. 1 below.

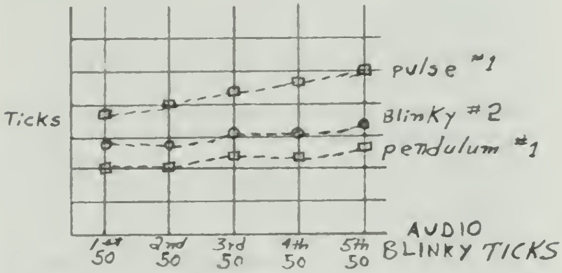


Fig. 1 The regular events are those that show similar curves on a graph.

If a light, even that which comes in the window, is allowed to fall on the blinky bulb, the rate will change by 4% or more. This is especially true for some of the early experimental models. All recent models have had a radioactive gas added to trigger the bulb in total darkness and to maintain stability. It might be a good idea to intentionally cause the rate to change during the run. The relativity of regularity would thus be emphasized, since the "good clocks" will all show common curved records on the graph. (However, since we want the students to accept the blinky as a reasonably good clock, the disturbance should be accounted for afterwards. If one or two blinkies are used as unaltered "controls," the explanation will be more convincing.)

Answers to questions

1. Answer depends upon results. In general it is not possible when comparing two isolated sets of events to state which is more regular.
2. Two events, here B and C, are compared to a third one, A; therefore A is taken as being the standard. If one event is <u>defined</u> as being "regular" then all other events can be compared to it. Thus, for example, if C is more consistent than B in the number of recurrences in "equal" time periods as marked by A, then C is the more regular.
3. There is no measure of absolute regularity. Timing is always a matter of comparison. Whatever is taken as the standard of comparison is assumed to be regular for the purposes at hand, whether it be rotation of the hands on a wall

clock, the apparent annual north–south movement of the sun in the noonday sky, the vibrations of a crystal, or any other seemingly periodic phenomenon.

E1–3 VARIATION IN DATA

Equipment needed:

A wide variety of objects to count, measure, and weigh

The student should become familiar with different kinds of variation in measurement by doing this experiment. While it is possible to introduce significant figures, the intention here is only to make students comfortable with variation in its simplest terms.

The general plan outlined below is intended to start the student on familiar ground, where it is firmly believed the variation is in the things measured. Then the experiment progresses through situations in which variation is in the measuring process to those in which the source of variation is uncertain.

You may wish to give these classifications of variation to the students after they have finished making the measurements but before the discussion; or you may feel it will be more valuable if these or similar categories are discovered through discussion. A few examples follow.

1. Situations where the variation is unquestionably due to differences among the things being measured.
   (a) Students' heights or weights
   (b) Family size
   (c) Number of pieces of candy, raisins, or other objects in different boxes
2. Variation unquestionably due to changes in the thing being measured.
   (a) Temperature of a beaker of warm water
   (b) Weight of a chunk of dry ice
   (c) Weight or length of a burning candle
3. Variations unquestionably due to the process of measuring.
   (a) Separation of blinky dots on a photograph using a ruler
   (b) Separation of blinky dots on a photograph using magnifier
   (c) Diameter of a piece of wire measured with a ruler
   (d) Diameter of wire using micrometer or magnifier
   (e) Diameter of a puck
4. Sources of variation uncertain.
   (a) Rotation rates of students' phonograph turntables
   (b) Height as measured in the morning compared to height at night.

These classifications are not the only ones possible. One important class of variation not really covered here is the statistical variation of random events (such as background radiation count). This classification will be considered in more detail in Unit 6, *The Nucleus*. For the lab work in this course, Class 3, which includes variations due to the process of measurement, is the most important, and is emphasized in some of the experiments.

Station suggestions

More ideas are listed here than can probably be used and you may have other ideas that you wish to substitute. Variety is the keynote, of course.

All students should visit every station but they do not have to begin at the same point in the cycle.

Object	Measuring Instrument	Quantity
Marble	Vernier caliper	Diameter
Large steel ball	Vernier caliper	Diameter
Beaker of water colder than room temperature	Thermometer*	Temperature
Beaker of water at room temperature	Thermometer	Temperature
Beaker of water warmer than room temperature	Thermometer	Temperature
Empty beaker	Thermometer	Temperature
Metal cylinder	Ruler	Length
Puck	Common calipers	Diameter
Puck	Ruler	Diameter
Blinky dots on photo	Ruler	Distance between
Blinky dots on photo	Magnifier	Distance between
Wire	Ruler	Diameter
Any object	Stopwatch	Time of fall from indicated height
Bottle of water	Graduated cylinder	Volume
Dry ice	Balance	Weight
Rotating wheel (slow)	Stopwatch	Rotation rate
Burning candle	Balance or ruler	Weight or length
Line circuit	Voltmeter	Line voltage
Dry cell	Voltmeter	Voltage

*Note: Students may realize that all thermometers do not read exactly alike under like circumstances. Carefully select the thermometers to reduce or eliminate this.

The purpose of this laboratory is not to achieve unanimous agreement on the sources of variation in the measurements, but to make students aware of the issue and how critical the issue can be in experiments. It is important for students to realize that variation problems are not confined to school laboratories. The best of scientists with the most expensive equipment are often faced with the interpretation of variation.

Advance preparation

The stations around the room must be set up before class starts. Suggestions for stations are listed below. About 10 or 12 stations will be needed if students are to gain a variety of experiences.

If each student makes every measurement, one 50-minute period will be needed. From one-half to a full period will be needed to write the results on the board and discuss them.

Answers to questions

1. Differences in use of instruments and especially in estimating between marks on scales. Also, similar objects, including instruments, for example, a set of meter sticks, are not exact duplicates. Sometimes the objects being measured can change between measurements in response to changing conditions.
2. No, there are no absolutely correct measurements (aside from the trivial case of counting a few discrete objects).
3. No, since the average will reflect all of the measured values, some of which could be far from "the correct" one, whereas some one of the individual measurements could be quite close. The trouble is that there is no way of knowing for sure which one of those values is most correct. Therefore, one uses the average on the assumption that "on the average" it will be closer than any other value. This assumption is reasonable if it seems likely, in a given set of measurements, that errors are as apt to be in one direction as another, that is, are comparably distributed on both sides of the "true" value.
4. By indicating the range of variation. One might indicate the average value and add a statement indicating the range of values that includes some proportion (say two-thirds) of all the measured values. Statistics textbooks provide methods for expressing distributions.

E1–4 MEASURING UNIFORM MOTION

Major equipment for version described:

Flat smooth surface
Plastic beads
Puck or other smooth-bottomed disk
Polaroid camera
Rotating disk strobe
Light source
Millimeter ruler for measuring picture
Blinky
Uniform Motion Device

Encourage students to study uniform motion in a variety of ways. For example, two students working together can

1. photograph a puck sliding on bead-covered glass ripple tank or some other smooth enclosed surface
2. photograph a glider coasting on a level air track
3. photograph a toy tractor pushing a blinky
4. measure motion of an object in a film loop projected on the chalkboard

One student alone can measure

5. a transparency showing what is asserted to be uniform motion
6. a strobe photograph, such as the momentum-conservation collision photos or the photo on page 12 of the *Text*

If not enough apparatus is available for the whole class to do the same experiment, perhaps the class can be broken up into small groups, each of which will use a different method.

Instructions for operating the Polaroid camera and for using the rotating disk stroboscope are found in the *Equipment Notes* section of this *Resource Book.*

In the *Handbook* we describe the experiment as done by Method 1 above. Other methods differ only slightly and in obvious ways. The procedure for using the data from any of the methods is identical.

If an air puck or a puck sliding on beads, is used, it should have a large white X or a rubber stopper painted white for easy reference in the photograph. Since the puck will probably rotate, the white indicator must be at the center, not on the edge, of the puck.

We assume students have studied the *Text* through Sec. 1.4, in which case they will end their write-ups after the section entitled "Graphing motion and finding the slope."

If they have studied graphs (Sec. 1.5), however, it may be desirable to have them go on to subsequent sections of this experiment in which they graph their data. In this case it may be worthwhile to take two runs at different speeds in order to show how the two resulting graphed lines differ in slope.

Answers to questions

1. Yes; because it is straight.
2. Answer depends on student graphs.
3. Yes, the same general method can be used, but the technique will vary with circumstances.
4. (a) $\pm$ 2.5 km/hr.
   (b) No, not reliably. The changes are smaller than the uncertainty. For the 2-km/hr change the reliability is greater than for the others.

E1–5 A SEVENTEENTH CENTURY EXPERIMENT

Major equipment for seventeenth century experiment:

- Grooved incline about 2 m long
- Supporting ringstands
- Ball to roll in groove
- Water clock

The next three experiments deal with the acceleration of gravity. Since *E1–7* is divided into six parts, there are eight possible attacks upon a_g. Some thought should be given to which experiments should be selected.

Only one phase of Galileo's investigation has been selected for this experiment. A full description of it can be found in *Dialogues Concerning Two New Sciences*, the "Third Day." See also the references to the Crew-de Salvio translation partially reproduced in Chapter 2 of Unit 1. A Dover Publishing Company reprint of this enjoyable book is available. Also a careful modern repetition of this experiment is described by Thomas B. Settle, "An Experiment in the History of Science," in *Science*, Vol. 133, January 6, 1961. Historians and philosophers of science are still hotly debating whether or not Galileo took actual experimentation very seriously, and whether he actually did some of the experiments he described in such graphic detail.

At least two students are needed for each setup, one to handle the rolling ball and the other to operate the water clock and record data. By dividing the jobs further, as many as four can be usefully employed.

Apparatus

An inclined plane about 2 m long is needed. It should have a groove or channel down one edge in which a ball runs very smoothly.

If only one inclined plane is available, it can be operated by one or two students while the rest of the class, individually or in pairs, are operating water clocks.

Distances marked on the incline are arbitrary, but should be chosen to work well with the rate of flow of the water clock. We find that 12 marks 15 cm apart serve well. Students should not convert to present-day standards of length, but should merely record the distances as units of length, 1, 2, 3, ...

The right size of tube and of collecting vessel for the water clock must be found by trial and error. The flow should last at least three or four seconds without overflowing the collector. The water clock does not work as well if it is started and stopped with a pinchcock on a rubber exit tube below the funnel.

It is recommended that the students perform a minimum of four trials for each distance and that the average time value recorded be used in the calculations. For longer distances fewer trials may be used if the times seem to be in close agreement. Four different distances, for example, 3, 6, 9, and 12 units should be sufficient for each angle of the ramp. It is probably not practical for any one group to attempt to take measurements at more than two different angles of inclination. An exact judgment of the slope of the channel is not critical. The results for heights over 30 cm may show considerable scatter, depending upon the skill of the students and quirks of the equipment.

Recording data

It is a good idea at this early stage in the course to firmly insist on a neat data table. If students are always quick to record all their data in ink directly into their final report, it does wonders to develop clear careful thinking as the year goes on. Mistakes will be made, of course, but should be crossed out neatly.

It is useful to plot d versus t first to show that the graph is not a straight line. Point out that there is no way of recognizing with the unaided eye any curve except a circle and a straight line. Only by plotting in such a way as to generate one of these shapes can we identify the relationship between d and t.

In graphing the results, plot (time)^2 along the horizontal axis. Not only is this conventional, but also, when d is plotted along the vertical axis, the resulting slope is equal to twice the acceleration.

If students suggest that the $d \propto t$ curve *does* look like a parabola and is therefore a $d \propto t^2$ relationship, challenge them to show that it isn't a $d \propto t^3$ relationship, which may have the same general form (as can be verified by trial).

It may be useful to have a pair of students do the twentieth century version of Galileo's experiment (*E1–6*) by photographing a glider sliding down a tilted air track. This yields more precise data, which may be reassuring when the final conclusion of the experiment is discussed.

Possible extensions

After each group has completed its investigation, one of two possible procedures is recommended:

(a) Each group may report its findings orally and comparisons may be made during a discussion period.

(b) Composite findings may be tallied on the chalkboard, and, using these combined results, all students may plot the entire family of curves for the different inclinations.

When comparing results make the point that the linear relation between d and t^2 appears to hold for the rolling ball, at least for small angles of inclination of the channel (within the variation expected). As an aside, mention that for any given angle of inclination the distance intervals rolled down the incline, in successive units of time, will follow the pattern 1:3:5:7. . . .

The value of a_g found by extrapolating data from this experiment, will be too low because only part of the ball's increasing kinetic energy as it descends is in the form of energy of motion along the plane. The remainder is in the form of energy of rotation. Secondly, friction is a very large factor in reducing the acceleration. Both these important effects are reduced or eliminated by photographing a glider descending an air track (see *E1–6*).

Derivation of a_g

Some of the better students may want to use their data to calculate a_g, the acceleration of gravity. This is not easy.

The potential energy of the motionless ball at the top of the track equals its kinetic energy at the bottom if friction is so small as to be ignored. But the kinetic energy is not merely energy of linear motion, $mv^2/2$; some energy is also in the form of rotational motion, described by the expression $I\omega^2/2$. Thus the conversion of energy from potential to kinetic is described by the equation

$$PE_{top} = KE_{bottom}$$
$$ma_gh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

The angular velocity of the ball ω at the bottom of the ramp is defined (in radians/sec) by $\omega = v/r$ where r is the radius of the ball and v is its velocity along the track. I is the moment of inertia of a rotating object. For a rolling solid sphere of mass m and radius r, $I = 2/5(mr^2)$.

Putting these expressions for ω and I into the rotational energy term, $\frac{I\omega^2}{2}$, the energy equation becomes

$$ma_gh = \frac{mv^2}{2} + \frac{2}{5}mr^2 \cdot \frac{v^2}{2r^2}$$
$$= \frac{mv^2}{2} + \frac{mv^2}{5}$$
$$ma_gh = \frac{7}{10}mv^2$$

But $v^2 = 2ad$ where d is the length of the plane and a is the acceleration parallel to the incline, so

$$a_g = \frac{7}{10} \times \frac{2ad}{h}$$
$$a_g = \frac{7}{5}\frac{ad}{h}$$

where h is the height of the plane and d is its length. The acceleration a is measured by means of the water clock or by other more precise methods.

Notice that m and r cancel out of the final expression for a_g. Hence the acceleration does not depend upon the size of the ball, which refutes Aristotle's assertion.

It would be a mistake to present this analysis to a class at this stage. Only a very special student might be able to follow this.

Answers to questions

1. The graph, d versus t^2, should be a straight line.
2. Student answer.

Going further

1.-2. Student answers.
3. A student probably cannot do as well as Galileo. However, estimating is a skill that can be improved with practice, and it is not out of the question that Galileo could have attained the claimed accuracy.
4. Ratio should be 1:3:5:7 . . . .

E1–6 A TWENTIETH CENTURY VERSION OF GALILEO'S EXPERIMENT

Major equipment for twentieth century version:

Air track and glider
Polaroid camera
Rotating disk strobe
Light source
Blower for air track

The modern version of Galileo's inclined-plane experiment with an improved clock and an air-track glider gives the same results, namely, that d/t^2 is a constant. However, the precision is improved. The idea behind the improved experiment is still Galileo's. It is a test of a logical consequence from the assertion that things accelerate in the physical world.

Measurement of a_g

A simple procedure for students who have not taken trigonometry is to have them calculate the accleration (a_1, a_2, a_3, etc.) for different angles of inclination between the air track and the horizontal. Next plot a graph of acceleration versus angle of inclination. Finally extrapolate this graph to 90° in order to estimate the free-fall acceleration, a_g.

Students who have had trigonometry may calculate a_g by the following relationship: $a_g = \frac{a}{\sin\theta}$ or $a_g = a\left(\frac{l}{h}\right)$. Refer to the diagram below for an explanation of the algebraic symbols in these relationships.

$$\sin\theta = \frac{a_\theta}{a_g}$$
$$a_g = \frac{a_\theta}{\sin\theta}$$
$$a_g = a_\theta\left(\frac{l}{h}\right)$$

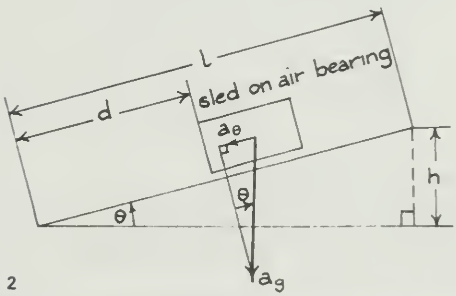


Fig. 2

θ = the angle between the horizontal and air track
l = length of air track
h = The height of the air track
a_θ = the acceleration parallel to air track at angle θ
d = the distance through which the sled is allowed to slide

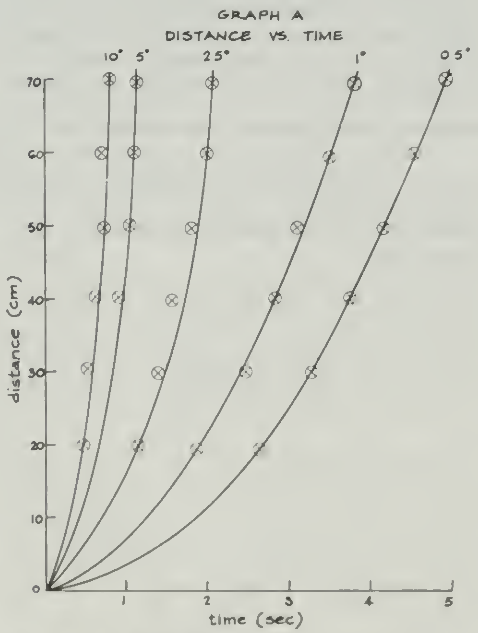
Sample Data and Results

θ d (cm)	.5° t (sec)	1.0° t (sec)	2.5° t (sec)	5.0° t (sec)	10° t* (sec)
20	2.7	1.9	1.1		.4
30	3.3	2.5	1.4		.5
40	3.7	2.8	1.6	0.9	.6
50	4.2	3.1	1.8	1.0	.7
60	4.5	3.5	2.0	1.1	.7
70	4.9	3.8	2.1	1.2	.8

*Data for 10° was obtained from a strobe photo. All others were obtained using a stopwatch.

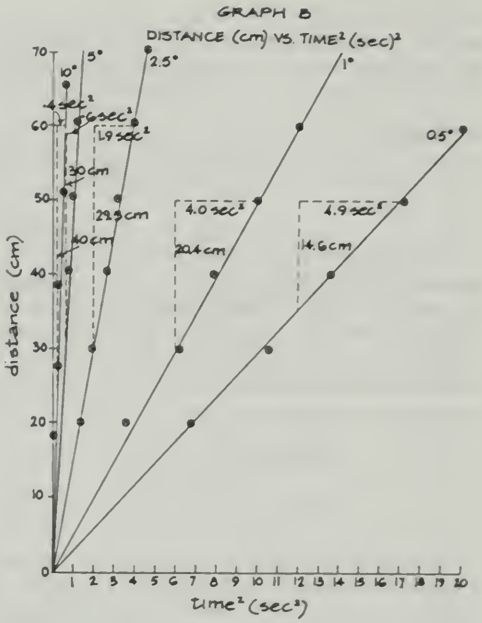
Calculations for θ = 5°

t^2 (sec²)	$\frac{d}{t^2}$ $\left(\frac{cm}{sec^2}\right)$	$2\left(\frac{d}{t^2}\right) = a_\theta \sin\theta$ $\left(\frac{cm}{sec^2}\right)$	$a_g = \frac{a_\theta}{\sin\theta}$ $\left(\frac{m}{sec^2}\right)$
0.8	slope		
1.0	from		
1.2	Graph B		
1.4	$\frac{30}{0.6} = 50$	100	$0.087 \; \frac{100 \text{ cm sec}^2}{100 \text{ cm/m } (0.087)} = 11$



Extension of laboratory

The above calculation indicates that the free-fall acceleration was 11 m/sec² rather than 9.8 m/sec². Consider the following points regarding this experimental result.



1. Graph B shows that the acceleration down the incline is directly proportional to the square of the time. This supports Galileo's notion that a body does accelerate.

2. This is in disagreement with Aristotelian physics, which has no way of talking about acceleration. This result is, therefore, revolutionary.

3. One more remarkable aspect of either version of the seventeenth century experiment is that Galileo mentally eliminated friction and that he thought forward to the possibility that there might be such a constant as free-fall acceleration near the surface of the earth. There is no evidence that he attempted to calculate this value.

4. The teaching of some mathematics from Graph B might be more important than arriving at 9.8 m/sec². That is, $a \propto t^2$ and $a = kt^2$ where the constant of proportionality k is the slope of a line and represents the acceleration of the sled at a certain angle of inclination. Finally, if the slopes are plotted against the angles, an extrapolation to 90° gives a new idea, namely, free-fall acceleration.

5. The fact that 11 m/sec² was calculated in comparison with 9.8 m/sec² is not *bad*. It is not bad because *Project Physics* is more than the verification of the data in the *Handbook of Chemistry and Physics*.

There is an opportunity here to guide the students toward operationalism. Ask them to describe in detail the measuring instruments and the scales on these instruments. Have them calculate the uncertainties of measurements. Have them calculate the uncertainty that can be expected in the result 11 m/sec².

Refer students who want to be more precise to E1–7.

Answers to questions

1. Student answer.
2. If the students claim that the graph is a straight line within the limits of uncertainty of their measurements, then the significance is that the air-track glider is accelerating.
3. Student answer.
4. $\% \text{ of error} = \dfrac{\text{error}}{\text{accepted value}} \times 100$
5. (a) Human reaction time. In measuring small time intervals the reaction time in operating a stopwatch becomes an increasingly significant factor. Notice the absence of data for the shorter distances on the 5° slope. Strobe photography is recommended for the steeper slopes.
   (b) In spite of the very low friction with the air track, friction is not eliminated completely.
   (c) The technique of releasing the glider is very critical to a valid measurement. Several dry runs are needed to achieve successful operation of the equipment.
   (d) The air blasts may exert an impulse upon the glider.

E1–7 MEASURING THE ACCELERATION OF GRAVITY a_g

Introduction

Acceleration due to gravity is a crucial topic that has been illustrated indirectly in the two previous experiments. Since there are six methods of finding a_g in this experiment, a decision must be made regarding which of these should be done. Ideally different groups of students should use different methods and then compare results in class discussion. This will provide an opportunity to raise questions about variations and error and about how "standard values" are arrived at.

Method A: a_g by Direct Fall

In any direct measurement of a_g, a falling object has to be timed accurately as it falls through several precisely measured distances. Ordinarily the distance of fall must be kept small in order to avoid the appreciable air resistance encountered at high speeds. But a short fall is usually too brief to time accurately without elaborate equipment. In this experiment with very simple equipment these two limitations cause an error of less than 2%.

If a recording timer is available, it may be more convenient than a tuning fork for marking the moving tape. The experiment is otherwise the same.

Clamp the timer at the edge of a table in such a way that the paper passes freely through it vertically and clears the edge of the table. Since it is difficult to measure the frequency of the clapper accurately when operated by 1.5 V dc you might short-circuit the breaker gap inside the timer and operate the timer on 60 Hz ac. Use a short length of wire with a small battery clip on each end. Be careful that your short circuit connections do not interfere with the free motion of the clapper.

You can provide the necessary low voltage from a bell-ringing transformer, or in some cases from the 6-V ac tap of your power supply. Use a small rheostat, such as the one used to control a ripple tank wave generator, in series with the power supply. It is important that the current be adjusted until the vibrator action is loud, firm, and regular. A skipped beat or two can completely spoil your results and occasionally does. Since you are overloading the coils of the timer, you should leave the current on as briefly as possible.

The clapper is now vibrating at either 60 or 120 Hz. To discover which, you need merely pull 1–2 m of tape through the timer by hand at a speed sufficient to resolve the dots being made by the clapper, and count the number of dots made in approximately one second. The choice between 60 and 120 Hz will be obvious and no other frequencies are possible.

To measure a_g hold the weighted tape in the timer, start the timer, and release the tape. The series of carbon-paper dots on the tape can then be analyzed in the same way as the waves formed by the tuning fork.

Answers to questions

1. Student answer.

$$\left(\% \text{ of error} = \frac{\text{error}}{\text{accepted value}} \times 100\right)$$

Method B: a_g from a Pendulum

Although this is an indirect method for measuring a_g, it is probably the simplest method that can be considered accurate.

The derivation of the equation for T, the period of a pendulum, draws upon concepts of simple harmonic motion that students at this stage are unable to follow. Most first-year college texts in general physics give the derivation. The practical considerations, however, are very simple.

The clamp that holds the top of the pendulum suspension must not have rounded edges to its jaws, for if it does, the suspension will, in effect, be shortened slightly as its sideways motion wraps the top few millimeters around the rounded edges. The clamp must also be very rigid; any back-and-forth wobble will increase the period.

Since the formula is only correct for very small amplitudes of swing (certainly no more than 10°), the timing should be done with the smallest swings that can still be seen after 20 trips.

If 20 round trips lasting 12.0 sec are timed with starting and stopping errors of 0.2 sec each, the total timing error is 0.4 sec. Since this error is shared among 20 swings, the timing error per swing is only 0.02 sec. Because each swing takes $12.0/20 = 0.60 \text{ sec} = T$, the uncertainty in T due to timing is 3%. This is very large indeed compared with other possible sources of error. To reduce it, time a larger number of swings, say 50, whereupon the same error in timing leads to only about 1.3% uncertainty in T.

The length of a pendulum whose period T is 1 sec is 24.8 cm. Remember that T is the time for a *round trip.* The pendulum that takes 1 sec to swing one way only will be 99.4 cm long.

More values are in Table 1.

TABLE 1
Period of Various Pendulums

l	T
20 cm	0.75 sec
40	1.26
60	1.54
80	1.78
100	1.98

Answers to questions

1. Student answer.
2. Answer should agree with accepted value within 1%.
3. Student answer.

Method C: a_g with Slow-Motion Photography (Film Loop)

A successful motion picture will depend upon the control of light. The black backdrop should be in shadow. It is worthwhile to arrange the lights so that they will cast a shadow on the backdrop but bathe the meter stick and falling ball in the foreground. It is best to always shine the light from the side, that is, about 90° from the direction of the movie camera. This will result in excellent contrast.

Most teachers will want to use the film loop. The film loop projector is one of the most effective pieces of laboratory equipment. Try it.

Method D: a_g from Falling Water Drops

The simplicity of the apparatus and the clever manner of finding t recommends this accurate method for determining a_g. Be sure that students maintain the water source at a constant level.

Answers to questions

1. Student answer.
2. (% of error $= \dfrac{\text{error}}{\text{accepted value}} \times 100$)
3. Student answer.
4. Student answer.

Method E: a_g with Falling Ball and Turntable

The distance between the marks made by the falling balls indicates the *difference* in fall times. The expression for a_g is manageable only if the lower ball is hung very close to the turntable.

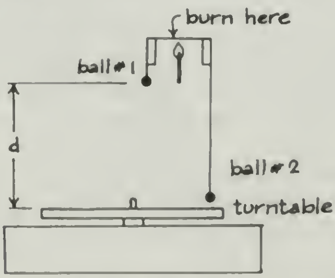


Fig. 3

Use a turntable frequency of 33⅓ rpm. At this speed, if the difference in heights of the balls is about 20 cm, the turntable will turn through about 40°.

Answers to questions

1. Student answer.
2. (% of error $= \dfrac{\text{error}}{\text{accepted value}} \times 100$)
3. Student answer.

Method F: a_g with Strobe Photography

It is helpful to illuminate the ball from the side while a black cloth is draped in the background. Be sure to photograph a meter stick. For this work a meter stick with white calibrations on a dark background is best.

With 12 slots open and a 300-rpm strobe disk motor, the period between consecutive peaks at the free-falling object is 1/60 sec.

If a xenon strobe flash is used refer to the *Equipment Notes* for calibrating information.

Answers to questions

1. Student answer.
2. (% of error $= \dfrac{\text{error}}{\text{accepted value}} \times 100$)
3. Student answer.

E1–8 NEWTON'S SECOND LAW

Major equipment:

- Dynamics cart
- Blinky
- Spring scale taped to cart
- Table-corner pulley
- Weights (hooked) and string
- Polaroid camera
- Either rotating disk stroboscope and light source or accelerometer

It is assumed that students have recently completed Sec. 3.7 in the *Text* on Newton's second law.

Purpose 1

In this experiment students familiarize themselves with the relationship between F_{net}, m, and a. In no sense do they prove or even verify the law.

Using stroboscope photographs will certainly not afford enough time for a single student (or group of students) to take a series of data on a versus F_{net} and also on a versus m. If the task is distributed among several students or groups, however, two graphs can be drawn and each student can contribute a point or two to one of the graphs.

The graph of a versus F_{net} with m held constant will be a straight line through the origin.

The graph of a versus m with F_{net} held constant will be a hyperbola. The graph cannot be recognized as a hyperbola, however. Students should be challenged on this point to show that it is not part of a circle, an ellipse, or a parabola. Only by finding how to convert it into a straight line (or a circle), which are identifiable by simple inspection, can one then work backwards to discover the original shape.

Thus in the case of a versus m, a graph of $1/a$ versus m yields a straight line through the origin. All such lines must have the equation $y = kx$, or, in this particular case,

$$\frac{1}{a} = km$$

Thus,

$$ma = \text{constant}$$

which *is* the graph of a hyperbola.

Moreover this behavior is consistent with Newton's second law. Using a calibrated liquid-surface accelerometer on the dynamics cart a single student (or group of students) may be able to gather all the data alone, if this seems desirable. Certainly the work can be done faster than it can from photographs.

The action of the accelerometer is described under *Activities* in the *Handbook*.

Purpose 2

A second major purpose of this experiment is the study of experimental errors.

It may be desirable to pursue this treatment in a subsequent laboratory or class period. The ideas developed here will be assumed in future discussions of experimental error.

The question is asked: "Does your measured value of F_{net} really equal your measured value of ma?" Since all three quantities are the results of measurements that have inherent uncertainties, the measurement of F_{net} will almost certainly not equal ma. This discrepancy does not necessarily mean disagreement with Newton's second law. It does mean that experimenters must consider uncertainties of measurement and the propagation of error.

If students do find that F_{net} is equal to ma, within the experimental uncertainty, then all is well. If the uncertainty is large, they may justifiably point out that it is a poor experiment. If the difference between the measured value of F_{net} and the calculated value of ma is greater than the experimental uncertainty, the most likely explanations (apart from miscalculation, or the use of inconsistent units) are:

(a) the force measured by the spring scale is not the only force acting (friction), and

(b) the spring scale has a significant error, or is inaccurately calibrated.

Discussion of error propagation

The *Handbook* points out that the uncertainty in the *difference* between two measured quantities is the *sum* of the uncertainties in the two measurements. The same is true for the *sum* of the two measurements. Students may ask about the uncertainty in a product. This is a general rule: The percentage uncertainty in a product is equal to the sum of the percentage uncertainties in each measurement, however many terms there are in the product. Similarly, for a quotient, the percentage uncertainty is equal to the sum of the percentage uncertainties of all the terms.

While the simplifications outlined above are useful for an introductory exercise, a much more general approach to uncertainty and its analysis is required for most experimental situations. This generality is needed because (a) there are a variety of mechanisms responsible for introducing uncertainties, and (b) there are, of course, other kinds of functions through which uncertainties are to be propagated during the course of the year. A brief outline summary of these factors follows.

Mechanisms responsible for uncertainties

1. Scale-reading uncertainties. Finite space between marks on scales.

2. Object irregularities
   (a) Obvious variations that can be identified and have predictable effects.
   (b) Perturbations requiring a statistical treatment of the final results (for example, population surveys, radioactive disintegrations).

3. Systematic discrepancies introduced by
   (a) Bias, due to poor experimental design.
   (b) Use of oversimplified theory.

Propagation rules to calculate maximum uncertainties

1. Sums and differences
   Add absolute uncertainties to obtain absolute uncertainty in result.
   Example:
   If $A = 2.51 \pm 0.01$
   and $B = 3.33 \pm 0.02$
   then $A + B = 5.84 \pm 0.03$

2. Products and quotients
   Add % uncertainties to obtain % uncertainty in result.
   Example:
   If $A = 2.51 \pm 0.01$ (or $\pm$ 0.4%)
   and $B = 3.33 \pm 0.02$ (or $\pm$ 0.3%),
   then $AB = 8.36 \pm 0.06$ (or $\pm$ 0.7%)

3. Power and roots
   Multiply % uncertainty by power or root (exponent) to obtain % uncertainty in the result.
   Example:
   If $A = 2.51 \pm 0.01$ (or $\pm$ 0.4%)
   then $A^2 = 6.29 \pm 0.05$ (or $\pm$ 0.8%)

Exercises could be invented to provide drill and practice on any of the items listed in the tables. But it is probably more appropriate to call attention to them as they are needed.

Remember not to give the whole bottle of medicine in one sitting; parcel it out in gentle doses over the whole year. The therapy takes time!

Answers to questions

1. F_{av} (as measured) $\neq ma_{av}$ (as computed).
2. Observations may support Newton's second law if the uncertainties of measurements are taken into consideration.

3–4. Student answers. Refer to sample results below.

Sample calculations:

1. Predicted acceleration

F = 1.6 N (Av of 5 runs)

$F = 1.6\ \text{N} \pm 0.2\ \text{N} = 1.6\ \text{N} \pm 12\%$

$m = 1.032\ \text{kg} \pm 0.050\ \text{kg} = 1.032\ \text{kg} \pm 5\%$

$$a = \frac{F}{m} = \frac{1.6\ \text{N} \pm 12\%}{1.032\ \text{kg} \pm 5\%} = 1.55\ \text{m/sec}^2 \pm 17\%$$

or $1.55\ \text{m/sec}^2 \pm 0.25$

Range: 1.30 to 1.80 m/sec^2

2. Actual acceleration

projected distance =
$2.0\ \text{m} \pm .05 = 2.0\ \text{m} \pm 2.5\%$
actual distance =
$(2.0\ \text{m} \pm 2.5\%) \times 4 = 8.0 \pm 2.5\%$

$$\text{actual speed}_{av} = \frac{d}{z} = \frac{8\ \text{m} \pm 2.5\%}{{}^{1}/_{20}\ \text{sec} \pm 2.5\%} =$$

$$1.6\ \text{m/sec} \pm 5.0\% = 1.6\ \text{m/sec} \pm 5\%$$

$$\text{actual acceleration} = \frac{v_2 - v_1}{t_2 - t_1} =$$

$$\frac{1.6\ \text{m/sec} \pm 5\% - 0}{{}^{22}/_{20}\ \text{sec} \pm 2.5\% - 0} = 1.4\ \text{m/sec}^2 \pm 7.5\%$$

It is logical that actual $a <$ predicted a because of friction.

Newton's second law $\rightarrow F_{net} = ma$

$$F_{net} = F_{tension} - F_{friction}$$

$F_{friction}$ may be too small to measure accurately.

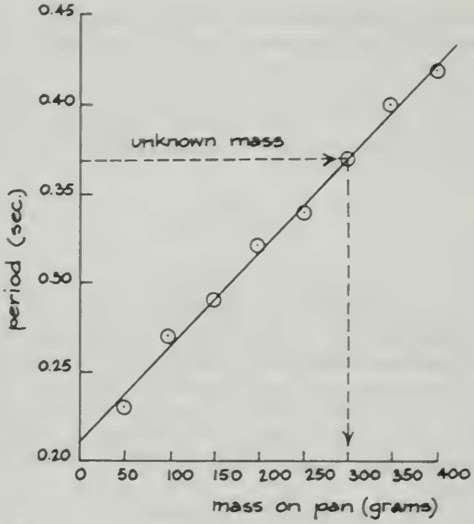


Fig. 4

Sample data

Mass on balance g	Time for 50 oscillations (Av of 4–5 trials) sec	T sec
50	11.3	0.23
100	13.6	0.27
150	14.6	0.29
200	16.0	0.32
250	17.1	0.34
300	18.6	0.37
350	19.9	0.40
300	21.0	0.42
unknown mass (a)	18.4	0.37
(b)	18.4	0.37

(a) unknown mass resting on the pan
(b) unknown mass supported by string, independent of balance

Answers to questions

1. They are the same to within the errors of measurement.
2. Compare the masses in the same way as before. Compare the magnetic forces by supporting each mass on beads or on a practically frictionless puck or other bearing, and use a spring scale to measure the pull of the large magnet acting horizontally on each one in turn.

E1–9 MASS AND WEIGHT

This is a very subtle demonstration. The importance of it may be lost on all except the most perceptive students. Students should be given the idea that the calculations are not the primary purpose of the experiment. The principle that inertial mass is distinctly different from gravitational mass is the significant concept. Furthermore, the difference is inherent in the *different operations* used to find gravitational and inertial mass. The fact that different operations based on quite different concepts of mass give equivalent results (that is, 3 times the mass according to one concept is exactly 3 times the mass according to the other) is an extraordinary and significant idea.

E1–10 CURVES OF TRAJECTORIES

Major equipment:

Trajectory-plotting equipment
Onion skin paper
Carbon paper
Steel ball
Graph paper

In the short version of this experiment students can stop after recording the path of the ball, before the section entitled "Analyzing your data." By this point they have plotted the trajectory for themselves, which may be sufficient.

However, another important result of this experiment is an understanding of the principle of superposition, and for this the students must go on to analyze their data. The principle can be made particularly clear if the vertical displacements are graphed against time squared. This graph and the graph of horizontal motion against time should both be straight lines, as would be expected of the two motions if they took place separately.

Notice that there are two tracks that can be fixed to the pegboard. One track will launch a sphere about 45° upwards from the horizontal. If the first track is used, the experiment will be easier. However, it is worthwhile to have some students use both tracks so that projectile motion can be dealt with more generally in a postlab discussion. It is wise to be sure that the students understand *Text* Secs. 4.2 and 4.3 before doing this experiment.

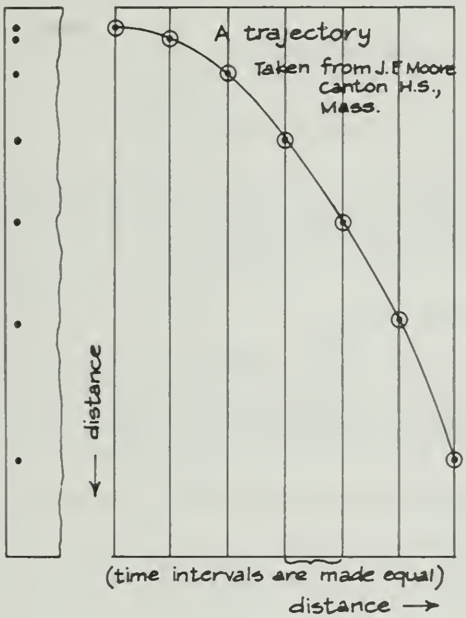


Fig. 5

Answers to questions

1. The graph of horizontal distance against time is a straight line beginning at the origin.
2. The vertical motion can be described as uniform acceleration. One way to show this is to plot the vertical distance fallen against the square of the time. If the plot is a straight line, it demonstrates uniform acceleration. Another method is to show that Δd, the *change* in distance traversed, in consecutive equal time intervals, is constant.
3. The horizontal and vertical motions are independent of one another.
4. $\Delta x = v\Delta t$
5. $\Delta y = \frac{1}{2}a_g(\Delta t)^2$

Try these yourself

1. You can expect nearly the same results with the glass marble as with the steel sphere of the same size. Very lightweight balls are slowed down by the roughness of the track and air resistance.
2. The horizontal component of its velocity will be different. The curve of the trajectory will be parabolic, but it will be a different parabola.
3. To the degree that friction can be ignored, different-size balls will have the same trajectories if started from the same positions on the ramp.
4. The descending half of the curve is similar to the first trajectory.

E1–11 PREDICTION OF TRAJECTORIES

Major equipment:

- Steel ball
- Meter stick
- Clock with sweep second hand (preferably stopwatch)
- Support stand
- Ramp

Students should understand Secs. 4.2 and 4.3 of the *Text* before doing this experiment.

In a postlab discussion of this experiment, it is worthwhile to point out to the students the power of logic and the drama of prediction. If one assumes the equations in the *Handbook* to be correct, then $x = v_0\sqrt{\frac{2y}{a_g}}$ is true, for it is a logical consequence of the previous assumptions. Here it is interesting to have students argue whether this logical consequence gives us anything new. The value of the equation, $x = v_0\sqrt{\frac{2y}{a_g}}$ lies in the fact that the prediction of the landing point can be made without knowing the time of flight.

Answers to questions

1. This is the same situation as that examined in the laboratory. If the slingshot is held at a distance y above the ground, the range x will be

$$x = v_0\sqrt{\frac{2y}{a_g}}$$

But what is v_0? To find this, shoot the same projectile vertically upward and time its flight. Since $v_0 = a_g t$ for each half of the flight, the time T for the round trip will be twice t, or

$$T = 2t = \frac{2v_0}{a_g}$$

when $v_0 = \frac{a_g T}{2}$ and our expression for the range x becomes

$$x = \frac{a_g T}{2}\sqrt{\frac{2y}{a_g}} = T\sqrt{\frac{a_g y}{2}}$$

2. We assume that the ball is launched with the same horizontal velocity, v_h, and vertical velocity, v_v, as on the earth.

   Consider the first half of the ball's flight on *earth,* in which it rises to the top of its trajectory. It will reach this high point in a time t defined by $v_v = a_g t$; therefore,

$$t = \frac{v_g}{a_g}$$

   During this same time the ball is also traveling horizontally with a velocity v_h and will therefore have covered a horizontal distance $d = v_h t$, which becomes on substitution

$$d = v_h \cdot \frac{v_v}{a_g}$$

   The ball will cover an additional equal distance during the descending half of its trajectory, so its total range R on earth will be

$$R = 2d = \frac{2v_h v_v}{a_g}$$

   On the moon a_g is only one-sixth as great as on the earth, and hence R must be six times as great.

3. The assumptions hold as long as we can ignore the effect of air resistance and as long as we assume the force of gravity is constant in magnitude and direction.

   If the earth had no atmosphere, therefore, the answer to the questions would be "yes," but in fact air resistance will reduce both the horizontal and the vertical distances traveled in a time t. The quickest way to appreciate this is to play "catch" with a ping-pong ball.

E1–12 CENTRIPETAL FORCE

This experiment assumes that students have *not* studied *Text* Secs. 4.6 and 4.7, in which the formula for centripetal force is derived. Instead the experiment leads the students to discover that F is proportional to m, f^2, and R.

Apparatus

The equipment is easy to assemble if no ready-made device is available. One needs:

- A spring scale calibrated in newtons or dynes
- String
- Rubber stoppers for weights
- A stick (meter stick) around which the weighted string can be rotated
- An audible timing device (metronome)
- Medicine dropper (or plastic or metal tube)

A scale calibrated in grams can be converted to a force scale by placing a piece of tape along one edge and marking the corresponding force units on it in newtons.

1 newton = 102 grams weight
1 kg weight = 1 × 9.8 newtons

If you use a glass medicine-dropper tube for the bearing, be careful to tape it completely so that if it cracks it will not shatter. You may also use a plastic or metal tube.

A student with a watch counting out the time aloud may replace the metronome.

An assumption

As the stoppers are swung in a circle at low speed the string is by no means horizontal, and the stoppers' distance from the vertical stick, R', grows less than R, the length of the string. Students may wonder whether the centripetal force is determined by R or by R'. The answer is that R is still the correct length to measure, as the following analysis shows.

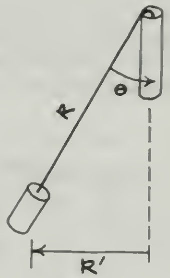


Fig. 6

When the string sags, the mass moves in a smaller circle whose radius is

$$R' = R \cos \theta$$

Its velocity becomes

$$v' = \frac{2\pi R'}{T} = \frac{2\pi R \cos \theta}{T} = v \cos \theta$$

and the centripetal force is reduced to

$$F' = F \cos \theta.$$

Substituting these expressions for R', v', and F' into

$$F' = \frac{mv'^2}{R'} \qquad (1)$$

gives us

$$F \cos \theta = m \frac{v^2 \cos^2 \theta}{R \cos \theta}$$

which simplifies to

$$F = \frac{mv^2}{R} \qquad (2)$$

Thus formula (1), which describes the centripetal force when the string sags, is really the same as formula (2), which students have been seeking to verify on the assumption that the string was horizontal.

Students who have not had trigonometry should be cautioned to measure R.

Discussion

Groups of students can be assigned different sets of conditions; then the data can be pooled. The students can then proceed to compare the data collected.

Because the error terms associated with the variables in this experiment range from very small (for the mass) to very large (for the period), a discussion of error and its estimation would be appropriate here.

As a second topic for discussion present the students with the height of a satellite orbit and its velocity (for Alouette I, $h = 1{,}040$ km and $v = 26{,}400$ km/hr), and ask them either how much faster it would have to go to boost its orbit 100 km, or how much its orbit would be increased if it added 100 km/hr to its velocity. This is a simplified problem quite similar to those which astronauts solve during maneuvering.

To solve the problem, one can assume that the gravitational force does not change appreciably over relatively small distances such as 100 km. Set

$$F_1 = \frac{mv_1^2}{R_1} \text{ for the initial orbit, and}$$

$$F_2 = \frac{mv_2^2}{R_2} \text{ for the final orbit.}$$

When any three of the values are known, the fourth can be calculated. The radius of the earth is 6,334 km. So $R_1 = 6{,}334 + h$ km.

Answers to questions

1. $F \propto m$
2. $F \propto f^2$
3. $F \propto R$, if f and m are kept constant.
   Note that if f is kept constant and the radius of orbit is not decreased, the speed decreases because the stopper travels a smaller distance in the same time. Remember that according to the relation $F = \dfrac{mv^2}{R}$, the decrease of radius will increase the force, but this will be counteracted by the decrease in speed, which is a second power dependent variable. The net effect of decreasing R and decreasing v^2 will be a decrease in force. All this is a consequence of the students' keeping the frequency the same.
4. $F \propto Rmf^2$
   $F = kRmf^2$, where $k = 4\pi^2$
   (actually $F = 4\pi^2Rmf^2$)

E1–13 CENTRIPETAL FORCE ON A TURNTABLE

Major equipment:

Turntable with large masonite top
Weights
Spring scale and string

The instructions for this experiment assume that the students have already studied the subject of circular motion through Sec. 4.7.

The previous experiment, "Centripetal Force" (*E1–12*), assumes that the students have *not* yet studied circular motion, and they discover $F = 4\pi^2 mRf^2$ for themselves in the lab.

Whichever circular motion experiment is used, the teacher should notice that it uses insights derived from the preceding work on Newton's second law; that is, an acceleration, v^2/R, results from a force, F. It is also very important to notice that the study of circular motion is central to the study of planetary motion in Unit 2.

The object of this experiment is to predict the maximum radius at which an object can be located on a rotating platform as a function of the period and the friction force. If student predictions are within 10% of the experimental results, you can consider them a success.

The friction force needed to get the object started differs from the sliding friction. You might want some students to investigate this curious difference.

Remember that R must be measured to the *center* of the mass on the turntable. Have the students mark the inner and the outer edges of the mass in the position where it begins to slip, and then later measure R to the midpoint between them. Also when we measure R to the center of mass we assume that R is several times larger than the radius of the weight. Typical data are in the table below.

Typical data for brass weight on masonite turntable

Mass g	Force to start slipping	16 rpm	Radius 33⅓ rpm cm	Radius 45 rpm cm	Radius 78 rpm cm
1000	1.5 to 2N	no slip	21.5	9.7	3.0
500	0.9 to 1.1	no slip	20.9	9.3	2.5
300	0.5 to 0.6	no slip	14.5	7.8	2.2
200	0.4 to 0.5	no slip	21.0	9.7	2.5
100	0.2 to 0.3	no slip	19.2	10.0	3.2

Note: Measurements of radius include the radius of the weight.

The students can be asked to determine the frequency of the turntable. The table may not be turning at 33⅓, 45, or 78 rpm. You may have to review the difference between frequency and period and also make sure that periods are expressed in *seconds* not minutes.

Answers to questions

1. The percentage difference should not be more than 5%. Have the students use the frequency found experimentally.
2. When the mass is made smaller it might seem that the radius R would have to be smaller, since R appears to be proportional to m in the expression $F = mv^2/R$.
   But F is also a function of v, and v is a function of R, so the answer to the question is not obvious.
   In particular, the centripetal force F is equal

to the force of friction at the moment of slipping, which means that

$$F = kma_g$$

where k is the coefficient of friction and ma_g is the weight of the object on the turntable, assumed to be horizontal.

$$\text{Also } v^2 = \left(\frac{2\pi R}{T}\right)^2$$

Putting these expressions for F and v^2 into the centripetal force equation, we get

$$kma_g = m \cdot \frac{4\pi^2 R^2}{RT^2}$$

and simplifying

$$R = \frac{ka_g T}{4\pi^2}$$

Since m has now vanished from the expression for R, it follows that the value of R is independent of m. This means that for a given solution for R you can use any mass.

3. Changing the mass of the object should have no effect because the friction force should increase with the mass as will the centripetal force, and the two effects will cancel.

$$F_t = \tfrac{1}{3}ma_g = \frac{mv^2}{R}$$

$$R = \frac{3v^2}{a_g}, \text{ which does not contain } m$$

$$100 \text{ km/hr} = 27 \text{ m/sec}$$

$$R = \frac{3 \times (27 \text{ m/sec})^2}{9.8 \text{ m/sec}^2}$$

$$= 200 \text{ m (wide arc)}$$

Film Loop Notes

L1 ACCELERATION DUE TO GRAVITY. I

Slow-motion photography in one continuous sequence allows measurement of the average speed of a falling bowling ball during two 50-cm intervals separated by 1.5 m. In case the question comes up, an iron plug was inserted into the top of the bowling ball to allow for magnetic release of the ball.

The key operating assumption in this measurement loop is that for uniformly accelerated motion the average speed v equals the instantaneous speed at the midtime of the interval. A formal proof of this statement is as follows:

If the acceleration is constant during a time interval of duration T, the speed at the midtime is

$$v_m = v_1 + a\left(\frac{T}{2}\right)$$

But the average speed is

$$\bar{v} = \left(\frac{v_1 + v_2}{2}\right) = \left[\frac{v_1 + (v_1 + aT)}{2}\right] = v_1 + a\left(\frac{T}{2}\right)$$

Hence $v_m = \bar{v}$.

The simplifying assumption that average speed equal instantaneous speed at the midtime is valid in uniformly accelerated motion for any size time interval. The statement is true for an arbitrary motion only as the time interval approaches zero.

Deviations of projector speed from the nominal 18 frames/sec are usually no greater than ± 1 frame/sec but this exceeds 5%. This is why it is suggested to students that they calibrate their projector.

An error of ± 0.04 in the value of a_g (compared to the accepted value at Montreal, Canada) would still give some "significance" to a final digit in a result such as 9.76 or 9.81 m/sec^2. This would require a student's measurements to be within half of one percent, which is very unlikely. A more reasonable expectation would be to obtain a to within ± 0.1 m/sec^2 (to within 1%).

L2 ACCELERATION DUE TO GRAVITY. II

Slow-motion photography allows measurement of the average speed of a falling bowling ball as it passes through four 20-cm intervals spaced 1 m apart.

Remind students of the need to calibrate projectors if they wish precise results. The Technicolor projectors are unlikely to have speeds in error by more than ± 1 frame/sec. This is, however, more than 5%.

When Unit 3 has been studied, the student will see that the equation $a_g = \dfrac{v_f^2}{2d}$ can also be derived from the law of conservation of energy. If the initial speed is v_i and the final speed is v_f, then

$$E_p + E_k = E_p + E_k$$

$$ma_g d + \tfrac{1}{2}mv_i^2 = 0 + \tfrac{1}{2}mv_f^2$$

$$2a_g d = v_f^2 - v_i^2$$

and for the case of $v_i^2 = 0$

$$2a_g d = v_f^2$$

$$a_g = \frac{v_f^2}{2d}$$

At the end, the students are asked a very difficult question, namely "Is there any evidence for a systematic trend in the values?" In theory, a systematic trend exists because of the approximation made. The ball speeds up as it passes through any interval; at the midtime it is slightly above the midpoint of the interval. Hence each value of d should be decreased very slightly, and the effect is largest for small values of d where the speed changes by a larger fraction during the interval. In practice, the error is negligible and will not be observed.

For the worst case, consider the motion of the ball from d = 0.90 m to d = 1.10 m.

From $d = \frac{1}{2}a_g t^2$, the time to fall 1.10 m is

$$t = \sqrt{\frac{2d}{a_g}} = \sqrt{\frac{2(1.10\ \text{m})}{9.80\ \text{m/sec}^2}} = 0.473\ \text{sec}$$

and the time to fall 0.90 m is

$$\sqrt{\frac{2(0.90\ \text{m})}{9.80\ \text{m/sec}^2}} = 0.43\ \text{sec}$$

The midtime is thus at t_m = 0.45 sec and the displacement at midtime is

$$d = \tfrac{1}{2}a_g t_m^2 = \tfrac{1}{2}(9.80\ \text{m/sec}^2)(0.45\ \text{sec})^2 = 0.98\ \text{m}$$

So we see that the value of d that corresponds to the measured v is 0.98 m, only 0.02 m or 20 mm above the midpoint of the interval. The error in d is 2% and therefore the error in a_g is also only 2%. The percent error is even less for the measurements at d = 2 m, 3 m, and 4 m.

Some typical results:

d	Δt	v_f	a_g
1 m	4.5 sec	4.45 m/sec	9.85 m/sec^2
2 m	3.2	6.25	9.75
3 m	2.5	8.00	10.5
4 m	2.3	8.70	9.5
			Av: 9.9 ± 0.4 m/sec^2

L3 VECTOR ADDITION: VELOCITY OF A BOAT

A motorboat is viewed from above as it moves upstream and downstream and as it heads across stream and at an angle upstream. Vector triangles can be drawn for the various velocities.

The student notes are somewhat more detailed than usual, because vector addition as such is not discussed very fully in the text.

At the time this film was made the physical conditions were not ideal. You can easily verify that the river's speed is not as uniform as would be desirable. There is about a 25% variation between the speeds at the extreme left and the extreme right of the picture. The average speed at the middle of the frame should be used. Careful observation also shows that the direction of the river flow is a few degrees off from being perpendicular to the line connecting the markers: in Scene 4 the flow is 265° and in Scene 5 it is 268°. Scene 1 is really superfluous, since the river speed can be measured well enough in the other scenes using patches of foam floating on the surface. However the pieces of wood may be easier to see.

As with all measurements of speed using film loops, it is essential to repeat each time measurement several times to average out errors (or to allow one to discard an obviously wrong value). If this is done, surprisingly good results can be obtained.

As an indication of the consistency of results obtainable with this loop, we give some typical results of measurements by the techniques described. In the four scenes (using foam as reference points) the water speed was 2.0, 2.0, 2.1, and 2.0 units. On the same scale, the values of $\vec{v}_{BW}$ were 4.0, 3.9, 4.3 and 4.5 units. The agreement between calculated and observed boat headings was ± 1° for Scene 4, and ±5° for Scene 5. Probable reasons for variations in $\vec{v}_{BW}$ were the inability of the operator to maintain exactly constant motor power, and some errors in steering.

L4 A MATTER OF RELATIVE MOTION

A collision between two equally massive cars is viewed from various stationary and moving frames of reference.

This is a qualitative demonstration loop for repeated classroom use by the teacher. The concepts used are (a) relative velocity and Galilean relativity (Unit 1); and (b) principles of conservation of momentum and conservation of energy in elastic collisions (Unit 3). It is suggested that the teacher stop the projector near the beginning of the loop when a message on the screen asks, "How did these events differ?" Encourage the students to describe the events they have just seen, without attempting to speculate on the ways in which the events were photographed. Then project the rest of the loop and initiate a discussion of relative motion and frames of reference. Come back to the loop when the conservation laws are studied in Unit 3.

In a technical sense, the word "event" implies knowledge of both places and times. A student walks from home to school between 8:00 and 8:20 on Monday, and again between 8:00 and 8:20 on Tuesday. These are two different events. They are similar events, one being a repetition of the other. In the loop, three events not only occur during different time intervals, but also appear to be physically different. The student should be encouraged to describe what he or she sees, and the events do seem to require different descriptions.

The principle of Galilean relativity is discussed in Sec. 4.4 of the *Text*: "Any mechanical experiment will yield the same results when performed in a frame of reference moving with uniform velocity as in a stationary frame of reference." In other words, the form of any law of mechanics is independent of the uniform motion of the frame of reference of the observer. Einstein broadened the principle to include all laws of physics, not just the laws of mechanics. Thus, Einstein relativity includes the laws of electromagnetism, which describe the propagation of light, as well as the mechanical laws of conservation of momentum and conservation of energy, which are sufficient for our study of colliding carts.

The student may be able to get some clues as to what is "really" happening by closely observing the rolling wheels of the carts, and will perhaps see the apparent motion of wrinkles in the pale blue background cloth. The teacher should ask the student what he or she means by "really" happening; it should become clear that one frame of reference (the earth) is being subconsciously identified as the "real" frame. The point of the film is that the other two (moving) frames are just as "real," and events taking place in them are described by the same laws of mechanics.

When using this loop in Unit 3, a discussion of the "laws" can be given. We are dealing with a collision. This is governed by the "law of mechanics," which we call the "law of conservation of momentum." In each event, momentum is conserved.

event	before collision	after collision	total momentum
A	$0 + (-mv)$	$(-mv) + 0$	$-mv$
B	$(+mv) + 0$	$0 + (+mv)$	$+mv$
C	$(+\frac{1}{2}mv) + (-\frac{1}{2}mv)$	$(-\frac{1}{2}mv) + (+\frac{1}{2}mv)$	0

The total momentum of the pair of carts has a different magnitude and direction in each of the three frames of reference, but the "law" or "principle" of conservation of momentum is equally valid in each frame of reference.

The collisions of the carts are, moreover, of the type called "perfectly elastic." In this type of collision, the "law of mechanics" that also applies is: Kinetic energy is conserved. Again we find that this "law" is equally valid in the three frames of reference. The total kinetic energy in each case is indeed the same before and after collision. But its value needn't be the same in each frame of reference. It is $\frac{1}{2}mv^2$ in Event A, $\frac{1}{2}mv^2$ in Event B, and $\frac{1}{4}mv^2$ in Event C.

L5 GALILEAN RELATIVITY: BALL DROPPED FROM MAST OF SHIP

This film loop is a realization of the experiment suggested in Galileo's *Dialogue on the Two Great World Systems*. A ball dropped from the mast of a moving ship lands at the base of the mast, just as it would if the ship were not moving.

Descriptions of Events 2 and 3 in the frames of reference of the boat and the earth might run something like these:

Event 2, boat frame: "A ball is moving horizontally toward the right and, at the moment it is opposite an observer, it is allowed to move freely as a projectile. The path of the ball is a parabola, and the ball moves downward and to the right."

Event 2, earth frame: "A ball is allowed to fall vertically from rest, and it strikes a point directly below the point of release."

Event 3, boat frame: "A ball, initially moving toward the right, is stopped by the muscular action of a student who is stationary on the mast. The student lets go of the ball, which then falls vertically downward."

Event 3, earth frame: "A stationary ball is given a forward velocity to the left by the muscular action of a student. The ball is then released, and its motion, that of a projectile, takes the ball to a point downward and to the left of the starting point."

L6 GALILEAN RELATIVITY: OBJECT DROPPED FROM AIRCRAFT

A flare is dropped from an aircraft which is flying horizontally. The parabolic path of the flare is shown, and freeze frames are provided for measurement of the position at 10 equally spaced intervals.

For the student who likes the challenge of careful quantitative work, the most interesting question raised is whether the effect of air resistance is noticeable in the horizontal and vertical components of the flare's motion. If air resistance were negligible, the horizontal displacement graph would be a straight line passing through the origin, $x = v_x t$, assuming that the correction B is 0. The vertical displacement graph would be a parabola, $d = \frac{1}{2}at^2$. In fact, careful measurement and plotting show that the horizontal motion experiences a decided "droop" due to air resistance. The vertical motion is surprisingly good, for the graph of $\sqrt{s}$ versus t remains almost straight for the whole motion, even in Scene 2, which is the longer of the two. To explain this, note that air resistance depends on speed. The flare is moving at large horizontal speed from the instant it is released, but it has large vertical speed only for the latter part of the trajectory.

L7 GALILEAN RELATIVITY: PROJECTILE FIRED VERTICALLY

A flare is fired vertically from a Ski-doo that moves along a snow-covered path. Events are shown in which the Ski-doo's speed remains constant, and also in which the speed changes after firing.

According to Galilean relativity, the flare retains the velocity (if any) of the Ski-doo. Relative to the earth, each event is the usual parabola of projectile motion. Relative to the Ski-doo, Event 2 is a vertical motion; the flare falls down again into the Ski-doo. In Event 3, the Ski-doo comes to a halt after the flare is fired, so the flare lands ahead of the Ski-doo. In Event 4, the Ski-doo accelerates in the forward direction after the flare is fired, so the flare lands behind it.

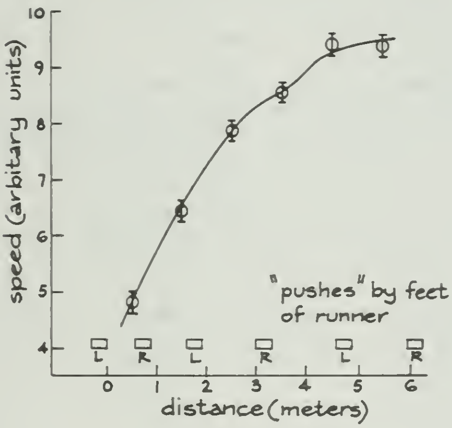
L8 ANALYSIS OF A HURDLE RACE, I

Slow-motion photography allows measurement of speed variations during a hurdle race. This loop, along with *Film Loop 9*, is intended to give students a feeling for the power of careful measurement to reveal "structure" in motion that seems to a casual observer to be nearly uniform. It also encourages them to speculate on the causes of the changes in motion that they observe.

A student may suggest that a systematic error has occurred because of perspective. This was taken into account when the meter marks were located on the wall behind the runner. The camera was positioned opposite the middle of the 6-m interval, and the markers were "spread" somewhat so that the runner's positions are correctly indicated when his image coincides with those of the markers.

The front of the runner's shorts would be unreliable because he straightens up after the start. Using the forward edge of the vertical meter marker is helpful because it gives the observer time to anticipate the moment of tangency.

Careful measurements give a speed graph for the first 6 m similar to the one below. The "droop" at 5 m can be related to Newton's second law, as suggested in the *Handbook*. A good student should be encouraged to study the film closely, perhaps plotting regions of "push" as in the graph shown here for your information. It is evident that the runner is practically coasting as his hip moves from "5" to "6." The initial acceleration can be found from $d = \frac{1}{2}at^2$: for d = 1.4 m and t = 38/80 sec, a is calculated to be 12.5 m/sec^2. The average acceleration during the first 1.4 m is about 1.3 times the acceleration due to gravity. Thus, the ground pushes on the runner, and the runner on the ground, with a force about 1.3 times his own weight (about 100 kg). If this seems unreasonable, note that the world record for weight-lifting, using arm muscles only, is about 180 kg. The acceleration is even greater at the very start, during the first 0.1 m of motion. A frame-by-frame analysis of the film gave an acceleration of about 40 m/sec^2 during this interval, corresponding to a momentary force of more than 2,600 N.



L9 ANALYSIS OF A HURDLE RACE. II

A continuation of the analysis of motion begun in *Film Loop 8*.

The graph from *Physical Review Letters* is shown to reassure the student who may be unhappy with a graph whose plotted points show considerable scatter. It is not necessary to go into the details of the experiment summarized by that graph, except to point out that this graph is a real-life example of published work by a team of five highly capable physicists.

In Scene 1, the speed increases just after the runner clears the hurdle, while he is still in the air. This paradoxical result is explained by the fact that the runner straightens up after clearing the hurdle. If his center of mass maintains constant speed then his hip must come forward as his knee and torso come back relative to the center of mass. This unexpected result is clearly shown in a typical student measurement and should provoke a valuable discussion. A similar effect explains the continued rise of speed in the 2-m to 3-m interval of *Film Loop 8;* the runner is still straightening his torso following the start of the run. In Scene 2, the measurements are less precise than in Scene 1 because the magnification is less. There is a modest rise in speed as the runner approaches the finish line at 50 m.

Equipment Notes

POLAROID PHOTOGRAPHY

CAMERAS

Almost any Polaroid Land camera can be used in classroom demonstrations and experiments in physics. The notes refer in detail to (A) the modified model 320 camera, and (B) the older models 95, 150, and 800 that can be bought relatively inexpensively and are used in many classrooms. A third section of these notes (C) on photographic techniques refers to all models.

A. The modified model 320 Polaroid Land Camera

This camera is a modified version of the model 210 camera, in which exposure time is controlled automatically by the electric eye. The manufacturer's instruction booklet describes the normal use of the 320 (loading the film pack, processing, etc.).

The modifications consist of:

1. a cover for the electric eye that makes it pos-

sible to take bulb exposures. When the eye is covered the camera shutter remains open as long as the shutter release (or cable) is held depressed. There are very few, if any, experiments for which you will use the eye to control exposure time automatically. Always keep the eye covered when the camera is not in use to prevent rapid rundown of the internal battery.

2. a cable release clamped semipermanently on the shutter release button.

3. a base plate with locking thumb screw. For most classroom work the camera is used as a fixed-focus camera. It is convenient to use the camera at a distance that gives a 10:1 photographic reduction. The locking screw is used to fix the camera bellows at the correct extension. The base plate also has a screw hole that takes a standard ¼"–20 screw for mounting it on a camera tripod or motor strobe disk unit.

4. a close-up accessory lens, which clips onto the camera lens to give an approximately 1:1 reduction for photographing traces on an oscilloscope screen, etc.

5. a clip-on slit, to be used in conjunction with the motor strobe unit (see notes on strobe photography)

6. a focusing screen of ground glass mounted in a frame that has the same dimensions as the film.

FOCUSING

The camera has a nonautomatic range finder. Look through the finder window: the position of the arrow on the scale at the left of the window indicates the focal distance in feet. Focus is adjusted by pushing the buttons marked "1" back and forth.

For most classroom use, it is convenient to work at a standard distance from the event being recorded. A distance of about 1.2 m gives a 10:1 photographic reduction. We recommend that one of the first things students do with the camera is establish precisely what the 10:1 distance is.

Camera model 320 provides a focusing screen. If you have another model, such a screen can be made very simply as follows. Take a discarded film pack apart into its three component pieces. One of these pieces is a frame that encloses an area the size of the picture. Fix a piece of ground glass in the frame, so that the ground surface faces toward the lens when the frame is put into the camera. If ground glass is not available, a satisfactory screen can be improvised by sticking tape (not the clear variety) on a piece of flat glass, or using tracing paper.

Insert the frame in the camera, just as if it were a film pack, ground glass surface toward the lens. Leave the camera back open. Set up a well-illuminated meter stick about 1.2 m in front of the camera. Cover the electric eye and set the speed selector to 75. Open the shutter and keep it open, by keeping the "2" button or the cable release depressed. (The cable release can be locked by tightening the set screw.) Look at the image on the focusing screen and adjust the range finder until the image is sharply focused. Measure the image of the meter stick on the screen. Adjust the camera–stick distance and focus until the sharply focused image of the meter stick is 10 cm long.*

Once the 10:1 distance has been found and the camera focused, use the thumb screw provided to lock the camera bellows in this position. Measure the lens-to-object distance. It will now be easy to set up and photograph an object or event at 10:1 reduction. Do not refocus the camera or loosen the locking thumb screw unnecessarily.

This preliminary exercise can be extended to establish two important points about using the camera to record events at the 10:1 distance.

(a) What is the field of view at this distance? (It should be just under 1 m.)

(b) Is the photographic reduction uniform over the print? Is it the same near the edge as at the center, or is there some distortion? (There is in fact very little distortion; the 10:1 factor can be used on all parts of the print.)

EXPOSURE

(a) *Aperture.* Students' attention may also be directed at this time to the effect of the "Film Selector" (manufacturer's instruction booklet). Remove the screen, open the camera shutter, and look through the lens with camera back open. At the 3000 setting the lens aperture is small; at 75 the aperture is 40 times larger in area.

For most strobe photography, use the 75 setting, even though the camera is loaded with 3000-speed film. The numbers refer to the ASA "speeds" of the two types of film. For normal outdoor use the selector is set to 300 for 3000-speed black-and-white film and to 75 for 75-speed color film. But this does not apply to our special classroom use of the camera. Although 3000-speed film will be used in our experiments, in many instances the 75 setting is needed. The lighten darken control (manufacturer's instruction booklet) is effective only when the electric eye is open.

(b) *Time.* If the electric eye is open, the exposure time is controlled automatically. If the electric eye is closed, the shutter will remain open as long as the cable release (or shutter release) is held depressed. For strobe work, cover the electric eye and control the exposure time manually. Try to keep the shutter open for the minimum time necessary to record the event. The longer the camera is open the poorer will be the contrast in the picture.

The electric eye will not work when the battery has lost most of its charge.

LIGHTING

The strobe photography experiments and demonstrations that are described in detail in the *Re-*

*Unfortunately, the screen is just less than 10 cm long. Therefore the meter stick must be set up obliquely. Alternatively, adjust until a 90-cm-long part of the meter stick gives an image 9 cm long.

source Book and *Handbook* do not require a darkroom. In many cases it is not even necessary to turn off the room lights, unless there is a light directly over the lab table.

Since a dark background is essential in strobe photography, a black cloth screen works well (see Fig. 1).

Fig. 1 Blinky photograph taken with modified model 210 Polaroid Camera. Room lights were on and a black cloth screen was used.

It is often useful to record both the strobe event and a scale (meter stick) in the same picture. Table 1 summarizes conditions for the various strobe techniques.

When working at 75 aperture, a small decrease in exposure can be effected by adding the clip-on slit over the camera lens.

TABLE 1
Suggested exposure conditions using modified model 320 Polaroid Land Camera

Strobe Technique	Lighting	Film Selector (aperture)	Procedure
light source and disk strobe	normal—but not directly overhead	75	Single-bulb exposure records both event and scale
xenon strobe	normal—but not directly overhead	3000	Single bulb exposure records both event and scale
blinky	darkened room—but not dark room	75	Single bulb exposure records both event and scale

CLOSE-UP ACCESSORY LENS

With the accessory lens clipped in place over the regular camera lens the camera can be used for close-up work. Focusing with this lens is quite critical and must be done with a focusing screen. The object should be between 12 and 14 cm from the front surface of the accessory lens, depending on the magnification you want. With the camera focus set to infinity the ratio of image size to object size is about 0.85, and with the bellows fully extended, the ratio is about 1.2.

For most classroom work the camera is used at a bellows extension that gives a 10:1 reduction (without the clip-on lens), and it is convenient to keep the bellows fixed in this position. You can use the camera for close-up work without changing the bellows extension. Add the accessory lens, insert a focusing screen in the camera back, and focus on the object by moving the whole camera toward or away from it. The magnification will be approximately 1 ×.

PHOTOGRAPHY OF TRACES ON THE OSCILLOSCOPE SCREEN

Remove any colored plastic window that may be in front of the screen. Clip on the close-up lens and focus the camera as described above. For stationary patterns set the film selector to 3000 and give a bulb exposure of about 1 sec duration. It is not necessary to darken the room. For single-trace work it may be necessary to set to 75 and darken the room or add a light shield around the oscilloscope face, long enough to reach to the camera. Keep the shutter open for the minimum time necessary to record the trace.

B. Models 95, 150, 160, 800 Polaroid Land Camera

These cameras all use roll film and give a picture size just under 7.5 cm × 10 cm. Models 150, 160, and 800 have a range finder; model 95 does not. On these cameras *one* adjustment determines both the lens opening and the time the shutter stays open. Speed and aperture combinations corresponding to the EV numbers of the various cameras are given in Table 2.

Notice that to convert EV numbers given for a model 95B, 150, or 800 to values for a model 95 or 95A, one must subtract 9, and vice versa. A setting of 15 on one series gives the same exposure as a setting of 6 on the other series. In this note and others in this *Resource Book*, we will give both settings, for example, EV 15 (6).

TABLE 2

Models 95A, 95B, The 700, 150, 160, and 800 Cameras				Model 95 Camera		
Shutter No. Models 95A, The 700	Shutter No. (EV Scale) Models 95B, 150, 800	Lens Opening	Shutter Speed	Shutter No.	Lens Opening	Shutter Speed
1	10	f/8.8	1/12 sec	1	f/11	1/8 sec
2	11	f/8.8	1/25 sec	2	f/11	1/15 sec
3	12	f/8.8	1/50 sec	3	f/11	1/30 sec
4	13	f/8.8	1/100 sec	4	f/11	1/60 sec
5	14	f/12.5	1/100 sec	5	f/16	1/60 sec
6	15	f/17.5	1/100 sec	6	f/22	1/60 sec
7	16	f/25	1/100 sec	7	f/32	1/60 sec
8	17	f/35	1/100 sec	8	f/45	1/60 sec

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A decrease of one unit in EV number means that twice as much light reaches the film. This is true for "instantaneous" photographs, but not necessarily so for time exposures. Note from the table that at all settings below EV 13(4) the camera lens is wide open. For time exposures any further decrease in EV will not affect the amount of light reaching the film.

All these roll cameras have a little knob on the camera face close to the lens. This can be set to either "I" for "instantaneous" exposures (exposure times as given in Table 2), or to "B" for "bulb" exposures (shutter remains open as long as the shutter release or cable release is held depressed). This knob returns to the "I" position automatically after every bulb exposure, and must be reset to "B" for each time exposure. Failure to reset it is the most common cause of unsuccessful exposures. (Possibly the second most common cause is forgetting to check that there is film in the camera.)

FILM

The most useful type of film for classroom use is the 3000-speed, type 47. It is the most sensitive and has the shortest development time (10 sec). The two transparency films are useful occasionally but are less sensitive. One of them (46–L) also needs longer development time.

TABLE 3

Film	ASA Speed Value	Development Time	Format
47	3000	10 sec	prints
46–L	800	2 min	half-tone transparency, for slides
Polaline 146–L	120	10 sec	black-and-white transparency (high contrast for line drawings)

If prints are to be kept more than a few days, they should be coated soon after exposure with the squeegee supplied with each roll of film. Prints are normally somewhat curled; flatten prints by pulling over a straight edge, picture side up, before coating them. Transparencies are preserved by immersion in "Dippit" liquid for at least 20 sec, and can then be mounted in easily assembled frames for projection. Read the instructions supplied with film, with "Dippit," and with slide frames for more details.

EXPOSURE

It is impossible to give hard-and-fast rules about exposures, as these will vary according to local conditions. Exposure values given in the notes on particular experiments and demonstrations must be regarded as suggestions only. In all kinds of multiple-exposure photography (blinky, strobe), it is important to increase contrast as much as possible. It is not necessary to have a completely blacked-out room. Regular opaque shades are quite adequate; some venetian blinds are satisfactory. A black background such as the black cloth screen mentioned earlier in these notes will improve the contrast enormously. In the particular conditions of the laboratory at Harvard, the following values were found to be useful starting points.

Photography of moving blinky: EV 15(6)

Photography of moving light source (pen-light cell and bulb) with 300 rpm disk strobe: EV 14(5)

Xenon strobe photography: falling steel ball: EV 16(7)

Xenon strobe photography: white mast on dynamics cart: EV 15(6)

C. Photographic techniques (all models)

All blinky or strobe photographs could be called multiple-exposure. By multiple-exposure we mean the recording of more than one image (of the same or different bodies) on one photograph. An example is the Unit 1 *Demonstration 7*, "Two Ways to Demonstrate the Addition of Vectors." Usually it is necessary to move either the object or the camera slightly between each exposure or to tilt the camera a bit, to prevent successive images from overlapping. The shutter must be recocked (model 200) or the knob returned to "B" (model 95, etc.) each time. The background light level is more important in this sort of work, but up to 20 blinky traces have been recorded on a single print.

Fig. 2 A "multiple-trace-" exposure blinky photograph. Shutter setting for 3000 film was EV 16(7) on model 210 and 75 on the experimental camera.

How to use the pictures

Ideally, each student team will be able to make and analyze its own photographs.

Students can probably best make measurements using a 10× magnifier and a transparent scale. Even quite dark prints can be measured with the magnifier in good light. Hold the print against a window pane, put it on the stage of an overhead projector, or use a reading lamp close to the print. The scale is made much more visible by backing it with white sticky tape (ACS tape).

Also satisfactory is a technique using dividers and millimeter scale.

Because the protective coating takes several minutes to dry, it will save time to measure photographs before coating; however, the uncoated emulsion is soft and easily scratched.

Students can also use the "negative" to take measurements that halves the number of exposures needed. To preserve the "negative" wash it with a damp sponge and coat it in the usual way.

If it is impossible for each team to produce its own print, the information on one print can quickly be passed on to the class by projection. Carefully make a pin prick at each dot on the print and use the overhead projector to project onto a sheet or pad of paper pinned to the wall. You may need a sheet of glass or corner weights to keep the print flat. A trial may be needed to find the best pinhole size for your projector. Each student makes a separate copy of the print by making a mark at each point of light on the projected image. He or she then takes down or tears off the sheet, and the next student in turn makes an enlarged copy of the print. These enlarged copies can be measured with rulers.

It is possible to make projection transparencies from black-and-white prints on a copying machine. Do not coat the print before make a copy. A high-contrast print and careful adjustment of the lighter darker knob on the copier are important.

For demonstrations it may be useful to project at high-projection magnification directly onto a meter stick and read off the positions of the dots.

When using projection techniques, make sure that the projected image is not distorted. The projector must be set perpendicular to the wall so that no "keystoning" exists. A quick way to check your projector for keystoning is to place a transparent ruler in the position later to be occupied by the photograph and to see if the scale in the projected image remains similar to the original one. Measure distances between equivalent points, for example, cm marks. Most projectors introduce some distortion near the edge of the picture area.

Opaque projection of prints is only marginally successful. Most opaque projectors do not have a lamp that is bright enough.

Polaroid transparency film (types 46–L and 146–L) can be projected, using either an overhead projector or a slide projector. This is more successful than opaque projection of prints, but in general has been found less useful than the projection of pricked-through prints. Transparency film is not available in pack form and so you cannot use the technique with the modified model 320 camera. Transparencies can be made from Polaroid negatives. Buford L. Williams of Kimball County High School, Nebraska, describes the procedures in *The Science Teacher*, March 1974, p. 41.

Scale

For many experiments and demonstrations, distance measurements can be made in arbitrary units. Millimeters on the film is most convenient. Similarly, time intervals can often be expressed in multiples of an arbitrary unit, flashes of the blinky or the strobe. But there are instances in which it is necessary to know actual distance and time values in conventional units. In the determination of the acceleration of a freely falling body, for example, you must convert distances and times into some familiar units to compare your result with known values.

It is quite easy to take a picture that shows both a moving object and a scale, such as a meter stick. For the scale image to be useful, the scale must be in the same plane as the motion being photographed.

With the modified model 320, both the moving object and the scale can be photographed in a single exposure (Fig. 3). See Table 1 on page 61 for recommended exposures and lighting conditions. If you are using one of the older cameras (95, 150, etc.) a double exposure may be necessary (Fig. 4).

It may be worth bearing in mind that if the $10 \times$ magnifier is used to measure photographs taken at 10:1 reduction, each millimeter on the print is 1 cm in real space.

Fig. 3 Single exposure, 3000 film, model 210 camera. Falling light source, disk strobe. Selector at 75, room lights on, electric eye covered.

Fig. 4 Double exposure photograph, 3000 speed film. (a) To photograph scale: EV 13(4), instantaneous exposure, with room lights on. This type of scale, with 1-cm wide bars is easier to photograph than a scale with millimeter divisions. (b) To photograph falling light: EV 13(4), bulb exposure, darkened room. Disk strobe with 18 slots, 300 rpm. For explanation of the elongation of the images see notes on stroboscopic photography.

If two points in real space are known to be a certain distance apart when photographed, it is possible to reestablish the real scale by projection. Move the projector to or from the screen until the images of the two points are the same distance apart as the objects were.

Checklists of the actual operations involved in using the two types of camera appear in the *Handbook*.

Illuminated scale for Polaroid photography

Often you want to include a scale (meter stick) in a strobe photograph so that you can convert measurements taken on the photo into real units. Although you can do this by double-exposing a real meter stick, the special illuminated scale described here makes this sort of photography easier and produces very impressive pictures.

Take a piece of 0.5-cm thick lucite, about 3.75 cm wide and 1 m long. Use an engraving tool to inscribe lines at exactly 1-cm intervals. The lines should be about 1 mm deep. Make every tenth line the full width of the rule. Number the 10, 20, 30 ... cm marks IN REVERSE, scribing the numbers carefully and being careful not to scratch adjoining surfaces of the lucite. The scale (engraved side down) should now look like this:

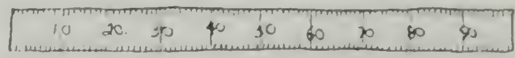


Fig. 5

To use the scale, shine light *into* the stick from the two ENDS. Light is scattered in all directions wherever there is a scribed line on the stick, causing the numbers and lines to show brightly against a dark background. Set up the stick so that you view it from the UNSCRATCHED face. You will then see the numbers in proper position. (If the numbers were on the front of the stick, some light would be scattered from them back into the ruler, and be reflected from the back, causing a double image.)

STROBOSCOPIC PHOTOGRAPHY

INTRODUCTION

Many of the experiments and demonstrations described in this *Resource Book* require stroboscopic photographs. There are several reasons why we use this technique so often.

1. The strobe photograph can sometimes give at a glance a qualitative idea of the time-displacement relationship in a particular motion. Uniform circular motion, free fall, and trajectories are examples.
2. The strobe photograph is a permanent record. Measurements made on a permanent record can be more precise and unambiguous than those made during the fleeting moment while the event is occurring. The measurements can be checked several times if necessary. Strobe photographs are by no means the only permanent records that will be used in this course. See, for example, the experiment on uniform motion, the photography of spectra, and the use of a strip chart recorder. This corresponds to a very modern tendency in the research lab—namely, to let the event "record itself" on an *xy* plotter or on-line computer.
3. Measurements can be made over rather short time intervals, so that rapidly moving objects and events of short duration can be analyzed.

Someone familiar with strobe techniques can often very quickly take a photograph to illustrate a point discussed in class. The more familiar one is with the camera and stroboscope equipment and their use in the particular local conditions of background illumination, etc., the more easily these demonstrations can be performed and the more effective they become.

TECHNIQUES

It is convenient to classify three kinds of stroboscopic photography. Most of the experiments and demonstrations described in this *Resource Book* can be done by any of the three methods.

(a) The moving object is illuminated intermittently by an intense light such as a *xenon strobe*.

(b) The moving object carries a flashing light source, for example, a blinky (relaxation oscillator with neon bulb).

(c) The moving object carries a steady light source, and light from this source to the camera is interrupted by a chopper in front of the camera lens (a motor-driven *disk strobe*).

Xenon strobe

Xenon strobe photography has the advantage that often nothing needs to be added to the moving object. Xenon stroboscopes (Stansi model 1812W, Stansi Scientific Co., 1231 N. Honore Street, Chicago, Illinois; or Strobotac by General Radio Co., West Concord, Massachusetts), are readily available. The Strobotac is calibrated and can be set to rates between 100 and 25,000 flashes/min. The Stansi strobe gives much more light, but is uncalibrated (see notes on "Calibration of Stroboscopes"). Of course, once a xenon stroboscope is available, much more can be done with it than simple strobe photography. For example, the measurements of rates of rotation and some very effective visual demonstrations that depend upon the "freezing" of various motions may be carried out.

As in all strobe photography, a suitable background is very important: Black cloth or a surface painted flat black are good. A clean chalkboard or cheap paper used in roofing and flooring can be used. But even these surfaces will give a surprisingly bright and troublesome reflection if the stroboscope is not carefully placed. It should light the background at a glancing angle, if at all.

Fig. 1 Xenon strobe light reflected from black cloth background.

The moving object is illuminated from the side (or occasionally from above or below). The background should, if possible, be some distance behind the event or screened to make sure that the background is in shadow while the object remains well lighted. The black cloth in Fig. 1 should have been moved backwards to prevent the poor contrast on the light. The stroboscope must not be in front of the object near to the camera. Make sure that the object is illuminated by strobe light throughout the motion that you want to photograph. Sometimes it is helpful to have a student hold the stroboscope and follow the moving object. Figures 2 and 3 show typical arrangements.

Objects to be photographed using xenon stroboscope:

(a) A golf ball will look more like a ball than any other object due to its surface texture.
(b) A ping pong ball, if clean, will give a white disk.
(c) A steel ball provides a sharp, bright point of light due to the convex mirror effect of the spherical surface. These points are ideal for taking measurements from a photograph, but the focusing effect may introduce a small error. The camera "sees" the virtual image of the light source reflected in the polished surface of the sphere. As the relative positions of light source and ball change, the virtual image will shift. The maximum possible error that can be introduced in this way is one ball radius. For most setups the error is less, and for any ball of less than 2.5-cm diameter it can usually be ignored. The size of the virtual image also depends on the radius of the ball. For very small balls the image may be so small that is is hard to photograph.
(d) Dynamics carts can be strobed. It is important, however, to have some bright object to serve as a reflector. A pencil painted black, except for the sharpened end which is painted white, can be fixed to the cart in a vertical position. Reflective tape (Scotchlike silver), knitting needles, and metallized drinking straws are good also.

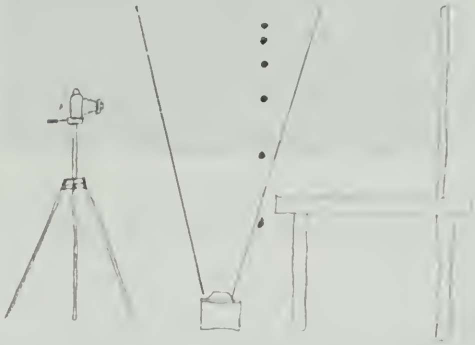


Fig. 2 In this set-up for a free-fall demonstration, the xenon strobe on the floor illuminates the falling steel ball, but not the background.

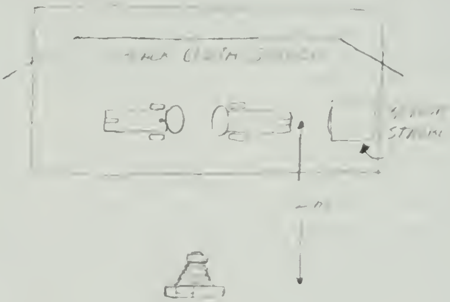


Fig. 3 Xenon strobe photography of dynamics carts. Note position of strobe and cloth screen, which is not immediately behind event to be photographed.

As always, optimum camera setting will depend upon local conditions. The photographs shown here (Figs. 4 and 5) were taken using a Stansi strobe, with a black cloth background behind the moving object.

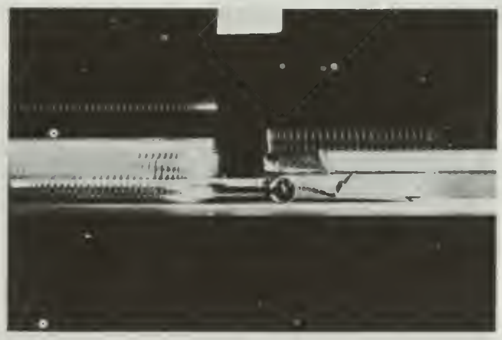


Fig. 4 Xenon strobe photo of dynamics carts. In the particular conditions of our laboratory a setting of EV 5 on model 800 camera was used. For model 320, set film selector to 3000. Strobe rate about 60/sec.

Fig. 5 Xenon strobe photograph of trajectory of steel ball. Strobe rate about 20/sec. Aperture setting of EV 16 on models 800 and 150; EV 7 on model 95. Film selector to 3000 on model 320.

Blinky

The battery-powered blinky can be made to flash at rates between about 20 and 200/min and it is fairly massive. But the principle of strobe photography is probably most easily explained using the blinky. The so-called "ac blinky" is certainly lightweight and it flashes at a known frequency (line frequency). However, it is not self-contained and it must always be attached to an ac outlet of at least 90 V. Because of its higher flash rate the ac blinky is suitable for faster moving objects, such as pendulums. It is possible to make a simple variable-frequency blinky (20–2000/second) using an audio-oscillator, amplifier, transformer, and neon bulb.

Although the blinky is not always the most convenient of the three strobe methods discussed here, students will probably find a photograph taken with the blinky technique the easiest to understand. The blinky is the first choice for demonstrations early in the course (uniform motion, vector addition of velocities, etc.). Because the light output of the blinky is rather low, it is important to keep the background illumination low so that fairly wide apertures (low EV numbers) can be used without losing contrast. The data given in Fig. 6 should be regarded as only a starting point from which to establish optimum conditions for your own local situation.

Fig. 6 Blinky photograph: three traces. Model 150 camera was set to EV 17 (model 95 setting would be 8). With model 320, set film selector to 75.

Strobe-disk photography

The small light sources supplied by Damon have a mass of about 25 g, and so their mass can often be ignored if they are used on the 1-kg dynamics carts. But their mass can very definitely not be ignored if they are added to air-track gliders, the smallest of which has a mass of about 30 g.

The heart of the motor strobe kit is a 300 rpm synchronous motor. If the disk supplied by Damon has 12 equally spaced slots, this gives a maximum strobe rate of 3,600/min or 60/sec. By taping over some of the slots (so that the open slots are equally spaced), the rate can be reduced to as low as 300/min (5/sec) when only one slot is open. (The requirement that the open slots be equally spaced limits the possible rates to submultiples of the maximum frequency.)

Fig. 7 Disk-strobe photograph of dynamics carts: 1.5-V light source on each cart. Six slots, 300 rpm (30/sec). Shutter setting EV 14(5) on old cameras; film selector to 75 on modified model 320.

Fig. 8 Disk-strobe photograph of uniform acceleration. One-slot disk, 300 rpm (5/sec). 1.5-V light source; EV 14(5), or film selector to 75 on the Polaroid camera.

Of course, by changing the motor or the disk, the range can be extended. Synchronous motors of various speeds are available from most radio-supply houses (Lafayette, Allied, Radio Shack, etc.). Extra disks can easily be made of cardboard.

Strobe rates for 300-rpm motor

Number of Slots Open	Rate	Time between Successive Images
12	3,600/min	1/60 sec
6	1,800/min	1/30 sec
4	1,200/min	1/20 sec
3	900/min	1/15 sec
2	600/min	1/10 sec
1	300/min	1/5 sec

Fig. 9 Free-fall, disk-strobe technique, showing elongation of images.

Fig. 10 Free-fall, disk-strobe technique. Slit on camera lens reduces elongation of images.

An important point to remember when using the disk-strobe technique is illustrated by the pair of prints shown in Figs. 9 and 10. A slot 0.5 cm wide in a disk of 10-cm radius rotating at 300 rpm takes about 0.005 sec to pass in front of the camera lens (diameter about 1.5 cm). The camera lens will be "open" for this time. In 0.005 sec a body moving at 4 m sec (the speed of a freely falling body 80 cm below release) will move about 2 cm. This explains the elongation of the images, which increases with the speed of the object, in Fig. 9. This elongation is reduced, but not completely eliminated, by taping a fixed slit (supplied with the kit) to the camera lens (Fig. 10). This slit should be parallel to the slot in the rotating disk as it passes in front of the lens. The length of the streak could be further reduced by using a narrower slit on the lens, but image brightness will be reduced by lens slots narrower than the disk slot.

The duration of a blinky flash is about 0.010 sec, but since the blinky is unlikely to be used for fast-moving objects, the problem of image elongation is unlikely to occur. The duration of a xenon stroboscope flash can be several orders of magnitude less, ranging from about 5 μsec in more expensive strobes to around 10 μsec or more in others. In all experiments that one is likely to do in the classroom, the object will be effectively "frozen" by stroboscopic illumination.

CALIBRATION OF STROBOSCOPES

Inexpensive stroboscopes are usually uncalibrated; that is, the numbers on the frequency-control dials don't correspond to actual frequencies. Below are several methods for finding the dial readings that correspond to a set of known frequencies. A calibration graph is constructed by plotting the dial values against the known frequencies and drawing a smooth curve through the plotted points. Dial readings can then be converted to frequencies by referring to the calibration curve.

OSCILLOSCOPE METHOD

1. "Linear trace" on oscilloscope

Connect a phototube (such as the IP39 tube, which is part of the phototube module recommended by *Project Physics*) to the vertical input terminals of the oscilloscope. Notice that no voltage source is needed in this circuit.

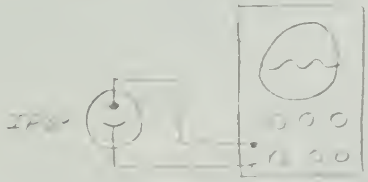


Fig. 1

Set the horizontal sweep rate to about 10 sec. Adjust the vertical gain until you see a 60-cycle trace on the oscilloscope (the phototube has a very high impedance, and the wires to it act as an antenna picking up 60-cycle noise). Adjust "sync" control of oscilloscope until this 60-cycle pattern is stable.

Position the stroboscope so that light from it falls on the phototube. Each flash will produce a sharp vertical line on the trace (Fig. 2). Adjust the flash rate until there is one flash per cycle of the 60-cycle pattern.

With the flash rate slightly above 60 sec the lines will be slightly less than one "wavelength" apart, and will move to the left, and vice versa. Only when the strobe rate is exactly 60 sec will the vertical lines be stationary on the 60-cycle trace.

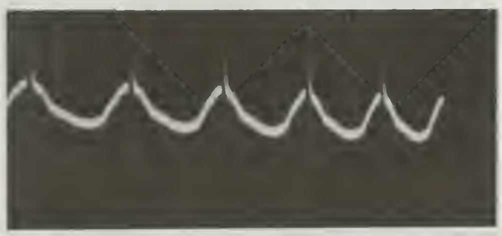


Fig. 2

Now reduce the strobe rate. The next simple frequencies to recognize are 30 sec (Fig. 3) and 20 sec. See Fig. 4 on the next page.

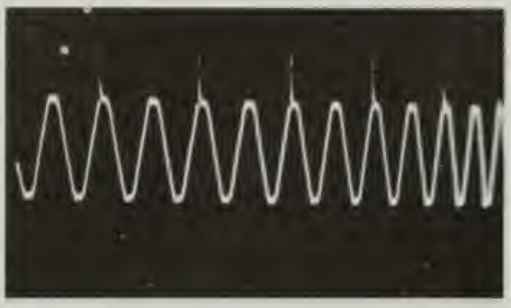


Fig. 3 $f = \frac{1}{2} \times 60 = 30/\text{sec}$

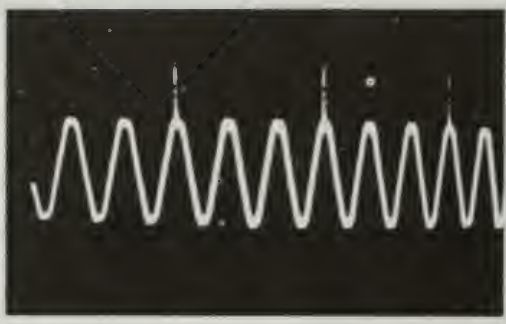


Fig. 4 $f = \frac{1}{3} \times 60 = 20/\text{sec}$

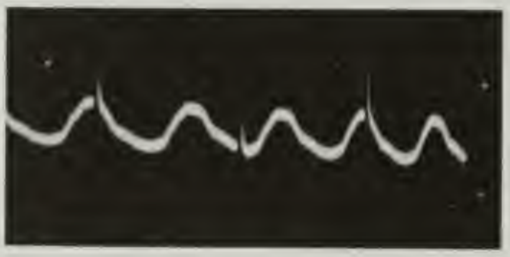


Fig. 5

In Fig. 5 there are two flashes for every three 60-cycle periods. The time between flashes is therefore $\frac{1}{2} \times 3 \times \frac{1}{60} = \frac{1}{40}$ sec. So the frequency is 40 flashes/sec.

Patterns for other fractional frequencies of 60/sec can also be recognized and interpreted.

On the low range of the Stansi strobe the stationary patterns shown in Figs. 6 and 7 were observed.

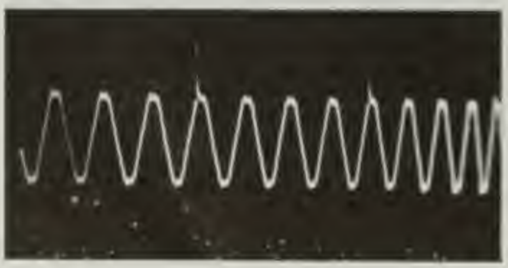


Fig. 6 $f = \frac{1}{4} \times 60 = 15/\text{sec}$

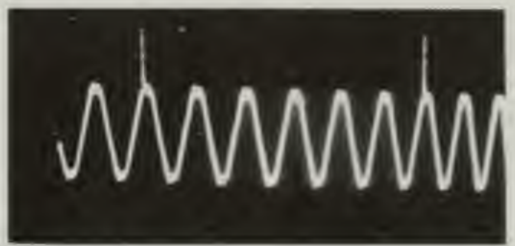


Fig. 7 $f = \frac{1}{6} \times 60 = 10/\text{sec}$

2. "Circular trace" on strobe

Connect the phototube to the vertical input as described above.

Establish a circular or elliptical trace on the oscilloscope face either by (a) setting the horizontal frequency selector to line sweep or (b) setting to external input and connecting the horizontal input terminal to the 60 vibrations/sec calibration signal available on the scope (or simply attach a short wire to the horizontal input that will act as an antenna to pick up 60-cycle noise). Adjust horizontal and vertical gain as necessary to obtain an open figure.

The electron beam is now tracing out one revolution of this figure in $\frac{1}{60}$ of a sec. Turn on the stroboscope. Every flash will cause a sharp vertical peak. Adjust the flash rate until this peak is stationary. The simplest figure to interpret is one flash/cycle.

Fig. 8

As the flash rate is reduced other stationary patterns will be produced and can be interpreted. For instance, 30 flashes/sec will produce a peak every second revolution of the spot.

Fig. 9

Notice the subtle difference between this pattern and the previous one. Here the vertical spike is superimposed on a closed ellipse.

As the flash rate is reduced further this pattern will recur at $f = \frac{60}{n}$/sec where n is an integer, that is,

$$f = \frac{60}{2} = 30;\ \frac{60}{3} = 20;\ \frac{60}{4} = 15;\ \frac{60}{5} = 12;$$
$$\frac{60}{6} = 10;\ \frac{60}{7} = 8.6;\ \frac{60}{8} = 7.5;\ \frac{60}{9} = 6.7;\ \ldots$$

Other series of stationary patterns can be produced.

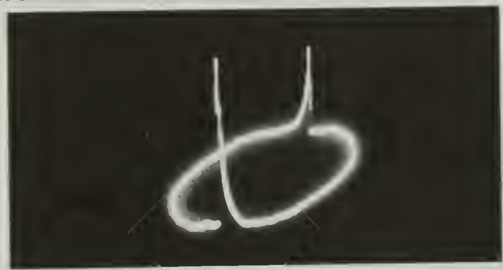


Fig. 10 Two spikes in $\frac{1}{60}$ sec indicate a frequency of 120/sec but few strobes can flash at this rate—the Stansi strobe cannot.

The pattern in Fig. 11 will occur if the strobe flashes twice in every 3, 5, 7, . . . cycles, corresponding to flash rates of $\frac{60}{3} = 20;\ \frac{60}{5} = 12;\ \frac{60}{7} = 8.6, \ldots$/sec.

Fig. 11

Patterns containing 3 and more spikes per cycle can also be obtained.

Linear versus circular trace method

Clearly the circular technique needs more careful interpretation than the "linear trace" method described above. However, it *is* particularly useful at low flash rates. It may not be possible to get more than 6 cycles of 60-cycle signal on the oscilloscope face, and this puts a lower limit on the frequency at which the "linear trace" method can be used. One spike in 6 cycles means $f = 60/6 = 10$/sec. The circular trace method can be used down to the lowest frequencies.

The "circular trace" method has the advantage that it is easier to obtain a stationary pattern on the circular trace than on the linear trace. On the other hand the linear trace is much easier to interpret. A combination of the two methods is useful. Use the circular trace to establish a stationary pattern. Then at the same flash rate switch to linear trace for interpretation.

ROTATING-DISK METHOD

Any rotating object with a known rate of rotation can be used. A synchronous motor with a suitable disk is the most reliable. Some electric fans and other rotating machines that have speed ratings given may also be used. Rotation rates of less than about 300/min are not very satisfactory, as explained below.

The method will be described here in terms of a specific example. Be quite careful about generalizing to other disks rotating at different rates.

Mount a disk with 12 equally spaced marks on the shaft of a 300-rpm synchronous motor (the motor strobe kit supplied by Damon). Add another single mark, such as a white star or a piece of masking tape, between two of the slots (Fig. 12). Start the motor, darken the room, and turn on the strobe. As the strobe rate is changed, different stationary patterns will appear.

Fig. 12

The simplest pattern to interpret is one that shows 12 slots and 12 stars (Fig. 13). The strobe is flashing 12 times for each revolution of the disk and the strobe rate is 12 times the rotation rate of the disk: $12 \times 300 = 3{,}600$ flashes/min.

Fig. 13

Fig. 14

Reduce the strobe rate slowly until a stationary pattern showing 12 slots and 6 stars is observed. The strobe rate is now six flashes/revolution, or 1,800 flashes/min.

Other patterns that are easy to interpret are shown in Figs. 15–17.

Fig. 15

Flash rate = 4/revolution
(= 1,200/min)

Fig. 16

Flash rate = 3/revolution
(= 900/min)

Fig. 17

Flash rate = 2/revolution
(= 600/min)

Figures 16 and 17 correspond to flash rates of 10/sec and 5/sec, respectively, which bring us down to rates slow enough to be counted directly.

This really completes the simple calibration of a stroboscope by this method. However, it is probably worthwhile mentioning some of the other stationary patterns that can be observed, and their interpretation.

Fig. 18

Flash rate = 1/revolution
(= 300/min)

Figure 18 is the pattern observed if the lamp flashes once per revolution. The same pattern would also be seen if the lamp flashed once for every <u>two</u> revolutions of the disk, and once for every <u>three</u> revolutions, and so on. But in fact there need be no confusion, for two reasons. First, if one works from high to low flash rates in this calibration, the <u>first</u> time that Fig. 18 is observed it will correspond to one flash/revolution, the next time to one flash per two revolutions, and so on. Second, in the particular case of a 300-rpm motor the flash rates concerned are low enough (5, $2\frac{1}{2}$, $1\frac{1}{4}$ flashes/sec) to be identified by direct counting.

The same sort of thing will happen at other flash rates too. Consider, for instance, Fig. 16. The strobe flashes 3 times for each revolution of the disk. If it flashed once for every 1⅓ (or 4/3) revolutions of the disk the same pattern would be obtained. Similarly, Fig. 15 would be obtained with one flash for 1¼ (or 5/4) revolutions of the disk as well as at four flashes/revolution. And in general a figure with n stars (which is obtained at a flash rate of n flashes/revolution) is also obtained when the rate is one flash for every $\frac{n+1}{n}$ revolutions.

Other stationary patterns can be observed in which more than 12 slots are seen. For instance, a flash rate of 8/revolution (2,400/min with 300-rpm motor) will give a pattern showing 24 slots and 8 stars (Fig. 19).

Fig. 19

Flash rates of more than 12/revolution will give more than 12 slots, of course. At a flash rate of 16/revolution, 16 stars and 48 slots are seen (Fig. 20).

Fig. 20

Disks that rotate at 78 rpm (called phonograph turntables) are easy to obtain, but their usefulness for strobe calibration is limited. They can only be used for slow flash rates.

With a turntable rotating at 78 rpm carrying a disk with 6 symmetrical radii, a stationary image of the disk is observed for flash rates of 156 (= 2 × 78); 234 (= 3 × 78); 468 (= 6 × 78)/min. At 936 flashes/min a disk with 12 radii is seen (Fig. 21). At higher flash rates the number of radii grows and counting them becomes difficult.

Fig. 21

Sample results

Figures A and B (on the next page) illustrate calibration curves for a Stansi strobe obtained by the oscilloscope method. The vernier adjustment was kept at its upper limit. The plot of flash rate against scale reading is quite nonlinear as in Fig. B.

A plot of $\frac{1}{f}$ (the period, or time interval between flashes) against scale reading is linear. (This is because the period is determined by the time constant of an RC circuit in the stroboscope, and turning the knob adjusts the resistance, evidently in a linear manner.) The plot of $\frac{1}{f}$ against scale reading makes interpolation and particularly extrapolation easier, and fewer points are needed to complete the graph (see Fig. A).

Note that a plot of f against $\frac{1}{\text{scale reading}}$ is not linear. This is because there is a fixed resistor in the circuit, as well as the variable one controlled by the knob.

$f \propto \frac{1}{R_{\text{variable}} + r_{\text{fixed}}}$, so f is not proportional to $\frac{1}{R_{\text{variable}}}$.

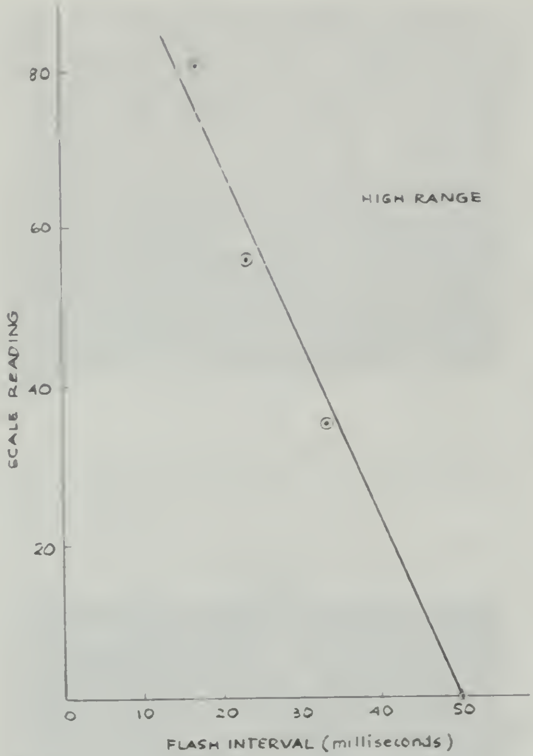


Fig. A

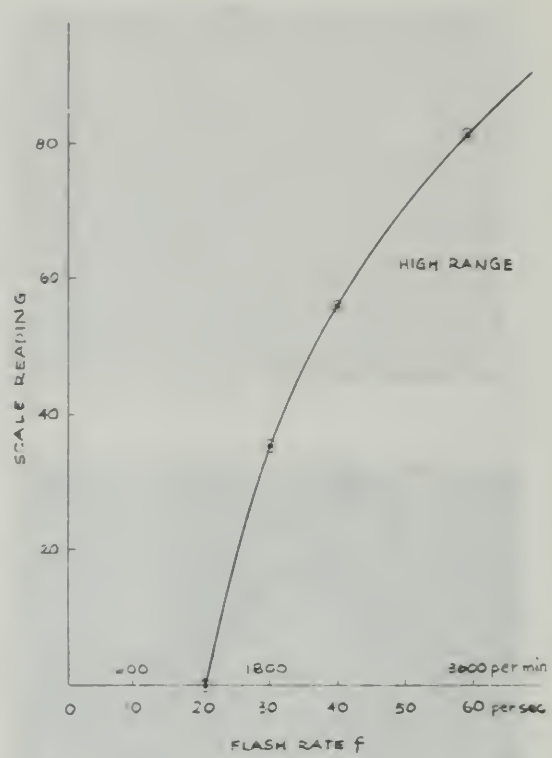


Fig. B

THE BLINKY

A simplified circuit diagram of the blinky is shown in Fig. 1.

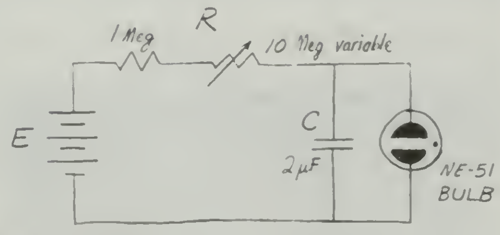


Fig. 1 The blinky

Since it goes through a certain sequence of actions periodically on its own initiative, the blinky is an oscillator. It is one of a class known as relaxation oscillators.

The three 30-V batteries E charge the capacitor C through the resistance R.

The neon lamp remains nonconducting as long as the potential difference across it remains below the "break-down" voltage which is about 70 V. The voltage across the neon lamp is, of course, equal to the voltage across the capacitor. The capacitor continues to charge up until the neon lamp becomes conducting at 70 V.

Once the neon lamp becomes conducting, the capacitor begins to discharge through it. The neon lamp continues to discharge even when the potential difference across it has fallen below the "breakdown" voltage. In fact, it continues to conduct and the capacitor continues to discharge through it until the potential difference across them reaches about 53 V. This all happens very quickly; the whole process just described takes on the order of 10 milliseconds (0.01 sec).

The capacitor now begins to charge up again from the batteries, and the neon remains nonconducting until the "breakdown" voltage is reached again. Then the neon bulb glows briefly as the voltage drops down to about 53 V.

The knob on the front of the blinky box adjusts the variable resistance that controls the rate at which the capacitor charges between discharges.

Do not worry about the batteries running down. The current drawn from them is very small. It is the "shelf life" of the batteries that determines how long they will last. This can be extended by keeping the blinky cool (as in the refrigerator) during the summer.

The most likely reason for a blinky not to blink is poor contact between one of the 30-V batteries and its holder.

AC BLINKY

This is easy to make and is a useful piece of equipment for motion studies and photographs.

An ac blinky is a neon glow lamp circuit that operates directly from the 100–120 V ac line. The intensity and duration of the flashes can be varied, but the flash rate (frequency) cannot: It is fixed at 60/sec.

Two factors make the ac blinky especially useful:

(a) The flash rate is accurately known since line frequency is usually maintained very precisely at 60/sec.

(b) The flash rate is high, making it useful for rapidly moving objects.

But, unlike the regular (dc) blinky, the ac blinky is *not* a self-contained unit. It must always be plugged into the line.

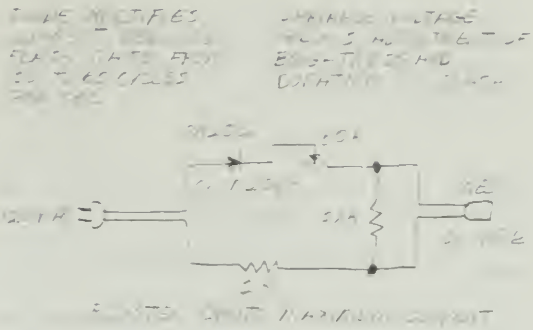


Fig. 2 Schematic of ac blinky circuit

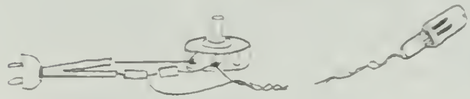


Fig. 3 Physical layout of ac blinky

AIR TRACKS

The linear air track recommended by *Project Physics* is an inexpensive model. Although it is quite adequate for many demonstrations and experiments, it is not a high-precision device.

Any medium-to-large household vacuum cleaner that can be used as a blower should be adequate. The air flow will be increased if you remove the dust-bag from the cleaner. If you use a large industrial-type cleaner (for instance, one borrowed from the school shop or from the janitor), you may find that it helps to plug it into a variac. Too strong an air flow will cause the gliders to float too high. We have found that the compressed air supply sometimes available in laboratories is generally not enough to operate the air track.

To test the track, raise one end a few centimeters and release a glider from the top. The glider is running satisfactorily if it rebounds from the rubber band at the lower end of the track to within 25 cm of its starting point.

Use the leveling screws to adjust the track so that a glider, released from rest, has no tendency to move toward one end or the other. Because of the slight drop in air pressure along the track, this balance will not necessarily be achieved when the track is perfectly horizontal.

The two small gliders supplied by the manufacturer have equal masses. The one large glider has twice the mass of either small one ($\pm 2\%$). The three gliders allow you to perform equal-mass elastic collisions, unequal-mass elastic collisions, and unequal-mass inelastic collisions. Note that if the gliders are carrying light sources for strobe photography, the mass ratios will not be 1:1 or 2:1.

The range of mass ratios can be extended by taping extra mass to the gliders. Be sure that the added mass is distributed symmetrically. It is important to keep the center of mass low and therefore it is better to add mass equally to the two sides of the glider than to the top. Check the glider for free running after you have added extra mass by doing the rebound test described above. The large glider should support an extra load of at least 40 g.

The following setup can be used to impart the same initial velocity to a glider on consecutive trials: Attach a small block to the glider. Draw the pendulum bob back and let it strike the block. If the pendulum is always released from the same point and the glider is in the same position (so that the bob hits it at the bottom of its swing), the glider will always acquire the same initial velocity.

QUANTITATIVE WORK WITH LIQUID-SURFACE ACCELEROMETER

Theory predicts that the slope of the liquid surface is given by

$$\tan \theta = a\,a_g$$

Figure 1 shows an accelerometer moving horizontally with constant acceleration *a*.

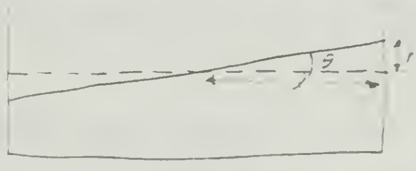


Fig. 1

If the cell has length $2l$ and the liquid rises to a height h above its rest position at the end of the cell, then the angle θ that the surface makes with the horizontal is given by

$$\tan\theta = \frac{h}{l}$$

So

$$\frac{h}{l} = \frac{a}{a_g}$$

and

$$a = \frac{h}{l}\,a_g$$

That is, the ratio of the two lengths h to l gives the acceleration in a_g.

The matter can be simplified further. Since a_g is almost 10 m/sec^2, if we make the length l = 10 cm, then

$$a = \frac{h \text{ cm}}{10 \text{ cm}} \times 10 \text{ m/sec}^2 = h \text{ m/sec}^2$$

The height h, in centimeters, is equal to the acceleration in meters/sec^2.

To read h it is convenient to stick some centimeter tape to the front surface of the cell, with the scale vertical, and exactly 10 cm from the center of the cell. The zero mark of the scale should be at the height of the undisturbed horizontal level of the liquid, usually about halfway up the cell. It also helps to stick a slightly wider piece of white paper or tape on the back of the cell, opposite the scale. This gives a definite background against which to observe the liquid level (Fig. 2).

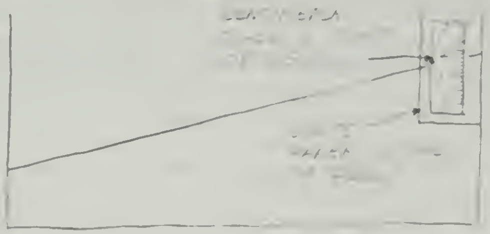


Fig. 2

CALIBRATION OF THE ACCELEROMETER

The theoretical derivation described above can be confirmed experimentally by the following procedure. Use a conventional string, pulley, and mass setup to produce uniform acceleration of a dynamics cart carrying the accelerometer. The actual acceleration can be measured from a strobe photograph. (The strobe rate and photographic reduction must be known, of course. The calculation is much simplified if the strobe rate is 10/sec, and the reduction is 10:1.) From the same photograph the height h can be measured on successive images and the average value of $\frac{h}{l}$ calculated. (A variation of less than 10% was found.)

This is repeated with several different falling weights (or masses on the carts) to produce a range of values of a. The average value of $\frac{h}{l}$ is plotted against the average value of a for each photograph. A typical result is shown in Fig. 3.

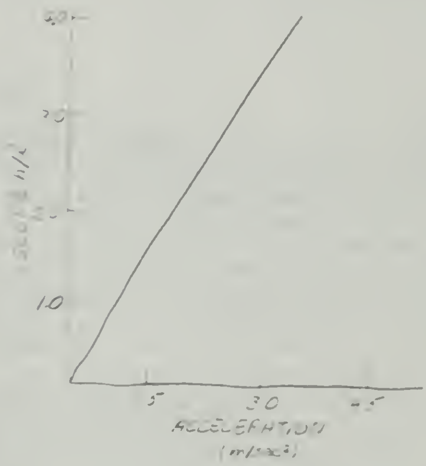


Fig. 3

As an alternative (which is less precise but involves more students), have several students stationed along the cart's path and let each one observe the value of h as the cart passes by.

For further details and theoretical derivation of the formula mentioned above, see the article by J. Harris and A. Ahlgren, *Physics Teacher*, Vol. 4, pages 314–315 (October 1966).

A VERSATILE "CANNON"

A very versatile and inexpensive rubber-band-powered "cannon" can be built, either as an individual activity, or as a mass-production class activity. Four of the immediate uses we have tried are:

1. a launcher for range of projectile demonstrations
2. a launcher for the "Monkey in the Tree" demonstration
3. a device for reproducible forces for accelerating carts, air-track gliders, etc.
4. a sighting tube for astronomy (made more accurate by taping a plastic soda straw along the top of the barrel since a paper straw gets soggy and bends in damp night air)

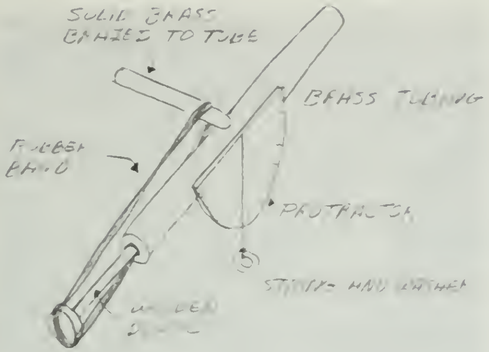


Fig. 1

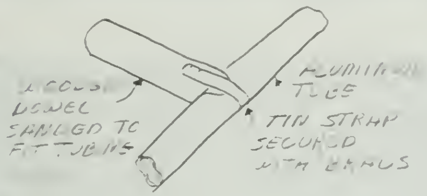


Fig. 2

Our model (see Fig. 1) consisted of a 20-cm length of 8-mm bore brass tubing, with a piece of solid brass brazed to the middle. To avoid brazing an alternative would be a length of aluminum tubing with a wooden dowel fastened to it with epoxy cement and a small metal strap around the tube (see Fig. 2). The plunger consists of a wooden dowel with a larger piece of wood screwed to the end. A slot cut across the end of the wooden piece keeps the rubber band from slipping off the end of the plunger. A plastic protractor is glued to the side of the tube. A short pin is glued in the reference hole in the protractor, and a thread and washer are attached to it for determining a plumb line. For use as a sighting instrument, the handle can be put through a hole in a piece of wood that is pivoted on a flat board marked off in degrees.

RANGE PREDICTION EXPERIMENT

Procedure 1

(a) Determine the muzzle velocity by firing the cannon vertically, measuring h, and substituting in $v = \sqrt{2gh}$.

(b) Then estimate the horizontal range, knowing v from the above calculation, and h, height above the floor, from the relations:

$$h = \tfrac{1}{2} a_g t^2,\ t = \sqrt{\frac{2h}{a_g}},\ \text{and range} = vt$$

Mark the expected range on the floor and try to hit the mark.

Procedure 2

For more advanced students, develop (or have them derive) the general range formula, $R = \frac{v^2 \sin 2\theta}{a_g}$, and then try the experiment.

Procedure 3

In Unit 3 the energy concept can be used for the same situation:

(a) Make a graph of force versus length for the rubber band.

(b) From the graph, find maximum F and minimum F when the rubber band is used for a particular shot.

(c) Find the mass of the plunger and cannon ball.

(d) Find the estimated velocity, using

$$F_{av} \times d = \text{kinetic energy } (\tfrac{1}{2}mv^2)$$

Sample results

Using *Procedure 2*, a measured value of 4.34 m was obtained for an estimate of 4.20 m.

The discrepancy was slightly larger for *Procedure 3*. The predicted muzzle velocity (from force-extension curve) was 6.9 m/sec; the measured velocity (from the height to which the ball rises when fired vertically) was 6.4 m/sec. A direct check with a strobe photo or two photocells and an oscilloscope would be excellent.

Note that the force versus extension curve for *Procedure 3* (Fig. 3) does not pass through the origin because the rubber band is already stretched before the plunger is pulled back at all. There is a finite force for a zero extension (on our scale). The energy given to ball and plunger is the total "area" under the graph.

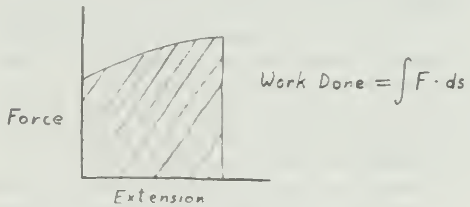


Fig. 3

CATHODE-RAY OSCILLOSCOPE

The cathode-ray oscilloscope (CRO) is one of the most versatile laboratory instruments. This note can only summarize its different capabilities and functions in the lab and some of its uses as a teaching aid. The approximate numerical values given in this note refer to a typical inexpensive 'scope such as the Heathkit Model IO–12 (wired from Heath, Benton Harbor, Michigan).

FUNCTIONS OF THE OSCILLOSCOPE

The CRO is a voltmeter. It can measure voltages down to about 10^{-2} V (depending on the amplifier). It can measure short voltage pulses (down to about 10^{-6} sec). Because it has a high input impedance, it draws little current from the voltage source being measured.

As well as being used to measure voltages, the CRO is useful as a null detector. Examples are:

(a) a phototube (illuminated by a pulsed light source) is connected through an amplifier to a CRO and the reverse voltage across the phototube is increased until the pulses on the CRO trace disappear, indicating that the "stopping voltage" for the photoelectrons has been reached.

(b) an ultrasound detector or microphone is connected to the CRO and moved through an interference or standing-wave pattern until the signal falls to zero, indicating that the detector is at a node (point of zero intensity).

The CRO can be used to show the wave form of a voltage signal (sinusoidal, square, saw-tooth, etc.) and to measure the phase difference between two signals. It can be used to measure time intervals (10^{-1} to 10^{-6} sec) and frequency (10 to 10^6 cycles/sec).

These and other functions make the CRO a valuable "trouble shooting" tool in the lab, in the repair of radio and TV sets, and in electronics work generally.

THE OSCILLOSCOPE AS A TEACHING AID

The CRO also has many applications in the teaching of physics, some of which are listed here.

Electricity: demonstration of the effect of capacitance and inductance in a circuit; phase relationships between voltages across different elements in an LCR circuit; oscillations in tuned circuits.

Sound: demonstrations of the wave forms of pure and impure tones; beats.

Simple harmonic motion: addition of two sine curves to show amplitude modulation (beats) and Lissajous figures, measurement of phase and frequency; Fourier synthesis.

Electronics: display of the function and characteristics of devices, such as diodes, transistors, and vacuum tubes.

Time measurements: time-of-flight measurement of projectiles; pulse of sound; display of pulses from Geiger counter.

The CRO can also be used to set up some very effective attention-getting displays, corridor demonstrations, science fair projects, and so on. Examples of interesting traces are given in Figs. 1, 2, and 3.

Fig. 1

Fig. 2

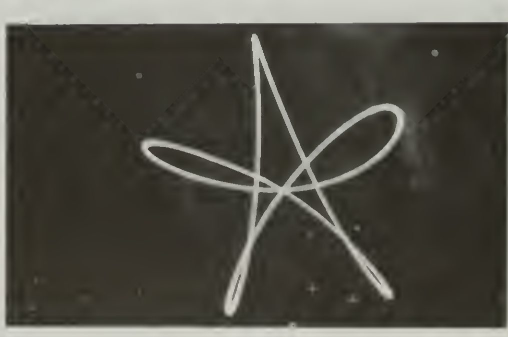


Fig. 3

OPERATION

These notes are necessarily of a very general nature. Refer to the manufacturer's instruction manual for detailed notes on the operation of a particular oscilloscope.

The ON/OFF switch is often combined with the INTENSITY control. Wait a minute or two after turning on before trying to get a trace on the screen. Adjust intensity and the FOCUS knob until a bright, sharp spot or line is obtained. Its position on the screen can be varied by means of the VERT(ICAL) POS(ITION) and HOR(IZONTAL) POS(ITION) controls. It is bad practice to leave the 'scope turned on with a high-intensity stationary spot since it may burn a hole in the phosphor coating of the screen.

Vertical deflection: voltage measurement

The cathode-ray tube itself is sensitive to both dc and ac voltages, but its sensitivity (displacement of the spot per volt of potential difference between the deflecting plates) is low—typically of the order of 0.2 mm/V. Amplifiers are therefore added to increase the sensitivity. In most simple oscilloscopes these are ac amplifiers, and so these oscilloscopes cannot be used for dc signals.* A dc oscilloscope has a dc/ac switch that must be set to the appropriate position.

A voltage signal to be measured is applied between the VERT INPUT terminal and GROUND terminal. (Make sure that the connection to ground is consistent with the circuit or device providing the signal: that is, beware of "crossed grounds.") Voltages applied here deflect the beam up and down on the screen. This terminal is sometimes referred to as the Y INPUT, and the deflection as Y DEFLECTION. The amplification of this signal is controlled by two knobs that may be called VERT INPUT, VERT GAIN, VERT ATTENUATOR, VERT AMPLIFIER, etc. Usually one knob provides coarse control in three or more steps (1×, 10×, 100×) and the other gives fine control. In more expensive oscilloscopes these controls are calibrated in volts per centimeter deflection of the spot on the screen. With simpler 'scopes it is necessary to calibrate the sensitivity at a given setting by applying a signal of known voltage and measuring the deflection. Such a calibrating signal may be provided at one of the terminals on the 'scope itself. On the Heath IO–12, for example, the 1-V P–P terminal provides a 60-cycle signal with a peak-to-peak amplitude of 1 V. (Note that the legends 100×, 10×, 1× may refer to how much the input signal is attenuated rather than to how much it is amplified, so the 1× is the range of highest sensitivity.)

Pickup

Sometimes you may find that a signal is seen on the oscilloscope face even if no obvious voltage is applied to the oscilloscope input. To see some of the characteristics of this "pickup," try the following procedure. Set the FREQ SELECTOR to about 10 sec, and turn up the VERT GAIN to the maximum setting. Attach one end of a short length of wire to the VERT INPUT, leaving the other end unconnected. An approximately 60-cycle sinusoidal trace will appear on the oscilloscope. Its amplitude increases if you touch the end of the wire, or if you use a longer piece of wire.

The wire is acting as an antenna and is picking up the 60-cycle electromagnetic field that exists, to a greater or lesser extent, in the vicinity of any 60-cycle current. The field is particularly strong near transformers, fluorescent lamps, etc. Although the ac voltage due to the varying field is small, the large amplification and high input impedance of the oscilloscope can result in an appreciable trace amplitude.

Connect a resistor ($R \sim 1$ megohm) between the "antenna" and the ground terminal of the CRO. The amplitude of the signal decreases, but is still appreciable. If the value of R is decreased, the pick up becomes smaller.

Because of this spurious "pick-up" signal, shielded cable must be used to connect the CRO to high-impedance, low-voltage sources. The same considerations apply to all high-gain amplifiers. Note that the phototube supplied by Damon ($R \sim 5$ megohms) is mounted in a grounded metal box, and shielded cable is used to connect it to the amplifier.

Horizontal deflection: measurement of time and frequency

With the HOR FREQ SELECTOR (or SWEEP SELECTOR) set to EXT(ERNAL), a signal applied to the HORIZ INPUT (X input) terminals causes the beam to move left or right across the tube face (horizontal or X deflection). As with the VERT INPUT, the signal is amplified and except on dc oscilloscopes a steady (dc) voltage does not produce a deflection. The amplification is controlled by the HOR(IZONTAL) GAIN or HOR(IZONTAL) AMP(LITUDE) knob.

When the FREQ SELECTOR or SWEEP SELECTOR is in the LINE SWEEP position, a 60 cycle-per-sec sinusoidal voltage is applied to the horizontal deflection plates. If there is no vertical deflection, the spot will move back and forth across the screen in simple harmonic motion. Note that the deflection is <u>not</u> linear with time in this setting. If another sinusoidal voltage is applied to the vertical deflection plates, the resultant motion of the spot will be the combination of two perpendicular SHM's, i.e., straight line, circle, ellipse, Lissajous figure, depending on the relative amplitude, phase, and frequency of the two signals.

With the SWEEP SELECTOR at LINE SWEEP, the PHASE knob is used to shift the phase of the sweep voltage with respect to the input signal. The traces shown in Figs. 4 and 5 were both made with the selector on LINE SWEEP and 60-cps signal on the vertical plate. The phase is shifted 90° between Fig. 4 and Fig. 5.

For other settings of the HOR FREQ SELECTOR or SWEEP SELECTOR control, an internal circuit applies a varying voltage to the plates that control the horizontal position of the spot. This voltage has a saw-tooth wave form: . The spot moves across the screen from left to right at a uniform rate while the voltage is increasing and very rapidly flies back to its starting position when the voltage drops to its minimum value. In this setting, deflection *is* linear with time. (Automatic "retrace

*In ac oscilloscopes it is sometimes possible to bypass the amplifier and apply a signal directly to the tube, thus getting a deflection for a dc input. This usually involves removing a panel at the back or side of the instrument to expose the appropriate terminals. PROCEED WITH GREAT CAUTION: THESE TERMINALS MAY BE AT VOLTAGES AS HIGH AS 1500 V. Be sure to unplug the instrument before you expose the terminals. Because of the low sensitivity of the cathode-ray tube itself the deflection will probably be small.

blanking" reduces the intensity of the spot so that it is not seen as it flies back to the left of the screen.) The sweep frequency is controlled by two knobs. The HOR/FREQ SELECTOR, or SWEEP SELECTOR, provides coarse control in steps. Typically, one setting will cover a "decade" of frequencies, for example 10–100, 100–1000, etc., cycles/sec. The FREQ VERNIER or SWEEP VERNIER gives fine control within these ranges.

Fig. 4

Fig. 5

On expensive oscilloscopes, these controls may already be calibrated. Otherwise, they can be calibrated by the following procedure. A signal of known frequency is applied to the VERT INPUT. By counting the number of cycles of the known-frequency signal on the trace, one can establish the sweep frequency. That is, if there are exactly n cycles of a 60 cycle-per-second signal on the trace, then the sweep frequency is $\frac{1}{n}$ 60/sec (see Fig. 6.). For low sweep rates, a 60-cycle signal can be used. On the Heath oscilloscope IO–12, changing the FREQ SELECTOR one step will change the sweep rate by approximately a factor of 10, for instance from 20/sec to 200/sec. For more accurate calibration at higher sweep rates, use a calibrated audio oscillator (signal generator) to provide a signal of known frequency. Some expensive oscilloscopes have a built-in oscillator that can be used to apply known frequency signals to the vertical deflection plates. Others have an output terminal that gives a 1-V 60-cycle signal.

The length of the trace is controlled by the HOR GAIN knob; this does not affect the sweep frequency (number of sweeps per second)—one full sweep still represents the same time interval—but it does, of course, change the sweep rate (cm/sec), and 1 cm will represent a different time interval.

Synchronization of the horizontal sweep frequency with the signal applied to the vertical input is important. If the two are synchronized, then the same pattern will be repeated for successive sweeps, and what appears to be a stationary trace will be obtained on the screen (as in Fig. 6).

Fig. 6

If the signal and sweep frequencies are not synchronized, then the traces obtained for successive sweeps of the screen will not coincide (Fig. 7).

Fig. 7

Synchronization is achieved by fine adjustment of the FREQ VERNIER control until the sweep frequency is an exact fraction of the signal frequency. By setting the SYNC SELECTOR to INT+ or INT− the start of the sweep can be synchronized with either the positive or negative slope of the input signal (Figs. 8a, b).

The sweep can also be synchronized with a signal applied to the EXT(ERNAL) SYNC terminal, by setting the SYNC SELECTOR knob to EXT SYNC. Adjust the EXT SYNC AMPLITUDE control until the sweep is synchronized with the signal. The EXT SYNC amplitude setting has no effect unless the SYNC SELECTOR is set to EXT SYNC.

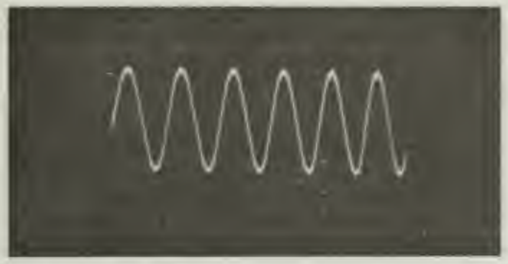


Fig. 8a

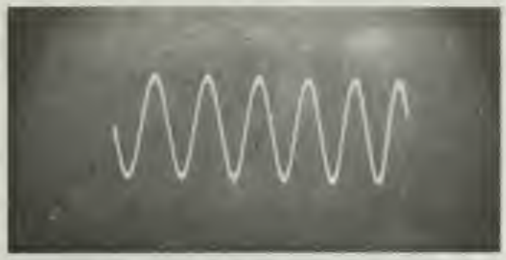


Fig. 8b

There may also be a "LINE" setting of the SYNC SELECTOR. In this position, the horizontal sweep is synchronized with the (60-cycle) line frequency.

A useful feature present on some oscilloscopes is a "trigger." (The Heath IO–12 does not have this feature.) The horizontal sweep can be triggered by a signal applied from an external circuit to the trigger input. Until the triggering signal is applied, the spot remains stationary. This is particularly useful, for example, in time-of-flight measurements. If one wants to measure the time interval between two signals (such as, the interruption of two beams of light to two photocells), it is desirable (though not essential) that the two signal pulses occur on the same horizontal sweep. This can be achieved by triggering the sweep on the rise of the first signal pulse. If the CRO has no trigger facility, then it may happen that the first signal will occur toward the end of one sweep and the second signal will occur on the next sweep. This makes measurement of the time interval between the signals difficult.

Some, but by no means all, oscilloscopes have a two-beam display: there are two Y inputs and it is possible to apply different signals to the two beams. This makes it very easy to compare the amplitudes and frequencies of two different signals. In reality, there is only one electron beam that is switched up and down so rapidly that two apparently continuous beams are seen (Fig. 9). The sweep rate for the two beams must be the same, but the amplifications of the two signals can be adjusted independently.

On a "two-beam oscilloscope" the switching is done internally. External switching circuits are available that enable one to make a two-beam display of two independent signals on a regular oscilloscope (Heathkit Electronic Switch ID–22, unassembled).

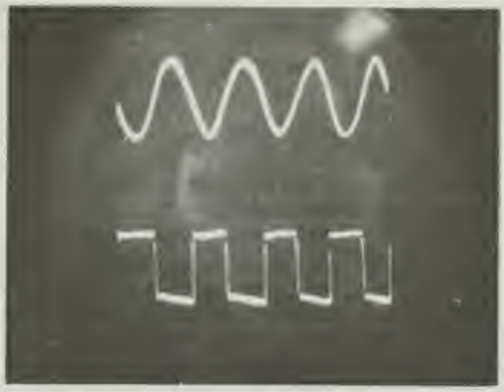


Fig. 9

Intensity modulation (Z modulation)

Some oscilloscopes have an input terminal that is connected (through a capacitor) to the intensity-control grid of the cathode-ray tube. This terminal is commonly marked Z-AXIS. On some oscilloscopes it is necessary to remove the back panel to uncover this terminal. Always unplug the oscilloscope before removing the panel.

The potential of the grid (with respect to the cathode) controls the intensity of the electron beam, and hence the brightness of the spot or trace on the screen. It is this grid potential that is adjusted by the INT(ENSITY) control knob. If the grid is made more positive, the spot becomes brighter. If a varying voltage is applied to the grid, the beam intensity will be modulated at the frequency of the applied signal. Typically, about 10 V is required for complete blanking of the trace.

Intensity modulation may be used to provide accurate time markers on the trace. The same intensity modulation, by the way, creates the light and dark areas in the picture on a TV screen.

PHOTOGRAPHY OF CRO TRACES

The use of fast (3000-speed) Polaroid film makes it possible to photograph the trace. A close-up auxiliary lens to give an approximately 1:1 object-to-image ratio is necessary. If possible, remove the camera back and insert a ground-glass or other focusing screen in the plane of the film. With the sutter open, adjust the camera position to sharp focus. A rigid support for the camera is needed, of course. Turn up the oscilloscope intensity control until a bright trace is obtained. However, if the intensity is increased too far on some oscilloscopes, the whole screen may begin to glow faintly and there will be a loss of contrast. If there is a colored screen or filter mounted in front of the oscilloscope face, removing the screen or filter may improve the image. Background illumination should be low, but it is certainly not necessary to work in a darkroom.

The appropriate aperture and time settings can quickly be found by trial and error. Don't forget that if the shutter speed is faster than the sweep rate, only part of the trace will be photographed; for example, at 1/50 sec exposure, you cannot photograph a complete 1/30-sec trace.

The photographs used to illustrate this note were taken with the modified model 210 Polaroid Land camera, using the clip-on auxiliary lens.

EXAMPLES OF THE USE OF A CRO IN TEACHING *PROJECT PHYSICS*

1. The ultrasound transducers used in the wave experiments in Unit 3 have a very sharp resonance at 40 kilocycles. Before attempting any experiment with them, the oscillator driving the source transducer must be carefully tuned to the resonant frequency. Set up the equipment as shown in Fig. 10, with the receiver transducer a few centimeters in front of the source. Set the CRO to a sweep rate of about 10 kilocycles/sec. Slowly adjust the frequency control on the audio oscillator until the trace on the oscilloscope screen "peaks" to a maximum signal.

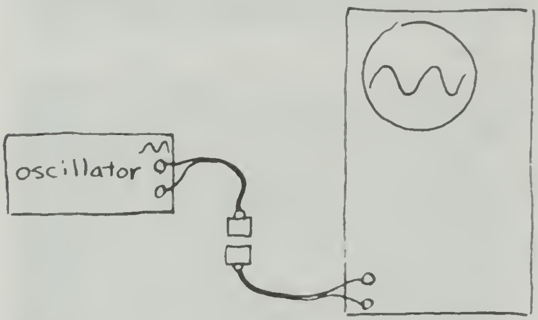


Fig. 10

In the experiments themselves, the amplitude of the trace on the oscilloscope screen is used to estimate the effectiveness of various materials as reflectors and absorbers of ultrasound, and to locate the positions of nodes (zero amplitude) and antinodes (maximum amplitude) in various interference and standing-wave patterns.

2. The oscilloscope is used as a current meter (or, more accurately, as a null detector) in the investigation of the photoelectric effect, one of the Unit 5 experiments. The output of the phototube is fed (via an external amplifier) to the oscilloscope. As the counterpotential across the phototube is increased, the photocurrent, and thus the amplitude of the trace on the CRO, decreases. The experiment consists of finding what "stopping voltage" is needed to reduce the photocurrent to zero for light of different frequencies.

3. Although these functions are not included in *Project Physics*, the CRO can be used to make quantitative measurements of ac voltages and currents. Comparison of the peak-to-peak voltage with the reading given by an ac voltmeter, which is the root-mean-square voltage $\left(\frac{1}{2}V_{p-p} = \frac{V_{RMS}}{\sqrt{2}}\right)$, is also possible.

To measure ac current, connect the oscilloscope across a known resistance (noninductive) and use $I = V/R$ to calculate current.

Wave-form display

The CRO can be used to show the difference between sinusoidal and square waves, to show half- and full-wave rectification of ac and the effect of a smoothing capacitor. It can also be used to show the wave forms of the sounds produced by various musical instruments, or by students' voices. (Use a microphone as a detector. A small speaker can also be used as a microphone; amplification may be necessary.)

Fig. 11 Recorder plays a high C

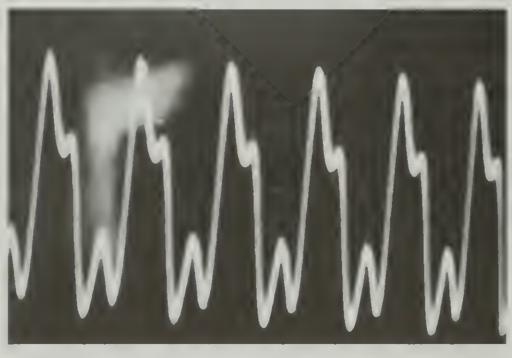


Fig. 12 Harmonica plays a high C

Two or more audio oscillators can be set to fundamental and harmonic frequencies to synthesize tones approaching those of various musical instruments. The higher frequencies must be set to exact multiples of the fundamental to get a stable trace.

To demonstrate the formation of beats, set the two oscillators to frequencies that are only slightly different.

Specific examples of wave-form display in *Project Physics* work:

1. To show that the electron-beam tube, like a vacuum diode, is a rectifier. Even if an ac voltage is applied between filament and plate, the current is dc (half-wave rectified), corresponding to electrons moving from filament to plate. (A two-beam display would be useful here.)

2. To show the action of the transistor switch

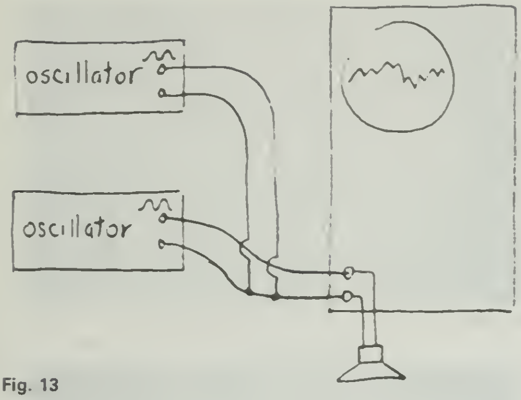


Fig. 13

used in various standing-wave demonstrations in Units 4 and 5.

3. To show the damped oscillations in an *LCR* circuit (Demonstration on Induction, Resonance—Unit 4).

4. To "see" the signal broadcast by a radio station (Demonstration on Induction, Resonance—Unit 4).

Time measurements

(a) Timing moving objects

Use two photocells in series and two light beams. The phototube units (PV100) and light sources from the Millikan equipment supplied by Damon can be used. Notice that no voltage supply is needed for the phototubes. This arrangement can be used to time falling objects, bullets, etc. Sweep rate must be known, of course. A rough idea of the speed of the object will make it easier to choose a suitable sweep rate and distance between photocells.

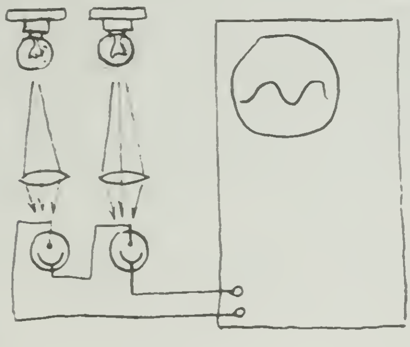


Fig. 14

In some situations, the photocell can be replaced by a simple switch that is momentarily closed by the object as it moves past. (For instance, a steel ball making contact between two pieces of aluminum foil as it passes.)

Some care is needed in interpreting the trace. The signals from the two phototubes (or switches) will have slightly different shapes due to differences in illumination. Establish which signal is from which tube by intercepting first one light beam, then the other. Now examine carefully the trace record obtained when the moving object crosses both light beams or switches. If the signal due to the interruption of the first light beam occurs toward the beginning of the sweep and is followed by the second signal, then there is no special problem. In this case the distance between the two signals represents the time between the two events. But it can happen that the *first* signal occurs toward the *end* of one sweep and the *second* signal occurs on the *next* trace. In this case the sum of the distance from the first signal to the end of the trace, plus the distance from the beginning of the trace to the second signal, represents the time interval between the events.

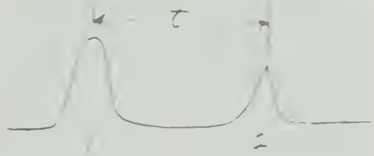


Fig. 15a

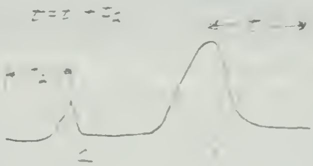


Fig. 15b

If a triggered sweep circuit is available, this complication need not occur. The triggering circuit can be used to start the sweep just as the object crosses the first beam, or closes the first switch.

(b) Stroboscope calibration

A 60-cycle signal is used as a reference. See the notes on calibration of a xenon stroboscope in this *Resource Book.*

Frequency measurements

The precision of a frequency measurement depends upon the accuracy of the reference source available. If the unknown frequency is a simple multiple or submultiple of 60 cycles, then 60-cycle line frequency, which is usually very closely controlled, can be used.

Set the HORIZ SELECTOR to LINE SWEEP and the SYNC SELECTOR to LINE, or apply a 60-cycle signal from a stepdown transformer to the HORIZ INPUT.

Apply the signal whose frequency is to be measured to the VERT INPUT. Adjust the HORIZ and VERT gains if necessary. Figures 16 through 19 are typical of the patterns that can be obtained. They are called Lissajous figures.

Only if there is a simple whole number ratio between f_{vert} and f_{horiz} will stationary figures of this type be obtained.

Fig. 16

The pattern observed depends on the relative phase of the two signals as well as their frequency ratio. The circle shown in Fig. 16 is obtained from two perpendicular sinusoidal signals 90° ($\pi/2$ radians) out of phase. If the phase difference between the two signals is 0° or 180° (π radians), the resultant trace will be a straight line. Intermediate values of phase difference will give ellipses. The PHASE knob can be used to vary the phase difference between the signal applied to the Y plates and the line frequency sweep.

If the two frequencies are not equal, then the phase difference will vary continuously. The trace will change from straight line (0°) to ellipse (45°) to circle (90°) to ellipse (135°) to a straight line perpendicular to the original one (180°), and through ellipse, circle, ellipse, back to the original straight line. The frequency at which this change occurs is equal to the frequency difference between the two signals. For example, if the two signals are 60 and 61 cycles, the trace pattern will go through one complete cycle of transformations in 1 sec.

This technique can be used to calibrate an oscillator against a 60-cycle signal, at frequencies that are, or are very close to, multiples or submultiples of 60/sec. But if the problem is to measure a frequency that does not happen to be equal or close to a multiple or submultiple of 60 cycles, then this method cannot be used. Instead, it is necessary to use a variable-frequency oscillator whose calibration is accurately known as a reference.

Fig. 17

Fig. 18

Fig. 19

Demonstrations of complex motions

In the previous section on frequency measurement by means of Lissajous figures, two independent signals were applied: one to the Y and one to the X input.

It is also possible to produce circular and elliptical traces using only one ac voltage by making use of the fact that in an *RC* circuit there is a 90° phase difference between the voltage across the resistor and the voltage across the capacitor. (See Fig. 20.)

Note that the midpoint of the *RC* circuit is connected to the ground terminal of the oscilloscope:

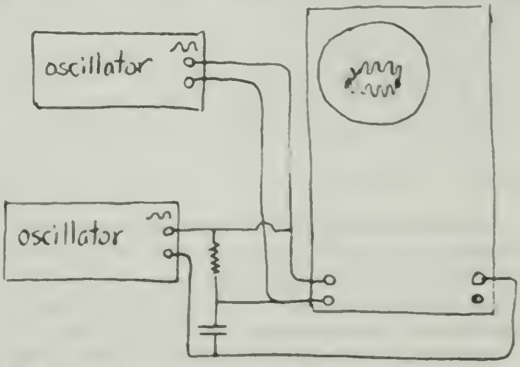


Fig. 20 The HORIZ SELECTOR is set to EXT.

It is important that neither output terminal of the oscillator be grounded. If both the oscilloscope and the oscillator are connected to the line by a three-wire cable and three-pin plug, you may have to use a three-to-two adapter plug to isolate the oscillator from the ground.

The trace will be a circle or an ellipse, depending on the two voltages and the horizontal and vertical gains.

Suitable values of R and C for 1000 cycles/sec are 1000 ohms and 0.1 microfarads. Note that as the frequency is increased the impedance of the capacitor drops, the voltage-drop across it drops, and what was a circle becomes elliptical.

More complex patterns can be made if two oscillators are available. For example, the trace shown in Fig. 3 at the beginning of this article was produced by the circuit shown in Fig. 21. (The HORIZ SELECTOR is set to EXT.)

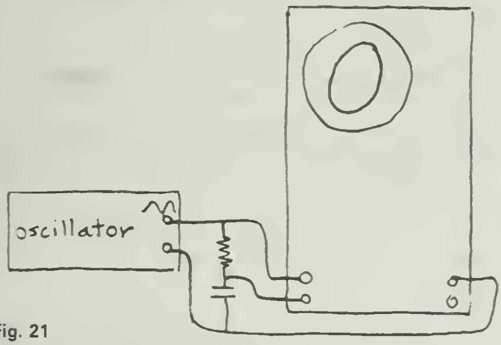


Fig. 21

Set up a circular or elliptical trace as described above, at, say, 60 cycles. Then apply a higher frequency, say, 1,200 cycles, sine- or square-wave voltage to the VERT INPUT.

Intensity modulation

1. Time markers. Set up circular, elliptical and epicycle traces, as described in the section "Demonstrations of Complex Motions." Apply a sinusoidal signal to the Z-axis to provide intensity modulation at a frequency that is at least 10 times higher than the frequencies applied to the X and Y inputs in order to provide at least ten time markers per cycle. This modulation frequency must be adjusted carefully. Only when it is an exact multiple of the trace frequency will a stationary pattern be obtained. These time markers show that the spot moves around the circle with constant speed. In the ellipse it moves most quickly when it is close to the center; but note that this motion, unlike planetary motion, is symmetrical about the center of the ellipse. (So in this case the "equal areas" law can be applied to the motion about the center, not to motion relative to a focus. The same is true for the motion of a conical pendulum.)

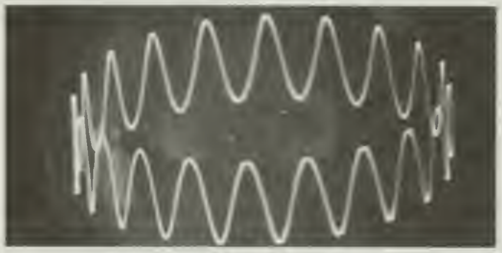


Fig. 22

Fig. 23

2. "Television." Set the HORIZ FREQ to about 15 kilocycles. Apply a 60-cycle sinusoidal signal of a few volts peak to peak to the Y input. This can be from an audio oscillator, a step-down transformer, or the 60-cycle calibration signal provided by the oscilloscope itself. Adjust the horizontal and vertical gain to obtain a square or rectangular area that fills most of the tube face.

Apply an ac voltage of about 10 V peak to peak (for instance, from the *Project Physics* oscillator unit) to the Z axis. If the frequency of this signal is a few times greater than the sweep frequency, a sweep pattern of vertical bars will be formed as the trace is blanked out several times in each sweep. If the modulation frequency is several times less than the sweep frequency, then a pattern of horizontal stripes is formed. Stabilize the pattern by setting the SYNC SELECTOR to EXT, connecting the EXT SYNC terminal to the Z-axis oscillator, and turning up the EXT SYNC AMP control until the pattern "freezes."

With two oscillators at different frequencies, it is possible to combine the vertical bars and the horizontal stripes to form a checkerboard pattern.

Suggested Solutions to Study Guide Problems

CHAPTER 1

2. *Speed* is the ratio of a change in distance to the time taken by the moving body to effect the change. Symbolically, if:

$d_1 - d_0 = \Delta d$

$t_1 - t_0 = \Delta t$

$v = \text{speed}$

then $v = \dfrac{\Delta d}{\Delta t}$

Uniform motion: If the ratio of change in distance to time taken is constant for some successive intervals, regardless of how close the intervals are, the moving body is said to move with uniform motion.

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$v_{av} = \frac{d_{total}}{t_{total}}$$

$$\text{Slope} = \frac{\text{change in vertical distance } (\Delta y)}{\text{change in horizontal distance } (\Delta x)}$$

Let change in vertical distance $= y_2 - y_1 = \Delta y$
Change in horizontal distance $= x_2 - x_1 = \Delta x$

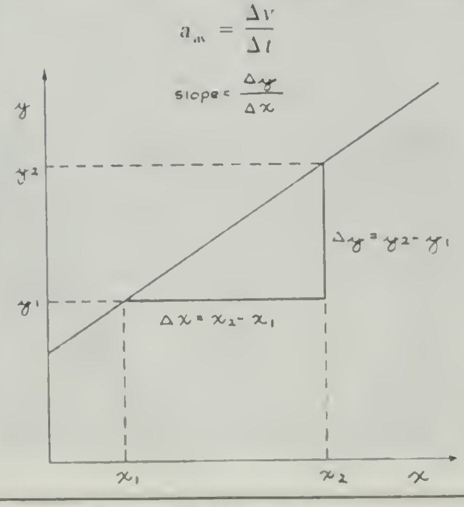
Then slope is denoted by $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$

Instantaneous speed is the rate of change of distance at a particular instant.

$v_{inst} = \dfrac{\Delta d}{\Delta t}$, as Δt becomes very small.

$$\text{average aceleration} = \frac{\text{change in velocity}}{\text{time taken for change}}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$



3. distance $= d = 30$ m

time $= t = 1.5$ sec

$$\text{average speed} = v_{av} = \frac{d_{total}}{t_{total}} = \frac{30 \text{ m}}{1.5 \text{ sec}} =$$

20 m/sec

4. (a) $\text{speed} = \dfrac{\text{distance traveled}}{\text{time elapsed}} = \dfrac{72 \text{ cm}}{12 \text{ sec}}$

$= 6.0$ cm/sec

(b) $\text{distance traveled} = 60 \dfrac{\text{km}}{\text{hr}} \times \dfrac{20 \text{ min}}{60 \text{ min/hr}}$

$= 20$ km

(c) $\text{time elapsed} = \dfrac{9.0 \text{ m}}{36 \text{ m/min}} = 0.25 \text{ min}$

(d) The data given indicate that the speed was probably uniform at 3 cm/sec; so at 8 sec we expect the speed to be 3 cm/sec and the position to be 24 cm.

(e) $v_{av} = \dfrac{\Delta d}{\Delta t} = \dfrac{240 \text{ km}}{6 \text{ hr}} = 40 \text{ km/hr}$

(f) On a trip as long as this it would, no doubt, be impossible to maintain a constant speed so the best we can do is estimate the position to be about 120 km on the assumption that the entire trip was made at 40 km/hr.

(g) $\Delta t = \dfrac{\Delta d}{v_{av}} = \dfrac{418 \text{ cm}}{76 \text{ cm/sec}} = 5.5 \text{ sec}$

(h) $\Delta d = v_{av} \Delta t = 44 \dfrac{\text{m}}{\text{sec}} \times 0.20 \text{ sec} = 8.8 \text{ m}$

5. If the diver falls with uniform speed, it means that the effect of gravity has been counterbalanced by air resistance. Therefore, acceleration due to gravity will not be considered while the diver falls.

$v = 12$ m/sec
distance of fall $= d_f = 228$ m

$$\text{time of fall} = t_f = \frac{d_f}{v} = \frac{228 \text{ m}}{12 \text{ m/sec}} = 19 \text{ sec}$$

Additional time of fall $= t_s = 25$ sec
Additional distance fallen $= d_s = vt_s = (12$ m/sec$)(25$ sec$) = 300$ m

Total distance fallen $= d_{total} = d_f + d_s$

$= (228 \text{ m} + 300 \text{ m})$

$= 528$ m

6. (a) First, we need to find the total time taken Δt

$$\Delta t = \Delta t_1 + \Delta t_2$$
$$= \frac{\Delta d_1}{v_1} + \frac{\Delta d_2}{v_2}$$
$$= \frac{100\ m}{5.0\ m/sec} + \frac{100\ m}{1.0\ m/sec} = 120\ sec$$
$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{200\ m}{120\ sec} = 1.7\ m/sec$$

(b) $\Delta d = \Delta d_1 + \Delta d_2$

$$= 5\ m/sec \times 100\ sec + 1.0\ m/sec \times 100\ sec$$
$$= 500\ m + 100\ m = 600\ m$$
$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{600\ m}{200\ sec} = 3\ m/sec$$

Note: In one case, the average speed is the average of the individual speeds; in the other case it is not.

7. $d_{practice} = 30$ m
practice time for rabbit $= t_{rabbit} = 5$ sec
practice time for turtle $= t_{turtle} = 120$ sec
practice velocity for rabbit $= \frac{d_p}{t_{rabbit}} = \frac{30\ m}{5\ sec} =$ 6 m/sec
practice velocity for turtle $= \frac{d_p}{d_{turtle}}$
$$= \frac{30\ m}{120\ sec} = 0.25\ m/sec$$
Total distance for race $= d_R = 96$ m
Average velocity of rabbit = 6 m/sec
Therefore, total time used by rabbit $= \frac{d}{v_r} = \frac{96\ m}{6\ m/sec} = 16$ sec
Total time used by turtle $= \frac{d}{v_t}$
$$= \frac{96\ m}{0.25\ m/sec} = 384\ sec$$
Difference in time = (384 sec − 16 sec) = 368 sec

8. Δt = 4 hr 34 min = 4.57 hr
$\Delta d = v_{av}\Delta t = 790\ km/hr \times 4.57\ hr = 3.6 \times 10^3$ km

9. (a) Compare the speed of the ball with the speed of a runner from third base. (The times are about the same.) A runner does 90 m in 10 sec or 9 m/sec, so it will take about 3 sec to run the 27 m from third base to home plate. The distance to the outfield is about 90 m so the speed of the ball will be about 30 m/sec.

(b) 1) Determine the speed of a leaf, for example, being blown by the wind.
2) Hang a plumb line from the center of a protractor and calibrate this by holding it out the window of a car on a calm day. Notice how far away from the vertical it is displaced as you travel at various speeds. If, then, you find the same displacement on a windy day when the instrument is held fixed, the wind must have the corresponding speed.

(c) Sight on an edge of the cloud and determine the time required to move through a certain angle. Assuming the cloud to be 3–5 km above the ground, you can estimate the actual distance moved from a scale diagram involving the height and the measured angle.

10. (a) It is necessary to determine first the number of seconds in a year:

Δt = 365 day/yr × 24 hr/day × 60 min/hr × 60 sec/min = 3.18×10^7 sec/yr
$\Delta d = 3.0 \times 10^8\ m/sec \times 3.18 \times 10^7\ sec = 9.5 \times 10^{15}$ m

(b) Total distance to and from Alpha Centauri $= 8.12 \times 10^{16}$ m

$$\Delta t = \frac{8.12 \times 10^{16}\ m}{3.0 \times 10^8\ m/sec} = 2.7 \times 10^8\ sec$$
or
$$= \frac{2.7 \times 10^8\ sec}{3.18 \times 10^7\ sec/yr}$$
$$= 8.5\ yr$$

(c) One major problem is the short time intervals because of the high speed. Use a light pulse reflected from a known distance to excite a photocell. Amplify output of cell to excite the lamp giving light pulse. Measure high frequency of pulses. The light beam couples the lamp to the cell. If the action of lamp and cell takes appreciable time, note change in frequency as reflecting mirror is moved a known distance.

(d) Confine the ant to a definite path.

(e) Analyze the length of the trace on film left by a fast-moving bright light. (You need to determine the scale of the picture.)

(f) A high-speed motion picture camera could give the duration of the blink. Another way would be to bounce a beam of light off the eyeball into a photocell whose output is amplified and connected to an oscilloscope.

(g) Determine how many hours it takes the whisker to grow to a measurable length.

11. $d_{total} = 500$ m
Let t_b be the time taken by the blue bicycle and t_r the time for the red bicycle to finish the race.
$t_r = (t_b + 20\ sec)$
speed of red bicycle $= v_r = 10$ m/sec
$$t_r = (t_b + 20\ sec) = \frac{d_{total}}{v_r} = \frac{500\ m}{10\ m/sec}$$
$$= 50\ sec$$

Therefore, $t_b = (50\text{ sec} - 20\text{ sec}) = 30\text{ sec}$

$$\text{average speed of blue bicycle} = v_b = \frac{d}{t_b}$$

$$= \frac{500\text{ m}}{30\text{ sec}} = 16.7\text{ m/sec}$$

12. (a) Initial speed $= v_i = 0$
Speed after 5 sec $= v_5 = 30$ m/sec
Time taken $= t = 5$ sec

$$\text{Average acceleration} = a_{av} = \frac{v_f - v_i}{t}$$

$$= \frac{(30\text{ m/sec} - 0\text{ m/sec})}{5\text{ sec}} = 6\text{ m/sec}^2$$

(b) If $v_i = 0$
$a_{av} = 6\text{ m/sec}^2$
total time taken $= t = 10$ sec

$$\text{Using } a_{av} = \frac{v_f - v_i}{t};\ v_f = (a_{av}\, t) + v_i$$

$$v_f = (6\text{ m/sec}^2)(10\text{ sec}) + 0 = 60\text{ m/sec}$$

13. (a) (1) Average speed from starting line (0 sec) to 6 sec is given by $v_{av} = \dfrac{\Delta d}{\Delta t} = \dfrac{(30\text{ m} - 0\text{ m})}{(6\text{ sec} - 0\text{ sec})} = 5\text{ m/sec}$

(2) Average speed from starting line to 10 sec is given by $v_{av} = \dfrac{\Delta d}{\Delta t} = \dfrac{(50\text{ m} - 0\text{ m})}{(10\text{ sec} - 0\text{ sec})} = 5\text{ m/sec}$

(3) Average speed from 6 sec to 10 sec is given by $v_{av} = \dfrac{\Delta d}{\Delta t} = \dfrac{(50\text{ m} - 30\text{ m})}{(10\text{ sec} - 6\text{ sec})}$
$= 5$ m/sec

(4) Average speed from 5 sec to 8 sec is given by
$v_{av} =$
$\dfrac{\Delta d}{\Delta t} = \dfrac{(40\text{ m} - 25\text{ m})}{(8\text{ sec} - 5\text{ sec})} = 5\text{ m/sec}$

(b) A straight line obtained from a distance–time graph indicates uniform speed. Hence, no matter how small the interval chosen, the same speed will be found.

(c) Since the speed is uniform, the instantaneous speed will be 5 m/sec. The speed line (distance versus time) has the same slope at all times.

14. (a) The slope of the line is proportional to the speed. By looking at the graph, one observes that the ball traveled fastest in section CD and slowest in section BC.

(b) Average speed between A and B

$$v_{AB} = \frac{175\text{ m} - 0\text{ m}}{4\text{ sec} - 0\text{ sec}} = 44\text{ m/sec}$$

Average speed between B and C

$$v_{BC} = \frac{200\text{ m} - 175\text{ m}}{11\text{ sec} - 4\text{ sec}} = \frac{25\text{ m}}{7\text{ sec}} = 3.6\text{ m/sec}$$

Average speed between C and D

$$v_{CD} = \frac{500\text{ m} - 200\text{ m}}{15\text{ sec} - 11\text{ sec}} = \frac{300\text{ m}}{4\text{ sec}}$$

$$= 75\text{ m/sec}$$

(b) Average speed between A and D

$$v_{AD} = \frac{500\text{ m} - 0\text{ m}}{15\text{ sec} - 0\text{ sec}} = 33.3\text{ m/sec}$$

(c) Instantaneous speed at point f in section CD = 75 m/sec as for the section CD.

15. (a) $v_{inst} = \dfrac{\Delta d}{\Delta t}$

$$= \frac{30\text{ m} - 25\text{ m}}{15\text{ sec} - 5\text{ sec}}$$

$= 0.5$ m/sec (at 10-sec mark)

$$v_{inst} = \frac{50\text{ m} - 35\text{ m}}{30\text{ sec} - 20\text{ sec}}$$

$= 1.5$ m/sec (at 25-sec mark)

(b) $a_{av} = \dfrac{\Delta v}{\Delta t}$

$$= \frac{1.5\text{ m/sec} - 0.5\text{ m/sec}}{25\text{ sec} - 10\text{ sec}}$$

$= 0.06\text{ m/sec}^2$

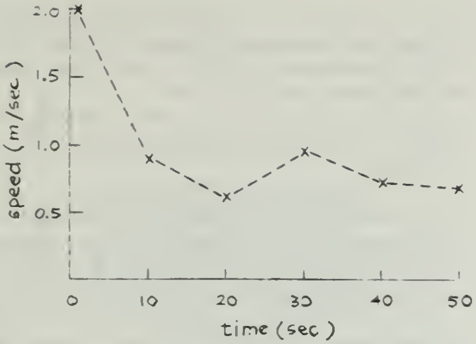
16. Randall wins over Weissmuller by 19 sec. Weissmuller's speed is $\dfrac{400.0\text{ m}}{297\text{ sec}}$ so in 19 sec he would still have $\dfrac{400 \times 19.0}{297} = 25.6$ m to go.

In making the graph for purposes of extrapolation, plot the number of seconds above 4 min versus dates.

17.

t	v
0 sec	2.0 m/sec
10	0.75
20	0.57
30	0.85
40	0.63
50	0.57

She was swimming fastest at the beginning, then slowing down but with a spurt at about 30 sec. Note: The actual values for v obtained by each student will show wide variation due to difficulty in estimating intervals on the graph.



18. The error, which Mark Twain was fully aware of, was in assuming that physical processes remain unchanged. In this case, as in so many, one effect of a phenomenon (shortening of the river) is to change the rate at which that phenomenon takes place. Also, it often happens that other events (e.g., geologic uplifting) can alter the circumstances enough to radically alter the phenomenon of interest. Since 176 years is miniscule compared to geologic time, many such events are likely to have occurred, making Twain's extrapolation as erroneous as it is humorous.

19. (a) The speed was greatest in the interval from 1 to 4.5 sec.

$$v = \frac{\Delta d}{\Delta t} = \frac{5.4 \text{ m} - 1.0 \text{ m}}{4.5 \text{ sec} - 1 \text{ sec}} = \frac{4.4 \text{ m}}{3.5 \text{ sec}} = 1.3 \text{ m/sec}$$

(b) The speed was least in the interval from 6 to 10 sec.

$$v = \frac{6.7 \text{ m} - 6.2 \text{ m}}{10 \text{ sec} - 6 \text{ sec}} = \frac{0.5 \text{ m}}{4.0 \text{ sec}} = 0.13 \text{ m/sec}$$

(c) A tangent drawn to the curve at $t = 5$ sec can be made the hypotenuse of a triangle with legs $\Delta d = 6$ m; $\Delta t = 8$ sec, thus, $v = 0.75$ m/sec.

(d) A similarly drawn triangle at $t = 0.5$ sec gives

$$\frac{\Delta d}{\Delta t} = \frac{8 \text{ m}}{8 \text{ sec}} = 1 \text{ m/sec}$$

(e) Reading directly from the graph, distance $= 6.6 \text{ m} - 6.2 \text{ m} = 0.4$ m. Using the value for the speed determined in (b), the distance $= 0.13 \text{ m/sec} \times 2.5 \text{ sec} = 0.33$ m.

Which method do you think is more precise?

20. (a) DE was covered fastest.
BC was covered slowest.

(b) EF was supposed to be a resting interval; therefore, it should have been drawn parallel to the horizontal axis.

(c) $$v_{av} = \frac{d_{total}}{t_{total}} = \frac{600 \text{ km} - 0 \text{ km}}{8 \text{ weeks} - 0 \text{ week}} = 75 \text{ km/week}$$

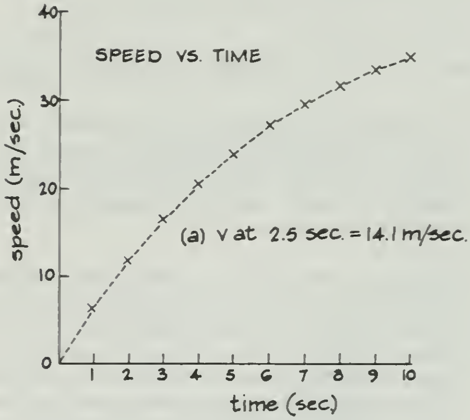
(d) Instantaneous speed at P $= \frac{\Delta d}{\Delta t}$ at P

$$v_P = \frac{200 \text{ km} - 0 \text{ km}}{2.5 \text{ weeks} - 0 \text{ week}} = 80 \text{ km/week}$$

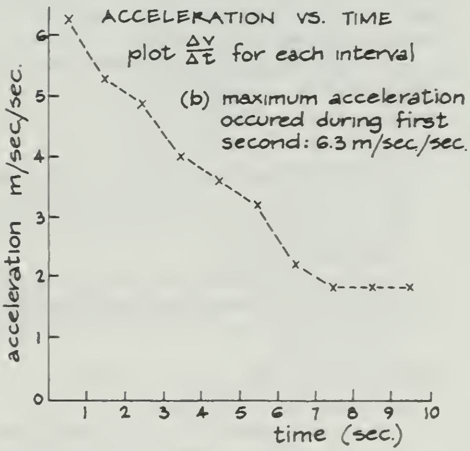
Instantaneous speed at Q $= \frac{\Delta d}{\Delta t}$ at Q

$$v_Q = \frac{500 \text{ km} - 300 \text{ km}}{6 \text{ weeks} - 5 \text{ weeks}} = 200 \text{ km/week}$$

21. (a)



(b)



22. Total distance = 525 lines/picture × 50 cm/line = 26,250 cm/picture

$$\text{Speed} = \frac{26{,}250 \text{ cm/picture}}{0.03 \text{ sec/picture}} = 875{,}000 \text{ cm/sec}$$

23.

Data for graphs taken from photographs

Position read on scale	d	Δd	Δt	v
1.5	0 cm	9.0 cm	0.20 sec	45 cm/sec
10.5	9	13	0.20	65
23.5	22	17.5	0.20	87.5
41	39.5	21	0.20	105
62	60.5	25.5	0.20	127
87.5	86			

24. Fire bullets through two rotating thin paper discs spaced a short distance apart and rotating at a high, known speed. The bullet hole in the second disc will be displaced a certain angle relative to that in the first. The fraction this angle is of a whole revolution, times the period of one revolution, gives the time for the bullet to travel between discs, from which the speed may be computed by $v = d/t$.

    An optional method is to use a ballistic pendulum, using the law of conservation of momentum.

25. In line with the ideas developed in this chapter, it would seem appropriate to start by making a graph of distance versus time, plotting the location of the horse's nose, for example, at each instant photographed. Inspection of the varying slope of the graph will give information of the speed that can be correlated with the motions being made by the horse's body and legs.

CHAPTER 2

2. A stone released at the surface of water will go to its natural position below the water. Raindrops fall through the air seeking their natural position. (Accept any other imaginative examples from students.)

3. Very heavy body with no resistance: Aristotle would predict an infinite speed (due to dividing by zero); Philoponus would have the speed depend on weight alone. (Note, however, that Philoponus has a basic difficulty for all weights and resistances in that he did not indicate how these could be expressed quantitatively in the same units; that is essential for the subtraction process.) Very light body with great resistance: Aristotle would expect it to fall very slowly; for Philoponus, the difficulty described above is paramount with at least the mathematical possibility that the resistance be greater than the weight, giving a negative value to the speed (that is, it would travel upward!).

4. (a) *Simp:* Both pieces slow down to half-speed and fall together, taking twice the time that the 1-kg rock would have taken to fall the remaining distance.

    Salv: Both pieces continue to fall at the same rate as before fracture and strike the ground at the same time as the 1-kg rock would have.

    (b) *Simp:* The 5-kg rock would fall at a faster rate than the 4.5-kg rock.

    Salv: They would fall at the same rate.

    (c) *Simp:* The sack containing the 100 rocks would speed up to fall the remaining distance in 1/100 the time required by uncaptured rocks.

    Salv: The sack would reach the bottom in the same time as that taken by the separate rocks.

5. If the objects are of comparable density, they will fall with the same acceleration, and the string will hang limp between them. It will be similar to the limp umbilical cord which attaches an astronaut to a space capsule for the same reason; both have the same acceleration. In an extended situation where there is appreciable air resistance, an object with a greater cross-sectional area per unit mass will be retarded more, accelerate more slowly, and cause the string to become taut.

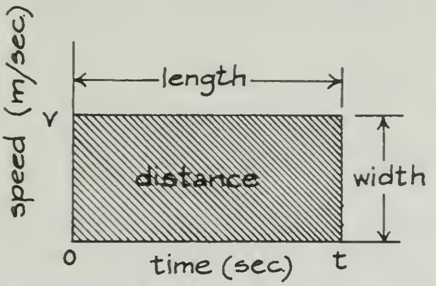
6. (a) $a = 2\text{ m/sec}^2$ for 6 sec

$$d = \tfrac{1}{2}at^2 \text{ (initial velocity } = 0)$$
$$d = \tfrac{1}{2}(2\text{ m/sec}^2)(6\text{ sec})^2$$
$$= 36\text{ m}$$
$$v_{av} = \frac{(v_f - v_i)}{t}$$
$$= \frac{36\text{ m} - 0\text{ m}}{6\text{ sec}}$$
$$= 6\text{ m/sec}$$
$$d_{av} = v_{av}t$$
$$= (6\text{ m/sec})(6\text{ sec})$$
$$= 36\text{ m}$$

    (b) The equations used assume that the acceleration is uniform (as with gravity). A varying acceleration would require a different (more complex) mathematical model.

7. (a) Constant speed on a speed–time graph is represented by a horizontal line at a height above the t-axis corresponding to the particular speed v. We know that algebraically the distance traveled in the time t is given by $d = vt$. The rectangle shown on the graph has length $= t$ (sec) and width $= v$

(m/sec), so its area = vt, which is simply the distance traveled measured in meters.



(b) and (c) v_f = final speed after t seconds

v_{av} = average speed = $\frac{1}{2}(v_f + 0)$

$= \frac{1}{2} v_f$

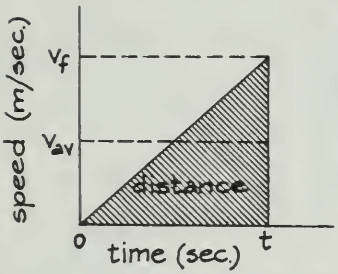
Area of triangle = ½ base × height

$= \frac{1}{2} t \times v_f$

or $\frac{1}{2} v_f t$

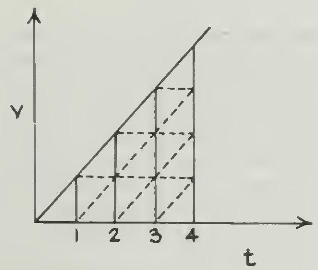
Area of the rectangle formed by v_{av} and $t = v_{av} t$

$= \frac{1}{2} v_f t$



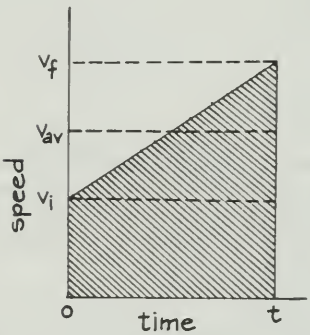
8. (a), (b), (c) are correct answers.

9. Students may see that each successive trapezoid may be broken up into a number of congruent triangles that may then be added up as 1:3:5:7. They may also notice that the total area is in the ratio 1:4:9:16, or is proportional to t^2.
 This problem may also be done by computing the area of each trapezoid as $\frac{B(h_1 + h_2)}{2}$ using arbitrary units, but the purely geometric approach seems preferable.



10. Photograph a falling object illuminated by a stroboscopic light. The distance between the last two images divided by the time between flashes of the light will give the *average speed* in the final interval. The shorter the time between light flashes, the closer this average speed will be to the instantaneous speed.

11. The plan for showing the equivalence of the given expression with the Merton theorem is to logically deduce the expression from the rule.
 1. Distance traveled at average speed = distance traveled when speed is changing uniformly.
 2. Distance traveled is represented by the area under the curve of a speed–time graph.
 3. The area of the rectangle above determined by v_{av} and t = the area of the crosshatched trapezoid. (Refer to the argument in the answer to 7C.)
 4. The algebraic statement of this relationship is
 $$v_{average}\, t = \left[\frac{v_{initial} + v_{final}}{2}\right] t$$
 5. $$v_{average} = \frac{v_{initial} + v_{final}}{2}$$



12.

$$\text{Average} = \frac{15 + 16 + 17 + 18 + 19}{5}$$

$$= \frac{85}{5} = 17 \text{ years}$$

$$\text{Average earning power} = \frac{\$8{,}000 + \$12{,}000}{2}$$

$$= \frac{\$20{,}000}{2} = \$10{,}000$$

13. (a) $a_{av} = \frac{v}{t}$

v = 284 m/sec

t = 5.0 sec

$$a_{av} = \frac{284 \text{ m/sec}}{5.0 \text{ sec}}$$

a_{av} = 57 m/sec^2

(b) $d = \frac{1}{2}at^2$

$a = 57 \text{ m/sec}^2$

$t = 5.0 \text{ sec}$

$d = \frac{1}{2} \times 57 \text{ m/sec}^2 \times (5.0 \text{ sec})^2$

$d = 710 \text{ m}$

(c) $a_{av} = \frac{v}{t}$

$v = v_f - v_i = 0 \text{ m/sec} - 284 \text{ m/sec}$

$t = 1.5 \text{ sec}$

$$a_{av} = \frac{-284 \text{ m/sec}}{1.5 \text{ sec}}$$

$a_{av} = -190 \text{ m/sec}^2$

Since 10 m/sec^2 is approximately the acceleration of gravity, this means Col. Stapp was subject to an average acceleration 19 times gravitational acceleration. (The fact that he was subjected to a maximum of 22 g indicates that the actual acceleration was not constant.)

14. (a) true
(b) true
(c) true
(d) true
(e) true

15. (a) Given: $\frac{d}{t^2} = K$

Show: "... that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity (namely 1:3:5:7 ...)..."

$d = Kt^2$

Substitute equal time intervals of any arbitrary but equal units of time as follows:

$t_0 = 0$ units, $t_1 = 1$ unit, $t_2 = 2$ units, $t_3 = 3$ units, and $t_4 = 4$ units.

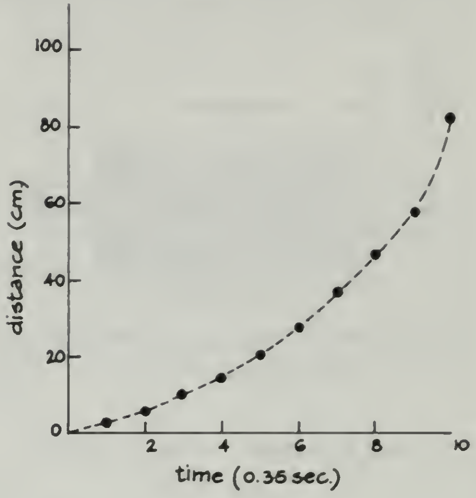
Calculations of total distances traversed

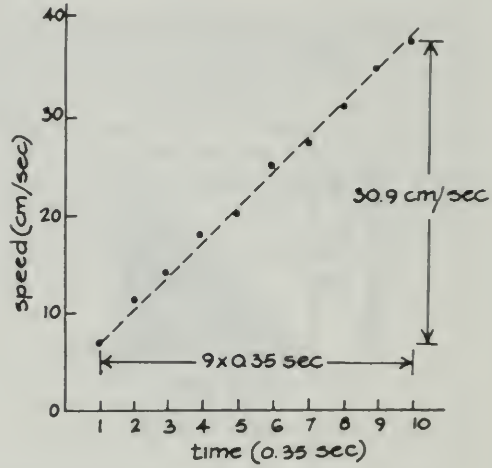
$d_0 = K0^2 = 0K$* units of length
$d_1 = K1^2 = 1K$
$d_2 = K2^2 = 4K$
$d_3 = K3^2 = 9K$
$d_4 = K4^2 = 16K$
*K is constant of proportionality

Calculation of the difference between consecutive distances traversed

$d_1 - d_0 = 1K$ units of length
$d_2 - d_1 = 3K$
$d_3 - d_2 = 5K$
$d_4 - d_3 = 7K$

(b)





$$\text{acceleration} = \frac{\Delta v}{\Delta t} = \frac{30.9 \text{ cm sec}}{9 \times 0.35 \text{ sec}}$$
$$= 9.8 \text{ cm sec}^2$$

16. (a)

Position	d	v
A	+	+
B	+	+
C	+	0
D	+	−
E	−	−

(b) When the ball is in its upward trajectory from A to C, there is a decrease in velocity in the upward or positive direction that, in effect, is an increase in velocity in the earthward or negative direction, so Δv_{AB} is negative. When the ball is in its downward trajectory form C to E, its velocity is increasing in the earthward or negative direction, so that Δv_{DE} is negative. It should be noted that time is not taken to have a dimension of direction; time is scalar. Therefore, the acceleration is negative, as is indicated by the definition $a = v/\Delta t$. There is within this question the notion that when a vector quantity is divided by a scalar, the scalar quantity has no effect upon direction.

Note that a vector analysis of this question could be equally instructive.

(c) The acceleration due to gravity would be (+) in all instances.

17. The points would have a pattern corresponding to the photo in 16 turned upside down. A strong magnet held over a small nail would produce an upward acceleration. Also, a piece of wood released below the surface of water would accelerate upward.

18. (a) $d = \frac{1}{2}at^2$

$a = -10 \text{ m/sec}^2$

$t = 1.0 \text{ sec}$

$d = \frac{1}{2} \times -10 \text{ m/sec}^2 (1.0 \text{ sec})^2$

$d = -5.0 \text{ m}$

(b) $v = at$

$a = -10 \text{ m/sec}^2$

$t = 1.0 \text{ sec}$

$v = -10 \text{ m/sec}^2 \times 1.0 \text{ sec}$

$v = -10 \text{ m/sec}$

(c) $d = \frac{1}{2}at_2^2 - \frac{1}{2}at_1^2$

$d = \frac{1}{2}a(t_2^2 - t_1^2)$

$a = -10 \text{ m/sec}^2$

$t_2 = 2.0 \text{ sec}$

$t_1 = 1.0 \text{ sec}$

$d = \frac{1}{2} \times -10 \text{ m/sec}^2 (4.0 \text{ sec}^2 - 1.0 \text{ sec}^2)$

$d = -15 \text{ m}$

19. By definition,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time elapsed}}$$

or

$$a = \frac{v_{\text{final}} - v_{\text{initial}}}{t}$$

Note $a_a \cong 10 \text{ m/sec}^2$

(a) $v_f = v_i - at$

$v_i = 20 \text{ m/sec}^2$

$a = -10 \text{ m/sec}^2$

$t = 1.0 \text{ sec}$

$v = 20 \text{ m/sec} - 10 \text{ m/sec}^2 \times 1 \text{ sec}$

$v = 10 \text{ m/sec}$

(b) $d = vt$

$$v = \frac{v_i + v_f}{2}$$

$$d = \left(\frac{v_i + v_f}{2}\right) \times t$$

$v_i = 20 \text{ m/sec}$

$v_f = 10 \text{ m/sec}$

$t = 1 \text{ sec}$

$$d = \left(\frac{20 \text{ m/sec} + 10 \text{ m/sec}}{2}\right) \times 1 \text{ sec}$$

$d = 15 \text{ m}$

(c) $v_f = v_i + at$

$at = v_f - v_i$

$$t = \frac{v_f - v_i}{a}$$

$v_f = 0 \text{ m/sec}$

$v_i = 20 \text{ m/sec}$

$a = -10 \text{ m/sec}^2$

$$t = \frac{-20 \text{ m/sec}}{-10 \text{ m/sec}^2}$$

$t = 2 \text{ sec}$

(d) $d = vt$

$$d = \left(\frac{v_f + v_i}{2}\right) t$$

$v_f = 0 \text{ m/sec}$

$v_i = 20 \text{ m/sec}$

$t = 2 \text{ sec}$

$$d = \left(\frac{0 \text{ m/sec} + 20 \text{ m/sec}}{2}\right) \times 2 \text{ sec}$$

$d = 20 \text{ m}$

(e) Since the object falls the same distance it rises and undergoes the same acceleration, we can immediately say that the downward trip will be similar to the upward one. Then it will take the same time and Δv will be the same. We can also show this mathematically:

Since $d = \frac{1}{2}at^2$, the time to fall a distance d is

$$t = \left(\frac{2d}{a}\right)^{\frac{1}{2}}$$

The velocity to which the object will accelerate in this time is

$$v = at = a\left(\frac{2d}{a}\right)^{\frac{1}{2}} = (2ad)^{\frac{1}{2}}$$

$a = -10 \text{ m/sec}^2$

$d = -20 \text{ m}$

(d is negative because the object is going in a negative direction)

$v = (2 \times -10 \text{ m/sec}^2 \times -20 \text{ m})^{1/2}$

$v = +20 \text{ m/sec}$

$v = -20 \text{ m/sec}$

(The negative root is the one with meaning in this situation.)

20. (a) $v_f = v_i + at$

$v_i = 40 \text{ m/sec}$

$a = -10 \text{ m/sec}^2$

$t = 2 \text{ sec}$

$v_f = 40 \text{ m/sec} - 10 \text{ m/sec}^2 \times 2 \text{ sec}$

$v_f = +20 \text{ m/sec}$

(b) $v_f = v_i + at$

$t = 6 \text{ sec}$

$v_f = 40 \text{ m/sec} - 10 \text{ m/sec}^2 \times 6 \text{ sec}$

$v_f = -20 \text{ m/sec}$

(c) The ball reaches its highest point when $v_f = 0$.

$v_f = 0 \text{ m/sec}$

$v_i = 40 \text{ m/sec}$

$a = -10 \text{ m/sec}^2$

$$t = \frac{0 \text{ m/sec} - 40 \text{ m/sec}}{-10 \text{ m/sec}^2}$$

$t = 4 \text{ sec}$

(d) $d = v_i t + \frac{1}{2}at^2$

$= 40 \text{ m/sec} \times 4 \text{ sec} + \frac{1}{2}(-10 \text{ m/sec}^2)16 \text{ sec}^2$

$= 160 \text{ m} - 80 \text{ m}$

$= 80 \text{ m}$

(e) The speed is zero, since by symmetry the ball must reach the ground (and hence come to a stop) in just 8 sec.

(f) We can say immediately from the symmetry of the problem that the speed will be of the same magnitude when it gets back to the ground as it was when it left. This may be proved in the following manner:

$v_f = v_i + at$

The time t is the time it takes the distance d to return to zero.

$d = v_i t + \frac{1}{2}at^2$

Letting $d = 0$ and solving for t,

$t(v_i + \frac{1}{2}at) = 0$ (The root $t = 0$ describes the initial conditions)

$v_i + \frac{1}{2}at = 0$

$$t = \frac{-2v_i}{a}$$

Substituting,

$$v_f = v_i + a\frac{-2v_i}{a} = -v_i$$

Since $v_i = 40 \text{ m/sec}$, $v_f = -40 \text{ m/sec}$

21. (a) $a = \frac{\Delta v}{\Delta t}$

$\Delta v = v_f - v_i = 0 \text{ m/sec} - 4 \text{ m/sec}$

$\Delta v = -4 \text{ m/sec}$

$\Delta t = 2 \text{ sec}$

$$a = \frac{-4 \text{ m/sec}}{2 \text{ sec}}$$

$a = -2 \text{ m/sec}^2$

(b) $v_{av} = \frac{v_i + v_f}{2}$

$$v_{av} = \frac{4 \text{ m/sec} + 0 \text{ m/sec}}{2}$$

$v_{av} = 2 \text{ m/sec}$

(c) $v_f = v_i + at$

$v_i = 4 \text{ m/sec}$

$a = -2 \text{ m/sec}^2$

$t = 1 \text{ sec}$

$v_f = 4 \text{ m/sec} - (2 \text{ m/sec}^2 \times 1 \text{ sec})$

$v_f = 2 \text{ m/sec}$

(d) $d = v_i t + \frac{1}{2}at^2$

$v_i = 4 \text{ m/sec}$

$a = -2 \text{ m/sec}^2$

$t = 2 \text{ sec}$

$d = 4 \text{ m/sec} \times 2 \text{ sec} + \frac{1}{2}(-2 \text{ m/sec}^2)4 \text{ sec}^2$

$d = 8 \text{ m} - 4 \text{ m} = 4 \text{ m}$

or

$d = v_{av} t$

$= 2 \text{ m/sec} \times 2 \text{ sec} = 4 \text{ m}$

(e) $v_f = v_i + at$

$t = 3 \text{ sec}$

$v_f = 4 \text{ m/sec} - 2 \text{ m/sec}^2 \times 3 \text{ sec}$

$v_f = -2 \text{ m/sec}$

(f) The total time will be 4 sec, just twice the length of time needed to reach the highest point.

22. (a) The proposal is to determine whether different masses fall with the same acceleration. This experiment involves a direct measurement of the ratio d/t^2. However, variations in t for the same event will probably be relatively large; thus, there will be variations in a_g.

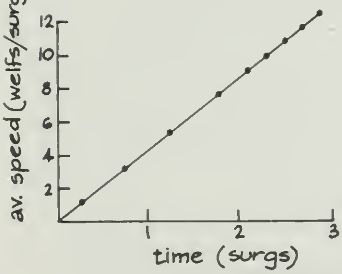
A student might propose to determine the discrepancies among the falling times, and then compare the small discrepancies with the total time to fall. This would involve a change in the design of the experiment to a "null method," which is more sensitive.

(b) This project looks interesting and will allow us to determine whether d/t^2 is constant for different values of d. However, the proposal states that each student will obtain instantaneous speed of the ball as it passes the window. The procedure is not outlined and may prove difficult or impossible to do.

(c) The effect of air resistance on the cotton balls is going to be appreciable. If this is the case, we cannot expect the balls to have a constant acceleration. The experiment might provide interesting information, but it would not be pertinent to Galileo's problem. However, a student might justify accepting the proposal on the basis that the effect of air resistance might not be known until someone had performed this experiment.

23. (a) We can find the acceleration due to gravity on Arret by constructing a graph of speed versus time for the freely falling body and finding its slope. To do this we could first construct a distance–time graph and measure the slope of the curve at various points; or, we can approximate the velocity at different points by finding the average velocity between two points on either side of the desired points. This can be done with a table. Note that the average velocity between points A and B approximates the instantaneous speed at the time 0.25 sec. Also note that the time interval in the given data changes from 0.5 sec to 0.2 sec.

Position	Time (surgs)	d (welfs)	Interval	d	v_{av} $\left(\frac{\text{welfs}}{\text{surg}}\right)$
A	0.0	0.00	AB	0.54	1.1
B	0.5	0.54	BC	1.61	3.2
C	1.0	2.15	CD	2.69	5.4
D	1.5	4.84	DE	3.76	7.5
E	2.0	8.60	EF	1.81	9.0
F	2.2	10.41	FG	1.98	9.9
G	2.4	12.39	GH	2.15	10.7
H	2.6	14.54	HI	2.32	11.6
I	2.8	16.86	IJ	2.47	12.3
J	3.0	19.33			



$$\text{acceleration} = \frac{\Delta v}{\Delta t} = \frac{12.3 - 0}{2.9} = 4.3 \text{ welfs/surg}^2$$

(b) 1 welf = 0.633 m
1 surg = 0.167 sec

$$1 \text{ welf/surg}^2 = \frac{1 \text{ welf } (0.633 \text{ m/welf})}{1 \text{ surg}^2 \ (0.167 \text{ sec}^2/\text{surg}^2)} = 2.27 \text{ m/sec}^2$$

$$4.3 \text{ welf/surg}^2 \times \frac{2.27 \text{ m/sec}^2}{1 \text{ welf/surg}^2} = 9.8 \text{ m/sec}^2$$

The accleration caused by gravity on Arret is 9.8 m/sec^2, about the same as that on earth.

24. Special conditions implied in given equations
1) object starts from rest
2) acceleration is uniform

(a) Derivation:
Given: $v = at$

$$t = \frac{v}{a}$$

Given: $d = ½at^2$

$$d = ½a\left(\frac{v}{a}\right)^2$$

$$d = \frac{v^2}{2a}$$

$$v^2 = 2ad$$

(b) As the ball returns to earth it will have the same speed downward that it was given upward initially. Thus, the height h from which it must fall to attain the speed v is determined as follows:

$$v^2 = 2a_gh$$

$$h = \frac{v^2}{2a_g}$$

25. The equations we have available are as follows:

1) $a = \dfrac{v_f - v_i}{t}$ from which we can say $t = \dfrac{v_f - v_i}{a}$

2) $d = v_it + ½at^2$

3) $d = v_{av}t$

4) $v_{av} = ½(v_i + v_f)$

Combining equations 1, 3, and 4 we get:

$$d = ½(v_i + v_f)\frac{v_f - v_i}{a}$$

$$2ad = v_f^2 - v_i^2$$

$$v_f^2 = v_i^2 + 2ad$$

The same result is obtained if we combine equations 1 and 2.

$$d = v_i\frac{v_f - v_i}{a} + ½a\left(\frac{v_f - v_i}{a}\right)^2$$

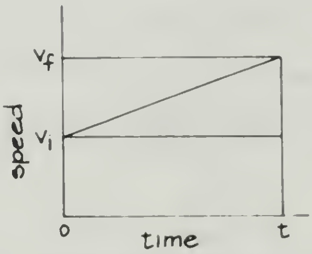
$$ad = v_iv_f - v_i^2 + \tfrac{1}{2}(v_f^2 - 2v_iv_f + v_i^2)$$
$$ad = v_iv_f - v_i^2 + \tfrac{1}{2}v_f^2 - v_iv_f + \tfrac{1}{2}v_i^2$$
$$= \tfrac{1}{2}v_f^2 - \tfrac{1}{2}v_i^2$$
$$2ad = v_f^2 - v_i^2$$
$$v_f^2 = v_i^2 + 2ad$$

26. The area under the graph line is composed of the rectangle of sides corresponding to v_i and t plus the triangle of base t and height $v_f - v_i$

Total area $= v_it + \tfrac{1}{2}(v_f - v_i)t$

but $a = \dfrac{v_f - v_i}{t}$ or $v_f - v_i = at$

Thus, total area = distance traveled $= v_it + \tfrac{1}{2}at^2$. Thus,



27. The steps in Galileo's investigation may be identified as follows:
   1) Definition: define *uniform acceleration* as constant increase in velocity with time.
   2) Hypothesis: Freely falling bodies are uniformly accelerated, as defined above.
   3) Deduction: $\dfrac{d}{t^2} = \dfrac{a}{2}$ = constant for balls falling from rest.
   4) Deduction: $\dfrac{d}{t^2} = \dfrac{a}{2}$ = constant for balls rolling down an inclined plane.
   5) Observation: 4) is verified by experiment.
   6) Conclusion: 2) is verified by the above process.

   The argument was limited by Galileo's ability to measure time intervals accurately and by his idealization that rolling motion was simply a slowed-down falling motion. He ignored the rotational motion of the ball about its center.

28. (a) The average speed is equal to the distance interval traveled divided by the time interval over which the distance was measured.

   The average acceleration is equal to a change in velocity divided by the time interval over which the velocity change was measured.

   The distance traveled by a uniformly accelerating object is equal to one-half the acceleration measured from the start multiplied by the square of the time since the start.

   (b) A wide variety of equally good problems can be expected here.

   (c) For the problem suggested, the answer is:

$$\Delta t = \frac{\Delta d}{v_{av}}$$
$$= \frac{3{,}200\text{ km}}{1{,}000\text{ km/hr}}$$
$$= 3.2\text{ hr}$$

29. Although his followers may have relied too heavily on handed-down information, Aristotle himself did observe nature and successfully classified many plants and animals in addition to recognizing that stones, for example, do fall faster than leaves. Galileo showed that it is necessary to question authority in science, especially when contradictions are observed in nature. He was not able to show "everyone directly," but was able to convince many by indirect and mathematical arguments that Aristotle's analysis of free fall was wrong. If by "science" we mean the study of nature as it can be observed, coupled with an attempt to correlate separate events into a coherent pattern, then Galileo was certainly not the first to do this. His principal contribution to our modern scientific method was the recognition of the key role played by mathematics in describing nature.

30. (a) $v_i = 5$ m/sec, $v_f = 30$ m/sec, $t = 10$ sec

$$a = \frac{(v_f - v_i)}{t} = \frac{30\text{ m/sec} - 5\text{ m/sec}}{10\text{ sec}} = 2.5\text{ m/sec}^2$$

(b) $t = 9.0$ sec, $v_i = 0$

$$d = \tfrac{1}{2}at^2 = \tfrac{1}{2}(9.8\text{ m/sec}^2)(9.0\text{ sec})^2 = 397\text{ m}$$

(for $a = 10$ m/sec^2, $d = 405$ m)

(c) $a = 2$ m/sec^2, $d = 20$ m

$$d = \tfrac{1}{2}at^2$$
$$t^2 = \frac{2d}{a}$$
$$t^2 = \frac{2(20\text{ m})}{2\text{ m/sec}^2}$$
$$t^2 = 20\text{ sec}^2$$
$$t = 4.5\text{ sec}$$

If $v_i = 0$, $v_f = a\Delta t$
$$= (2\text{ m/sec}^2)(4.5\text{ sec})$$
$$= 9\text{ m/sec}$$

(d) $v_i = 8$ m/sec north
$a = 5$ m/sec^2 north
$t = 10$ sec

$$d = v_i t + \tfrac{1}{2}at^2$$
$$= (8\text{ m/sec})(10\text{ sec}) + \tfrac{1}{2}(5\text{ m/sec}^2)(10\text{ sec})^2$$
$$= 80\text{ m} + 250\text{ m}$$
$$= 330\text{ m north}$$
$$v_f = v_i + at$$
$$= 8\text{ m/sec} + (5\text{ m/sec}^2)(10\text{ sec})$$
$$= 58\text{ m/sec north}$$

(e) $a = 2\text{ m/sec}^2$ $d = v_i t + \tfrac{1}{2}at^2$

$d = 4\text{ m}$ $4\text{ m} = 0 + \tfrac{1}{2}(2\text{ m/sec}^2)t^2$

$v_i = 0$ $4\text{ m} = 1\text{ m/sec}^2\, t^2$

$$\frac{4\text{ m}}{1\text{ m/sec}^2} = t^2$$
$$4\text{ sec}^2 = t^2$$
$$2\text{ sec} = t$$

$$v_f = at$$
$$= (2\text{ m/sec}^2)(2\text{ sec})$$
$$= 4\text{ m/sec}$$

(f) $v_i = 6\text{ m/sec}$

$a = 1\text{ m/sec}^2$

$d = 2\text{ m}$

$$v_f^2 = v_i^2 + 2ad$$
$$v_f^2 = (6\text{ m/sec})^2 + 2(1\text{ m/sec}^2)(2\text{ m})$$
$$v_f^2 = (36 + 4)\text{ m}^2/\text{sec}^2$$
$$v_f = \sqrt{40}\text{ m/sec}$$
$$v_f = 6.3\text{ m/sec}$$

(g) $v_i = 5\text{ m/sec}$ $d = \tfrac{1}{2}(v_f + v_i)t$

$v_f = 55\text{ m/sec}$ $100\text{ m} = \tfrac{1}{2}(55\text{ m/sec} + 5\text{ m/sec})t$

$d = 100\text{ m}$ $100\text{ m} = \tfrac{1}{2}(60\text{ m/sec})t$

$$100\text{ m} = (30\text{ m/sec})t$$
$$3.3\text{ sec} = t$$

$$a = \frac{v_f - v_i}{t}$$
$$= \frac{55\text{ m/sec} - 5\text{ m/sec}}{3.3\text{ sec}}$$
$$= \frac{50\text{ m/sec}}{3.3\text{ sec}}$$
$$= 15.2\text{ m/sec}^2$$

(h) $d = 45\text{ m}$

$t = 4\text{ sec}$

$$d = v_i t + \tfrac{1}{2}at^2$$
$$45\text{ m} = v_i(4\text{ sec}) + \tfrac{1}{2}(-9.8\text{ m/sec}^2)(4\text{ sec})^2$$
$$45\text{ m} = v_i(4\text{ sec}) + \tfrac{1}{2}(-9.8\text{ m/sec}^2)(16\text{ sec}^2)$$
$$45\text{ m} = v_i(4\text{ sec}) - 78\text{ m}$$
$$\frac{45\text{ m} + 78\text{ m}}{4\text{ sec}} = v_i$$
$$31\text{ m/sec} = v_i$$

31. (a) $v_{av} = \dfrac{\Delta d}{\Delta t} = \dfrac{10\text{ m}}{4\text{ sec}} = 2.5\text{ m/sec (AB)}$

$v_{av} = \dfrac{30\text{ m} - 15\text{ m}}{2\text{ sec}} = \dfrac{15\text{ m}}{2\text{ sec}} = 7.5\text{ m/sec (CD)}$

(b) $v_{av} = \dfrac{15\text{ m} - 10\text{ m}}{2\text{ sec}} = 2.5\text{ m/sec (BC)}$

$a_{av} = \dfrac{7.5\text{ m/sec} - 2.5\text{ m/sec}}{2\text{ sec}} = 2.5\text{ m/sec}^2$

(c) Discussion

32. (a) graph : constant velocity
graph B: acceleration
graph C: acceleration
graph D: negative acceleration (deceleration)

(b) graph A: backward
graph B: forward
graph C: backward
graph D: forward

CHAPTER 3

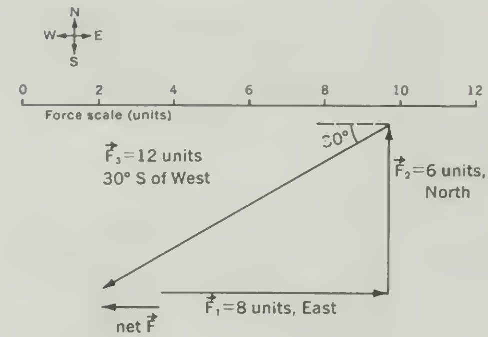
2. We do in fact always observe that a force is required to keep an object moving with constant speed across a table, and if a massive rock and a ping pong ball are dropped from the same height above the ground, the rock will reach the ground first. We tend to remain Aristotelian since Newtonian force analysis requires us to consider ideal situations or to deal with invisible forces like friction.

3. (a) Mechanics is the branch of physics that deals with the study of forces on objects. Dynamics is the study of forces that produce motion in objects (cause objects to move). Kinematics is the study of motions without reference to the causes of the motions.

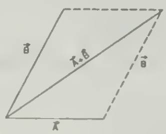
(b) (1) d (scalar)
(2) $\vec{f}$ (vector)
(3) v (scalar)
(4) f (scalar)
(5) m (scalar)
(6) $\vec{d}$ (vector)
(7) t (scalar)
(8) $\vec{a}$ (vector)
(9) $\vec{v}$ (vector)

4. (a) Three blocks east of the starting point.
   (b) (3 + 4 + 5 + 1 + 2) blocks = 15 blocks
   (c) Part (a) is a vector problem; part (b) is a scalar problem.

5. (a) The forces do not balance. The tip of F_3 does not coincide with the tail of F_1.
   Note that the direction system and the force scale are needed before attempting a solution.
   (b) Net force = 2.4 units west (shown just below the diagram).



6. $\vec{A} + \vec{B}$ has the same magnitude and direction whether it is obtained as the diagonal of the parallelogram having $\vec{A}$ and $\vec{B}$ as adjacent sides or as the third side of the triangle having $\vec{A}$ and $\vec{B}$ as the other two sides. The essential procedure in both methods is to make sure that the given magnitudes *and directions* of the original vectors are carefully preserved before attempting to complete either the parallelogram or the triangle.



7. The parachutist's weight is 750 N down. Air resistance is therefore 750 N up, since the parachutist is falling with uniform speed. (Air resistance just balances the effect of gravity.)

8. Another example is a Lunar Module approaching the surface of the moon at a constant speed of 24 m/sec. In this example, the downward gravitational force is equal to the upward thrust of the module in magnitude but opposite to it in direction. Both of the forces increase as the module approaches the surface of the moon.

9. (a) Equilibrium is the state in which the net force acting on a system is zero.
   (b) Two possible states of motion for an object in equilibrium are: (1) the object is at rest; (2) the object is moving uniformly.

10. (a) We would say that the force is not large enough to overcome the force of starting friction. An Aristotelian would say that rest is a natural state and that any motion requires a force; the force is not enough to change the box from its natural state.
   (b) We would say that the applied force is now greater than the force of friction, resulting in an unbalanced force. Consequently, the box accelerated according to Newton's second law. An Aristotelian probably would have maintained that the force producing the motion was now great enough to displace the box from its natural state. However, the Aristotelians had no clear concept of acceleration.
   (c) We would say that the frictional force between the box and the table is directed opposite to the motion, accelerating it in this reverse direction until it comes to rest. Aristotle would say that the box was returning to its natural state of rest.

11. (a) The ice puck will move with a uniform velocity.
   If the laboratory itself is moving with a uniform velocity, the puck's motion will appear unaffected to an observer inside the laboratory. An observer outside the laboratory will see the puck moving with a velocity equal to the sum of the laboratory's velocity and the velocity imparted by the push.
   In a laboratory undergoing uniform linear acceleration, an observer inside could not determine by observing *only* the puck's motion whether the puck was being accelerated in one direction or the laboratory was being accelerated in the opposite direction.
   Motion of a puck in a curved path can be explained either by assuming a force acting on the puck in a direction that makes an angle with the velocity it was originally given, *or* by assuming the laboratory to be accelerating at the supplementary angle.
   (b) The man will see the puck curve away from him. Relying on Newton's laws, he will think that the puck is being subjected to a force that accelerates it in the curved path. He will be wrong, of course, because he is in an accelerated frame of reference. (This fictitious force is called a Coriolis force.)

12. (a) The brakes slow the car but not the passengers, since they are not rigidly attached to the car. Their inertia causes their forward motion to continue unchanged momentarily while that of the car is reduced.
   (b) Velocity is a vector quantity. When the force of the road against the tires changes the direction of the car, it fails to change the direction of the passengers immediately. They continue in the original direction until the force of the seat and the side of the car

on their bodies changes their direction of motion.

(c) The centripetal force needed to hold the coin in "orbit" increases as the rotation rate of the turntable increases. The frictional force that links the coin to the turntable remains constant. When the frequency of the turntable rotation has increased so that the centripetal force equals the frictional force, a further increase in the centripetal force required to keep the coin in its "orbit" cannot be provided by friction. The coin will then slip toward the rim of the turntable. Note: This anticipates the discussion in the next chapter.

13. One way would be to use a rubber band (or a spring) stretched to the same extent and attach it to each of the different masses successively.

14. (a) Newton's second law says that acceleration is (1) directly proportional to the magnitude of the force, (2) in the same direction as the force, and (3) inversely proportional to the mass.

(b) $a = 10 \text{ m/sec}^2$ north

$F = ma$

$a = \frac{f}{m}$

Therefore, a is directly proportional to F and inversely proportional to m. If the force becomes $\frac{1}{2}F$ and the mass becomes $\frac{1}{3}m$:

$a \propto \frac{1}{2}F \therefore 10 \text{ m/sec}^2 \times \frac{1}{2} = 5 \text{ m/sec}^2$

$a \propto 3m \therefore\ 5 \text{ m/sec}^2 \times 3 = 15 \text{ m/sec}^2$

The new acceleration is 15 m/sec^2 east (since the new force is to the east).

15. That k must have the dimension hr/sec is seen as follows:

$\Delta d = kv\Delta t$

$(\text{mi}) = k\,(\text{mi/hr})(\text{sec})$

We need to compensate for the sec/hr found in the v and Δt terms. The value of k is then the number of hours in a second, which is the fraction 1/3,600 or 2.78×10^{-4}. Thus, $k = 2.78 \times 10^{-4}$ hr/sec.

16. As explained in Sec. 3.10, students should find that their weight seems to increase as the elevator accelerates upward. The new weight will be equal to the original weight plus the added force caused by the upward acceleration ($F = ma$). As the elevator slows down, a student's weight will gradually seem to decrease to the original weight. If the elevator moves up and down at constant speed, the student's weight will also appear to remain constant. Although your weight does not really change during an elevator ride, the scale shows a difference when the elevator is accelerating because the scale measures the net force acting on your body. This force increases when the elevator is accelerating and decreases when it is decelerating.

In a space vehicle, your weight would seem to decrease as you got farther from earth.

17. Provide yourself with suitable standard masses and measure the accelerations associated with particular values of the extension of the spring. For each case, F can be determined by multiplying the mass by the acceleration and marked at the place on a scale indicating the extent to which the spring was stretched. To actually do this it would be difficult (1) to maintain a uniform stretch of the spring, (2) to eliminate frictional forces, and (3) to measure the accelerations precisely.

18. (a) A simple experiment could be set up by hanging different masses from a spring and noting the extension of the spring for each mass. Since the force of gravity on each mass can be calculated, we can plot force versus extension. If Hooke is correct, the points will lie along a straight line. This law does not hold when the spring is stretched beyond its elastic limit; that is, when the spring fails to resume its original length when the mass is removed. This permanently damages the spring and should be avoided. (Note: Modern usage requires the substitution of the word "force" where Hooke used "power." Today, power has a different meaning.)

(b) A static method of calibration may now be used. When the spring is stretched by a known mass, the force on the spring is just equal to the gravitational force on the mass (its weight) and is equal to the mass times the value of a_g at the particular location.

19. (c) 24 N out
(d) 15 N left
(e) 0.86 N north
(f) 9.0 kg
(g) 0.30 kg
(h) 0.20 kg
(i) 3.00 m/sec^2 east
(j) 2.5 m/sec^2 left
(k) 2.50 m/sec^2 down

20. (a) $a_{av} = \frac{F}{m}$

$F = 8.9 \times 10^5 \text{ N}$

$m = 4.44 \times 10^3 \text{ kg}$

$a_{av} = \frac{8.9 \times 10^5 \text{ kg m/sec}^2}{4.44 \times 10^3 \text{ kg}}$

$a_{av} = 2.0 \times 10^2 \text{ m/sec}^2$

$v = at$

$t = 3.9 \text{ sec}$

$v = 2.0 \times 10^2 \text{ m/sec}^2 \times 3.9 \text{ sec}$

$v = 7.8 \times 10^2 \text{ m/sec}$

(b) 2.0×10^2 m/sec^2 is about 20 g. Since the maximum acceleration is 30 g, the acceleration varied. Note that 20 g is the average.

(c) $v^2 = 2ad$

$$a_{av} = \frac{v^2}{2d}$$

$$v = 860 \text{ m/sec}$$

$$d = 1530 \text{ m}$$

$$a_{av} = \frac{(860 \text{ m/sec})^2}{2 \times 1530 \text{ m}}$$

$$a = 2.4 \times 10^2 \text{ m/sec}^2$$

The average acceleration and the maximum speed turn out to be higher than that obtained by using the equation for Newton's second law of motion, as in (a). The discrepancy is explained by the fact that the rocket mass is constantly decreasing, and hence it is incorrect to use the initial mass for the whole run.

A good student might like to try to calculate the mass lost during the 3.9-sec run.

21. To determine the unknown mass, we first calibrate the spring balance. This may be done by accelerating the 1-kg standard with a constant force indicated on the spring balance. The time to cover a measured distance from rest can be determined and the acceleration calculated:

$$d = \tfrac{1}{2}at^2 \qquad a = \frac{2d}{t^2}$$

From the known values of m and a, F can be calculated using Newton's second law, $F = ma$.

The unknown mass can then be accelerated with this same force and its acceleration measured. If values for F and a are substituted into $F = ma$, the unknown mass can be calculated.

It may be noted that since the same force is used each time, it is not necessary to compute the value of the force to find the mass:

$$F_1 = F_2$$

$$m_1a_1 = m_2a_2$$

$$m_2 = \frac{m_1a_1}{a_2}$$

22. Since the balance reading is 0.40 N when the block is dragged at any constant velocity, this must be the frictional force. The net force is the applied force less the frictional force.

$$F_{net} = F_{applied} - F_{friction}$$

$$F_{net} = 2.1 \text{ N} - 0.40 \text{ N} = 1.7 \text{ N}$$

$$F_{net} = ma$$

$$m = \frac{F_{net}}{a}$$

$$a = 0.85 \text{ m/sec}^2$$

$$m = \frac{1.7 \text{ kg m/sec}^2}{0.85 \text{ m/sec}^2}$$

$$m = 2.0 \text{ kg}$$

23. This question is intentionally phrased in personal terms of "you" and "your." The students may propose a variety of explanations, such as the following:
   (1) Because all parts of the body are accelerating downward at the same rate, those below do not support those above them as they normally do. There are no upward forces being exerted that compensate for the downward gravitational forces.
   (2) This is only an "apparent" weightlessness, because gravitational forces certainly are acting on the body, making it fall.
   (3) "True" weightlessness, as in deep space, we can only imagine; but we understand in Newtonian terms, that there would be no appreciable forces among the various parts of the body.

24. (a) (i) The mass will be 1 kg in both places.
   (ii) $F = ma_g$

$$m = 1.000 \text{ kg}$$

$$a_g \text{ (Paris)} = 9.81 \text{ m/sec}^2$$

$$F = 1.000 \text{ kg} \times 9.81 \text{ m/sec}^2$$

$$F \text{ (Paris)} = 9.81 \text{ N}$$

$$a_g \text{ (Washington)} = 9.80 \text{ m/sec}^2$$

$$F = 1.000 \text{ kg} \times 9.80 \text{ m/sec}^2$$

$$F \text{ (Washington)} = 9.80 \text{ N}$$

(b) The change in any student's weight ma_g can be calculated as follows:

$$\Delta F = F \text{ (Paris)} - F' \text{ (Washington)}$$

$$\Delta F = ma_g - ma_g'$$

$$\Delta F = m(a_g - a_g')$$

where a_g = acceleration at Paris

a_g' = acceleration at Washington

25. (a) Since the pound is a unit of force (weight) and the kilogram is a unit of mass, they cannot be directly converted. Weight is a measure of the earth's gravitational attraction at its surface and therefore comparisons can only be made on earth.
   (b) Student answers will vary.
   (c) Student answers will vary. (For each 1 kg of mass lifted, 9.8 N of force are required.)

26. This question anticipates the discussion on circular motion in later chapters, but it may have already been raised by students in connection with 23. When in orbit a few hundred kilometers above the earth, "weightlessness" cannot be due to a very small value of a_g. It can be

shown that $a_g = \frac{Gm_E}{R^2}$ where G is the Cavendish constant, m_E the mass of the earth, and R the distance to the center of the earth. Since the radius of the earth is about 6,400 km, a few hundred more kilometers will not make a large change. The correct explanation lies in the fact that the astronauts, their capsule, and all its contents are in a constant state of centripetal acceleration = a_g; a kind of free fall.

27. (a)
$$F_R = -F_E$$
$$m_R a_R = -m_E a_E$$
$$(60)(5) = -(60 \times 10^{23})(a_E)$$
$$a_E = -5 \times 10^{-23} \text{ m/sec}^2$$

(b)
$$v_R = a_R t$$
$$v_R = (5)(2)$$
$$v_R = 10 \text{ m/sec}$$

(c)
$$v_E = a_E t$$
$$v_E = (5 \times 10^{-23})(2)$$
$$v_E = 10 \times 10^{-23} \text{ m/sec}$$

28. $F = ma$
$$a_{girl} = \frac{F}{m_{girl}}$$
$$= \frac{80 \text{ kg} \cdot \text{m/sec}^2}{40 \text{ kg}}$$
$$= 2 \text{ m/sec}^2$$

According to Newton's third law, the force on the boy (F_{boy}) is 80 N (kg · m/sec^2). Therefore,
$$a_{boy} = \frac{80 \text{ kg} \cdot \text{m/sec}^2}{70 \text{ kg}}$$
$$= 1.14 \text{ m/sec}^2$$

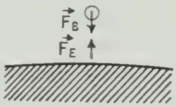
29. (a) True, but this would also be true without the condition of standing perfectly still.
(b) True. The propeller exerts a force on the air. The air exerts an equal and opposite force on the propeller, enabling the plane to move forward.
(c) True. Both are numerically equal to the weight of A.

30. Think what the tractor must do to bring about its motion. As power is applied, the tracks push backward against the surface of the earth. Some loose earth may be pushed away. The locomotion of objects commonly involves pushing backward, opposite to the direction of motion. But according to the third law, if the treads of the tractor push backward on the surface of the earth, the earth must simultaneously push forward on the treads. Whether or not the tractor moves depends solely on the balance of forces impinging on the *tractor*: the tractor will accelerate if, and only if, there is an unbalanced force on *it*. The force of the log on the tractor opposes the motion of the tractor, as does the friction in the moving parts of the tractor and between the tractor and the ground. It is only when the force of the earth on the tractor becomes greater than these retarding forces that it will begin to move.

Another way of answering the question about why the tractor moves is to say that the force it exerts on the ground is greater than that which is exerted by the log. Therefore, the accelerating force of the earth is greater than the retarding force of the log.

For a different presentation of the tractor–log paradox refer to *Transparency T8.*

31. (a)



F_b = the force with which the earth pulls on the ball

F_e = the force with which the ball pulls on the earth

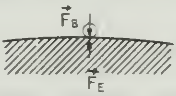
(b) $F_b = m_b a_g$
$$F_b = (1.0)(10)$$
F_b = 10 N of force acting on ball

F_e = 10 N of force acting on the earth
$$F_e = m_e a_e$$
$$10 = (6.0 \times 10^{24})(a_e)$$
$$a_e = 1.7 \times 10^{-24} \text{ m/sec}^2$$

(c) $$\frac{a_b}{a_e} = \frac{10}{1.7 \times 10^{-24}} = \frac{6 \times 10^{24}}{1}$$

The ratio of the accelerations is just the inverse of the ratio of the masses.

(d)



32. (a) (i) To accelerate a 75-kg person at 1.5 m/sec^2 requires an unbalanced net force that has a magnitude equal to the product of mass and acceleration.

$$F_{net} = ma$$
$$m = 75 \text{ kg}$$
$$a = 1.5 \text{ m/sec}^2$$
$$F_{net} = (75 \text{ kg})(1.5 \text{ m/sec}^2) = 112.5 \text{ N}$$

Gravity exerts a constant downward force on the person equal to weight F_w. In order that the person experience an upward acceleration, the elevator floor must exert an upward force F_e that is greater than the weight. The net force will equal the excess of F_e over F_w.

$$F_{net} = F_e - F_w$$

$$F_e = F_{net} + F_w$$

$$F_{net} = 112 \text{ N}$$

$$F_e = ma_g$$

$$F_w = 75 \text{ kg} \times 10 \text{ m/sec}^2 = 750 \text{ N}$$

$$F_e = 112 \text{ N} + 750 \text{ N}$$

$$F_e = 862 \text{ N upward}$$

(ii) The net force on any body is zero if it moves with constant velocity. Therefore, the elevator floor must exert an upward force F_e equal in magnitude to the person's weight F_w. F_w has already been found to be 750 N.
(iii) A person accelerating downward experiences a net force downward. Again, the net force will equal the difference of F_e and F_w. However, in this case, the person's weight F_w must be greater than the upward force F_e exerted by the elevator floor.

$$F_{net} = F_w - F_e$$

$$F_e = F_w - F_{net}$$

$$F_{net} = 112 \text{ N}$$

$$F_w = 750 \text{ N}$$

$$F_e = 750 \text{ N} - 112 \text{ N}$$

$$F_e = 638 \text{ N upward}$$

(b) According to Newton's third law, for every force there is an equal and opposite force. When the elevator floor exerts a certain force on the person, the person will in turn exert an equal force (in the opposite direction) on the floor or scale. The bathroom scale would read the values calculated in (a) for each of the three cases.

(c) As a result of the different forces in the conditions examined above, it does appear that the person's weight changes, since we are accustomed to associating weight with the force we exert against the floor (or vice versa according to Newton's third law). We should remember, however, that since we defined weight as $F_w = ma_g$, the actual weight does not change. The apparent change was due to the accelerated frame of reference.

33. *See Text* page 99.

34. (a) $F_{net} = F_R - F_L$

$$= 20 \text{ N} - 5 \text{ N}$$

$$= 15 \text{ N right}$$

$$a = \frac{F_{net}}{m}$$

$$= \frac{15 \text{ kg} \cdot \text{m/sec}^2}{5 \text{ kg}}$$

$$= 3 \text{ m/sec}^2$$

$$d = \tfrac{1}{2}at^2$$

$$= \tfrac{1}{2}(3 \text{ m/sec}^2)(10 \text{ sec})^2$$

$$= 150 \text{ m}$$

(b) $F_{net} = F_2 - F_1$

$$= 1{,}200 \text{ N} - 200 \text{ N}$$

$$= 1{,}000 \text{ N}$$

$$a = \frac{F_{net}}{m}$$

$$= \frac{1{,}000 \text{ kg} \cdot \text{m/sec}^2}{50 \text{ kg}}$$

$$= 20 \text{ m/sec}^2$$

$$v = at$$

$$= (2 \text{ m/sec}^2)(20 \text{ sec})$$

$$= 400 \text{ m/sec}$$

(c) $\Delta v = v_2 - v_1$

$$= 80 \text{ m/sec} - 40 \text{ m/sec}$$

$$= 40 \text{ m/sec}$$

$$a = \frac{\Delta v}{t}$$

$$= \frac{40 \text{ m/sec}}{10 \text{ sec}}$$

$$= 4 \text{ m/sec}^2$$

$$F = ma$$

$$= (4 \text{ kg})(4 \text{ m/sec}^2)$$

$$= 16 \text{ N}$$

(d) $F_{net} = F_2 - F_1$

$$= 40 \text{ N} - 15 \text{ N}$$

$$= 25 \text{ N}$$

$$a = \frac{\Delta v}{t}$$

$$= \frac{55 \text{ m/sec}}{11 \text{ sec}}$$

$$= 5 \text{ m/sec}^2$$

$$F = ma$$

$$m = \frac{F}{a}$$

$$= \frac{25 \text{ kg} \cdot \text{m/sec}^2}{5 \text{ m/sec}^2}$$

$$= 5 \text{ kg}$$

(e) $a = \dfrac{F}{m}$

$$= \frac{80 \text{ kg} \cdot \text{m/sec}^2}{5 \text{ kg}}$$

$$= 16 \text{ m/sec}^2$$

When m is reduced by one-half (2.5 kg)

$$a = \frac{80 \text{ kg} \cdot \text{m/sec}^2}{2.5 \text{ kg}}$$

$$= 32 \text{ m/sec}^2$$

(f) $$a_1 = \frac{F_{net}}{m_1}$$

$$= \frac{40 \text{ kg} \cdot \text{m/sec}^2 - 4 \text{ kg} \cdot \text{m/sec}^2}{3 \text{ kg}}$$

$$= 12 \text{ m/sec}^2$$

$$a_2 = \frac{F_{net}}{m_2}$$

$$= \frac{40 \text{ kg} \cdot \text{m/sec}^2 - 4 \text{ kg} \cdot \text{m/sec}^2}{9 \text{ kg}}$$

$$= 4 \text{ m/sec}^2$$

$$a_{total} = a_1 + a_2$$

$$= 12 \text{ m/sec}^2 + 4 \text{ m/sec}^2$$

$$= 16 \text{ m/sec}^2$$

$$d = \tfrac{1}{2} at^2$$

$$200 \text{ m} = \tfrac{1}{2} (16 \text{ m/sec}^2) t^2$$

$$25 \text{ sec}^2 = t^2$$

$$5 \text{ sec} = t$$

(g) $$F_{net} = F_R - (F_L + F_f)$$

$$= 18 \text{ N} - (15 \text{ N} + 3 \text{ N})$$

$$= 0$$

$$a = \frac{F}{m}$$

$$a = 0$$

CHAPTER 4

2. $F_{net} = ma$; when the rocket rises vertically,

$$F_{net} = \text{thrust} - \text{weight}$$

$$a = \frac{F_{net}}{m} = \frac{7.37 \times 10^6 - 5.4 \times 10^5 \times 9.8}{5.4 \times 10^5}$$

$a = 3.8 \text{ m/sec}^2$ = the acceleration at lift-off.

$d = \tfrac{1}{2}at^2$ As the fuel burns, the mass m decreases. Therefore, $\frac{F}{m}$ decreases.

$$t^2 = \frac{2d}{a}$$

$$t^2 = \frac{2 \times 50}{3.8} = 26.4 \text{ sec}^2$$

$$t = 5.1 \text{ sec}$$

3. No, the bullet will not follow the line of sight along the barrel. It will start to drop as soon as it clears the end of the gun.

Yes, the bottle will be hit. It and the bullet both fall with the same vertical acceleration, a_g.

In an analytical argument it will simplify the algebra to assume that the line of sight along the gun barrel is horizontal; that is, that the gun and the bottle are initially at the same height above the ground. Then it is quite easy to show that for an initial bullet speed v_b in the horizontal direction and an initial distance d_h between gun and bottle it will take d_h/v_b seconds for the bullet to reach the bottle. However, during this time the bullet and the bottle will both be accelerating at the same rate vertically and so both will fall a vertical distance $d_v = \tfrac{1}{2}a_g t^2$. Analysis of the more general case when the gun must be pointed at some angle with the horizontal requires resolving the initial bullet velocity into horizontal and vertical components. Perhaps some of the better students may wish to do this.

4.

when t =	2 sec	5 sec	10 sec
(a) if v_x = 4 m/sec, Δx =	8 m	20 m	40 m
if v_y = 3 m/sec, Δy =	6 m	15 m	30 m
(b) total distance $= \sqrt{x^2 + y^2}$ =	10 m	25 m	50 m
(c) $v = \frac{d}{t}$, $v = \sqrt{v_x^2 + v_y^2}$	5 m/sec	5 m/sec	5 m/sec

5. The general equation for a parabola is $y = ax^2 + bx + c$ when a, b, and c are constants. In this case

$x = v_x t$ and $y = v_y t + \tfrac{1}{2}a_g t^2$

Since $t = \frac{x}{v_x}$ we can substitute $\frac{x}{v_x}$ for t in the expression for y

$$y = v_y \left(\frac{x}{v_x}\right) + \tfrac{1}{2}a_g \left(\frac{x}{v_x}\right)^2$$

$$y = \left(\frac{v_y}{v_x}\right) x + \left(\frac{a_g}{2v_x^2}\right) x^2$$

Comparing this with the general equation for a parabola, we find that

$$a = \frac{a_g}{2v_x^2}, \quad b = \frac{v_y}{v_x}, \quad c = 0$$

Since a_g, v_x, and v_y are all constants, the trajectory is indeed a parabola.

6. $d_x = v_x t = 1.0 \text{ m/sec} \times 0.5 \text{ sec} = 0.5 \text{ m}$

The horizontal displacement therefore 0.5 m.

$d_y = \tfrac{1}{2}a_g t^2$

$$a_g = 10\ \text{m/sec}^2$$

$$d_y = \tfrac{1}{2}(10)(0.5)^2 = 1.25\ \text{m}$$

The vertical displacement is 1.25 m.

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{0.25 + 1.56} = \sqrt{1.81}$$
$$= 1.3\ \text{m}$$

The resultant displacement is 1.3 m.

The direction of d relative to the horizontal can be determined either by measurement of the angle formed by the 1.3-m hypotenuse with the horizontal 0.5-m leg of a right triangle on a carefully drawn scale diagram, or by trigonometry, since

$$\tan\theta = \frac{d_y}{d_x} = \frac{1.25}{0.5} = 2.5$$

Either method gives an angle of about 67° below the horizontal.

$$v = \sqrt{v_x^2 + v_y^2}$$
$$v_x = 1.0\ \text{m/sec}$$
$$v_y = a_g t = 5\ \text{m/sec}$$
$$v = \sqrt{1 + 25} = \sqrt{26} = 5.1\ \text{m/sec}$$

Either a scale diagram or trigonometry gives an angle of about 79° below the horizontal.

7. (a) $v = \dfrac{d}{t}$

$$t = \frac{d}{v}$$
$$= \frac{25\ \text{m}}{10\ \text{m/sec}}$$
$$= 2.5\ \text{sec}$$

(b) $d = \tfrac{1}{2}at^2$

$$= \tfrac{1}{2}(-9.8\ \text{m/sec}^2)(2.5\ \text{sec})^2$$
$$= -30.6\ \text{m}$$

(c) The horizontal velocity v_x must be large enough so that the vertical distance d_y does not exceed 20 m:

$$d_y = \tfrac{1}{2}at^2$$
$$20\ \text{m} = \tfrac{1}{2}(9.8\ \text{m/sec}^2)\ t^2$$
$$2\ \text{sec} = t$$
$$v_x = \frac{d_x}{t}$$
$$= \frac{25\ \text{m}}{2\ \text{sec}}$$
$$= 12.5\ \text{m/sec}$$

8. $v_f^2 = v_i^2 + 2ad$

$$= (-1\ \text{m/sec})^2 + 2(-9.8\ \text{m/sec}^2)(-45\ \text{m})$$
$$= -29.7\ \text{m/sec}$$
$$v_f = v_i + at$$
$$t = \frac{v_f - v_i}{a}$$
$$= \frac{-29.7\ \text{m/sec} - (-1\ \text{m/sec})}{-9.8\ \text{m/sec}^2}$$
$$= 2.93\ \text{sec}$$

9. (a) $d = \tfrac{1}{2}at^2$

$$80\ \text{m} = \tfrac{1}{2}(9.8\ \text{m/sec}^2)\ t^2$$
$$4.1\ \text{sec} = t$$

(b) Since time of fall is independent of the horizontal velocity, t does not change if v_x is doubled.

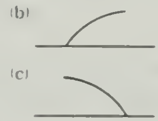
(c) $v_y = at$

$$= (-9.8\ \text{m/sec}^2)(4.1\ \text{sec})$$
$$= -40\ \text{m/sec}$$

(d) The horizontal velocity $v_x = 8$ m/sec.

10. They increase at the approximate rate of 1:3:5:7.

11. (a) The ball would move straight down.
(b) It would seem to travel along a parabola curving backward.



(c) It would seem to travel along a parabola curving forward.
(d) It would move along a straight line tending toward the rear of the van, the angle depending upon the magnitude of the van's acceleration relative to a_g.
(e) same as (b)
(f) same as (c)

12. The condition described could take place if one person in a train traveling at a uniform velocity let an object drop to the floor. To that person, the path would be a straight line. An outside observer watching the train go by would see the object fall along a path which, seen in that frame of reference, is a parabola.

13. None of the given alternatives describes the pilot's observations.
(a) The pilot will see the bullets move away eastward at 1,000 km/hr.
(b) The pilot will see the bullets move away westward at 1,000 km/hr.
(c) The pilot will see the bullets move straight down. In each of these cases, the plane's actual speed relative to the ground has no effect on the pilot's observations. (a), (b), and (c) would be the observations made when the earth is the frame of reference.

14. $t_1 = 1/f_1 = 1/16.6 = 6.0 \times 10^{-2}$ min
$t_2 = 1/f_2 = 1/33.3 = 3.0 \times 10^{-2}$ min
$t_3 = 1/f_3 = 1/45 = 2.2 \times 10^{-2}$ min
$t_4 = 1/f_4 = 1/78 = 1.3 \times 10^{-2}$ min

15. The passengers tend to move in a straight line at a uniform speed, while the car is being accelerated by a centripetal force toward the left. The door exerts a centripetal force on the passengers causing them to move contrary to this straight-line motion at a constant speed. The door is "thrown against the passengers." The passengers, of course, exert an equal and opposite force against the door.

16. (a) The loose surface may not be able to provide the force required to keep the car on the road.
 (b) Softer tires would give a larger surface-contact area. Therefore, less frictional force per square centimeter of road surface would be required.
 (c) A banked road exerts a force on the car as a reaction to the car's weight and its speed as it travels on a curved path. The force exerted perpendicular to the road surface now has a component directed inward toward the center of the curve, thus providing part of the required centripetal force.

Note: A complete discussion of all the forces and their angles relative to each other will be found on pp. 246–249, Vol. 1 of the book, *Physical Science: Its Structure and Development* by Edwin C. Kemble (M.I.T. Press, Cambridge, Mass. 1966).

17.

Name of Concept	Symbol	Definition	Example
Total distance	d	Length of a path between any two points as measured along the path	**The speedometer reading recorded on a trip from Los Angeles to San Diego and return**
Displacement	$\vec{d}$	**The straight-line distance and direction**	Straight-line distance and direction from Detroit to Chicago
Average speed or constant speed	v	**Time rate of change of total distance**	**A car drives 8 km through traffic in 20 min: v = 26 km/hr**
Instantaneous speed	v_i	**The value of the average speed taken for a very small time interval. If the calculation is made for a smaller time, v_i will not change.**	**Measurements from a high-speed strobe photograph of a pendulum show that Δd = 1.3 cm and Δt = 0.10 sec. Thus, v_i = 13 cm/sec.**
Velocity	$\vec{v}$	**Time rate of change of displacement**	An airplane flying west at 640 km/hr at constant altitude
Acceleration	$\vec{a}$	Time rate of change of velocity	**A car accelerates at 3 m/sec² toward the north**
Acceleration of gravity	$\vec{a}_g$	**The acceleration of a freely falling body**	**The acceleration of gravity in San Francisco is 9.800 m/sec² toward the center of the earth.**
Centripetal acceleration	$\vec{a}_c$	**Time rate of change of velocity toward the center of a circle**	**A child on a merry-go-round**
Frequency	f	**The number of complete cycles per unit of time**	The drive shaft of an automobile turns 600 rpm in low gear.
Period	T	The time it takes to make one complete revolution	**The period of a drive shaft turning 600 rpm is 0.1 sec.**

Note: Answers to be supplied by student are in bold type.

18. (a) $a_c = \dfrac{v^2}{R}$

$v = 2.5 \times 10^5$ m/sec

$R = (3 \times 10^4$ light year$)(9.46 \times 10^{15}$ m/light year$)$

$= 2.84 \times 10^{20}$ m

$$a_c = \frac{(2.5 \times 10^5 \text{ m/sec})^2}{2.84 \times 10^{20} \text{ m}}$$

$a_c = 2.2 \times 10^{-10}$ m/sec²

(b) $F_s = ma_c$

$m = 1.98 \times 10^{30}$ kg

$a_c = 2 \times 10^{-10}$ m/sec²

$F_s = 1.98 \times 10^{30}$ kg $\times 2 \times 10^{-10}$ m/sec

$F_s = 4 \times 10^{20}$ N

(c) $F_e = \dfrac{mv^2}{R}$

$m = 5.98 \times 10^{24}$ kg

$R = 1.495 \times 10^{11}$ m

$v = \dfrac{2\pi R}{T}$

$v = \dfrac{2\pi \times 1.495 \times 10^{11} \text{ m}}{1 \text{ yr} \times 365 \text{ day/yr} \times 24 \text{ hr/day} \times 3600 \text{ sec/hr}}$

$v = 2.98 \times 10^4$ m/sec

$F_e = \dfrac{5.98 \times 10^{24} \text{ kg } (2.98 \times 10^4 \text{ m/sec})^2}{1.495 \times 10^{11} \text{ m}}$

$F_e = 3.55 \times 10^{22}$ N

F_e is about 100 times greater than F_s.

19. (a) From the photograph, the radius of the circle seems to be about equal to the athlete's height, which can be estimated as 1.8 m. During the Olympic coverage of the hammer throw on TV, it appeared that the period of the swing was about 1 sec.

$$F_c = \frac{m\, 4\pi^2 R}{T^2} = \frac{7.27 \times 4 \times 9.86 \times 1.8}{1}$$

$$= 5.2 \times 10^2 \text{ N}$$

Because of the estimations involved, we give an order of magnitude value at 10^3 N.

(b) There must be an upward component of force sufficient to balance the downward force of gravity on the hammer. Also, there would be some air resistance to overcome.

20. Rectilinear motion is motion along a straight line. Example: a car moving along a straight road. The velocity at any instant will depend on the object's initial velocity and the length of time it has been subjected to an acceleration. In rectilinear motion, the only two possible directions for acceleration are in the direction of the original velocity or in opposition to it.

Projectile motion is the motion of a body that is not self-propelled and that has been launched with a specific initial velocity and then comes under the influence of a gravitational force (its weight).

Example: any object hurled into the air at any angle. If we neglect air resistance, as was done in this chapter, the object maintains a uniform horizontal velocity as long as it is in flight while being accelerated downward.

Uniform circular motion is motion at a constant speed along a circular path.

Example: a point on a rotating turntable. Although moving at uniform speed, the direction of the object's velocity is continually changing. At any instant the velocity is directed along a tangent to the circular path at the location of the object. It is subjected to an acceleration always at right angles to its direction of motion; that is, the acceleration is directed toward the center of the circular path. The acceleration in this case does not speed up or slow down the object but serves only to change its direction.

21. (a) $v = \dfrac{2\pi R}{T}$

$= \dfrac{2\pi(2 \text{ m})}{2 \text{ sec}}$

$= 6.3$ m/sec

(b) $a = \dfrac{v^2}{R}$

$= \dfrac{(6.3 \text{ m/sec})^2}{2 \text{ m}}$

$= 19.8 \text{ m/sec}^2$

(c) $F = ma$

$= (2 \text{ kg})(19.8 \text{ m/sec}^2)$

$= 36.9$ N

22. $F = \dfrac{mv^2}{R}$

$v^2 = \dfrac{FR}{m}$

$v^2 = \dfrac{(5 \text{ kg} \cdot \text{m/sec}^2)(5 \text{ m})}{2.5 \text{ kg}}$

$v^2 = 10 \text{ m}^2/\text{sec}^2$

$v = 3.2$ m/sec

23. $a = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$

$T^2 = \dfrac{4\pi^2 R}{a}$

$T^2 = \dfrac{4\pi^2 \text{ (3 m)}}{10 \text{ m sec}^2}$

$T^2 = 11.8 \text{ sec}^2$

$T = 3.4$ sec

The mass of the object is not involved since force is not considered.

24. $a = \dfrac{4\pi^2 R}{T^2}$ $\qquad f = \dfrac{1}{T}$

$= \dfrac{4\pi^2 \text{ (0.5 m)}}{(0.1 \text{ sec})^2}$ $\qquad = \dfrac{1}{0.1 \text{ sec}}$

$= 1{,}970 \text{ m sec}^2$ $\qquad = 10/\text{sec}$

25. (a) Syncom 2 has the most nearly circular orbit since the distance from the surface (and also from the earth's center) varies by only 8 km. If the radius of the earth is taken as 6,400 km, this is a difference of only about 0.02%. (8 km/35,520 km × 100% = 0.02%.)

(b) Without actually calculating the eccentricity, it would be reasonable to estimate which satellite has the greatest percentage variation in its greatest and least distance from the *center* of the earth. This is Lunik 3.

Eccentricity is explained in detail in Unit 2. The actual calculations for the two most obviously eccentric satellites are:

Lunik 3 $e = c/a = 208{,}800/263{,}200 = 0.80$
Luna 4 $e' = c'/a' = 303{,}200/399{,}200 = 0.76$

Note that the percentage of variation is the obvious method.

(c) Luna 4

(d) The earth rotates once in 1,440 min and Syncom 2 orbits once in 1,460 min. If the satellite begins directly overhead, it will be only 5° to the west in 24 hr. That is, it will take 20 additional minutes to reach the position directly overhead.

The following relationships between degree and time measurement were used:

$$24 \text{ hr} = 360°$$
$$1 \text{ hr} = 15°$$
$$20/60 \text{ hr} = 5°$$

It is recommended that "star time" and "sun time" not be discussed. The point of the problem is to appreciate a near-synchronous orbit.

26. $a_g = \dfrac{v^2}{R}$

$$v^2 = a_g R$$
$$v^2 = (8.7 \text{ m/sec}^2)(6.8 \times 10^6 \text{ m})$$
$$v^2 = 59.2 \times 10^6 \text{ m}^2/\text{sec}^2$$
$$v = 7{,}690 \text{ m/sec}$$

The mass of the satellite is not important.

27. $F = \dfrac{mv^2}{R} = \dfrac{4\pi^2 mR}{T^2}$

$$= \frac{4\pi^2 (500 \text{ kg})(18 \times 10^6 \text{ m})}{(22{,}800 \text{ sec})^2}$$
$$= 683 \text{ N}$$

28. This problem is the same as question 27, except that the center of motion is the moon rather than the earth. Therefore, the central (accelerating) force is one-sixth that of earth (as found in question 27):

$$F = \tfrac{1}{6}(683 \text{ N})$$
$$= 114 \text{ N}$$

For the same distance R (18,000 km) $a_{\text{moon}} = \frac{1}{6}\, a_{\text{earth}}$:

$$(aT^2)_{\text{moon}} = (aT^2)_{\text{earth}}$$
$$T^2_{\text{moon}} = \frac{(aT^2)_{\text{earth}}}{a_{\text{moon}}}$$
$$T^2_{\text{moon}} = \frac{(1)(380 \text{ min})^2}{1/6}$$
$$T_{\text{moon}} = 931 \text{ min}$$

29.

$a_c = \dfrac{4\pi^2 R}{T^2}$. But $a_c = a_g = 9.8 \text{ m/sec}^2$ at the earth's surface.

$$9.8 \text{ m/sec}^2 = \frac{(4)(9.9)(6.38 \times 10^6 \text{ m})}{T^2}$$
$$T = \sqrt{26 \times 10^6 \text{ sec}^2}$$
$$T = 5.1 \times 10^3 \text{ sec}$$
$$T = 85 \text{ min}$$
$$a_c = \frac{v^2}{R}$$
$$9.8 = \frac{v^2}{6.38 \times 10^6}$$
$$v = \sqrt{62 \times 10^6}$$
$$v = 7.9 \times 10^3 \text{ m/sec}$$

30. A satellite is held in its orbit only by the pull of gravity. As problem 29 shows, the shortest possible period for a satellite is 85 min. A shorter period would require a centripetal acceleration greater than that of gravity.

Yes, it is impossible.

31. $R = 110 \text{ km} + 1{,}730 \text{ km moon radius} = 1{,}840 \text{ km}$

$R = 1.84 \times 10^6$ m from the center of the moon

$a_c = 1.43 \text{ m/sec}^2$ is the acceleration 110 km from the surface of the moon.

Then:

$$a_c = \frac{4\pi^2 R}{T^2}$$
$$1.43 = \frac{(4)(9.9)(1.84 \times 10^6)}{T^2}$$
$$T = \sqrt{\frac{(4)(9.9)(1.84 \times 10^6)}{1.43}}$$
$$T = \sqrt{51 \times 10^6}$$
$$T = 7.1 \times 10^3 \text{ sec or } T = 1.2 \times 10^2 \text{ min}$$

32. Given:

a_g at moon's surface $= 1.5 \text{ m/sec}^2$
a_g at 100 km from moon's surface $= 1.43 \text{ m/sec}^2$
$d_v = 1.0 \times 10^2 \text{ km} = 1.0 \times 10^5 \text{ m}$
$v_x = 1.0 \times 10^2 \text{ m/sec}$

(a) $d_v = \frac{1}{2} a_g t^2$

$$1.0 \times 10^5 = (\tfrac{1}{2})(1.5)\, t^2$$
$$t = \sqrt{\frac{1.0 \times 10^5}{0.75}} = \sqrt{1.3 \times 10^5} = \sqrt{13 \times 10^4}$$
$$t = 3.6 \times 10^2 \text{ sec}$$

(b) $d_x = v_x t$

$$d_x = (1.0 \times 10^2)(3.6 \times 10^2)$$
$$d_x = 3.6 \times 10^4 \text{ m}$$
$$d_x = 36 \text{ km}$$

(c) All that can be estimated is that the braking must start at a distance greater than 36 km from the landing target. In order to answer the question in more detail, one would have to gather the following information:

(1) lunar preorbital injection speed at 100 km
(2) thrust value of the engines in newtons
(3) the mass of Apollo 8
(4) the desired time of burning in seconds

33. Given:

v_0 = speed necessary for orbit
v = preinjection speed
F = thrust
m = mass

To calculate time, Δt, for engine to burn

$$a = \frac{v_0 - v}{\Delta t}$$

$$\Delta t = \frac{v_0 - v}{a}$$

$$F = ma$$

$$a = F/m$$

$$\Delta t = \frac{v_0 - v}{F/m}$$

$$\Delta t = (m/F)(v_0 - v)$$

34. **1. Simplest motions**

(c) *car going from 50 km/hr to a complete stop* The car moves along a straight line. It is not clear, however, that acceleration is constant. We will assume that it is.
(f) *rock dropped 3 km* It moves along a straight line with a constant acceleration. Assume that air resistance has little effect.
(g) *person standing on a moving escalator* He or she moves at a constant speed in a straight line.

2. More complex motions

(b) *"human cannon ball" in flight* This is an example of projectile motion. Ideally, the path is a parabola and the velocity changes in magnitude and direction. This motion assumes that the horizontal component of the velocity is constant, which may not be exactly true due to air friction.
(e) *child riding a ferris wheel* We assume that the child travels in a circle at a constant speed. The magnitude of the acceleration is constant, while its direction changes uniformly.
(i) *person walking* The motion may have a regular rhythm and may be in a plane parallel to the earth. However, the direction and speed components of velocity are continually changing.
(h) *climber ascending Mt. Everest* The velocity will undergo many complicated changes.

3. Very complex motions

(a) *helicopter landing* This is a complex motion when one considers: the motion of each rotor; the motion of the helicopter as a whole. The rotors exhibit uniform circular motions at right angles to one another. Each rotor exerts a controlled force on the vehicle. The velocity of the whole helicopter is only at times constant and in straight lines.
(d) *tree growing* If this motion is considered for a short time period, such as 1 sec, it is a simple motion. However, if the motion is considered over a long time period of 25 yr, the motion is complicated.
(j) *leaf falling from a tree* This is the most complicated motion. The mass is so small that frictional and gravitational forces will produce large and varied accelerations. An additional complication lies in the three-dimensional nature of the motion due to wind and tumbling effects.

35. Some ideas that might well be included in the essay are:

1. Identification of probable details of how the photograph was made. For example how was the camera shutter controlled? Was the shutter open for a long or a short time? Was the shutter opened more than once to produce the final photo? If more than once, what was the probable order of magnitude of time between exposures?
2. How can each motion be identified as examples of uniform velocity and or uniform acceleration?
3. What forces seem to be acting in each case?
4. Could this picture be interpreted in more than one way?

Motion in the Heavens

Organization of Instruction

THE MULTI-MEDIA SCHEDULE

Day 1
Make assignments for student debates to be held on Day 6. You will need debaters, timekeepers, and judges. This is a good opportunity to work with the English department.

Day 2
Small groups plot data from *E1–1* and discuss questions from the experiment. If data were not available, use the data provided in *E2–1*.

A visit to a planetarium or an evening star-gazing session would be useful.

Day 3
Teacher presentation on Aristotle and Plato. Present and discuss the scientific and philosophical viewpoints of the ancients.

Organize debate activity for Day 6.

Day 4
Lab stations: Ptolemy

1. Epicycle machine (*Handbook*, Activities section)
2. *D28* (phases of the moon)
3. Film strip, "Retrograde Motion of Mars"
4. *L10* (retrograde motion—geocentric model)
5. Celestial sphere (*Handbook*, Activities section)
6. Making angular measurements (*Handbook*, Activities section)

Design activities so that students either move from station to station or select one station.

Day 5
Class discussion: Ptolemy and Copernicus

Answer questions that will arise with regard to geocentric and heliocentric celestial mechanics. *T13* and *T16* should be helpful.

Day 6
Student debate: The nature of the universe as described by Ptolemy and Copernicus. Students should present both viewpoints in standard debate form.

Day 7
Students collectively do *E2–6*, "The Shape of the Earth's Orbit." Several students read measurements of solar diameters from projected photographs and every student makes an orbit plot. Large sheets of graph paper are very helpful.

Day 8
Some students can assist the teacher in running help sessions to clear up all questions regarding *E2–6*.

Some students can work in small problem-solving groups.

Day 9
Divide class into small groups to discuss *Study Guide* questions. Circulate among these groups.
Take about 20 min to summarize Chapters 5 and 6. Explain evaluation procedure.

Day 10
Give a quiz and then discuss it. (Some other evaluation procedure as indicated in notes for Days 22–24 may be used instead.)

Day 11
Lab stations: Kepler
1. *T17* (orbit parameters)
2. *L11* (retrograde motion)
3. *E2-7* (using lenses)
4. Three-dimensional model
5. *D30* (heliocentric model)
6. Drawing ellipses (use pins and string)

Day 12
Teacher presentation or class discussion
Possible discussion topics:
1. models of the universe (Aristotle to Kepler)
2. the changing nature of physical laws
3. separation of celestial physics and terrestrial physics in history
4. Kepler's law

Day 13
E2–8 (orbit of Mars)
Students plot Mars' orbit on the graph they made on Day 7.

Day 14
Teacher can discuss with the class the details of *E2–8*. In addition to answering questions and giving individual help, point out some possible choices for student activities on Days 20 and 21. Refer to Days 20 and 21 for ideas.

Day 15
Divide the class into small problem-solving groups. They might discuss the assigned problems, work on others of their own choice, help one another, or work independently. Give concrete help to each group as you circulate. Teach to the point of a specific question.

Day 16
Lab stations: Newton
1. *T18* (motion under central forces)
2. *L12* (Jupiter satellite orbit)
3. Photometry: With a light meter measure intensity at various distances from a small light source.
4. Radioactivity: Measure intensity of radiation at various distances.
5. Sound: Microphone and amplifier drive a "Vu" meter. Measure intensities at various distances.

Suggestions 4, 5 and 6 are intended to illustrate the inverse-square law. (Consult Unit 3 for details of apparatus.) Arrange equipment so that students may stay in one group all period.

Day 17
Students demonstrate activities carried on during Day 16 and show results to class.

Day 18
Teacher presentation: The Newtonian synthesis
At this time Kepler's laws, Galileo's observations, and terrestrial physics are combined into one law. See Holton and Roller, *Foundations of Modern Physical Science*, Chapters 11 and 12; Kemble, *Physical Science, Its Structure and Development*, Chapter 9; and Andrade, *Sir Isaac Newton*.

Day 19
By equating the centripetal force to the gravitational force, show how one can calculate the mass of Jupiter. This is a startling achievement of Newton's work. Refer to *Text* page 228.
Organize optional activities for the next two days.

Day 20
Student option
In small groups or individually, students may plan their own activity for this day. Possibilities include:
1. *E2-2* (size of the earth)
2. *E2-4* (the height of Piton)
3. *E2-7* (using lenses to make a telescope)
4. *E2-11* (stepwise approximation to an orbit)
5. *E2-9* (Inclination of Mars' orbit)
6. *E2-12* (Model of the orbit of Halley's comet)
7. *L12* (Jupiter satellite orbit)

Day 21
Student option
Some possibilities:
1. field trip to a planetarium
2. essay about Unit 2 topics
3. PSSC Film #0309 "Universal Gravitation" followed by discussion

Days 22-24
One method of evaluation is to review, test, and discuss the test. Devote a day to each activity.
Another method of evaluation is through individual student-teacher conferences during a period of three days. Evaluation can be based upon laboratory reports, essays, poems, equipment design, sets of *Study Guide* answers, etc.
Note that two of these three days of testing could be done at other times during the 24 days.

Unit 2 SAMPLE MULTI-MEDIA SYSTEMS APPROACH

Note: This is just one path of many that a teacher may take through Unit 2. In this system, the teacher is a manipulator of environment and a tutor.

Day	Activity	Assignment
1	Small-group discussion	Text: Prologue to Unit 2 Handbook: Survey Ch. 5
2	Exchange and plot data from E2-1: Naked-Eye Astronomy	Text: 5.1–5.4
3	Teacher presentation: Plato's and Aristotle's Views	Text: 5.5–5.9
4	Lab stations: Ptolemy	Text: 6.1–6.5
5	Class discussion: Ptolemy and Copernicus	Prepare for debate
6	Student debate	Handbook: E2-6
7	Filmstrip demonstration E2-6, The Shape of the Earth's Orbit	Finish orbit plot Text: 6.6–6.8
8	Help session on E2-6 or Problem-solving session	Selected Study Guide quest.
9	Small-group problem solving	Review previous work in Unit 2
10	Quiz on Ch. 5 & 6 or Other evaluation	Handbook Survey Ch. 7
11	Lab Stations: Kepler	Text: 7.1–7.4
12	Teacher presentation: Brahe versus Kepler Kepler's laws	Bring earth plot to class Text: 7.5–7.9
13	E2-8 Orbit of Mars	Handbook: E2-8 Finish Mars plot
14	Class discussion on E2-8 Orbit of Mars	Selected Study Guide questions
15	Small-group problem solving	Text: 8.1–8.4
16	Lab stations: Newton	Handbook: Survey Ch. 8 for options days 20 & 21
17	Demonstrations from day 16	Text: 8.5–8.7
18	Teacher presentation: The Newton Synthesis	Text: 8.8–8.10
19	Teacher presentation: The Mass of Jupiter; and Organization of Student option	Prepare for optional activity
20	Student option	Student assignment
21	Student option	Text: Unit 2 Epilogue Review Unit 2
22	Review or Other evaluation	Review Unit 2
23	Test or Other evaluation	
24	Discuss test or Other evaluation	

Unit 2 SUGGESTED SCHEDULE BLOCKS AND TIMETABLE

Each block represents one day of classroom activity and implies approximately a 50-min period.

CHAPTER 5 WHERE IS THE EARTH? THE GREEKS' ANSWERS — **CHAPTER 6**

	Text: Prologue HB: Survey Ch. 5	Text: 5.1–5.4	Text: 5.5–5.9	Text: 6.1–6.5
Small-group discussion	Lab E2-1: Naked-Eye Astronomy Lab E2-3: Distance to the Moon	Teacher presentation: Plato and Aristotle	Lab stations: Ptolemy (See day 4.)	Class discussion: Ptolemy and Copernicus

DOES THE EARTH MOVE?

Prepare debate	HB: E2-6	Text: 6.6–6.8	Selected SG Quest.	Review
Student debate on Ptolemaic versus Copernican models	Lab E2-6: The Shape of the Earth's Orbit	Discussion: Lab E2-6	Small-group problem solving	Quiz Ch. 5 and Ch. 6 or Other evaluation

CHAPTER 7 A NEW UNIVERSE APPEARS

HB: Survey Ch. 7	Text: 7.1–7.4	Bring earth plot Text: 7.5–7.9	HB: E 2-10 The Orbit of Mercury Finish Mars Plot	Selected SG Quest.
Lab. stations: Kepler (See day 11.)	Teacher presentation: Brahe versus Kepler Kepler's Laws	Labs E2-8 and E2-9: Orbit of Mars	Class discussion: Lab E2-8	Small-group problem solving

CHAPTER 8 THE UNITY OF EARTH AND SKY: THE WORK OF NEWTON

Text: 8.1–8.4	HB: Survey Ch. 8	Text: 8.5–8.7	Text: 8.8–8.10	Student assignment
Lab stations: Newton (See day 16.)	Student demonstrations	Teacher presentation: Newtonian Synthesis	Teacher presentation: Mass of Jupiter and student options	Student option

Student assignment	Text: Epilogue Review Unit 2	Review Unit 2	
Student option	Evaluation or Review	Evaluation or Test	Evaluation or Discuss test

CHAPTER 5 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
5.1 Motions of the sun and stars	2 4	3 5		E 2-1 Naked-eye astronomy E 2-2 Size of the earth E 2-3 Distance to the Moon	T 13 Stellar Motion R2 Roll Call T 14 Celestial Sphere F 6 Universe—NASA (Prologue) F 7 Mystery of Stonehenge—McGraw-Hill (Prologue)	How long is a sidereal day? Build a sundial	5.1
5.2 Motions of the moon	6			E 2-4 The height of Piton, a mountain on the moon D 28 Phases of the moon		Stonehenge	5.2
5.3 The "wandering" stars	7				Film Strip—Retrograde motion of Mars	Making angular measurements Scale model of the solar system	5.3
5.4 Plato's problem							5.4
5.5 The Greek idea of "explanation"							5.5
5.6 The first earth-centered solution				D 29 Geocentric epicycle model	T 13 Stellar motion T 14 The celestial sphere R 11 The Garden of Epicurus	Celestial sphere model	5.6
5.7 A sun-centered solution				D 30 Heliocentric model	T 15 Retrograde motion		5.7
5.8 The geocentric system of Ptolemy	10	8	9		T 16 Eccentrics and equants L 10 Retrograde motion—geocentric model	Epicycles and retrograde motion	5.8
5.9 Successes and limitations of the Ptolemaic model	11 12						5.9

CHAPTER 6 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
6.1 The Copernican system	3	2		E 2-5 Retrograde motion E 2-6 The shape of the earth's orbit	L 11 Retrograde motion—heliocentric model		6.1
6.2 New conclusions	5	4 6 8	7				6.2
6.3 Arguments for the Copernican system							6.3
6.4 Arguments against the Copernican system	10 11	9			F 8 Frames of reference—PSSC	Two activities on frames of reference	6.4
6.5 Historical consequences	12				F 9 Planets in orbit—EBF		6.5
6.6 Tycho Brahe		13			R 16 The Great Comet of 1965		6.6
6.7 Tycho's observations				E 2-7 Using lenses to make a telescope	R 3 A Night at the Observatory	Note suggestions related to E 2-7, such as observing sunspots.	6.7
6.8 Tycho's compromise system	14						6.8

CHAPTER 7 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
7.1 The abandonment of uniform circular motion		2			R 7 Kepler R 8 Kepler on Mars R 6 Kepler's Celestial Music		7.1
7.2 Kepler's law of areas	3 4			D 31 Plane motions E 2-8 Orbit of Mars	F 10 Elliptic orbits—PSSC	Three-dimensional model of two orbits	7.2
7.3 Kepler's law of elliptical orbits	5 6 11	7 8 9 10 13 14	12	E 2-9 Inclination of Mars orbit E 2-10 Orbit of Mercury D 32 Conic-sections model	T 17 Orbit parameters F 11 Measuring large distance—PSSC		7.3
7.4 Kepler's law of periods	18 22	16 17 19	15 20 21		T 17 Orbit parameters		7.4
7.5 The new concept of physical law							7.5
7.6 Galileo and Kepler		24 25 26			R 5 The Starry Messenger R 7 Kepler		7.6
7.7 The telescopic evidence			23		L 12 Jupiter satellite orbit R 3 A Night at the Observatory R 15 The Boy who Redeemed His Father's Name		7.7
7.8 Galileo focuses the controversy							7.8
7.9 Science and freedom	27				F 12 Of stars and men (about Galileo)—Columbia University Press	Galileo (in modern drama)	7.9

CHAPTER 8 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R),	Student Activities	
8.1 Newton and seventeenth century science					R 10 The Laws of Motion and Proposition One		8.1
8.2 Newton's *Principia*					R 9 Newton and The Principia	Haiku	8.2
8.3 The inverse-square law of planetary force	4	2 6	3	E 2-11 Stepwise approximation to an orbit	L 13 Program orbit I L 14 Program orbit II L 15 Central forces—Iterated blows		8.3
8.4 Law of universal gravitation	7	5			R 12 Universal Gravitation R 17 Gravity Experiments		8.4
8.5 Newton and hypotheses							8.5
8.6 The magnitude of planetary force							8.6
8.7 Planetary motion and gravitational constant	8 12		9 10 11			Forces of a pendulum	8.7
8.8 The value of G and the actual masses of planets			13 14 22		L 12 Jupiter satellite orbit		8.8
8.9 Further successes	17 18	15 16 19	20 21 23	E 2-12 Model of the Orbit of Halley's Comet	L 16 Kepler's laws L 17 Unusual orbits R 16 The Great Comet of 1965		8.9
8.10 Some effects and limitations of Newton's work	26 27 28 29 30		24 25			Other comet orbits Discovery of Neptune and Pluto	8.10
Epilogue					R 18 Space the Unconquerable R 19 Is There Intelligent Life beyond the Earth? R 25 Three Poetic Fragments about Astronomy	Trial of Copernicus	Epilogue

Background and Development

OVERVIEW OF UNIT 2

Unit 2 is a brief story of the physics that developed as people attempted to account for the motions of heavenly bodies. It is *not* a short course in astronomy. The prologue to Unit 2 gives a brief overview of the unit.

The climax of the unit is the work of Newton. For the first time in history, scientific generalizations to explain earthly events were found to apply to events in the heavens as well. This remarkable synthesis, summarized in Chapter 8, produced echoes in philosophy, poetry, economics, religion and even politics.

The early chapters are necessary to establish the nature and magnitude of the problem that Newton solved. They also show that observational data are necessary to the growth of a theory. Thus, Chapters 5, 6, and 7 are a prelude to Chapter 8 and constitute a case history in the development of science.

Chapter 5 constructs a model of the universe based upon the kinds of observations made by the ancients, the Greeks, and by people in modern times. It describes the motions of sun, moon, planets and stars as seen from a fixed-earth frame of reference. It relates Plato's model and Ptolemy's geocentric universe.

Chapter 6 describes the work of Copernicus and Tycho Brahe, discusses the arguments that developed, and cites the historic consequences of this radical view of the universe. The diligent observations and records kept by Tycho show the importance of such work to science.

The work of Tycho's successor, Johannes Kepler and that of Galileo are related in Chapter 7, as well as the world-shaking consequences of these works, which changed the course of religion, philosophy, and science.

Chapter 8 presents a profile of Newton, the individual, and an insight into the tremendous power of the synthesis of earthly and celestial mechanics. The cement of his synthesis, the law of universal gravitation, is developed, and several tests of the law are discussed.

CHAPTER 5 / WHERE IS THE EARTH? THE GREEKS' ANSWERS

5.1 | MOTIONS OF THE SUN AND STARS

When the change from the Julian to the Gregorian calendar was made in England in 1752, September 2 was followed by September 14 for a correction of 11 days. Many peasants are reported to have claimed they "wanted their eleven days back."

George Washington was actually born on February 11, 1732, according to the Julian calendar then used by the British. Scholars must be careful to distinguish Julian (Old Style) dates from Gregorian (New Style) dates on original documents from the latter half of the eighteenth century.

There are 88 official constellations. By international agreement all the boundaries have been defined along north–south or east–west lines, although older star maps show curved boundaries.

In the *Text* we have avoided referring to the zodiac and to sidereal time. Sidereal time is star time that gains on mean solar time by 3 min 56.6 sec per day, due to the motion of the earth about the sun.

Students may wish to read "Stonehenge" by Jacquetta Hawkes (*Scientific American*, June 1953, Vol. 188, No. 6) or *Stonehenge Decoded* by Gerald Hawkins, Doubleday, 1965.

5.2 | MOTIONS OF THE MOON

As recently as the Revolutionary War, before marine chronometers were developed to keep accurate time at sea, navigators depended upon the position of the moon among the stars to determine their longitude. Because the position of the moon among the stars changes rapidly, a precise observation of the moon could be used for this purpose. The procedure, known as "lunars," is mentioned in various historical novels as well as in more official documents.

Longitudes shown on old maps are often much in error. This reveals the difficulties of fixing the longitude of a place. The need for precise predictions of the moon's position greatly stimulated astronomical observations and theory.

The motions of the moon could be predicted more accurately after Newton's studies of gravitation. Today many of the very small residual variations are forecast from corrections based on observations of eclipses and the orbits of earth satellites rather than from theory.

5.3 | THE "WANDERING" STARS

Sections 5.1, 5.2, and 5.3 review the basic observations to be explained by a theory. If the students can complete the following table correctly, they know the major motions to be explained.

Motion	Stars	Sun	Moon	Planets
Daily motion from eastern horizon to western horizon	x	x	x	x
Generally move eastward among the stars		x	x	x
Moves N–S–N while moving eastward		x	x	x
Moves N–S–N in one year		x		
Moves N–S–N in one month			x	
Retrograde motion				x

5.4 | PLATO'S PROBLEM

In *Archimedes in the Middle Ages,* Vol. 1, Univ. of Wisconsin Press, 1964, Marshall Clagett reviews the history of the manuscripts available on the works of Archimedes. Because Archimedes was one of the giants of Hellenistic Greece, we might expect that many manuscripts would be available. However, modern knowledge of Archimedes is actually based on three Byzantine Greek manuscripts, two of which were copied in the Middle Ages, but are lost.

The discovery of the third manuscript in 1906 by Heilberg is itself an exciting story. This manuscript is of the type known as palimpsest; that is, a parchment used a second time after the initial writing has been erased. The first writing can be revealed with infrared light. Careful scientific detective work by Heilberg revealed that a religious tract covered the precious copy of Archimedes' *On the Method,* previously not available.

Some pupils might wish to investigate how dates are derived for old manuscripts and what means, in addition to infrared light, are used to study them.

5.5 | THE GREEK IDEA OF "EXPLANATION"

It may be useful to refer back to these three features of an "explanation" as the course proceeds.

In Unit 3, the kinetic-molecular theory of gases is developed in a very similar way. That discussion explains the nature of a gas by summarizing the laboratory observations of its properties in a series of mathematical statements. Each statement is then "explained" by inventing assumptions about gases that are the simplest possible ones consistent with the facts. This collection of assumptions finally suggests an imaginary model of a gas which, if correct, should exhibit all the properties that are actually observed. The model should also suggest properties of gases not yet observed. If further experiment confirms these suggested properties, our faith in the accuracy of the model is increased.

The study of electricity and magnetism in Unit 4 leads to the electromagnetic theory in a similar way. In Unit 5 a like series of steps leads to an understanding of the outer structure of the atom.

5.6 | THE FIRST EARTH-CENTERED SOLUTION

To the ancients the earth was very large, immobile and at the center of all motions. It seemed easy to explain the motions of the fixed stars with the earth at the center. It is mentioned that Eudoxus, Plato's pupil, needed only 26 spheres to explain the general observations. Aristotle added 29 more mainly to provide enough motions to account for the various cycles observed. Thus, as more cycles were included for greater precision, more motions were needed.

From our point of view, one big drawback of the earth-centered scheme was its failure to precisely predict the positions of planets in the sky. But Greek science had different purposes than modern science; its theories were, at first, only intended to account for the general changes observed. The desire for greater precision came later.

Students should understand that it is impossible to describe *the* theory of the Greeks. There were many variations. Plato believed that the earth was spherical because of the shape of its shadow thrown on the moon at lunar eclipses. Heraclides of Pontus, who, like Aristotle, was a pupil of Plato, believed that the earth was at the center and rotated while the heavens were at rest.

Students will probably be amazed to find that in the thirteenth century most astronomical explanations were still those of Greek antiquity. To elaborate is to trace western civilization; perhaps some students will want to present a capsulated history to the class. Or perhaps a student will want to explain to the class how Dante in the *Divine Comedy* (1300 A.D.) described the spherical earth in the center of the world and the planets and stars moving in celestial spheres.

The Greek view of the arrangement of the planets has come down to us in the names of the days of the week. Students might note that many of the names we use (for example, "Thor's day") refer to characteristics of various gods and goddesses in the Greek and Teutonic mythologies. Language students will note that the day names in French, Spanish, and Italian are close to the original Greek names.

5.7 | A SUN-CENTERED SOLUTION

Through the *Almagest,* which circulated among scholars and students in the Middle Ages, the idea of a heliocentric system was known. Copernicus tried to defend a sun-centered system (see Chaper 6) and to refute the argument of Ptolemy in the *Almagest.*

The drawing on *Text* page 146 illustrates retrograde motion. It shows the earth and a planet moving in circular orbits around the sun. Both move at constant speed, but in this example the earth moves faster. When the earth is at points 1 and 2 the projection of the line of sight below shows that the planet appears to be moving to the right, ahead of the earth or eastward. Between points 3 and 5 the velocity of the earth perpendicular to the line of sight becomes greater than that of the planet, so the planet appears to be moving backward (westward or retrograde). At points 6 and 7, the earth's velocity perpendicular to the line of sight again becomes less than that of the planet (although its actual speed remains unchanged, of course) and hence the planet is seen to be moving eastward again. See *Transparency T15*.

See *Additional Background Articles* for a note on the sizes of and distances to the sun and moon by Aristarchus.

5.8 | THE GEOCENTRIC SYSTEM OF PTOLEMY

During the 500 years between Plato and Ptolemy the Greeks had made great achievements in geometry. Ptolemy applied some of these results in his attempt to find ways to predict the positions of the planets precisely. He was willing to sacrifice Plato's assumption of uniform angular motion around the centers of circles for greater precision in his predictions. Emphasis was upon the longitudes, or positions along the ecliptic, rather than upon the latitudes, or positions perpendicular to the ecliptic. The latitudes could be predicted, at least roughly, by tilting the planes of the epicycles a bit from the plane of the ecliptic.

The intent is *not* to sress the details of the various geometrical devices used by Ptolemy, but rather to indicate his ability to introduce many different types of motions to satisfy the increasingly more precise observations. To satisfy a variety of cycles found in planetary motions, Ptolemy introduced a variety of geometrical models: the eccentric, epicycle, and equant. He used geometry to solve problems for which we would now use trigonometric equations composed of terms containing sines and cosines of various angles.

See *Additional Background Articles* for notes on epicycles.

5.9 | SUCCESSES AND LIMITATIONS OF THE PTOLEMAIC MODEL

In summary, it appears that the Ptolemaic model meets the requirements for successful explanation discussed in Sec. 5.5. Even before starting the next chapter it may be worthwhile to consider why this apparently successful model is no longer accepted.

CHAPTER 6 / DOES THE EARTH MOVE? THE WORK OF COPERNICUS AND TYCHO

6.1 | THE COPERNICAN SYSTEM

Since both the Copernican and the Ptolemaic systems had to account for the same observations, the two systems had about the same number of motions. Copernicus also used Ptolemy's numerical constants, which described the magnitude of the motions. Consequently, the Copernican system was no more precise than the Ptolemaic system it proposed to replace. But increased precision was not really Copernicus' primary intention. He wished to purify the model, to describe all the motions on the basis of combined uniform circular motions.

Copernicus had been requested by Pope Paul III to assist in the reform of the calendar, a procedure which resulted later in the Gregorian calendar in current use. But Copernicus declined, claiming that a better calendar should be based on an improved system for predicting celestial events. Some idea of the complexity of forming a calendar for civil and religious purposes is included in the article "Calendar" in the *Encyclopaedia Britannica*.

About 1512 Copernicus prepared and circulated to a few friends the *Commentariolus*, which was a sketch of his proposed system. Through it a number of people learned a bit about the ideas he was developing. Later in his *Revolutionibus* Copernicus made some changes in the argument and added other, small cyclic motions, perhaps as a result of criticisms from his friends.

6.2 | NEW CONCLUSIONS

The orbital distances for Mercury and Venus were found from the maximum angular elongations from the sun. The orbit of Venus is almost circular. An optional experiment, *E2-10*, uses such observations to yield an orbit for Mercury.

As the table below suggests, students can work out their own approximate values from the diagram on *Text* page 163. (The data are not intended to be exact.)

Derivation of Planetary Orbits

Object	Diameter of Epicycle cm	Ratio Sun/Planet	Radius* of Deferent cm	Radius AU
Sun	2.00	1.00	1.00	1.0
Mars	3.25	0.61	2.62	1.6
Jupiter	2.15	0.93	4.40	5.0
Saturn	1.45	1.38	7.25	10.0

*The *diameters* of the deferents are not shown in the figure

While the details of these calculations will interest some students, it is more important for all students to realize that the heliocentric model allowed such results to be obtained for the first time in history.

At the end of this chapter we raise questions about the reality of these orbits. Certainly they seem much more "real" than the computing devices used by Ptolemy or the transparent crystalline spheres proposed earlier.

6.3 | ARGUMENTS FOR THE COPERNICAN SYSTEM

In some ways, the Copernican system was relatively simple, but its details were just as complex as the Ptolemaic system. The simplicity is essentially aesthetic or philosophical; that is, the *basic* idea as shown in the bottom diagram on *Text* page 163 is simple. Yet the computations needed to make precise predictions were just as complex as ever. In this "messiness" lay the motivation of Tycho Brahe and of Kepler to find a simpler model.

Useful background reading would be Chapter 1 of *The Origins of Modern Science* by Herbert Butterfield (Free Press, NY; 1951).

6.4 | ARGUMENTS AGAINST THE COPERNICAN SYSTEM

Members of all the religious groups attacked Copernicus; the attacks and ridicule were not limited to any one group. As the religious leaders realized, if the sun were the center of the system, the stars must be very far away and very luminous, perhaps even themselves suns. If they were suns, they might have planets, and these planets might support intelligent life. This idea, that there might be other planets around other stars, was called "the plurality of worlds." It presented the notion that the earth and our religious experiences here might not be unique. The possibility of life existing on bodies other than the earth was voiced only slowly, in England by Thomas Digges and on the Continent by Giordano Bruno, who was burned at the stake for heresy in 1600.

6.5 | HISTORICAL CONSEQUENCES

Follow *Text*.

6.6 | TYCHO BRAHE

Some background in regard to the "new" star observed by Tycho in 1572 will be helpful. A star is "new" only in the sense of becoming more observable or conspicuous. Current explanations describe the process as one in which the star's hydrogen content has gradually been consumed so that the outward radiation pressure within the gaseous star no longer balances the gravitational attraction toward the center. Then the star collapses. Very high central pressures are then devloped and the star flares up for a few years as a nova. Part of the outer envelope is blown off and, for fairly nearby novae, may later be detected as wispy filaments of outward moving gas. Apparently novae eventually settle down as "white dwarfs," still gaseous but having surface temperatures around 10,000°C and internal specific gravities of 10^5 or 10^6.

Other stars seem to undergo much greater outbursts and become supernovae. For a few years their luminosity, or actual output of radiant energy, may be as great as 10^8 times that of our sun. When a sun-sized star becomes a supernova, it blows off much of its mass and appears to change into a neutron star only a few kilometers in diameter. A larger star seems to turn into a black hole and "disappear" except for its intense gravitational field. Much of the information about these strange ex-supernovae is obtained with radio telescopes.

The novae observed in 1572 by Tycho, and in 1604 by Kepler and Galileo, were both supernovae. The supernova observed in Taurus in 1054 A.D. has resulted in the Crab Nebula. There is evidence that the Indians in western America saw this event since they cut in a rock face the symbols of a star and the crescent moon. What else could have been so impressive that it produced this record? Probably a star like this supernova was visible during the day. From oriental records and recent computations, it has been found that the nova appeared in July 1054 near the crescent moon.

Tycho's Uraniborg might be likened to one of our present large research centers supported by the government; perhaps the Brookhaven Laboratory, the Lawrence Laboratory, the Argonne Laboratory, or CERN in Switzerland. An article in *Scientific American*, February 1961, page 118, discusses Tycho's observatory.

Interested students might read about comets and how they are studied. *Sky and Telescope* for December 1965 and for 1973–74 included many articles and photographs of bright comets.

Perhaps some discussion of the many superstitions surrounding the unexpectedness of comets and novae would be useful to suggest the variety of ways in which people interpret unexpected events. The writings of Shakespeare contain many allusions to astronomical events as omens. Although we still have many rudiments of such superstitions with us, we may be increasingly conscious that we are reacting fearfully on the basis of unwarranted assumptions about the world.

6.7 | TYCHO'S OBSERVATIONS

As we saw earlier, Copernicus relied mainly upon the records of Ptolemy, which were inaccurate. These old observations had been made by different people at different times. Scholars still discuss the extent to which the observations in the *Almagest* were made by Ptolemy or were in part adopted and corrected from earlier work by Hipparchus about 150 B.C. Tycho concluded that new and more precise observations made over a number of years were essential before any new description of planetary motions could be created or judged.

The sections on Tycho's equipment might stimulate some students interested in mechanics and equipment design. The inherent limitations of our eyes also could be investigated through readings, or by a project for those interested.

6.8 | TYCHO'S COMPROMISE SYSTEM

Tycho's observations were planned as the basis for the development of a new model of planetary motions. Although he died before much of the analysis could be completed, his general idea of a system is that indicated by the lower drawing on *Text* page 175. Tycho's system had the planets moving around the sun as in the Copernican system. However, because he could not observe any parallactic shift that should have resulted from the motion of the earth around the sun, Tycho assumed a fixed earth with the sun orbiting it. In all other ways, Tycho's system was essentially the same as the Copernican system. Since no stellar parallax had been observed, many people accepted the Tychonic model rather than the Copernican.

At the end of the section we introduce the important question: "Are scientific models descriptions of reality, or only convenient computational devices?" The Ptolemaic system permitted computations of the positions of the planets: it did not attempt to describe reality. The heavens were visualized by the Greeks and by the medieval world in terms of crystalline spheres. This vision is described by Dante in the *Divine Comedy* (A.D. 1300). But Copernicus and Tycho were concerned with the *real* motions of the planets. Here, as well as elsewhere in the text, we raise the question about the reality of conclusions based on scientific theories. The point should be included in class discussion.

CHAPTER 7 / A NEW UNIVERSE APPEARS: THE WORK OF KEPLER AND GALILEO

7.1 | THE ABANDONMENT OF UNIFORM CIRCULAR MOTION

Kepler was a strange blend of mystic and scientist with a deep Pythagorean feeling for the numerical perfection in the world and the music of the spheres. His early paper on the spacing of planets and his later work on the third law suggest that sometimes scientists begin with aesthetic or artistic premises. The recent stress upon "symmetry" in particle physics is a similar example.

Due largely to the fact that Kepler inherited all of Tycho's data on Mars and had access to the writings of preceding astronomers, the time was ripe for new ideas *not* prejudiced by the assumption of uniform circular motion. You might remind students that in many instances in life one may be forced to reexamine early assumptions and perhaps to replace them.

Seen in historical perspective, Tycho and Kepler made an ideal pair. Tycho stressed the importance of improved observations and devoted his life to obtaining such observations. Without them, Kepler would have had the same difficulties as did his predecessors.

After more than 70 unsuccessful trials Kepler found that he could not fit the observations with any combination of circular motions. You might wish to dramatize the situation in which Kepler found himself. He felt that some satisfactory solution could be found. Since Mars continued to move across the sky, oblivious to Kepler's efforts, the trouble must lie with the theory. Therefore, he was obliged to look at the problem in a new way. This is always difficult for us, but Kepler did it.

His work, both unsuccessful and eventually successful, was laborious because the mathematical techniques of his time were cumbersome. Kepler was one of the first scientists to use logarithms.

Kepler was caught in the religious conflicts of the Thirty Years' War and the struggles between the Catholics and Protestants. At best he had a difficult time earning a living, despite the promises of the king. The trial of his mother for witchcraft might be paralleled with similar occurrences a bit later in the American colonies. It indicates the cultural and social context within which Kepler, like Galileo, was working. The popular book entitled *Kepler, 1571–1630* by Max Caspar might interest some students.

7.2 | KEPLER'S LAW OF AREAS

Kepler noticed that the speed of Mars changed as it moved through its orbit. His challenge was to find something constant about this changing speed. Students should be sure that they clearly understand this problem.

His first discovery, that a line drawn from the sun to Mars sweeps out equal areas during equal time intervals, reveals something unchanging or constant about this orbital speed. Such an unchanging mathematical construct is an empirical law. It is based upon observations; and it must only satisfy the constraint of accounting for those observations. As a matter of fact, all of Kepler's laws are *empirical*.

Note that *E2-6*, "The Shape of the Earth's Orbit," and *E2-8*, "Orbit of Mars," might be done even before reading the text.

7.3 | KEPLER'S LAW OF ELLIPTICAL ORBITS

The *Text* quoted Kepler's comment that, "Mars alone enables us to penetrate the secrets of astronomy which otherwise would remain forever hidden from us." This statement almost surely refers to the sizable eccentricity of the orbit of Mars ($e = 0.09$). Of the outer planets, only Mars is near enough to be studied accurately by Kepler's triangulation method. Although Mercury has a more eccentric orbit ($= 0.21$), studies of it were practically impossible by his method. Mercury is seen only in the twilight when few stars can be observed to determine accurate positions. Today telescopic observations of the position of Mercury and even of bright stars can be made in the daytime. Of course, Uranus, Neptune, and Pluto were unknown at the time of Kepler.

An excellent background on the mathematics of conic sections appears in the SMSG publication *Intermediate Mathematics, Teacher's Commentary*, Unit 19, Yale University Press, New Haven, 1961.

7.4 | KEPLER'S LAW OF PERIODS

Previously, Kepler discovered by trial and error laws that accounted for the shape of an orbit and the speed of a single planet. His next problem was to determine what is invariant about a set of planets that have different elliptical orbits and therefore different speeds. He discovered that the ratio of the square of each period to the cube of each average distance is constant. This is the law of periods.

Two insights might be stressed in class. One is that this last law is general since it deals with all of the satellites of the sun. The second insight is that the law does not show the relationship among satellite systems. That is, each satellite system has a different constant of proportionality between the square of the periods and the cube of the average distances.

$$\frac{T^3}{R^3} = K_{\text{Sun}}, \quad \frac{T^2}{R^3} = K_{\text{Earth}}, \text{ and } \frac{T^2}{R^3} = K_{\text{Jupiter}}$$

$$K_{\text{Sun}} \neq K_{\text{Earth}} \neq K_{\text{Jupiter}}$$

7.5 | THE NEW CONCEPT OF PHYSICAL LAW

Kepler's work reflects the change from a mystical interpretation of how the world ought to be to a reliance upon observations as the final basis for decisions. He had a growing feeling that some mechanism was essential to move the planets. We know that he often wrote to Galileo but that after a few letters from Galileo the correspondence was one-sided. Why Galileo did not accept the elliptical orbits of Kepler is difficult to understand. Perhaps it was because, as one authority noted, Kepler wrote in a flowery style that was often most difficult to understand. Unfortunately, his major contributions are buried in masses of words. (Is there a moral in this for your students? Have they examined some scientific and technical writing in professional journals?)

In the historical study of science, it is often difficult to establish who actually had an idea first. Ideas often grow as various people consider them and their consequences. The idea of the universe operating like a clockwork or a giant machine was implicit in the sequence of invisible celestial spheres proposed by Eudoxus. However, Kepler's analogy is important because, as Chapter 8 shows, this idea became firmly entrenched. Perhaps the ultimate form of the idea was the statement of one later scientist to the effect that, if he knew the initial positions and velocities of all the bodies in the universe, he could accurately predict the future of the universe. The great success of Newtonian mechanics, discussed in Chapter 8, supported such a mechanistic view. Only within the current century have physicists been obliged by new types of observations to abandon such sweeping general assertions (see Units 4, 5, and 6).

7.6 | GALILEO AND KEPLER

A number of lines of evidence, including Galileo's work in mechanics and the astronomical models of Copernicus and Kepler, were undermining the premises on which the Aristotelians based their arguments. Even such figures as the poet and writer John Milton in England were aware of what was happening. Milton visited Galileo in southern Europe during the summer of 1638. In the quoted section of *Paradise Lost*, the poet raises the question that had been rejected by the Ptolemaics.

Galileo was incensed that his contemporaries would not even use the telescope and try to refute his observations. They remained entrenched in their own ideas and wouldn't consider either challenging them by looking for themselves or accepting his reports. We all experience great difficulty in making a major shift in concepts. Certainly the shift from an earth-centered system was gigantic in its implications. Can students suggest other comparable shifts that have caused us to reinterpret the world and our place in it? Do not restrict the list to those shifts that seemed to appear abruptly. It required nearly 1,800 years for the sun-centered system to be considered seriously. The students might list such reinterpretations as determination of the age of the earth, the Darwinian theory of evolution, relativity, and Freud's psychoanalytic theories.

7.7 | THE TELESCOPIC EVIDENCE

The photograph on *Text* page 193 shows two of Galileo's telescopes, with which he saw and interpreted many new objects. His conclusions are even more important than his drawings of what he saw. Others might have viewed the moon but not have found the mountains he recognized. The difference between raw data and interpretation might be developed with the students. Although new instruments permit new observations, instruments only provide data that must then be interpreted.

7.8 | GALILEO FOCUSES THE CONTROVERSY

Follow *Text*.

7.9 | SCIENCE AND FREEDOM

Students may want to report on the history of the Catholic Church in the seventeenth century and to compare it with the Church in the twentieth century. Others may want to discuss the rise of the Protestant groups. Is it likely that a community hospitable to the followers of Martin Luther or John Calvin would be hospitable to new ideas in science?

Do not attempt to create a "hero and villain" image of Galileo and the Church. Rather try to have students examine the known facts objectively and conclude that there probably was error and provocation on both sides. *The Crime of Galileo* by de Santillana presents Galileo as a martyr. Is it a fair position?

CHAPTER 8 / THE UNITY OF EARTH AND SKY: THE WORK OF NEWTON

8.1 | NEWTON AND SEVENTEENTH-CENTURY SCIENCE

The emphasis in this section is upon the growing acceptance in northern Europe of the "new philosophy" of empirical, experimental science. In addition to the Royal Philosophical Society of London, there was in France the Academie des Sciences, and in Italy, the Accademia de Lincei (Lynxes) at Rome and the Accademia de Cimento at Florence. These scientific societies, first in Italy and then in England and France, were important because they allowed scientists to work and argue together and to publish journals that could be sent to their colleagues in other countries. Stress how the work of many people, as illustrated by the quote from Lord Rutherford on *Text* page 209, is demonstrated by the achievement of Newton.

The origins of the great generalizations of science can be traced in preceding decades. Any study of Newton's analysis should emphasize the importance of the historical background leading to Newton's great synthesis of his laws of motion and of universal gravitation. The barrier between celestial and terrestrial motions set up by Aristotle was gradually being broken down. Tycho Brahe located comets beyond the moon. Kepler replaced "perfect" circular motions by motions in elliptical orbits. Jeremiah Horrocks, born in the year Kepler's third law was published, entered Emmanuel College of the University of Cambridge at the age of 13. When 19 and curate of Hoole in Lancashire, he applied Kepler's first law to the motion of the moon around the earth. He even showed that the eccentricity of its orbit changed periodically and the major axis of the ellipse slowly rotated. This was 25 years before the youthful Newton conceived his ideas on universal gravitation, which were not published for yet another 20 years.

The persistent question that might be raised is whether this great theory (universal gravitation) is ever "proved." Students should conclude that a rigorous proof is not possible. Yet in spite of this, the theory seems to work well. It explains much that is known and predicts many other phenomena and quantities.

The arguments in this chapter follow Newton's rather closely, although some have been modified or reworded in the spirit of Newton for this discussion.

8.2 | NEWTON'S *PRINCIPIA*

The first edition of the *Principia* was published in 1686. The second edition in 1713 included many corrections to the first printing, some new arguments, and considerably more data on comets based mainly on Halley's work.

Various authors have repeatedly pointed out that Newton did not attempt to explain gravitation. He postulated an inverse square force of attraction between bodies and it worked. He did not know how it worked or why it seemed to be associated with masses. In his famous "General Scholium" at the end of the *Principia*, he observed that "he framed no hypotheses" on the nature of gravity. He was concerned, but had no conclusions that he wished to publish.

At this point you may wish to ask students about the usefulness of an undefined concept such as gravity. We can measure its effects, predict the outcome of certain experiments, and in general make some use of gravity, yet we do not know what it is. Einstein was working on a unification of several aspects of gravity at the time of his death. People involved in this research today have still not explained gravity.

8.3 | THE INVERSE-SQUARE LAW OF PLANETARY FORCE

Point out the shift in Newton's assumptions from the Greek notion of circular motion as perfect, to the inertial circular motion of Galileo discussed in Chapter 4, and then to the definition of inertial motion in an optically straight line. Also, Newton's idea was that circular motion is caused by a force in action, and he extended this to include the elliptical motion of Kepler's laws.

A Blend of the Laws of Kepler and Newton

The synthesis of Kepler's laws and Newton's laws to reach basic conclusions about the nature of the central force acting on the planets provides an excellent example of logical reasoning. The following material, taken from earlier editions of the *Text*, is included here so that the teacher can gradually develop the argument step by step with the class.

Newton's laws	*Kepler's laws*
1. A body continues in a state of rest, or of uniform motion in a straight line, unless acted upon by a net force (law of inertia).	1. The planets move in orbits that are ellipses and have the sun at one focus.
2. The net force acting on an object is directly proportional to and in the same direction as the acceleration.	2. The line from the sun to a planet sweeps over areas that are proportional to the time intervals.
3. To every action there is an equal and opposite reaction.	3. The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun ($T^2 = k R^3$).

According to Newton's first law, a change in motion, either in direction or amount, requires action of a net force. But, according to Kepler, the planets move in orbits that are ellipses, that is, curved orbits. Therefore, such a force is acting to change their motion. Notice that this conclusion does not specify the type or direction of the net force.

Combination of Newton's second law with the first two laws of Kepler clarifies the direction of the force. According to Newton's second law, the net force is exerted in the direction of the observed acceleration. So, what is the direction of the force acting on the planets? Newton showed by a geometrical analysis, which is developed in the *Text*, that a body moving under a central force will, when viewed from the center of the force, move according to Kepler's law of areas. But Kepler's law of areas relates the planets to the sun. Therefore, Newton could conclude that the sun at one focus of each ellipse was the source of the central force acting on the planets.

Newton then found that motion in an elliptical path (or a path defined by any of the conic sections mentioned in Chapter 7) would occur only when the central force was an inverse-square force, $F \sim \frac{1}{R^2}$. Thus, only an inverse-square force exerted by the sun would result in the observed elliptical orbits of the planets described by Kepler. Newton then clinched the argument by showing that such a force law would also result in Kepler's third law, the law of periods, $T^2 = kR^3$.

From this analysis Newton concluded that one general law of universal gravitation that applied to the earth and an apple also applied to the sun, planets, comets, and all other bodies moving in the solar system. This is the central argument of Newton's great synthesis.

Motion Under a Central Force

Your students may encounter difficulty in the geometric development of Newton's argument because they are unable to see how the method of measuring the triangular areas changes. To minimize this difficulty, make use of the drawings on *Text* pages 218 and 219, which demonstrate how each side of a triangle may be used as a base and how a perpendicular may be dropped from each vertex.

It might also be a good idea to emphasize the unexpectedness of the conclusion that the law of areas holds even when no central force is acting.

It might be useful to see Newton's original development of this argument in Book I of the *Principia*. While the students may not be able to follow the text, the wording of the propositions and scholiums and the illustrations will serve to demonstrate how neatly Newton tied his argumentative package together.

The universal law of gravitation was a very bold proposal. Dramatize the audacity of Newton to propose the universality of physical laws whose action could generally only be observed on the earth. The people of Newton's time were still bound by the concepts of separate worlds and other Aristotelian doctrines.

8.4 | LAW OF UNIVERSAL GRAVITATION

One of the high points of the text is the philosophy of the Newtonian synthesis. This asserts that graviation applies throughout the universe. Thus one law explains observations on the earth as well as in the heavens. Furthermore, universal gravitation is a synthesis in the sense that it accounts for all three of Kepler's laws.

Students are often fascinated by Descartes' alternate argument that a fluid causes the planets to stay in their orbits. Also, such basic questions in philosophy as the meaning of "explanation" and of "cause" can be the topics of discussions.

The French philosopher Descartes (1596–1650) proposed an alternate theory that all space was filled with a subtle, invisible fluid that carried the planets around the sun in a huge whirlpool-like motion. Descartes' theory was first published in 1644 and received wide acceptance on the continent. An English edition was finally published in London in 1682 before the *Principia* was published.

This theory was a popular nonmathematical statement read by large numbers of people and readily accepted as a better explanation than none. It sounded good and was not too radically different from the Aristotelian attitudes that the people had previously learned. Descartes' theory was widely taught, even at Cambridge, long after the publication of the *Principia*!

It might be interesting to point out that Voltaire's famous essay, "Elements of Newtonian Philosophy," was banned in France because the man in charge of permissions to publish was a Cartesian.

8.5 | NEWTON AND HYPOTHESES

The discussion raises the question of action at a distance. Note the quotation from Newton on *Text* page 222. Direct the students' attention to the fact that, from the observations of Tycho and the empirical relations of Kepler and of Galileo, Newton had been able to fashion an exceedingly general and abstract description of heavenly motions. But in the process he had been obliged to postulate the gravitational force that he could not explain. In much of science, as in mathematics, there are some postulates and axioms that cannot be analyzed within the problems considered. Occasionally someone can interpret one or more of these axioms by a more basic proposition.

8.6 | THE MAGNITUDE OF PLANETARY FORCE

Note that the discussion of geometric points in the case of Kepler's law of areas changes to a discussion about the masses of stones and planets. The idea of "mass" has already been introduced by Newton's second law of motion. Note particularly the argument on *Text* pages 224–226 that it is the *mass* of a body that is associated with the notion of gravitational force. This argument really marks Newton's great contribution, a leap in understanding from a consideration of the *direction* of the force to that of the *amount* of the force.

The gravitational constant G serves the same function as any constant that changes a proportion into an algebraic equation. In the case of equations involving physical quantities, the constant also serves as a "balancer of units." It might be worthwhile to remind the students at this point that symbols that stand for physical quantities are not sacred cows, and that they only mean what you want them to mean. It could also be pointed out, for example, that this same kind of operation involving a constant turns up in the algebraic form of Kepler's third law, where $T^2 \sim R^3$ becomes $T^2 = kR^3$.

8.7 | PLANETARY MOTION AND THE GRAVITATIONAL CONSTANT

The experiments that we can carry out in order to determine that G is a *universal* constant are limited in number. The extension to universality must be carried out in terms of a kind of well-sustained faith. Such a conclusion may come as a shock to those students who feel that science is a rational process that has little or no room for imagination or statements based upon "revelation." You have a ready-made situation for an interesting discussion.

The reason that the masses of the sun and Jupiter can be compared is that Jupiter acts like a miniature solar system. (Galileo had this thought upon identifying the satellites of Jupiter.) The only difference between the sun system and the Jupiter system, insofar as Kepler's law is concerned, is that each k involves a different central mass. As long as we can measure the R and T for a revolving body in each system, the two central masses can easily be compared. Note that the power of the law of universal gravitation is that one does not have to know the value of G in order to make such a comparison! By forming ratios of the equations (refer to *Text* pages 228–229), the constants, including undetermined G, cancel.

8.8 | THE VALUE OF *G* AND THE ACTUAL MASSES OF THE PLANETS

As long as Newton had to depend upon the use of ratios, in which G did not enter into the quantitative results, his statement about gravitation was really a *hypothesis*. Once G was measured, the hypothesis could really be called a *law*, since all quantities in the statement were now measurable.

Although the *Text* defines mass as the quantitative measure of the inertia of an object, we should distinguish the inertial mass of an object from its gravitational mass. The fact that we must exert a force F to give an object an acceleration a is a property of the inertial mass m_i. By Newton's law, we know that $F = m_i a$. This law presumably would hold whether or not we perform the experiment in a gravitational field. The weight W of an object and the attractive gravitational force F_g between objects, however, depend on the gravitational mass m_g. These phenomena are independent of the inertial properties of matter, but they could not exist without a gravitational field. In describing them, we should write

$$W = m_g a_g$$

and

$$F_g = \frac{GM_g m_g}{R^2}$$

8.9 | FURTHER SUCCESSES

Tides

Some of your students may have difficulty in understanding the concept of differential forces in the case of the pulls exerted on the earth by the sun and the moon. They ought to see that this difference is really a function of distance. Even though the gravitational pull of the sun on the earth is much greater than that of the moon, the sun is so far away that it does not "distinguish" between the near and far side of the earth. The moon, which is much closer, does.

If one had the data for high and low tides from all different parts of the earth, would that information be enough for the formation of a general predictive theory for the tides? In what way does the principle of universal gravitation become a "breakthrough" here?

Comets

Students will be able to find allusions to comets as omens in Shakespeare, Chaucer, Julius Caesar, and in all kinds of folklore. Remind your students to look carefully at the reproduction of the Bayeux tapestry in the *Text*.

Students might also be asked to refer back to the portion of the *Text* where the contribution of Tycho Brahe to the understanding of comets is discussed.

Halley's comet is forecast to return in 1986 and pass perihelion on February 9 of that year. Probably the comet will be detected early in 1985 and possibly earlier (when it is as far as Jupiter's distance from the sun). This will be a disappointing appearance of the comet: Perihelion passage will be on the far side of the sun and far south in the sky. Bright moonlight before and after perihelion passage will also blot out most of the faint tail when it is likely to be brightest. The best observations may be made in April 1986 when the comet is farther north in the sky, at about the earth's distance from the sun, and seen 90° from the sun (see *The Physics Teacher* 15, 260, 1977).

The model developed in the Unit 2 *Activity* can be used for the 1986 appearance as well as for that of 1910. The date of perihelion can be changed to February 9, 1986, and the calendar for the comet's motion then worked out before and after perihelion passage. The calendar for the earth's positions remains the same. With a simple model made of cardboard, students can see for themselves why the 1986 appearance of Halley's comet will be disappointing.

8.10 | SOME EFFECTS AND LIMITATIONS OF NEWTON'S WORK

It is worthwhile to discuss Newtonian physics at some length. Appreciation of the achievements of universal gravitation should be one of the foremost goals of *Project Physics*. Students have a right to see the "big picture."

Part of the big picture is that universal gravitation accounts for observations on the earth as well as in the heavens. Another aspect is that universal gravitation is a great theory based upon numerous laws (such as those of Kepler and of Newton), *a priori* statements (force acts through a distance), observations, and arbitrary definitions. Of course the most dramatic result of a great theory is its predictive power, for example, the expectation that Halley's comet is a satellite of the sun that will return every 75 years.

Finally, and perhaps most important, is the relationship of universal gravitation to small things (leading to quantum mechanics) and to large things (leading to relativity). This consideration will allow the teacher to give a preview of coming attractions in Units 3–6. Applying orbital ideas from planets to the tiny electron going around the nucleus gives rise to the quantum constraint of certain specified allowable orbits. Moving to large-scale considerations, Einstein saw the space around Jupiter as warped rather than as a simple Newtonian gravitational field ($g = \frac{F}{m}$). Perhaps it is wise to only hint at these relationships at this time.

EPILOGUE

Some students may want to look up the Encyclopedist movement in France, begun by Denis Diderot. The influence of Newton's work upon the great Voltaire is also worth some research. A good encyclopedia will certainly have much to say about this particular period in France and England, when ideas like "the rights of man" and "democracy" were emerging from the minds of people who were beginning to use logic rather than revelation as their approach to a philosophy of life.

The limitations of the Newtonian analysis of certain phenomena will become apparent later, when we discuss relativity and quantum mechanics.

The eighteenth century saw the rise of iatromechanics, that is, the concept that the human body is a machine whose parts run according to the laws of Newtonian dynamics. About 1745, a French doctor, La Mettrie, published a physiology text whose title was "L'Homme Machine" ("Man the Machine").

CONCEPT FLOW CHARTS

These charts are designed to help you follow the development and interconnection of ideas in the *Project Physics* course.

Three kinds of terms appear on each chart, differentiated by the form of the letters. Lower case (for example, "observations") refers to phenomena, observations, or experiments. Upper case (for example, "MODELS") refers to well-defined concepts, models, or theories. Italics (for example, "*Themes*") refers to sweepingly general ideas, themes, or viewpoints, which are often closely related to philosophy. There is sometimes room for argument as to which should be used, but for the most part observations are distinct from models, and both are distinct from general viewpoints.

Some of the conceptual themes are so general that it would be impractical to show all the arrows on the chart. For example, the idea of "mathematics as the proper language of nature" affected almost every development in physics from Galileo's time and perhaps long before. The ancient conceptual themes at the top of the chart were, of course, derived from observation and are all part of the even broader viewpoint of nature as knowable.

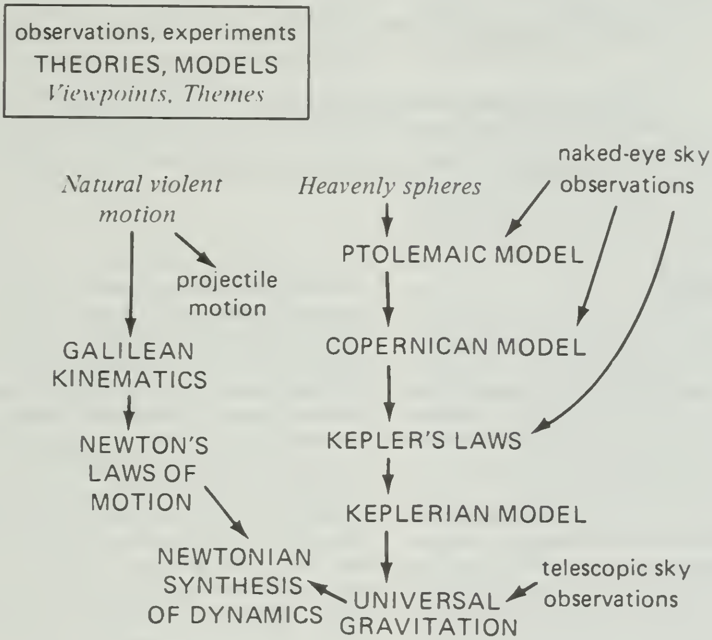
The arrows do not all mean exactly the same thing. Some imply a direct derivation, while others imply only a suspected influence. In any case, they represent only those connections that are discussed in the *Project Physics Text*.

There is a rough progression in time from top to bottom, but this is not rigorous and ought not to be taken too literally.

A word of caution! The interconnections of concepts, viewpoints, and experimental data in the development of physics are subtle, complex, and numerous. For every connection we know about, there are many more of which we may be ignorant or which we misunderstand. Even the necessarily simplified story that is sketched in the *Project Physics Text* is complicated and to some extent speculative. These charts are intended only as one way of outlining the connections as they are treated in the *Text*. The charts are, then, an incomplete and somewhat arbitrary skeleton of a *Text* that itself is incomplete. Furthermore, hardly any indication is given of the exceedingly important relations of physics to philosophy and social developments. These reservations keep us from calling the charts "The Story of Physics."

Nonetheless, the *Text* does tell a story something like that depicted on the chart, and some kind of map through the forest seems to be needed. However clear the account may be in the *Text*, it is necessarily cut into somewhat arbitrary chunks that are presented one after the other. The charts, if they do nothing else, show cross-connections between concurrent developments.

CONCEPT FLOW CHART UNIT 2



Additional Background Articles

BACKGROUND INFORMATION ON CALENDARS

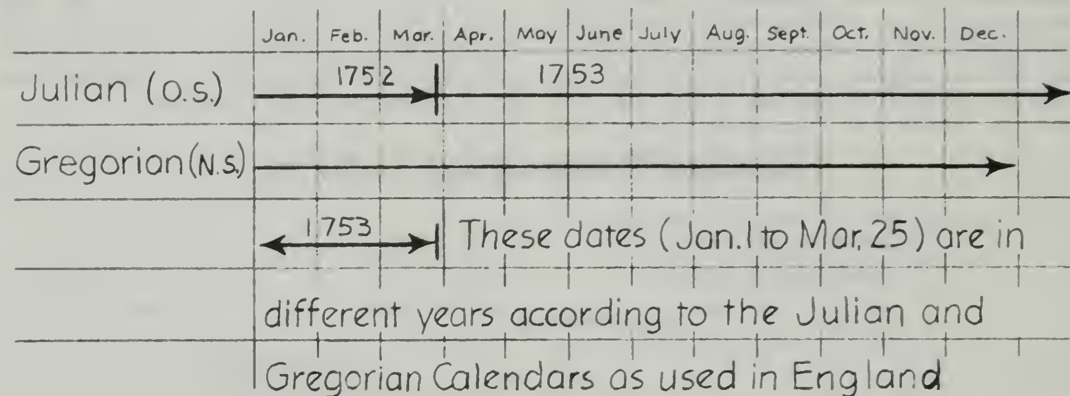
(Secs. 5.1 and 8.1)

In 45 B.C., Julius Caesar decreed a new civil calendar of 365¼ days. As the *Text*, Unit 2, indicates, this "Julian" year exceeded the actual motion of the sun by 11 min 14 sec per year. As a result, the Julian calendar was slow by one day in 128 years. By 1582 A.D., the Julian calendar was in error by 10 days and the sun passed the vernal equinox on March 11 rather than on March 21, as required by church canons. In 1582, Pope Gregory XIII abolished the old calendar and replaced it with a new civil calendar now known as the Gregorian or New Style Calendar. October 4, 1582 was followed by October 15th. The new calendar was immediately adopted by all Catholic countries but England and some other non-Catholic countries would not adopt the new calendar because it was established by Catholics. Not until 1752 did the Gregorian calendar finally become official in England, when September 2 was followed by September 14.

Some examples of the confusion presented to historians by the difference between the Julian and Gregorian calendars can be illustrated by the birth and death dates of Newton. Often Newton is said to have been born in the year that Galileo died. Galileo died in Italy on January 8, 1642 (New Style and Newton was born in England on December 25, 1642 (Old Style). When the date of Newton's birth is changed to New Style (Gregorian) it becomes January 5, 1643.

Newton's death is generally reported as occurring on March 20, 1727, yet the year carved on his tomb in Westminster Abbey is 1726. When the Calendar Act of 1750 went into effect in 1752, not only were 11 days dropped from the time record, but also the date of New Year's Day was changed from March 25 to January 1. Actually Newton died on March 20, 1726 (Old Style), but on the new calendar, which was adopted later, this became March 20, 1727 (New Style).

Schematically the change looked like the diagram below:



ARMILLARY SPHERE

(Secs. 5.1 to 5.3)

An armillary sphere is a mechanical device that shows the various coordinate systems used in the sky. Metal arcs are used to represent the horizon, the celestial equator, and the ecliptic, as well as north–south and east–west coordinates. You will find such a device very helpful as you try to visualize these imaginary lines in the sky.

Armillary spheres, and plastic spheres which can serve the same function, are available from several scientific equipment companies (for example, the Welch Scientific Company). However, you can make a reasonably satisfactory substitute from a hemispherical hanging-plant basket purchased from a garden supply store.

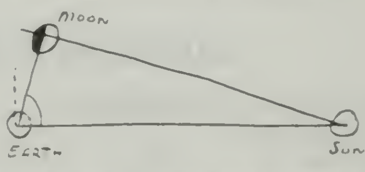
A wire basket 25 or 30 cm in diameter would probably by most useful. Two would make a sphere. They will have a great circle with ribs going toward the bottom (pole). One or two small circles of wire paralleling the great circle help support the ribs (meridians). You can add wire circles for other coordinates. For example, if the great circle represents the equator, add another great circle tipped at 23½° to show the ecliptic. Use bits of paper to locate some of the brighter stars.

NOTE ON THE SIZES AND DISTANCES TO THE SUN AND MOON, BY ARISTARCHUS

(Sec. 5.7)

This summary is based on a section in *A Sourcebook in Greek Science*, M. R. Cohen and I. E. Drabkin (McGraw-Hill, New York, 1948).

Aristarchus assumed that the moon was a sphere shining by reflected sunlight. As the figure shows in an exaggerated manner, when the moon appeared to be just half-illuminated, it would be located less than 90° from the sun. Aristarchus measured the angle at the earth between the sun and moon when the moon appeared to be exactly at first quarter (half-illuminated) as 87°. (Actually the angle is about 89°50′.) By a complicated geometrical analysis, he concluded that the sun must be between 18 and 20 times farther from the earth than the moon. But the distance to the moon was known approximately to be several hundred thousands of kilometers. Therefore, the distance to the sun must be several million kilometers.



The analysis also provided information on the sizes of the moon and sun. The moon was found to have a diameter about one third of that of the earth. Then the sun, having the same angular size at 18 times the moon's distance, must be at least 18/3, or 6 times the diameter of the earth, and 216 times the volume of the earth. To some philosophers, this raised a question of whether the larger body would move around the smaller one. Note that there was no evidence of concern for the masses of these bodies.

EPICYCLES

(Sec. 5.8)

The epicycle sketch on *Text* page 149 has a radius about half that of the deferent. Ptolemy's values for the planets are almost the same as those used by Copernicus and shown in Tables 6.1 and 6.2

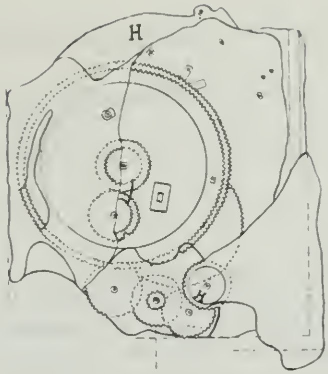
The rates of angular motion obtained by the use of epicycles did not agree well with the observations at certain sections of the orbits. As the bottom drawing on *Text* page 151 indicates, in a series of oppositions of Mars no two occurrences were identical. To provide a better fit between theory and observations, Ptolemy introduced another geometrical device called the equant. A planet P moved at a constant distance from the center, C. (Epicycles around P could also be added.) P moved at a uniform angular rate about an off-center point C′, while the observer was located on the earth, offset equally but oppositely to C′. The search for stability and uniformity, or predictability, required increasingly complex descriptions.

There is little evidence that anyone believed that the planets actually moved through space in paths described by Ptolemy. His analysis was strictly mathematical for the prediction of precise positions of each planet separately.

The drawings on *Text* page 151 illustrate a simplified scale diagram of the Ptolemaic system. The simplification results from the omission of the eccentrics and equants, and of the several motions of the moon. Notice that each planet had only one epicycle. All other cyclic motions were represented by eccentrics and equants. The very large epicycle for Venus occupies about three fourths of the space between the earth and sun. To Copernicus this was of special interest. With a protractor, students can check the angles subtended at the earth by the epicycles of Mercury and Venus to see if they agree with those shown in the drawing on *Text* page 141.

The lower drawing on *Text* page 151 shows that the radii of the epicycles for Mars, Jupiter, and Saturn had a period of 1 year and were always in line with the earth–sun line. This diagram will become important in Chapter 6 when we discuss how Copernicus replaced all these large epicycles by one annual motion for the earth, and derived distances to the orbits of the planets. In the Ptolemaic system, each planet was considered to be at a distance such that its motion did not quite overlap that of the adjacent planets. Actually, to Ptolemy these planetary distances were not important.

Our awareness of the advanced degree of Greek mathematics and technical skill was sharply increased by the discovery of the so-called Antikythera machine, named for an island near which it was found. About the size of a large book, this device apparently contained at least 20 gears and a crown wheel, as well as pointers moving over dials. While the use of this complex, but badly corroded, machine is still to be unravelled, it is suspected that it was used to compute the positions of the



sun and moon, and possibly of the planets too. The machine was recovered from the remains of a ship that sank about 65 B.C. (See diagram above.) Some students may want to read "An Ancient Greek Computer" by Derek J. De Solla Price in *Scientific American*, June, 1959.

NOTE ON THE "CHASE PROBLEM" (Sec. 6.2)

The motions of the hands of a clock provide a commonplace illustration of the "chase problem" described in the *Text*, Sec. 6.2. Because the hands are moving in the same direction and because the hour hand continually moves ahead, the minute hand must "chase" the hour hand in order to overtake and pass it. The questions listed below might be used to stimulate class discussion before or during a demonstration with a clock.

1. How many times does the minute hand overtake and pass the hour hand during an elapsed time of 12 hours? (11 times)

2. Starting with the hands in the 12 o'clock position, at what time will the minute hand overtake the hour hand? ($1^h05^m27^s$)

3. Can we derive an expression to show the relationship between the period of the minute hand, the period of the hour hand and the *synodic period* (the interval between successive overtakes)?

The rate of synodic motion, $\frac{1}{T_S}$, is the difference between the other two rates, or $\frac{1}{T_S} = \frac{1}{T_E} - \frac{1}{T_P}$. For T_S, this becomes $T_S = \frac{T_E T_P}{T_E - T_P}$.

Observe the hands of a clock or watch through 12 revolutions of the minute hand. Students may predict 11, 12, or 13 "overtakes," but there will be only 11.

Now find the relationship between the period of the hands and the synodic period; let T_m be the "sidereal period" of the minute hand. (The "sidereal period" is the time required for the revolving object to make one complete revolution, 60 min in this case.) Let T_h be the "sidereal period" of the hour hand (12 h or 720 min) and let T_s be the "synodic period" of the hands. In 1 min, the minute hand advances $\frac{1}{T_m}$ revolution; its rate of motion is $\frac{1}{T_m}$ revolution/min. Therefore, in time T_S, it must make $\frac{T_S}{T_m}$ revolutions. In the same time, the hour hand must make $\frac{T_S}{T_h}$ revolution at a rate of $\frac{1}{T_h}$ revolution/min.

Turn the hands of the clock through one synodic period. Note that the minute hand makes one complete revolution, then goes an additional fraction of a revolution, or angle, that is the same as that traversed by the hour hand. In symbols, $\frac{T_S}{T_m} = \frac{T_m}{T_h} + \frac{T_S}{T_h}$. This expression can be rearranged to find the value of any one of the quantities. Solving for T_S gives the synodic period:

$$T_S = \frac{T_m T_h}{T_h - T_m}$$

Substituting 60 min for T_m and 720 min for T_h gives T_S = 65.45 min, $1^h 05^m 27^s$, or 1.019 h.

This clock analogy is very close to the situation for earth and Jupiter. The synodic period is about 1.092 years, and Jupiter's sidereal period is then about 11.8 years.

A simple device to demonstrate these revolutionary relationships on an overhead projector can be constructed from a "dollar" pocket watch, a piece of 0.5-cm plastic and a few bits of wire. Remove the crystal from the watch and cement the watch, face up, to the center of a 30-cm × 30-cm sheet of plastic. Cement a 5-cm length of wire to the minute hand and a 10-cm length of wire to the hour hand. Cement small disks of paper to the ends of the wires to represent planets. Solder or braze a 17.5-cm length of heavy wire to the winding stem of the watch. Place the plastic on the stage of an overhead projector and rotate the hands slowly by twisting the wire soldered to the winding stem.

With a little more effort, the device described here can be used to show how the retrograde motion of the planets occurs. Instead of paper disks on the wires, cement thumbtacks with their points up. Then cut a very thin pointer from balsa wood so that it will ride on the two thumbtacks as the hands revolve. The pointer will show clearly the apparent backward motion of the outer "planet" as seen from the inner one, each time the minute hand overtakes the hour hand.

For the synodic period P_S of a planet of heliocentric period P_P seen from the earth having a period P_E, the equation above becomes $T_S = \frac{T_E T_P}{T_P - T_E}$. Reformed as $\frac{1}{T}$ to give the rate of motion, this becomes

$$\frac{1}{T_S} = \frac{1}{T_E} - \frac{1}{T_P}$$

But the synodic period T_S can be found by observation, while the earth's period T_E is known as 1 year. Then

$$\frac{1}{T_P} = \frac{1}{T_E} - \frac{1}{T_S}$$

which becomes

$$T_P = \frac{T_E}{\left(1 - \frac{T_E}{T_S}\right)}$$

For Mars, T_S = 780 days, while T_E = 365 days. Then

$$T_{Mars} = \frac{365 \text{ days}}{\left(1 - \frac{365}{780}\right)} = \frac{365 \text{ days}}{1 - .469} = 687 \text{ days}$$

If the planet moves inside the earth's orbit, the planet gains on the earth and the equation becomes

$$\frac{1}{T_P} = \frac{1}{T_E} + \frac{1}{T_S}, \text{ or } T_P = \frac{T_E}{\left(1 + \frac{T_E}{T_P}\right)}$$

Consider Venus, which comes to maximum elongations at an average interval of 584 days. Then

$$T_{Venus} = \frac{365 \text{ days}}{\left(1 + \frac{365}{584}\right)} = \frac{365}{1 + 0.626} = 225 \text{ days}$$

ATMOSPHERIC REFRACTION

(Sec. 6.7)

Any ray of light entering the earth's atmosphere at a slant is bent downward (refracted), with the result that the source appears to us to be higher above the horizon than it really is. The farther the body is from the observer's zenith (straight overhead), the greater is the length of the air path and the greater the angle at which the ray enters the atmosphere. As a result the amount of deviation due to refraction increases rapidly near the horizon. The curved atmosphere acts like a thin lens.

Because the amount of the refraction increases rapidly near the horizon, the observed image of the setting sun is distorted. The bottom limb of the sun is half a degree farther from the zenith than is the upper limb of the sun. Light from the lower limb is refracted upward more, and the sun takes on an elliptical, or oval, appearance.

An interesting consequence of this refractive effect is that the actual sun (no atmosphere) has set below the horizon before the lower limb of the apparent (refracted) sun touches the horizon. Notice also that most of the blue and much of the green light from sunlight is scattered by the atmosphere. The only light not strongly scattered (Rayleigh scattering) is red, the color of the setting sun.

If you wish to expand on this refractive effect and the color due to scattering, consider the appearance of the moon in total eclipse. Students may be surprised to learn, and perhaps can confirm from their own observations, that the eclipsed moon does not "go black." Instead it appears coppery red, even in the middle of the earth's shadow. With a bit of suggestion, students can conclude that the thin edge of the earth's atmosphere perpendicular to sunlight is acting like a thin lens. Thus, some sunlight is refracted into the earth's shadow. The path of this light through the earth's atmosphere is twice as long as the light we see from the setting sun. Therefore, only red light remains in the rays refracted into the earth's shadow.

Clearly, atmospheric refraction would result in errors in star positions unless corrected. Long sequences of careful observations, such as those made by Tycho, are needed before the corrections can be determined.

RELATIONS IN AN ELLIPSE

(Sec. 7.3)

a = major axis

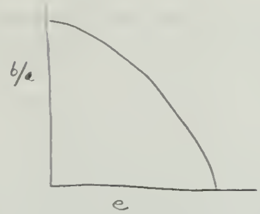
b = minor axis

c = distance between foci

e = eccentricity

$e = \frac{c}{a}$	$\frac{b}{a} = (1 - e^2)^{1/2}$
0.1	0.995
0.2	0.98
0.3	0.955
0.4	0.916
0.5	0.866
0.6	0.800
0.7	0.715
0.8	0.600
0.9	0.435
0.95	0.313

Give the equation $\left(\frac{b}{a}\right) = (1 - e^2)$ and suggest that someone work out this table and graph it. Students will be surprised to see how rapidly e changes with shapes, or how "round" is an ellipse of large e.



ABOUT MASS

(Sec. 8.8)

In Unit 1, Sec. 3.7, the concept of mass was introduced as something that is measured by inertia, the resistance to a change in motion. This sort of mass is called *inertial mass*, and it is used in Newton's second law, $F = ma$.

We measured the inertial mass of a body by seeing how its motion changes under the action of a known force.

In Sec. 3.8 the concept of mass was connected with gravitational forces of attraction. Here the mass of a body is a measure of the gravitational force that other bodies exert on it, or that it exerts on other bodies. Assigning a value to the mass of one particular body, we can, in principle, find the relative mass of any other body by measuring the gravitational force between the two. That sort of mass is called *gravitational mass*.

The question now arises whether the inertial and gravitational masses of a body are linearly proportional to each other. If we know how bodies accelerate when experiencing only a gravitational force, then we can tell whether inertial and gravitational masses are proportional. Consider two bodies A and B with inertial masses m_{A_i} and m_{B_i} and gravitational masses m_{A_g} and m_{B_g}. We put body A a distance R from a fixed third body with gravitational mass M, and we ask for the acceleration of body A when it experiences only the gravitational attraction due to M. Using Newton's second law, we have

$$\frac{Gm_{A_g}M}{R^2} = m_{A_i}a_A$$

$$a_A = \left(\frac{m_{A_g}}{m_{A_i}}\right)\frac{GM}{R^2} \qquad (1)$$

If we do the same with body B, we get

$$a_B = \left(\frac{m_{B_g}}{m_{B_i}}\right)\frac{GM}{R^2} \qquad (2)$$

Now if a body's gravitational mass is linearly proportional to its inertial mass; that is, $m_g = km_i$ in general, where k is a universal constant, then

$$m_{A_g} = km_{A_i}$$

and

$$m_{B_g} = km_{B_i}$$

If we put the first of these expressions into Equation (1) and the second into Equation (2), then we get

$$a_A = a_B = k\frac{GM}{R^2}$$

The two accelerations will be equal. To check this analysis all we need do is perform the experiment and see if the accelerations are equal.

If we use the earth as M, we would conclude that, at a given distance from the earth's center, all freely falling bodies should have the same acceleration. Therefore, experimental verification that this is true would be proof that inertial mass is proportional to gravitational mass. Unfortunately, the experiment is very difficult to perform with high precision.

Isaac Newton devised an experiment that tested the proposition in an indirect way. A pendulum bob is not a freely falling object, but in its motion to and fro it does accelerate, and the value of its acceleration governs the rate of oscillation. Newton was able to show that only if inertial mass is proportional to gravitational mass will the rate of oscillation be the same for pendulum bobs of different masses. Newton made a hollow pendulum bob in the form of a thin metal shell into which he put different materials, always being careful to see, by using an equal arm balance, that the weight of the material was the same each time. Since weight is a measure of gravitational mass, any difference in the rate of oscillation of the pendulum would be due to a difference in inertial mass. No such difference appeared, and Newton concluded that inertial and gravitational masses are equivalent.

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{But } \frac{m_i a}{m_g g} = \frac{x}{l}, \therefore \frac{x}{a} = \frac{l}{g}$$

$$\text{if } m_i = m_g$$

One can test for <u>each</u> bob separately, then test for various materials.

Professor Dicke and his co-workers at Princeton University believe they have shown the equivalence of inertial and gravitational mass to within 1 part in 10^{11} parts.

THE MOON'S IRREGULAR MOTION

(Secs. 8.3 and 8.9)

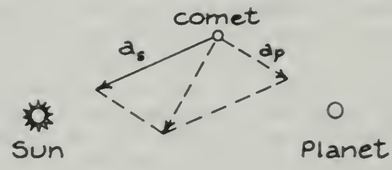
The observed motion of the moon contains many small variations that cannot be predicted by the simple assumption of a gravitational force between two mass points m_t and m_M.

Newton's investigations accounted for some of these discrepancies, but he studied only a few. Nevertheless, his theoretical results were reasonably close to the observed values of his time.

Though the process of applying the law of universal gravitation to separate sets of two-mass systems may seem to allow relatively easy solutions for motions, what happens when a <u>third</u> body gets involved? Thus, the sun–earth and earth–moon systems appear to be simple gravitational phenomena, but the reality is a single sun–moon–earth system that becomes so complex that a solution of its motions by gravitational theory becomes possible only under very limited conditions.

What if another large body (like Jupiter at 5 AU from the sun) were on the other side of the comet in the figure below? The student could then realize that the prediction of orbital motion would have to be painstakingly worked out by adding up all the acceleration vectors concerned. This suggests the real problem of computing paths or orbits for space probes, moon missions, and Mars and Venus fly-bys, where all the planets are attracting the space ship.

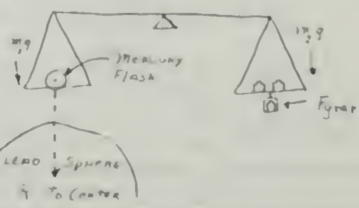
The computations are so lengthy and complex that precision orbits, docking, and other maneuvers would be impossible without high-speed computers.



MEASURING G

(Sec. 8.8)

It might also be easier for some students to understand a method of measuring G that was designed and carried out by a German physicist, Von Jolly, in the mid-nineteenth century. He used an equal-arm balance instead of the rather complicated torsion apparatus of Cavendish. On one side, Von Jolly put a spherical flask filled with mercury that he balanced with weights in the other pan. Then he put a large lead sphere below and close to the flask of mercury. He could determine the distance between the two spheres. The gravitational force between the two spheres caused the side of the balance with the flask to dip down slightly. Thus, the weights necessary to rebalance the equivalent were a measure of the F_{grav} between the spheres.



Here is a set of figures typical of the Von Jolly experiment that your students can use to calculate G for themselves:

m_1 (mass of mercury) = 5 kg
m_2 (mass of lead sphere) = 5775 kg
R (between sphere centers) = 0.57 m
F_{grav} = 0.59 mg (added for balance)
$= 5.9 \times 10^{-7}$ kg $\times$ 9.8 m/sec^2
$= 57.8 \times 10^{-7}$ N

Then

$$F_{grav} = \frac{Gm_1m_2}{R^2}$$

and

$$G = \frac{FR^2}{m_1m_2}$$

$$G = \frac{57.8 \times 10^{-7} \times (5.7 \times 10^{-1})^2}{5 \times 5.775 \times 10^3}$$

$$G = \text{about } 6.5 \times 10^{-11}\ \text{m}^3/\text{kg} \cdot \text{sec}^2$$

You might ask the students to indicate which of the above measurements present probable sources of error, and why. For example, does the lead ball also attract the weights in the other pan?

Another important point that could be made here is that the Cavendish experiment represents an *inertial* method of measuring G, while the Von Jolly experiment uses the *gravitational* method. You might wish to refer back to Sec. 3.8, Unit 1.

THEORIES (AN EXTENSION)

(Sec. 8.10)

Theories often have important practical applications. This is less apparent in the astronomical context, although the development of instruments and mathematics were influenced. Many other examples of more direct practical consequences will appear throughout the course. Currently, rocket development is having a major impact upon the design of many commerical products: Such events often occur without public notice.

As human creations, theories are produced, developed, judged, and applied by people with personal prejudices and frailties. Therefore, the combined judgment of many scientists is safer than the reaction of one. Yet history may show that, for a time, the one might be right and the majority wrong. It is important here to try to replace the all-too-common snap-judgment, good-or-bad evaluation of new ideas by a critical interest in theories as *possibilities*. One can be informed about and interested in a new theory without necessarily accepting or rejecting the theory. Suspended judgment is often a mark of maturity.

Can the students suggest other theories in science, government, art, or economics, which at first seemed shocking, yet have become commonly accepted? Perhaps impressionistic art, now commonly used in advertising, would be an example. What was the public reaction to Manet and Picasso? Or have someone interested in music report on the initial reception of the compositions of Wagner, Brahms, or Stravinsky. (The latter's ballet, *The Rite of Spring*, was loudly booed when first performed in Paris in 1913). Or perhaps consider the acceptance of James Joyce's *Ulysses*.

Contest among ideas, the trial by combat, is essential in science and every other field of human creation. Supine acceptance or adulation, or casual rejection, of a great individual's creation is naive.

Theories are changed over time. They are not fixed and permanent to be idolized, but rather are working tools to be used and resharpened. Rarely is a theory completely abandoned. Most are modified, but some are replaced. Scientists, like other people, cannot tolerate a complete absence of some sort of explanation. They will not completely abandon an old theory, even if it is known to have serious limitations. At least it worked in some cases, and still satisfies some phenomena.

In many ways, scientists are artists. Each is a specialist in the study and interpretation of some set of phenomena. Each brings to his or her work a general sense of what types of theories and explanations are satisfying. That is, scientists have personal styles. Some are mainly concerned about the precision of measurement and the design of equipment. Others look at theories as the bases for predictions. Still others try to imagine a variety of possible explanations, some of which are more daring than others. Einstein and Fermi are revered because they were very imaginative and would play with possibilities, turning them this way and that to see what consequences might result. In this way, the individual characteristics of the scientist are more apparent. At least initially, the possible line of a theory is always qualitative and often pictorial. The "sort-of-like-this" imagery comes first and reveals the basic aesthetic approach of the individual's vision of the world in which we live.

Brief Descriptions of Learning Materials

SUMMARY LIST OF UNIT 2 MATERIALS

Experiments

E2-1 Naked-Eye Astronomy
E2-2 Size of the Earth
E2-3 The Distance to the Moon
E2-4 The Height of Piton, A Mountain on the Moon
E2-5 Retrograde Motion
E2-6 The Shape of the Earth's Orbit
E2-7 Using Lenses to Make a Telescope
E2-8 The Orbit of Mars
E2-9 Inclination of Mars' Orbit
E2-10 The Orbit of Mercury
E2-11 Stepwise Approximation to an Orbit
E2-12 Model of the Orbit of Halley's Comet

Demonstrations

D28 Phases of the moon
D29 Geocentric epicycle machine
D30 Heliocentric model
D31 Plane motions
D32 Conic-sections model

Film Loops and Filmstrips

Retrograde Motion of Mars (Filmstrip)
L10 Retrograde Motion: Geocentric Model
L11 Retrograde Motion: Heliocentric Model
L12 Jupiter Satellite Orbit
L13 Program Orbit. I
L14 Program Orbit. II
L15 Central Forces: Iterated Blows
L16 Kepler's Laws
L17 Unusual Orbits

Reader Articles

R1 *The Black Cloud* by Fred Hoyle
R2 *Roll Call* by Isaac Asimov
R3 *A Night at the Observatory* by Henry S. F. Cooper, Jr.
R4 *Preface to De Revolutionibus* by Nicolaus Copernicus
R5 *The Starry Messenger* by Galileo Galilei
R6 *Kepler's Celestial Music* by I. Bernard Cohen
R7 *Kepler* by Gerald Holton
R8 *Kepler on Mars* by Johannes Kepler
R9 *Newton and The Principia* by C. C. Gillispie
R10 *The Laws of Motion and Proposition One* by Isaac Newton
R11 *The Garden of Epicurus* by Anatole France
R12 *Universal Gravitation* by Richard P. Feynman, Robert B. Leighton, and Matthew Sands
R13 *An Appreciation of the Earth* by Stephen H. Dole
R14 *Mariners 6 and 7 Television Pictures Preliminary Analysis* by R. B. Leighton and others
R15 *The Boy Who Redeemed His Father's Name* by Terry Morris
R16 *The Great Comet of 1965* by Owen Gingerich
R17 *Gravity Experiments* by R. H. Dicke, P. G. Roll and J. Weber
R18 *Space The Unconquerable* by Arthur C. Clarke
R19 *Is There Intelligent Life Beyond the Earth?* by I. S. Shklovskii and Carl Sagan
R20 *The Stars Within Twenty-Two Light Years That Could Have Habitable Planets* by Stephen Dole
R21 *Scientific Study of Unidentified Flying Objects from Condon Report with introduction* by Walter Sullivan
R22 *The Life-Story of a Galaxy* by Margaret Burbidge
R23 *Expansion of the Universe* by Hermann Bondi
R24 *Negative Mass* by Banesh Hoffman
R25 *Three Poetic Fragments about Astronomy* by William Shakespeare, Samuel Butler and John Ciardi
R26 *The Dyson Sphere* by I. S. Shklovskii and Carl Sagan

Sound Films (16 mm)

F6 Universe
F7 Mystery of Stonehenge
F8 Frames of Reference
F9 Planets in Orbit
F10 Elliptic Orbits
F11 Measuring Large Distance
F12 Of Stars and Men
F13 Tides of Fundy
F14 Harlow Shapley
F15 Universal Gravitation
F16 Forces
F17 The Invisible Planet
F18 Close-up of Mars
F19 Of Stars and Men
F20 Newton's Equal Areas

Transparencies

T13 Stellar Motion
T14 The Celestial Sphere
T15 Retrograde Motion
T16 Eccentrics and Equants
T17 Orbit Parameters
T18 Motion under Central Forces

FILM LOOPS

Quantitative measurements can be made with *Film Loops* marked (Lab), but these loops can also be used qualitatively.

L10 RETROGRADE MOTION: GEOCENTRIC MODEL

A machine was constructed in which the planet is represented by a lamp bulb on an epicyclic arm revolving around a deferent. The camera is at the position of the stationary earth, pointing in a fixed direction in space.

11 RETROGRADE MOTION: HELIOCENTRIC MODEL

The epicycle machine is used with the camera on an arm revolving around the sun. The camera points in a fixed direction in space.

L12 JUPITER SATELLITE ORBIT

Time-lapse photography, at 1-min intervals, of the motion of Jupiter's satellite Io. The period of revolution can be measured, the scale is given, and hence Jupiter's mass found. (Lab)

L13 PROGRAM ORBIT. I

A computer is programmed to calculate the same orbit that a student calculates in the laboratory when doing *E2-11*, "Stepwise Approximation to an Orbit." The result is displayed on an *X–Y* plotter. Because of th stepwise approximation used, the orbit fails to close up exactly.

L14 PROGRAM ORBIT. II

The computer calculates an orbit using many more points than in the preceding loop: This time the orbit closes up. The display on the *X–Y* plotter is repeated on the face of a cathode-ray tube (CRT). All other computer loops in this series use CRT display.

L15 CENTRAL FORCES: ITERATED BLOWS (COMPUTER PROGRAM)

The computer is programmed to give sharp blows to a mass at equal time intervals. The blows are directed (at random) toward and away from a center of force, and the magnitude of the blows is also random. The law of areas can be verified. (Lab)

L16 KEPLER'S LAWS (COMPUTER PROGRAM)

Two planetary orbits in an inverse-square force field are programmed for display on the CRT. The positions of the planets are shown at successive, equally spaced time intervals. All three of Kepler's laws can be verified. (Lab)

L17 UNUSUAL ORBITS (COMPUTER PROGRAM)

The computer is programmed to display two motions taking place in central fields that are not exact inverse-square fields. One perturbation gives an advance of perihelion, as for Mercury's orbit. The other perturbation gives a catastrophic orbit in which the planet spirals into the sun.

Note: A fuller discussion of each *Film Loop* and suggestions for its use will be found in the section entitled "Film Loop Notes."

FILMSTRIP

RETROGRADE MOTION OF MARS

Three sequences of photographs taken at irregular intervals show Mars and Jupiter in retrograde motion. The angular size and durations of retrograde motions can be determined. This filmstrip defines retrograde motion for students.

SOUND FILMS (16mm)

F6 UNIVERSE

B & W, 26 min, National Film Board of Canada, available from NASA Films. A triumph of film art, creating on the screen a vast, awe-inspiring picture of the universe as it would appear to the voyager through space. Realistic animation takes one out beyond our solar system, into far regions of space perceived by the modern astronomer. Beyond the reach of the strongest telescope, past moon, sun, Milky Way, into galaxies yet unfathomed, one travels on into the staggering depths of the night, astonished, spellbound at the sheer immensity of the universe. The starting point for this journey is the David Dunlap Observatory, Toronto. Seventeen film awards, including International Film Festival, Cannes, France; International Film Festival, Edinburgh, Scotland; British Film Academy, London, England.

F7 MYSTERY OF STONEHENGE

B & W, 58 min (two parts), available from McGraw-Hill. This film could be shown to awaken interest in the explanation of such structures built long ago. It was filmed by The Columbia Broadcasting System and shown on television in the United States and in Britain. The vigorous conflict of interpretations between Professor Hawkins and others is notable.

F8 FRAMES OF REFERENCE

B & W, 28 min, PSSC, Modern Learning Aids. If you haven't shown it previously, Sec. 6.4 might be a good place. Although it presents much more information than is necessary, it is an excellent film. It does give students the idea that the appearance of events may depend upon the frame of reference.

F9 PLANETS IN ORBIT

B & W, 10 min, EBF. This film presents animated representations of some of the differences between the Ptolemaic and Copernican systems.

F10 ELLIPTIC ORBITS

PSSC, Dr. A. V. Baez, Cat. #0310, Modern Learning Aids. This film might be used to make clear to students what area is being discussed in the law of areas.

F11 MEASURING LARGE DISTANCE

PSSC, Dr. F. G. Watson, Cat. #0103, Modern Learning Aids. With a series of models, the film stresses the use of triangulation as the primary means for determining large distances. Toward the end of the film other techniques based on photometry are illustrated as a means of extending the distance scale when triangulation is no longer possible.

F12 OF STARS AND MEN (ABOUT GALILEO)

This biography is available from Center for Mass Communication, Columbia University Press, New York, NY, 10025.

F13 TIDES OF FUNDY

Color, 14 min, National Film Board of Canada, available from NASA Films. A fascinating study of the phenomenal tides in the Bay of Fundy on Canada's Atlantic coast and how they affect the life of the region.

Animated pictures explain the forces of moon and ocean and the earth's rotation that together create in the Bay of Fundy the highest tides in the world.

Filmed with an eye for the dramatic, this film brings to the screen scenes that are truly amazing. It shows, in this tiny pocket of the sea, a sequence of cause and effect that involves the very forces of the universe. It is a film that will appeal to every audience.

F14 HARLOW SHAPLEY

30 min, Encyclopedia Britannica Films, #1806. This film discusses major astronomical discoveries and how they have influenced philosophy, religion, and our orientation to the world.

F15 UNIVERSAL GRAVITATION

PSSC, 31 min, available from Modern Learning Aids, #0309. In this film, the law of universal gravitation is derived for an imaginary solar system of one star and one planet.

F16 FORCES

PSSC, 23 min, Modern Learning Aids. This film is relevant to Unit 2. It introduces mechanics in general and shows a qualitative Cavendish experiment, in which the gravitational force between two small masses is demonstrated.

F17 THE INVISIBLE PLANET

NET Film Service. As this film opens, students meet Peter Van de Kamp, director of the Sproul Observatory at Swarthmore College, and learn of his interest in Barnard's star, a small star near us in the solar system. With Dr. Van de Kamp and Mr. Herbert as guides, the student learns about the operation of the large refractor telescope, the use of photographic plates, the recording and analysis of data, and the results of data carefully recorded for over 25 years. From this data, Dr. Van de Kamp and his colleagues were able to determine the apparent presence of a small planet near Barnard's star that causes a small perturbation or wobbling. The precision, time, and care in astronomical observations are portrayed with impact in this film. Recommended for use in physics or in earth science.

F18 CLOSE-UP OF MARS

NET Film Service. This is the story of the development of the camera system aboard the spacecraft Mariner IV that took the historic photographs of the surface of the planet Mars in mid-July, 1965. The audience follows Robert Leighton, professor of physics at the California Institute of Technology, as he and the scientist-engineers working with him tackle the problem of designing, building and using a camera system that can weigh no more than 11 pounds and use only 10 watts of electricity. In viewing this film, students can sense the difficulties surrounding the assignment and the excitement of success as the first films are relayed back to earth from 520 million km out in space. Particularly recommended for students of physics or electronics.

F19 OF STARS AND MEN

Color, 53 min, available from Brandon Films Inc. Produced and adapted by John and Faith Hubley from the book by Harlow Shapley. The film helps the audience to locate our place in the universe of atoms, protoplasm, stars, and galaxies. Our relationships to space, time, energy, and matter are explored.

F20 NEWTON'S EQUAL AREAS

Color, 8 min. Bruce and Katherine Cornwell. Alfred Bork. Available from International Film Bureau. This animated film is based on Isaac Newton's simple geometrical proof of the law of areas for any central force. It first established the laws of motion in the form needed by Newton, goes through Newton's proof for several different cases, including the limit considerations, and then shows several examples (first simple then complex) of equal areas being traced out with a central force.

ADDITIONAL FILMS

Several astronomical computer-produced, color and sound films, 7 to 8 min, have been produced by Dr. M. L. Meeks and are available for sale or rental by the Houghton-Mifflin Company, Department M, 1 Beacon Street, Boston, Massachusetts 02107. Those related to Unit 2 are: "The Motion of Attracting Bodies," and "Planetary Motions and Kepler's Laws." Silent loops are available for both Technicolor super-8 and Kodak spool projectors.

TRANSPARENCIES

T13 STELLAR MOTION

Displays a two-sphere universe explanation of apparent stellar motion as observed at mid-northern latitudes, the equator and north pole. (Secs. 5.1, 5.5)

T14 THE CELESTIAL SPHERE

Illustrates the scheme of the celestial sphere, indicates the meaning of equinoxes and solstices, shows the sun's path in relation to the zodiac, and gives the meaning of declination, right ascension, and celestial longitude and latitude. (Secs. 5.1, 5.5, *E2-6, E2-8*)

T15 RETROGRADE MOTION

Explains apparent retrograde (westward) motion of an outer planet by means of heliocentric model. (Sec. 5.6)

T16 ECCENTRICS AND EQUANTS

Displays features of geocentric schemes of Ptolemy in accounting for observed planetary motion. (Sec. 5.8)

T17 ORBIT PARAMETERS

Illustrates the six elements that define any orbit. (Secs. 7.2, 7.3, *E2-8, E2-10*)

T18 MOTION UNDER CENTRAL FORCE

Illustrates in geometric steps that objects subject to a central force obey Kepler's law of areas (Sec. 8.3, *E2-11*)

Note: A fuller discussion of each *Transparency* and suggestions for its use will be found in the Visu-Book containing Unit 2 *Project Physics Transparencies*.

Demonstration Notes

D28 PHASES OF THE MOON

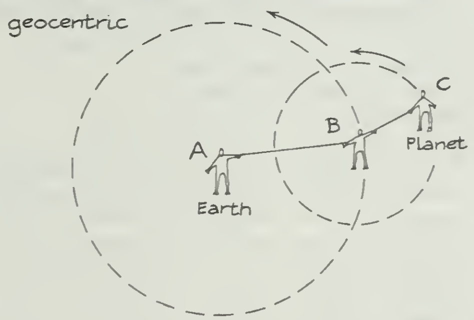
The following model will help to clarify the phases of the moon. Attach a ping pong or tennis ball to a thread. Then in a darkened room have students watch the phases of this moon as you swing it:

(a) around a single lamp bulb (not too bright)

(b) around their heads with the lamp bulb a few meters to one side.

The latter matches best with our observations of the moon's phases.

D29 GEOCENTRIC-EPICYCLE MODEL

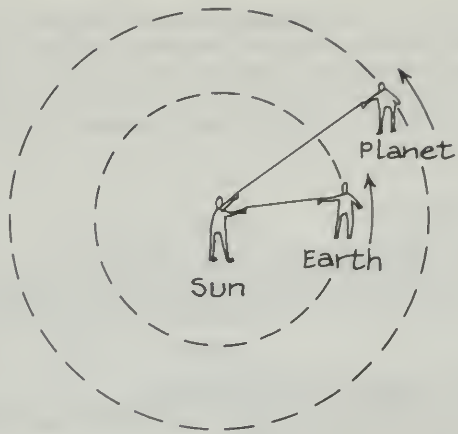


In this demonstration, students themselves play the part of planets. As well as showing that the geocentric-epicycle mode of the solar system gives retrograde motion, it demonstrates the effect that an observer's own motion can have on his or her view of the motion of another object.

Student A, representing the earth, stands still while the two others, B and C, move around: B in a circle, and C, representing a planet, in an epicycle. A length of string (about 5 m) between A and B, and a shorter one (about 2.5 m) between B and C keep the radii of the circles constant. Student C will have to move fairly fast to make one or more revolutions about B, while B, walking at a steady rate, makes one revolution about A. Once they have established appropriate speeds, they must try to keep them as constant as possible. In this demonstration, A is the earth, B is merely a point in space, C is a planet. "Earth" observes the motion of the "planet" with reference to a distant background, such as trees, goalposts, school buildings, which represent "the fixed stars." Does the planet always appear to be moving in the same direction? When does it retrograde? How long does the retrograde motion last?

D30 HELIOCENTRIC MODEL

In this model, the stationary student represents the sun. The earth, now displaced from its position at the hub of the universe, moves around in a circle (radius about 4 m). The third student, representing Mars, moves around the sun in the same sense in a larger circle (6 m). If earth and Mars walk in step, but Mars takes a shorter step, earth's period will



be considerably less than that of Mars and their relative motions will approximate fairly well the actual movement of the two planets. Again, earth is asked to describe the relative motion of Mars as it appears against the distant background of fixed objects. Does the motion appear uniform? Is there retrograde motion? When does it occur? Retrograde motion of an inner planet may be more difficult to spot. Try these parameters: earth's orbit, 10-m radius; Mercury's orbit, 4-m radius; Mercury takes two paces for every one taken by earth.

D31 PLANE MOTIONS

The importance of Kepler's use of motion in a plane cannot be overstressed. With Unit 1 in mind, have students make fists with their left hands to represent the sun. Have them hold pens or pencils in their right hands to represent a point in space and a velocity vector. You can anticipate Chapter 8 and ask students what forces are acting on the body. (The only force is the central pull of the sun.) What initial motion does the body have? (The initial velocity vector is represented by the pen or pencil.) But one point and a line define a plane. What would you infer if the body did NOT move in a plane? (Some other force is acting from a place not in the orbital plane.) This planar assumption is applied in the activity in which the orbital inclination of Mars is derived from the observations of the positions of Mars north or south of the ecliptic, *E2-9*.

D32 CONIC-SECTIONS MODEL

If the mathematics department has a model of a cone, use it to let the student see the natural occurrence of ellipses and other conic sections.

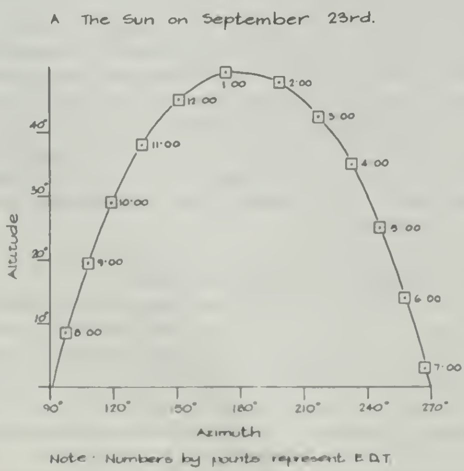
Experiment Notes

E2–1 NAKED-EYE ASTRONOMY

Equipment:

Astrolabe
Constellation Chart

It is best to have students make their own observations rather than use the results in *E2–1* exclusively. It is also most important to observe direct motion, that is, eastward motion with respect to the fixed background of stars.



Answers to questions

Part A

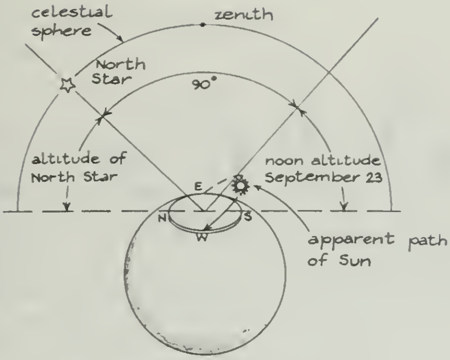
The graph was drawn from the data in *E2–1* Part A. Note that noon occurs after 1:00 P.M. because of:

1. the equation of time
2. the observer's location west of the 75th meridian
3. use of daylight time

Answers are taken from the graph.

1. 49½°
2. Since the sun was on the equator September 23, the latitude of the observer is 90° minus sun's noon altitude, or 90° − 49½° = 40½° North. Refer to the drawing on the right.
3. The sun was highest at about 1:18 P.M. E.S.T.
4. When the sun was highest.
5. Just after sunrise or just before sunset.
6. Since the speed of the earth orbiting around the sun is not constant, it happens that on September 23 the event of smallest shadows, that is, so-called noon, will be 8 min before 1:00 P.M. E.S.T. or 12:52, on the 75th meridian. This 8 min of time is called the equation of time correction.

The observer sees noon at 1:18 E.S.T., or 26 min later than the person on the 75th meridian. Consequently, the observer must be west of the 75th meridian. This is because the 75th meridian rotates under the sun first and then 26 min later the observer rotates under the sun and observes that

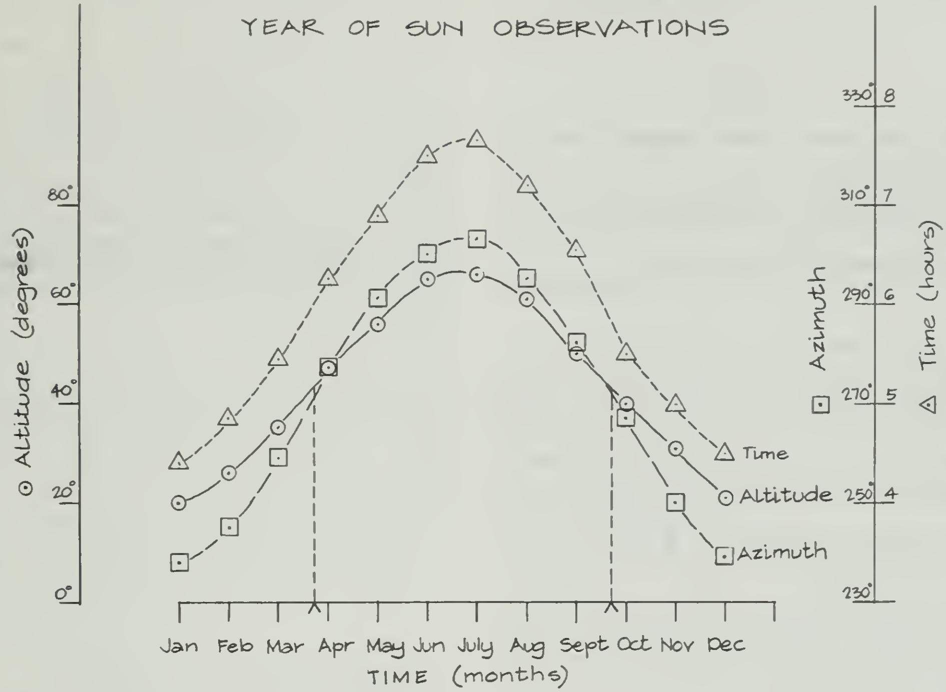


shadows are shortest. Since 1° of longitude is equivalent to 4 min of time, 26 min is equivalent to 6½° of longitude. Thus, the observer is 75° + 6½° = 81½° West. The observer's latitude is 40½° N. On a map this point is close to Canton, Ohio.

Part B

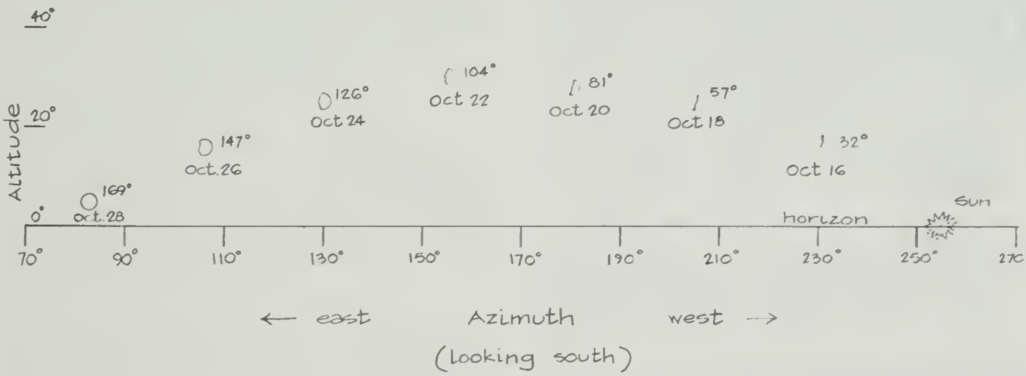
The data in part B are plotted in the figure below.

7. 43°
8. Latitude = 90° − 43° = 47° N
9. Québec, Canada
10. 303° − 238° = 65° range
11. $\Delta t = 7.6^h - 4.4^h = 3.2^h = 4^h12^m$
12. Shortest day: $8^h\ 48^m$
    Longest day: $15^h\ 12^m$



Part C

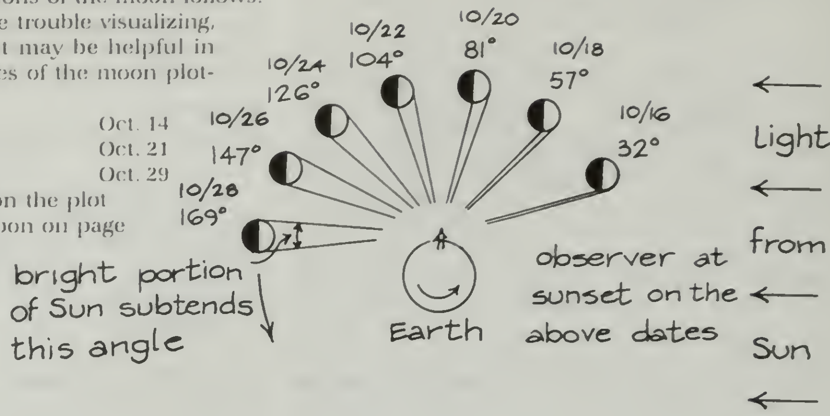
13. The plot of the positions of the moon follows.



Part C

13. The plot of the positions of the moon follows.

For students who have trouble visualizing, the drawing at the right may be helpful in accounting for the phases of the moon plotted on page 137.



14. New moon Oct. 14
First quarter-moon Oct. 21
Full moon Oct. 29

15. Sketches are shown on the plot of positions of the moon on page 137.

Part E

The sun will have the same longitude after 364½ more days.

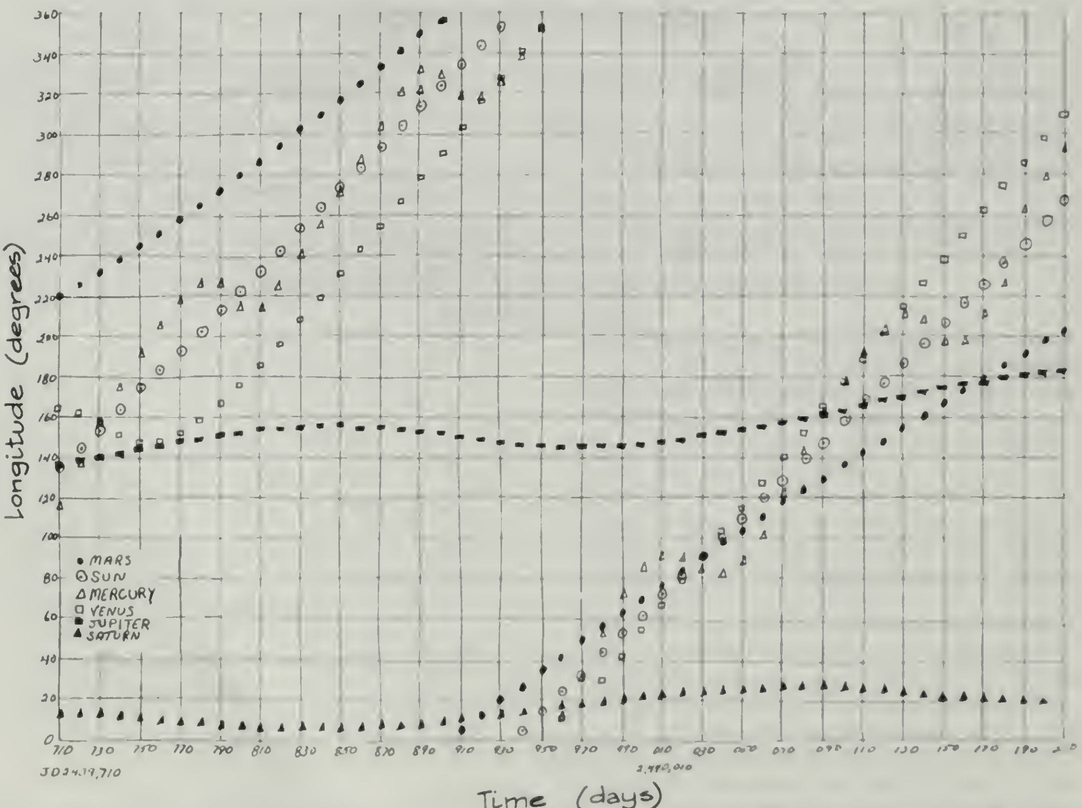
Since the period of orbit of the earth is constant, the date when the question is asked does not matter.

The sun moves about 1° per day.

The sun moves fastest along the ecliptic in January.

Mercury is separated from the sun by no more than 28°.

About every 88th day Mercury will pass in front of the sun and then about 44 days later it will pass in back of the sun (superior conjunction). Student answers will differ from this because of the uncertainity of the plot and because of the eccentricity of the orbit of Mercury.



E2–2 SIZE OF THE EARTH

Equipment:

Astrolabe

This is an excellent experiment for a few students to do as a special laboratory, because the students must communicate with another school at least 300 km away on the same longitude. Some practice with the astrolabe (or a more refined instrument) is advisable in order to measure the altitude of a star or the sun accurately. All this takes time. However, the payoff is monumental, for is not the size of the earth a rather dramatic measurement?

An entire class may work on *E2–2*, but, if so, it will be a class effort that takes weeks to arrange and perform, while requiring only a small portion of a class period each day. Let the students do the whole experiment, including contacting the second observing school. If the teacher is enthusiastic and is able to catch the imagination of a group of students about finding the dimensions of the earth, this single activity could inspire a great spirit within a class.

It is best to find the altitude of a heavenly body when it is on the meridian because there is less atmospheric refraction at that time. Be sure that the star chosen is bright. A good star to use is Sirius.

An interesting historical note is that Eratosthenes used two points in Egypt: Alexandria, on the Mediterranean Sea; and Syene, near the present location of the Aswan Dam. He measured the distance between these points in "stadia." Unfortunately, we do not know the proper conversion from his stadia to our kilometers. According to one interpretation, Eratosthenes' value for the circumference of the earth was about 20% larger than the presently accepted value. However, according to another interpretation, his value agrees with the modern value to about 1%.

Some additional points of interest might be mentioned before or after this experiment. It is often said that the voyage of Columbus to the New World was a daring feat because it was not known in 1492 that the earth was spherical. However, by 250 B.C., Greeks such as Plato and Aristotle had concluded that the earth was a sphere. What observations may have caused the Greeks to make that conclusion? How might you explain the popular concept at the time of Columbus that the earth was flat and that one could fall off the edge?

Since about 1673, scientists have known that the earth is slightly oblate; that is, the polar diameter is slightly less than the equatorial diameter. Near the pole there are about 111 km along the earth's surface per degree of latitude; while there are only about 110 km per degree at the equator.

Answers to questions

1. The uncertainty of the distance of 300 km is perhaps 1 part in 200 or 0.5%. The uncertainty in measuring the angle of a star is considerable. At 300 km apart the angular difference of a star's altitude would be 2.9°. A 1° uncertainty would be maximum; a percentage error of 30% can be expected. Consequently, the uncertainty of θ could be 60 times the uncertainty of the distance between the two observers.

   Of course, one way to improve the accuracy of the experiment is to increase the baseline and improve the technique of measuring angles. If one location is 800 km north or south of the other, the difference in altitude of a given star will be 7¼° instead of 2.9°.
2. At best, students will be able to calculate the circumference of the earth to be about 40,000 km. The uncertainty will probably be no less than 30%.
3. $\% \text{ of error} = \dfrac{\text{student's error}}{40{,}000 \text{ km}} \times 100$
4. A 30% error is acceptable.

   Students should not be discouraged about a 30% error. This is a difficult measurement to make with crude instruments, so that to be able to obtain the order of magnitude of the circumference of the earth is an achievement. However, encourage them to refine their angle-measuring procedure.

E2–3 THE DISTANCE TO THE MOON

This is a straightforward triangulation. By observations of their thumb with first one eye and then the other, students notice the parallax angle.

Answers to questions

1. If the object is moved farther away, the parallax angle becomes smaller.
2. If the baseline is made longer, the parallax angle increases; if shorter, the parallax angle decreases.
3. Since the moon moves rapidly among the stars, about 13° per day, or one moon's diameter per hour, the photographs should be nearly simultaneous. Earthshine is sunlight reflected from the earth to the moon and back to the earth. Since the moon's displacement on these enlarged photographs is less than 1 cm, some care is needed in the plotting and measuring. The images of the moon, Venus, and Jupiter are overexposed and distorted by light scattered within the camera lenses and the films. The displacement of corresponding points should be measured—at least both cusps. Our parallax angle averaged about 0.26°.
4. The positions of Venus and Jupiter do not shift compared to the stars.

   Our results with the technique described average around 408,000 km. How triangulation can be used to get *any* answer is much more important than agreement with a "correct answer."
5. The linear diameter of the moon, just less than 0.5° in the sky, comes out to be close to 3,500 km. Intercomparisons of results among students

would show the scatter of values and raise questions about experimental errors.

6. Only two significant figures seem to be justified.
7. If the observed parallax from widely separated places on the earth is very small, or zero, the objects must be much farther than the moon. This conclusion was reached by Tycho Brahe about the comet of 1577. See Chapter 6.
8. Every observation has inherent errors. Every analysis involves approximations. Those made in this analysis are relatively small, but worth the attention of curious students.
   (a) The baseline is about 15° from being perpendicular to the direction to the moon. Thus, the foreshortening of the effective baseline is proportional to cos 15° (0.970), an error of 3%.
   (b) The difference in the baseline between the arc over the surface and the chord through the earth is about 13 km; the resulting systematic error is less than 1%.
   (c) Overexposure of the moon's images expands them. Therefore the systematic error is toward measuring extra-large displacements. For the diameter of the moon the best measurements are made between cusps on the lower picture.
   (d) The displacement of the moon was nearly parallel to the horizontal so that differential refraction is not significant.

E2–4 THE HEIGHT OF PITON, A MOUNTAIN ON THE MOON

Equipment:

Ruler
Handbook
10× magnifier
Moon photograph
Piece of string 1 m long

This experiment also uses indirect measurement. Our ability to make measurements by direct methods (direct application of our senses) is severely limited. We can't even see very small things, and a mountain is just too large to measure by direct means. Some objects, such as a cloud, are too inaccessible to measure. In each of these cases and in countless others, we use indirect methods to "estimate" dimensions. In this exercise students combine observations with a geometrical model to measure a very large, very inaccessible object.

The photographs at the beginning of the experiment show the large crater Copernicus, which is near the equator in the moon's eastern hemisphere during the third quarter.

The discussion of Figs. 2-11 and 2-12 in the *Handbook* concerns a change in viewpoint in observing the shadow of Piton. This change of view can be made clear with a basketball. Mark the north pole and the central meridian (terminator). Use a golf tee to represent Piton. Show the basketball to the class so that they see it as in the photograph and Fig. 2-11. Now rotate it by bringing the north pole up toward you (away from the class) until the class sees Piton on the edge of the moon's disk. This is the view seen in Fig. 2-12.

Answers to questions

Students may measure distances either on a mosaic photograph of the moon or on the photographs in the *Handbook*. The following solutions are from data taken from the large photograph in the *Handbook*.

1. Data: $l = 1.0$ cm
   $d = 2.6$ cm
   $2r = 81.0$ cm

   Calculations:
   $r = 40.5$ cm
   $$h = \frac{l \times d}{r} = \frac{(1.0)(2.6)}{4.05 \times 10^1} = 6.4 \times 10^{-2}\text{ cm}$$
2. $$\text{Scale} = \frac{\text{Actual diameter of moon in km}}{\text{Measured diameter of moon on photograph in km}}$$
   Change 81 cm to 8.1×10^{-4} km
   $$\text{Scale} = \frac{3.476 \times 10^3\text{ km}}{8.1 \times 10^{-4}\text{ km}} = 4.3 \times 10^6$$
   A linear dimension of the moon is 4.3 million times as great as a linear dimension of the photograph.
3. Therefore, the estimate of the actual height of Piton is $4.3 \times 10^6\ (6.4 \times 10^{-7})$ km $= 2.7$ km
4. and 5. Questions 4 and 5 are not easy ones for students to answer but they should agree that the error in their measurements is less than 25% (see discussion below). The method itself has been somewhat simplified, resulting in an additional possible error of about 10%.

   Encourage students to use a magnifier. Some may discover for the first time that a halftone reproduction is made up of dots.

   A question that each student must answer individually is: "From which point in the illuminated part of the mountain should I measure: the center or the western (left) edge?" It is probably correct to measure from the left-hand edge on the assumption that it is that part of the mountain that casts the longest shadow.

   Uncertainty in l is about 10%. Uncertainty in r is comparatively insignificant. Maximum uncertainty of the final result is therefore 15% + 10% = 25%.

   Percentage error of our result is
   $$\frac{2.7 - 2.1}{2.7} \times 100 = \frac{0.4}{2.7} \times 100 = 15\%$$

   As the example above shows, it is unlikely that results obtained by the students will differ from 2.7 km by more than the experimental uncertainty. The correct answer to "Can you suggest why?" is probably "I underestimated the uncertainty in locating the terminator, etc."

Students should not be disappointed with the rather high percentage error of their results. To have measured the height of Piton even to within 30% is an impressive achievement.

This is a good time to emphasize the importance and value of order of magnitude measurements in physics. Estimation seems to contradict the cliché "Physics is an exact science," but an order of magnitude value is often all that can be obtained, particularly if the quantity being measured is very large or very small. Of course, it is essential to have an estimate of uncertainty.

E2–5 RETROGRADE MOTION

The notes that accompany the *Film Loop* might well be inserted here in your *Resource Book*. The filmstrip of sky photographs on the same scale and carefully positioned to show the same star field in each frame allows students to readily see the motion of Mars in retrograde. The table below gives the dates that students may estimate as the turning points in Mars' motion and the approximate duration of the retrograde motion in days. At best, these are crude estimates because the photographs were taken at irregular intervals and often fail to show the critical times. In addition to seeing the actual retrograde motion, students should realize that the durations and displacements differ between recurrences of the retrograde motion.

Retrograde Motions of Mars from the Filmstrip

	Begins	Ends	Duration
1941	Sept. 1	Nov. 16	76 days
1943–44	Oct. 29	Dec. 20 ?	53 days
1945–46	Dec. 8	Feb. 23	77 days

The retrograde motion of Jupiter is much longer, about 120 days, and extends beyond the intervals covered by the 1943–44 and 1945–46 photographs. Of interest is the relative motions of Mars and Jupiter and the difference in duration of the retrograde motions

E2–6 THE SHAPE OF THE EARTH'S ORBIT

Equipment:

- Filmstrip projector (35mm)
- Screen
- Meter stick
- Graph paper—20 × 20 size (desirable), or smaller sheets
- Filmstrip of sun photos
- Ruler
- Compass
- Protractor
- *Handbook*

E2–6, "The Shape of the Earth's Orbit," should be done before *E2–8, 9,* and *10,* in which the students determine the orbits of Mars and Mercury.

You may decide to conduct this experiment as a group activity in which the whole class participates in collecting the data. Each student should make a separate plot, however. Once the data have been collected the plotting could be done at home since the only tools needed are protractor and ruler.

The Changing Size of the Sun

This experiment is based on the assumption that the angular size of the sun changes because our distance from it changes. Before beginning, you may want to challenge students to offer other explanations that fit the facts.

For example, perhaps the sun periodically expands and contracts, as many stars do. (The most famous type of periodic "variable stars" are called *cepheid variables*. One famous cepheid is Polaris, which has a period of just less than four days.) You might ask, if the sun does vary in size, does it also vary in other ways, for example, in brightness? Could this be an explanation for the seasons? Does it seem reasonable that the sun's period of variation would coincide exactly with the earth's period of revolution about the sun? What other effects would we observe on earth and on other planets if the sun does vary in size or brightness?

There are other possible sources of systematic or cyclic error that could account for the observations. The effective focal length (and therefore amount of magnification) of the telescope might vary with temperature, and therefore with the seasons. To test this idea a study of solar diameter versus air temperature could be made. A more conclusive study would involve similar photos from the southern hemisphere where the seasons differ from ours.

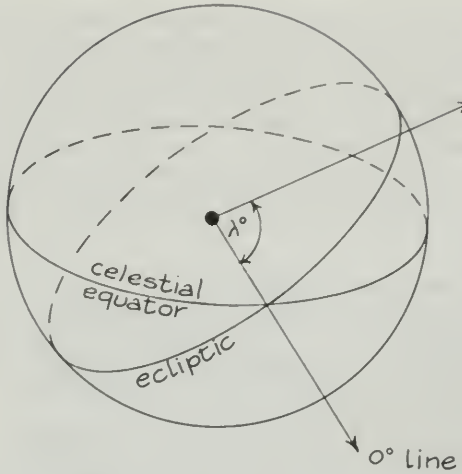
After a variation in the sun's physical size has been ruled out as a probable explanation, return to the idea that this is an apparent variation only and that probably it results from a variation in the distance from earth to sun.

Students may be surprised to learn that the sun is actually closer to the earth in the northern hemisphere during winter than it is in summer. This contributes little to seasonal differences, which are mainly caused by the inclination of the earth's axis of rotation to its plane of revolution.

The Coordinate System

In locating the 0° direction on the graph paper remember that the point where the ecliptic crosses the equator on March 21 is called the vernal equinox. (See *T17*). The sun is then close to Pisces. A line from the earth to the vernal equinox is the reference line from which celestial longitudes are measured along the ecliptic. The angles are measured eastward from this 0° line.

This system is used in *E2–8, 9,* and *10,* as well as in this experiment, and students should become



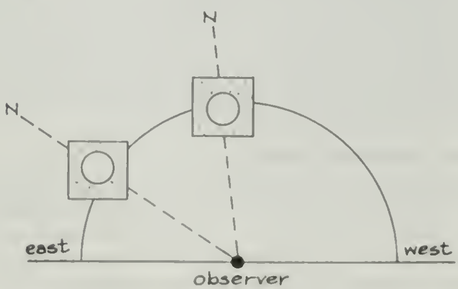
familiar with it. Part of *Transparency T14* can be used to help explain the system.

Gathering Data

Project the filmstrip onto a wall or other flat surface to give an image of the sun with a diameter between 50 and 100 cm. Measure the diameters with a meter stick. Do not move or refocus the projector after the measurements start.

If you have more filmstrips than projectors, some students can measure on the film directly, using a 10× magnifier.

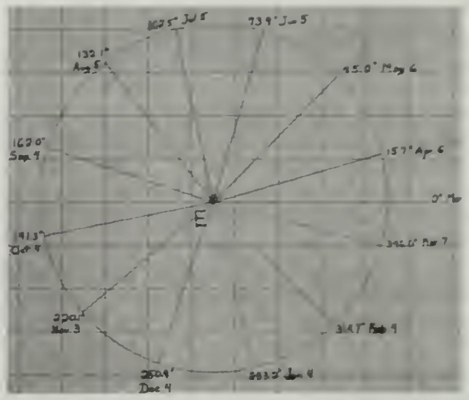
In frames 7 through 18 the directions marked N and E refer to the directions as seen in the sky and not to directions on the sun itself. Note that the direction marked "north" varies from frame to frame, principally because the photographs were made at different times of the day. The frames always have the same alignment with respect to the horizon. On plates taken early in the morning, the north direction is tipped toward the left. We recommend measuring along the horizontal diameters, which are parallel to the bottoms of the frames since this lessens the effect of atmospheric refraction. REMEMBER: Do not move the projector or refocus.

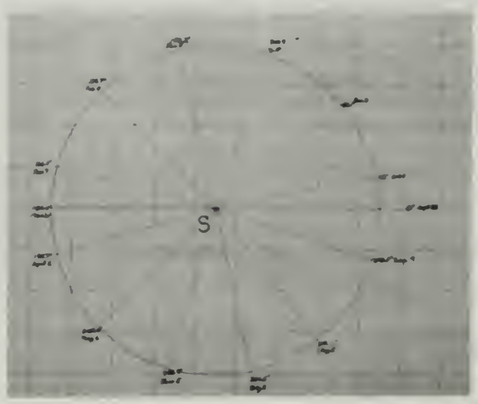


A 10-cm radius gives an orbit that is convenient to use as the starting point for plotting the orbit of Mars *E2–8*.

In discussing the change from an earth-centered to a sun-centered plot, remember that if the plot is turned through 180° it now represents the orbit of the earth around the sun. Some students may see this; many probably won't. Rather than try to convince them by intellectual arguments that the same plot can be used for either orbit we suggest that they begin to plot the motion of the earth around the sun. If they are encouraged to compare the new plot with their first one as they go along, they should soon realize that they do not need to make a whole new plot. Rotating the first plot is sufficient.

Here are examples of the two plots.





Answers to questions

1. The orbit is best drawn as a slightly off-center circle. In fact it is an ellipse whose eccentricity is 0.017.
2. Perihelion is January 3. Aphelion is July 5. The ratio of aphelion distance to perihelion distance is about 1.04.
3. See Fig. 2-20 in the *Handbook*.

E2–7 USING LENSES TO MAKE A TELESCOPE

Equipment:

- Large lens
- 10× magnifier
- Small lens mounted in wooden cylinder (optics kit)
- 2 telescoping cardboard tubes
- Plastic cap for mounting large lens
- *Handbook*

Optional equipment:

- Wooden saddle
- Rubber bands
- Camera tripod
- Telescope kits

Through some simple experiments students can learn enough optics to understand how a telescope works.

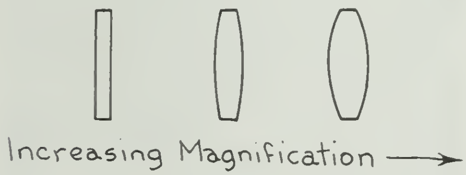
Use your own stock of assorted lenses. Include lenses that have the same diameter but different powers, some negative lenses, and some plane glass disks if you have them. If necessary, use lenses from the telescope kits; otherwise, keep the kits in the background. Caution students to handle lenses by the edges and to keep their fingers off the surfaces.

The Simple Magnifier

If the object distance is held constant at its value for maximum magnification, the angular size of the image will remain constant as the eye is moved away from the lens.

Answers to questions

1. Magnifers are thicker in the middle than at edges. The lens thickest in the middle has the highest magnifying power.
2. Curvature is the important feature. The more convex (fatter) a lens is, the greater the magnification it is capable of, given an appropriate choice of object distance and image distance.



3. Only lenses thicker in the middle than at the edges (convex lenses) produce an image that can be projected on a screen. Such an image is called a *real image*. Real images are always inverted.
4. Whether the real image is larger or smaller than the object depends on the relative distances of the object and the image from the lens. If the object is farther from the lens than the image is, the image will be smaller than the object and vice versa.
5. If you want to look at a real image without using the paper screen, you have to put your eye behind the image and at least 25 cm from it. To a student who has trouble finding the image ask: "How far would your eye have to be from a real object to see it clearly?"
6. The image seen in this way may be quite small. In order to inspect it more closely use a second lens as a magnifier held at the same distance from the image as it would be held from a printed page for clear viewing. If students have difficulty in placing the magnifier in the right place, let them focus the image formed by the first lens on a sheet of paper. Having thus located the real image, they can adjust the distance of the magnifier from it accordingly and then remove the paper.
7. The telescope produces the largest images if the lowest-power lens is used as the *objective* (the first lens the light goes through) and the highest-power lens is used as the *eyepiece* (the magnifier).

E2–8 ORBIT OF MARS

Equipment:

- Booklet of Mars photographs
- Transparent overlays
- Graph of earth's orbit *E2–6*
- Protractor
- Straightedge
- *Handbook*

The Photographs

None of the photos used in this experiment are retouched. They may show lint or dust marks or the edges of dried watermarks that mar the originals. The center is often heavily exposed while the edge is barely exposed. This is due to vignetting (shadowing) at the edges of the field caused by camera design. The student is as close to the "data" of observational astronomy as are research astronomers themselves.

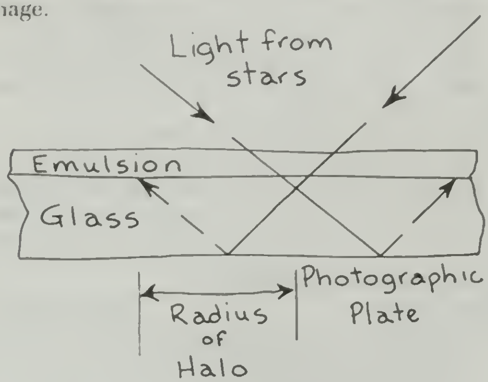
Some photographs show fewer or fainter stars than others because:

1. In some parts of the sky, well away from the Milky Way, star density is low.
2. Some of the photographs may have been made through thin clouds, smoke, or haze, and will not show the fainter stars.

The size of a focused image of Mars on the original plate is about 1/40 mm, as are the diffraction disks of star images. Consequently, the images of planets are theoretically indistinguishable from those of stars. Actually, light scattered within the photographic emulsion and "twinkling" causes brighter images to grow larger than fainter images.

The best image formed by a lens is on the optical axis (in the center of the picture). Distortion becomes more pronounced toward the edges of the field. One kind of distortion, which makes the shape of images seem triangular, is clearly evident in some of the frames taken from the edge of a plate. Nothing can be concluded about real shapes and sizes from these photos: They are valuable because they record relative positions of Mars and the stars.

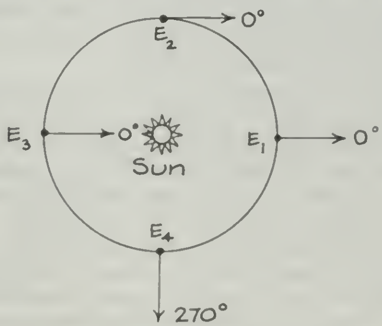
Almost surely some questions will arise about the "halo" around the bright object (Mars) in plates F and O. This occurs because the photograph emulsion is on a glass plate about 1 mm thick. As the sketch below indicates, light from a very bright object will penetrate through the thin emulsion, be reflected by the back of the glass, and strike the emulsion from below in a ring around the initial image.



The Coordinate System

Before beginning this experiment be sure students have the dates and angles properly recorded upon the earth plot from *E2–6*. If 0° is on the right, this should be labeled September 23. That is, on this date the earth has a heliocentric longitude of 0° as seen from the sun. On March 22 the earth has a heliocentric longitude of 180° and the sun has a geocentric longitude of 0°.

Refer to the typical Mars orbit on page 67 of the *Handbook* (Fig. 2-27). Some students have trouble visualizing that a line drawn horizontally to the right from the earth takes a direction of 0°. It does not matter where the earth is in its orbit; to the right on the typical diagram is 0°, to the left is 180°, and directly downward is 270°. For further clarification refer to the diagram below.



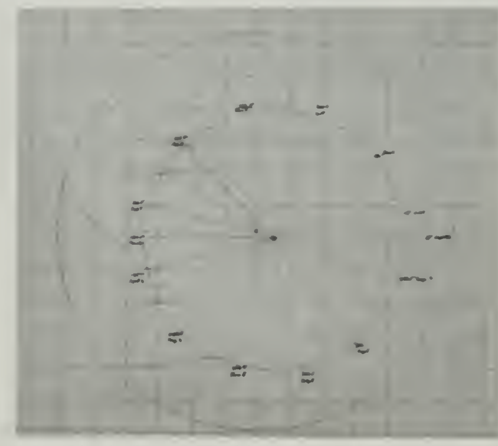
Because of the changing attractions of the moon and sun on the oblate earth, the earth's rotation has several wobbles. All except one are too small to be important in this analysis. The one large change is called *precession* and has a period of 26,000 years. During this time the direction in which the north pole of the earth points moves through a large circle on the sky. The result of this motion is to cause the location of the vernal equinox to slide westward about 50 sec of arc each year. Since our coordinate grids are based upon the vernal equinox as the zero point, the longitudes we assign to the stars change very slowly with the years. For this study, we have adopted the coordinate system of 1950.0.

Longitude of Mars, seen from Earth

Plate	Date	Longitude
A	Mar 21, 1931	118.6°
B	Feb 5, 1933	169.0°
C	Apr 20, 1933	151.4°
D	Mar 8, 1935	204.4°
E	May 26, 1935	186.7°
F	Apr 12, 1937	245.7°
G	Sep 26, 1939	297.5°
H	Aug 4, 1941	016.5°
I	Nov 22, 1943	012.1°
J	Oct 11, 1943	080.1°
K	Jan 21, 1944	065.6°
L	Dec 9, 1945	123.2°
M	Mar 19, 1946	107.6°
N	Feb 3, 1948	153.4°
O	Apr 4, 1948	138.3°
P	Feb 21, 1950	190.7°

A Typical Mars Orbit

In this plot, we have drawn a circle of radius 15.5 cm (1.55 AU) that passes through or close to most of the positions of Mars. The center of the circle is above and to the left of the sun's position.



Data for Mars' orbit:

Mean distance a = 1.52 AU
Eccentricity e = 0.09

Two articles on measuring areas with planimeters have appeared in *Scientific American*, August 1958 and February, 1959. Mechanically minded students might wish to make and calibrate their own planimeters and verify Kepler's law of areas.

Answers to questions

1. Student answers will vary.
2. The sun-to-earth distance is often referred to as "one astronomical unit" (AU). The distance from sun to Mars in AU:

$$\begin{aligned}\text{Maximum distance} &= 1.7\ \text{AU}\\ \text{Minimum distance} &= 1.4\ \text{AU}\\ \text{Mean Distance} &= 1.55\ \text{AU}\end{aligned}$$

3. As seen from the sun, the directions (longitude) of perihelion and of aphelion of Mars are approximately 340° and 160°, respectively.
4. The earth is about 0.4 AU from the orbit of Mars in September. This is the closest that the two planets can approach each other.
5. The eccentricity of the Mars orbit is $e = 0.09$.

6–10. Student answers will vary.

E2–9 INCLINATION OF MARS' ORBIT

Equipment:

Booklet of Mars photographs
Transparent overlays
Graph paper
Handbook

If some students are interested in carrying further the analysis of Mars' orbit, they can use the same star-field photographs and coordinate overlays to derive the inclination of the orbit.

The elements of an orbit are discussed again in *E2–12* "model of the orbit of Halley's Comet."

Transparency 18 shows the details of Fig. 2-37 more clearly than a single drawing can.

Data for the completion of Fig. 2-31 (*E2–8*) follow:

Position of Mars

		As seen from earth		As seen from sun	
Plate	Date	long.	lat.	long.	lat.
A	Mar 21, 1931	118.6°	3.2N	150°	1.8N
B	Feb 5, 1933	169.0	4.0N	150	1.8N
C	Apr 20, 1933	151.4	2.5N	183	1.4N
D	Mar 8, 1935	204.4	3.0N	183	1.4N
E	May 26, 1935	186.7	0.6N	220	0.3N
F	Apr 12, 1937	245.7	0.7N	220	0.3N
G	Sep 16, 1939	297.5	4.6S	335	1.7S
H	Aug 4, 1941	016.5	4.1S	335	1.7S
I	Nov 22, 1941	012.1	0.6S	043	0.2S
J	Oct 10, 1943	080.1	0.5S	043	0.2S
K	Jan 21, 1944	065.6	2.7N	096	1.3N
L	Dec 9, 1945	123.2	2.9N	096	1.3N
M	Mar 19, 1946	107.6	3.0N	142	1.9N
N	Feb 3, 1948	153.4	4.4N	142	1.8N
O	Apr 4, 1948	138.3	3.1N	169	1.6N
P	Feb 20, 1950	190.7	3.5N	169	1.6N

The plot of latitudes (north or south of the ecliptic) is a sine wave. The ascending node (where the path crosses from south to north as the longitude of Mars increases) is near 050° and the descending node is near 230°. The maximum latitude as seen from the sun is 1.8°, halfway between the nodes; this is the angle of orbital inclination.

You may wish to discuss the way in which a small number of data points can be interpreted i.e. when you know that the curve will be a sine wave.

E2–10 ORBIT OF MERCURY

Equipment:

Graph of earth's orbit (*E2–6*)
Table of planet positions
Handbook
Protractor
Straightedge

This simple exercise provides additional experience with the concepts of orbit theory. Orbital eccentricity and Kepler's second law can both be studied. Although this is a relatively brief activity that some pupils have completed at home in less than 20 min, the results can be surprisingly accurate.

Because the orbit of Mercury is not a circle, the tangent to the orbit is not perpendicular to the line joining sun to planet; that is, the assumption suggested here is only an approximation. Students may find that they cannot draw a smooth curve to join all the points located in this way. In this case it is quite legitimate to move some of the points slightly. The final orbit should be represented by a smooth curve that touches, without crossing, all the sight lines.

Calculating R_{av} and the Orbital Eccentricity

Mercury's perihelion point occurs at a longitude of about 78° (in about the direction of the earth's position on December 10), and the major axis of the orbit lies along this line. From the data given in the experiment, the major axis of Mercury's orbit is about 7.8 cm (0.78 AU) long.

$$\begin{aligned}2a &= 0.78\ \text{AU}\\ a &= 0.39\ \text{AU}\end{aligned}$$

The accepted value for a is 0.387 AU.

From the plot, $R_p = 4.60$ cm and $a = 3.9$ cm. Thus,

$$\begin{aligned}e &= \frac{4.60}{3.9} - 1\\ e &= 1.18 - 1\\ e &= 0.18\end{aligned}$$

The accepted value of e is 0.206.

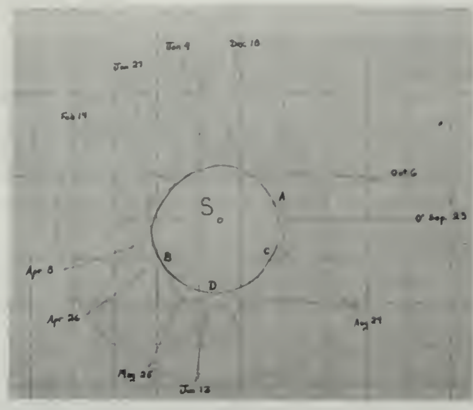
Calculating the accepted value should not be a major concern. The emphasis should be on determining if the orbits of the earth and Mercury have the same shape. If they do not, then how can one describe the difference? Eccentricity, a mathematical device, describes this difference in shape. However, it is more important for students to be able to describe the difference between elliptical shapes in their own way than to feel compelled to calculate $e = 0.206$ from their data.

Kepler's Second Law

Label the sun S and the positions of Mercury as follows:

Jan. 4, 1963	A
Feb. 14, 1963	B
June 13, 1963	C
Aug. 24, 1963	D

Count the number of squares in the areas SAB and SCD. Note that each of these areas is more than half the orbit area.



With our data:

$$\frac{SAB}{t_1} = \frac{274 \text{ squares}}{41 \text{ days}} = 6.7 \text{ sq/day}$$

$$\frac{SCD}{t_2} = \frac{525 \text{ squares}}{72 \text{ days}} = 7.3 \text{ sq/day}$$

$$\text{Average} = \frac{274 + 525}{41 + 72} = \frac{799}{113} = 7.07 \text{ sq/day}$$

If the law of areas holds, the two ratios should be equal. Our experimentally determined ratios agree within 10%. Perhaps your students, using a smoother orbit, will obtain greater accuracy.

E2–11 STEPWISE APPROXIMATION TO AN ORBIT

Equipment:

- 20 × 20 graph paper
- Straightedge
- 45° or 60° triangle
- Dividers or compass
- *Handbook*

This experiment has two results. The students will come to understand that if a force is central toward the sun and if it is inversely proportional to the square of the distance from the sun, the resulting orbit is an ellipse. Equally important, after the plot is completed, the student will have a greatly improved understanding of the infall of a body toward the sun.

The velocity of a body in a gravitational field is continuously changing, whether the body is a projectile near the earth's surface, a satellite in orbit around a planet, or a comet in orbit about the sun. For a projectile near the earth's surface the gravitational force is constant and the trajectory can be found fairly easily. (See Unit 1, Chapter 4.) However, for a body in space the gravitational force changes with distance from the sun, earth, and other bodies. Precise prediction of the orbit of a body in a varying force field is complicated. However, if one assumes that the force acts intermittently at equal time intervals, like hammer blows, an orbit can be approximated rather quickly.

See Feynman's *Lectures on Physics, Vol. I,* Chapter 7 for an algebraic equivalent of the geometric method used in this experiment for computing orbits. Newton used this method to prove that Kepler's second law follows from a central-force hypothesis. (See *Principia,* page 40 in the paperback edition and also *Text* Sec. 8.4.) The experiment described here is based on one developed by Dr. Leo Lavatelli, University of Illinois, and printed in the *American Journal of Physics,* 1965, *33,* p. 605.

In the *Project Physics Film Loops 13* and *14* entitled "Program Orbit. I" and "Program Orbit. II" a computer works out the same orbit by iteration. In the first loop the time interval between blows is 60 days and the result is close to what the students should obtain. In the second loop a 3-day iteration interval is used; the orbit is smoother. Both loops should probably be used after the students have made their plots.

The "thought experiment" suggested in *E2–11* ("Imagine a ball rolling . . .") can form the basis of a demonstration in which the teacher applies repeated lateral (sidewise) blows, all directed toward the same point, to a heavy ball or air puck. It is helpful to try a demonstration involving all the students. Each one gives a centrally directed blow to the ball or puck as it passes. The ball's initial velocity must be fairly high and the blows not too strong.

Answers to questions

1. The ball will continue to move in a straight line with the same velocity.
2. The path direction would change.
3. Its speed may change depending on initial speed and acceleration imparted by the blow. (In circular motion only direction changes, not speed.)

The ball will move in a path made up of a series of straight-line segments.

Effect of the Central Force

Answers to questions

4. The force is greater if the comet is nearer to the sun:

$$\vec{F} \propto \frac{1}{R^2}$$

5. The greater the force of the blow, the greater the velocity change ($\Delta \vec{v} \propto \vec{F}$).

Because all blows have the same duration (Δt) and m is constant, Newton's second law $F = m\, \Delta v/\Delta t$ can be simplified to $\Delta \vec{v} \propto \vec{F}$.

Because the time interval between blows is constant (60 days) the comet's displacement along its orbit during a 60-day interval is proportional to its velocity.

$\Delta d = \vec{v} \times 60$ days, becomes $\Delta \vec{d} \propto \vec{v}$.

Scale of the Plot

The particular orbit chosen is similar to that of the short-period comets, like Encke's comet, which stay entirely within the orbit of Jupiter. These parameters, and the 60-day interval, give an orbit that is completed in about 25 steps. Half of the orbit can be obtained in 12 steps.

The earth's average orbital speed is about 96,000 km/hr.

Computing Δv

The following alternative may be preferable to using Fig. 2-50 as a computer by the method described at the end of this section. Plot the graph of Δv versus R on transparent paper. With the orbit plot on a drawing board, stick a pin through the origin of the graph and the sun point of the orbit plot. It is also a good idea to reinforce the graph with a piece of masking tape in the area of origin. Swing the graph around the pin as a pivot to establish the value of R and point d_1.

In either method of making the plot, 12 or 13 steps should bring the comet to perihelion. If time is short, students can complete the orbit by assuming that the halves are symmetrical.

The *Film Loops*, "Program Orbit. I" and "Program Orbit. II," illustrate that a shorter iteration interval results in an orbit that is more nearly a smooth ellipse. A student can draw a smooth ellipse on the plot by using the two-pin-and-loop-of-string technique.

Answers to questions

Answers are based on a sample plot.

6. Perihelion distance = 1.1 AU
7. e = 0.54
8. Period of revolution = 24 × 60 days
 = 1,440 days
 = 4 years
9. The closer the comet is to the sun the greater the speed.
10. The path has the approximate shape of an ellipse with the sun occupying one of the two foci. If Δt were decreased from 60 days, the curve would be smoother, and the curve would tend to close. This provides a straightforward application of calculus, for as we imagine Δt approaching zero we model a continuously changing force rather than a single blow every 60 days. This question can lead to much class discussion and to *Film Loops 13* through *17*.
11. The area is approximately 27 cm². Students can find this by calculating $A = \frac{1}{2}ab$ or by counting squares. Consider the students' inaccuracies due to the iteration process. Discuss what order of deviation might be considered a constant area in view of this approximation of iteration.

More things to do

1. The student who completed a superior graphical representation of the orbit might be encouraged to repeat the experiment using a different initial speed but retaining the same direction. Another student might change the initial direction and retain the same speed, that is, keep the length of the vector the same.

   Have these students present their findings to the class. Some interesting results include the sensitivity of orbits to small changes in the initial velocity.
2. An example of such a repulsive force is that between two like charges. The path will be hyperbolic and this will help the students appreciate alpha-particle scattering in Unit 6. A special kind of student will be interested in this and benefit from it.

E2–12 MODEL OF THE ORBIT OF HALLEY'S COMET

Equipment:

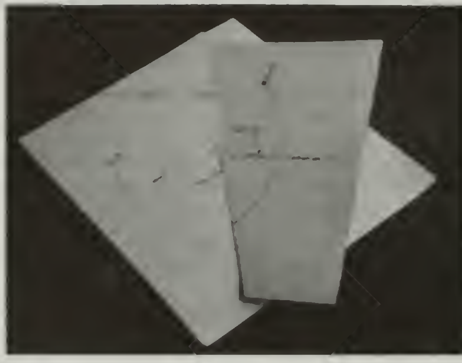
Cardboard or stiff paper, two sheets
Ruler
Protractor
Data for Halley's comet (*Handbook*)
Compass
Scissors

In constructing the orbits of Halley's comet and the earth it may be useful to refer to *Transparency 17*, which illustrates the elements of an orbit more clearly than a single drawing can.

The orbit of Halley's comet is an ellipse with $e = 0.967$; the portion close to the sun is very nearly a parabola. All parabolas have an eccentricity of 1.

When fitting the two orbits together, remember that the *ascending node* is the point at which the comet's orbit crosses the ecliptic plane going from south to north. The *descending node* is the point at which the comet crosses the ecliptic from north to south.

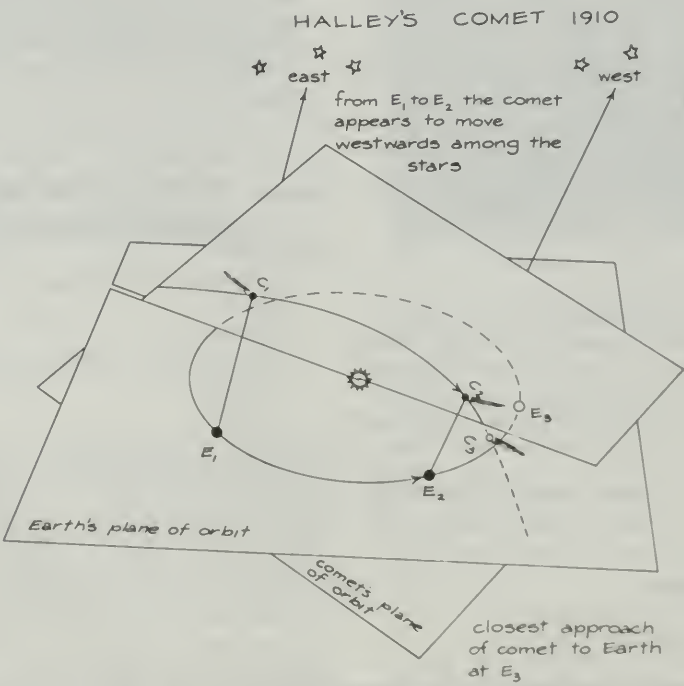
Halley's comet moves in the opposite sense to the earth and other planets. The earth and planets move counterclockwise when viewed from above (north of) the ecliptic; Halley's comet moves clockwise.



Pictures showing how the two planes go together.

Answers to questions

1. Refer to the sketch below. As the comet moves from C_1 to C_2, the earth moves from E_1 to E_2. Due to these relative motions, the comet appears to move westward among the stars.
2. When the earth is at E_2 the comet is at C_2. Furthermore, the motion of the comet is in a direction directly down the sight line C_2E_2, a collision course!
3. Refer to the sketch. As the comet moved from C_2 to C_3 and as the earth moved from E_2 to E_3, the comet moved between the earth and sun fairly near the earth, so its angular motion was great.
4. On May 19 the comet crossed the earth's orbital plane between the sun and the earth.
5. Yes, the earth passed through the tail. Refer to the sketch and visualize the tail sweeping from C_2 and then cutting through the plane of the ecliptic at the line of nodes. The earth passed through this tenuous tail.
6. Research question.
7. Nothing unusual happened. The tail is very tenuous. The comet's tail is much less dense than the air near the surface of the earth. Therefore one could not expect anything to have happened.



Film Loop and Filmstrip Notes

Filmstrip RETROGRADE MOTION OF MARS

Because photographs are the most honest evidence we have of the actual retrograde motions of Mars and Jupiter, the filmstrip should be shown as soon as motions of the planets are mentioned.

The Photographs

The frames were made from unretouched contact prints of sections of the original photographs. The photographs were taken with the short-focus camera (focal length 15 cm) shown in one of the first frames. Because Mars was never in the center of the field, but sometimes almost at the edge, the star images show distortions from limitations of the camera lens. During each exposure the camera was driven by clockwork to follow the western motion of the stars and hold their images fixed on the photographic plate. Because the sky was less clear on some nights and the exposures varied somewhat in duration, the images of the stars and planets are not of equal brightness on all pictures. However, some of the frames show beautiful pictures of the Milky Way in Taurus (1943) and Gemini (1945).

These three series of photographs were selected as the most extensive available for recent oppositions of Mars. The photographs were taken as part of the routine Harvard Sky Patrol, and were not made especially to show Mars. The planet just happened to be in the star fields being photographed. (See Notes on *E2–5*.)

L10 RETROGRADE MOTION: GEOCENTRIC MODEL

Using a large "epicycle machine" as a model of the Ptolemaic system, this film illustrates the motion of a planet, such as Mars, as seen from the earth.

First, the machine is viewed from above, with the characteristic retrograde motion during the "loop" when the planet is closest to the earth. When the studio lights go up, it becomes discernible that the motion is due to the combination of two circular motions. One arm of the model rotates as an epicycle at the end of the other (the deferent).

The earth, at the center of the model, is then replaced by a camera that points in a fixed direction in space. The camera sees the motion of the planet relative to the fixed stars, so we are ignoring the rotation of the earth on its axis. For an observer viewing the stars and planets from the earth, this is the same as always looking toward one constellation of the zodiac, such as Sagittarius.

Imagine that you are facing south, looking upward toward the selected constellation. East is on your left and west is on your right. The direct motion of the planet, relative to the fixed stars, is eastward toward the left. The planet (represented by a white globe) is seen along the plane of motion. A planet's retrograde motion does not always occur at the same place in the sky, so some retrograde motions are not visible in the chosen direction.

Three retrograde motions use smaller bulbs and slower speeds to simulate greater distances.

Note the changes in apparent brightness and angular size of the globe as it sweeps close to the camera. Actual planets appear only as points of light to the eye, but a marked change in brightness can be observed. This was not considered in the Ptolemaic system, which concentrated only upon positions in the sky.

Retrograde Motion: Heliocentric Model (L11) shows a similar model based on a heliocentric theory.

L11 RETROGRADE MOTION: HELIOCENTRIC MODEL

This film is based on a large heliocentric model. Globes representing the earth and a planet move in concentric circles around the sun. The earth (represented by a light blue globe) passes inside a slower-moving outer planet, such as Mars (represented by an orange globe). A yellow globe indicates the sun. The orbits of both planets are assumed to be circular.

The "earth" is replaced by a camera having a 25° field. The camera points in a fixed direction in space, indicated by an arrow, thus ignoring the daily rotation of the earth and concentrating on the motion of the planet relative to the sun.

Scenes are viewed from above and along the plane of motion. Retrograde motion occurs whenever Mars is in opposition; that is, when Mars is opposite the sun as viewed from the earth. But not all these oppositions take place when Mars is in the sector the camera sees. The time between oppositions averages about 2.1 years. The film shows that the earth moves about 2.1 times around its orbit between oppositions.

It is possible to calculate this value. The length of a year is 365 days for the earth and 687 days for Mars. In 1 day the earth moves $1/687$ of 360°, and the motion of the earth relative to Mars is $(1/365 - 1/687)$ of 360°. Thus, it will take 780 days for the earth to catch up to Mars again. The average phase period of Mars is 780 days, or 2.14 years. The view from the moving earth is shown for more than 1 year. First Mars is seen in direct motion, then it comes to opposition and undergoes a retrograde motion loop, and finally we see Mars again in direct motion.

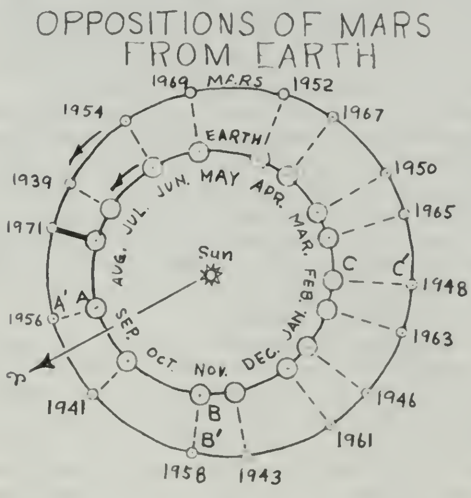
Note the increase in apparent size and brightness of the globe representing Mars when it is nearest the earth. Viewed with the naked eye, Mars shows a large variation in brightness but always appears to be only a point of light. With the telescope, we can see that the angular size also varies as predicted by the model. The heliocentric model is in some ways simpler than the geocentric model of Ptolemy and gives the general features observed for the planets: angular position, retrograde motion, and variation in brightness. However, detailed numerical agreement between theory and observation cannot be obtained using circular orbits. Kepler's elliptical orbits gave better agreement with observation.

Retrograde Motion: Geocentric Model (L10) shows a similar model representing the geocentric theory of Ptolemy.

Some of the finer details of the motion of Mars are related to the planet's rather strongly elliptical orbit (eccentricity 0.093 compared with 0.017 for the earth's orbit). Some oppositions are more "favorable" (closer) than are others. The drawing below shows that the closest oppositions occur if the earth is at A (in late August). A little over 2 years later the next opposition is not so close (earth at B, in November). The following distances illustrate this point:

most favorable opposition, AA′ = 56,000 km
least favorable opposition, CC′ = 100,800,000 km
least favorable superior conjunction, AC′ = 398,400,000 km

The model used in the film is based on the approximation of circular orbits.



L12 JUPITER SATELLITE ORBIT

The purpose of the loop is to give students a feeling for the motion of a celestial body; in this case, a satellite of Jupiter moving under the influence of gravitational force. The loop is primarily intended for qualitative use. However, some simple measurements of period and size of orbit can help a student appreciate the nature of astronomical observations in a "real" situation.

Jupiter was in opposition on Jan. 20, 1967, and was therefore closest to the earth (at about 4.2 AU) and maintaining a relatively constant size as viewed from day to day. The entire satellite orbit could not be photographed, because on Feb. 8, 1967 (when the missing part of the orbit was being traversed by Io), the image was blurred because of high-altitude jet streams in the earth's atmosphere. The next return of Io to this part of its orbit during darkness in Arizona was on Feb. 24, and by then Jupiter would have been farther away and its image would have been smaller. Also, use of a large telescope must be tightly scheduled, and our project had already used major amounts of telescope time on seven nights. For all these reasons, we settled for a film showing 84% of a complete orbit, including all the portions needed for calculations.

Exposures were for 4 sec on 35 mm black and white film of the type used for aerial mapping and reconnaissance. A green filter (Wratten 58) was used to give maximum sharpness of the images. A decision was made to use the best exposure to show the satellites, thus overexposing the image of the disk of Jupiter. For this reason, the surface markings due to atmospheric storms on Jupiter are only glimpsed occasionally, during moments of haze or cloudiness.

First, the film shows a segment of the orbit as photographed at the telescope; a clock shows the passage of time. Due to small errors in guiding the telescope, and atmospheric turbulence, the highly magnified images of Jupiter and its satellites dance about. To remove this unsteadiness, each image was optically centered in the frame. Thus, the stabilized images were joined to give a continuous record of the motion of Io. Some variation in brightness was caused by haze or cloudiness.

The satellites move nearly in a plane that we view almost edge-on. Thus, they seem to move back and forth along a line. The position of Io in the last frame of the January 29 segment matches the position in the first frame of the February 7 segment. However, since these were photographed nine days apart, the other three satellites had moved varying distances, so they pop in and out while the image of Io is continuous.

Jupiter is not seen as a perfect circle because its rotation causes it to flatten at the poles and bulge at the equator. The effect is quite noticeable for Jupiter is large (equatorial diameter of 142,720 km and polar diameter of 133,440 km) and rotates rapidly (period of 9^h55^m).

The flattening of Jupiter is about 1/15, compared with the flattening of 1/330 for the earth. The centripetal force at the equator is $mv^2/r = mr\omega^2$, so the effect depends on r as well as on ω, the angular velocity of rotation of the planet. For Jupiter, r is 11.2 times that of the earth, and ω is 2.42 times that of the earth. The centrifugal field (artificial gravity tending to lift a mass off the surface) is, therefore, $(11.2)(2.42)^2$ as much, i.e., 64 times as great on Jupiter as on the earth.

Before the development of marine chronometers about 1750, the motions of the satellites of Jupiter were used as a "clock" for the determination of longitudes. From many observations and theory, the times at which the satellites would transit Jupiter or be eclipsed could be predicted for some standard place like Greenwich. A distant observer seeing the event would then know the time at Greenwich and could compare it with the local time, from the time of observed noon or sunset. The differences in the two times was then the observer's longitude east or west of Greenwich.

Measurements

1. Period of orbit

(Refer to pages 96 and 97 of the Unit 2 *Handbook.*) In measurement of T and R, a circular orbit is assumed. Io's orbit is perhaps the most nearly circular in all of astronomy (see Table 2-12). A student may raise the point that the earth is not infinitely far away, hence the points B and D (in Fig. 2-87) are not on parallel lines tangent to Jupiter. This makes the observed time interval slightly less than half a revolution. The effect is negligibly small, only 11 sec of real time. In the film, this corresponds to only 0.01 sec of apparent time.

2. Radius of orbit

The simple method of orbit radius described in the *Handbook* gives results even more precise than the measurement of the period.

3. Mass of Jupiter

As discussed in the *Text*, the mass of Jupiter and the mass of the sun are related as follows:

$$\frac{m_j}{m_s} = \left(\frac{R_{\text{Io's orbit}}}{R_{\text{earth's orbit}}}\right)^3 \times \left(\frac{T_{\text{Io around Jupiter}}}{T_{\text{earth (1 year)}}}\right)^2$$

The student knows the values for the earth's orbit and has measured the values for Io's orbit. Hence, the ratio of the mass of Jupiter to that of the sun can be calculated. Using values similar to the ones the student will obtain by measurement of the film, we have

$$\frac{m_j}{m_s} = \left(\frac{422 \times 10^3 \text{ km}}{150 \times 10^6 \text{ km}}\right) \times \left(\frac{365 \times 24 \text{ hr}}{42.5 \text{ hr}}\right)^2$$

$$= (2.813 \times 10^{-3}) \times (206.1)^2$$

$$= 94.9 \times 10^{-5}$$

$$= \frac{1}{1059}$$

If a student wishes to go still further, the *density* of Jupiter relative to that of the sun or that of the earth can now be calculated. The respective volumes are proportional to the cubes of the diameters, and the ratio of masses has been found. The average diameter of Jupiter is 139,000 km; that of the sun is 1,390,000 km. The result is that Jupiter's density relative to that of the sun is

$$\left(\frac{1}{1048}\right)\left(\frac{1{,}390{,}000}{139{,}000}\right)^3 = 0.95$$

Thus, Jupiter is only slightly less dense than the sun. The actual densities in gm/cm^3, based on knowledge of the gravitational constant G are: earth, 5.52; sun, 1.42; Jupiter, 1.34.

L13 PROGRAM ORBIT. I

In this film, a student is plotting the orbit of a planet, using a stepwise approximation. Then, the computer is instructed to solve the same problem. The computer and the student follow a similar procedure with 60-day intervals.

The computer "language" used was FORTRAN. The FORTRAN program (contained in the stack of punched cards) consists of the "rules of the game": the laws of motion and of gravitation. These describe precisely how the calculation is to be done. The program is translated from FORTRAN and is stored in the computer memory before it is executed.

The calculation begins with the choice of initial position and velocity. The initial vaues of X and Y are selected and also the initial components of *XVEL* and *YVEL*. (*XVEL* is the name of a single FORTRAN variable, not a product of four variables.)

The program instructs the computer to calculate the force of the sun on the planet from the inverse-square law of gravitation. The calculational procedure can be thought of as a "blow" applied toward the sun. Newton's laws of motion are used to determine how far and in what direction the planet moves after each blow.

The computer output, the result of the calculation, can be presented in many ways. A table of X and Y position values can be typed or printed. An X–Y plotter can draw a graph from the values similar to the hand-constructed graph made by the student. The computer results can also be shown on a cathode-ray tube (CRT), similar to that in your television set.

The numerical values for initial position and velocity are entered at the computer typewriter by the operator after the computer types messages requesting the values. The dialogue for trial 1 is as follows:

GIVE ME INITIAL POSITION IN AU . . . .

$X = 4$

$Y = 0$

GIVE ME INITIAL VELOCITY IN AU/YR . . . .

XVEL = 0

YVEL = 2

GIVE ME CALCULATION STEP IN DAYS . . . .

6

GIVE ME NUMBER OF STEPS FOR EACH POINT PLOTTED . . . .

1

GIVE ME DISPLAY MODE . . . .

X–Y PLOTTER

Students see that the orbit displayed on the X–Y plotter is like their own graphs. Both orbits fail to close exactly. This result is surprising, as it is known that the orbits of planets are closed. Discussion should bring out that perhaps the stepwise approximation is too coarse. The blows may be too infrequent near perihelion, where the force is largest, to be a good approximation to a continuously acting force. In the *Film Loop* "Program Orbit. II" the student can see if this explanation is reasonable.

L14 PROGRAM ORBIT. II

This film is a continuation of "Program Orbit. I" A computer is used to plot a planetary orbit with a force inversely proportional to the distance. The computer program uses Newton's laws of motion. Blows act on the body at equal time intervals.

Presumably the orbit calculated in the previous film failed to close because the blows were spaced too far apart. To test this assumption we need only specify a smaller time interval between the calculated points, and use the program previously stored in the computer memory.

A portion of the "dialogue" between the computer and the operator for trial 2 is as follows:

GIVE ME CALCULATION STEP IN DAYS ....

3

GIVE ME NUMBER OF STEPS FOR EACH POINT PLOTTED ....

7

GIVE ME DISPLAY MODE ....

X–Y PLOTTER

Points are now calculated every 3 days (20 times as many calculations as for trial 1 in "Program Orbit. I") but only one out of every seven of the calculated points is plotted to avoid a graph with too many points.

The computer output in this film can also be displayed on the face of a cathode-ray tube (CRT). The CRT display has the advantage of speed and flexibility. On the other hand, the permanent record afforded by the X–Y plotter is sometimes very convenient.

We will use the CRT display in the other *Film Loops* in this series, *L15, L16*, and *L17*.

L15 CENTRAL FORCES: ITERATED BLOWS (computer program)

A body acted on by a central force, one always directed toward or away from one point, moves so that equal areas are swept out in equal times. The force can be constant or variable, attractive or repulsive. The law of areas is a consequence of the laws of motion and of force directed always toward or away from a fixed point.

This film was made by photographing the face of a computer-driven cathode-ray tube. The computer program uses Newton's laws of motion to predict the result of applying blows. The basic program remains the same for all parts of the film, but we can program the computer to vary the force.

The computer generates a random number that determines the magnitude of the blow. Blows are applied at equal time intervals. The direction, toward or away from the center, is also selected at random, with a slight preference for attractive blows to keep the pattern on the screen. The dots show the particle's position after equal intervals of time. The intensity and direction of each blow is represented by a line at the point of impact.

Have students project the film on paper and mark the center, the points where blows were applied, and the direction of each blow. They should then measure the areas of several triangles to determine if the law of areas applies to this motion.

If a weight on a string is pulled back and released with a sideways shove, it moves in an elliptical orbit with the force center (lowest point) at the center of the ellipse. The force is proportional to the distance from the center. The computer approximates a smooth orbit for such a force by delivering the blows at shorter time intervals. In scene 2a, four blows are used for a full orbit; in scene 2b there are nine blows. In scene 2c, 20 blows give a good approximation to the ellipse that is observed with this force.

A similar program uses two planets with a force on each that varies inversely as the square of the distance from a force center. It is assumed that no force acts from one planet to the other. For the resulting ellipses, the force center is at one focus (Kepler's first law), not at the center of the ellipse as in the previous case.

L16 KEPLER'S LAWS (computer program)

A computer program similar to the one used in the *L15*, "Central Forces: Iterated Blows," causes the computer to display the motion of two planets. Impulsive blows directed toward a center (the sun) act on each planet at equal time intervals. The force exerted by the planets on each other is ignored in the program. Each planet is acted upon only by gravitational force of the sun, which varies inversely as the square of the distance from the sun.

Initial positions and initial velocities for the planets were selected. The positions of the planets are shown as dots on the face of the cathode-ray tube at regular intervals. (Many more points were calculated between those displayed.) This film is a true "loop," since the motion is continuous. There is no beginning and no end!

Students can check Kepler's three laws by projecting the film on paper and marking the successive positions of the planets. The law of areas can be verified in this situation by drawing triangles and measuring areas. Students should find the areas swept out in at least three places: near perihelion, near aphelion, and at a point approximately midway between perihelion and aphelion.

To check Kepler's third law, students measure the distances of perihelion and aphelion for each body and measure the periods of revolution. They determine whether the orbit is an ellipse with the sun at a focus by using a string and thumbtacks to draw an ellipse, as shown on page 100 of the *Handbook*. The empty focus should be symmetrical with respect to the sun's position.

Another method students may use to test whether their plot approximates an ellipse is to place the plot on an inclined plane and shine a flashlight on it. Adjustment of the distance of the light source and the inclination of the plane should be possible so that the boundary of the light matches that of the elliptical plot closely.

Advanced students might seek an algebraic method.

L17 UNUSUAL ORBITS (computer program)

In this film we use a modification of the computer program described in the notes for *L15* "Central Forces: Iterated Blows." The forces are still central, always directed toward or away from one point, but they are no longer only inverse-square forces.

The planet Neptune was discovered because of its gravitational pull on Uranus. The main force on Uranus is the pull of the sun, but the force exerted by Neptune changes the orbit of Uranus very slightly. Astronomers predicted the position of the unknown planet from this small effect on Uranus.

Typically, a planet's orbit rotates slowly because of the small pulls of other planets and the retarding force of friction due to dust in space. This effect is called "advance of perihelion." Mercury's perihelion advances about 500 sec of arc/century. Most of this was explained by perturbations due to the other planets. However, about 43 sec/century remained unaccounted for. When Einstein reexamined the nature of space and time in developing the theory of relativity, he developed a new gravitational theory. Mercury's orbit is closest to the sun, and relativity was successful in explaining the extra 43 sec/century.

The first sequence shows the advance of perihelion due to a force proportional to the distance R, added to the usual inverse-square force. The "dialogue" between operator and computer starts as follows:

PRECESSION PROGRAM WILL USE

ACCEL = $G/(R * R) + P * R$

GIVE ME PERTURBATION P

$P = 0.66666$

GIVE ME INITIAL POSITION IN AU

$X = 2$
$Y = 0$

GIVE ME INITIAL VELOCITY IN AU/YR

$XVEL = 0$
$YVEL = 3$

The symbol * means multiplication in the FORTRAN language used in the program. Thus $G/(R * R)$ is the inverse square force, and $P * R$ is the perturbing force, proportional to R.

In the second part of the film, the force is an inverse-cube force. The orbit resulting from the inverse-cube attractive force, as with most forces, is not closed. The planet spirals into the sun in a "catastrophic" orbit. As the planet approaches the sun, it speeds up, so the points are separated by a large fraction of a revolution. Different initial positions and velocities would lead to quite different orbits.

Equipment Notes

EPICYCLE MACHINE

Turning the handle of the epicycle machine shown in the photograph on the left, causes the lighted bulb to move back upon its own path and describe a retrograde loop, as in the photograph on the right. (The battery can be fixed to the machine with

tape that is sticky on both sides.) The radius of the epicycle loop can be adjusted by moving the battery and light bulb toward the center of the epicycle. Furthermore, the relative speeds along the deferent and epicycle can be changed by changing the elastic drive to different positions on the drive wheels. The consequences of these changes are retrograde loops of different shapes and durations. In short, the epicycle machine may be used to demonstrate how retrograde motion may be explained by the Ptolemaic model.

PLANETARIUM USE FOR *PROJECT PHYSICS*

Advantages of the planetarium

The opportunity to easily point to a planet with an arrow projected on a star field will in itself make a trip to the planetarium worthwhile. In addition, time may be speeded up to illustrate the consequences of the rotation, revolution, and precession of the earth. Most planetariums have auxiliary projectors for slides, overlays, and binary motions. Perhaps the greatest value to a class will be to see the projection of various coordinate systems on the star field.

The emotional impact of music used during a planetarium show has teaching advantages if the music is selected appropriately and used with discretion.

Modes of operation

Most planetarium curators are primarily lecturers. This mode of teaching can assist you in presenting ideas related to Unit 2.

Other modes of teaching that you may use are worksheets, oral questions and answers, arguing, moving back and forth from the star dome to out-of-doors, and challenging groups of students with special projects.

Worksheets can be used during twilight conditions or by rigging red lights around the cove of the planetarium. The red lights are brought up only during an answer period and do not disturb night adaptation of the eyes.

The question and answer technique is difficult when working in the dark. However, if preplanning is done with the students, questioning can be effective. A certain amount of argument results in a high degree of student involvement, but it must be done skillfully.

Moving groups of students back and forth from the star dome to the out-of-doors will result in application of the knowledge gained in the planetarium to field observations. However, this requires a good viewing night and a planetarium located away from city lights.

Preparation

Discuss and plan the program with the planetarium curator. The first program is often a compromise between what you want to teach in Unit 2 and what the curator is equipped to do in the planetarium. Often the curator will modify one of several successful programs to fit your needs. Later you can try one of the other modes of teaching described previously.

Sample programs

Each program should have a clear but limited relationship to what is being taught in Unit 2. Those that follow require about 30 min in the planetarium. Leave plenty of time for discussion, testing, and questions and/or permit students to ask questions that are not on the subject of the program. These additional activities mean that a 30-min planned program will require 60 min. More than one program could be given during a visit.

PROGRAM I THE MOON AND THE SUN

Program	*Comments*
A. The moon 1. young crescent 2. diurnal motion to west 3. 13° eastward motion per 24^h 4. phasing 5. 1 month of observations 6. eclipses 7. daily predictions of position and phase	If this can be followed up by having students observe the actual motion of the moon toward the east every 24 hr, then one can meaningfully define the direct motion of planets.
B. The sun 1. diurnal motion 2. north–south motion 3. eastward motion among stars 4. the equinoxes 5. the solstices 6. looking at the sun at different latitudes	Most planetarium machines can change the latitude of the viewer to demonstrate part 6.

C. Discussions, questions, arguments, test, or other activities.

PROGRAM II THE PLANETS

Program	*Comments*
A. Venus (or Mercury) 1. twilight observation 2. greatest angle of eastern elongation 3. greatest angle of western elongation 4. direct motion 5. * phase change 6. intensity change 7. * inferior and superior conjunctions 8. * transits	Try to schedule this planetarium visit when Venus is at its greatest angle of elongation (refer to *Sky and Telescope*). Then have students make independent daily observations. Also, it is advantageous to schedule this visit close to doing *E-10*, (Orbit of Mercury). *Supplement with slide projections

B. Mars (Jupiter or Saturn)
   1. identification by
      (a) intensity
      (b) location
      (c) motion among stars
   2. direct motion
   3. retrograde motion
   4. out-of-doors observation through a telescope

Refer to the magazine *Sky and Telescope* to determine when a superior planet is up in the evening sky and plan the planetarium visit accordingly.

Most planetariums have a telescope and its use will inspire students.

C. Discussion, questions, arguments, test, or other activities.

PROGRAM III THE EVENING SKY AND COORDINATES SYSTEMS

Program	*Comments*
A. The Evening sky 1. What can be seen at twilight? 2. moon 3. planets 4. circumpolar stars 5. one constellation 6. outside observation if possible	Most planetariums have programs planned on this subject. This program should always be given before any of the others on a given evening because the operator must set the sky carefully. During all other programs the sky will be changed from the date of the program.
B. Coordinate systems 1. altitude and azimuth 2. local meridian 3. declination 4. celestial equator 5. right ascension 6. ecliptic 7. examples of star locations	It is helpful to do classwork on coordinate systems both before and after this planetarium program. Use *T13*, *T14*, and *T17*.

C. Discussions, questions, arguments, test, or other activities.

Suggested Solutions to Study Guide Problems

CHAPTER 5

2. (a) Local noon will occur when the shadow is shortest, or midway between A.M. and P.M. times of equal length
   (b) The noon shadow on June 21st will be shorter than at any other noon.
   (c) The solar year is the number of days between days, in the same season, when the shadow lengths are equal. For example, this could be June to June. The more accurate observations would be made close to the equinoxes when the sun's N–S position is changing most rapidly.

3. (a) 365.25000 days − 365.24220 days = 0.00780 day = $(7.80 \times 10^{-3}\text{ days})(8.64 \times 10^{4}\text{ sec/day}) = 674\text{ sec}$

   (b) $$\frac{7.80 \times 10^{-3}\text{ days}}{3.6525 \times 10^{2}\text{ days}} \times 100 = 0.0021\%\text{ of a year}$$

4.

(a) *Observations*	(b) *Reason for Importance*
Apparent motions of the sun:	
(a) daily westward motion	day–night determination
(2) annual north–south motions	seasonal changes
(e) annual eastward motion through the stars	length of the year, basis of the calendar
Apparent motions of the moon:	
(a) phase changes	tie-in with other physical phenomena
(2) continuous motion eastward among the stars	basis of the month; related to eclipses of sun and of moon
Apparent planetary motions:	
(1) retrograde motions westward at opposition (Mars, Jupiter, Saturn)	a seemingly contrary motion that should be explained
(2) periodic motions of Venus and Mercury near the sun	planets are different in some ways
Apparent fixed positions of the stars:	
(1) continuous circumpolar rotation of the celestial sphere	the most uniform of all observed motions

5. The nightly and annual changes could be explained for an observer at a fixed location. However, one could not explain the changes in angular elevation of the North Star when traveling in the N–S direction, nor the variation in rising and setting times when traveling in the E–W direction.

6. In each 24-hour period the moon rises in the east and sets in the west, rising later on each

successive date. It also appears to be moving eastward among the stars and relative to the sun. It goes through the full cycle of phases in a month.

7. After Mercury and Venus move westward past the sun, they appear in the early morning sky and soon reach their maximum brightness. At maximum elongation, east or west, they will appear in quarter phase. Thereafter, they will fade slowly and move in toward the sun, pass it, reappear in the evening sky, and brighten slowly as they approach maximum eastward elongation.

8.

Quadrant	No. of Degrees
1	102
2	78
3	78
4	102

9. (a) $\frac{360^\circ}{24^h} = 15^\circ$ per hour

(b) (1) Check latitudes on a map of the U.S.A. or compare the length of noon shadows of posts of equal height.

(2) The airplane goes 5×800 km or 4,000 km between Washington, D.C. and San Francisco.

(3) From the difference in sunset times we know that the longitudes of Washington, D.C. and San Francisco differ by 3 hours, or 45°, or one half of the distance around the earth.

Then the distance around the earth at the latitude of these cities (about 38°) is $8 \times$ 4,000 km or about 32,000 km. The value at the equator would be larger by about 20%. The equatorial circumference would be $32{,}000 \times 1.2$, or about 38,000 km. The diameter of a circle is the $\frac{\text{circumference}}{\pi}$. So the diameter of the earth is about $\frac{38{,}000 \text{ km}}{3.14} = 12{,}100$ km.

10. Ptolemy assumed: (a),(b),(c),(d),(e),(f)

11. (a) They both predicted the location of the stars and planets reasonably well.

(b) According to Greek science, the geocentric system had the advantage of conforming to the dogma of the perfect planets moving only in perfect circles and the earth being stationary. The disadvantage of the Greek system was that too many epicycles were necessary to explain the heavenly motions.

Although the heliocentric system explained retrograde motion more simply, it did not conform to the Greek dogma and it predicted stellar parallax, which could not be confirmed during ancient times.

(c) Ptolemy did provide a practical system based on celestial events as viewed from the earth, which served quite well for many centuries.

12. The phenomena of interest in ancient astronomy are mostly slowly changing and cyclic. Therefore, a few relatively simple assumptions such as circular motions, could lead to a fairly accurate theory.

The phenomena of meteorology and zoology are far more numerous and rapidly changing.

CHAPTER 6

2.

3. Copernicus argued that having the sun at the center of the planetary system was "more reasonable," and that the planets would "move uniformly about (their) proper center—as the rule of absolute motion requires." Also, the distances of the planets from the sun increased with their periods, which was "pleasing to the mind."

4. If you start with the hands together at 12 o'clock, you will observe that the first pass occurs a little after 1:05; the second a little more after 2:10; the sixth at about 6:33, etc. The eleventh pass occurs as the hands return to 12:00. Thus, the frequency of passing is 11/12 or 0.92 cycle/hr, which represents the apparent frequency of the long hand as viewed by an observer riding on the short hand. The actual frequency of the short hand is 1/12 cycle/hr, since it takes 12 hr to come back to the original numeral on the clock face. The actual frequency of the long hand is 1 cycle/hr. Since $11/12 + 1/12 = 1$, we are led to the statement that:

actual frequency of the long hand = apparent frequency of the long hand + actual frequency of the short hand

This statement correlates with the discussion about the planets as viewed from the earth. (See *Text* page 160.) Once the frequency is known, the period follows immediately as $\frac{1}{\text{frequency}}$.

5. Copernicus calculated the distances of the planetary orbits from the sun and the periods of planetary motion around the sun. The Copernican system was more simple and harmonious than that of Ptolemy. Also, these orbits began to seem like the paths of real planets, rather than only mathematical combinations of circles that were useful for computing positions.

6. The Copernican system involved a reordering of the relative importance of the sun and the earth. The sun became dominant while the earth became "another planet." These philosophical results were more important than the shift in geometry.

7. (a) When Mercury and Venus are moving from farthest east to farthest west relative to the sun, they are overtaking the earth and passing between the earth and sun. In the case of Venus, only about one-quarter of its orbital period is required for the planet to move from farthest east to farthest west.

   (b) To find a period for Mercury's motion relative to the sun, we have a choice of which intervals to measure. Probably the most significant would be the times required for Mercury to pass the sun. The three intervals for motion from west to east are: 110, 105, and 130 days. For motion from east to west, the intervals are 127 days and 112 days. The average of the five intervals is 115 days. The variations result from the eccentric orbit of Mercury.

   (c) The mean cycle compared to the sun is 115 days, or 0.315 year. This is T in the equation of the chase problem. N is 1. Then

$$\text{period of Mercury} = \frac{T}{(T + N)} = \frac{0.315}{1.315} = 0.240 \text{ yr} = 87.5 \text{ days}$$

   (d) The major sources of uncertainty are:

   (1) Only three cycles for Mercury are shown. With more cycles a better average would result.

   (2) The orbit of Mercury is not circular, but is rather eccentric. Therefore, the observed intervals depend upon the direction from which we on the moving earth see Mercury in its orbit.

   (e) Only a little more than half a cycle is shown for Venus. We assume that the motion is symmetrical. Since a half cycle takes 289 days, a full cycle would be 578 days. In one year we observe $\frac{365}{578}$ of a cycle; this is N = 0.632. T is 1 year. Then

$$\text{period of Venus} = \frac{1}{(1 + 0.632)} = 0.613 \text{ yr} = 224 \text{ days}$$

8. The sequence 0.39 AU to 9.5 AU shows no obvious regularity. A natural step for the student to take would be to plot average orbital radius versus sequential order and then extrapolate to $n = 7$. This yields a value of about 14 AU. The next planet, Uranus, actually has an average orbital radius of about 19.2 AU, so extrapolation as above is not satisfactory. An empirical formula, known as the Bode–Titus law, is surprisingly successful not only in correlating the known data but also in predicting the correct orbit for Uranus. However, it breaks down completely for Neptune and Pluto. (This law is discussed on pages 198–200 of *Foundations of Modern Physical Science* by Holton and Roller.)

9. $$\tan\frac{\theta}{2} = \frac{1 \text{ AU}}{x}$$

$$x = \frac{1 \text{ AU}}{\tan\frac{\theta}{2}}$$

$$\theta = \left(\frac{1}{2400}\right)^{\circ} = (4 \times 10^{-4})^{\circ}$$

$$\frac{\theta}{2} = (2 \times 10^{-4})^{\circ}$$

Consulting trigonometric tables we find

1. for small angles the tangent of the angle is proportional to the angle

2. $\tan 1^{\circ} = 0.018 = 1.8 \times 10^{-2}$

thus

$$\tan\frac{\theta}{2} = \tan(2 \times 10^{-4})^{\circ} = 3.6 \times 10^{-6}$$

then

$$x = \frac{1 \text{ AU}}{3.6 \times 10^{-6}} = 2.8 \times 10^{5} \text{ AU}$$

10. Each star could have been assigned a very small epicycle with a period of 1 year. This would be similar to the larger epicycles with a 1-year period assigned o Mars, Jupiter, and Saturn.

11. Some that might be suggested are
    (a) we do not feel the earth's rotation on its axis and its revolution around the sun
    (b) it is difficult to imagine the wave–particle duality of light and matter
    (c) time dilation from relativity theory
    (d) atomic description of solids

12. If the earth is not the center of the universe but just one of many planets, we may not be unique. Projects that scan the skies for evidence of some kind of extraterrestrial intelligence in the form of radio signal patterns have been attempted. Our knowledge of the other planets of our solar system virtually rules out th possibility of life being found on any of them. The possible discovery of such extraterrestrial life raises such questions as: How does their state of civilization compare with ours? How do we all fit into a cosmic plan, if indeed there is one?

13. The motion is difficult to explain. From Nov. 1, 1909, until May 3, 1910, the comet moved westward (retrograde). During April 1910, its position changed very little, although the earth's position relative to the sun changed about 30°. During May, the comet moved more than 90° eastward, with the most rapid motion occurring around May 19th.

    Students who do *E2–11* will find that they can, with the three-dimensional model, explain the comet's observed motion. Also, they will notice some other interesting results.

14. The phrasing of the question, of course, calls for a personal reaction. The Copernican system did not seem to most people any more "real" than he Ptolemaic, but it did lay the groundwork for the later work of Kepler and Newton. So we develop the feeling that a heliocentric scheme is not only simpler but lends itself to a dynamic explanation through the law of gravitational attraction.

CHAPTER 7

2. By definition, 1 min of arc is 1/60°. Therefore, 8 min is 8/60° or about 1/8°. The moon's apparent diameter is close to 30 min; 8 min is therefore about 1/4 of the moon's apparent diameter.

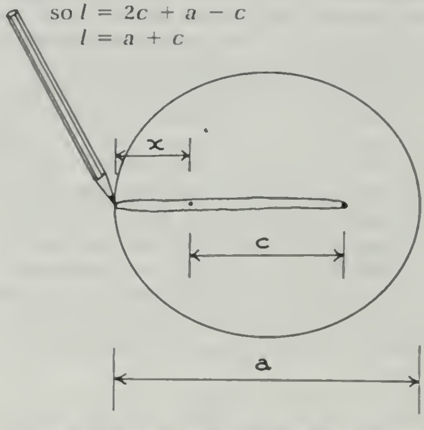
3. (a) Compare an observation of Mars at opposition to another observation 687 days later, that is, when Mars is back at the same place in its orbit.
    (b) Use the observed direction toward Mars and the sun on these two observation dates to triangulate the second position of the earth relative to the first position.
    (c) Repeat this procedure for different sets of observations 687 days apart.

4. Since the earth's orbital speed is inversely proportional to its distance from the sun, the percentage change in speed will be the same as the percentage change in distance. 1.02 AU/0.98 AU = 1.04, so the distance 1.02 AU is 4% greater than the distance 0.98 AU. The speed at 0.98 AU will be 4% greater than the speed at 1.02 AU.

5. (a) First plot the orbit of the earth.
    (b) Compare observations of Mars made 687 days apart (that is, when Mars is at the same position in its orbit) and draw sight lines from the position of the earth in its orbit. The point where the two sight lines to Mars intersect is a point on the orbit of Mars.
    (c) Repeat this process on pairs of observations for different positions of Mars in its orbit.

6. The length l of the string in the loop is most easily found by considering the extreme case at one end of the ellipse. If x is the distance from a focus to the nearest point on the ellipse then,

$$l = 2c + 2x$$
but $2x = a - c$
so $l = 2c + a - c$
$$l = a + c$$



7. The shape is shown in the figure above. If the tacks (foci) are together, c is zero and the figure is a circle of radius a. As the tacks are moved farther apart, the ellipse becomes thinner and the eccentricity (e) greater.

8. $e = a/c$

$$e = \frac{5\ \text{cm}}{9\ \text{cm}} = 0.56$$

9. (a) Your sketch should look like the one in question 6 where x equals the perihelion distance while the distance from the right focus to the ellipse is the aphelion.

(b) When P is the perihelion distance and A is the aphelion distance,

$$c = (A - P) \text{ and } a = (A + P)$$

$$P = \frac{a - c}{2} \text{ and } A = \frac{a + c}{2}$$

(c) $R_{av} = \dfrac{a}{2} = \dfrac{A + P}{2}$

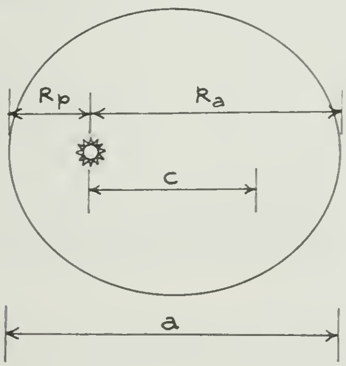
10. The second focus is empty.

11. (a) apocrypha, apostasy, apostrophe, apojove, periodontal, pericardium, perijove
    (b) Apogee is the position of the satellite farthest from the earth, perigee the position closest to the earth.
    (c) apolune and perilune

12. (a) Given: $A = 5$ cm and $P = 2$ cm
    $c = (A - P) = (5\text{ cm} - 2\text{ cm}) = 3\text{ cm}$
    $a = (A + P) = (5\text{ cm} + 2\text{ cm}) = 7\text{ cm}$
    $e = a/c = 7\text{ cm}/3\text{ cm} = 0.43$
    (b) Given: $e = 0.5$ and $a = 10$ cm
    $c = a \times e = 0.5 \times 10\text{ cm} = 5\text{ cm}$
    $P = 2/(a - c) = 2/(10\text{ cm} - 5\text{ cm})$
    $= 2.5\text{ cm}$
    $A = 2/(a + c) = 2/(10\text{ cm} + 5\text{ cm})$
    $= 7.5\text{ cm}$
    (c) Given: $P = 5$ cm and $e = 0.8$
    $e = a/c = (A + P)/(A - P)$
    $A(1 - e) = P(1 + e)$
    $A(1 - 0.8) = 5\text{ cm}(1 + 0.8)$
    $0.2\,A = 9\text{ cm}$
    $A = 45\text{ cm}$
    $c = (A - P) = (45\text{ cm} - 5\text{ cm})$
    $= 40\text{ cm}$
    $a = (A + P) = (45\text{ cm} + 5\text{ cm})$
    $= 50\text{ cm}$

13. By definition, $e = \dfrac{c}{a}$

$$a = R_a + R_p = 70.0 \times 10^6\text{ km} + 45.8 \times 10^6\text{ km}$$
$$= 115.8 \times 10^6\text{ km}$$
$$c = R_a - R_p = 70.0 \times 10^6\text{ km} - 45.8 \times 10^6\text{ km}$$
$$= 24.2 \times 10^6\text{ km}$$
$$e = \frac{24.2 \times 10^6\text{ km}}{115.8 \times 10^6\text{ km}}$$
$$e = 0.209$$



14. The ratio of speeds is the inverse of the ratio of distances.

$$R_p = \tfrac{1}{2}(a - c)$$
$$R_a = \tfrac{1}{2}(a + c)$$
$$\frac{R_p}{R_a} = \frac{a - c}{a + c}$$

but $e = \dfrac{c}{a}$, so $c = 0.254a$

$$\frac{R_p}{R_a} = \frac{a - 0.254a}{a + 0.254a}$$
$$= \frac{0.746a}{1.254a}$$
$$= 0.594$$

Therefore, $\dfrac{V_a}{V_p} = 0.594$

15. (a) From Kepler's law, for bodies orbiting the sun

$$\frac{T^2}{R_{av}{}^3} = 1\,\frac{\text{yr}^2}{\text{AU}^3}$$

For Halley's comet, $T = 76$ yr, so (with the understanding that T is expressed in yr and R_{av} in AU)

$$\frac{(76\text{ yr})^2}{R_{av}{}^3} = 1\,\frac{\text{yr}^2}{\text{AU}^3}$$
$$\frac{76^2}{R_{av}{}^3} = 1$$
$$R_{av}{}^3 = 76^2$$
$$= \sqrt[3]{76^2}$$
$$= \sqrt[3]{57{,}700}$$
$$= 17.9\text{ AU}$$

(b) $R_a = \tfrac{1}{2}a + \tfrac{1}{2}c$ but $\dfrac{c}{a} = e = 0.97$

so $R_a = \tfrac{1}{2}(a + 0.97a)$

$= \dfrac{1.97a}{2}$ or, since $\dfrac{a}{2} = R_{av}$,

$= 1.97\,R_{av} = 1.97 \times 17.9\text{ AU}$

$= 35.3\text{ AU}$

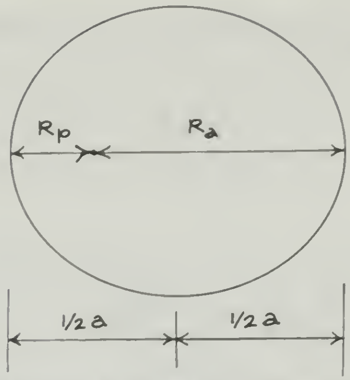
(c) $R_p = \tfrac{1}{2}a - \tfrac{1}{2}c$ but $\dfrac{c}{a} = e = 0.97$

so $R_p = \tfrac{1}{2}(a - 0.97a)$

$= \dfrac{0.03a}{2}$ or, since $\dfrac{a}{2} = R_{av}$,

$= 0.03\,R_{av} = 0.03 \times 17.9\text{ AU}$

$= 0.54\text{ AU}$



(d) The ratio of speeds is the inverse of the ratio of the distances.

$$\frac{R_a}{R_p} = \frac{35.4}{0.54} \approx 66$$

The greatest speed is about 66 times the least speed.

16. For planets orbiting the sun, $\dfrac{T^2}{R_{av}^{\ 2}} = 1\ \dfrac{\text{yr}^2}{\text{AU}^3}$.

Thus, $T = \sqrt{R_{av}^{\ 3}}$ when T is expressed in yr and R_{av} in AU.
For Pluto, $R_{av} = 39.6$ AU, so

$$T_{\text{Pluto}} = \sqrt{39.6^3} \approx \sqrt{62{,}000} \approx 249 \text{ yr}$$

17. For Uranus $\dfrac{T_U^{\ 2}}{R_U^{\ 3}} = \dfrac{84.013^2}{19.19^3} \approx \dfrac{7{,}060}{7{,}070} \approx 1.00$

For Neptune $\dfrac{T_N^{\ 2}}{R_N^{\ 3}} = \dfrac{164.783^2}{30.07^3} \approx \dfrac{27{,}150}{27{,}200} \approx 1.00$

For Pluto $\dfrac{T_P^{\ 2}}{R_P^{\ 3}} = \dfrac{248.42^2}{39.52^3} \approx \dfrac{61{,}800}{61{,}600} \approx 1.00$

18. The figures available to Kepler were less accurate than those used in question 17, but must have given values of k close to 1.0. He wanted such a regularity and was willing to overlook the small variations in k.

19. The radius of the satellite orbit is measured from the center of the earth: 380 min is equal to 22,800 sec and 18,000 km is equal to 18×10^6 m. Therefore

$$\frac{T^2}{R^3} = \frac{(22{,}800 \text{ sec})^2}{(18 \times 10^6 \text{ m})^3} = 8.9 \times 10^{-14} \text{ sec}^2/\text{m}^3$$

20. $R_1^3/T_1^2 = k = R_2^3/T_2^2$
Therefore,

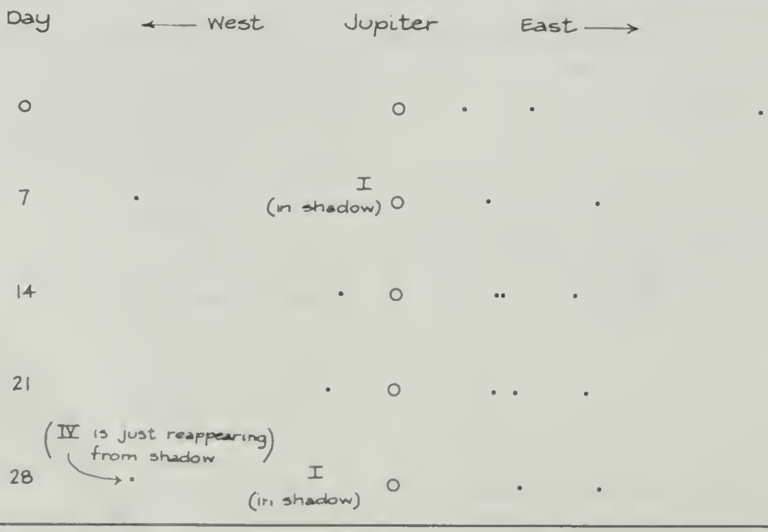
$$(T_1/T_2)^2 = (R_1/R_2)^3$$
$$(T_1/T_2)^2 = (1/4)^3$$
$$(T_1/T_2)^2 = 64$$
$$T_1/T_2 = \sqrt{64}$$
$$T_1/T_2 = 8 \ (T_2 \text{ is 8 times } T_1)$$

21. The period of the satellite is 28 days (24×10^5 sec); $T^2/R^3 = 9.9 \times 10^{-14}$ sec^2/m^3.

$$R^3 = 9.9 \times 10^{-14} \text{ sec}^2/\text{m}^3/T^2$$
$$R^3 = \frac{(24 \times 10^5 \text{ sec})^2}{9.9 \times 10^{-14} \text{ sec}^2/\text{m}^3}$$
$$R^3 = 5.91 \times 10^{25} \text{ m}^3$$
$$R = 3.9 \times 10^8 \text{ m}$$

22. T^2/R^3 is equal to 9.9×10^{-14} sec^2/m^3 within the accuracy of the data given and the calculations.

23. (a) Note that, in addition to East being on the right, North is downward. It may be of special interest to a very few students that the sun is northeast of Jupiter; therefore, a shadow is cast toward the southwest. Consequently, on days 7 and 28 Io is in a shadow. Furthermore, on day 28 Calistro is just emerging from a shadow. These are worthwhile three-dimensional thoughts.



(b), (c)

	Satellite	R_{av} (mm)*	T days	hours	T (× 10^2 hours)	T^2 (× 10^4 hours^2)	R_{av}^3 (× 10^2 mm^3)	$T^2/R^3 = k'$ (hr^2/mm^3)**
I	Io	3.4	1	20	0.44	0.193	0.398	485
II	Europa	5.2	3	12	0.84	0.703	1.42	495
III	Ganymede	8.2	7	0	1.68	2.81	5.56	505
IV	Calistro	14.6	16	16	3.84	14.7	31.3	470

*The measurements were made by means of a 10 × view finder with a millimeter scale attached to it. Therefore the scale distance of 3.4 mm for Io represents 419 000 km. This is perfectly proper and does not prevent one from determining the similarity of the constant of proportionality between T^2 and R^3.

**Students will wonder whether these numbers adequately demonstrate invariance. They should answer their own question by analyzing the error involved in each measurement and the inaccuracy of the drawing from *Sky and Telescope*.

24. Guide students to a discussion of science coverage in various media (newspapers, magazines, TV, etc.).

25. Kepler's persistence (or stubbornness) is remarkable. The discussion of this question could be quite interesting.

26. Kepler expected a theory to predict new observations with accuracy; began to seek physical causes for motions; switched from geometrical to algebraic mathematics; examined with care the limits of accuracy of the observations of Brahe and accepted the observations to that accuracy (that is, he trusted the instruments and the observers).

27. An empirical law is a generalization based on observations. Empirical laws are inherently limited since phenomena can never be observed completely. Their value is that (1) they are directly related to experience and (2) they serve as a foundation for theoretical speculation.

CHAPTER 8

2. (a) along a straight line at uniform speed.
   (b) caused the planets to deviate from motion in a straight line.
   (c) directed to a center; and that center was the sun.
   (d) varies inversely with the square of the distance.

3. (a) The fall of the apple caused Newton to contemplate the fall of the moon toward the earth.
   (b) Because the moon is 60 times farther than the apple from the center of the earth, the acceleration on the moon should be $(1/60)^2$ of that on the apple, or 9.8 m/sec^2 × $(1/60)^2$ = 2.7×10^{-3} m/sec^2.
   (c) The centripetal acceleration
   $a_c = R/v^2$

$$a_c = \frac{(2\pi R)^2}{T^2} \times \frac{1}{R}$$

$$= \frac{4\pi^2\,(3.9 \times 10^8)\text{ m}}{(28 \times 24 \times 3600)^2\text{ sec}^2}$$

$$2.71 \times 10^{-3}\text{ m/sec}^2$$

4. The real question is "What holds the moon down?" The Newtonian view requires a force to prevent the moon from flying off in a straight line.

5. Every object in the universe attracts every other object with a gravitational force directed between the centers of the objects. $F = G(Mm)/R^2$ where F is the force between objects, M and m are the masses of the objects, R is the distance between centers of the objects, and $G = 6.67 \times 10^{-11}$ N · m^2/kg^2. Available evidence shows no change in G with time or position.

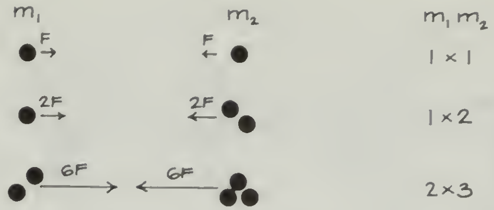
6.

	T (days)	T^2	D	D^3	$T^2/D^3 (\times 10^{-2})$
I	1.77	3.13	6.04 r	222	1.41
II	3.55	12.55	9.62	900	1.396
III	7.15	51.0	15.3	3610	1.407
IV	16.7	278	27.0	20000	1.39

The results are in agreement to an accuracy of about 1%, which is about the accuracy of the data given. The law of periods appears to hold.

7. (a) Descartes' theory directed further attention to the question "Why do planets move according to Kepler's laws?"
   (b) Having space filled with a fluid avoided the problem of "action at a distance" associated with Newtonian gravitation.
   (c) The theory conceived of planetary motions as explainable in terms of phenomena familiar to us on earth.

8. (a) Since the designation of which body is m_1 or m_2 is arbitrary, the general relation appears to be $F \propto m_1 m_2$.



(b) (1) $(m_1 + m_2) \propto F$

If either mass vanished, a force would still exist, but this is contrary to the definition of a force that must produce an acceleration. It is absurd.

(2) $\dfrac{m_1}{m_2} \propto F$

If m_1 vanishes, the force F becomes zero, which seems to be consistent with the definition of a force. But either body could be identified as m_1. If the denominator vanishes, the force becomes infinite, which is absurd.

This type of analysis uses the "limiting case" in an imaginary solution. This is a powerful technique in the study of possible solutions.

9. The acceleration due to gravity at the surface of a body $a_g \propto m/R^2$ when m is the mass of the body and R its radius.

$$\frac{a_g \text{ (moon)}}{a_g \text{ (earth)}} = \frac{m_M/R^2_M}{m_E/R^2_E}$$
$$= \frac{m_M}{m_E} \times \frac{R^2_E}{R^2_M}$$
$$= \frac{0.012}{1.00} \times \left(\frac{3.95 \times 10^2}{1.08 \times 10^3}\right)^2$$
$$= 0.012 \times \frac{15.6}{1.16} = 0.162$$

10. $$F_c = m_p a_c = m_p \left(\frac{4\pi^2 R}{T^2}\right)$$
$$F_g = G\frac{m_p m_s}{R^2}$$

But $F_c = F_g$, so $\left(\dfrac{4\pi^2 R}{T^2}\right) = G\dfrac{m_s}{R^2}$

Then $T^2 = \dfrac{4\pi^2 R^3}{Gm_s}$, or $T^2 = \left(\dfrac{4\pi^2}{G}\right)\dfrac{R^3}{m_s}$

11. The gravitational force between the two spheres is given by

$$F = G\frac{Mm}{R^2}$$
$$= 6 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2 \left(\frac{1{,}000 \text{ kg} \times 100 \text{ kg}}{(10 \text{ m})^2}\right)$$
$$= 6.67 \times 10^{-8}\ \text{N}$$

The acceleration on the large sphere is

$$a_M = M/F$$
$$= 1{,}000 \text{ kg}/6.67 \times 10^{-8}\ \text{N}$$
$$= (6.67 \times 10^{-11} \text{ kg m/sec}^2)/\text{kg}$$
$$= 6.67 \times 10^{-11} \text{ m/sec}^2$$

The acceleration on the small sphere is 10 times greater: $a_m = 6.67 \times 10^{-10}$ m/sec^2.

12. The predicted positions and discoveries of Neptune and later Pluto were dramatic applications of the Newtonian gravitational theory. The predicted position of Neptune was based on small unexplained irregularities in the motion of Uranus. By a complex analysis, the magnitude and direction of the disturbing forces were calculated. Then the direction to the center of those forces (Neptune) was derived. On the basis of Bode's law, the distance of Neptune from the sun had been assumed. Actually, the planet was not so distant nor so massive as assumed.

During the years when Uranus was showing the unexpected motions, it was passing between Neptune and the sun. Thus, the attractions caused by Neptune were at a maximum.

The predicted position of Pluto was based on irregularities in the motions of Neptune, Uranus, and Saturn, which were all at the limits of observational accuracy. Although Pluto was discovered near one of the predicted positions, its mass is probably too small to exert the suspected forces.

13. (a) $$k_{Jup} = \frac{T^2}{R^3}$$
$$= \frac{\left(\dfrac{16.7^d}{365 \text{ d/yr}}\right)^2}{\left(\dfrac{1}{80}\text{ AU}\right)^3} = \frac{(4.57 \times 10^{-2})^2}{(1.25 \times 10^{-2})^3}$$
$$= \frac{20.8 \times 10^{-4}}{1.97 \times 10^{-6}} = 1.05 \times 10^3 \text{ yr}^2/\text{AU}^3$$

(b) $T_s^2 = \left(\dfrac{4\pi^2}{G}\right)\dfrac{R_s^3}{m_s}$, for a planet moving about the sun, S. (See 10.)

Similarly, $T_J^2 = \left(\dfrac{4\pi^2}{G}\right)\dfrac{R_J^3}{m_J}$, for a satellite moving about Jupiter, J.

Then $m_s = \dfrac{4\pi^2 R_s^3}{G\,T_s^2}$ and $m_J = \dfrac{4\pi^2 R_J^3}{G\,T_J^2}$

Thus $\dfrac{m_s}{m_J} = \left(\dfrac{R_s}{R_J}\right)^3 \left(\dfrac{T_J}{T_s}\right)^2$

$$\frac{m_s}{m_J} = \left(\frac{5.2}{1.26 \times 10^{-2}}\right)^3 \left(\frac{45.8 \times 10^{-3}}{11.9}\right)^2$$

(in years and AU)

$$= (4.12 \times 10^2)^3 (3.85 \times 10^{-3})^2$$
$$= 7.12 \times 10^6 \times 14.8 \times 10^{-6}$$
$$= 1.05 \times 10^3 \text{ yr}^2/\text{AU}^3$$

Note: In this calculation, the values used for R_S and T_S were those associated with Jupiter. We could have used the set associated with any of the sun's planets, for example the earth. Would this have been an advantage?

Alternate Solution

Since $T_S^2 = \left(\frac{4\pi^2}{G}\right)\frac{R_S^3}{m_S}$, we have

$$m_S = \left(\frac{4\pi^2}{G}\right)\left(\frac{R_S^3}{T_S^2}\right), \text{ but } k_S = \frac{T_S^2}{R_S^3},$$

$$\text{so } m_S = \frac{\left(\frac{4\pi^2}{G}\right)}{k_S}$$

Similarly, for Jupiter $m_J = \dfrac{\left(\frac{4\pi^2}{G}\right)}{k_J}$

Then $\dfrac{m_S}{m_J} = \dfrac{k_J}{k_S} = 1.05 \times 10^3$, as found in part b.

(c) The distance to Jupiter could be found by triangulation in the same way that Kepler found the distance to Mars. At that distance, each unit of angle corresponds to a specific distance. The angular radius of the satellite's orbit can be observed and then converted into kilometers.

14. From 10 we have

$$T = \left(\frac{4\pi^2}{G}\right)\left(\frac{R^3}{m_E}\right)$$

as applied to an earth satellite.

$$T = 24 \text{ hr} = 24 \times 3600 \text{ sec} = 8.65 \times 10^4 \text{ sec}$$

$$R^3 = \frac{G\, m_E}{4\pi^2}T^2$$

$$= \frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{4 \times 3.14^2} \times (8.65^2 \times 10^8)$$

$$= \frac{6.67 \times 5.98 \times 8.65 \times 8.65}{4 \times 3.14 \times 3.14} \times 10^{21}$$

$$= 75.8 \times 10^{21}$$

$$R = \sqrt[3]{75.8} \times 10^7 \text{ m} = 4.2 \times 10^4 \text{ km}$$

Note: This is the distance from the center of the earth, not from the surface of the earth.

Alternate Solution

From the motion of the moon we can derive k_E:

$$k_E = \frac{T^2}{R^3} = \frac{(27.3 \text{ days})^2}{(3.80 \times 10^5 \text{ km})^3}$$

$$= \frac{743}{55.0 \times 10^{15}} = 13.5 \times 10^{-15}$$

For a satellite to stay above the same place on the earth it must have a period T of 1 day. Then

$$R^3 = \frac{T^2}{k_E}, \text{ or } R = \left(\frac{T^2}{k_E}\right)^{1/3}$$

$$R = \left(\frac{1 \text{ day}}{13.5 \times 10^{-15}}\right)^{1/3} = (0.0740 \times 10^{15})^{\frac{1}{3}} = 0.424 \times 10^5 \text{ km}$$

$$R = 42{,}400 \text{ km}$$

15. $F_g = G\left(\dfrac{m_1 m_2}{R^2}\right)$

Let $m_1 = 1$ kg and $m_2 = m_E =$ mass of earth

$$m_E = \frac{R^2 F_g}{G\, m_1} = \frac{(6.4 \times 10^6)^2 [\text{m}^2] \times 9.8\,[\text{N}]}{6.67 \times 10^{-11}\left[\frac{\text{Nm}^2}{\text{kg}^2}\right] \times 1\,[\text{kg}]}$$

$$= \frac{(40.7 \times 10^{12}) \times 9.8}{6.67 \times 10^{-11}}$$

$$= 59.8 \times 10^{23} \text{ kg, or } 5.98 \times 10^{24} \text{ kg}$$

16. $m_E = \left(\dfrac{4\pi^2}{G}\right)\dfrac{R^3}{T^2}$, where R and T refer to the moon

mean distance to moon $= 3.84 \times 10^8$ m
mean period of moon $= 27\frac{1}{3}$ days $\times$ (86,400 sec/day) $= 2.36 \times 10^6$ sec

$$m_E = \frac{4\pi^2}{6.67 \times 10^{-11}}\left[\frac{\text{kg}}{\text{Nm}^2}\right] \times \frac{(3.84 \times 10^8)^3 [\text{m}^3]}{(2.36 \times 10^6)^2 [\text{sec}^2]}$$

$$= \frac{4 \times 9.88 \times (56.8 \times 10^{24})}{(6.67 \times 10^{-11}) \times (5.57 \times 10^{12})}$$

$$= 6.04 \times 10^{24} \text{ kg}$$

This agrees with the answer to 15 to within 1%.

17. The moon's gravitational attraction on the fluid waters on the far side of the earth is less than its attraction on the solid earth. Therefore, the solid earth is pulled away from the water on the far side and a high tide results.

18. Discussion. The moon's inertial motion is uniform motion in a straight line; its displacement x would be $x = vt$. Combined with that moion is the gravitational displacement y toward the earth given by $y = \frac{1}{2}at^2$.

19. (a) density $= \dfrac{\text{mass}}{\text{volume}}$

For the earth,
mass $= 5.98 \times 10^{24}$ kg
radius $= 635 \times 10^6$ m

$$\text{volume} = \frac{4}{3}\pi R^3 = 1.33 \times 3.14 \times (0.635)^3 \times 10^{21}$$

$$= 1.08 \times 10^{21}\ \text{m}^3$$

$$\text{density} = \frac{5.98 \times 10^{24}}{1.08 \times 10^{21}} = 5.52 \times 10^3\ \text{kg/m}^3$$

$$= 5.52\ \text{g/cm}^3$$

(b) Somewhere within the earth, not at the surface, there must be a large volume with a density well above the mean value of 5.52 g/cm³. Because the earth is almost a symmetrical sphere, the large, dense mass may be a central core.

20. Height above moon = 112 km
Period of orbit around moon = 120.5 min.
= 7.23×10^3 sec

$$m_m = \left(\frac{4\pi^2}{G}\right)\frac{R^3}{T^2}$$

$$R = (1{,}740 + 112)\ \text{km} = 1{,}852\ \text{km} = 1.85 \times 10^6\ \text{m}$$

$$m_M = \frac{4 \times 3.14 \times 3.14}{6.67 \times 10^{-11}} \times \frac{(1.85 \times 10^6)^3}{(7.23 \times 10^3)^2}$$

$$m_M = 5.92 \times 10^{11} \times \frac{6.40 \times 10^{18}}{5.20 \times 10^7}$$

$$m_M = 7.30 \times 10^{22}\ \text{kg}$$

21. (a) $T^2 = \left(\frac{4\pi^2}{G}\right)\frac{R^3}{m_{Mars}}$

R = radius of Mars
= 3,385 km, or $3{,}385 \times 10^6$ m

The mass of Mars is 0.11 of the earth's mass, or $0.11 \times 6.0 \times 10^{24}$ kg = 6.6×10^{23} kg.

$$T^2 = \frac{4 \times 9.88}{6.67 \times 10^{-11}} \times \frac{(3.385 \times 10^6)^3}{6.6 \times 10^{23}}$$

$$= 35.2 \times 10^6\ \text{sec}^2$$

$$T = 5.9 \times 10^3\ \text{sec}^2$$

$$T = 1.65\ \text{hours of earth time.}$$

(b) In a circular orbit, the satellite's velocity would be

$$v = \frac{2\pi R}{T}$$

$$= 2\pi\frac{3.385 \times 10^6\ \text{m}}{5.99 \times 10^3\ \text{sec}} = 3{,}550\ \text{m/sec}$$

$$v = 3.55\ \text{km/sec}$$

(c) Mars is not smooth. The slightest gravitational effect of another satellite, for example, would cause the satellite to collide with the surface of Mars.

22.

	Actual mass	Mass relative to earth = 1
Sun	1,980,000 × 10^{24} kg	330,000.
Mercury	0.328	0.055
Venus	4.83	0.81
Earth	5.98	1.0
Mars	0.637	0.107
Jupiter	1900.	318.
Saturn	567.	95.
Uranus	88.0	14.7
Neptune	103.	17.2
Pluto	1.1	0.185

23.

$$\frac{F_g\ (\text{sun on earth})}{F'_g\ (\text{moon on earth})} = \frac{G\dfrac{m_S m_E}{R^2_{SE}}}{G\dfrac{m_M m_E}{R^2_{ME}}}$$

$$= \frac{m_S R^2_{ME}}{m_M R^2_{SE}}$$

R_{ME} = distance moon to earth

R_{SE} = distance sun to earth

$$= 2.7 \times 10^7 \times \left(\frac{1}{400}\right)^2$$

$$= \frac{27 \times 10^6}{16 \times 10^4}$$

$$= 1.69 \times 10^2$$

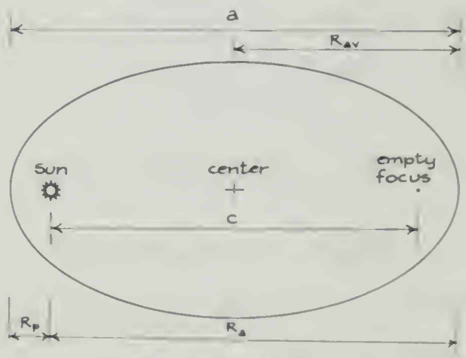
The gravitational force exerted on the earth by the sun is about 170 times as great as the force exerted by the moon.

24.

$k = \frac{R^3}{T^2}$, according to Kepler's harmonic law. k = 1, for motion about the sun in years and AU.
Then, $R^3 = kT^2$

$$R = \sqrt[3]{75^2} = \sqrt[3]{5{,}600} = 17.7\ \text{AU}$$

The average distance of Halley's comet from the sun is 17.7 AU.



a = major axis
Rav = average distance from Sun to comet
c = distance between foci
Rp = perihelion distance
Ra = aphelion distance

Closest to sun:

$a = 2R$, where a is the major axis of ellipse, so

$a = 35.4$ AU

$e = \frac{c}{a}$, where e is eccentricity and c is the distance between the two foci, so

$c = ea = 0.967(35.4 \text{ AU}) = 34.2$ AU

$a = c + 2R_p$, where R_p is the perihelion distance

$$R_p = \frac{a - c}{2} = \frac{35.4 - 34.2}{2} = \frac{1.2}{2} = 0.60 \text{ AU}$$

The closest approach of Halley's comet is 0.60 AU from the sun.

Farthest from the sun:

Let R_a = the aphelion distance

$R_a = a - R_p = 35.4 - 0.60 = 34.8$ AU

At the farthest point in its orbit, Halley's comet is 34.8 AU from the sun.

25. (a) $F_g = m_1 a_g = G \dfrac{m_1 m_E}{R^2}$

Since m_1 appears on both sides of the equation and R is constant for a given position on the earth, nothing changes. Thus, a_g for a particular place should be constant.

(b) Since the earth is not a perfect sphere, R may be different at different places on the earth. Then a_g would also be different for those places.

(c)

Since $F_g = (a_g m_1) = G \dfrac{m_1 m_2}{R^2}$, where $(a_g m_1)$ is the weight of the mass m_1 then

$$(a_g m_1) = m_1 \left(\frac{Gm_2}{R^2}\right)$$

Since G, m_2, and R are constant at a position on the earth's surface, the weight can be seen to vary as the mass.

(d) The gravitational force on a body is F_g, while the force producing centripetal acceleration for a circular orbit is F_c. These forces must be equal.

$$F_c = m_1 a_c = m_1 \frac{v^2}{R}$$

But $v = \dfrac{2\pi R}{T}$ so $F_c = m_1 \dfrac{4\pi^2 R}{T^2}$

$$F_g = G\left(\frac{m_1 m_2}{R^2}\right)$$

Then, since $F_c = F_g$, $\dfrac{4\pi^2 R}{T^2} = G\dfrac{m_2}{R^2}$ and

$$T^2 = \left(\frac{4\pi^2}{Gm_2}\right) R^3,$$

which has the form $T^2 = kR^3$, of Kepler's harmonic law.

$\dfrac{R^3}{T^2}$ is a constant; in this case $k = \dfrac{4\pi^2}{Gm_2}$

(e) At any given time there will be two tides: one toward the moon, the other away from the moon. Due to the differences in R in the gravitational formula, the water is pulled away from the solid earth on the near side; the earth is pulled away from the water on the far side. As the earth rotates under the moon any given location will experience a high tide about every 12½ hours.

26. Discussion.

27. Kepler replaced Plato's problem with a new way of explaining planetary motions. Plato's problem, stated in Chapter 5, was never solved, for no system of uniform circular motions could describe the motions of the planets with satisfactory accuracy.

28. Today we accept a heliocentric system because the observed motions of planets, comets, etc., are accurately described by the theory of universal gravitation applied to a sun-centered system.

Today's concept of a heliocentric system differs considerably from that of either Copernicus or Kepler.

The geocentric system is not disproved, but replaced by another system that is simpler, more elegant, and more precise.

29. Newton's basic assumptions and conclusions are used every day in scientific work.

30. Discussion.

The Triumph of Mechanics

Organization of Instruction

THE MULTI-MEDIA SCHEDULE

Day 1
Lab stations: Conservation of Mass
Students do only one of the following:

1. Alchemical. Lead filings are heated in an open test tube. The system gains weight
2. Boyle. Lead filings are heated in an open tube. After heating, the tube is sealed, cooled, and weighed.
3. Lavoisier. Lead filings are heated in a closed tube.
4. Antacid. Weigh a thick 2-L flask, stopper, water, and tablets before and after interaction. Use a balance sensitive to 0.1 g.
5. Precipitate. Put 20 g of lead nitrate dissolved in water in a 1-L flask and 11 g of potassium dichromate in water in a small test tube placed inside the flask in an upright position. Stopper and weigh. Invert to mix and reweigh.

Assignment: Prepare to give the class a report on your experiment.

Day 2
Students reports: Conservation of Mass
After a 10-min rehearsal, groups of students explain to the class the experiment they did on the previous day. Students might explain what has happened as the scientists who originally performed that type of experiment might have explained it.

Day 3
Teacher demonstration. Inelastic Collisions (*D33*)
Use lab carts with bricks to show one-dimensional inelastic collisions. Use double-sided tape on carts.

Day 4
Qualitative lab stations: Conservation of Momentum
Students experience one- and two-dimensional collisions using balloon pucks, Dylite beads, disc magnets, and dynamics carts. In each case, they are to look for conservation of momentum.

Day 5
E3–1: Collisions in One Dimension
A quantitative measurement is made of momentum exchange in a collision. An air track or colliding dynamics carts may be used. Data are recorded with strobe and camera. Each student will need a record of a collision for analysis.

Day 6
Student presentation of *E3–3* or *E3–4*: Collisions in Two Dimensions
Have two better students do this and analyze

conservation of momentum in two dimensions for the class.

Day 7
Work session: Two-Dimensional Collisions

First discuss *E3–3*. Then solve *T20*, Equal Mass Two-Dimensional Collisions, while the students work on the solution of Event 8 in the *Handbook*. Supplement this activity with *L22*, Two-Dimensional Collisions, II. The best students can proceed to another collision event. Give individual help.

Day 8
Film: Energy and Work (PSSC #0311, 28 min)

Small group discussions should follow the film. Provide groups with questions to discuss.

Day 9
Qualitative lab stations: Kinetic and Potential Energy

1. simple pendulum
2. Galileo's pendulum
3. ball on inclined plane
4. energy stored in compressed spring
5. *Film Loops: L32, L34,* or *L35*

Students are to look for changes in *PE* and *KE*.

Day 10
Discuss results of previous day's experiments (about 10 min).

Problem solving: Use photos taken on Day 5 to check for conservation of kinetic energy (about 40 min). See *Handbook*, page 160.

Day 11
Teacher demonstration: Conservation of Energy in Inelastic Collisions

Activity: Mechanical Equivalent of Heat (see *Handbook*, page 149), or

D33; An inelastic collision. Consult the *Demonstration Notes* in this *Resource Book*. This requires careful preparation.

Day 12
E3–10, E3–11, or *E3–12*

Permit students to choose which experiment they wish to complete.

Day 13
Quiz or other evaluation

Day 14
Teacher presentation: Carefully discuss the kinetic molecular theory of gases. Point to the need for statistical mathematics and describe the choice between the two games in *E3–13*, Monte Carlo Experiment on Molecular Collisions.

Day 15
E3–13: Monte Carlo Experiment on Molecular Collisions

Note that students complete either Game I or Game II.

Day 16
Class discussion: Work through the details of how to estimate the dimensions of a molecule

Have students discuss the *Study Guide* questions in small groups. Circulate among groups giving assistance and making certain that the students are working effectively.

Day 17
E3–14: Behavior of Gases

Day 18
Student activity day: Students pick activities from *Handbook* or other sources. Make arrangements in advance for needed materials.

Day 19
Teacher discussion: The Second Law of Thermodynamics (about 35 min)

Assignment: Encourage students to exercise the freedom of reading any articles. This takes some selling in order to have a successful discussion on Day 20.

Day 20
Student discussion

Students should sit in a circle and the teacher should be careful to say as little as possible. Ask leading questions, however, and encourage students to express opinions.

Days 21 and 22
Lab stations: Waves *E3–15, E3–16,* or *E3–17*

1. pulses on a rope or rubber tube
2. pulses on a slinky
3. pulses in a ripple tank
4. sound waves in air
5. ultra-sound
6. microwaves
7. continuous waves in a ripple tank

Students are asked to look for and make observations of velocity of propagation, wavelength, frequency, diffraction, absorption, reflection, superposition, energy transfer, standing waves, etc.

Day 23
Discuss laboratory observations from Days 21 and 22 (15 min).

Day 24
Teacher presentation: Present the details of superposition and two-source interference. Note that *T25* through *T29* are very useful for this purpose.

Day 25
E3–18: Sound and *E3–19:* Ultrasound

A student should do only one of the various parts of this experiment. A quantitative analysis should be completed rather than the type of qualitative survey with waves that was done on Days 21 and 22.

Day 26
Small-group problem solving

Have each group decide upon an activity for tomorrow.

Day 27
Student activities:

Some of the possible activities are:

1. *E3–5* and *E3–6* Conservation of Energy
2. *E3–7* Measuring the Speed of a Bullet
3. Standing waves on a drum (*Handbook,* page 152)
4. Moiré Patterns (*Handbook,* page 153)
5. Mechanical wave machines (*Handbook,* page 155)
6. *Film Loops 36* through *43*

Days 28–30
Evaluation

One method of evaluation is to review, test, and discuss the test. Devote a day to each activity.

Another method of evaluation is through individual student–teacher conferences during a period of three days. Evaluation can be based upon laboratory reports, essays, poems, equipment design, sets of *Study Guide* answers, etc.

Note that two of these three days of testing could be moved to other positions in the 30-day plan.

Unit 3 SAMPLE MULTI-MEDIA SYSTEMS APPROACH

Note: This is just one path of many that a teacher may take through Unit 3.

1 Lab stations: Conservation of mass	2 Student reports from day 1	3 Teacher demonstration: Inelastic collisions	4 Lab stations: Conservation of momentum
Text: Unit 3 Prologue and 9.1	Text: 9.2	Text: 9.3–9.4	Handbook: E3-1 and E3-2
5 Labs E3-1 and E3-2: Collisions in one dimension	6 Student demonstrations: E3-3 or E3-4	7 Work session: Two-dimensional collisions	8 P.S.S.C. Film: Energy and Work Small-group discussion
Write up E3-1 or E3-2	Text: 9.5–9.7	Handbook: Finish analysis of events 8 & 9	Text: 10.1–10.4
9 Lab stations: Kinetic and Potential Energy	10 Discuss day 9 lab Problem solving: Energy conservation	11 Teacher demonstration: Conservation of energy in inelastic collisions	12 Lab E3-10, E3-11, or E3-12
Selected Study Guide questions	Text: 10.5–10.8	Handbook: E3-10, E3-11 or E3-12	Write up Lab Text: 10.9–10.11
13 Quiz or other evaluation	14 Teacher presentation: Gas models (D35 and D36)	15 Lab E3-13: Monte Carlo experiment on molecular collision	16 Class discussion of E3-13 Small-group problem solving
Text: 11.1–11.4	Handbook: E3-13	Selected Study Guide questions	Handbook: E3-14
17 Lab E3-14: Behavior of gases	18 Student activity day	19 Teacher discussion: Second law of thermodynamics	20 Student-centered discussion
Handbook: Survey Ch. 1 for activities	Text: 11.5–11.8		Text: 12.1–12.4
21 Lab station: Waves and wave behavior	22	23	24 Teacher presentation: Interference
Handbook: Survey Ch. 12	Write up observations	Text: 12.5–12.8	Text: 12.9–12.11
25 Lab E3-18: Sound or E3-19: Ultrasound	26 Small-group problem solving	27 Small-group activities	28 Review or individual evaluation
Selected Study Guide questions	Handbook: Survey and select activity	Write up activity	Review
29 Test or Individual evaluation	30 Discuss test or Individual evaluation		

Unit 3 SUGGESTED SCHEDULE BLOCKS AND TIMETABLE

The sample schedule outlined on this page is one idea of how to present the chapter. Each block represents one class session (approximately 50 min) and the spaces between blocks are used to indicate homework. More specific suggestions can be found under Multi-media.

CHAPTER 9 CONSERVATION OF MASS AND MOMENTUM

Lab stations: Conservation of mass (See day 1)	Text: Prologue to 9.1	Student reports	Text: 9.2	Teacher demonstration: Inelastic collisions	Text: 9.3–9.4	Lab stations: Conservation of momentum (see day 4.)	Handbook: E3-1 & E3-2	Labs E3-1 and E3-2: Collisions in one dimension

CHAPTER 10 ENERGY

Write up E3-1 & E3-2	Student demonstrations	Text: 9.5–9.7	Work session: Collision events 8 and 9		Film: Energy and Work Discussion	Text: 10.1–10.4	Lab stations: Kinetic and potential energy (See day 9.)	Selected Study Guide questions	Problem-solving session

CHAPTER 11 THE KINETIC THEORY OF GASES

Text: 10.5–10.8	Teacher demonstration: Energy conservation	Handbook: E3-10, E3-11, or E3-12	Lab E3-10, E3-11, or E3-12: Temperature, calorimetry or ice calorimetry	Write up E3-10, E3-11, or E3-12 Text: 10.9–10.11	Quiz	Text: 11.1–11.4	Teacher presentation: Gas models	Handbook: E3-13	Lab E3-13: Monte Carlo experiment on molecular collisions
Selected Study Guide questions	Discussion of E3-13 Study Guide questions	Handbook: E3-14	Lab E3-14: Behavior of gases	Handbook survey Ch. 11	Student activity day	Text: 11.5–11.8	Discussion: 2nd law of thermodynamics		Student-centered discussion

CHAPTER 12 WAVES

Text: 12.1–12.4	Lab stations: Wave properties Waves in a ripple tank (see days 21-22.)	Handbook: Survey Ch. 12	Lab stations: Measuring wavelength	Write up observations	Programmed instruction: Waves I, II	Text: 12.5–12.8	Teacher presentation: Interference	Text: 12.9–12.11	Lab E3-18: Sound Lab E3-19: Ultrasound
Selected Study Guide questions	Small-group problem solving	Handbook: Survey and select activity	Small-group activities	Review	Review or Individual evaluation	Study for test	Test or Individual evaluation		Discuss test or Individual evaluation

CHAPTER 9 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
Prologue					R 1 Silence, Please		Prologue
9.1 Conservation of mass	2 3 4 6	5 7			F 17 Elements, compounds and mixtures F 18 The perfection of matter	Is mass conserved?	9.1
9.2 Collisions		8		E 3-1 and E3-2 Collisions in one dimension	L 18 One-dimensional Collision I L 19 One-dimensional Collision II L 20 Perfectly inelastic one-dimensional collisions		9.2
9.3 Conservation of momentum	11 12	9 10 14	16	E 3-3 and E3-4 Collisions in two dimensions	T 20 Equal mass two-dimensional collisions T 21 Unequal mass two-dimensional collisions L 23 Inelastic two-dimensional collisions	Interesting exchange of momentum devices Interesting case of elastic impact	9.3
9.4 Momentum and Newton's laws of motion	17 18 24	15 20 23 25	19 21 22		L 28 Recoil		9.4
9.5 Isolated systems	26 28 29	27 30			L 24 Scattering of a cluster of objects L 25 Explosion of a cluster of objects		9.5
9.6 Elastic Collisions	31 33 38 39	35 36	32 34		L 21 Two-dimensional collisions I L 22 Two-dimensional collisions II F 19 Elastic collisions and stored energy (PSSC)		9.6
9.7 Leibniz and the Conservation Law							9.7

CHAPTER 10 RESOURCE CHART

Text	Study Guide E	M	H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
10.1 Work and kinetic energy	2 4 5 6	7 8 9	3	E 3-7 Measuring the speed of a bullet D 33 An inelastic collision	T 22 Inelastic two-dimensional collisions L 31 A method of measuring energy: nails driven into wood L 33 Kinetic energy F 20 Energy and work (PSSC)		10.1
10.2 Potential energy	11 16	10 12 13	14	D 34 Range of a slingshot	T 23 Slow collisions L 32 Gravitational potential energy	Predicting the range of an arrow	10.2
10.3 Conservation of mechanical energy	15			E 3-5 and E 3-6 Conservation of energy	L 34 Conservation of energy. I: pole vault L 35 Conservation of energy. II: aircraft take-off L 26 Finding the speed of a rifle bullet. I L 27 Finding the speed of a rifle bullet. II		10.3
10.4 Forces that do no work		17 18					10.4
10.5 Heat as energy			19	E 3-10 Temperature and thermometers	R 2 The Steam Engine Comes of Age		10.5
10.6 The steam engine and Industrial Revolution	24 25 26 27	20 21 30 32	22 23 31 33		T 24 The Watt engine		10.6
10.7 The efficiency of engines		34 35		E 3-11 Calorimetry E 3-12 Ice Calorimetry		Steam-powered boat Mechanical equivalent of heat	10.7
10.8 Energy in biological systems		37 38 39			R 17 The Seven Images of Science R 18 Scientific Cranks	Student horsepower	10.8
10.9 Arriving at a general conservation law	40			E 3-8 Energy analysis of a pendulum swing	L 29 Colliding freight cars L 30 Dynamics of a billiard ball L 4 A matter of relative motion F 21 Conservation of energy (P.S.S.C.) R 3 The Great Conservation Principles		10.9
10.10 The laws of thermodynamics	41 42	44 45					10.10
10.11 Faith in the laws of thermodynamics	46					Problems of scientific and technological growth	10.11

CHAPTER 11 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
11.1 An overview of the chapter					F 22 Mechanical energy and thermal energy (PSSC)	Mechanical equivalent of heat	11.1
11.2 A model for the gaseous state	2 4			E 3-14 Behavior of gases D 35 Diffusion of gases	F 23 Demonstrating the gas laws (Coronet) F 24 Gas laws and their applications (EBF)		11.2
11.3 The speeds of molecules	3 6		5	D 36 Brownian motion E 3-13 Monte Carlo experiment on molecular collisions	R 8 The Law		11.3
11.4 The sizes of molecules	7	8	9		F 25 Molecular theory of matter (EBT)		11.4
11.5 (Optional) Predicting the behavior of gases from the kinetic theory	13 17 18 21 22	10 11 12	14 15 16 19		R 5 The Great Molecular Theory of Gases	A diver in a bottle Drinking duck How to weigh a car with a pressure gauge	11.5
11.6 The second law of thermodynamics and the dissipation of energy	26	28 30			R 4 The Barometer Story	Rockets	11.6
11.7 Maxwell's demon and the statistical view of the second law of thermodynamics	31				R 7 The Law of Disorder R 10 James Clerk Maxwell	Perpetual motion mechanics	11.7
11.8 Time's arrow and the recurrence paradox	33 36 38	32 34 37 39	35		L 36 Reversibility of time R 9 The Arrow of Time R 12 Randomness and the Twentieth Century		11.8

CHAPTER 12 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
12.1 Introduction					R 13 Waves		12.1
12.2 Properties of waves				E 3-15 Wave properties	R 14 What Is a Wave? R 15 Musical Instruments and Scales	Mechanical wave machines	12.2
12.3 Wave propagation		2		D 37 Wave propagation D 38 Energy transport	F 26 Progressive waves, transverse and longitudinal (McGraw-Hill)		12.3
12.4 Periodic waves		3		E 3-16 Waves in a ripple tank E 3-17 Measuring wavelength			12.4
12.5 When waves meet	4 5 6 8	7		D 39 Superposition D 46 Two turntable oscillators (beats)	T 25 Superposition T 26 Square-wave analysis L 37 Superposition of waves		12.5
12.6 A two-source interference pattern	10	9 12	11	D 43 Interference patterns	T 28 Two-slit interference	Moiré patterns	12.6
12.7 Standing waves		14 15 16 17	13 18	D 45 Standing waves	T 27 Standing waves T 29 Interference pattern analysis L 38 Standing waves on a string L 41 Vibrations of a rubber hose L 40 Vibrations of a wire F 27 Stationary longitudinal waves F 28 Stationary transverse waves	Standing waves on drum and violin	12.7
12.8 Wave fronts and diffraction		19 21 22 23	20	D 44 Diffraction			12.8
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Background and Development

OVERVIEW OF UNIT 3

In Units 1 and 2 we have developed the basic principles of Newtonian (or "classical") physics and shown their successful application to the astronomy of the solar system. Historically, this success led physicists in the eighteenth and nineteenth centuries to use these same principles in explaining many other natural phenomena.

In Unit 3 we will focus on the *generalization* of Newtonian mechanics by means of conservation laws for mass, momentum, and energy, and the *application* of Newtonian mechanics to collisions of objects, heat, gas theory, and waves.

In presenting the conservation laws, we stress the metaphysical and semitheological origin of these laws in the seventeenth century: the idea that the world is like a machine, which God has created with a fixed amount of matter and motion. In the discussion of the generalized law of conservation of energy, we point out connections with steam-engine technology and other historical and philosophical factors that lay in the background of the simultaneous discovery of this law by several scientists in the middle of the nineteenth century. Our purpose here is to make students aware of the relationships between physics and other human activities.

In selecting certain applications for detailed presentation while ignoring many others that are parts of the traditional physics course, we have been guided by two main criteria: (1) to give the bare minimum of technical detail that is needed to illustrate the main principles and (2) to develop material that will be needed in later units. The climax of the unit is really in Chapter 11, where the concepts of momentum, energy, and heat are combined in the kinetic theory of gases to yield the first definite information about molecular speeds and sizes (as well as an explanation of the macroscopic properties of gases). This is about as far as Newtonian mechanics can take us into the atomic world: we will have to wait for the quantum concepts introduced in Unit 5 before we can go further. Chapter 12 begins the study of wave phenomena, still in a mechanical framework but breaking the ground for the study of light and electromagnetic waves in Unit 4.

CHAPTER 9 / CONSERVATION OF MASS AND MOMENTUM

SUMMARY OF CHAPTER 9

The law of conservation of mass expresses accurately the ancient conviction that the amount of matter in the universe does not change. A sound experimental basis for this belief was provided in the eighteenth century by the experiments of Lavoisier. It has since been demonstrated with great precision that the mass of any closed system always remains the same.

Seventeenth-century philosophers also believed that the "quantity of motion" in the universe always remained the same; the problem was to formulate a definition of "quantity of motion" such that it would be conserved. Descartes proposed the product of mass and speed as the correct measure of quantity of motion. By considering collisions between two carts, however, it is seen that Descartes was wrong. It is the product of mass and *velocity* that is conserved. This vector quantity is called "momentum."

The law of conservation of momentum is perfectly general. The resultant momentum of any number of bodies exerting any kind of forces on one another is conserved, so long as the net force on the entire system of bodies is zero.

The law of conservation of momentum is a logical consequence of Newton's second and third laws of motion. Yet it allows us to solve problems that could not be solved by direct application of Newton's laws. In particular, use of the law of conservation of momentum enables one to find out about the motion of interacting objects even when the details of the forces of interaction are unknown.

A system on which the net force is zero (and the momentum of which therefore is conserved) is called an isolated system. It is nearly impossible to arrange a system that is truly isolated. Many systems can be regarded as very nearly isolated, however, and the law of conservation of momentum can be applied to them with little error.

A description of a demonstration witnessed by the Royal Society in 1666 in which two hard pendulum bobs repeatedly collided with one another leads to the introduction of another quantity that Huygens showed to be conserved in collisions between very hard objects (perfectly elastic collisions). It was called *vis viva*. The German philosopher Leibniz was convinced that *vis viva* was always conserved, even in collisions between soft objects (inelastic collisions).

Just as Descartes' "quantity of motion" (mv) developed into the modern concept of momentum ($m\vec{v}$), so *vis viva* is closely related to the modern concept of kinetic energy (mv^2).

9.1 | CONSERVATION OF MASS

The main point of this section is the faith that natural philosophers had in the conservation of mass, even in the absence of good experimental verification. The work of Lavoisier, which finally gave acceptable experimental support to the conservation of mass, was not a source of surprise or even of much information; it was rather somewhat of a relief.

There are any number of visually appealing demonstrations that could be done in a closed flask to show conservation of mass. The one using antacid tablets is described in some detail in the *Handbook* and can be used to make the more subtle point of accuracy limitations to conservation laws.

The discussion on *Text* page 250 may seem to be inconsistent with the caption under the figure illustrating changes in mass during burning. Burning leads to an increase in mass if all products are included. However, if burning takes place in an open pan, as shown in the photograph, initial heating can drive off gases, and smoke can escape, which leads to a net loss in mass. If time permits, this would be a good time to discuss some chemical reactions that seem to increase mass by addition of reactants and some that seem to decrease mass by subtraction of products when an open container is used.

Although substances seem to disappear when dissolved, this does not constitute a violation of the law of conservation of matter. Solution may occur through dispersion of molecules of the dissolved substance in the solvent or through a chemical reaction.

During World War II, Niels Bohr, the physicist who developed the atomic theory discussed in Chapter 19, had to flee his laboratory in Denmark. Two of his colleagues gave him their gold Nobel Prize medals, which he dissolved in a jar of acid and left in his office. Because of his experience with the conservation of matter, he was certain that the reaction between the acid and the gold would convert the gold into a soluble chemical compound. He felt that the German invaders would never suspect the jar of acid to be valuable. When he returned after the war he used chemical methods to recover the gold from the acid and had the medals recast.

The distinction between mass and weight was made in Unit 1, but it might be well to treat it briefly again. Changes in weight with changes in distance from the earth were treated extensively in Unit 2. Because mass conservation was, until recently, studied only with analytical balances, the operation of balances and what they measure might be discussed.

The ratio of the weights of two bodies is equal to the ratio of their masses if they are weighed at the same place or at places where a_g has the same value. (Recall Galileo's work, discussed previously.) The proportionality has been experimentally established to better than 1 part in 10^{10}. In general relativity, the proportionality is assumed on the basis that acceleration and gravitational fields are mathematically indistinguishable.

After suggestions that very small changes in mass might accompany radiation, precision chemical experiments, especially those by Hans Landolt, were made about the beginning of the twentieth century. These experiments established conservation of mass to 1 part in 10^6.

In some cases, 10^{-6} g can be quite a potent quantity. (For example, that amount of LSD has terrifying effects on humans.) A cube with an edge 0.10 mm long has a volume of 10^{-6} cm^3. With a density near that of water, such a cube has a mass of about 10^{-6} g. On any desk top, one can probably see several specks of dust with very similar masses.

It has now been observed that in some nuclear reactions the rest mass of all the particles after the reaction is detectably different from what it was before the reaction. However, if the mass equivalent of the energy involved in the reaction is included as calculated by $m = E/c^2$, the total mass is still conserved.

If 18 g of methane (CH_4) are burned with 64 g of oxygen (one gram molecular weight of each), 211 kilocalories of heat are released. This amounts to a change in rest mass of about 10^{-8} g or about one part in 10^{10}, as the following equation shows.

$$\Delta m = \frac{\Delta E}{c^2}$$

$$= \frac{(211\ \text{kcal})(4.19 \times 10^{10}\ \text{ergs/kcal})}{(3.00 \times 10^{10}\ \text{cm/sec})^2}$$

$$= 9.8 \times 10^{-9}\ \text{g}.$$

This is a relatively energetic reaction, so that this fractional mass change is very much larger than for the reactions Landholt studied.

Try to avoid any extended discussion of mass–energy relationships. The important point to emphasize is that, with increasing knowledge, conservation laws may need some adjustment in terms of just what it is that is being conserved. In Chapter 17, return to a discussion of $E = mc^2$.

If questions are raised about reactions involving the annihilation of electrons and positrons, where the rest mass of the particles disappears completely and is turned into the energy of two gamma rays, it is more accurate to say that mass "disappears" only in the sense that it cannot be measured on a balance. What "disappears," of course, is the electron and the positron as identifiable particles having rest mass.

Suggestions for Quiz or Class Discussion

1. A conservation law can be confirmed experimentally only to within a certain margin of error. Yet conservation of energy and momentum are believed to be exact laws, and were believed to be exact even before extremely precise experiments were done. There are many quantities that are nearly constant (such as the number of

pianos in Boston during a one-week period), yet we do not believe that there are exact conservation laws for these quantities. Can you think of anything that influences our belief in conservation laws besides quantitative experimental evidence?

2. Give a brief explanation of why physicists are interested in conservation laws.
3. A person observes a burning tree and makes the following two statements:

   (1) The wood disappears, heat and light are given off, and ashes remain.

   (2) Molecules of wood combine with molecules of oxygen in the air to form molecules of the gas carbon dioxide; the minerals in the wood are left in the form of solid oxides, and heat and light are given off.

   (a) Which of the statements is the description of what happens?

   (b) Which statement depends upon a theory?

   (c) Which, if either, would make sense to a person who did not believe in atoms?

   (d) In the light of further evidence, which statement is more likely to remain unchanged? Will both remain unchanged?

   (e) Which statement is "true"?

9.2 | COLLISIONS

This entire section is designed to show how people were using incomplete and vague concepts in their search for order and fundamental principles. It may well represent the sort of thing that always goes on when one is trying to find order amid the seemingly chaotic; a sort of private, prephysics stage in which guess work, hunch, and intuition are stirred in about equal parts with experimental observation, good existing theory, and hope. Looking back from our present vantage point, it seems extraordinary that anything came from it all. One hesitates to draw current parallels in other areas of study for fear of being misunderstood, but to some degree the same sort of searching goes on today in physics as we try to understand the role of fundamental particles. Of course, now there is a good bit less reference to what is on God's mind than there was in Descartes' time.

Note that while the text treats inelastic collisions, the experiments and demonstrations deal primarily with elastic collisions. This should present no problem if no distinction is made at this time. Section 9.6 deals with this matter.

Beginning with an equal-mass elastic collision (one body initially at rest), students can be led to suggest that speed is conserved. Then, to establish the generality of a "conservation of speed" principle, the principle must be confirmed in other collisions. An unequal-mass elastic collision will show that "conservation of speed" does not hold. But, if the masses are in simple ratio, like 2:1, for instance, it will be fairly obvious that the product of mass and speed is conserved. Again, to claim the generality of the "conservation of mass × speed" principle, other types of collisions must be tried. Note that the product of mass and speed squared is also conserved, but not m^2v, mv^3, $m\,v$, $m\sqrt{v}$, etc. In the two types of collisions considered thus far, mv and mv^2 have both been conserved. Students can be advised that the simpler relation will be taken up first, and that mv^2 will be taken up later. The linear "explosion" or head-on inelastic collision will show that even mass × speed is not conserved. Then the conservation principle can be saved only if the direction of the speed is considered. A "conservation of mass × velocity" principle holds in these two cases as well as in the others. The use of the vector velocity should suggest that nonlinear collisions might be tried. The collisions of pucks on plastic beads or of air-pucks can be photographed stroboscopically with very satisfactory results. The film loops on two-dimensional collisions give excellent results. An inexpensive air-table provides excellent two-dimensional analyses.

9.3 | CONSERVATION OF MOMENTUM

A wealth of material, including experiments and demonstrations, film loops, stroboscopic photographs, and overhead transparencies, is available for teaching conservation of momentum. The topic is, however, not so important to warrant the use of all parts of all these media. What is important is that students get some intuitive feel for the kind of outcome to be expected in a collision and that they get this feel before the corresponding bald claim about conservation is made in the text. The collection, or examination, of data in the laboratory could be coordinated to allow the progressive "discovery" of conservation of momentum independent of (and preferably before) the reading of the text treatment.

This is an appropriate time to introduce or review the summation symbol Σ if you have not done so. Here Σ means *vector addition*. The use of different objects and subscript numbers 1 and 2 to mean before and after collision is not very generalizable. If we let subscript numbers identify the different objects and let unprimed and primed v's mean before and after, respectively, then we can write in general

$$\sum_i m_i v_i = \sum m_i v'_i$$

or

$$\Delta \sum_i m_i v_i = 0$$

Notice that the summation is *vectorial*. Actually, it might be more satisfactory to write *algebraic* summation for each of the three components of momentum:

$$\Delta \sum_i m_i v_{x_i} = 0$$

$$\Delta \sum_i m_i v_{y_i} = 0$$

$$\Delta \sum_i m_i v_{z_i} = 0$$

These principles are true generally, both independently and all together.

9.4 | MOMENTUM AND NEWTON'S LAWS OF MOTION

Attention should be given to the question: "How does momentum enter or leave a system?" The only way we can change the linear momentum of a system is to exert a force on part or on all of the system from outside. The force that one part of a system exerts on another part cannot change the system's total momentum. Such a force might redistribute the momentum among the parts, but it cannot change the total.

The operation of a rocket engine may be discussed in terms of conservation of linear momentum. The analogy of the recoil of a gun suggests that the forward momentum of the bullet as it leaves the muzzle of the gun is equal to the recoil momentum of the gun in the opposite direction. If we then picture the rocket as a continuously firing gun, we can derive the "rocket equation."

9.5 | ISOLATED SYSTEMS

An isolated system need not be completely isolated, only isolated with respect to the quantities we are interested in. "Entering" or "leaving" a system implies passing through a spatial boundary surrounding the system. It should be made clear that something created or destroyed within the boundary of a closed system is not entering or leaving the system.

A conservation *law* must hold true for any closed system. We do not speak of "the conservation-of-energy-in-an-insulated-calorimeter law." It is not enough to conserve something in a particular isolated system; that something must be conserved in every isolated system; otherwise there is no law.

It is very difficult indeed to think of good conservation laws dealing with concepts outside of physics. That is one reason it is so remarkable that they occur in physics at all. Since all around us we see change, it is surprising and wonderful to find that amidst all the turmoil some things sometimes do not change.

Physicists cling to the conservation laws, sometimes in the face of seemingly contrary evidence. At first the conservation laws of linear momentum and energy seemed to be violated in the β decay of nuclei. Physicists then invented the neutrino in order to preserve the conservation laws.

The students have already studied a very good conservation law: Kepler's second law (Unit 2, Sec. 7.2). The *Text* states: "The line from the sun to the moving planet sweeps over areas that are proportional to the time intervals." Of course, for this to be true, it must be that the line sweeps over an area at the same rate no matter where the planet is in its motion. That is, the rate at which the line sweeps out an area is conserved throughout all the changes in the planet's velocity and distance from the sun.

Since we do not discuss angular momentum in the *Text*, the student will not recognize Kepler's second law as just a special case of the conservation of angular momentum. It does have the advantage of being a physical example in which the quantity being conserved is not a substance. Also, in this illustration the quantity that is conserved has different values for different sun–planet systems.

9.6 | ELASTIC COLLISIONS

The main point of this section is the necessity of introducing another conservation law in order to successfully account for the motion of the bobs. Since we have an interaction involving two unknown velocities, we must have two equations to arrive at their final values.

9.7 | LEIBNIZ AND THE CONSERVATION LAW

This section illustrates the evolution of science through controversy and the transition from mv^2 to heat, the subject of the next chapter.

Here students could be encouraged to argue out the Leibniz–Descartes controversy regarding what it is about motion that is conserved. This is worthwhile even though the two men were not contemporaries. Furthermore, the desire of Leibniz to assure the conservation of mv^2 is an example of great intuition and faith in an idea. Such faith and intuition is a part of science.

Demonstration D33 is appropriate for showing the students an example of the transfer of mechanical energy to heat or internal energy. This activity is timely between Chapters 9 and 10 or between Secs. 10.4 and 10.5. Refer to the *Demonstration Notes.*

CHAPTER 10 / ENERGY

SUMMARY OF CHAPTER 10

The concept of work, defined as the product of the force on an object and the distance the object moves while the force is exerted on it, is interpreted as a measure of energy changed from one form to another. With this interpretation, expressions can be derived for the kinetic energy $\frac{1}{2}mv^2$ of an object and for the change in gravitational potential energy $F_{grav}d$ of an object of weight F_{grav} that moves through a vertical distance d. Other forms of potential energy are mentioned. If friction is negligible, the sum of the kinetic energy and the potential energy does not change; this is the law of conservation of mechanical energy.

Work is more accurately defined as the product of the *component* of the force on the object *in the* *direction of motion* of the object and the distance the object moves while the force is being applied. Thus, if an object moves in a direction perpendicular to the direction of the force on it, the force does no work.

The present-day view of heat as a form of energy was established in the nineteenth century, partly because of knowledge of heat and work gained in the development of the steam engine. The first practical steam engine was invented by Savery to pump water from flooded mines. A considerably better engine was that of Newcomen, which was widely used in Britain and other European countries in the eighteenth century.

The invention of the separate condenser by Watt in 1765 resulted in a vastly superior steam engine, one that could do more work than the Newcomen engine with the same amount of fuel. Watt charged a fee for use of his engines that depended on the rate at which they could do work; that is, on their power. The Watt engine was quickly adapted to a variety of tasks and was instrumental in pushing forward the Industrial Revolution.

One of the scientists who helped establish the idea that heat is a form of energy was Joule, who performed a variety of experiments to show that a given amount of mechanical energy (measured, for example, in joules) is always transformed into an equivalent amount of heat (measured, for example, in kilocalories).

Living systems require a supply of energy to maintain their vital processes and to do work on external objects. Plants obtain energy from sunlight and, by the process of photosynthesis, convert it into chemical energy stored in the molecules of the plant. An animal that eats plants, or that eats other animals that have eaten plants, releases the stored chemical energy in the process of oxidation, and uses the energy to run the "machinery" of its body and to do work on its surroundings. Different activities use food energy at different rates. A healthy college-age person needs at least 1,700 kilocalories of food energy a day merely to keep the body functioning, without doing any work on that person's surroundings. Yet there are countries where the average individual intake of food amounts to less than 1,700 kilocalories a day.

In the early nineteenth century, developments in science, engineering, and philosophy contributed to the growing conviction that all forms of energy (including heat) could be transformed into one another and that the total amount of energy in the universe was conserved.

The newly developing science of electricity and magnetism revealed many relations between mechanical, chemical, electrical, magnetic, and heat phenomena, suggesting that the basic "forces" of nature were related.

Since steam engines were compared on the basis of how much work they could do for a given supply of fuel, the concept of work began to assume considerable importance. It began to be used in general as a measure of the amount of energy transformed from one form to another and thus made possible quantitative statements about energy transformations.

The German "nature philosophers," concerned with discovering through intuition the inner meaning of nature, stimulated the belief that the various phenomena of nature were different manifestations of one basic entity that came to be called energy.

Of the large number of scientists and philosophers who proposed a law of conservation of energy in some form, it was von Helmholtz who most clearly asserted that any machine or engine that does work cannot provide more energy than it obtains from some energy source. If the energy input to a system (in the form of work or heat) is different from the energy output, the difference is accounted for by a change in the internal energy of the system.

The law of conservation of energy (or the first law of thermodynamics) has become one of the very cornerstones of physics. It is practically a certainty that no exception to the law will ever be discovered.

10.1 | WORK AND KINETIC ENERGY

It may seem strange that we have brought our discussion of mechanics so far with hardly a mention of energy, which now holds such an important place among the concepts of physics. We might mention that the logical development of mechanics is quite possible without it. All the problems of classical mechanics can be solved without reference to energy. The "idea" of energy is historically much older than the name. It goes back to Galileo's work with machines, in which the concept of "work" was involved.

While there was confusion between the words *force* and *work*, due to Leibniz's interpretation, Leonardo da Vinci had remarked 200 years before that work implied that an object moved in the direction of the force. In the literature of physics today, work means just this: force multiplied by the distance traversed in the direction of the force. The approach taken in the *Text* is to present the concept of work in an easily digestible form first, and to qualify it later as necessary. Even using the *Text* approach, you might want to hedge the initial "definition" by pointing out to students that both force and displacement are vectors and that it isn't immediately obvious what to do when the force and displacement aren't along the same line. As long as they are in line, it is correct to take work as *Fd*.

Many physicists use the concept of *negative work* because it allows one to treat these problems from a more general point of view with one equation instead of two. We have avoided doing this in the *Text* to avoid excessive abstraction and stay closer to familiar concepts. Thus, we say that B does work on A, rather than that A does negative work on B.

As we saw in Sec. 9.6, Huygens made prominent the concept of *vis viva or living force*, a quantity varying as the mass multiplied by the square of the velocity. The term *energy* was not attributed to the *vis viva* concept until the nineteenth century. The product Fd is called the "work" done by the force during the displacement, while the quantity $\frac{1}{2}mv^2$ is one-half that which used to be known as *vis viva*. We now call $\frac{1}{2}mv^2$ the *kinetic energy*.

If a body moves a distance d directly against a resisting net force F, it loses kinetic energy. The amount of kinetic energy lost can be shown as follows, starting from $F = ma$:

$$\begin{aligned} F &= ma \\ Fd &= ma \cdot d \\ &= m\frac{(v_f - v_i)}{t} \cdot \frac{(v_i + v_f)}{2}t \\ &= \tfrac{1}{2}m(v_f^2 - v_i^2) \\ &= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \end{aligned}$$

10.2 | POTENTIAL ENERGY

Experiment *E3-5* should be done before (or closely in conjunction with) the beginning of this section. The "dip" in the kinetic energy graph for the slow collision is a nice way to introduce ("discover") potential energy. Some of the kinetic energy disappears and then reappears. Where was it? The "explosion" from the tied steel loop makes a good follow-up demonstration; there is no kinetic energy in the system before and much afterwards, but there was something related to energy in the tied spring. If *E3-5* can't be done until after the gravitational force of *PE* has been considered, the dip in *KE* can be predicted instead of discovered.

If we accept the idea that a falling body is continually gaining kinetic energy due to the previous work we did in raising it, we must then accept the idea that the "raised" body, before it started to fall, possessed the energy that is appearing in kinetic form. Energy due to motion is fairly obvious, which is why kinetic energy (*vis viva*) came upon the scene relatively early. Energy due to position is less obvious. In fact, it eluded both Leibniz and Descartes. The first mention of "position" energy was in a book by Carnot in 1803 in which he stated:

Vis viva can figure as the product of a mass and the square of its velocity, or as the product of a moving power and a length or a height. In the first case it is a vis viva properly called; in the second it is a latent vis viva.

It was not until 1853 that the name *potential* energy was first used by Rankine, and it has been used ever since.

See the article "Energy Reference Levels" in the section of this *Resource Book* entitled "Additional Background Articles." Note that its content is not used in the *Text* development.

10.3 | CONSERVATION OF MECHANICAL ENERGY

The nature of energy leads to two fundamental concepts: kinetic and potential energy. Kinetic energy is possessed by reason of motion while potential energy is possessed by reason of position or condition, as in a raised weight or stretched spring. It should be clear that either kind of energy may come into being in consequence of the performance of work. In the first case, the work is devoted to producing speed; in the second case, work involves some reversible process that subsequently can be made to yield the energy thus stored.

In using the momentum and energy conservation principles for collisions it should be noted that prediction is possible only for rectilinear collisions of two balls. As many students may remember, a set of equations, all of which are true, can be solved for as many unknowns as there are independent equations. Energy and momentum conservation principles constitute a pair of such independent "simultaneous" equations and so can be solved for only two unknowns: the two velocities after collision of two balls. If the collision is in two dimensions, there are two unknowns for each ball after collision (x and y velocity components or speed and angle) giving four unknowns in all. Thus, two other values would be needed in order to be able to solve for the remaining two velocities. The situation of a single ball striking a row of balls in contact has, after the collision, as many parameters as there are balls: one velocity value for each ball. Energy and momentum conservation are not sufficient to find a unique set of "after" values and, as a matter of fact, all the balls will have finite velocities after the collision, even if the balls are perfectly elastic. If the balls are not in contact, but separated enough so that the collision of the first and the second is over by the time the second touches the third, then the traditional all-zero-except-the-last-ball result is predictable, but the event is no more than a series of two-ball collisions.

Historically, the energy conservation generalization was employed by Leonardo da Vinci near the end of the fifteenth century; Simon Stevin used it in 1605 as the basis for his development of the laws of statics, while Galileo employed a similar argument in his analysis of a frictionless inclined plane.

10.4 | FORCES THAT DO NO WORK

The idea of zero work being done when heavy bodies are held or transported horizontally is troublesome to students and should probably be qualified as follows. When someone stands and holds a heavy body, no work is being done on the body, but this is not to say that there is not a great deal of chemical transfer of energy (work, in the technical sense, on the cellular or molecular scale) required to maintain the muscle contractions that result in the body being held motionless. Work can be going on inside you (transferring chemical energy ultimately into heat) without any work being done on anything outside the closed system "you." In this sense, it is quite reasonable to say that you can get tired and hot without doing any outside work.

It does take a force to move something horizontally, even without friction. If it is an "immeasurably small force," then it will take an immeasurably long time for the body to make the change in position. A more reasonable treatment would be to admit that a finite force is required, and that therefore work must be done on the body to get it moving. Point out that when the body reaches the new position it will still be moving. The work that went into getting it moving went into kinetic energy, which it still has after reaching the new position. No energy was "used up" in changing position. (If a small spring gun were used to start the body, the spring gun could be moved around and used to stop the body as it arrived at the new position, and the gun would be cocked in the process. Everything then would be the same as before except that the body had changed position.)

"No-work" forces, then, are those that are perpendicular to the direction of a body's motion and so do not contribute to kinetic energy changes. A motion that is restricted to a prescribed path, like a track or wire, or to a prescribed plane, is called a *constrained* motion. In such motion, no-work forces are present but knowledge of them is not necessary for the calculation of changes in kinetic energy.

10.5 | HEAT AS ENERGY

One must be aware that the development of principles of heat and mechanics progressed along quite different paths: the study of motion and the *mechanical interaction* of bodies on the one hand and the study of temperature and the *thermal interaction* of bodies on the other. The work of Joule joined these two almost entirely independent disciplines. This joining of heat and energy, along with early developments in chemistry, brought about the kinetic theory. This will be developed further in Sec. 10.9. We do not describe the long controversy over the nature of heat that extended from the time of the early Greek philosophers until the middle of the nineteenth century. In the long run, however, the multitude of physical processes in which heat seems to be generated by the expenditure of mechanical work must be reckoned with.

The detailed operation of steam engines is not an important part of the story line of Unit 3. Accordingly, it is necessary only that students learn enough about engine operation to make sense of the extended treatment of steam engine improvement and its technological and social effects. If students understand well the principle of Savery's first engine, no further detail is important. Subsequent improvements can be adequately understood in terms of the operation of this fairly simple device. The Newcomen engine, for example, was different from the Savery engine only in that a piston was moved instead of water being moved directly. There was also a mechanical linkage that sprayed the water, inside, at just the right time. Watt's principal improvement was to condense the steam in a separate container, so that the main cylinder did not need to be reheated by the steam for each cycle. The development of steam engines was primarily a commercial rather than a scientific enterprise. The early stages of development, it should be noted, were the work of amateurs; there were as yet no professionals of engine design. Sec. 10.6 discusses the steam engine's impact on the Industrial Revolution.

10.6 | THE STEAM ENGINE AND THE INDUSTRIAL REVOLUTION

The Industrial Revolution was a vast and complex set of phenomena; the bare account given here can be treated as a reading assignment. One point that should be made is that distaste for mechanistic science could now be based on objections to its practical consequences as well as on its philosophical implications.

"The Steam Engine," from *Technics and Civilization*, by Lewis Mumford, Harcourt, Brace Jovanovich, New York, 1934, provides background material that may be useful for a class discussion on the social and cultural effects of the development of the steam engine. The following definitions may be useful to the reader:

eotechnic civilization: the pre-machine age
paleotechnic civilization: the period beginning about 1750 that followed the Industrial Revolution.

In the Watt engine, the use of a separate condenser provided a great gain in efficiency. However, it is even more efficient to use high-pressure steam. Early attempts to do this were frustrated because the materials available for constructing the boiler could not withstand high pressures and temperatures. But advances in metallurgy later in the nineteenth century made it possible to construct practical high-pressure engines. Eventually, the separate condenser became unnecessary, but by this time it had already served its purpose in getting steam engines established as the major source of energy in industry.

Watt's working model of Newcomen's engine had a problem of scale, which is a tangential topic of considerable interest in itself. If you have time, PSSC Chapter 4, "Functions and Scaling," and the PSSC film "Change of Scale," could be used.

An additional historical sidelight: The first U.S. Government research grant was made in the nineteenth century to the Franklin Institute of Philadelphia to investigate the causes of explosions in steam engine boilers.

The section "The Steam Engine after Watt" (pp. 42–55), in I. T. Sandford's *Heat Engines* (Doubleday, 1962), is a nice account of some of the first steps toward modern engines. Internal combustion, steam turbines and gas turbines, turbojets, and rockets are taken up in Chapter 7 of Sandford's book. The discussion includes some thermodynamics, which is not difficult to skip over.

10.7 | THE EFFICIENCY OF ENGINES

A single experiment cannot, of course, establish that there is a constant conversion factor between, say, work and heat. Only when the same proportionality constant is found in a number of different experiments with different substances can it be called the conversion factor. The student activity "Mechanical Equivalent of Heat" (*Handbook*, page 149) is, accordingly, a way of finding a proportionality constant for internal energy and gross mechanical energy of lead shot. That this is the conversion factor for work and heat is an unsupported generalization.

If lab time is available, you may wish to pursue the calorie in *Experiment E3-11*, "Calorimetry."

The distinction between calorie and kilocalorie (Calorie) should be emphasized to avoid confusion later.

10.8 | ENERGY IN BIOLOGICAL SYSTEMS

The main point of this section is that a meaningful connection can be made between principles developed in the abstract world of physics (where a "body" is a "body" whether it is a molecule of hydrogen, Caesar's corpse, or a red giant star) and the familiar world of widely differentiated living organisms. The physical principles are no less true for bacteria and high school students than for asteroids and roller carts. The details of the section are not important to the story line of Unit 3.

It should be pointed out that the "energy available in food" must be measured; thus, a known mass of the food substance is oxidized in a closed container of a calorimeter.

Throughout the section there is mention of inefficiency and energy loss. The dissipation of energy is discussed in Sec. 11.6. If Secs. 10.7 and 11.6 are brought together as a reading assignment, students can be asked in the subsequent class discussion to explain what is meant by the "loss" of energy in the food chain. The point, of course, is that only a small amount of food energy is available for growth and movement. Most of it is either dissipated internally as heat or excreted in the form of undigested foods.

See Additional Background Articles for notes on "Food Energy."

10.9 | ARRIVING AT A GENERAL CONSERVATION LAW

See Additional Background Articles for a note on "Classifications of Energy."

The philosophical material in this section should be treated as was done elsewhere, the details of the philosophy or their correctness is not important. The extent to which the philosophical concern influenced the development of physics is important.

The majority of those people who first comprehended the full import of the generality of energy conservation were young, and many were outside the field of physics at the time they made their contributions: Mayer, a German physician aged 28; Carnot, a French engineer who preceded all the rest in the discovery (and who will be discussed in Sec. 11.6), aged 34; Helmholtz, a German physiologist, aged 32; Joule, an English physicist, aged 25; and Colding (not mentioned in the *Text*), a Danish engineer who made the same discovery independently of the others and almost simultaneously, aged 27.

10.10 | THE LAWS OF THERMODYNAMICS

This section implies that total energy consists of something more than mechanical energy, heat, and potential energy. Examples of other forms of energy are the energy of excitation of an atom (discussed in Unit 5) and nuclear energy (treated in Unit 6). Students may ask about these other forms. However, the point to stress here is that the statement of the first law of thermodynamics, $\Delta E = \Delta W + \Delta H$, includes all special cases discussed in the past and will cover energy situations discussed in the future.

ΔH can represent either the addition of heat or the absorption of electromagnetic radiation to excite an atom. The fact that the source of ΔH does not matter renders the law of conservation of energy general and therefore very powerful.

A discussion of this idea will illustrate the coherence of physics. However, do not begin a full discussion of atomic and nuclear physics at this time.

10.11 | FAITH IN THE LAWS OF THERMODYNAMICS

Students may think it curious that faith plays such a crucial role in this great law. Whenever acceptance of conservation of energy is endangered, a new motion is postulated to preserve the law. When mechanical energy disappeared (*D33*), internal energy was invented; when energy disappeared in a nuclear reaction, the neutrino was postulated to protect this law. Thus, faith in energy conservation becomes fruitful, for it predicts new observations.

In the discussion of kinetic theory (Chapter 11), students will observe evidence of the postulated internal energy of gases. Do not miss the opportunity to stress this triumph of mechanics. Newtonian physics not only explains the properties of gases but also predicts new properties.

SUMMARY OF CHAPTER 11

The development of the kinetic theory of gases in the nineteenth century led to the last major triumph of Newtonian mechanics. A simple theoretical model of a gas was adopted: a large number of very small particles in rapid disordered motion. Since the size of the particles is assumed to be very small compared to the space occupied by the gas, each particle moves most of the time at constant velocity, occasionally colliding with other particles or with the sides of the container. By applying Newtonian mechanics to this model, scientists could deduce equations that related observable properties of gases (such as pressure, density, and temperature) to the sizes and speeds of molecules. With these equations, kinetic theorists could: (1) explain the known relations between observable properties of gases, such as Boyle's law; (2) predict new relations, such as the fact that the viscosity of a gas increases with temperature but is independent of density; and (3) infer the sizes and speeds of the molecules.

The chapter first discusses the model and its consequences qualitatively; then details and derivations are collected in an optional section (11.5), which can be omitted by most students. The rest of the chapter is devoted to exploring the relation between the second law of thermodynamics, or principle of dissipation of energy, and the kinetic theory. It would appear that, if Newtonian mechanics were strictly applicable on the atomic level, then all molecular processes would be time reversible, and any initial arrangement would eventually recur. Boltzmann's attempts to reconcile these reversibility and recurrence paradoxes with experience involved statistical reasoning and introduced interesting speculations about the direction of time.

11.1 AN OVERVIEW OF THE CHAPTER

This section should be treated primarily as a reading assignment. Discussion could include the following. The kinetic-hypothesis-of-heat idea is of much longer standing than is ordinarily realized. Disregarding Greek speculations, one can find that in the scientific era (1706) John Locke said:

> Heat is a very brisk agitation of the insensible parts of the object, which produces in us the sensation from whence we denominate the object hot: so what in our sensation is heat, in the object is nothing but motion.

In 1738, Daniel Bernoulli had remarked that:

> it is admitted that heat may be considered as an increasing internal motion of particles.

In 1780, Lavoisier and Laplace were even more explicit. They said in their "Memoire sur La Chaleur":

> . . . heat is the vis viva resulting from the insensible movements of the molecules of a body. It is the sum of the products of the mass of each molecule by the square of its velocity.

In 1798, Rumford, drawing conclusions from an experiment, stated that it appeared to him to be:

> extremely difficult if not quite impossible to form any distinct idea of anything, capable of being excited and communicated, in the manner the Heat was excited and communicated in these Experiments, except it be MOTION.

All these pronouncements, except Rumford's went considerably beyond any really justifiable evidence from experimental data.

Again, most of the central ideas here will be developed later. Treat this section as introductory material, but always be prepared to add to a discussion. The "equal-sharing" principle (equipartition) and the problem of "irreversibility" are mentioned only to show contradiction between Newtonian mechanics and observable properties of matter.

11.2 A MODEL FOR THE GASEOUS STATE

Historically, suggestions along the lines of a particle theory of gases had been made in 1738 by Bernoulli. Detailed theory was developed during the latter half of the nineteenth century by such men as Clausius, Maxwell, Boltzmann, Kronig, and Joule. Joule is better known for his experimental confirmations of the principle of conservation of energy than he is for his work in developing the kinetic theory of gases. The theory stands very much as it was left by these men, but is refined primarily to include knowledge of intermolecular forces.

The theory can be looked at systematically in historical perspective in the following manner: (1) 1738, suggestions by Hooke and Bernoulli; (2) 1848, fresh attack by Joule in light of his mechanical equivalent of heat experiments; (3) 1857, chief difficulties encountered by Joule solved by Clausius; (4) 1859, application of statistical mechanics by Maxwell.

What do we mean when we say *model?* This idea should be discussed so that it can be applied fully to the kinetic theory of gases.

11.3 THE SPEEDS OF MOLECULES

The Newtonian mechanics of collisions can be used to derive the relation between pressure and volume (P and V) for a collection of perfectly elastic, vanishingly small particles. The product of pressure and volume is proportional to the total kinetic energy of the particles. This idealized model does not appear to match real gases, however, because Newtonian mechanics shows also that the kinetic energy, and therefore the product PV, changes when the volume is changed. The model might still be correct, however, if something happens during volume changes that continually acts to return the total kinetic energy of the particles to the initial value. This something will later be shown to be heat exchange with the surroundings; keeping the kinetic energy constant is the same thing as keeping the temperature constant.

Although Newton's static $1/r$ repulsion model does account for the empirical relation of P and V, and Bernoulli's kinetic model apparently does not, we have dismissed the former because it isn't necessarily correct and pursued the latter because it is "reasonable." Students may need an optimistically sly "now just wait and see."

The second part of this section touches on the very rich field of statistical description. The essential point is that normal distributions are likely to result when a very large number of independent small effects produce a measurable result.

11.4 | THE SIZES OF MOLECULES

There was no real need to consider the collisions between molecules in developing the gas laws and the specific heat of a gas. However, many effects and properties of gases and molecules cannot be understood on a molecular basis without a quantitative study of molecular collisions and primarily the mean free path. With a molecular radius of about 10^{-8} cm, we find that the mean free path is 10^{-5} cm in a gas at atmospheric pressure and at a temperature of about 300°K. This is 1,000 times larger than the assumed molecular dimensions. To obtain a mean free path of 1 m, the pressure must be only 1.5×10^{-5} mm Hg. Many effects and properties of gases and molecules, such as heat conduction and viscosity, cannot be understood on a molecular basis without a quantitative study of molecular collisions.

11.5 | PREDICTING THE BEHAVIOR OF GASES FROM THE KINETIC THEORY (Special topic—optional)

Consider the options available at this point in the text. One option is to delete Sec. 11.5 and therefore move from the sizes of molecules (Sec. 11.4) to the second law of thermodynamics (Sec. 11.6). The other option is to teach kinetic theory by covering Sec. 11.5. The teacher must decide upon an appropriate investment of time and effort. The mathematical derivation is not needed as a foundation for later work in this course, but would be most useful to students who will continue the study of physics.

Those who want to invest a few days' time can lead students toward an appreciation of the empirical versus the theoretical dimensions of physics. Two equations are discussed:

$P = kDT$ Empirical equation
$P = \frac{1}{3}Dv^2_{av}$ Theoretical equation

"Empirical" suggests knowledge found after experience. In this case, the experience is that of measuring P, D, and T and by trial and error finding a mathematical relationship. Refer to *E3-14*, "The Behavior of Gases."

The theoretical equation is based upon the *a priori* statement "air consists of molecules." "A priori" suggests knowledge found before experience. Therefore, the theoretical equation differs from the empirical equation since the "molecule notion" comes from a person's mind. The molecule is postulated to preserve the success of Newtonian physics, momentum in particular.

Note that these equations have P and D in common. If the remaining quantities are equated, the startling discovery is that temperature is equal to the average kinetic energy of the molecules of a gas.

The statement in the *Text*, page 332, that "the height of the mercury column that can be supported by air pressure does not depend on the diameter of the tube" may need demonstrating.

You can vividly demonstrate the truth of the statement by making mercury barometers out of transparent hoses of various diameters and showing that the column height is the same for all of them. Barometers made with water are even more dramatic if you can manage hoses about 10 m long, suspended perhaps in a stairwell.

Emphasize that, as the diameter of the tube changes, the weight of the column changes, but so does the area upon which the column is supported. Thus, the upward force on columns of different diameters is always proportional to the weight of liquid being supported. The balance of upward and downward forces cannot, therefore, be altered by changing the column diameter, but it can be altered by changing the pressure exerted on the air at the bottom of the column (weather changes) or by changing the density of the liquid used (water instead of mercury).

11.6 | THE SECOND LAW OF THERMODYNAMICS AND THE DISSIPATION OF ENERGY

Note that Carnot's work led to principles with sweeping generality, even though his theory treated heat as a fluid. His conclusions went beyond the first law of thermodynamics (the conservation of energy).

The first law states that the total output of any heat engine must equal the total energy input. It follows directly that the useful work cannot exceed the total input energy. Carnot's theory placed a further restriction, stating that the useful work output can never exceed a certain fraction of the total input energy. The fraction is

$$\frac{T_i - T_o}{T_i}$$

where T_i is the absolute temperature in the working part of the engine and T_o is the absolute temperature of the exhaust. A heat engine can approach 100% efficiency only as the temperature of its exhaust approaches absolute zero, or as its working temperature becomes infinitely great! Because of design limitations, heat engines always fall short of even this theoretical fraction of useful work output.

The principle of dissipation of energy can be applied to all processes, mechanical or organic. The principle can be stated in many different ways and each way can be called the second law of thermodynamics. Because it was first formulated with regard to engines, many of these statements of the second law refer to restrictions on the operation of heat engines. The statement "second law of thermodynamics" is not particularly better than any other. Indeed, it leaves unclear what the "maximum amount of work" is. (It is the maximum allowed by the first law of thermodynamics, an amount equal to the input energy.)

It may not be particularly pleasing to have a major principle stated so negatively. Physicists have invented the concept of *entropy*, defined as $\Delta S = \Delta H/T$, an abstract quantity that can be calculated for any process of heat transfer. In any real process, the entropy total for all participating systems will increase. This is a pleasantly positive statement of the second law of thermodynamics even if it lacks much intuitive meaning. Some intuitive meaning can be attached to the "entropy" quantity by associating it with the degree of disorder in a system. (This association has even led the developer of modern "information theory" to refer to the "entropy of a message," a quantitative measure of the disorder, and therefore information, in a set of symbols.) So another statement of the second law of thermodynamics could be: In any real process, the total disorder for all participating systems tends to increase. The topic of entropy increase is taken up again in terms of kinetic molecular theory, and it will be seen then that these simple statements need to be qualified to allow for brief statistical "fluctuations" during which total entropy can decrease. This qualification has been attempted by the use of "tends to." It won't be clear yet to students what "tends to" means, but it should be pointed out that it is a qualifier that will be discussed further later.

Reference is sometimes made to a third law of thermodynamics, one statement of which is: "It is impossible to reduce, in a finite number of steps, the temperature of any system to absolute zero." By using a very large number of cooling steps, temperatures of 0.001°K or less can be produced. (This restriction is not the same as the "zero-point energy" of quantum theory.)

The "entropy death," also called the "heat death," appears to be a "cold death." The principle of dissipation predicts only that all ordered motion will eventually disappear in any closed system and only random (completely disordered) thermal motion will remain. Whether this is "hot" or "cold" depends on how much energy there is to go around in how much space. The statement about the possibility of avoiding the entropy death refers to the recurrence theorem. If we could wait long enough, we might find that entropy decreases again and we come back to where we started from.

The principle of dissipation of energy is not best characterized as a consequence of the second law of thermodynamics. A better expression of the relation is that the second law is a quantitative but somewhat restricted version of the dissipation principle. The statement at the end of the paragraph suggests that heat performs work while flowing, without being used up. An alternate statement would be as follows: Mechanical work can be derived from internal energy only when there is a temperature difference between two parts of a system. As a system approaches a uniform temperature, the possibility of producing mechanical work from the internal energy approaches zero.

11.7 | MAXWELL'S DEMON AND THE STATISTICAL VIEW OF THE SECOND LAW OF THERMODYNAMICS

The main point of this section is very important to the overall goals of the course. It suggests that laws of physics need not be absolute descriptions of what must or must not happen: they can just as well give probabilities for what might or might not happen. An aspect of this kind of law, not mentioned explicitly in the *Text*, is that the accuracy of statistical description increases with the number of particles. In other words, the expected percentage of fluctuation from the most probable value decreases as the number of particles increases. (Even a dust speck contains some billion billion atoms, so virtually all directly observable events are very, very close to being completely determined by statistical laws.) Also, the longer the time interval over which an average value is taken, the less the expected percentage of fluctuation from the statistically most probable value. For relatively small numbers of particles over relatively short time intervals (for example, a thousand particles over a billionth of a second) the statistical description predicts pronounced fluctuations from the most probable value. Note, however, that the fluctuations do not constitute error in the statistical description; the statistical description includes the expectation of fluctuation. It is the expected fluctuation that decreases as the number of particles and the time interval increase. The Steinberg cartoon from *The New Yorker* might be labeled "a very improbable fluctuation." The statistical form of the second law of thermodynamics would give an exceedingly small (but not zero) probability to so great a fluctuation as the boulder rolling up the hill as it and the hill cooled off.

Maxwell's demon is important to the development. It is an example of how the second law could be violated, on the basis of Newtonian mechanics, if only one could get information about molecular arrangements and sort out the molecules. In this respect, it illustrates the connection between entropy and information mentioned in Sec. 11.6. Feynman gives the example of a mechanical demon consisting of a tiny spring door of so small a mass and so loose a spring that it would open under the impact of fast molecules only. The impacts would result in the door becoming increasingly hot, so that its own random thermal agitation would progressively disrupt its function.

11.8 | TIME'S ARROW AND THE RECURRENCE PARADOX

Your students will probably be feeling anxious about the statistical turn that the course has taken. Statistics may be all right for describing the chaotic organic world, but the immutable laws of matter are something else. They may find some comfort in knowing that many scientists in the nineteenth century were not willing to accept as a basic law of physics one that gave only probabilities instead of certainties. They argued that if kinetic theory didn't lead to a completely deterministic second law of thermodynamics, the theory must be wrong.

Another way of stating this objection is the "reversibility paradox." The kinetic theory involved only reversible collisions of molecules so that events could just as easily happen backwards as forwards. For instance, the hill could cool slightly and propel the boulder up it as easily as the boulder could roll down and warm the hill: concentrating energy instead of dissipating it, increasing order instead of disorder. But all observed events in the universe go in just one direction, that of energy dissipation and increasing disorder. So it seems that the kinetic theory must be wrong. The sequence of photographs on *Text* page 344 is intended to suggest this very sticky question. The sequence of events in the very nearly perfectly elastic collision photographed stroboscopically in the top picture could be almost equally good physics whether beginning at the right or at the left. In the perfectly elastic molecular collisions of the kinetic model, there would be no way of choosing between a left-to-right and a right-to-left time sequence. In the other pictures there is no doubt about which way the process is going; if a motion picture of one of them were reversed, it would easily be spotted as "backwards." *Film Loop 36*, "Reversibility of Time," could be used to develop this point.

CHAPTER 12 / WAVES

SUMMARY OF CHAPTER 12

Chapter 12, perhaps more than any other chapter in the unit, depends upon experiments and/or demonstrations to give substance to the material discussed in the *Text*. The purpose of the demonstrations and experiments is to provide a general understanding of waves.

We are concerned with wave phenomena in a mechanical framework in order to set the stage for the study of light and electromagnetic waves in Unit 4. It is important that students become familiar with wave phenomena, particularly interference, so that a wave-versus-particle model will make sense to them. This will also be useful in Unit 5 when they study the atom.

12.1 | INTRODUCTION

Our previous work has been concerned, on one hand, with gross motion of particles and of rigid bodies and, on the other hand, with some of the internal properties and constituents of matter. Throughout our study of rigid bodies, we have assumed that the various parts of a body remained fixed distances apart. That is, we ignored the compressible nature of the solid form of matter. We shall now ask how a compressible body responds to externally applied forces and see that this leads directly to the study of waves. In chapters to come, we shall see that these phenomena (waves) are particularly descriptive of many events in nature, such as light, sound, radio, and water-surface phenomena. We shall see in the units called "Models of the Atom" and "The Nucleus" that atoms, electrons, and nuclear and subnuclear particles have wave-like, as well as particle-like, properties.

12.2 | PROPERTIES OF WAVES

Wave motion implies the transmission of a *state*. If we stand dominoes on end in a long line and then knock over the first one, we start a chain of events that leads eventually to all the dominoes being knocked down. There was no net mass transport along the line of dominoes; rather, it was their *state* of falling that traveled. In this simple case, the speed at which the state of falling traveled is called *wave speed*.

All wave pulses possess momentum and energy, which can be determined without much difficulty. This could be assigned to "better" students with some guide lines.

The first serious suggestion that polarization might be due to vibrations of light transverse to the direction of propagation came from Thomas Young in 1817. The name "polarization" was first used in this connection in 1808 by a French investigator named Malus.

12.3 | WAVE PROPAGATION

Energy can be transmitted over considerable distances by wave motion. The energy in the waves is the kinetic and potential energy of the matter, but the transmission of the energy comes about by its being passed along from one part of the matter to the next, not by any long-range motion of the matter itself. The properties of a medium that determine the speed of a mechanical wave through that medium are its inertia and its elasticity. Given the characteristics of the medium, it is possible to calculate the wave speed from the basic principles of Newtonian mechanics.

12.4 | PERIODIC WAVES

It is most important for students to have a thorough understanding of the important definitions in wave motion. The wavelength "idea" is an obvious, imperative, and prerequisite concept for an understanding of *phase*. Since the period T is the time required to travel one wavelength λ, it follows that $\lambda = vT$. This velocity is called the *phase velocity* and is the only velocity involved in a simple harmonic wave. In contrast to this is a *group* velocity, which is important when several wavelengths and phase velocities travel through a medium. We shall limit our discussion to phase velocity and simple harmonic waves. The relationship $v = \lambda f$ is important and applies to all waves impartially: water waves, waves on springs or ropes, sound waves, and electromagnetic waves.

12.5 | WHEN WAVES MEET: THE SUPERPOSITION PRINCIPLE

The superposition principle appears so obvious that it might be worthwhile to mention when it does not apply. When the equations of the wave motion are not linear, superposition fails. This happens physically when we have relatively large wave disturbances for which the laws of mechanics are no longer linear. An example is the linear relation in Hooke's law. Beyond the elastic limit, $F = -kx$ no longer applies. For this reason, shock waves in sound behave differently than ordinary sound waves and hence superposition does not hold. A quadratic equation governs the behavior of shock waves. Very loud tones will not add linearly, thus giving rise to a distortion known in high-fidelity jargon as "intermodulation distortion." Ripples in water waves can travel independently across gentle swells but not across breakers.

It would be instructive to stress the fact that, if two waves act independently of one another, the displacement of any particle at a given time is simply the sum of the displacements it would receive from the individual waves alone.

12.6 | A TWO-SOURCE INTERFERENCE PATTERN

It is interesting to note that Thomas Young (1773–1829) was a London physician who became interested in the study of light through his medical studies, particularly his discovery of the mechanism of accommodation (focusing) of the eye. He had done extensive experiments with sound and was impressed with the study of beats. Two sounds can combine so as to produce silence and this is most easily explained on the basis of a wave picture. It isn't surprising then, that he presented reports to the Royal Society concerning experiments on light, namely, his now famous two-slit interference experiment.

The node and antinode lines are actually hyperbolas. The hyperbola, since it is the curve for which the difference in distance from two fixed points is constant, obviously fits the condition for a given fringe, namely, the constancy of the path difference. Although this deviation from linearity may become important with sound and other waves, it is usually negligible when the wavelengths are as short as those of light.

12.7 | STANDING WAVES

Standing waves are most commonly produced as a result of reflections. However, they can also be produced by two independent sources of disturbance, one of which produces a wave that travels to the right and the other a wave that travels to the left. In terms of energy, the difference between a traveling and a standing wave is that in the traveling wave there is a net flow of energy, whereas in a standing wave there is none. The energy is "trapped" in the standing wave.

Each characteristic frequency of the one-dimensional spring, string, or any other similar elastic system, corresponds to a certain characteristic mode of oscillation. Similar characteristic frequencies and modes of oscillation are found also in two-dimensional patterns. An important difference exists between the one- and two-dimensional cases. In the one-dimensional case, there is a one-to-one correspondence between the characteristic frequencies and the characteristic modes of vibration. In the two-dimensional case, however, there may be several modes of oscillation with the same frequency.

Standing waves are of first importance in the study of sound, since, without exception, every musical instrument depends on this principle. In 1807, Fourier showed that any periodic function can be expressed as the sum of a number of sine and cosine functions with appropriate amplitudes. This mode of analysis is of great value in almost every branch of physics.

Reference to Sec. 12.6 can now show that the pattern for two-slit interference between the sources is a standing wave pattern.

12.8 | WAVE FRONTS AND DIFFRACTION

Huygens' theory simply assumes that light is a wave pulse rather than, say, a stream of particles. It says nothing about the nature of the wave, and gives no hint of the electromagnetic character of light. This theory is based on a geometrical construction, called Huygens' principle, which allows us to tell where a given wave front will be at any time in the future if we know its present position.

Today this principle can be stated somewhat succinctly as follows: Every point on an advancing wave front acts as a source from which secondary waves continually spread. The passage of a ripple through an aperture illustrates this effect. (Refer to ripple tank experiments such as *E3-16* or *D44*). It was a similar spreading of beams of light that gave rise to the phenomena that Grimaldi had classified under the name *diffraction* in his publication of 1665. Newton not only referred to Grimaldi's experiments but he repeated and improved upon them. The preoccupation with one theory blinded even a scientist of Newton's caliber to the significance of the evidence pointing toward another. It was left to Thomas Young to find the most convincing evidence for the wave hypothesis.

12.9 | REFLECTION

The law of reflection was known to Euclid. It can be derived from Maxwell's equations, which means it should hold for all regions of the electromagnetic spectrum.

The reflection of waves is familiar from such events as the echo of a sound wave, the reflection of a ripple on a water surface, or the reflection of waves on a rope. When waves are incident on a boundary between two media in which the velocity is appreciably different, the incident wave train is divided into reflected or refracted (or transmitted) trains. The amount of energy reflected will be relatively greater the larger the change in velocity.

12.10 | REFRACTION

Observation of the bending of light waves goes back to antiquity. Cleomedes, in the first century A.D., was one of the first to suggest that the sun remains visible for a time after it has actually set. Aristotle, in his *Book of Problems*, correctly described the appearance of an oar dipped in water. Ptolemy (about A.D. 150) tabulated angles of refraction for air–water–glass media. He concluded that the angle of incidence and refraction were the same. Alhazen (965–1039), an Arabian investigator, pointed out the error of Ptolemy's generalization. Ptolemy's tables were extended (though not correctly) by Vitellio about 1270 and by Kircher (1601–1680). Kepler, in addition to his astronomical studies, made important contributions to optics and investigated reflection and refraction angles in media with somewhat more success than Ptolemy. It was not until 1621 that the legitimate relation was experimentally discovered by Willebrod Snell and deduced from an early corpuscular theory of light by René Descartes. The law of refraction is known as Snell's law or (in France) Descartes' law. There is reason to believe that Descartes had seen Snell's manuscript, though he subsequently published the law of refraction as his own discovery. Descartes' plagiarism upon Snell points out, perhaps unfortunately, that unethical practices in the pursuit of personal ambition are not entirely unheard of in the history of science.

12.11 | SOUND WAVES

Sound waves are longitudinal mechanical waves. They can be propagated in solids, liquids, and gases. The material particles transmitting such a wave oscillate in the direction of propagation of the wave itself. The distinction between the subjective and objective attributes of sound has not always been recognized, but John Locke, the seventeenth century philosopher, said

> That which is conveyed into the brain by the ear is called sound, though in truth until it comes to reach and affect the perceptive part, it is nothing but motion. The motion which produces in us the perception of sound is a vibration of the air caused by an exceedingly short but quick tremulous motion of the body from which it is propagated, and therefore we consider and denominate them as bodies sounding.

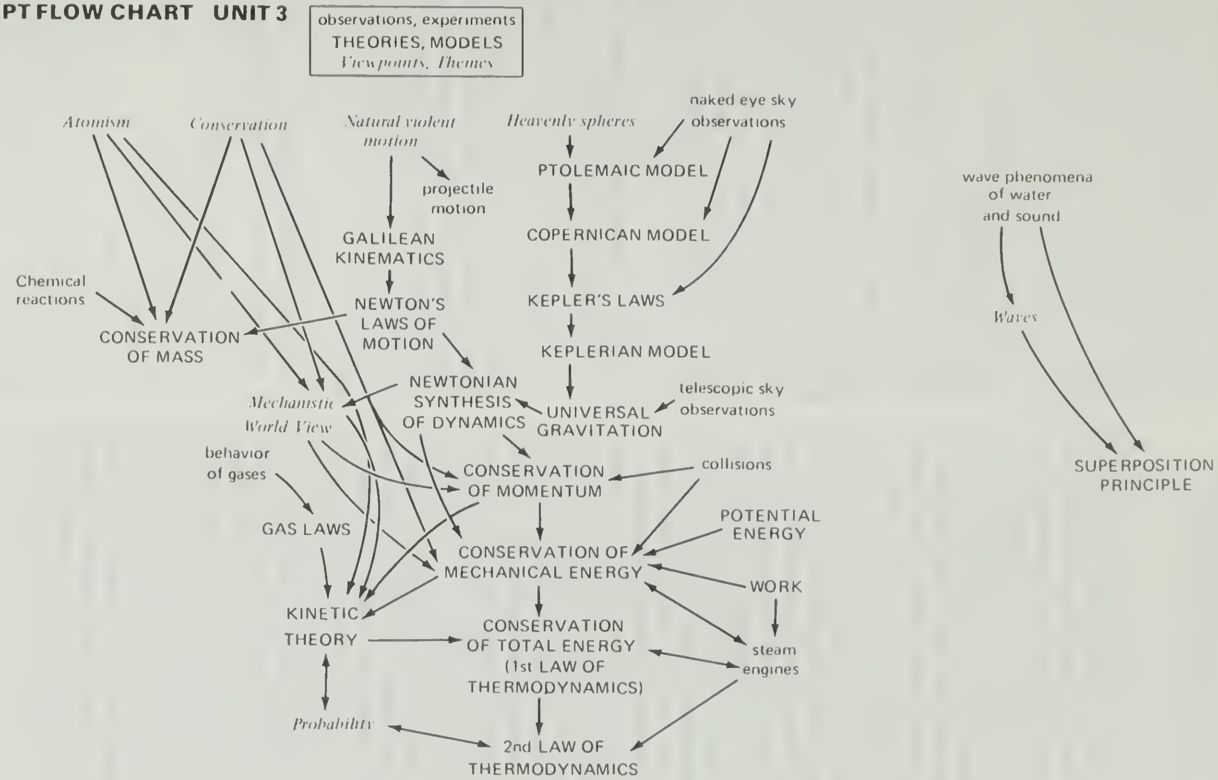
But in any case the origin of sound can be traced to motion of some kind. Again, historically speaking, Aristotle made this observation when he said

> All sounds arise either from bodies falling on bodies or from air falling on bodies. It is due to air . . . being moved by expansion, contraction and compression.

Robert Boyle in 1660 improved the air pump recently invented by von Guericke and with it performed a bell-in-vacuo experiment where it is reported he said

> We silently expected the time when the alarm should begin to ring . . . and were satisfied that we heard the watch not at all.

CONCEPT FLOW CHART UNIT 3



Additional Background Articles

NOTE ON THE STATE OF PHYSICS AS A SCIENCE AT THE BEGINNING OF THE NINETEENTH CENTURY

Until the beginning of the nineteenth century, practical sources of electrical current had not been devised, so the study of electricity was not well developed. Heat and electricity were both considered to be weightless fluids, in accordance with the mechanical point of view. An adequate theory of heat was not developed until the middle of the nineteenth century. The nature of light was not known at the beginning of the nineteenth century; the particle hypothesis of Newton was favored against the wave hypothesis of Huygens, but both were mechanical theories.

It should be noted that those who contributed to the development of mechanics, like Newton and others in the following generations, were great mathematicians. This put a definite stamp of precision and logic on the development of mechanics. Mathematics seemed to be a science of unlimited possibilities. One of the French mathematicians once said that, given sufficient time, it will become possible to express human thoughts in the form of a mathematical formula. Mechanics was considered to be an applied aspect of mathematics and as such also as a science of unlimited possibilities. (Remember the full title of Newton's *Principia: Philosophiae Naturalis Principia Mathematica*, "Mathematical Principles of Natural Philosophy.") This was true both for "pure" mechanics (for instance, laws of motion) and technological or engineering applications of mechanics. As we will see later in this unit, Heron of Alexandria was able to make certain mechanical devices utilizing the power of steam in about A.D. 100. In the seventeenth and eighteenth centuries, many sophisticated mechanical devices and automata were built, such as Besancon's duck that could swallow food, or the mechanical boy who could play a musical instrument. The Swiss watchmakers produced ingenious devices, some of them able to give a whole theatrical performance staged by mechanical dolls. One needed only to wind the spring and the mechanism started to operate. The possibilities of these mechanical devices (from the clocks of the thirteenth century on) greatly stimulated human ingenuity in the West. (See L. Mumford, *Technics and Civilization*.)

This development of mechanics and mathematics on one side, and the construction of mechanisms of high complexity on another side, induced scientists to think of God as the greatest of all scientists. God had built the world-machine, the universe in which we live, and had wound it up for all time. As Kepler put it, the task of a scientist consisted therefore in following the thoughts of God at the moment of creation. Of course, not all scientists accepted this view. This lack of consensus is illustrated by the famous anecdote involving the French scientist Laplace, who had suggested a mechanical account of the origin of the earth and our solar system. Napoleon I, to whom Laplace explained his theory, asked him, "Where is God in your theory?" to which Laplace is said to have replied, "I have no need of that hypothesis." ("Je n'ai pas besoin de cette hypothèse-là.")

NOTE ON CONSERVATION LAWS IN PHYSICS

The discovery of conservation laws has been one of the most important achievements of science. These laws, which are perhaps the most powerful and certainly the most prized tools of analysis in physical science, say in essence that, whatever happens within a system of interacting bodies, there are certain measurable quantities that can be counted on to remain constant so long as the system remains isolated.

The list of conservation laws has grown in recent years, particularly as a result of work in the area of fundamental (or "elementary") particles. Some of the newer laws are imperfectly and incompletely understood. There are others that are on tenuous ground and are still being argued.

A list of conservation laws is given here. It would be foolhardy to say the list is complete or entirely accurate. Only too recently we have had to surrender long-held, cherished conceptions that appeared almost self-evident. But this list includes those conservation laws that comprise the working tool kit of physicists today. Those which are starred are discussed in the basic units of this course; the others are treated in supplementary (optional) units.

1. Linear momentum*
2. Energy (including mass)*
3. Angular momentum (including spin)
4. Charge*
5. Electron-family number
6. Muon-family number
7. Baryon-family number
8. Strangeness number
9. Isotopic spin

Those listed as numbers 5 through 9 result from work in nuclear physics, high-energy physics, or elementary or fundamental particle physics. If these laws are unfamiliar, you will find Kenneth Ford's "Conservation Laws" interesting, enlightening, and worth reading at this stage. The selection is a chapter from Ford's book *The World of Elementary Particles* (Blaisdell Publishing Co., 1963). The book is a well-written introduction that should appeal to anyone who wants to learn about one of the leading frontiers of current physics. Ford discusses the first seven laws in that selection, and, even for those uninitiated into the mysteries of elementary particles, he gives a clear presentation of many ideas basic to contemporary physics.

ELASTIC AND INELASTIC COLLISIONS

The terms "elastic" and "inelastic" appear in Chapter 9. It is difficult to discuss these terms fully without using the concept of energy. For an *elastic* collision between bodies A and B, B's velocity relative to A after collision is just the negative of what it was before collision. The distance between A and B afterward will increase at the same rate at which it decreased before collision. For a completely or perfectly inelastic collision, the relative velocity is zero after collision. For a collision somewhere between completely elastic and completely inelastic, the bodies will separate more slowly than they approached.

More usually, an elastic collision is defined as one for which the kinetic energy after the collision is the same as the kinetic energy before. Below is a proof that this is equivalent to the above definition.

In either treatment, an elastic collision is "reversible"; that is, it would be impossible to tell whether a motion picture of the collision was running forwards or backwards.

NOTES ON THE EQUIVALENCE OF THE DEFINITIONS OF "ELASTIC COLLISION"

<u>Before</u>	<u>After</u>
$A \xrightarrow{v_A} \qquad B \xrightarrow{v_B}$	$A \xrightarrow{v_A'} \qquad B \xrightarrow{v_B'}$
$(v_B - v_A)$ is the velocity of B relative to A before.	$(v_A' - v_B')$ is the velocity of B relative to A after.

If kinetic energy before and after are equal, then

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2. \qquad (1)$$

Since the linear momentum is always conserved

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'. \qquad (2)$$

We can rearrange equation (1)

$$m_B(v_B'^2 - v_B^2) = m_A(v_A'^2 - v_A^2) \qquad (3)$$

and equation (2)

$$m_B(v_B' - v_B) = -m_A(v_A' - v_A). \qquad (4)$$

If we divide equation (4) into equation (3), we get

$$v_B' + v_B = v_A' + v_A$$

or

$$v_B' - v_A' = -(v_B - v_A), \qquad (5)$$

which says that the velocity of B relative to A after collision is equal to the negative of what it was before.

ENERGY REFERENCE LEVELS

The zero level for measuring potential energy (or even for measuring kinetic energy!) is arbitrary. Because the conservation principle really deals only with changes, the zero levels can be chosen as whatever is most convenient at the moment. This is more easily seen if the general formulation used for momentum is used again here:

$$\sum_i (\tfrac{1}{2}m_i v_i^2 + F_i h_i) = 0.$$

Lest students conclude that playing with the zero levels is only "mathematical," some examples can be given to show that there is really no meaning to absolute zero levels for potential or kinetic energy. It is common, for example, to take the lowest point of the pendulum's swing as the zero potential level. But if the string is cut, the bob can fall to the floor and decrease its potential energy still more. The potential energy at the floor level could be taken as "negative," or the floor level could be chosen as a new zero level. But if a hole were to be dug next to the bob, it could decrease its gravitational potential still further by falling into the hole all the way to the center of the earth! (What would happen if it went beyond the center?) But neither will the center of the earth do as an absolute zero of potential energy, since there is still the possibility of falling all the way into the sun . . . and so on, ad infinitum.

The topic of "inertial" frames of reference is too subtle to take up here so it is not easy to discuss the "zero level" for speed. Thus, the "zero level" for kinetic energy is not so easy to handle. The change in v^2 in one frame of reference will <u>not</u> be the same in some other frame of reference moving with uniform translation relative to the first, but neither will the observed distance through which the force acts. Very often (for example, in nuclear physics) velocities are referred to the center-of-mass of the interacting bodies to simplify calculation.

After all this, the main point to be made is as follows: Intuitively, $\tfrac{1}{2}mv^2$ is not an obvious choice for "quantity of motion," but neither is it particularly objectionable. Likewise, weight times height is not objectionable as a measure of a "potential" something.

However, by making these choices, we can compute a quantity, the sum of $-mv$ and Fh, which is <u>conserved</u> (at least in frictionless rising and falling near the surface of the earth). It will turn out to be valid also for the frictionless "rising and falling" in the solar system and in many other situations (when some other distance measure than distance from the earth is used). We have given the sum the name "energy." When related to ideas of "heat" and "work," "energy" proves to be one of the most powerful ideas of physics, making many connections with other sciences as well as within physics.

NOTE ON FOOD ENERGY

Many foods combine with oxygen in the body to produce carbon dioxide and water. Since, in the body, the process of oxidation progresses through many steps, the energy is released much more slowly than when the same food is burned in air.

However, the chain of chemical reactions through which the process takes place does not alter the amount of energy released.

The energy value of a food can be ascertained by burning samples in a laboratory. This process is used as the basis for Calorie charts that tell us how much energy a particular food is capable of providing.

Measurements of the amount of food actually "burned" in the body may also be made. This is done by collecting the gas a person exhales for a short time and analyzing the sample to determine how much oxygen from the inhaled air has been replaced by carbon dioxide. Persons vary in the extent to which their bodies obtain and use energy from food. The minimum energy used to keep alive is called the "basal metabolism."

Different amounts of food energy are needed by the body for different types of activities. The more a person exercises or performs heavy physical work, the more energy is needed. In general, however, the body converts a large amount of the energy stored in food to heat energy and utilizes a much smaller portion of the total food energy for other life processes.

NOTE ON CLASSIFICATIONS OF ENERGY

For some purposes, there are advantages to classifying energies. But the kind of classification that one might best make depends upon the situation under consideration and is, in any case, arbitrary.

Suppose we are looking at a physical system: a bowl of soup, a soap bubble, or maybe our solar system. Someone asks, "What kinds of energy does the system possess?" We might answer that it has kinetic and potential energies; but that statement, while true, is likely to be too general, too all-inclusive.

We might go further. We might say that the system has kinetic energy as a consequence of the motion of the system as a whole. We may have dropped the bowl of soup; the soap bubble may be floating away; or our solar system may be moving relative to the center of our galaxy. The magnitude of this energy depends upon the reference frame in which we choose to measure the system's motion.

In addition, the system as a whole may possess potential energy as a consequence of its position relative to external sources of force. The soup and soap bubble have gravitational potential energy as a consequence of experiencing the earth's gravitational force. Similarly, the solar system may experience gravitational forces due to nearby stars.

For some purposes, these kinds of kinetic and potential energies are not very important. For example, we do not consider them at all when we ask about the soup's temperature or about the way the planets move relative to the sun. When that is the case, we generally say that all the rest of the energy is *internal energy*. For most thermodynamics problems, this is the sort of classification we make. (In fact, the laws of thermodynamics are independent of the detailed nature of the system.)

How we go about describing the internal energy depends upon our purposes. If we are dealing with helium gas at moderate temperature and pressure, it is reasonable to say that the internal energy is just the translational kinetic energy of the atoms, where we measure their speeds relative to the container. That is the approach taken in Secs. 12.4 and 12.6 of Unit 3. This works reasonably well for He, because the helium atoms exert only very weak forces on each other when not colliding, and because the He molecule is monatomic (composed of only one atom).

If we are dealing with a gas composed of more complex molecules, we need to consider other forms of internal energy; for example, the kinetic and potential energies associated with the vibrational motion of the atoms within the molecule and the kinetic energy associated with the molecule's rotation. We might even need to include the energy associated with the interactions of the molecules with each other, often referred to simply as chemical energy.

Even these forms do not exhaust the sources of internal energy. There is the energy involved in binding an atom's electrons to its nucleus and that associated with the forces holding the particles together within the nucleus. There are instances for which we might need to include the system's radiation energy (light).

It should be clear that there are many different ways of making energy classifications. There are times when such classifications as elastic, electrical, and chemical energies are helpful, even though we know that elastic and chemical energies are predominantly electromagnetic in character. (Think of a charged compressed spring in a jar of sulfuric acid.) It is probably worthwhile to give some thought to the sources of internal energy, but it is not the sort of thing one wants to bludgeon students with. Detailed classifications can sometimes lead to confusion, and are always somewhat arbitrary.

SOME NOTES ON WATT

(from *James Watt, Craftsman and Engineer,* H. W. Dickinson, Cambridge University Press, 1936.)

"The problem now was to make the apparatus into an engine capable of repeating its motion indefinitely. Watt started on the construction of a model with a cylinder 2 in. diam. While thus engaged, Robison's story is that he burst into Watt's parlour and found him with a 'little tin cistern' on his lap. Robison began to talk engines, as he had done previously, but Watt cut him short by saying: 'You need not fash yourself about that, man; I have now made an engine that shall not waste a particle of steam.' Robison put to Watt a leading question as to the nature of his contrivance but 'he answered me rather drily and vouchsafed no explanation.' If an artist ever wishes to paint a genre picture of Watt, instead of perpetuating the unfounded story of his playing as a boy with the steam issuing from the spout of a kettle, he might limn the young workman in his leathern apron with the separate condenser on his lap and Robison trying to quiz him.

"If Watt lacked experience in the construction of engines 'in great,' i.e. of full size, he had the advantage of being free from preconceived ideas of what engines should be like. In fact he had in view two engines, one reciprocating and the other rotary, entirely different in design from anything that had gone before. This very fertility of mind, and his resource in expedient, may almost be said to have delayed his progress. Of a number of alternatives he does not seem to have had the flair of knowing which was the most practicable, hence he expended his energies on many avenues that led to dead ends. In truth this is the attitude of mind of the scientist rather than that of the craftsman. Still, unless he had explored these avenues he could not be certain that they led nowhere."

DISCUSSION OF CONSERVATION LAWS

Conservation laws are important because they enable us to make predictions. This ability has a far wider significance than its application to colliding carts in the laboratory. For example, much of what is known about subatomic particles comes from analyzing scattering (collision) experiments. Students hear a great deal about bubble chambers and particle accelerators without understanding the way in which information from these devices is used.

A detailed account of Chadwick's discovery of the neutron and his calculation of its mass is found in Chapter 23 and could well be used here as an example in which both conservation laws are used in making new discoveries.

We suggest the following procedure as an effective way to show the use of conservation laws to make predictions. It will also introduce a method of using graphs that is very similar to that used by engineers and physicists. Although the computations and plotting may be assigned as homework, it is important that you go through the analysis of the graph carefully with the class so that they will appreciate the full significance of the curves they have drawn. Data from one of the collisions you photographed could be used. However, if you feel that students would get bogged down in arithmetic, you could give them easy round numbers, such as those used in the example below.

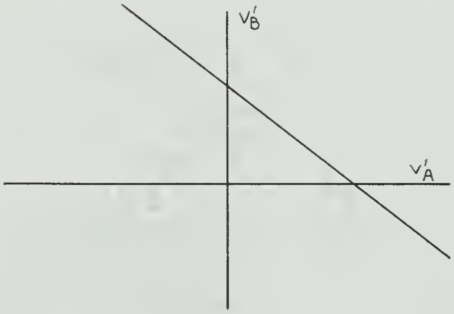
First, ask students to assume only that momentum is conserved in a collision. For example, let $m_A = 1$ kg, $m_B = 2$ kg, $v_A = 0$, and $v_B = 10$ cm/sec. Then,

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$0 + 20 = v_A' + 2v_B'$$
$$v_A' = 20 - 2v_B'$$

Have the students assign values for v_B' and compute v_A':

v_B'	v_A'
0	+20
5	+10
10	0
15	−10
−5	+30

Plot these points as in the figure below.



Point out that there is an infinite number of values for v_B' and v_A' that would satisfy the above equation, and ask if all of them might actually be observed if the experiment were done enough times. Make sure that the significance of positive and negative values is clear. For example, ask students to predict from their graphs what the velocity of the smaller cart would be if the large cart rebounded with a speed of 10 cm/sec ($v_A' = 40$ cm/sec).

Next, ask students to assume only that kinetic energy is conserved in this collision.

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2$$
$$0 + 200 = v_A'^2 + 2v_B'^2$$
$$v_A'^2 = 200 - 2v_B'^2$$

Constructing a table of values for v_B' and v_A' is now a little more time-consuming because of the square roots. However, a number of shortcuts are available. Students might plot values of x^2 as a function of x, draw a smooth curve, and use this as a simple computer.

Again, any number of values for v_B' and v_A' can be calculated, but most of them are physically meaningless. The table of values could be constructed as follows:

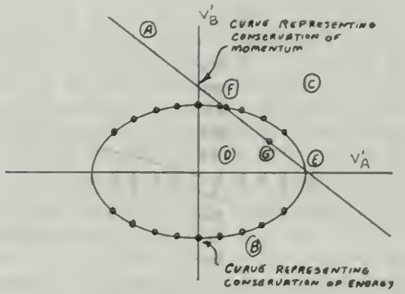
v_B'	$v_B'^2$	$2v_B'^2$	$v_A'^2$	v_A'
0	0	0	200	±14.1
± 2	4	8	192	±13.8
± 4	16	32	168	±12.9
± 6	36	72	128	±11.3
± 8	64	128	72	± 8.5
±10	100	200	0	± 8.5
±12	144	288	−88	(imaginary)

Plot these values of v_B' and v_A' on the same axes as the preceding graph. An equation in the form

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$$

results in the curve called an ellipse. The equation resulting from the conservation of energy in the collision has this form. Students may be familiar (from Unit 2) with the mathematics involved here, but may not appreciate that these curves can be used to infer a number of interesting facts about real physical events.

Use the key letters on the graph below to develop the following ideas:



Point A: Any point lying on the straight line represents values of v_B' and v_A' for which momentum is conserved.

Point B: Any point lying on the ellipse represents values for which kinetic energy is the same after the collision as before.

Point C: For any point lying outside the ellipse, the corresponding kinetic energy of the carts would have to be greater than before the collision. In other words, kinetic energy was added to the system. This could not happen in this kind of collision; it would require that an explosion be set off during the collision.

Point D: For any point inside the ellipse, the total kinetic energy of the carts after the collision is less than before. In other words, kinetic energy was lost from the system, for example, by friction between the bumpers.

Point E: This point on the ellipse represents the case in which the cart with velocity v_B goes past or through the other cart having velocity $v_A = 0$ without exerting any force on it. This case may seem trivial or physically meaningless, but a comment about neutrinos may be in order here.

Point F: This point on the ellipse represents the only values for v_B' and v_A' (other than case E) for which both momentum and kinetic energy are conserved, as shown by the intersection of the two curves. If students used starting data from one of their photographs, they should see where the values for v_B' and v_A' obtained from their photograph plot on the graph.

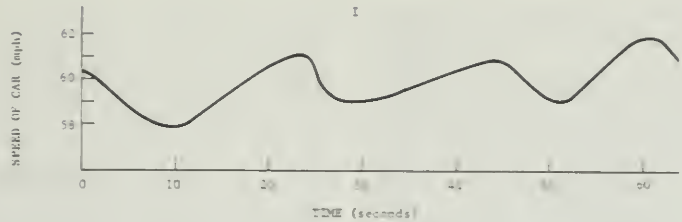
Point G: This point on the line shows v_B' and v_A' set for a collision in which the carts stick together after the collision. In other words, $v_B' = v_A'$. Since momentum is always conserved, the point must fall on the straight line, and so it falls inside the ellipse. Kinetic energy was lost, so this represents an inelastic collision. Points along the line segment FG represent all possible values of v_B' and v_A' for which the cart with $v_B = 10$ cm/sec pushes the other ahead. The line segment GE represents all values for which the first cart overtakes or passes through the second one.

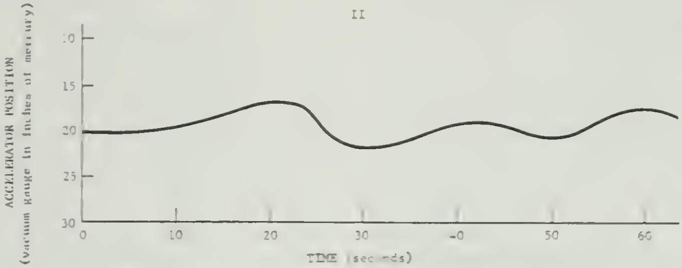
If the values of v_B' and v_A' obtained from the photograph do not plot at point F, ask the students for possible reasons. (It should fall in the region FG).

FEEDBACK

An enormous difference often seems to exist between the practical world of applied science and engineering and the idealized world of pure science and theoretical engineering. This is because it is often difficult to describe commonplace physical situations theoretically by exact mathematical relationships, while simple quantitative theoretical ideas, such as frictionless surfaces, constant speed, point sources, pure sine waves, and a host of other theoretical concepts may be difficult to produce experimentally. Usually, we must be satisfied to produce these basic physical or mathematical ideas to within acceptable limits of error. To do this, we often make use of a system of control called *feedback*. In attempting a definition of feedback, let us consider a situation that any motorist meets and solves daily.

Suppose the driver of an automobile on a lightly traveled superhighway is asked to hold a speed of 90 km/hr for 5 min. Assuming that the driver cooperates as much as possible, the chances are that it will take quite a bit of manipulation of the gas pedal to keep the car's speed constant to within a few kilometers per hour of 90 because of the different road conditions to be encountered in 5 min. For example, the car will tend to slow up when climbing hills unless it is given more gas and will tend to speed up on going downhill unless the driver lets up on the accelerator. Various road surfaces also require different accelerator settings. Thus, we realize by experience or observation that a motorist who attempts to maintain a constant speed is continually moving the accelerator by small amounts. In engineering terms, we describe this situation by saying that the motorist forms a feedback system for the car and attempts to keep the car moving with constant speed under a changing load (the hills and road surface) by observing the deviation from the desired output signal (the 90 km/hr on the speedometer) and then correcting the input signal (the flow of gas controlled by the accelerator) in such a way as to bring the output signal back to the desired value (90 km/hr). The process just described is the essence of what we mean when we use the term feedback in a technical sense.





Simulated graphs of (I) automobile speed and (II) vacuum gauge readings as a function of time. The vacuum gauge, an indicator of gasoline flow to the carburetor as governed by the accelerator, is a fast response instrument compared with the speedometer. This difference in response contributes to phase differences in the two curves. Though simulated, the values are realistic.

There are some other things about a feedback system that are characterized by this situation For one it is quite evident that the more closely the motorist tries to hold the car's speed to 90 km hr the more tiring it is. We can say that a tightly coupled feedback system one allowing only small deviations from a given value generally requires a greater expenditure of energy than a system in which larger deviations are allowed. Secondly we note that the fluctuations in the speedometer and the variations in the accelerator position do not occur together because it takes the driver and car a certain amount of time to react to the output signal. Thus we see that the simple mathematical statement v = 90 km hr, used to describe the speed of the car in a theoretical problem, may in practice require good equipment concentration and skill to fulfill.

In engineering, one often wishes to dispense with the human element replacing it with some physical device that doesn t tire yet performs its function in the feedback loop. To show in some detail how feedback operates in a physical system we shall choose a simpler and more direct example than that of the automobile and driver. Let us consider how we might meter and control the flow of a liquid in a completely automatic way. In this metering system the objective will be to maintain a constant flow of liquid under a pressure variation stemming from a change in the height of a liquid in a tank at the high pressure side of the tube.

In order to make a liquid metering device theory shows that all that is needed is a tube of constant length and uniform bore. For this tube and a specified liquid, the volume of liquid passing through the tube in a given time is directly proportional to the pressure difference across the ends of the tube. In a common arrangement one end of the tube is open to the atmosphere and the other end is fed from a column of liquid of some height H. As the pressure on the side where the liquid flows out of the tube is atmospheric pressure while the high-pressure side is at atmospheric pressure plus a term proportional to the height of liquid the pressure difference across the tube is directly proportional to the height H of liquid. From this analysis we see that by maintaining a constant height of liquid feeding the tube a constant flow of liquid through the tube is assured. The problem therefore resolves itself into finding a means for keeping H constant as liquid flows through the tube.

The classical way of solving this problem is through use of a weir. This arrangement is shown in Fig. 1. The weir is the open tank labeled W The metering system is the horizontal tube of length D. The weir maintains a constant liquid level at height H above the tube by means of an overflow pipe the curved tube inserted in the tank Liquid flows through the supply pipe entering the system at the point labeled input and flows out into the weir From the weir the liquid leaves the system at the output point after having first flowed through the metering tube the tube of length D After the supply pipe fills the weir to height H this level is maintained by allowing the excess liquid to run out of the overflow pipe as waste. Once this condition is

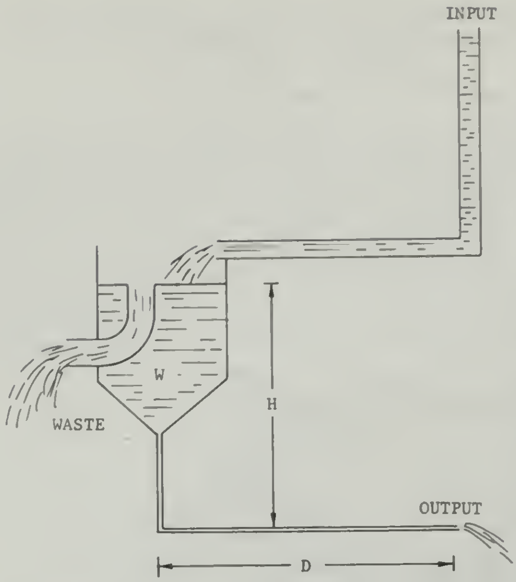


Fig. 1 Liquid metering system

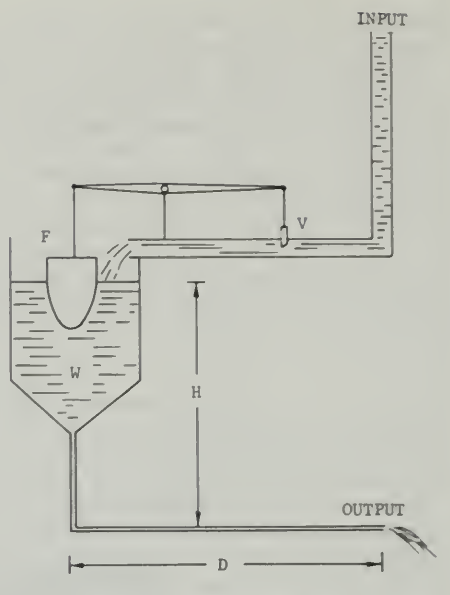


Fig. 2 Liquid metering system with feedback

reached, the output through the metering tube is constant for all variations of input through the supply tube, provided there is always some liquid flowing out the overflow pipe.

This system for metering a liquid is very precise and as an apparatus for research concerning the viscosity of liquids, for example, yields definite results. For less exacting uses, such as in the metering of fuel to the carburetor of an internal combustion engine, it is more convenient to dispense with the overflow pipe and devise a system where the height of liquid in the tank is controlled automatically by a valve in the supply line. Two ways of accomplishing this are suggested, both of which use feedback. One type is purely mechanical and the other is electromechanical feedback. The aim here is to illustrate the concept of feedback, not to design the most practical system possible.

In Figs. 2 and 3, the height of liquid is determined by a float-controlled valve. The simplest of the two arrangements is shown in Fig. 2, where a float is suspended at one end of a beam balance and valve V is suspended at the other end. The float and valve (shown here as a simple plate damming the liquid in the supply pipe) are adjusted at a certain level H for a fixed pressure on the input side of the supply line. The subsequent action of the float and valve is such that when an increase in pressure, and hence increase in flow of the liquid, occurs in the supply line, the resulting flow through the valve leads to an increase in height H in the tank. This causes the float to rise. Since it is connected to the opposite end of a beam balance, the rise of the float depresses the valve, decreasing the flow of liquids and leading to a fall in the liquid level H. When this level falls, the float falls and the valve opens. This compensating action tends to keep level H constant, because the float always moves the valve in such a way as to keep the liquid surface at a fixed height. The float and valve constitute a mechanical feedback system.

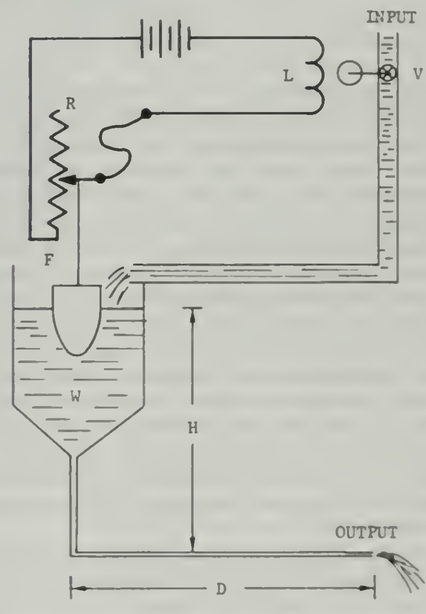


Fig. 3 Liquid metering system with electromechanical feedback

Figure 3 is an electromechanical adaptation of Fig. 2. In this feedback system, float F and valve V are linked electrically, not mechanically. The change in the height of the float changes the resistance, and hence the current, in the electrical circuit shown. A change in current also occurs in the coil L, changing the strength of the magnetic field in the coil. By controlling the magnetic field of a coil that is part of a motor or relay, one controls the position of a valve connected to the motor or relay and hence controls the flow of liquid in the supply line in the same sense as in Fig. 2. The advantage here, of course, is that F and V need not be close to one another, which makes this arrangement the more flexible feedback system.

Historically, feedback goes back to the fly-ball governor on James Watt's steam engine. In order to keep this engine running at constant speed under differing mechanical loads, it was necessary to control the amount of steam admitted to the cylinder containing the piston. This control was achieved by linking a valve in the steam line (a valve acting like the gas pedal of a car) to a fly-ball governor rotated by the shaft of the engine. This governor operates on the same principle as a stone on a string. That is, when a stone on the end of a string is swung in a horizontal circle at constant speed, the angle between the string and the vertical increases as the speed of the stone increases. On the fly-ball governor, this change in angle with speed can be used to control the action of the valve in the steam line.

Since Watt's time, and especially now, feedback has become an important and sophisticated part of the structure of modern devices, ranging from the oscillators in tiny radio sets to the automatic pilots of our largest jet airliners.

A METHOD FOR CALCULATING THE PRESSURE OF THE ATMOSPHERE

Assume any cross-sectional area A for a column of height h and liquid density ρ. The volume of the column of height h will be Ah, the mass will be $\rho V = \rho Ah$, and the weight will be $ma_g = \rho Aha_g$. The pressure on the bottom needed to support the column will be the weight divided by the area:

$$P = \frac{\rho Aha_g}{A} = \rho ha_g$$

Using mks units, $\rho = 1.4 \times 10^4 \text{kg/m}^3$, $h = 0.76$ m, and $a_g = 9.8 \text{ m/sec}^2$, so

$$P \cong 10^5 \frac{\text{kg /m}}{\text{sec}^2} \cdot \frac{1}{\text{m}^2} = 10^5 \text{ N/m}^2.$$

You may prefer to find numerical values for each step instead of waiting until the last step.

The detailed history of Boyle's law is not important in itself. It takes on importance in this chapter because it can be used to test models of gases. It should be made very clear that PV = constant is an empirical rule, summarizing the data of many experiments.

You might want to give students a rough idea of the range of pressures it is possible to obtain in earthly laboratories: from 10^{-11} atmosphere or even less in the best vacuum pumps to 10^7 atmospheres or more in special high-pressure apparatuses. The initial pressure produced by a hydrogen bomb is on the order of several billion atmospheres.

The Newtonian mechanics of collisions can be used to derive the relation between P and V for a collection of perfectly elastic, vanishingly small particles. The product of pressure and volume is proportional to the total kinetic energy of the particles. This idealized model does not appear to match real gases, however, because Newtonian mechanics shows also that the kinetic energy, and therefore the product PV, changes when the volume is changed.

It is essential that students distinguish between the *empirical* relation, PV = constant (Boyle's law), and the *hypothetical* relation derived from a simplified kinetic model, $PV = \frac{2}{3}N(KE)$. The latter appears not to agree with the former, because a very simple analysis of moving a piston to compress a sample of gas shows that *KE* does not remain constant.

We can claim that the two agree only if we can show why *KE* should remain constant. The problem is resolved when the "constant *KE*" of the hypothetical relation is claimed to be equivalent to the "constant temperature" of the empirical relation. This solution might be spotted by students, many of whom have long been exposed to the identification of molecular motion and temperature, especially if the conditions are emphasized in your presentation: "PV = constant if the temperature stays constant; $PV = \frac{2}{3}N(KE)$ if the *KE* of the particles stays constant."

The statements about absolute zero are correct but they fail to make explicit two qualifications that you may wish to present briefly to students. The first is sometimes called the third law of thermodynamics: It is impossible for any process to reduce the temperature of a system to absolute zero in a finite number of steps. The approach to absolute zero becomes progressively more difficult as the temperature nears zero, so that each successive step becomes smaller. Classically then, absolute zero may be approached as closely as desired, but can never be reached exactly. Another qualification, the result of quantum mechanics, requires that a system have a finite least energy, the "zero point energy"; thus, even at a theoretical temperature of absolute zero, the particles of the system would not have zero kinetic energy.

Brief Description of Learning Materials

SUMMARY LIST OF UNIT 3 MATERIALS

Experiments

E3-1 Collisions in One Dimension. I
E3-2 Collisions in One Dimension. II
E3-3 Collisions in Two Dimensions. I
E3-4 Collisions in Two Dimensions. II
E3-5 Conservation of Energy. I
E3-6 Conservation of Energy. II
E3-7 Measuring the Speed of a Bullet
E3-8 Energy Analysis of a Pendulum Swing
E3-9 Least Energy
E3-10 Temperature and Thermometers
E3-11 Calorimetry
E3-12 Ice Calorimetry
E3-13 Monte Carlo Experiment on Molecular Collisions
E3-14 Behavior of Gases
E3-15 Wave Properties
E3-16 Waves in a Ripple Tank
E3-17 Measuring Wavelength
E3-18 Sound
E3-19 Ultrasound

Demonstrations

D33 An inelastic collision
D34 Predicting the range of a slingshot
D35 Diffusion of gases
D36 Brownian motion
D37 Wave propagation
D38 Energy transport
D39 Superposition
D40 Reflection
D41 Wave trains
D42 Refraction
D43 Interference patterns
D44 Diffraction
D45 Standing waves
D46 Two turntable oscillators (beats)

Film Loops

L18 One-Dimensional Collisions. I
L19 One-Dimensional Collisions. II
L20 Inelastic One-Dimensional Collision
L21 Two-Dimensional Collisions. I
L22 Two-Dimensional Collisions. II
L23 Inelastic Two-Dimensional Collisions
L24 Scattering of a Cluster of Objects
L25 Explosion of a Cluster of Objects
L26 Finding the Speed of a Rifle Bullet. I
L27 Finding the Speed of a Rifle Bullet. II
L28 Recoil
L29 Colliding Freight Cars
L30 Dynamics of a Billiard Ball
L31 A Method of Measuring Energy: Nails Driven into Wood
L32 Gravitational Potential Energy
L33 Kinetic Energy
L34 Conservation of Energy: Pole Vault
L35 Conservation of Energy: Aircraft Takeoff
L36 Reversibility of Time
L37 Superposition
L38 Standing Waves on a String
L39 Standing Waves in a Gas
L40 Vibrations of a Wire
L41 Vibrations of a Rubber Hose
L42 Vibrations of a Drum
L43 Vibrations of a Metal Plate

Reader Articles

R1 *Silence Please*
by Arthur C. Clarke
R2 *The Steam Engine Comes of Age*
by R. J. Forbes and E. J. Dijksterhuis
R3 *The Great Conservation Principles*
by Richard Feynman
R4 *The Barometer Story*
by Alexander Calandra
R5 *The Great Molecular Theory of Gases*
by Eric M. Rogers
R6 *Entropy and the Second Law of Thermodynamics*
by Kenneth W. Ford
R7 *The Law of Disorder*
by George Gamow
R8 *The Law*
by Robert M. Coates
R9 *The Arrow of Time*
by Jacob Bronowski
R10 *James Clerk Maxwell*
by James R. Newman
R11 *Frontiers of Physics Today: Acoustics*
by Leo L. Beranek
R12 *Randomness and the Twentieth Century*
by Alfred M. Bork
R13 *Waves*
by Richard Stevenson and R. B. Moore
R14 *What is a Wave?*
by Albert Einstein and Leopold Infeld
R15 *Musical Instruments and Scales*
by Harvey E. White
R16 *Founding a Family of Fiddles*
by Carleen M. Hutchins
R17 *The Seven Images of Science*
by Gerald Holton
R18 *Scientific Cranks*
by Martin Gardner

Sound Films (16mm)

F17 Elements, Compounds, and Mixtures
F18 The Perfection of Matter
F19 Elastic Collisions and Stored Energy
F20 Energy and Work

Transparencies
T19 One-Dimensional Collisions
T20 Equal Mass Two-Dimensional Collisions
T21 Unequal Mass Two-Dimensional Collisions
T22 Inelastic Two-Dimensional Collisons
T23 Slow Collisions
T24 The Watt Engine
T25 Superposition
T26 Square Wave Analysis
T27 Standing Waves
T28 Two-Slit Interference
T29 Interference Pattern Analysis

FILM LOOPS

Quantitative measurements can be made with film loops marked (Lab), but these loops can also be used qualitatively.

L18 ONE-DIMENSIONAL COLLISIONS. I
Slow-motion photography of elastic one-dimensional collisions. (Lab)

L18 ONE-DIMENSIONAL COLLISIONS. II
A continuation of the preceding loop. (Lab)

L20 INELASTIC ONE-DIMENSIONAL COLLISIONS
Slow-motion photography of inelastic one-dimensional collisions. (Lab)

L21 TWO-DIMENSIONAL COLLISIONS. I
Slow-motion photography of elastic collisions in which components of momentum along each axis can be measured. (Lab)

L22 TWO-DIMENSIONAL COLLISIONS. II
A continuation of the preceding loop. (Lab)

L23 INELASTIC TWO-DIMENSIONAL COLLISIONS
A continuation of the preceding two loops; plasticene is wrapped around one ball. (Lab)

L24 SCATTERING OF A CLUSTER OF OBJECTS
In slow-motion photography, a moving ball collides with a stationary cluster of six balls of various masses. Momentum is conserved. (Lab)

L25 EXPLOSION OF A CLUSTER OF OBJECTS
A powder charge is exploded at the center of a cluster of five balls of various masses. One ball is temporarily hidden in the smoke. The position and velocity of its emergence can be predicted using the law of conservation of momentum. (Lab)

L26 FINDING THE SPEED OF A RIFLE BULLET. I
A bullet is fired into a block of wood suspended by strings. The speed of the block is measured directly by timing its motion in slow-motion photography. (Lab)

L27 FINDING THE SPEED OF A RIFLE BULLET. II
A bullet is fired into a block of wood suspended by strings. The speed of the block is found by measuring its vertical rise. (Lab)

L28 RECOIL
A bullet is fired from a model gun. Direct measurements can be made of the bullet's speed and the speed of recoil of the gun. (Lab)

L29 COLLIDING FREIGHT CARS
The collision of two freight cars is photographed in slow motion during a railroad test of the strength of couplings. (Lab)

L30 DYNAMICS OF A BILLIARD BALL
Slow-motion photography of a rolling ball striking a stationary ball. The target ball slides, then starts to roll. Linear momentum and angular momentum are conserved. (Lab)

L31 A METHOD OF MEASURING ENERGY: NAILS DRIVEN INTO WOOD
A nail is driven into wood by repeated identical blows of a falling weight. A graph of penetration depth versus number of blows can be made; the result is nearly a straight line. This loop establishes a criterion for energy measurement used in the next two loops. (Lab)

L32 GRAVITATIONAL POTENTIAL ENERGY
Dependence of gravitational potential energy on weight; dependence on height. (Lab)

L33 KINETIC ENERGY
Dependence of kinetic energy on speed; dependence on mass. Slow-motion photography allows direct measurement of speed. (Lab)

L34 CONSERVATION OF ENERGY: POLE VAULT
The total energy of a pole vaulter can be measured at three times. Just before takeoff, the energy is kinetic; during the rise, it is partly kinetic, partly gravitational potential, and partly elastic energy of

the distorted pole; at the top, it is gravitational potential energy. (Lab)

L35 CONSERVATION OF ENERGY: AIRCRAFT TAKEOFF

Flying with constant power, an aircraft moves horizontally at ground level, rises, and levels off. Kinetic and potential energy can be measured at three levels. (Lab)

L36 REVERSIBILITY OF TIME

After some introductory shots of real-life actions that may or may not be reversible, the film shows events of increasing complexity: a two-ball collision on a billiard table; a four-ball event. Finally, a ball rolls to a stop while making some 10^{23} (invisible) collisions with the molecules of the table surface.

L37 SUPERPOSITION

Amplitudes and wavelengths of two waves are varied; the resultant is shown. Display is in three colors on the face of a cathode-ray tube.

L38 STANDING WAVES ON A STRING

Production of standing waves by interference of oppositely moving equal waves is shown in animation. Then a tuning fork sets a string into vibration and several modes are shown as the tension is adjusted. The string's motion is also shown stroboscopically.

L39 STANDING WAVES IN A GAS

A loudspeaker excites standing waves in a glass tube containing air. Nodes and antinodes are made visible in two ways: by the motion of cork dust, and by the cooling of a hot wire inside the tube.

L40 VIBRATIONS OF A WIRE

A horizontal stiff wire is set into vibration. The driving force is supplied by the interaction of alternating current through the wire and a fixed magnetic field. Several modes of vibration are shown, both for a straight wire (antinode at free end) and for a circular wire (nodes equally spaced around the circumference). The patterns are shown in real time and also stroboscopically.

L41 VIBRATIONS OF A RUBBER HOSE

A long vertical rubber hose is agitated by a variable-speed motor at one end. The frequency is adjusted to show a succession of nodal patterns.

L42 VIBRATIONS OF A DRUM

A loudspeaker is placed beneath a horizontal circular rubber drum head. Several symmetric and antisymmetric modes are shown stroboscopically, in apparent slow motion.

L43 VIBRATIONS OF A METAL PLATE

Vibration patterns are made visible by sprinkling white sand on a vibrating plate; sand collects at the nodal lines.

Note: A fuller discussion of each *Film Loop* and suggestions for its use will be found in the section of the *Resource Book* entitled "Film Loop Notes

SOUND FILMS (16mm)

F17 ELEMENTS, COMPOUNDS, AND MIXTURES

Color, 33 min, Modern Learning Aids. A discussion of the difference between elements, compounds, and mixtures, showing how a mixture can be separated by physical means. Demonstrates how a compound can be made and then taken apart by chemical methods, with identification of components by means of their physical properties, such as melting point, boiling point, solubility, color, etc.

F18 THE PERFECTION OF MATTER

Color, 25 min, Nuffield Foundation. This film is mainly for atmosphere. A cameo treatment of medieval culture and science, principally alchemy. Some explicit discussion of closed systems and conservation of mass.

F19 ELASTIC COLLISIONS AND STORED ENERGY

B & W, 27 min, Modern Learning Aids. Various collisions between two dry-ice pucks are demonstrated. Cylindrical magnets are mounted on the pucks producing a repelling force. Careful measurements of the kinetic energy of the pucks during an interaction lead to the concept of stored or potential energy.

F20 ENERGY AND WORK

B & W, 28 min, Modern Learning Aids. Shows that work, measured as the area under the force–distance curve, does measure the transfer of kinetic energy to a body, calculated from its mass and speed. Several different methods establish work as a useful measure of energy transfer.

TRANSPARENCIES

T19 ONE-DIMENSIONAL COLLISION

Facsimiles of stroboscopic photographs of two events involving two-body collisions in one dimension are provided. Measurements may be made directly from the transparency to establish the principle of conservation of momentum

T20 EQUAL MASS TWO-DIMENSIONAL COLLISIONS

A stroboscopic facsimile of an elastic collision between spheres of equal mass is shown. Overlays show accurately drawn momentum vectors before and after collision, illustrating conservation of momentum.

T21 UNEQUAL MASS TWO-DIMENSIONAL COLLISIONS

A stroboscopic facsimile of an elastic collision between spheres of unequal mass, both of which are moving before collision. Overlays show accurately drawn momentum vectors before and after collision, illustrating conservation of momentum.

T22 INELASTIC TWO-DIMENSIONAL COLLISIONS

A stroboscopic facsimile of an inelastic collision between two plasticene-covered spheres of equal mass, both of which are moving before collision. Overlay shows accurately drawn momentum vectors before and after collision, illustrating conservation of momentum.

T23 SLOW COLLISIONS

A stroboscopic facsimile shows a collision between two dynamics carts equipped with spring bumpers and light sources. Analysis of momentum and kinetic energy before, during, and after the collision may be made directly from the transparency.

T24 THE WATT ENGINE

Overlays depict a schematic diagram of the Watt external condenser engine during the steam-expansion and condensation phases of operation.

T25 SUPERPOSITION

Shows two pulses crossing at four instants of time, first with both pulses above the equilibrium line, then with one on either side of the line. The superposed wave for each case is also shown.

T26 SQUARE WAVE ANALYSIS

Shows how first four Fourier terms add to begin to produce a square wave. May be used for variety of superposition problems.

T27 STANDING WAVES

A set of sliding waves permits a detailed step-by-step analysis of a standing wave pattern.

T28 TWO-SLIT INTERFERENCE

The first two overlays are concentric circles drawn from two sources. The third overlay suggests the resulting constructive and destructive interference positions.

T29 INTERFERENCE PATTERN ANALYSIS

Overlays illustrate crests and troughs for two independent sources. Other overlays show nodal and antinodal lines and geometry for deriving the wavelength equation.

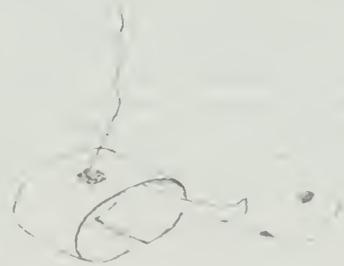
Demonstration Notes

D33 AN INELASTIC COLLISION

A perfectly elastic collision is one in which the total amount of kinetic energy present is the same before and after the collision. An inelastic collision, then, must be one in which kinetic energy is lost. What happens to it? You can show qualitatively by a very simple demonstration that a loss in kinetic energy is associated with a rise in temperature of the interacting bodies. Pound a nail into a piece of wood, and have students touch the nail. Remind students that when they analyzed a slow elastic collision they found that the total kinetic energy of the dynamics carts decreased temporarily, then went back to near its original value. Ask what would happen if the bumpers on the carts had instead been made from a soft metal, such as lead. Then demonstrate such a collision with the apparatus described below.

Bend the lead strip into a ring and tape it to the end of a dynamics cart.

Do this several minutes before you want to perform the demonstration so that the lead will have time to recover from the heating caused by handling and bending it.



If two carts, each loaded so that its mass is 2 kg, approach each other with speeds of 1 m/sec, their total kinetic energy is:

$$2 \times \tfrac{1}{2} \times 2\ \text{kg} \times (1\ \text{m/sec})^2 = 2\ \text{J}$$

If this energy is converted entirely into heat, it amounts to about 0.5 cal. The temperature rise in a 50-g strip of lead with a heat capacity of 0.03 cal/g C° will be

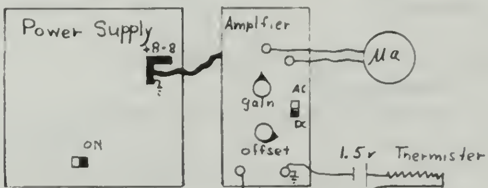
$$\Delta t = \frac{H}{mc} = \frac{0.5\ \text{cal}}{50\ \text{g} \times 0.03} = 0.3\ \text{C}^\circ$$

Although students are not ready for the quantitative treatment given here, the calculations are shown so that you will appreciate the difficulties involved in making the experiment quantitative.

A practical experiment to show the conversion of kinetic energy into heat in a collision requires a very sensitive thermometer; the rise in temperature will be only a few tenths of a degree.

A sensitive thermometer with very low heat capacity is made from a thermistor (a pellet of semiconductor material whose resistance drops markedly with increase in temperature) and an amplifier. A thermistor, already embedded in a thin strip, is supplied by Damon Educational. An identical thermistor, not embedded, is provided for other demonstrations and activities.

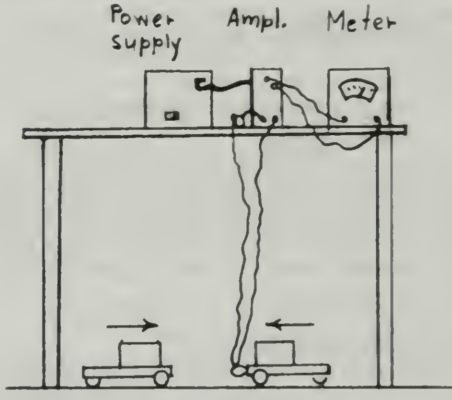
Briefly, an increase in temperature of the thermistor increases the input current to the amplifier, which increases the output current many times more. Small changes in the large output current (and, therefore, small changes in temperature) can be detected by blocking out most of the output current with the OUTPUT OFFSET control and connecting the output to a sensitive meter.



Connect the thermistor between a 1.5-V cell and the input terminals of the amplifier. Turn the ac/dc switch to dc. Set the OUTPUT OFFSET control to its maximum (full counterclockwise) and the gain to about half-way up. Connect a meter across the output of the amplifier. It is most convenient to use a multimeter with several voltage ranges. Start on the least sensitive (highest voltage) dc voltage scale. The reading should be near zero; if it is not, adjust the gain control until it is. Change to progressively more sensitive scales, using the gain control as necessary to keep the reading on scale.

Finally, with the meter on a scale of not more than 250 mV or 100 μA full scale deflection, adjust the gain or OUTPUT OFFSET so that the reading is approximately zero. Final adjustment is very sensitive, and it is not necessary to set to zero exactly. If you do not have such a sensitive scale on a multimeter, substitute an independent millivoltmeter or microammeter, but always use the coarser scales of the multimeter first, so that the sensitive meter will not be overloaded. Use a projection meter if you can.

With the meter reading approximately zero, increase the temperature of the thermistor by bringing your finger, a hot iron, or a match close to it (but not actually touching). There should be an appreciable increase in meter reading. Even blowing on the thermistor may send the reading off scale. Use one of these techniques to show that the system responds to temperature changes and that an increase in temperature increases the meter reading. Arrange the apparatus so that the wires from the amplifier–power supply unit to the cart are hanging freely. Wait for the meter reading to reach a steady value before doing the collision demonstration. Push the carts together so that the lead ring absorbs the kinetic energy impact. The meter deflection shows the increase in temperature of the lead. If you plan to calibrate the thermistor for quantitative work later, record the meter readings from this demonstration for future reference.



In a trial, we found that when a cart moving at about 1 m/sec collided with a stationary object (the wall), the temperature rise in the lead caused a change of 50 mV in meter reading on a 250-mV meter, or 30 μA on a 100-μA meter.

Suggestion for Quiz or Class Discussion

In order to show that the meter deflection is not simply due to mechanical shock (as one might suspect with such a sensitive instrument), you can try putting the lead strip on a brass block and hitting the block with a hammer. In this case, there will be no temperature rise.

D34 PREDICTING THE RANGE OF A SLINGSHOT

In this experiment, the impact point of a slingshot projectile is predicted from the drawing force and distance. The objectives are to provide (1) an exercise in energy conservation that both can (and will) engage the students' intuition, and (2) an experience of successfully predicting from the dry machinery of theoretical mechanics an event that is interesting to the students. The derivation of the expression for range requires some analysis not treated in the text: resolution of vectors and the work done by a varying force.

Procedure

A satisfactory sling can be made from a large rubber band. To insure a reasonably small error in measuring the length of draw, the rubber band should allow a draw of at least 20 cm without overstraining. The support can be almost anything that is sufficiently rigid; a pair of ring stands clamped to the table will suffice. The rubber band should be attached to the supports in a manner that will allow a minimum of friction during draw and release. (Alternatively, we have found that a toy shop model slingshot firing a 2-cm steel ball gives very satisfactory results.)

An excellent projectile can be made by twice folding a 2-cm × 12-cm piece of 0.2-cm thick lead sheet. Its mass should be about 50 g, great enough so that only a negligible fraction of the kinetic energy will appear in the rubber band upon release, but small enough to give an impressively long range. Be careful to fold the lead in such a way that it will not catch on the rubber band when it is released. The margin of error is adequately represented (and the drama is increased) by placing a wastebasket at the expected impact point. (See Fig. 1.)

Fig. 1

The draw and release can be made satisfactorily with the thumb and forefinger on the edges of the projectile, but some skill is required to avoid frictional losses on release. A thread-burning technique such as that suggested by Fig. 2 may prove to be better. The drawing force and distance are measured with a spring balance and meter stick, as indicated in Fig. 2. (The meter stick should be removed for launching.)

Fig. 2

Derivation of Range R:

$$R = v_{\rightarrow} t \qquad (1)$$

$$t = 2t_{\uparrow} \qquad (2)$$

$$R = 2v_{\rightarrow} t_{\uparrow}$$

$$t_{\uparrow} = \frac{v_{\uparrow}}{a_g} \qquad (3)$$

$$R = \frac{2v_{\rightarrow} v_{\uparrow}}{a_g}$$

$$v_{\rightarrow} = v_{\uparrow} = \frac{v}{\sqrt{2}} \qquad (4)$$

(for 45° launch angle*)

$$R = \frac{v^2}{a_g}$$

$$E_{K_{max}} = \tfrac{1}{2}mv^2 \qquad (5)$$

$$R = \frac{2E_{K_{max}}}{ma_g}$$

$$E_{K_{max}} = E_{p_{max}} \qquad (6)$$

$$= \bar{F}d_{max}$$

$$R = \frac{2\bar{F}d_{max}}{ma_g}$$

$$\bar{F} = \tfrac{1}{2}F_{max} \qquad (7)$$

(for linear force)

$$R = \frac{F_{max}d_{max}}{ma_g}$$

The derivation is lengthy; but, even so, if you know each input principle and your math is correct, you should have confidence in your prediction.

With the exception of equations (4) and (5), these relations all represent idealizations. As it happens, all the actual deviations from the ideal are in a direction that reduces the actual range. Only the step represented by equation (7) allows the deviation from the ideal to be accounted for; students can plot F against d and find $E_{p_{max}}$ (or $\bar{F}$) from the area under the somewhat nonlinear curve. But remember that the work done by a varying force is not considered in the *Text*. Probably the best way to treat a nonlinear force–extension curve is to replace it by an equivalent bar graph. (See Fig. 3.)

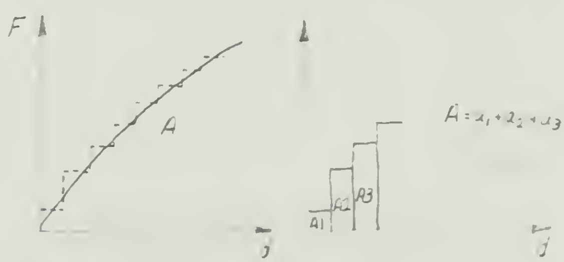


Fig. 3

*Vector resolution is not treated in the text, but should be familiar from problems.

The most appropriate approach to the derivation for a particular student must be determined by the teacher. Below are listed some approaches to suggest the broad range of possibilities. Any one approach would not have to be used for a whole class. For example, approach "C" might be planned for most of the class while several faster students could begin as in the approach "A." In any case, the rationale for bothering with prediction instead of just trying the experiment should be clear in this era of probing space with extremely expensive machinery.

Possible Approaches

A. Students are requested to derive an expression for the range of a projectile as a homework assignment, perhaps over a week or longer. The teacher is available to discuss problems.

B. As in A, but the problem is given better initial direction by a class discussion of "What would you have to know in order to figure out how far it will go?"

C. A discussion is begun as in B, but it is pursued through the entire derivation, with suggestions for steps coming almost always from the students. ("You say you want to know the initial speed. How could you find it?")

D. As in C, but with the teacher providing the structure by means of a concrete leading question. ("We can find the speed if we know the kinetic energy. What is the relation of v and KE?"

E. Lecture presentation of a derivation.

The more that students are able to come up with on their own, the more valuable the experiment is likely to be for them. On the other hand, painfully heavy demands on the students will spoil the effect of the demonstration and the time involved must always be weighed. In any approach, it should eventually be pointed out that the procedure is one very common to science. (See Fig. 4.)

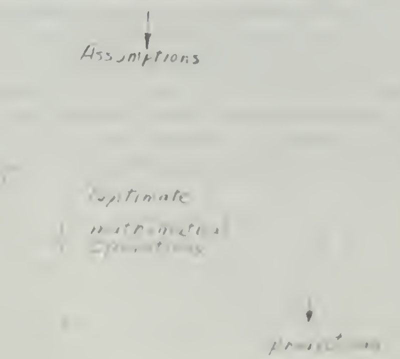


Fig. 4

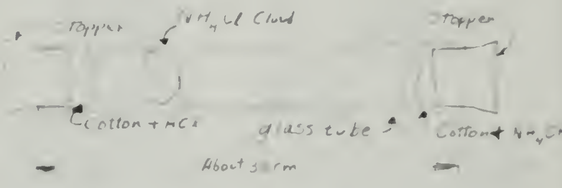
An interesting supplementary discussion might center on the question, "What do you do when the actual event doesn't match the prediction?" Any discussion of the uncertainty in the predicted location of the impact point should precede the actual launching. In addition to uncertainties in measurement, the effects of the idealizations should be considered.

The amount of derivation required of the students may vary a great deal: whatever treatment of the experiment is used, the intent is that the experiment be predictive and interesting. To this end, it is important that the actual launching be the very last step and that the anticipated range be as long as is practicable.

(Courageous teachers who choose approaches D or E might want to try a dramatic tack wherein an erroneous step is surreptitiously included in the derivation or calculation, causing the projectile to overshoot the target by a factor of two or more, perhaps proceeding out a door or window. The immediate drop in the stock of science must, of course, be quickly recouped, ideally by the students discovering the error themselves.)

D35 DIFFUSION OF GASES

The formation of a cloud of NH_4Cl vividly demonstrates the diffusion of HCl and NH_3 molecules through the air.



The demonstration also shows:

(a) The lighter NH_3 molecules diffuse more quickly than the heavier HCl molecules: The cloud of NH_4Cl is formed farther from the NH_3 source.

(b) Diffusion is comparatively slow at atmospheric pressure.

D36 BROWNIAN MOTION

The molecular-kinetic theory of matter developed in Chapter 11 as a model for a gas is consistent with many experimental observations (gas laws, specific heat, etc.). Brownian motion is the most direct evidence we have been able to present so far for the molecular-kinetic theory of matter. The phenomenon is called Brownian because the botanist Robert Brown, although not the first to observe it, showed (circa 1827) that it was found for a wide variety of particles, both organic and inorganic.

Historical Significance of Brownian Motion

Brown himself had no theory to account for the motion. He found that it existed for all kinds of inorganic particles, and various suggestions including the irregular molecular bombardment of the particle were made to account for it. (In fact, Brown used the persistence of the phenomena in isolated drops to "prove" that it could not be due to molecular activity.) But until work by Einstein and Smoluchowski and by Perrin early in the twentieth century, several eminent scientists still disputed the existence of atoms.

Einstein and Smoluchowski, using the mathematics of probability, were able to make quantitative predictions about the observed motion of the particles that were closely confirmed by Perrin's measurement. One of the achievements of the theory was the first accurate determination of Avogadro's number.

A NOTE CONCERNING DEMONSTRATIONS AND EXPERIMENTS IN CHAPTER 12

Chapter 12, perhaps more than any other chapter in the unit, depends upon demonstrations and experiments to give substance to the material discussed in the *Text*. If you examine the chart that follows, you will notice that the wave properties are carried entirely by either the experiments <u>or</u> the demonstrations. If you have the time and wish to do *Experiments E3–15* through *E3–19*, it is recommended that you only do demonstration *D41*. If only *E3–15* is done by the students, then *D38*, *D41*, and *D42* are recommended. If only *E3–18* and *E3–19* are carried out, *D37*, *D39*, *D40*, and *D41* should be considered. If no experiments are planned, the demonstrations can suffice. In any case, *D46* should be included.

The chart below lists various properties of waves that are demonstrated by the experiments and demonstrations for Chapter 12 and the types of equipment that may be used.

Equipment Key:

S—Slinky
B—Bell Wave Machine
T—Turntable Oscillator
R—Ripple Tank
P—*Project Physics* Equipment

Wave Property	E3-15 E3-16 E3-17	E3-18 E3-19	D37	D38	D39	D40	D41	D42	D43	D44	D45	D46
Pulse amplitude and length	S,R		S,B									
Pulse velocity	R		S,B,R									
Traveling pulse as energy transport				S,B,R								
Absorption at a barrier		P		B,R,P								
Superposition	S				S,B							T
Reflection from a free end	S					S,B						
Reflection from a fixed end (phase inversion)	S					S,B						
Partial reflection at an interface where $v_1 = v_2$	S,R					S,B,R						
Two-dimensional reflection from barrier	R	P				R,P						
Transmission (impedance, or index match)		P				S						
Wave trains (reflection, refraction, etc.)							B,R,P					
Refraction as a function of angle	R							R,P				
Refraction as a function of velocity	R							R,P				
Refraction through wave shaping elements (lenses)	R	P						R,P				
Interference patterns	R								R,P			
Young's experiment		R							R			
Diffraction around obstacles (frequency dependence)	R	P								R,P		
Single-, double-, and multiple-slit diffraction patterns	R									R,P		
Standing waves	R	P									S,B,R,P	
Longitudinal and transverse waves	S		S								S,B,R,P	

D37 WAVE PROPAGATION

Slinky

The slinky is employed to demonstrate wave propagation because pulses propagate slowly enough along it to be easily observed. It can be used either by partners or suspended horizontally by strings attached to a horizontal wire above it. If used by partners, pull the slinky out to a length of about 10 m on a smooth floor. Never let go of an end of the spring when it is stretched because the resulting snarl is almost impossible to untangle. The snarling problem is eliminated when the slinky is suspended properly. The strings suspending the slinky should be at least 1 m long, and spaced several centimeters apart along the entire length of the spring. One end of the spring may be tied to some support. Stretch the slinky slightly by fastening a light, 2-m string to the other end.

Pulse amplitude and length can be demonstrated by sending different-sized pulses along the spring. The amplitude of a pulse may be defined as the maximum displacement of any point on the spring. Ask questions such as: In what direction do the spring coils actually move as the wave motion travels along the spring? Does the shape of the wave change as it travels along the spring? What determines the length of the pulse? Is there a relation between the length of the pulse and its amplitude? Pulse velocity can be determined by measuring the time it takes the pulse to travel back and forth several times over a measured distance. How is the velocity affected by changing the amplitude? the pulse duration? the tension? What determines the pulse velocity?

Transverse waves can be demonstrated by grasping one end of the spring and snapping it rapidly at right angles to the length of the spring. Longitudinal waves are brought about by displacing the end of the spring in a direction parallel to its length. A logical question to pose is: As the wave travels along the spring, in what direction do the spring coils actually move?

Bell Wave Machine

Exp. 1, "Getting Acquainted with Waves," and Exp. 4, "Wave Speed," in the book *Similarities in Wave Behavior*. This book accompanies the Wave Machine, whether you rent or buy it.

Ripple Tank

Straight waves can be generated by placing a 2-cm dowel or section of broomstick handle along one edge of the tank, rolling it backward 1 or 2 cm, and then stopping, or by employing an electric rippler supplied by most scientific apparatus houses. Using a hand (or electronic) stroboscope, determine the speed of the waves and verify the relation $v = f\lambda$. Stress the fact that this relationship is applicable to waves both in the ripple tank and on a coil spring (slinky). Define the amplitude as the maximum displacement of any point on the surface. Make several measurements of frequency and wavelength when you determine the wave speed. How is the frequency of your strobe related to the frequency of the waves? What can be said about the accuracy of your determination of the wave speed?

D38 ENERGY TRANSPORT

Slinky

Energy transport is concerned with what happens to the amplitude of a traveling wave as the wave propagates. Under the assumption that it takes "work" to deform a medium, we associate the energy a wave possesses with its amplitude and ask such questions as: Does the "shape" of the wave change as it travels along the spring? Does the "size," or amplitude of the wave change? Why does it change? What determines the amplitude of the pulse? The progressive loss in amplitude as the pulse travels along the spring is called *damping*. What happens to the energy lost?

Bell Wave Machine

See Exp. 2, "Wave Damping," and Exp. 3 "Waves as Carriers of Energy," in the book *Similarities in Wave Behavior*, which accompanies the Wave Machine. Absorption at a barrier can be demonstrated in Exp. 10 where a dash-pot-and-piston arrangement is employed for a mechanical load.

Ripple Tank

Generate straight-fronted and circular-fronted waves. Does the amplitude of the straight-fronted wave change as the wave travels? If so, how? Why? Is any energy lost? Energy absorption at a barrier can be shown by allowing waves to strike the gauze fences and observing what happens to the amplitude. A discussion of reflection is not warranted at this time.

***Project Physics* Equipment**

The absorption of audible sound, ultrasound and microwave "waves" can be demonstrated with an assortment of materials, such as pieces of metal wood, glass, styrofoam, paraffin, masonite, etc The fact that energy is transmitted can be associated with the effect these waves have on various receivers.

D39 SUPERPOSITION

Slinky

The demonstration setup should be the same as for *D37*.

Superposition can be demonstrated by generating two simultaneous pulses, one from each end of the slinky.

Ask such questions as: What happens to the pulses as they collide? When the pulses meet how does the resulting amplitude compare with the amplitude of each individual pulse when the pulses are on the same side of the slinky? on opposite sides?

Bell Wave Machine

See Exp 5, "Criss-crossing of Waves" in *Similarities in Wave Behavior*.

D40 REFLECTION

Slinky

Reflection from a fixed end (phase inversion) is demonstrated by observing the reflected pulse when one end of the slinky is held rigidly in place (infinite impedance). The other case, reflection from a free end (zero impedance) can be observed by having the end of the spring connected only to a long thin thread. Observe these two cases to see whether the displacement of the reflected pulse is on the same side or on the opposite side of the spring from the incoming pulse. Partial reflection from an interface where $v_1 = v_2$ is then investigated by tying together two coil springs on which waves travel with different speeds. Send a pulse first in one direction and then in the other, asking what happens when the pulses reach the junction between the two springs. By employing different springs, and thus different media, and observing the amplitude of the transmitted and reflected waves, one can qualitatively demonstrate impedance, or index-match, in terms of the media velocities.

Bell Wave Machine

Reflection from free or fixed ends is demonstrated in Exp. 6, while partial reflection at an interface is contained in Exp. 11 of *Similarities in Wave Behavior*, which accompanies the Wave Machine.

Ripple Tank

The speed of water waves depends on the depth of the water. Two different depths of water therefore constitute two different media in which waves can be propagated. This situation can be brought about by a glass plate supported in the ripple tank. Ask what will happen if straight waves, generated in deep water, cross the boundary between the two media (water depths) involved. Here we are primarily interested in the "reflected" wave. The transmitted wave and its refraction is presented in *D42*. Two-dimensional reflection from an "opaque" barrier is always shown using paraffin blocks. The angle of incidence and angle of reflection can be measured.

Project Physics Equipment

The reflection of sound, ultrasound, and microwaves can be demonstrated by reflecting them from an assortment of materials. From *D38* you will have an idea of what materials provide optimal reflection.

D41 WAVE TRAINS

Demonstrations of reflection and refraction are presented by employing a constant frequency source so as to distinguish between a pulse and a wave train.

The Bell Wave Machine, Ripple Tank, or *Project Physics* equipment may be used.

D42 REFRACTION

Ripple Tank

Refraction in the ripple tank can be observed by laying a sheet of glass in the center of the tank to make a shallow area. To make the refraction quite obvious, the frequency of the wave should be low (less than 10 cycles/sec), and the water over the glass should be as shallow as possible. Use just enough water to cover the glass. The waves refract at the edge of this area because they travel more slowly where the water is shallow. The wave that passes over the plate is the refracted wave; the acute angle between its front and the boundary of the new medium is the angle of refraction, *r*.

Try varying the angle at which the pulse strikes the boundary between deep and shallow water. Measure the angles of incidence and the corresponding angles of refraction over a wide range of values and determine how the angle of refraction varies as a function of the angle of incidence. Determine the velocities of the waves in the deep and shallow parts of the tank. What is the ratio of their velocities? of their wavelengths? Compare with the data from angle measurements. Paraffin lenses can be cut or plastic ripple-tank lenses can be bought to demonstrate the focusing of waves.

Project Physics Equipment

Sound. Fill a "lab gas" balloon with carbon dioxide. (Less dense gases do not work well.) The resulting spherical "lens" will focus sound a few centimeters beyond the balloon. Explore the area near the balloon on the opposite side from the source. Try two or more frequencies.

Ultrasound. At the higher frequency of ultrasound, the gas lens may be too large. Experiment with various materials that are transparent to ultrasound and can be formed into a sphere, or hemisphere. Try the gas-filled balloon with this higher frequency.

Microwaves. From paraffin wax, cast a hemisphere or hemi-cylinder of about 3 cm radius, perhaps in a small frozen juice can. It will act as a short focal-length lens. Observe the area behind the lens with the lens in position and while removed.

D43 INTERFERENCE PATTERNS

Ripple Tank

The interference of waves from two point sources is demonstrated best with the ripple tank, since the student can see the development of the nodal lines as waves progress from the point sources. It is enlightening to observe first a single circular pulse, then two simultaneously generated pulses. Follow the path of the intersection of the two pulses. Next observe two pulses that originated at different points and at slightly different times. The locus of the intersection points is seen to curve away from the sources. Next produce two or three successive pairs of simultaneous pulses and observe their intersections. Finally observe the intersections of continuous waves. By marking the positions of the nodal lines as projected, it is possible to establish quantitatively the wavelength from the double-slit equation. For detailed suggestions for demonstrations consult the PSSC *Laboratory Guide*, Experiments II–8 through II–13; or Lehrman and Swartz, *Laboratory Experiments*, Nos. 36, 37, 39, 40, and 43; or Brinckerhoff and Taft, *Modern Laboratory Experiments in Physics*, Nos. 31 and 35.

***Project Physics* Equipment**

Sound. Connect the two loudspeakers in series to the oscillator and mount them at the edge of the table about 25 cm apart. Observe the signal strength as the ear is moved along a horizontal line in fromt of the sources. Move farther away from the sources and change the source separation to see what happens. Note the effect of changing the frequency.

This demonstration can be made quantitative by mounting a meter stick parallel to the line of the speakers so that the nodes can be located and their positions noted from the stick. Plot the positions (x) of maxima and minima; also record D and d.

Two-Source Interference (quantitative). Again the ripple tank and film loops are best for showing how the interference pattern is produced by two waves. The double sources used in the other experiments (except light) should be placed to minimize reflections from hard surfaces, otherwise spurious nodal points will be present. Set the source transducers at the edge of the tabletop and directed away from nearby walls.

Mount the two sources so that the distance from center to center can be measured, as well as the distance along the perpendicular bisector of the line connecting the sources.

Ultrasound. Plug the second source into the amplifier (the plugs "stack") and arrange the two sources about 5 cm apart. Explore the field with the detector about 25 cm in front of the sources, and plot the maxima and minima positions (x). Also record D and d.

Microwaves. A two-source extension horn is supplied with the generator. Fit the two-source horn into the horn of the generator. It should fit snugly, but if necessary support it with a block of wood or a rubber stopper. Explore the field about 25 cm in front of the two sources and plot the positions of maxima and minima (x). At least three maxima should be picked up on either side of the central antinode.

D44 DIFFRACTION

Ripple Tank

Demonstrate the behavior of pulses and waves at openings in barriers, around obstacles, and edges of barriers (diffraction).

***Project Physics* Equipment**

Diffraction around obstacles and edges

The obstacle must be at least a few wavelengths in size and yet not too large.

Sound

1. Stand on edge a piece of thick plywood or celotex about 25 cm high and at least equally long, with one of the vertical edges placed about 25 cm in front of the source. Explore slowly the area about 75 cm beyond the obstacle, along the x, y, and z axes. Try other obstacles (ear separation distances). Try other frequencies.
2. Similarly, use a piece of wood placed about 25 cm in front of the source, and with one vertical edge aligned with the center of the source. Explore the area in the "shadow zone" and immediately out of the shadow zone. How many "fringes" can be counted?

Ultrasound

1. Use an obstacle of about 3 cm width, placed about 10 cm in front of the source. The detector should be placed 5 to 10 cm beyond the obstacle. Probe on the x, y, and z axes.
2. Use a large screen to explore the edge diffraction pattern.

Microwaves

1. Use an obstacle about 4 or 5 cm wide such as the narrow aluminum screen provided, placed about 10 or 12 cm in front of the source. Explore the field at 5 cm behind the screen and at greater distances. Observe the "maximum" in the center of the shadow.
2. Mask one-half of the source with a large screen placed about 12 cm in front of the source. Explore the intensity of the field as the detector is moved parallel to the screen and about 5 cm behind it. You might use the meter to record and plot intensity as a function of x. You should be able to resolve at least two maxima. If the output is weak, use an amplifer to drive an ac (decibel) meter. Note that, at the first maximum intensity is greater than when there is no screen.

D45 STANDING WAVES

Slinky

The slinky, pulled out and held rigid at one end can show standing waves with one to several nodes.

Ripple Tank

Place a straight barrier across the center of the tank parallel to straight-fronted advancing waves. When the generator speed and barrier position are properly related, "standing waves" will be formed. How does the length of a standing wave appear to compare with the length of a moving wave? Can you measure the wavelength from the standing-wave pattern? Change the depth of the water and ask if a change in speed can be detected.

Bell Wave Machine

See Exp. 8, "Interference and Standing Waves," in *Similarities in Wave Behavior*.

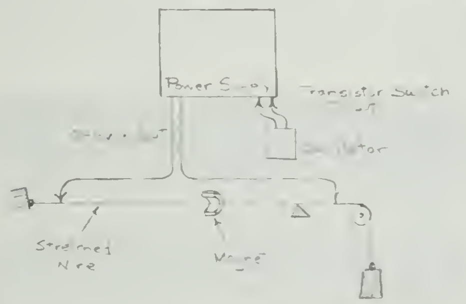
***Project Physics* Equipment**

These demonstrations, which are modified versions of "Melde's Experiment," show that a wire or spring of given length, mass–length ratio, and tension can be made to oscillate at only certain predictable frequencies that depend upon mass, length, and tension. These frequencies are related to each other by integers. This may be used as an introduction to the concept of characteristic (eigen) frequencies, normal modes, and eigenvalues. By analogy, this prepares the student for the concept of quantum numbers.

The demonstration may also be used to show how currents and magnetic fields interact. This concept will be covered in detail in Chapter 14 and should not be mentioned at this time. A current-carrying wire placed in a steady magnetic field has a force acting on it proportional to the current. An alternating current in the wire forces the wire to oscillate. Standing waves can be set up in the wire or spring if the frequency is adjusted to one of the resonant frequencies of the wire or spring.

***Project Physics* Equipment**

Transverse Waves. Standing transverse waves in a wire under tension are produced in the arrangement shown schematically in the figure below. Clamp one end of a copper wire (#18 or #20) 2–5 cm above the tabletop and stretch the wire over a pulley with a weight suspended on the other end.



A variable frequency alternating current of about 3 A is required for this demonstration. This current is provided by the *Project Physics* TRANSISTOR SWITCH, which pulses the current from a 6-V power supply. The switch is driven by an audio oscillator set for square wave output. On the DAMON equipment, connect the oscillator to TRANSISTOR SWITCH INPUT and move the slide switch to the TRANSISTOR SWITCH position. The switched current output is available at the "0→6V" terminals.

Alternatively, a power amplifier, rated at 20W or more, can be used. Add 30–60 cm of #30 nichrome wire in series with the output to provide a load of about 5 ohms.

The magnet should provide as large a field as possible, and a surplus magnetron magnet serves very well. Remember that the plane of oscillation is perpendicular to the field, so the magnetic field must be horizontal if the standing waves are to be in the vertical plane.

Hold the length and tension constant, and vary the frequency. The wire is fixed at both ends, and may vibrate in the fundamental mode and in many harmonic modes. Many of these modes are seen as the frequency is swept through several multiples of the fundamental frequency. The amplitude is smaller at the higher frequencies. Harmonics are easy to obtain if you position the magnet so that it is at an antinode. The frequency f_n of the nth harmonic is given by:

$$f_n = nf_1 \qquad (1)$$

where f_1 is the fundamental frequency. The frequency of the fundamental can be calculated from:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\sigma}} \qquad (2)$$

where L is the length of the wire in meters, T is the tension in newtons, and σ is the mass per unit length in kg/m.

You can do the same experiment without an audio oscillator. Use a current source with fixed frequency (the output of a 6-V 3-A filament transformer) and adjust either the length of the wire or the tension on the wire for maximum-amplitude standing waves. The tension is adjusted simply by changing the weights, and the length is easily changed by inserting a hardwood wedge under the string between the fixed edge and the pulley.

A typical experiment yields the following results: A 2-m length of #24 nichrome wire has a mass of about 3.42×10^{-3} kg, so σ is about 1.71×10^{-3} kg/m. The wire is stretched between its supports, and L is measured as 64.3 cm. The tension is varied until the wire resonates at 60 Hz with a mass of 1.020 kg attached to its free end. Substitute these values into Equation (2):

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\sigma}}$$

and you obtain a frequency of $f_1 = 59.5$ sec. This is within 1% of the expected 60.00 sec value.

Longitudinal Waves. You may also produce longitudinal standing waves in a stretched spring. The spring is mounted between two fixed supports with electrical connections made from each end to a power amplifier, a transformer, or audio switch. A surplus magnetron magnet (U-shape, not C-shape) has a cylindrical iron slug about 3 cm long held magnetically to one pole. Insert the slug a short distance into the end of the coil.

As the frequency of the current is varied, the spring responds at each of its resonant modes. Handwound coil "springs" are adequate for qualitative demonstrations. For quantitative work a brass SHM spring (Cenco #75490 or equivalent) is desirable. The spring maybe mounted horizontally on the stage of an overhead projector.

The harmonic series for a spring (or the series of frequencies at which standing waves appear) is given by Equation (1). The fundamental frequency f_1 is given by the equation:

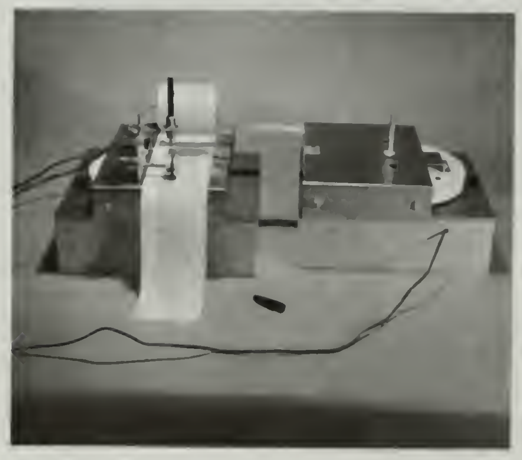
$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (3)$$

where k is the spring constant ($k = \Delta F/\Delta l$) found experimentally by Hooke's law; and where m is the total mass of the spring being subjected to oscillation. It is possible, therefore, to compare a set of predicted values with a set of experimental values.

You will find that if you use a fixed frequency, you cannot "tune" the spring to resonance by changing the tension or by changing the length of the spring by stretching. Once you have found the resonant condition, you can stretch the spring, and it continues oscillating in the same mode. You can only tune the spring by changing the mass of the portion of the spring that is oscillating. To do this, clamp the spring firmly at various points other than the end until resonance is found.

D46 TWO TURNTABLE OSCILLATORS

Beats. Two oscillators are set up so that the motions of the two platforms are parallel. One oscillator has a pen attached to the platform; the other carries a chart recorder positioned so that the paper moves perpendicularly to the oscillations of the platform. The pen writes on the moving paper.



If the first turntable only is switched on, the pen will draw a sine curve on the moving paper. If the second turntable only is switched on, the moving paper will be driven back and forth in simple harmonic motion under the stationary pen, and a sine curve will be drawn. If the two turntables are set to the same speed, the two curves will have the same wavelength. Now, if both turntables are switched on, the trace will be the result of the superposition of two sine curves. Even if both oscillators have been set to nominally the same frequency, there will almost always, in practice, be a detectable difference, which means that the resulting pattern will show beats.

With the turntable set to a given frequency (say 78 rpm), small adjustments in frequency can be achieved by loading down the platform to increase friction between the platform and its support. [Some phonograph motors can be adjusted over a small range by a Variac (Powerstat) in the supply line.] Thus, one can change the beat frequency by adjusting the frequency of one of the component oscillators.*

Approach to Harmonic Synthesis. Two oscillators are set up as before, so that a pen attached to one writes on a chart recorder mounted on the other. If the frequency of one is a multiple of the other, then the resulting trace illustrates in a simple manner the elements of harmonic synthesis. One particulary interesting trace that represents the addition of the first two terms in the Fourier synthesis of a square wave ($a \sin\theta + \frac{1}{3}a \sin 3\theta$) is shown in the figure below. The "coarse" tuning was done by setting the two turntable speeds to 16 and 45 rpm. See the section of this *Resource Book* entitled *Equipment Notes* for a description of the turntable oscillator.

*Incidentally, the discussion and analysis of beats provides a good opportunity to point out to students the low precision of a result that is the difference of two large numbers. Measure the wave number (reciprocal wavelength) of the two component oscillations, together with an estimate of the uncertainty. Calculate the wave number of the resultant beat by taking the difference of the two (nearly equal) component wave numbers. The percentage uncertainty of this result will be very large. On the other hand, the beat wave number can be measured directly with high precision, which demonstrates how sensitive the method of beats is in showing up small differences in frequency.

Experiment Notes

E3–1 COLLISIONS IN ONE DIMENSION. I

Equipment:

Method A

- Dynamics carts with a steel exploder spring for each pair
- 10 × magnifier with scale
- Weights for changing masses of carts
- Either bell-timers with batteries and ticker tape for each cart
- or Polaroid camera and motor strobe with 12-slotted disk
- or xenon strobe lamp

Method B

- Air track and two or three gliders
- Blower for air track
- Polaroid camera and tripod
- Either motor strobe with slotted disk
- or xenon strobe lamp
- White or metal straws or cardboard pointers to be attached to gliders as markers for photographic measurements
- 10 × magnifier with scale

General Discussion

Since the word momentum is never used and the concept of conservation of momentum is never assumed in the *Handbook*, the experiment may, if desired, be treated as a "discovery" lab.

Experiments 3–3 and *3–4* "Collisions in Two Dimensions," may be done concurrently if apparatus and student background permit. The instructions assume a knowledge of conservation of momentum in one dimension.

Several different procedures are described in these experiments. They could be combined into one lab with as many different procedures being followed simultaneously as apparatus permits. After all the working groups have finished, they can bring their findings to a common class discussion.

Let students develop an intuitive feeling for "frictionless" collisions by "playing" with balloon pucks, disk magnets sliding on plastic (Dylite) beads, balls, etc., before beginning the quantitative work with an air track, dynamics carts, or film loops.

Some of the procedures require strobe photography. Remember that the room need not be completely dark although a dark background is important. It is possible to have two groups working on photography in one part of the room, without making it impossible for the rest of the class to work.*

To save time, have the air track ready and the camera in position at the beginning of the period. Set about 1 m from the track, the camera makes an image reduced in size about 10:1. Demonstrate the techniques of simultaneously opening the shutter and launching the glider and then closing the shutter after the interaction.

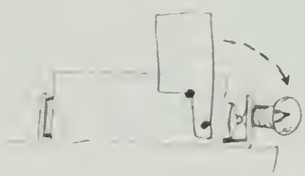
*Note: Techniques of stroboscopic photography are described in Unit 1 of this *Resource Book*.

Some limits must be set on the kinds of interactions to be photographed if the experiment is to be done in one class period. The photographs are less confusing to analyze when all interactions start with one glider stationary in the center of the track and the other launched toward it from the left. Tape the lights to the glider with one light higher than the other (bend up one lamp socket) so that their images can be distinguished on the photograph.

As an alternative to the light sources, mount a white or metallized drinking straw on each glider and use a xenon strobe. Make one straw taller than the other.

If the left-hand glider rebounds, the images will overlap and make measurement difficult. An inventive student could be encouraged to devise a way to distinguish the "before" pictures from the "after" ones. Otherwise, photograph only collisions in which the left-hand glider has a mass equal to or greater than the right-hand one so that the glider on the left does not rebound.

Ways to distinguish the images of the rebounding glider include the following: With a pivot fasten a small piece of colored transparent plastic or partly exposed photographic negative on the glider, so that, when the gliders collide, the plastic will fall in front of the light (see below). The images formed after the collision by this glider will then be fainter.



Alternatively, on the dynamics carts but not on the air-track gliders, the lamps themselves can be mounted vertically like the piece of plastic shown above, so that the entire lamp tips to a horizontal position upon collision.

A third possibility is to have a student drop a filter in front of the camera lens at the instant of collision, thus dimming all subsequent images.

Although students must determine the mass of each cart with the light taped to it, the actual mass in kilograms is not needed. Only the relative mass, expressed as a multiple of the smallest glider's mass, is important.

The notes on photography in Unit 1 suggest other ways of using the photographs.

The collision experiment between carts is possible as well, of course.

If bell timers and ticker tape are used to record the motion of the carts, notice that the tape attached to the left-hand cart passes <u>under</u> the right-hand cart to the timer. The timer lies far enough to the right of the right-hand cart to allow room for the recoil motion of the right-hand cart. Conversely, tape to the right-hand cart passes under the left-hand cart. The frequencies of the two timers may not be the same. Warn students to check this.

When students tabulate their data, have them record speeds to the right as positive and to the left as negative, and remind them about the difference between speed and velocity.

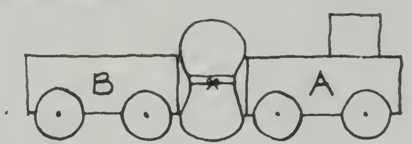
Assemble data from all the photographs in a table to enable students to see the pattern that emerges. With gliders of equal mass, students may conclude from a simple elastic collision that speed is a conserved quantity.

Use a linear explosion to show that the directions in which the carts travel must be considered if momentum is to be conserved.

The tabulated results make it clear to students that mv is the quantity that is conserved. Some students may notice that mv^2 is also conserved for elastic collisions; point out that it is not conserved for inelastic ones (or explosions, since mv^2 is a scalar, not a vector, quantity). Defer further discussion of this point until after *E3–5* is done.

Stroboscopic photographs of one-dimensional collisions are discussed in the Activity section of Unit 3 *Handbook* and later in this section of the *Resource Book*. *Film Loops 18* and *19* also deal with one-dimensional collisions as does *Transparency T19*.

Sample Results



P' = momentum after the explosion
$P' = m_A v_A' + m_B v_B'$
$P' = (2.2\ \text{kg})\ (0.39\ \text{m/sec}) + (1.1\ \text{kg})\ (-0.75\ \text{m/sec})$
$P' = 8.6\ \text{kg m/sec} - 8.2\ \text{kg m/sec}$
$P' \cong 0$
$P = 0$; there was no momentum before the explosion
$P = P'$; momentum was conserved

Answers to questions

1. Speed is not conserved.
2. Velocity is not conserved.
3. Momentum is conserved. The vector sum of the momentum is zero before and after the explosion. (See sample results above.)

E3–2 COLLISIONS IN ONE DIMENSION. II

METHOD A: Film Loop

See notes on *Film Loop 19, Handbook* page 157.

METHOD B: Stroboscopic Photographs

Equipment:
Film Loops 19, 20, and *21*
Technicolor loop projector
Graph paper, masking tape, ruler, stroboscopic photographs (in the *Handbook*)

STROBOSCOPIC PHOTOGRAPHS OF ONE-DIMENSIONAL COLLISIONS: EVENTS 1–7

I. MATERIALS PROVIDED AND THEIR USES

A set of stroboscopic photographs permits detailed quantitative study of seven two-body collisions in one dimension. In the discussion that follows, these will be called, simply, Events 1–7. The events are, primarily, illustrations of the principle of *conservation of momentum*.

The student may be assigned one or more of these events as take-home problems, or as study-period (or laboratory-period) tasks. It may be advantageous to assign a pair of students to a given problem. Prints and student notes are provided and may be kept by the student.

Two overhead *Transparencies, T19,* show the stroboscopic photos of Events 1 and 2. With these the teacher can, in a few minutes, describe the problems qualitatively to the class before distributing the assignment to students.

The *Transparencies* of the two collision events may also be used by the teacher to work out these examples in detail with the whole class, taking measurements directly from the wall (or chalkboard).

The better students may profit from working through more than one event. In fact, the series of events was so chosen that there are certain relations between events. In each pair, interesting discussion questions can be raised.

Events 1 and 2 form a pair because they are inverse events involving the same balls.

Event 3 is a good exercise as long as it is not assigned alone but as a second exercise with Event 1, 2, 4, or 5. This is due to the instructive character of error propagation in Event 3. A large relative error arises when subtracting two nearly equal quantities that are themselves known with small relative errors.

Events 3 and 4 involve the same balls. Yet, in the former only one-half the kinetic energy is conserved, while the latter is almost perfectly elastic.

Events 1 and 5 proceed similarly until the collision takes place. The collision in 1 is elastic (not perfectly elastic), whereas in 5 it is perfectly inelastic.

A calculation of the *kinetic energy* in the system before and after collision is also of interest. It reflects on the elasticity of the colliding balls. This energy is conserved if the collision is "perfectly elastic." None of the examples is a perfectly elastic collision, although kinetic energy is 98% conserved in Event 4.

II. APPARATUS USED TO OBTAIN THE PHOTOGRAPHS

The two colliding balls were, in each case, hung in bifilar suspension from thin piano wires as shown in Fig. 1. They were confined to move on circular paths in the same vertical plane. The radii of these paths were the same for both balls (about 10 m). Thus, the balls acted as pendulums of equal periods (about 2π sec) to excellent approximation. They were released simultaneously by relays from chosen initial positions and therefore collided at the bottom of their swing (point B) a quarter period (or about 1.6 sec) later.

A camera was placed in front of point B for a field of view as indicated. The (circular) path of the balls within this frame was illuminated by four synchronized General Radio Company Stroboscopes (not shown). The flash rate was always a simple integer submultiple of 60 Hz. This permitted very accurate calibration of this rate against the power company's time frequency by observing beats in a small neon bulb in the stroboscope circuits.

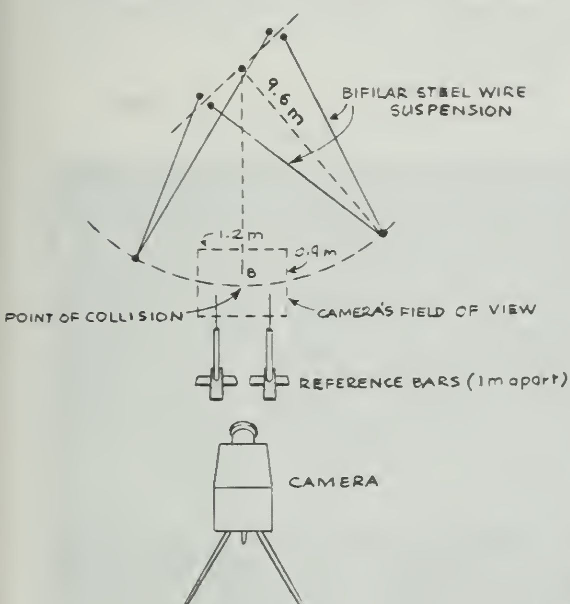


Fig. 1 Schematic diagram of the experiment (not to scale). The pendulums are very long. The height of the room in which the pictures were taken is 15 m.

Two vertical rods were placed in the field of view. The center points of the tops of these rods were, in all photos taken, 1 m apart (as precisely as possible). Therefore, the student can, by scaling, calculate actual distances from the measurements on the photographic prints.

Typical pictures can be found among the figures that follow. Clearly, the paths are not exactly straight. Nor should the velocities found be exactly the same for a given ball as it moves toward point B of collision, or away from B afterwards. But, if the amplitude of swing exceeds the portion within the camera frame (about 0.5 m on each side of B) by a sufficient factor, the velocity variation will be small.

The speed of the ball near the frame's edge is smaller than near the collision point B. (By the way, it is smaller by less than 1% if the full swing of the pendulum is 4 m; by 3% if it is 2 m; and so on.) But that need not be of great concern because the stroboscopic record allows one to check by how much the ball is slower near the frame's edge.

If precision is affected by this variation from the various exposures of ball position, then the best positions from which to calculate velocities are those nearest the center. If this rule is followed, the errors introduced by this aspect of the experimental setup are well within the estimated error of a reading of distance on the photograph by use of a good metric ruler with millimeter rulings. In none of the photographs is the "built-in" error greater than 2%.

III. DISCUSSION OF THE EVENTS

The seven collision events present similar analytical problems. For any given event, the steps of this analysis follow.

A. A schematic diagram is provided for each event. It specifies qualitatively what the conditions were before and after the collision. It also gives the masses of the colliding balls.

B. The student can then proceed to a qualitative study of the stroboscopic photographs provided for the event and can actually tell the order, in time, in which the stroboscope flashes occurred and number them.

C. The student then must find values for the speeds of the balls before and after collision, by measurement.

Here the student may discover more than one time interval from which measurement of displacement might be made. If so, "best choice" must be made.

Also, in some cases, conditions rule out an interval because the student can tell from a study of the picture that the collision occurred during that interval. The displacement of a ball in that interval occurred at different speeds; the speed before, during, and after collision.

A measurement of displacement is best made by measuring the distance between successive left (or right) edges of the ball in question.

Each photograph shows two vertical rods. The centers of these rods are 1 m apart, with precision of 2 mm, a fraction of 1%. Thus, measurements of displacement taken on the print can and should be converted to the actual values (in meters or centimeters). This can be done by simple scaling. The student then finds the "real speeds" of the balls before and after collision, which gives the exercise the flavor of the real experiment.

We recommend that students be provided with a good-quality, but simple, see-through plastic ruler marked in millimeters; that they take care in positioning this ruler for every measurement; and that they estimate to the tenth of the millimeter. They should be aware that this smallest significant digit may be in doubt.

D. They can now proceed to the calculations required by the problem.

The teacher is provided with an overhead *Transparency* for Events 1 and 2. It is most strongly recommended that the qualitative aspects of these events be discussed as prototypes before assigning events as problems for the student. Only the special aspects of Events 2 to 7 are discussed here.

Event 1

See also notes on *Film Loop 18* (first example, page 156).

Handbook, Fig. 3-4, page 105 shows, schematically, the conditions before and after collision. In the stroboscopic photograph (*Handbook,* Fig. 3-11, page 107), we see two balls of different size. To reduce confusion, two dark stripes have been painted on the smaller ball (ball B). The large ball (ball A) comes in from the right. The time and order of the flashes can therefore be analyzed. The photo at the bottom of this page corresponds to *Handbook,* Fig. 3-11, page 107.

Ball B clearly was at rest at center frame during flashes 1, 2, and 3. The collision must have occurred between flashes 3 and 4. Therefore, the displacements experienced by either ball in interval "3 to 4" must be ruled out from the measurements useful for calculating speeds before or after collision. Ball A was was considerably slowed down by the collision. At the time of flash 7, ball B was already out of frame. The camera shutter was closed before flash 8 occurred.

The student has two choices (intervals "1 to 2" and "2 to 3") in which to calculate the incoming speed of ball A and three choices ("4 to 5," "5 to 6," "6 to 7") to measure its speed after collision.

When using a ruler marked in millimeters on an 8 × 10 print of Fig. 3-4, the careful student will find that the displacement of ball A was about 0.5 mm greater during time interval "1 to 2" than in interval "2 to 3." The difference amounts to about 1%.

The student should be encouraged to decide whether there is a "best choice" in finding the speed before collision in view of the experimental setup illustrated in Fig. 1. A student may, in this particular case, even decide with reasonable arguments that the average of both displacements is adequate.

As a rule of thumb, of course, the interval closest to the instant of collision is the "best choice." By this reasoning, "4 to 5" is also "best choice" of interval to find the speeds of both balls after collision.

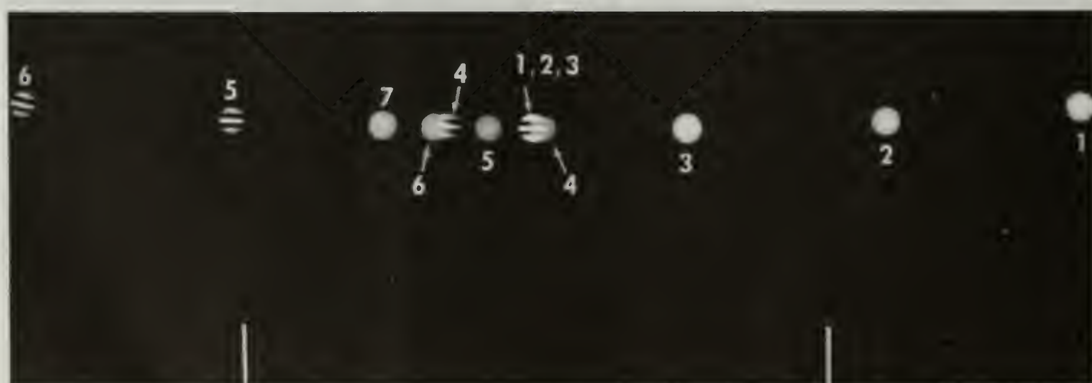
In any case, the student should convert each measurement of a displacement, taken from the print, to the actual displacement of the ball. This may be readily done by measuring the distance between the two vertical reference bars, which are 1 m (±2 mm) apart.

Event 1

Scale: 12.65 cm to 1 m on an 8 × 10 print
Ball A: 0.532 kg
Ball B: 0.350 kg
Flash rate: 10 per sec

Table 1

Item	Ball	Time	"Best Value"	Direction
velocity	A	before	3.37 m/sec	left
	A	after	0.917 m/sec	left
	B	before	0 m/sec	
	B	after	3.72 m/sec	left
momentum	A	before	1.79 kg·m/sec	left
	A	after	0.488 kg·m/sec	left
	B	before	0 kg·m/sec	
	B	after	1.30 kg·m/sec	left
kinetic energy	A	before	3.02 joules	
	A	after	0.223 joules	
	B	before	0 joules	
	B	after	2.42 joules	



(See *Handbook,* Fig. 3-11, page 107) Stroboscopic photograph of Event 1. 10 flashes per second. The numbers shown do not appear in the photograph given to the student. They correspond to the order in which the successive flashes occurred at 0.1 sec intervals. Each position of each ball can be associated with one of these numbers. The centers of the tops of the two rods appearing near the bottom of the picture are 1 m apart and serve as a scale reference. (See also *T19* and *L18,* first example.)

Table 1 shows the results obtained by careful measurement from a photographic 8 × 10 print. The "Best Value" is based on measurement of displacement with a good ruler, of 1 mm least count (estimating to the tenth of a millimeter), during time intervals "2 to 3" before collision and "4 to 5" after collision.

The momentum of the system of two balls before collision is equal to the momentum of the system after collision to the number of significant digits available: 1.79 kg·m/sec (directed to the left). On the other hand, the kinetic energy of the system goes from 3.02 J before collision to 2.64 J after collision. It is only 87.4% conserved owing to imperfect elasticity.

Event 2

See also notes on *Film Loop 18* (second example, page 156).

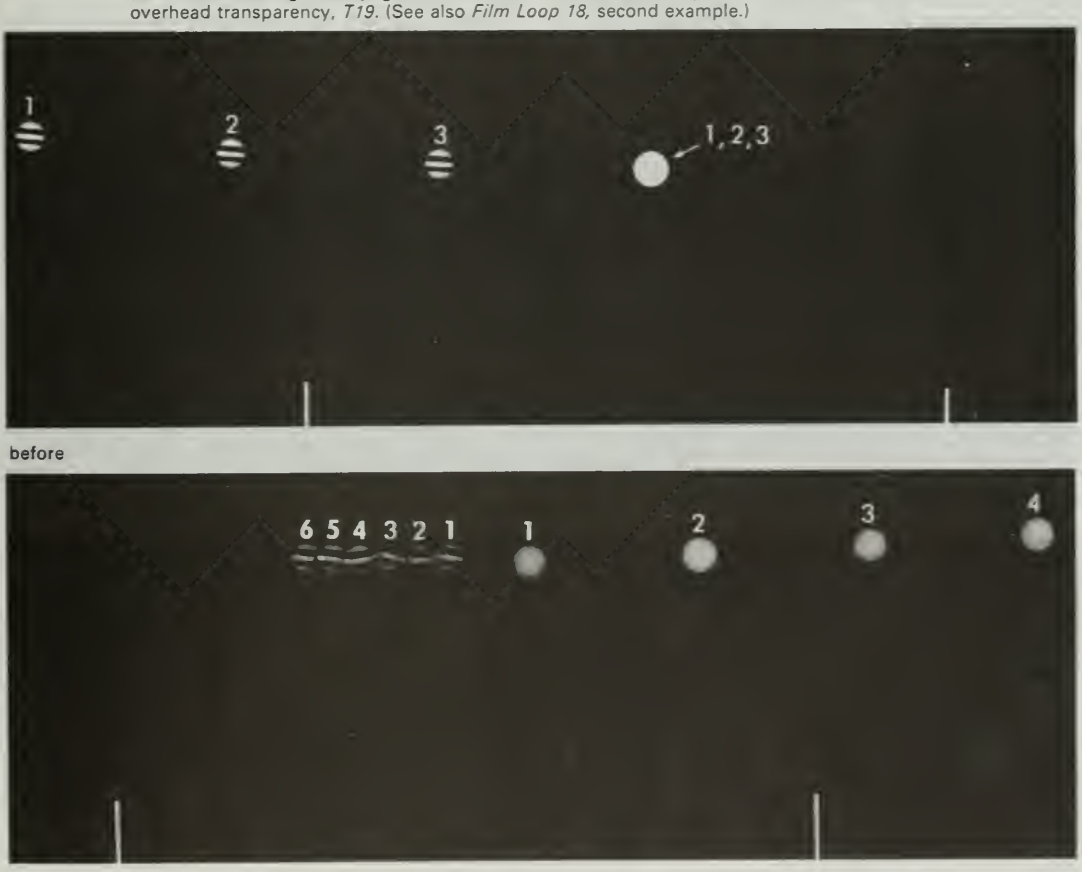
Refer to *Handbook*, Fig. 3-5, page 105 and notice that the balls have the same masses as in Event 1 and that the collision is the inverse of Event 1. This was intentional. In Event 2, ball B is striking ball A at rest from the left. The incoming speed of ball B is, in fact, roughly equal to that of ball A in Event 1.

Moreover, note that in this case the incoming ball is reflected by the collision. This is so because the ball it strikes has the greater mass.

There are two stroboscopic photographs recording this event in Fig. 3-12. The first shows the event before collision, the second after collision. The pictures were taken by allowing the event to control the camera shutter using electric relays. A relay closed the shutter before collision and opened it just after collision. If some such division of this event had not been made, it would be difficult to differentiate exposures of ball B because this ball retraces its path. The transparency overlay of Fig. 3-12 for overhead projectors provided to the teacher can be used to show this to the student.

The measurement of displacement for ball B after collision is difficult in this case. See Fig. 3-12 "after." It emerges from collision with small speed, which (as shown clearly by the picture) is decreasing visibly. Interval "1 to 2" in Fig. 3-12 is clearly "best choice" for both balls. Care is required to find the displacement of ball B in that interval.

(See *Handbook* Fig. 3-12, page 107) Event 2. 10 flashes/sec. Fig. 3-12 is also available as an overhead transparency, *T19*. (See also *Film Loop 18*, second example.)



before

after

Our values of total momentum before and after collision were 1.14 and 1.16 kg·m/sec, respectively (directed to the right). The difficulty in measuring the speed of ball B after collision and the large relative error arising from this difficulty do not greatly affect absolute errors. The reason is that the momentum of ball B after collision is so small. Discovery by students of these aspects of measurement is in itself a worthwhile goal of our laboratory instruction.

Our values showed the total kinetic energy to be 91% conserved.

Event 3

See also notes on *Film Loop 19* (first example, page 157).

As *Handbook* Fig. 3-6, page 105 and Fig. 3-13, page 108 illustrate, a massive ball A enters from the left, and ball B of considerably less mass comes from the right. The speed of ball B compared to the speed of ball A before collision, however, is so large that the net momentum vector of the system actually points to the left.

This has two consequences that we feel to be of pedagogical value. One of these concerns the aspects of measurement: The net momentum of the system is here a *small difference of two large numbers*. The two "large numbers" are the momenta of the balls before or after collision. Most students can find these momenta with a precision of 1 or 2%. However, in their small difference this becomes a large relative error. Matters are made worse because of the rule that states that, in additions or subtractions, the error in the result is the algebraic sum of the absolute errors of the two numbers involved.

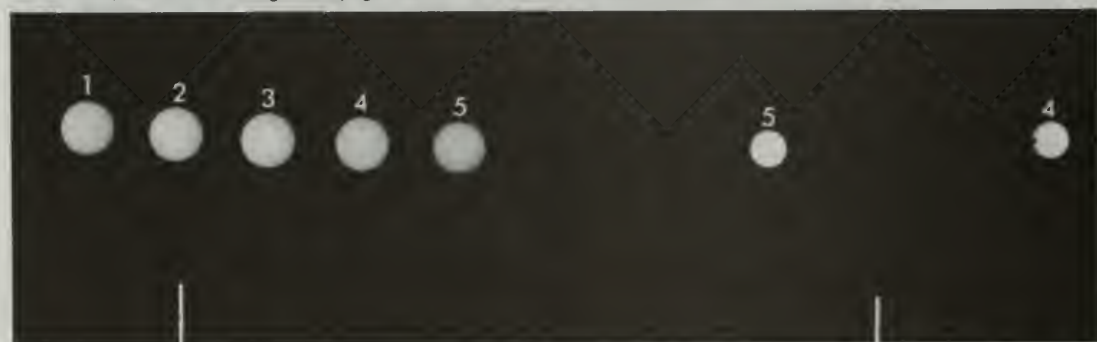
Our results were: 0.30 kg·m/sec and 0.27 kg·m/sec net momentum (directed toward the left) before and after. This is a 10% difference. We have only two significant digits in the result, although each ball's momentum was itself known to three digits.

The second consequence concerns a physical process. In this collision the impact of the steel balls was a violent one. Students can study this with their own data. Each ball's change in momentum, produced by the collision, was of the order of 2.8 kg·m/sec. (Change in momentum = (average force during collision) × (time of collision). With a conservative estimate of 0.01 sec for this time, the average force was 280 N, or the weight of 29 kg.)

Obviously, the hard and elastic steel case of these case-hardened balls was strongly deformed in this collision. This deformed the soft, and far less elastic, inner core. The result was a great deal of internal friction and heating. This explains why the initial kinetic energy of the system is dissipated more here than in other events. We found it to be only 51% conserved.

Event 4 involves the same balls in a weak collision, resulting in 98% conservation of kinetic energy.

(See *Handbook* Fig. 3-13, page 108.) Event 3. 10 flashes/sec (See also *Film Loop 19*, first example.)



before

after

Event 4
See also notes on *Film Loop 19* (second example, page 157).

Handbook Fig. 3-7, page 106 specifies this event. The massive ball A comes in from the left at high speed and overtakes the lighter ball B, which is going in the same direction. After collision both balls are still going in the same direction, but ball B is travelling much faster than before, whereas ball A is slowed down relatively little.

Since each ball crosses the entire field of view from left to right, a stroboscopic view of the whole event leads to superposition of images created by flashing of the bulb at different times. (See the following stroboscopic photograph of ball A and ball B.) As a consequence, such a picture is difficult to analyze.

We got around this difficulty by photographing Event 4 in its entirety twice. Once (Fig. 3-14, ball A) ball B was painted black while ball A was painted white. The second time (Fig. 3-14, ball B) the colors were reversed.

ball A and ball B

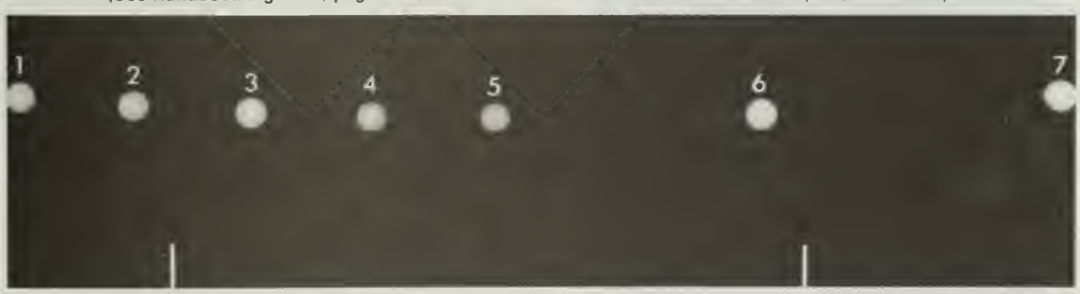
Stroboscopic photograph of the entire Event 4. 10 flashes sec. Both balls cross the field of view in the same direction (from left to right) as time goes by. It is not easy to analyze this picture. *Handbook* Fig. 3-14 shows how this difficulty was resolved.

Consider Fig. 3-14, ball A (above). It can be used to determine the speed of ball A before and after collision. However, position 3 is difficult to identify. It could, at first inspection, have been before or after collision. A good student will be able to show, by making measurements, that the flash which exposed position 3 occurred after collision. Distance "2 to 3" is slightly shorter than distance "1 to 2," while at the same time "3 to 4" is slightly longer than "4 to 5." If this is clear to the student then the "best choice" for ball A is before collision "1 to 2" and after collision "3 to 4."

(See *Handbook* Fig. 3-14, page 108.) Event 4. 10 flashes sec. Whole event. Ball A (large mass) black. (See also *Film Loop 19,* second example)

ball A

(See *Handbook* Fig. 3-15, page 109.) Event 5. 10 flashes sec (See also *Film Loop 20,* first example.)



ball B

(See *Handbook* Fig. 3-14, page 108.) Event 4. 10 flashes sec. Whole event. Ball B (small mass) black. See also *Film Loop 19,* second example.)

This is a fine point of the analysis that can be ignored. One can call position 3 doubtful. This rules out intervals "2 to 3" and "3 to 4." Equally precise results are obtained from Fig. 3-13 by choosing intervals "1 to 2" and "4 to 5."

Analogous difficulties arise in analyzing Fig. 3-14, ball B. The intervals before and after collision corresponding to safe choices are "3 to 4" and "6 to 7."

We find, from the safest choices, that the total momenta before and after collision are 8.35 and 8.38 kg·m/sec (to the right), respectively. The total kinetic energies before and after are 15.9 and 15.5 J, which correspond to 98% conservation of kinetic energy.

This weak collision is much more elastic than another stronger collision between these balls, namely, Event 3. Event 4 forms an instructive companion to Event 3. The *Handbook* contains questions about the surprising differences in apparent elasticity.

Perfectly Inelastic Collisions

In these examples, the colliding objects are steel balls covered by a thick layer of plasticene. They remain lodged together after collision.

Event 5

See also notes on *Film Loop 20* (first example, page 157).

Handbook Fig. 3-8, page 106 and Fig. 3-15, page 109 show ball A, coming in from the right, striking ball B, which is at rest. Our measurements yield momenta for the system of balls of 2.66 kg·m sec before and 2.67 kg·m sec after collision (directed to the right). The kinetic energy after collision was 61% of that before collision.

Event 6

See also notes on *Film Loop 20* (second example, page 157).

Handbook Fig. 3-9, page 106 and Fig. 3-16, page 109 tell the story. This case involves the same balls as Event 5. Momentum of the system before and after is 2.06 and 2.08 kg·m sec (directed to the left) by our measurements.

Event 6 is an instructive example, when considered together with Event 5, in which over 60% of the kinetic energy was conserved. Here only 24% is conserved. The collision was more violent in Event 6.

Event 7

A very massive ball A is coming in from the left (see *Handbook* Fig. 3-10, page 107) at a speed that is small when compared with the speed of ball B of small mass that enters from the right. When the balls become lodged together, they move off to the right, but the speed of ball A after collision is not very greatly reduced from what it was before. This is analogous to the case of a head-on collision of a truck with a small car.

Handbook Fig. 3-17, page 110 shows the stroboscopic records before and after collision.

Momenta before and after are 6.58 and 6.71 kg·m sec (directed to the right), respectively. Kinetic energy after collision is about 27% of that before.

E3–3 COLLISIONS IN TWO DIMENSIONS. I

Equipment:

Method A

- Ripple tank
- Plastic (Dylite) spheres
- Three or four balloon pucks without balloons, but with small (about 1-cm) white styrofoam hemispheres glued to center of *bottom* of two of them as markers
- Polaroid camera
- Mount for camera positioned vertically above center of the ripple tank (for example, on a step ladder)
- Either xenon strobe lamp
- or motor strobe with 12-slotted disc and strong spotlight
- 10× magnifier with scale

Method B

Exactly the same equipment and setup as Method A except that the air pucks are replaced by three or four disc magnets, two of which have their centers marked by 1-cm white styrofoam hemispheres.

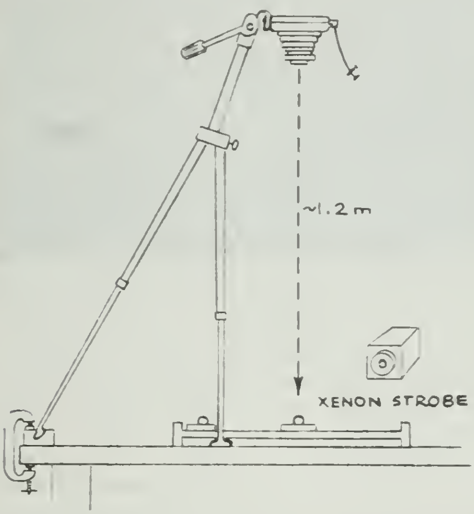
General Discussion

Even if your students do not do this experiment quantitatively, let them experiment with balloon pucks, pucks on beads, and magnet pucks on beads to get an intuitive feel for two-dimensional collisions.

Two sizes of puck are supplied. The mass ratio is 2:1. They may be used as balloon pucks on any flat and fairly clean surface. (Remove the stopper with the balloon when inflating.) You can improve sanitation, and avoid saliva on the pucks, by placing the puck tightly up against the exhaust of a vacuum pump to inflate the balloon.

The "new rule" is that momentum is a vector quantity.

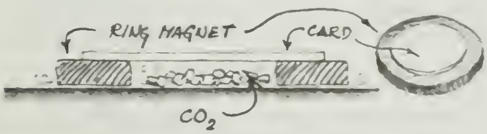
For strobe photographs, remove the balloons and slide the pucks on a low-friction surface made by sprinkling plastic (Dylite) spheres on a ripple tank. Use a small piece of clay or tape to fix a steel ball (such as that from the trajectory apparatus) or a white styrofoam hemisphere at the center of the puck. (Other reflectors, such as a white-painted stopper, are also possible, but give less satisfactory photographs.)



The xenon strobe must be at the side of the tank, not above it. This is to reduce reflections from the beads themselves, which may nevertheless be a nuisance: don't use too many beads!

To get the camera directly above the working surface, extend one leg of the tripod more than the other. The tripod will then be unstable so the long leg must be held down, as shown in the sketch.

Alternative procedure with ring magnets (which may be obtained from Damon): Glue a piece of card over the top of the magnet. Put a piece of dry ice inside, or fill with "dry snow" from a CO_2 fire extinguisher. Place the magnet on a flat surface (ripple tank); it will float on the film of CO_2 gas.



Another possibility here is an "explosion" between magnets. Would two magnets move apart in opposite directions along a straight line? If the masses are equal will the speeds be equal? What happens with three or more magnets held close together and released simultaneously?

Students can draw their vector diagrams by pricking holes and drawing on the back of the photograph, as shown below.

The conclusion that momentum is conserved in an interaction is not sufficient to enable us to predict the final velocities of the interacting bodies except in a few special cases. Given m_A, m_B, v_A, v_B the equation

$$m\vec{v}_A + m\vec{v}_B = m\vec{v}_A{}' + m\vec{v}_B{}'$$

has two unknown quantities, v_A and v_B.

Students should realize from their knowledge of algebra that two equations are needed to find these quantities. Since a given set of initial conditions always leads to the same pair of values of $v_A{}'$ and $v_B{}'$, some other requirement must be imposed on the system. Stress this point to prepare students to look for still another conservation law in the next experiment.

One special case is very well illustrated by the magnet disc experiments. In an (elastic) collision or interaction between two equal-mass bodies, one of which is initially at rest, the two velocities after the collision are perpendicular to each other. This is quite independent of the "off-centeredness" of the collision.

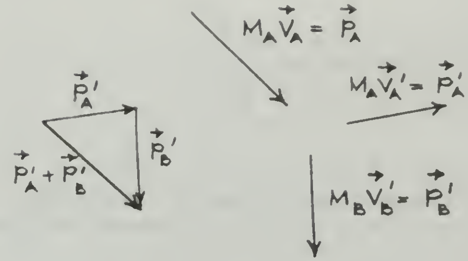
Analysis of Data

Three transparencies (*T20* through *T22*) analyze a two-dimensional collision in detail. Furthermore in this *Resource Book*, stroboscopic photographs

of two-dimensional events 8–14 are explained in the "Film Loop Notes" section. Use these references as samples of the results from *E3-3*.

An additional analysis of results follows:

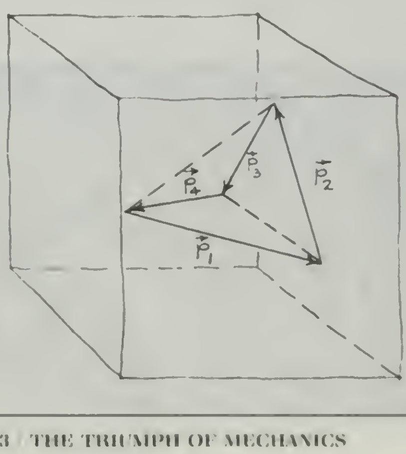
P_A = momentum of A before collision
P_B = 0
P_A' = momentum of A after collision
P_B' = momentum of B after collision

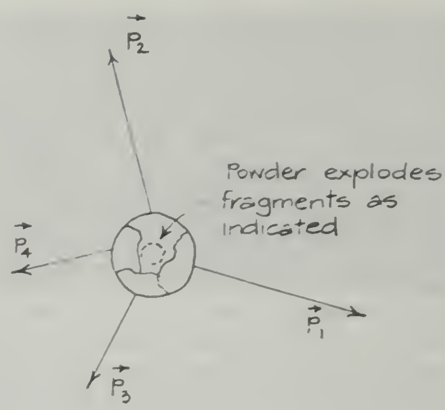


$$\vec{P}_A = \vec{P}_A' + \vec{P}_B'$$

Answers to questions

1. Student calculations.
2. The sum of the mass × speed before the collision is not equal to the sum of the mass × speed after the collision.
3. The vector sum of the momentum before the collision is equal to the vector sum of the momentum after the collision.
4. One-dimensional collision conservation of momentum is a special case of the two-dimensional problem. Two examples of three-dimensional conservation of momentum:





5. (a) $m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C = m_A\vec{v}_A' + m_B\vec{v}_B' + m_C\vec{v}_C'$
   (b) $m_A(\vec{v}_{Ax} + \vec{v}_{Ay} + \vec{v}_{Az}) + m_B(\vec{v}_{Bx} + \vec{v}_{By} + \vec{v}_{Bz}) = m_A(\vec{v}_{Ax}' + \vec{v}_{Ay}' + \vec{v}_{Az}') + m_B(\vec{v}_{Bx}' + \vec{v}_{By}' + \vec{v}_{Bz}')$
   (c) $m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C = m_A\vec{v}_A' + m_B\vec{v}_B' + m_C\vec{v}_C'$

Addendum to E3-3

With a small modification, the *Project Physics* trajectory apparatus (*E1-8*, Unit 1) can be used in a collision in a two-dimensional experiment.

The figure at the top of the next column shows how the target ball is positioned at the end of the launching ramp. The attachment can be swung about a vertical axis to vary the impact parameter (amount of "off-centeredness" of the collision).

The plotting board is laid flat at the corner of a table. On it are placed a sheet of carbon paper (carbon side up), and over the carbon paper a sheet of onionskin paper.

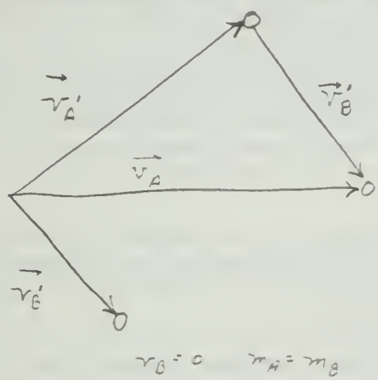
With no target ball in position, a ball is released from a point on the ramp and allowed to fall on the plotting board, which records its point of impact. Repeat a few times to get consistent results. A target ball is now placed on the support and the impact ball released again from the same point on the ramp. Both balls record the positions at which they land on the impact board.

Analysis of Data

The balls all fall through the same vertical distance from collision to the board, and are therefore in flight for the same time. The horizontal distance that each ball travels between collision and impact is thus proportional to its velocity. Distances are measured from the collision point.

Students can analyze for conservation of momentum in the horizontal plane by drawing vector diagrams as shown below the photograph. The launch position, mass ratio, and impact parameter can all be varied to provide a wide range of situations.

The same data can be analyzed for conservation of kinetic energy (in the horizontal plane).



E3-4 COLLISIONS IN TWO DIMENSIONS. II

Equipment:

Film Loops: 22, 23, 24, 25, and 26
Technicolor loop projector
Graph paper, masking tape, ruler
Stroboscopic photographs (in the *Handbook*)

Method A: Film Loops

See notes on *Film Loops* 22–26, beginning on page 158.

Method B: Stroboscopic photographs

STROBOSCOPIC STILL PHOTOGRAPHS OF TWO-DIMENSIONAL COLLISIONS: EVENTS 8–14

I. MATERIALS PROVIDED AND THEIR USES

Measurements in Events 8–14 are essentially of the same nature as in the one-dimensional problems. Analysis, on the other hand, requires construction of vector diagrams and is for this reason a bit more complicated.

The student may be assigned one or more of these events as take-home problems, or as study-period (or laboratory-period) tasks. It may be advantageous to assign a pair of students to a given problem.

Better students may profit from working through more than one example. Events 8 and 9 both involve collisions in which one ball is initially at rest. In Event 8, moreover, the balls have equal masses. Events 12 and 13 are perfectly inelastic collisions between balls of equal mass. The initial speeds are roughly the same in these two examples, but in one the angle between incoming paths is acute, whereas in the other it is obtuse. Event 14 is the most ambitious problem. Here, a rapidly moving ball scatters a cluster of six balls initially at rest. This problem should not be assigned unless the student has worked one of the simpler events (8–13).

The problems are, primarily, illustrations of the *principle of conservation of momentum.*

A calculation of the *kinetic energy* in the system before and after collision is also of interest. It reflects on the degree of violence of the collision and on the imperfect elasticity of the steel balls. (In Events 12 and 13, however, the colliding balls are plasticene.)

The stroboscopic photographs were made by the same procedures as those for Events 1–7.

II. DISCUSSION OF THE EVENTS

The vector diagram.

With the print securely taped to a corner of the large paper sheet, the student uses the drafting triangle to draw a line parallel to the incident direction of motion of one of the balls.

Choosing a convenient scale factor, the student measures off the momentum of this ball before collision. To the tip of this vector is now added the momentum of the next ball, before collision, by the same procedure of drawing a parallel line and using the same scale factor. Thus, by adding vectors, the total momentum before collision can be drawn and its magnitude measured.

The procedure is repeated to find the total vector momentum of the system after the collision.

It is recommended that the initial point of the procedure of adding vectors tip to tip be the same point for both the "before" and "after" phases of the collision event. It will then be relatively easy to find a value for the angle between the two vector sums. This angle is one of the two measures of error. The other is the percentage of difference in magnitudes.

Kinetic energy.

Total kinetic energy of the system before and after collision can be calculated from the given masses and from the speeds determined by the measurements.

In none of these events were the collisions perfectly elastic. Even though the hardened steel cases approach ideal elasticity, the soft steel inner cores were frequently permanently deformed by the collision. The loss in kinetic energy goes into the work of deformation and into heat.

Clearly, the percentage of loss of kinetic energy should be larger for more violent collisions. If a student works through more than one event (especially if the different events involved the same balls, such as in Events 8 and 10, 9 and 11, or 12 and 13) a comparison of the percentage of kinetic energy conserved in the collision can lead to interesting discussions.

Forget about rotation!

In Events 8–14, each ball was suspended by a single wire. It is not likely that the balls were spinning appreciably before collision. However, they may have been spinning considerably after collision. No provision was made (by marks on the balls, or other means) to permit measurement of these rates of spin. Only translational momentum and translational kinetic energy is accessible to measurement by our techniques.

In the perfectly inelastic collisions (Events 12 and 13) care was taken to use photographs in which these spins are essentially zero. See *Handbook* Figure 9, page 20 and Figure 10, page 21. Hence, the possible complications pertaining to angular momentum (and rotational kinetic energy) do not enter in Events 12 and 13.

Event 8

See also *T20* and notes on *Film Loop 20* (first example, page 157).

From the photograph of Event 8 (*Handbook* Fig. 3-27, page 115) note that the balls have equal mass. Ball A comes in from the upper left-hand corner while ball B is initially at rest. During the stroboscope flashes numbered 1, 2, and 3, ball B is strongly exposed, photographically, in its rest position. The collision occurred in the interval between flashes 3 and 4. This interval is therefore inappropriate for determining ball speeds since we are interested in speeds before and after collision.

In accordance with the idea that intervals nearest the collision points constitute "best choices," it is clear that interval "2 to 3" is "best" before and that interval "4 to 5" is "best" after collision for speed measurement.

Table 2 summarizes results for a typical measurement.

The kinetic energies of the two balls are scalars. They add numerically. By our measurements given in the table, the system possessed 1.39 J of translational kinetic energy before collision and 1.23 J afterwards (88.5% conservation).

Event 8

Scale: 23.1 cm to 1 m on an 8 × 10 print
Ball A_A = Ball B_B = 0.367 kg
Flashrate: 20 per sec

Table 2

Item	Ball	Time	"Best Value"
speed	A	before	2.76 m/sec
	A	after	2.16
	B	before	0
	B	after	1.44
magnitude of momentum	A	before	1.01 kg·m/sec
	A	after	0.774
	B	before	0
	B	after	0.527
kinetic energy	A	before	1.39 joules
	A	after	0.854
	B	before	0
	B	after	0.379

Because the masses of the balls are equal in this event, the photograph lends itself to an easy demonstration of the properties of the *center of mass of the system.*

The results illustrate the following theorem: *The center of mass of a system of particles, which is subject to zero net external force, travels uniformly along a straight line.*

It is not recommended that the teacher present this theorem and the construction to all students. Rather, we mention it here because of its intrinsic interest and because it may be appropriate to expose your best students to it.

Event 9

Our discussion of this and subsequent events will emphasize only special points of interest. It will be less detailed than for Event 8 because procedures are identical.

Event 9 (see *Handbook* Fig. 3-28, page 116) is similar to Event 8. Again, one ball is initially at rest. However, the ball masses are different in Event 9.

Our resultant momenta before and after collision coincided in direction. They differed in magnitude by 0.004 kg · m/sec (about 0.5%).

The kinetic energy of the system fell from 0.727 J before, to 0.481 J after collision. Only 66.2% of this energy is conserved. This should be compared to the 88% conservation in Event 8.

Notice that the balls A have the same mass in these two events. Why is the percentage loss of kinetic energy larger in Event 9? The answer is connected with the larger mass of ball B. In Event 9, each ball experiences a greater change in momentum during collision than in Event 8. Hence, the interaction forces during collision were larger on the average. The collision was more "violent." The balls were permanently deformed to a greater degree.

Event 10

See also notes on *Film Loop 22* (second example, page 158).

This event is slightly more complicated than the two discussed so far because both balls are moving before collision. Note, however, that the balls again have identical masses as in Event 8.

In our measurements, we found an angular separation of 0.6° and 1.5% difference in magnitude between the total momenta before and after collision.

Total kinetic energy was 2.17 J before and 1.73 J after collision (79% conserved).

Since Event 10 (see *Handbook* Fig. 3-29, page 117), like Event 8, uses balls of equal mass, it is again quite easy to locate the instantaneous images of balls A and B. It is clear that the path of the center of mass is a straight line and that its motion is uniform.

One merely measures the distance between two successive positions of the center of mass and converts to "real" distances by scaling. (Recall that the reference rods at bottom frame are 1.000 ± 0.002 m apart.) Successive positions are 0.05 sec apart in time. We find that the speed of the center of mass = 1.89 m/sec.

The center of mass of a system of particles can be treated as if it were a particle whose mass equals the total mass of the system and whose momentum equals the total momentum of the system. This is a general result of theoretical mechanics.

Since we know the speed of the "center-of-mass particle," the magnitude of its momentum is

$$\begin{aligned}&(0.367 + 0.367)\text{ kg} \times (1.89)\text{ m/sec}\\ &= 0.734\text{ kg} \times 1.89\text{ m/sec}\\ &= 1.39\text{ kg}\cdot\text{m/sec.}\end{aligned}$$

Notice that this comes close to the values found for the magnitude of the total momentum of the system by adding individual particle momenta vectorially.

Furthermore, one could easily verify that the direction of the straight-line path taken by the center of mass is parallel to the total momentum vector as found by the method of adding particle momenta.

Event 11

See also notes on *Film Loop 21* (page 157).

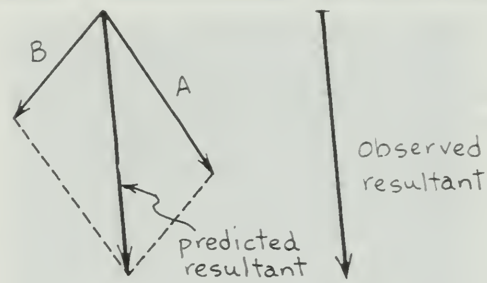
Event 11 (see *Handbook* Fig. 3-30, page 116) is analogous to Event 10 except that the masses of the two balls are not equal. We find a difference of 1% between the magnitudes and an angle of less than 1° between the directions of the vector sums. Total kinetic energy before and after collision is 3.10 and 2.65 J, respectively (85% conservation).

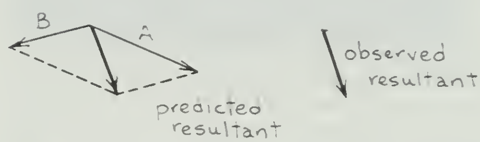
Events 12 and 13 (L23)

See also notes on *Film Loop 23* (page 158).

In these events, the steel balls were covered by a thick layer of plasticene. They remained lodged together after collision. The major difference between these two events was the fact that the angle between the initial directions of motion of the two balls was larger in Event 13 (see *Handbook* Fig. 3-32, page 117) than in Event 12 (see *Handbook* Fig. 3-31, page 117).

After collision, the "particle system" becomes a single, compound particle consisting of the coalesced balls. Its mass is the sum of the masses of the original balls. To find its speed and momentum, it is necessary to locate the center of mass. Even though the collision created noticeable deformation, it is possible to locate this point easily. Since the individual balls have identical masses, the "compound particle's" center of mass lies at its geometrical center. This can be judged by the naked eye and marked on the print by a pencil dot.





In Event 12, the total momenta before and after collision differed by 0.5% and subtended an angle of 0.5° according to our data. See Fig. 1. Kinetic energy fell from 3.10 J during collision (85% conservation).

As Fig. 2 shows, we found the directions of the total momenta for Event 13, before and after collision, to be the same. Their magnitudes differed by 3%. Kinetic energy dropped from 2.81 J to 0.33%. Only 12% of this energy was conserved.

Why is the relative error significantly larger in Event 13 than in Event 12? The principal reason can be found by an inspection of the vector diagrams in Figs. 25 and 26. In Event 13, the total momentum of the system before collision, compared to Event 12, is a short vector sum of two vectors. On the other hand, the vectors added together to produce these sums are, in fact, of nearly equal magnitudes in both events. They probably have nearly equal errors in them, too. When adding the vectors, the errors "pile up." And in the "shorter" vector sum of Event 13, the absolute error is about the same. So it is a larger relative error.

Why is there so much greater loss of kinetic energy in Event 13? The answer is obvious from an inspection of the photographs. The balls collide almost, but not quite, head-on in Event 13. Notice also, how much more flattening takes place at the points of contact in Event 13 compared to Event 12. The collision was more violent; more energy went into unrecoverable work of deformation.

Event 14

See also notes on *Film Loop 24* (page 158).

A stationary cluster of six balls is struck by a rapidly moving ball coming in from the right (see *Handbook* Fig. 3-33, page 117). The initial position of the cluster is seen at the center of the figure.

The total mass of the cluster is more than 4 kg. After the impact, ball speeds are relatively low. This explains why a flash rate of only 5 per sec was used and, also, why the speed of some of the balls after impact decreases noticeably as they move toward the edge of the field of view.

On the other hand, ball A is moving so fast before impact that the stroboscope, flashing only every 0.2 sec, did not capture within the frame more than one of its positions before collision. For this reason, a second picture was taken. See *Handbook* Fig. 3-34, page 118. The cluster of balls B to G was removed and ball A released from exactly the same initial point. The flash rate was greater (20 per sec). The "impact speed" of ball A can be determined from measurements of displacement in the center of the photograph.

The analysis of Event 14 is analogous to the analysis of the previous problems. It requires more measurements and comparison of the sum of seven momentum vectors (after collision) with the momentum of the incoming ball. We recommend that this problem be assigned only after one of Events 8–13 has been worked out by the student, and then only if he or she is one of the better students in your class.

Notice that balls A and C have identical radii (5 cm). The same is true of balls B and D (4.4 cm). To distinguish between them, a mark in the shape of a cross was taped to ball A, and a line-mark was taped to ball B.

In determining speeds, the appropriate displacements must not involve the initial cluster-positions of the balls. "Best choice" is the distance between the first and second "scattered" positions.

The reference laboratory distance of 1 m between tip-centers of the reference rods may not appear to be in the same scale as the two prints provided to the student. Hence, the scaling factors will not be equal. If this difference in scaling factors is taken into account by the students, the calculated "actual distance" will reproduce the laboratory conditions.

After determining the speeds and the magnitudes of the individual momenta, the student must add the seven individual momentum vectors together. The vector addition proceeds most simply if the head-to-tail method is used. The order of the vectors being added is immaterial because vector addition is "commutative."

In our measurements and calculations, system momentum, before and after, differed by 2.1% in magnitude and by 1.5° in angle. Kinetic energy fell from 5.22 J to 1.36% (26% conserved). This represents a very violent set of collisions, especially when we remember that we used case-hardened steel balls.

E3-5 CONSERVATION OF ENERGY. I

Equipment:

Method A

- One pair of dynamics carts with weak (0.025-cm) springs
- Four 1-kg masses
- Two light sources
- Polaroid camera and film
- Motor strobe unit
- Masking tape
- 10× magnifier and scale and or mm ruler
- Graph paper

Method B

- Disc magnets
- Plastic (Dylite) beads
- Ripple tank
- Polaroid camera and film
- Camera tripod, or other means of supporting camera <u>above</u> ripple tank
- Xenon strobe (or photo-flood lamps together with motor strobe unit)
- Steel or styrofoam balls
- 10× magnifier and scale and or mm ruler
- Graph paper

Method C

- Air track and glider, blower
- Meter stick
- Stopwatch
- (Optional: camera, light source, disc strobe, graph paper)

Students could analyze their data from *E3–3* for other conserved quantities as a homework assignment before beginning this experiment. For the elastic collisions, they will find that mv^2 is conserved. (It will not be conserved in the explosion of course. Other quantities like $m\,v$ and m^2v may be conserved in special cases; for instance, m^2v is conserved in an elastic collision between equal-mass objects.)

Method A. Dynamics Carts

Before the students start photographing collisions, demonstrate a slow collision as follows. Remove one screw from the spring bumper of each cart and arrange the carts as shown in Fig. 1. Add 1 or 2 kg of extra mass to each cart. The combination of large mass and weak spring produces an extremely slow interaction: students can see clearly that there is an instant when both carts are moving quite slowly. To take photographs, replace the screws so the bumpers are in their original shape. Two kinds of spring are supplied in the dynamics cart kits; use the lighter one (0.025 cm) for this experiment.

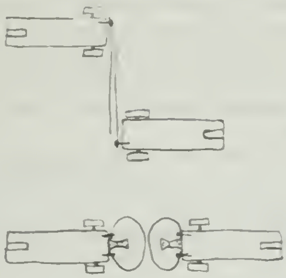


Fig. 1

You must limit the amount of compression of the bumpers by taping corks or rubber stoppers to the front of each cart, as shown in Fig. 1. Otherwise too violent a collision may bend the bumpers permanently and make them unusable for subsequent experiments. Of course, a collision in which the bumper touches the stoppers will not be elastic and should not be analyzed.

If the carts are moving too slowly, successive strobe images will be too close together to measure.

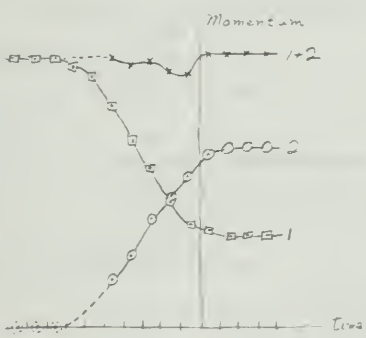


Fig. 2 Momentum of each cart, and total momentum, plotted against time.

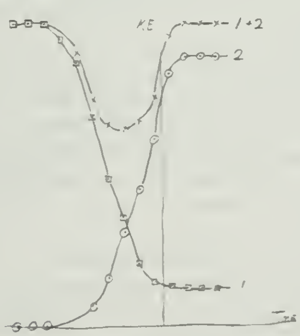


Fig. 3 Kinetic energy of each cart, and total kinetic energy, plotted against time.

It is essential to be able to identify simultaneous images of the two carts. The technique suggested to students in the *Handbook* is probably the simplest. Another is to close the camera shutter before either cart goes out of the field of view, and work backwards from the last image of each cart.

Transparency 23, slow collisions, is made from a photograph taken in this experiment. If your students don't succeed in getting a good photograph they could analyze data from the transparency in the same way.

Method B. Magnets

See *E3–3* for an alternative method of "floating" ring magnets.

The following photograph (Fig. 4) shows up the "right angle" law very nicely. In an elastic collision between two equal-mass objects, the angle between the two velocities after the collision is 90°. This is quite independent of the impact parameter (amount of "off-centeredness") of the collision.

Fig. 4 Photo of a two-dimensional "collision" between disc magnets. Strobe rate about 30 per sec.

Students should be aware of this right angle law, but need not necessarily understand the proof, which involves both conservation of momentum and of kinetic energy. The proof follows:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$m_A = m_B \text{ and } v_B = 0$$
$$\therefore v_A = v_A' + v_B'$$

The vectors v_A, v_A', v_B' can be represented by three sides of a triangle.

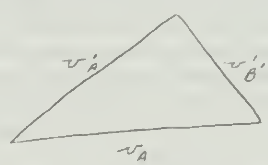


Fig. 5

When the masses are equal and the initial velocity of B is zero, the kinetic energy equation

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2$$

simplifies to

$$v_A^2 = v_A'^2 + v_B'^2$$

This is the Pythagorean equation for the three sides of a right triangle. If $v_A^2 = v_A'^2 + v_B'^2$, then the angle between v_A' and v_B' is 90°.

In fact there usually is a steady loss of kinetic energy as the magnet moves across. Notice from the curve plotted in Fig. 6 that the rate of energy loss is about constant. Although there is a significant drop in total kinetic energy during the interaction, the *KE* after the interaction is about what it would have been if no interaction had taken place.

This is not an easy experiment because the interaction is over so quickly, much more quickly than the collision between two carts with spring bumpers. In Fig. 6 (a plot of data from the photograph reproduced in Fig. 4), there is only one pair of values for velocities during the collision, but the kinetic energy minimum is nevertheless brief. The photograph was taken with the Stansi strobe rate at about 30/per sec.

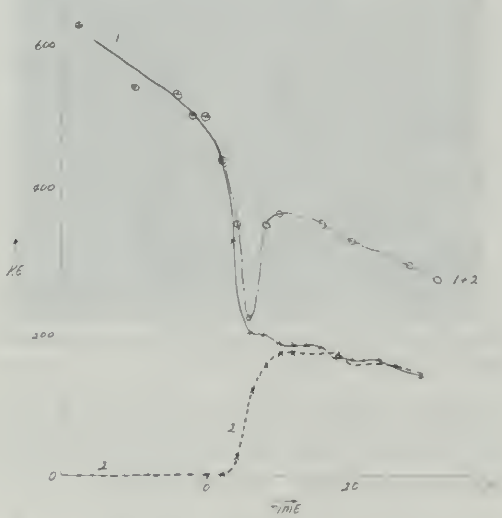


Fig. 6

Method C. Inclined Air Tracks

The expression $\Delta(PE) = -\Delta(KE)$ describes "ideal" behavior rarely met in practice. Students will probably find that the air track is far from perfect. There may be some loss due to poorly fitting gliders. There may even be a gain in energy! The air pressure is slightly higher at the inlet end of the track, which tends to "blow" the glider away from that end when the track is horizontal, or push it uphill when the inlet is at the low end. It is more important that students analyze their results honestly and suggest reasons for any difference between $\Delta(PE)$ and $-\Delta(KE)$ than that they "prove" that mechanical energy is conserved.

$$\tfrac{1}{2}mv^2 = ma_g h$$
$$\therefore v = \sqrt{2a_g h}$$

Note that v depends only on h, not on the slope of the track. In theory, therefore, v should be the same for all these trials. In practice we found that v_f is constant and is equal to $\sqrt{2a_g h}$ to within 10%.

Disc strobe technique with light source on the glider, or xenon strobe technique with a white or metallized drinking straw mounted on the glider may be used. For this experiment it is essential to measure velocity in meters per second. Therefore the strobe rate and the scale factor (preferably 10:1) must be known.

In one trial (with the air inlet at the low end of the track), we found a significant decrease in total energy. The kinetic energy increase was less than the potential energy decrease, presumably due to the air pressure effect mentioned above. When the air inlet was at the top of the track we found the reverse:

$$\Delta(KE) > -\Delta(PE)$$

When the glider is at rest at the bottom of the track, all the energy is stored (potential energy) in the stretched rubber band. The stretching of the rubber band as the glider slows down and stops can be seen much more easily if the band is made quite slack. This is analogous to the slow collision between two dynamics carts with spring bumpers.

The simplest way to measure the energy lost at each rebound would be to measure the height of successive rebounds.

$$E_1 = ma_g h_1$$
$$E_2 = ma_g h_2$$
$$\therefore \Delta E = ma_g \Delta h$$

These film loops could be used:

L18 One-dimensional collisions. I
L19 One-dimensional collisions. II
L21 Two-dimensional collisions. I
L22 Two-dimensional collisions. II
L31 A method of measuring energy: nails driven into wood
L32 Gravitational potential energy
L33 Kinetic energy
L34 Conservation of energy: pole vault
L35 Conservation of energy: aircraft takeoff

Discussion

The different observations made by the various student groups should be summarized in a class discussion after the experiments have been done. Some of the points to bring out are:

1. Unlike momentum, kinetic energy is not conserved in all the situations investigated. It is not conserved in elastic collisions or explosions, for example.
2. Although the kinetic energy after an interaction (for example, slow collision) may return to its initial value, there is a temporary disappearance of kinetic energy during the interaction, that is, while the springs are touching or while the magnets are close to each other. This is the time to

mention potential, or stored, energy and some of its forms, elastic and magnetic.

3. The air-track glider pushed up the track comes to rest momentarily near the top, increases speed coming down again, comes to rest momentarily at the rubber band, moves back up again, and so repeats the cycle. The original kinetic energy is converted to gravitational potential energy, back to kinetic energy, to elastic potential, back to kinetic energy, and so on.

Answers to questions

METHOD A

1. Yes.
2. The momentum was almost completely conserved.
3. Yes.
4. It would be displaced downward and have a smaller energy dip.
5. Yes. Student answers will vary.

METHOD B

1. Energy is almost conserved.
2. 10%.
3. Then energy is not conserved.
4. No. Some energy is stored in the magnetic fields.

METHOD C

1.-2 Note above discussion.

E3–6 CONSERVATION OF ENERGY. II

METHOD A: Film Loops

See notes on *Film Loops 19–36* beginning on *Handbook* page 157.

METHOD B: Stroboscopic Photographs

See previous notes on the photographs.

Equipment:

Any of *Film Loops 19–35*
Film loop projector
Stopwatch, or strip recorder ("dragstrip")
Stroboscopic photographs (in the *Handbook*)

E3–7 MEASURING THE SPEED OF A BULLET

Equipment:

Method A

Air track and glider
Gun and projectile
Juice can plus cotton wadding
Stopwatch
Meter stick

Method B

Can plus wadding, or soft block

If the bullet bounces back a bit, the cart will gain <u>more</u> momentum. From the relation

$$v = v'\frac{M + m}{m},$$

we have

$$\begin{aligned} v &= \frac{1.57 \times 10^{-1}}{3.5 \times 10^{-4}} \times 9 \times 10^{-2} \sqrt{\frac{10}{1.78}} \\ &= 0.404 \times 10^{1} \sqrt{5.6} \\ &= 4.04 \times 2.36 \\ &= 9.55 \\ &= 9.6 \text{ m/sec} \end{aligned}$$

Answers to questions

METHOD A:

1. Student answer.
2. If the bullet bounces back a bit, the cart will gain <u>more</u> momentum. Using $v = \frac{(M + m)v'}{m}$ would lead to too high a value of v.
3. (a) Use time-of-flight technique with two beams of light and two photocells as described in notes on the use of oscilloscope, *Resource Book*.
   (b) Fire horizontally and measure the horizontal distance traveled as it falls to ground.
   (c) Fire vertically upwards; measure maximum height attained.
   (d) Time-of-flight measurement described in *Handbook* Unit 1, page 30.

METHOD B:

1. Student answer.
2. A large fraction of the bullet's kinetic energy has been converted into work tearing apart the wood and generating heat.
3. Same as 3 above.

Using Method B, the Ballistic Pendulum, a light can be attached to a pendulum and photographed. From the photograph, the distance d may be measured easily and v computed, given the masses of bullet and block.

E3–8 ENERGY ANALYSIS OF A PENDULUM SWING

Equipment:

Pendulum (about 1 m long)
Polaroid Land camera
AC blinky, or light source and motor strobe

This is a relatively straightforward investigation that relates the kinetic and potential energies of a pendulum bob, and suggests that the total energy is constant. The potential energy of the raised bob is calculated from:

$$PE = ma_g \Delta h$$

Since the mass is constant during the experiment, the relative kinetic and potential energies can be derived by neglecting the mass.

The largest uncertainty is likely to be in the measurement of the velocity of the bob at the bottom of the swing.

E3–9 LEAST ENERGY

Equipment:

1 m of beaded chain
Graph paper, ruler
Weights and string

The principle of least energy, illustrated by this laboratory experiment, is a very useful principle. As Feynman states in his *Lectures*, Vol. 2, page 2, "The average kinetic energy less the average potential energy is as little as possible for the path of an object going from one point to another." Although sometimes termed the principle of least action, the more general idea of least energy applies to soap bubbles as well as hanging chains.

The hanging chain that has a uniform density, similar to a cable or telephone wire, takes the shape of a catenary. It is not a parabola. The article "Suspension Bridges" by Thomas B. Greenslade, Jr., in the *Physics Teacher* for January 1974 discusses the difference. When a dead weight like a roadway is supported at regular horizontal intervals along a suspended cable, the curve through the points of suspension do fall along a parabola. The two curves have similar but not identical shapes.

A catenary curve has the least potential energy.

The paper by Greenslade suggests a variety of interesting and relatively simple activities of potential interest to students. Building real as well as mathematical models may prove stimulating.

E3–10 TEMPERATURE AND THERMOMETERS

Equipment:

As many as possible of the following:

- Uncalibrated mercury in glass thermometer
- Gas-pressure thermometer
- Gas-volume thermometer
- Thermistor plus amplifier/power supply plus meter
- Thermocouple
- Large baths of boiling water, ice water, and four or five water baths at intermediate temperatures
- Millimeter scales (not plastic) to attach to uncalibrated thermometers
- As many different gases (CO_2, N_2, O_2, N_2O, etc.) as possible

Temperature

Students usually have not thought about what temperature means, are not aware that any problem of defining temperature exists, and may even be unwilling to admit that it does. Indeed many texts dismiss the problem with a statement like, "The temperature of a body is the scale reading on a suitable thermometer."

This experiment will have succeeded if the students have been brought, possibly for the first time, to think about the nature of the concept of *temperature*. The object of the experiment is NOT to have students calibrate other thermometers against the mercury-in-glass thermometer, but to realize that on first consideration the other devices are in themselves just as valid for use as thermometers (and for use when constructing temperature scales). It is only much later, when we have some theoretical basis for our ideas of heat and temperature, that we will have any reason other than convenience for choosing one device over another.

What follows is a more than usually lengthy outline of the sort of prelab and postlab discussion that we believe would make the students aware of the problem. The title we have given to the experiment reflects the order in which the subject has to be developed: from the crude subjective sensation of hotness and coldness, through the invention of some objective device sensitive to changes in hotness, to the establishment of a temperature scale.

The major point to bring out is this. Temperature, like all other ideas that have been of great value to science, is an invented concept. Acceleration is another example, which could have been defined as $\Delta v/\Delta s$ rather than $\Delta v/\Delta t$, and indeed Galileo considered this. But $\Delta v/\Delta t$ turns out to be a much more useful definition.

One point may summarize the whole problem. We frequently say and teach that Charles (or Gay-Lussac) found by experiment that the volume of a sample of gas is proportional to its absolute temperature. He probably used a mercury expansion thermometer. Suppose he had used a different sort of thermometer (one whose expansion was very nonlinear with respect to the expansion of mercury). What kind of "law" would he have discovered?

Galileo used a "thermoscope" consisting of a glass flask with a long neck dipping into water. The water level in the tube rose or fell as the air in the bulb was cooled or heated. You might want to add this to the variety of thermometers used in the experiment.

Any convenient, easily measured property of any substance that changes with hotness (the original, subjective sensation from which we start) could be used to construct a thermometer, to define temperature quantitatively. Notice that we cannot at this stage of the argument say that the expansion of mercury (for instance) is a good system to choose "because mercury expands linearly with temperature." Only the reverse is possible: We could use the mercury expansion system to define temperature. But the choice is not obvious and a wide variety of systems is available.

An optical pyrometer measures the temperature of hot bodies by comparing the light they emit with a standard (hot wire).

We could not put a gas bulb between two colliding carts (or insert it under a patient's tongue) and hope to measure a temperature rise. We cannot use a mercury thermometer below the freezing point or above the boiling point of mercury.

The original fixed points on the Fahrenheit scale were: 0°, the temperature of a mixture of equal parts of ice and salt, and 100°, human body temperature. Today, the Fahrenheit scale is defined by: 32°F, the temperature at which ice melts, and 212°F, the temperature at which water boils.

There is no reason at this stage, either theoretical or experimental, to suppose that the two thermometers should necessarily agree. There is a small but significant difference between a mercury expansion thermometer and an alcohol expansion thermometer. There is a very striking difference between the thermistor thermometer and almost any other thermometer.

Use as wide a variety of devices as possible: uncalibrated mercury, uncalibrated alcohol, gas volume, gas pressure, thermistor, thermocouple, etc. Assign a pair of devices to each group of students.

Plotting a graph helps students to distinguish between systematic differences and random ones.

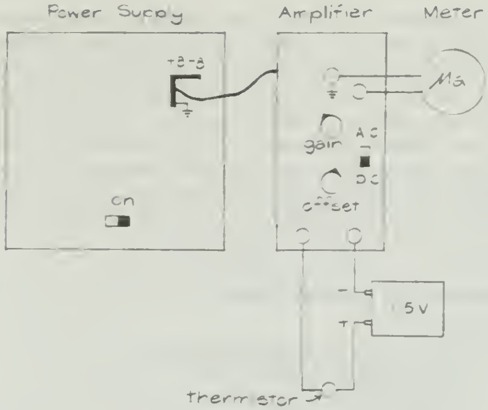
Have one ice bath and one boiling water bath and at least five other numbered baths covering the intermediate range. Note tha. except for the ice and boiling water, the baths need not be maintained at constant temperature. Each group must, of course, take the readings of its two devices at the same time.

Thermometers

A. Uncalibrated mercury-in-glass. (For example, Cenco cat. no. 77320). The volume of a confined sample of mercury is indicated by a thin thread of the sample that is free to rise in an empty tube. A centimeter scale should be fixed along the thermometer. A 30-cm wooden ruler (with the brass strip removed) is satisfactory when attached with rubber bands as in the figure below. The extra length provides a convenient handle. It is important that the scale be close enough to the mercury thread to facilitate reliable readings.

B. Uncalibrated alcohol-in-glass. (For example, Macalaster cat. no. 2666). The same principle is illustrated as in A. A 15-cm steel scale is long enough to serve here. Tape does not hold up well in boiling water. Rubber bands may be used, but care should be taken to bind them tightly enough to the thermometer and scale to prevent them from slipping out of alignment. A small stick or glass rod can be bound together with the thermometer and scale to help position the scale close to the liquid thread. With some maneuvering, the scale can be made to show through the glass directly behind the liquid thread.

C. Thermistor. The electrical conductivity of a semiconductor decreases rapidly with increase in temperature. The "100K" thermistor supplied by Damon has a resistance of about 400KΩ at 0°C, 100KΩ at 25°C, and 4KΩ at 100°C. Use a volt-ohm-milliammeter to measure resistance directly, or connect a 6-V dry cell and a milliammeter in series with the thermistor and use current as a measure

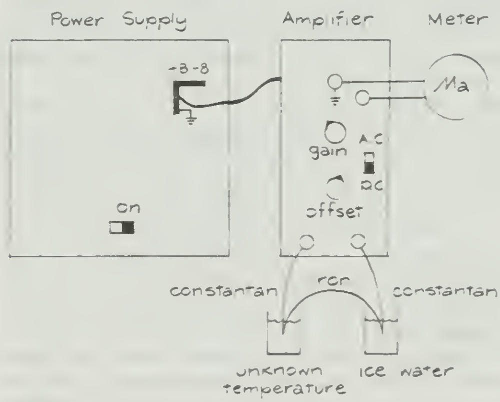


of temperature. The maximum current will be about 1.5 mA at 100°C.

It would also be possible to use the Damon amplifier along with the thermistor as is indicated in the above schematic diagram. Caution: Turn the ac-dc switch to dc, wire the microammeter across the terminals marked meter output, and use the 1.5-V battery used in the pen-light source. Then the OFF SET control of the amplifier can be adjusted so that the microammeter will read zero for any temperature sampling.

The thermistor and leads to it must be insulated to prevent conduction through the water. Slide a length of "spaghetti" or shrink tubing over it, or, if you use the thermistor embedded in lead, apply a coating of nail polish.

D. Thermocouple. The amplifier unit can also be used to detect the difference in voltage at the two junctions of a pair of wires made of different metals. An iron–constantan couple develops about 5 mV for a temperature difference of 100°C. With the amplifier GAIN set to 100 the maximum output voltage will be about 0.5 V.



You could simplify the circuit by connecting the iron wire directly to the amplifier terminal. This contact then becomes the reference junction and if it is less than room temperature, the voltage will be negative.

E. Gas Volume. Use a piece of capillary tubing closed at one end with a plug of silicone rubber cement. A mercury index traps a fixed amount of air (see Part B of *E3-14,* The Behavior of Gases). Use as many different gases as possible.

F. Gas Pressure. "John's law" apparatus (a toilet reservoir float plus absolute pressure gauge, for example, Welch Scientific, cat. no. 1602) can be used here. Use as many different gases as possible.

E3–11 CALORIMETRY

Equipment:

- Foam plastic (Styrofoam) cups
- Thermometer 0-100° C
- Hot and cold water
- Ice (cracked or small cubes)
- Balance
- Metal samples

This experiment introduces students to the idea that heat flow can be measured by observing the change in temperature of some standard substance, for example, liquid water. Using this method, students learn that measured amounts of heat produce differnt temperature changes in different substances. Also, students measure the latent heat of ice and the specific heat of a metal.

The foam plastic (Styrofoam) cups are extremely useful for heat experiments and are inexpensive. Keep a good supply on hand, and encourage students to use them to improvise experiments other than the ones specifically described here.

These cups are such good insulators that experiments can generally be done with the cup uncovered. For example, when water originally 15°C below room temperature is placed in an uncovered cup, it increases in temperature by less than 0.2 °C/min. Since the marks on thermometer scales are usually a degree or more apart, the error introduced by heat leakage in the brief experiments suggested here is not much more than the uncertainty in the temperature measurement. The cups are also very light (between 2 and 3 g) and therefore absorb very little heat. It is not necessary at this stage to require that students correct their calculations for this heat loss although the correction could be made later as a separate experiment.

The preliminary experiment establishes the approximate rate at which heat leaks to or from the calorimeters, and also prepares students for a discussion of latent heat later in the period. This preliminary work can be started in the beginning of the period and carried along with the other experiments, or can be done the day before. It will give students some feeling for the insulating characteristics of their cups and also impress them with the fact that a water-and-ice mixture remains at exactly 0° C until all the ice is melted. Caution students that they should stir the water gently with the thermometer before taking each reading.

Background

The calorimeter was first used quantitatively to measure heat by Joseph Black (1728–1799). He made the three assumptions about the nature of heat outlined in the *Handbook.*

Underlying the third assumption is the idea that temperature is a quantity that can be measured. The adoption of a temperature scale and basic improvements in thermometer design in the early eighteenth century by Fahrenheit and others made Black's work possible.

Around the beginning of the nineteenth century, the first assumption was shown to be inadequate to explain the relationship between work and heat. However, the caloric theory, as it was called, was extremely plausible; it served very well in developing early ideas about heat, and to this day is still implicit in many intuitive ideas used in calorimetry.

Mixing Hot and Cold Liquids

Students know from experience that when the two quantities of water are put together, the temperature of the resulting mixture will be between the two starting temperatures. Ask if they can predict the exact temperature of the mixture. This will be easy if the cups of hot and cold water have identical masses, but not so easy if their masses differ.

When unequal quantities of hot and cold water are mixed, the relationship

$$m_h \Delta t_h = m_c \Delta t_c$$

holds.

The calorie is the CGS unit of heat. It is a convenient unit for the calorimeter experiments described here, but is too small for most practical applications. The MKS unit, the kilocalorie (kcal or Cal), is the heat that enters or leaves 1 kg of water when the temperature changes by 1° C. This unit is the same as the Calorie used by dietitians.

Answers to questions

1.-17. Student answers.

Measuring Heat Capacity

The next reasonable question to ask is whether the constant of proportionality c in the heat equation is the same for other materials as it is for water. Provide students with small samples of various metals and ask them to predict the equilibrium temperature when a hot metal sample is mixed with cold water in a calorimeter.

The thread holding the metal samples should be tied to a wooden stick so that hands do not get too close to the steam.

Specific Heat Capacities of Metals

aluminum	0.22 cal g C°
brass	0.08
copper	0.093
iron	0.11
lead	0.031
mercury	0.033

Measuring Latent Heat

The ice-and-water mixture may still be at 0° C. Even if the ice has all melted, it is probably cooler than the cup originally containing only ice water. Since heat is leaking into both cups, what is happening to the heat that enters the ice-and-water mixture? Because this added heat does not produce a change in temperature of the water while the ice is melting, this heat is called latent, or hidden, heat.

After the ice melts, the resulting ice water also absorbs heat as it warms up to its final temperature. The necessity of considering this additional amount of heat in the equation seems to be difficult for many students. Perhaps a comparison to compounding interest in a bank account, getting interest on interest, would be helpful.

If students use ordinary ice cubes, their values will generally be less than the accepted value of 80 cal/g. This is probably due to the presence of air bubbles and impurities. Excellent results can be obtained using the plastic cups if you make ice cubes from distilled water. Start with the cup about half full of water at a few degrees above room temperature and record its mass. Add an ice cube that has been placed in a container of cold water for a few minutes so its temperature will be 0° C, rather than about $-10°$ C as it comes from the freezer. Dry off the ice with a paper towel to avoid transferring excess water to the calorimeter.

Rate of Cooling

Newton's law of cooling predicts that the rate of cooling is approximately proportional to the difference in temperature of a sample and its surroundings. This is an empirical result due to the combination of several physical processes. Students should be able to report qualitatively that the greater the temperature difference between the sample and its surroundings the quicker its fall (or rise) of temperature. But don't expect a formal statement of Newton's law.

Students who did not do the experiment on latent heat will find that the ice water with ice in it warmed up less than the ice water. Use this observation to introduce latent heat, or let those students who did that experiment report their results to the rest of the class.

Because the conditions of each experiment (quantity of water, amount of stirring, etc.) are different, students can only get an estimate of the error due to loss of heat from the calorimeter. In one trial, 100 g of water 25° C above room temperature lost about 2° C (ΔH = 200 cal) per minute.

Questions for Discussion

1. Students could be urged to look for other factors that can introduce errors in calorimeter experiments, and to suggest possible remedies. Here are a few examples:

(a) The thermometer must take up some of the heat in the calorimeter. If we knew the specific heat of mercury and glass and the mass of each that is immersed in the water, we could make allowance for this factor. This would be very difficult to do because the relative amounts of glass and mercury are unknown. Perhaps it would be possible to measure the heat capacity of a thermometer directly by experiment, but in any case the error resulting from neglecting this is quite small.

(b) Heat is lost while samples are being transferred: the hot water and metal samples will lose some heat while being transferred; some ice will melt after it has been dried but before it is put in the calorimeter. These will be difficult to estimate, but they make relatively small contributions to the total error.

(c) The major source of uncertainty is probably the thermometer reading. Note that the larger the change in temperature during the experiment, the smaller will be the fractional error in Δt due to uncertainty in reading the thermometer.

Another source of uncertainty in the thermometer readings is the way in which the thermometers are calibrated. Some are calibrated to read correctly when the entire thermometer is immersed, while others should have only the bulb in the liquid. In the latter case, it is possible to correct for the length of the exposed thread of mercury. This correction will, however, be very small.

2. After students have measured the latent heat of melting ice, they should consider ways to measure the latent heat of steam. Section 10.6 of the *Text* emphasizes the importance of Watt's improvement of the steam engine, which was based on his realization that the condensation of steam releases a large amount of heat. In fact, the latent heat of vaporization of water is about seven times the latent heat of fusion of ice. The latent heat is measured by bubbling steam through cold water in a calorimeter cup; the procedure is the same as for ice. However, you may not wish to expose students to the possibility of burns from the live steam.

The following notes on *Experiment 3–12* suggest a relatively safe way to estimate the heat of vaporization of water.

E3–12 ICE CALORIMETRY

Equipment:

Three Styrofoam cups
Small light bulb, wiring, electrical source
Dye (such as India ink)

In this experiment, the student becomes acquainted with the concept of heat of fusion, or melting. Only the ice–water system is explored, but students should expect that similar heat exchanges occur when other materials have a change of state.

Like Joseph Black, they may also conclude that a sizable quantity of heat is required to change water into steam. Black noted that a pan of water could be heated to boiling, but that continued heating was required to boil off the water. The water did not all instantly turn into steam when its temperature reached the boiling point.

Students might speculate on the history of the earth and possible technological consequences if water had no heats of fusion and of vaporization. Some rough estimate of this heat of vaporization for water can be made when a pan of water is put on a constant heat source. A time sequence of thermometer readings as the known quantity of water heats up will define the rate of heat input. The amount of time required to boil off the water then indicates the relative quantity of heat needed to vaporize each unit of water. Again, students should be encouraged to assume that other materials have similar heats of vaporization.

E3–13 MONTE CARLO EXPERIMENT ON MOLECULAR COLLISIONS

Equipment:

Game I

- 12 marbles
- Board studded with nails
- Board marked off with coordinates
- Some sticky wax

Game II

- Large piece of graph paper on a drawing board

Both the formula for mean free path and that for viscosity are approximate. A rigorous derivation may be found in an intermediate text on kinetic theory.

You cannot give a precise definition of randomness nor a clear-cut exposition of this technique.

You might get a random set of numbers by opening a metropolitan telephone directory and taking the last two digits from the call numbers as they occur. Another way is to take a linear expression, say $11x + 7$. Plug in a number for x, say 54. Compute the value of the expression, namely $11 \times 54 + 7 = 601$. Take the last two digits of this value for the next number of the random series and proceed as before. Thus, $11 \times 01 + 7 = 18$, $11 \times 18 + 7 = 205$, etc. The series is 54, 1, 18, 5, 62, 89, 86, 53 . . . etc. The table of random numbers given is from *Statistical Tables*, by Fisher and Yates. The first digit of the number taken from the table is the abscissa.

In Game I, both target molecules and bombarding molecule have the same finite diameter; but in Game II only the target molecules have a finite diameter. The bombarding molecule is a test particle with no diameter.

Sample Results (Game I)

Target marbles were set up, using random numbers, on a 6-by-6 grid (5-cm spacing). Bombarding marbles were released through the "pinball machine" array of nails illustrated in the *Handbook*. Three experiments of 50 trials each were made with N (number of target marbles) = 8. The measured length of the line of 8 marbles is 11.1 cm.

Thus, $d = \dfrac{11.1}{8} = 1.4$ cm. The target field should be at least 25 cm from the launching board, otherwise the bombarding marbles will not be moving in parallel and too many hits will result. The observed ratio, $\dfrac{H}{T}$ or R, of hits to total trials was:

$$\frac{37}{50}, \frac{31}{50}, \frac{35}{50}$$

The mean ratio is $\dfrac{34 \pm 3}{50}$.

$$\text{Then } d = \frac{HD}{2NT} = \frac{HD}{T2N} = \frac{(34 \pm 3)}{50} \times \frac{32.5 \text{ cm}}{2 \times 8}$$
$$= 1.38 \pm 0.13 \text{ cm}$$

Shielding effect becomes important as N is increased.

For $N = 12$, $\dfrac{H}{T} = \dfrac{88}{150}$

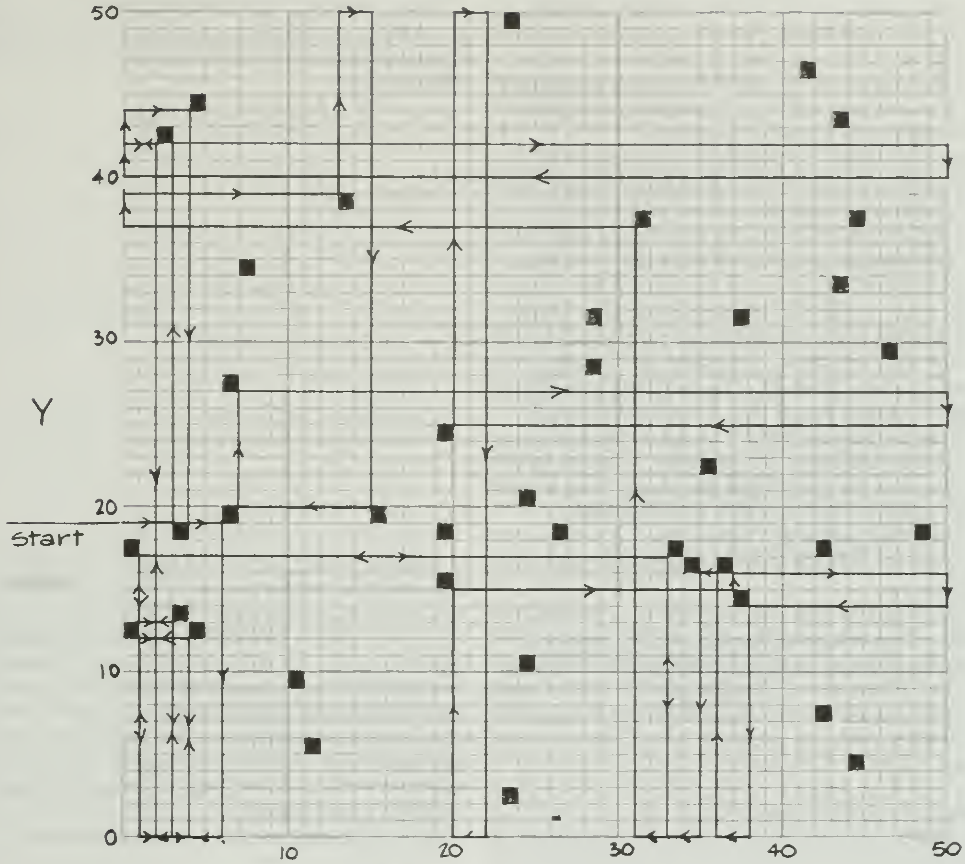
giving $d = \dfrac{88}{150} \times \dfrac{32.5 \text{ cm}}{2 \times 12} = 0.80$ cm.

For $N = 20$, $\dfrac{H}{T}$ was $\dfrac{43}{50}$

giving $d = \dfrac{43}{50} \times \dfrac{32.5 \text{ cm}}{2 \times 20} = 0.70$ cm.

Wandering horse in a snow-covered field.

Sample Results (Game II)



Random array of square target molecules.

Answers to questions

1.-2. Student answers.

3. $L = \dfrac{\text{sum of path lengths}}{\text{number of paths}} = \dfrac{1{,}354}{50} = 27.1$ units

4. $L = \dfrac{A}{Nd}$

$d = \dfrac{A}{NL} = \dfrac{(2{,}500)}{(27)(40)} = 2.31$ units

5. The approximation by means of the Clausius relationship is half as much as it should be. Perhaps what is wrong is that a dimensionless particle is used rather than another square molecule. Therefore, if $L = \dfrac{A}{Nd}$ is changed to $L = \dfrac{A}{N(2d)}$, the calculation in 4 will turn out to be 1 unit. Since the incident molecule must have a dimension, the result is a reduction in the mean free path. If the molecular diameters are equal, the mean free path is halved.

E3–14 BEHAVIOR OF GASES

Equipment:

I. Boyle's law apparatus: Either conventional J-tube,
or simple syringe type
(Macalaster #30220; Linco #6250; Damon #99129)
Set of weights for use with syringe, hooked or flat
Other gases such as CO_2, N_2, O_2, N_2O if possible

II. Capillary tubes
Mercury
Silicone rubber sealant
Millimeter scales (metal or wood, not plastic)
Beaker of water
Bunsen burner or hot plate
Thermometers 0–100° C

For an extended account of Boyle's work see *Harvard Case Histories in Experimental Science,* edited by J. B. Conant. For an edited, annotated version see *Great Experiments in Physics,* by Morris Shamos.

Detailed instructions for use of the conventional Boyle's law apparatus are found in most physics and chemistry laboratory manuals

The simple plastic syringes are preferable because they are inexpensive and easy to use; students can obtain data from them with a minimum of preliminary discussion. No mercury is used; nor need one explain the relationship between height of mercury column and pressure. Here pressure is simply force (weight on piston) divided by the area of the piston. You will probably need to remind students that weight is measured in newtons and is equal to mass times a_g. The numbers written on "weights" (100 g, 500 g, etc.) are actually masses.

The data obtained with this equipment are not very precise, but since the primary purpose of the experiment is to show how the data are analyzed and interpreted, the syringes are adequate.

Grease the piston with glycerine or vaseline to reduce friction. Make sure that no water gets into the cylinder.

Another way to improve the data is to record two readings for each force applied to the piston: one when the plunger is lifted slightly and released, and the other when the plunger is depressed slightly and released. The mean of the two readings is the value used.

I. Volume and Pressure

To get a P_w versus V plot that is convincingly not a straight line, one must work over a rather wide pressure range using weights of up to a few kilograms. But to get a good value for the intercept of the P_w versus $1/V$ plot, data at relatively low P_w values are needed (below 1 kg of added weight). Try to have some students work at both high and low pressures.

A common difficulty students encounter in working with gases is the tendency to confuse absolute pressure with gauge pressure. The procedure described is designed to show that Boyle's law as it is usually written,

$$PV = \text{constant}$$

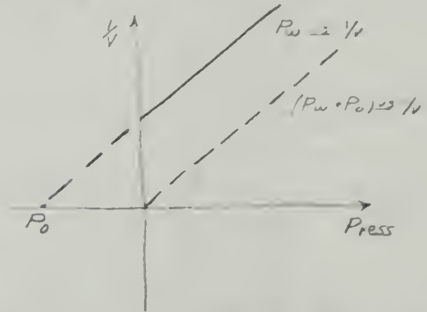
is only true if P is the *absolute* pressure. Students vary the force exerted on an air sample by placing weights on the piston. Then they are to convert these forces to pressure by measuring the diameter of the piston and computing its area. This step would not be necessary if the purpose of the experiment were simply to demonstrate a linear relationship between $1/V$ and P. However, when students use values for P obtained from their data, not taking into account atmospheric pressure, they find that their graph of $1/V$ against P does not pass through the origin. Instead it should pass through the P-axis at about -10 N/cm. Be sure students understand the significance of this intercept value.

Point out that if a constant term ($P_o = 10$ N/cm) is added to each value of the calculated pressure P_w, the graph would pass through the origin. This means that

$$(P_w + P_o) = k\frac{1}{V}$$

or

$$(P_w + P_o)V = \text{constant}$$



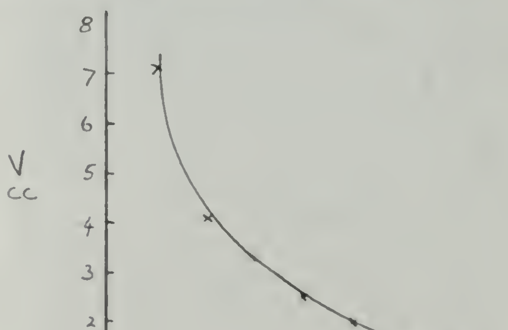
Explain that P_o is the additional, constant pressure exerted on the piston by the atmosphere, and that the total pressure of the gas in the syringe is ($P_w + P_o$). This is called the absolute pressure. The quantity P_w is usually called gauge pressure because it is the pressure that is most commonly measured by a pressure gauge.

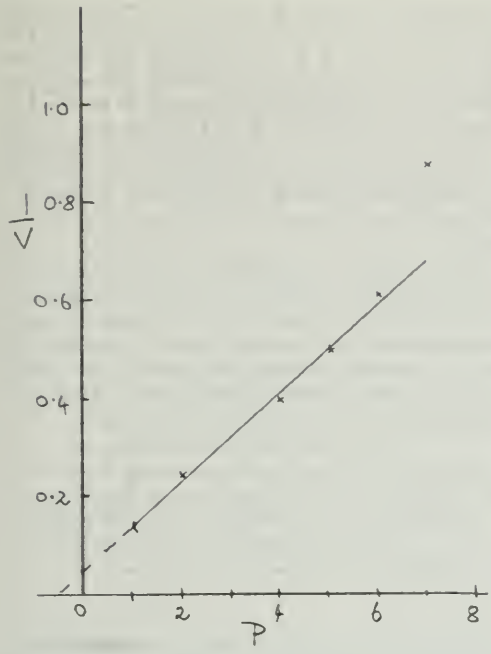
The similar behavior of different gases is an important point because it suggests that the same model (kinetic theory) could be used to explain the behavior of all gases.

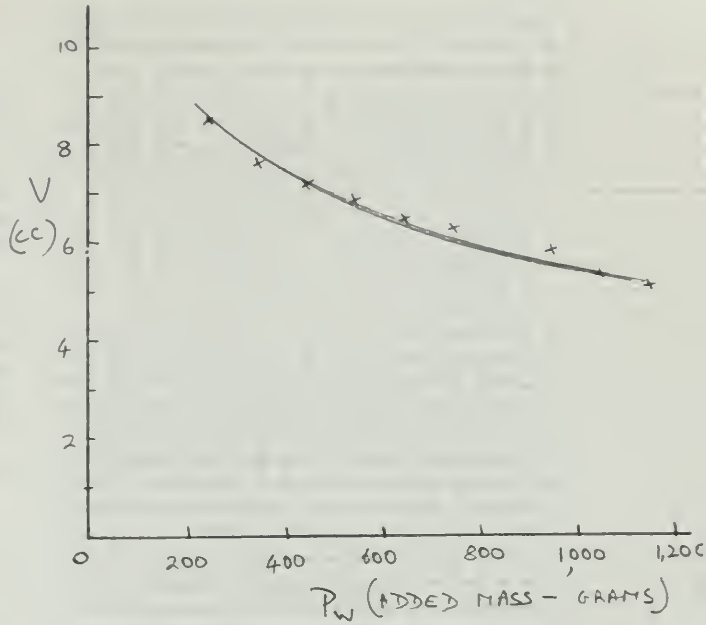
Have different students work with different gases if at all possible.

Answers to questions

Various Ways of Plotting the Results.





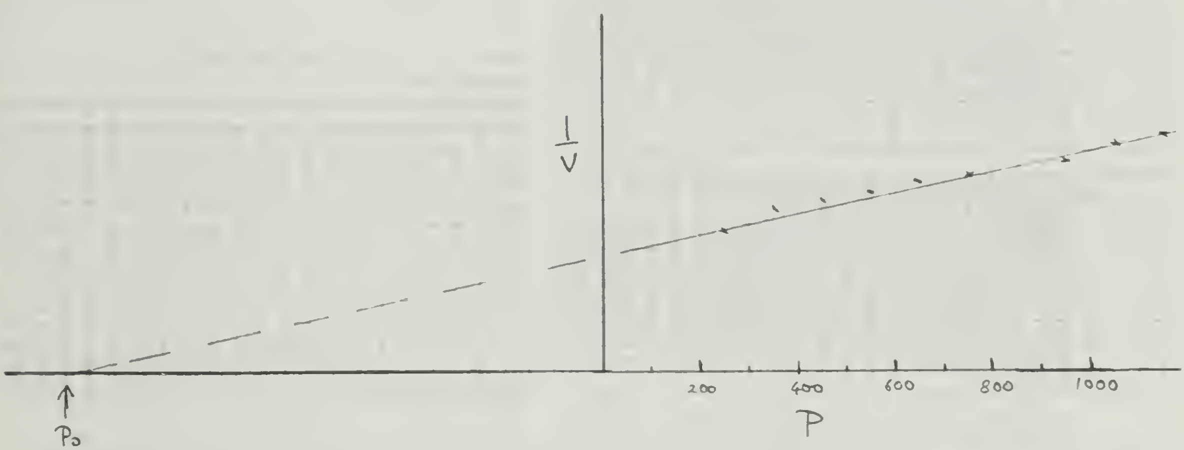


II. Volume and Temperature

In place of this experiment, you can substitute a quick and effective demonstration. A small flask or test tube is fitted with a one-hole stopper. A long capillary tube with a mercury pellet near one end is pushed through the stopper. The flask must be perfectly dry and the mercury pellet near the lower end of the tube at room temperature. The expansion of air is made vividly apparent as the flask is heated in a water bath.

Of course, if you want to make quantitative measurements of change in volume with temperature you must relate the volume of the flask to the volume (per unit length) of the capillary tube.

The plastic syringes are supported by a ringstand and immersed in a beaker of cold water. Students record temperatures and volumes as they slowly heat the water. Friction between the piston and syringe is again the major source of uncertainty in measuring the volume. The change in temperature must be very gradual and the water continuously stirred so that the air in the syringe will be at about the same temperature as the water. This apparatus will do little more than show that air expands as it is heated. Because of the large amount of friction in the syringe, no pretense should be made that students can determine the value of absolute zero by extrapolating their V–T curve to find the intercept on the T axis.



Detailed directions are given below for preparing the constant-pressure gas thermometers referred to in the *Handbook.*

Answers to questions

4. Even if a graph of V against T is a straight line, this shows only that air expands with increasing temperature in somewhat the same way that mercury in the glass thermometer does. If the optional experiment "Temperature and Thermometers" (*E3–10*) is not done, there should be a brief discussion of temperature scales as outlined earlier.
5. The behavior of a solid is usually linear over only a limited temperature range, which, for some solids, is too small to be of any use in thermometers. Within the limits of our experimental error any liquid will give a straight line if the temperature is not near its boiling or freezing point.

   The graphs for gases are more nearly straight than for liquids, if the temperature is not near the point where the gas liquifies.

   The failure of a material to expand or contract in a perfectly linear manner is largely a consequence of the forces of attraction acting between its molecules. Since these forces are least in gases and greatest in solids, gases provide the most nearly perfect thermometers, but not the most convenient in size. Liquids are more convenient, and for most purposes, sufficiently "linear."
6.-7. The lower limit is reached when the gas is so cold that the molecules are motionless and in contact with each other like marbles in a box. While we cannot reach this state of affairs, our straight-line graph of T versus $1/V$ will identify this temperature when it is extrapolated to very small volume.
8. When weights are added to the piston and the pressure increased, the temperature goes up because work has been done on the gas. However, since the sample in our case is small and is not in an insulated container, it quickly returns to room temperature.
9. The relationship between volume and temperature will continue to be a linear one within the accuracy of our experiment as long as the temperature is well above that at which the gas liquifies.

Equipment Note: Assembling a Constant-Pressure Gas Thermometer

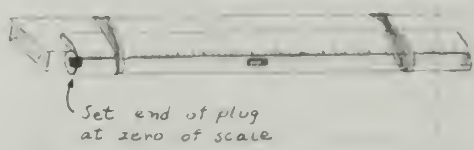
About 15 cm of capillary tube makes a thermometer of convenient size. The dimensions of the tube are not critical, but it is very important that the bore be dry. It can be dried by heating, by rinsing with alcohol and waving rapidly, or better still, by connecting it to a vacuum pump for a few moments.

The dry capillary tube is dipped into a container of mercury, and the end sealed with the fingertip as the tube is withdrawn, so that a pellet of mercury remains in the lower end of the tube.

The tube is held at an angle and the end tapped gently on a hard surface until the mercury pellet slides to about the center of the tube.

One end of the tube is sealed with a dab of silicone sealant. Some of the sealant will go up the bore, but this is perfectly all right. The sealant is easily set by immersing it in boiling water for a few moments.

A scale now must be positioned along the completed tube. The scale will be directly over the bore if a stick is placed as a spacer next to the tube and bound together with rubber bands. A long stick makes a convenient handle. The zero of the scale should be aligned carefully with the end of the gas column, that is, the end of the silicone steel.



In use, the thermometer should be completely immersed in whatever one wishes to measure the temperature of, and the end tapped against the side of the container gently to allow the mercury to slide to its final resting place.

To fill the thermometer with some gas other than air, connect a capillary tube to the gas supply by a short length of rubber tubing. Open the gas valve slightly to flush out the tube, and fill it with gas. Detach the rubber tube. Pick up a pellet of mercury as before. Keep your finger over the far end of the tube while you replace the rubber tube (gas valve shut). Lay the thermometer down flat. Work the mercury pellet to the center of the tube and, opening the gas valve slightly and very cautiously, release your finger for an instant.

Remove from the gas supply, seal off the end that was connected to the gas supply with silicone, and attach the scale.

E3–15 WAVE PROPERTIES

Special Note:

Experiment 3–15 begins a series of laboratory activities dealing with waves. These experiments are not only interesting in themselves, but they build toward a discussion of the nature of light as interference phenomena lead to a wave theory for light. Gradually the student progresses from obvious pulses and waves in springs to waves in a liquid, then to audible sound waves, and on to inaudible ultrasonic waves. At each step, increased reliance is made upon instrumental detection. The common properties of wavelength, standing waves, and interferences should be stressed. In Units 4 and 5, other discussions and experiments will treat the similar properties of light.

Equipment:

Waves in springs

"Slinky" spring

8–10 m of rope (clothesline) or a different spring that can be stretched to this length

Waves in a ripple tank

Ripple tank setup, complete with light source, beaches, and variable-speed wave generator with straight wave and two point sources

Paraffin blocks for wave barriers

Rubber tubing (about 50 cm) for use as a wave "mirror"

dowel (20 cm) or broomstick handle (30–50 cm long) to generate straight pulses

Sheet of glass with one edge 30–50 cm long to fit in tank, with washers as corner supports to adjust height (for refraction)

Hand-driven stroboscope (or motor stroboscope to be driven by hand)

Meter stick

Clock or watch with second hand

Large beaker or jar for filling and emptying ripple tank

Students should experiment with longitudinal pulses long enough to appreciate the difference between these and the transverse pulses with which the rest of the experiment deals.

Answers to questions

1. The amplitude changes because of friction: The amplitude of the pulse is a function of its energy and, as the energy is dissipated by friction, the amplitude of the pulse decreases.
2. That depends on whether the far end is free or fixed. Presumably the other person is holding it down, so that the reflected pulse will be upside down.
3. The speed of a wave along a slinky, rope, or similar device, is proportional to

$$\sqrt{[\text{tension } (T)/\text{mass per unit length } (\mu)]}.$$

Therefore, increasing the tension should increase the speed of the wave along the slinky.

4. The conclusions ought to be consistent with $v \propto \sqrt{T/\mu}$, but of course one would not expect quantitative results here.

5.-7. The pulses pass through one another without being altered. During their collision, the location of a point on the spring at any instant is simply the vector sum of the two separate locations that the point would have occupied if the pulses had passed over it separately.

8. They are inversely proportional. More precisely, their product, $f\lambda$, is a constant whose value is the velocity of the waves.

E3–16 WAVES IN A RIPPLE TANK

Answers to questions

1. Their directions make equal angles with the perpendicular to the barrier.
2. If the barrier is concave, the reflected straight waves become curved and converge on a small area or a point.
3. A pulse started at the focus should look like a movie of the reflected pulse run backward.

It might be interesting for some of your students to note that the intensity of the diffraction pattern has a graph of roughly the following shape:

4. The wave speed is less over the shallow area.
5. The wave direction is turned away toward a perpendicular to the boundary.
6. The angle with the perpendicular and the speed both decrease as the wave crosses the boundary into the shallower area. More precisely

$$\frac{V_1}{V_2} = \frac{\sin \theta_1}{\sin \theta_2}$$

where θ is the angle the wave direction makes with the perpendicular to the boundary.

7. Since $f\lambda$ is a constant, increasing one quantity decreases the other.
8. See figures on *Text* pages 364 and 373.
9. As the wavelength increases, the pattern emerging from the slits opens out like an unfolding fan, and the nodal lines make larger angles with the direction perpendicular to the barrier.
10. The fewer wavelengths along the width of the barrier the less the distortion of the wave train "downstream" from it.
11. The smaller the opening the greater the angle of spread.
12. The longer the wavelength the greater the angle of spread.

E3–17 MEASURING WAVELENGTH

Three methods of measuring wavelength are described. The first method (a) utilizes a stroboscope to "freeze" a pattern of waves. Measurement of the distance between crests will provide a value for the wavelength. The second method (b) employs standing waves.

The third method (c) utilizes the nodal and antinodal lines formed when two sources have the same frequency of vibration. Along antinodal lines, the distance to the two sources differ by a whole number of wavelengths. Students should start with a central antinodal line, such as point A in the *Handbook* illustration, and then measure the distances to the two sources at increasing angles off center.

The wavelengths obtained will vary with the setups.

E3–18 SOUND

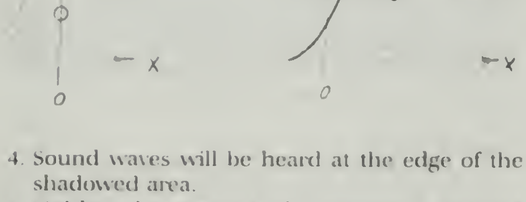
Equipment:

- Amplifier/power supply
- Oscillator plug-in unit
- 2 small loudspeakers
- Funnel (or thistle tube) and 50 cm of rubber tubing to fit it (for ear trumpet)
- Various sheets of Styrofoam, metal, glass, paraffin, masonite, wood, celotex, etc., for absorption tests
- Spherical balloon and CO_2 source
- 3 ringstands with adjustable clamps
- Meter stick

Again, there is a lot of material here. One could easily spend several lab periods on these experiments.

Answers to questions

1. The sound is absorbed. Heavy cloth, such as velvet or terrycloth, does very well.
2. Student answer.
3. The best patterns are probably obtained when the board is perpendicular to the line straight out from the speaker. The narrow board (two-edge diffraction) should be symmetrically placed, while the wide board (single-edge diffraction) should have its edge on that center line. The diffraction patterns are those familiar from optics. They should roughly have the following forms:



4. Sound waves will be heard at the edge of the shadowed area.
5. Neither the spacing of the minima nor the wavelength should depend on the loudness.
6. The wavelength increases when the frequency is decreased.

Note that these experiments parallel those done with a ripple tank in *E3–16* (much more qualitatively, of course).

7. The closer the two sources the more widely spaced are the nodes.
8. The positions of the maxima are separated by a distance

$$x = L\left(\frac{\lambda}{d}\right) = L\left(\frac{v}{df}\right),$$

where v is the speed of sound. Thus, changing d and changing f have completely equivalent effects.

9. The wavelength changes inversely with the frequency.
10. No. Speed is independent of intensity.
11. Yes

E3–19 ULTRASOUND

Equipment:

- Amplifier power supply
- Oscillator plug-in unit
- 3 ultrasonic transducers
- Oscilloscope <u>or</u>
- Amplifier/power supply, microameter, and diode (100,000 ohm resistor, optional)
- Sheets of test materials as listed under *Sound* above
- Meter stick

With invisible, inaudible ultrasound, this experiment parallels the activities of the previous experiments. Students explore the transmission, reflection, and diffraction of these waves. They then create standing waves and may estimate the wavelength as with audible sound waves. Finally, the students investigate the interference patterns they can establish.

Primary emphasis is upon the qualitative characteristics of the waves rather than upon quantitative results, although those may be of interest. A value for the speed of ultrasound permits comparisons with the speed of audible sound.

Answers to questions

1. Energy that is neither transmitted nor reflected is absorbed.
2. Ultrasound will be diffracted like audible sound, but the pattern of diffraction will be smaller because the wavelength is shorter.
3. The spacing of nodes for standing waves is not related to the intensity of the sound.
4. In the two-source (two-slit) setup, the spacing of the nulls will decrease as the separation of the sources increases.
5. Use of the equation assumes, among other things, that this is a wave phenomenon, that the frequency is stable, that the intensity is not significant, etc.
6. The speed of ultrasound is independent of the intensity.
7. The speed of ultrasound is the same as that of audible sound (about 333 m/sec). The actual values may vary somewhat with humidity. Also, the frequency of the oscillator may be uncertain by as much as 10%.

Film Loop Notes

SPECIAL NOTE ON THE UNIT 3 FILM LOOPS

In a number of cases, a single collision event is described by a film loop, a transparency, and a stroboscopic photograph. Each stroboscopic photo may be found in the Activity section of *Handbook* Unit 3 and a discussion of its use is included in this *Resource Book* in the section entitled *Experiment Notes*.

One way that a teacher might take advantage of this duplication is by solving a momentum conservation problem on a transparency while the students complete the same problem at their seats. Finally, by projecting the film loop on the chalkboard, the actual collision can be demonstrated or resolved. Other imaginative teaching strategies can be devised through the use of these three media to teach conservation of momentum and energy.

I. INTRODUCTION

Six different two-body collisions, occurring along one dimension, were filmed with a high-speed motion-picture camera. Each loop, *L18-L20*, contains two of these collision events. *L18*, in addition, shows establishing scenes, filmed at normal camera speeds, which instruct the viewer about the experimental setup used to produce these collisions. *L19* and *L20* contain high-speed sequences only. The projector shows the high-speed footage in slow motion.

Student notes for each loop are also provided in the *Handbook*. One or more events can be assigned as study-period (or laboratory-period) problems. Students should work in pairs.

The apparatus used in these filmed experiments is described briefly in the *Handbook* and in more detail in the *Experiment Notes* of this *Resource Book*.

The films should be projected upon a white sheet of paper taped to the wall. The 45-by-60 cm desk pads often found in offices are a good source of such sheets. Even better are 43-by-55 cm quadrille sheets, lightly ruled, 4 squares to the centimeter, available through drafting supply stores. The projected image should fill the paper sheet. Students also need a good plastic see-through ruler, marked in millimeters, and a stopwatch or other timing device precise to the tenth of a second. A pair of drafting triangles will be helpful for drawing accurately parallel lines. Precision up to three significant digits is possible if measurements are carefully made.

The experiments involve marking pairs of vertical lines down on the paper to serve as "timing bars." Their positions (and the distance between the lines) are to be chosen after initial viewings of the film and then accurately measured. The passage of the balls on their horizontal paths can now be timed. Care must be taken not to move the projector during the experiment.

Students calculate speeds of the balls before and after collision in relative units; for example, in terms of the apparent speed across the paper. Section II, which discusses the six events in detail, also describes further a typical series of such measurements and calculations. The masses of the balls are given.

Each collision is governed by the *principle of conservation of momentum.** But these loops may be used to teach students far more than mere verification of this principle, especially if a pair of events is assigned as problems. If only one event is assigned, this event should not be the first example in *L19*. In many cases the assignment of one event may be enough. But better students may well find time to study with profit a pair of events.

The events occurring in each loop do, in fact, constitute matched pairs that may be used to teach interesting additional lessons. In *L18* the two collisions are mutually inverse. In one collision, one ball is reflected, whereas this is not the case in the other, a matter which often surprises the novice. In each loop, the same balls collide strongly and weakly, respectively. Calculation of *kinetic energy*, before and after collision, can lead to interesting discussion on the more imperfectly elastic nature of the balls involved. Moreover, study of the two events in *L19* contains a useful lesson about error propagation. These aspects are discussed further in Section II, which follows.

II. THE SIX EVENTS, AND HOW TO TAKE DATA

The collision events present similar analytical problems. Measurements consist of timing each ball's motion past "timing bars" whose position and separation are chosen by the students. In our discussion, we give details for the first example in *L18* only. Only the special aspects of the other five events are discussed.

The simplest problem of the six is the first example of *L20*. There we deal with a perfectly inelastic collision of two balls with one ball initially at rest. After collision, the balls move together as one "compound" particle. Only two timings are required.

A schematic diagram is provided for each event both in the *Experiment Notes* of the *Resource Book* and in the *Handbook*. It specifies the masses of the balls and qualitative conditions before and after collision.

*Momentum is a vector quantity. Since the collisions in this series of film loops are one-dimensional, the vectors in any one problem are all parallel or antiparallel. This simplifies the calculations. (*Loops 21, 22, 23,* and *24* as well as one series of stroboscopic still photographs) involve two-dimensional collisions.

L18 ONE-DIMENSIONAL COLLISIONS. I

First example. See the Event 1 photograph on page 214. Ball B is initially at rest. After collision, it moves off to the left at roughly the same speed as the initial speed of ball A. Ball A comes in from the right and emerges from collision with unchanged direction of motion.

Three timings are required. Two of these (initial velocity of A, final velocity of B) are simple since the speeds are relatively large. The following describes a more detailed typical procedure than will be found in the student notes.

1. Align the projector with the paper sheet on the wall. This alignment must not be disturbed until measurements are completed. Run the loop at least once for orientation.

2. To find the initial velocity of A, two fine vertical lines must be drawn on the sheet. Here the rulings of graph paper sheets are useful. The two lines must be placed to the right of the collision point as close to that point as possible.

The separation between these parallel lines should be as large as possible so that the distance between them can be measured with reasonable precision. Since a ruler marked in millimeters is used, it is possible to estimate the tenth of a millimeter, although this digit is a doubtful figure. Hence, a distance of at least 10 cm is preferable, for then it can be measured with precision of three significant digits.

The separation between these lines, on the other hand, should be as small as possible because the ball, as it moves toward the collision point, gains slightly in speed. If the image from the projector fills the paper sheet as completely as possible, this source of error contributes only a small fraction of 1%. The students could measure the separation several times, estimating the nearest tenth of a millimeter, and use an average.

Similarly, 10-cm "timing bars" can be used for ball B after collision. But ball A moves slowly after collision.

When the speed of a ball is small, the amplitude of its swing (as a pendulum, see Event 1, page 214) is small compared to the field of view. Hence, it loses considerably in speed while still in the field of view. To reduce this source of error, the "timing bars" must be placed as near as possible to the collision and must be separated by as little as possible. This raises another source of error; now the separation cannot be measured with the same precision. We chose "timing bars" separated by 3 cm, and when this distance is measured by our ruler, we may have only two significant digits.

3. Three velocities must be determined, one for ball B and two (before and after) for ball A. One value for each of the three times of passage across the corresponding pair of "timing bars" is needed. With a stopwatch, obtain three values for each time interval and calculate the average. This will require repeated projection of the film loop.

Repetition of the measurements of distance with our ruler, in (2) above, may convert the tenths of the millimeter from doubtful to significant. A like conversion of digits from doubtful to significant may result from repeated time measurements.

4. Calculate velocities, momenta, and kinetic energies from the data. The table that follows lists these for a typical run of the experiment. Do not assume that the student will get the same numerical values as appear in the table. These values depend on how large the image is on the paper and on the frame rate delivered by a projector. The latter is not guaranteed equal by the manufacturers in all models. On the other hand, the conclusions reached (including those about errors) from the table are roughly those the student should reach.

5. In the table, we have intentionally omitted mean deviations to keep discussion simple. We have also, for simplicity, failed to take advantage of the gain in significant digits obtained through averaging repeated measurements. A good student can get better precision than is demonstrated here.

Loop 18

First Example: One-Dimensional Collisions
Ball A: 532 grams
Ball B: 350 grams
A line under a digit means "doubtful."

Item	Ball	Time	Average Values	Direction
velocity	A	before	0.813 cm sec	left
	A	after	$0.25\underline{2}$ cm sec	left
	B	before	0 cm sec	—
	B	after	0.885 cm sec	left
momentum	A	before	433. g·cm sec	left
	A	after	$13\underline{4}$ g·cm sec	left
	B	before	0 g·cm sec	—
	B	after	310. g·cm sec	left
kinetic energy	A	before	176. ergs	not
	A	after	$16.\underline{9}$ ergs	a
	B	before	0 ergs	vector
	B	after	137. ergs	

The small velocity of ball A after collision reduces the number of significant digits to two and affects the corresponding momentum in the same way. Total momentum after collision is 4.4×10^2 g·cm sec, and before collision it is 4.3×10^2 g·cm sec to the same number of significant digits. The difference is 2.3% of the average.

The average values depend on the size of the projected image. Students will not get these values, nor will different groups get equal values.

The kinetic energy calculation, however, does not suffer from a reduction from three to two significant digits. The kinetic energy of the system is 176 ergs before and 154 ergs after collision. Kinetic energy is 87.5% conserved. The collision is not perfectly elastic. The balls were case-hardened steel, which means that they have an inner core of soft steel. The collision is sufficiently strong to permanently deform this core slightly, with subsequent loss of mechanical energy into internal energy much of it heat.

When a student studies both examples in this loop, the percentages of energy conservation should be compared and will lead to an interesting discussion. See below.

Second example. Again, one ball comes in and strikes another at rest. (See Event 2 photo on page 215 of this *Resource Book.*) Again, three sets of timing measurements are required. But this time it is the more massive ball that is initially at rest. This reverses the direction of motion of the incoming ball after collision.

This event is, initially, the exact reverse of the previous example. The initial speed of ball B is the same as it was for ball A in the first example, to three significant digits.

After collision, ball B moves to the left very slowly. The "timing bars" need to be brought even more closely together than in the first example. The number of significant digits again drops from three to two.

But here there is no loss in significant digits in the calculation of total momentum. After collision our measurements gave:

$$\vec{P}_B = 23.\ \text{g·cm/sec to the left}$$
$$\vec{P}_A = 317.\ \text{g·cm/sec to the right}$$
$$\Sigma\vec{P} = 294.\ \text{g·cm/sec to the right}$$

(Numerical values given here are not representative of those in student data.)

The fact that there was no loss in significant digits does not, however, necessarily insure greater precision in the results. Our value for total momentum before collision is 285 g·cm/sec to the right. The difference is 3.1%.

This difference is larger than it was for the first example, but note that it is larger only in relative, not in absolute, value. The absolute difference is, in fact, 10 g·cm/sec in the first, 9 g·cm/sec in the second example. Since the measurements were equivalent in these two examples and equal in number, the fact that absolute differences are nearly equal should not surprise us.

Our data also revealed that only 82% of the kinetic energy survives the collision. Why was this percentage higher (87.5%) when the same balls collided in the opposite manner?

In our pair of examples, this question is answered in a surprising way. We should expect greater energy loss if two balls collide with each other "harder." The average collision force of interaction is proportional to momentum change experienced by either ball. In the first example, these changes shown in the table are 3.0×10^2 and 3.1×10^2 g·cm/sec respectively; in the second example (see data above), 3.08×10^2 and 3.17×10^2. Thus, the second collision was slightly greater, and, therefore, a higher percentage of energy was lost.

However, this conclusion must be discarded. For, if difference of momentum, which is in principle zero, was 3% on basis of the data, this indicates an experimental error of at least that size. In calculating kinetic energy, we square the measured speeds and this doubles the relative errors! The apparent difference of 5% in energy loss is explained by the errors of our observations and may be spurious.

L19 ONE-DIMENSIONAL COLLISIONS. II

The two examples of this loop involve the same pair of balls in different collision. One ball is more than three times as massive as the other.

In the first example, the two balls come at each other from opposite directions at considerable relative velocity. In the second example, both balls are moving across the field of view in the same direction, one ball catching up with the other, but the relative velocity is small.

In both cases, four velocities must be found from the film footage, and four pairs of "timing bars" are required.

The first example should not be assigned alone. Together with the second, it forms a highly instructive problem.

First example. See Event 3 on p. 216. Both balls reverse their direction of motion upon collision. Ball A is moving slowly before and after, and a separation of less than 10 cm may be needed between the "timing bars." Because ball A is massive, however, the magnitude of its momentum (before as well as after) is comparable to the corresponding values of momentum of ball B.

Since the momenta of the two balls are oppositely directed (both before and after the collision), the net momentum of the system can be calculated only as a small difference between large numbers. Hence, a large relative error may be expected.

When we performed these measurements, we found a difference of 8.7% between the net result for the momentum before collision and for the net momentum after. The actual numerical momentum values (in g·cm/sec) we are about to cite from our data will not be the same as those found by your students. Nevertheless, we quote them here: 133 net momentum before, 122 after, both directed to the left. But the individual momenta were much larger. Before collision: ball A, 448 to right; ball B, 581 to left. After collision: ball A, 454 to left; ball B, 332 to right.

The large relative error is completely accounted for by the circumstances of this collision. The large individual momenta are far more precisely known. Not so the difference! Had it been the sum, the relative error would be small.

The second point of interest here relates to the strength of the collision and is of particular value when this example is studied with the other event in this *Film Loop.* The student will find that only about 43% of the total initial kinetic energy survives the collision. In the other event, this energy is 97% conserved; that is the collision is almost perfectly elastic. Why? Because the soft core of these case-hardened steel balls is sorely tested in a real collision as occurs in the first example, but it is hardly touched in a weak collision. A good deal of any deformation of the inner core is permanent with resulting loss of the "work of deformation" from the initial supply of kinetic energy. The loss goes into "internal" energy (mostly heat).

Another way to explain the idea of "strength" in a collision is the following: The average force of interaction, multiplied by the time of contact in collision, equals the change in momentum of one of the balls. This is the "impulse theorem," a derived form of Newton's second law. Hence, change in momentum is a measure of the deforming force that occurs during contact in the collision. Note from the above data that ball A's momentum changes from 448 to the right to 454 to the left, that is, by 902 g·cm/sec (by our figures, not the student's). In the second example the change is only about 400 g·cm/sec.

Second example. See Event 4 on p. 217. All velocities and momenta have the same directions (to the right) in this event. Four sets of measurements must be made to find the four speeds. All the speeds are large enough to allow 10 cm separation between "timing bars." Hence, there is chance for three-digit precision.

Net momentum (before or after) is here calculated by addition of individual magnitudes (and not by subtraction, as was the case in the previous event). Therefore, the relative difference between momenta before and after will be small. Our value for it is 0.48%.

Kinetic energy is 97.2% conserved. The collision is almost perfectly elastic. This should be compared with the 43% conservation found when the same balls collided in the previous event.

L20 INELASTIC ONE-DIMENSIONAL COLLISIONS

In these events, the colliding objects are steel balls covered by a thick layer of plasticene. They remain lodged together after collision. Both collisions involve the same balls. The second collision is far stronger than the first.

The first example is the simplest of all six problems in this set of three film loops. It requires only two sets of timing measurements. The second example requires three.

Precision is excellent in both events.

First example. See Event 5 on p. 217. Ball A is initially at rest. Our value for the relative difference between net momentum before collision and after is 0.4%. Kinetic energy is 59.4% conserved. Three-digit precision throughout.

Second example. See Event 6 on p. 218. Here the balls come in from opposite directions. They move off to the left after collision. Three-digit precision throughout. To three significant digits, we found identical values for net momentum before and after. But kinetic energy is only 32.7% conserved because the collision is more violent here than in the first example.

NOTE:

Teacher notes for *Film Loops 21, 22, 23,* and *24* are included in the section of this *Resource Book* entitled *Experiment Notes*. The discussion deals with the use of the identical stroboscopic photographs that are reproduced both in this book and in the *Handbook*.

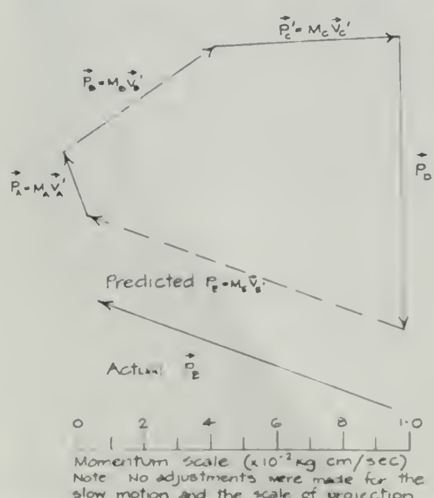
L21 TWO-DIMENSIONAL COLLISIONS. I

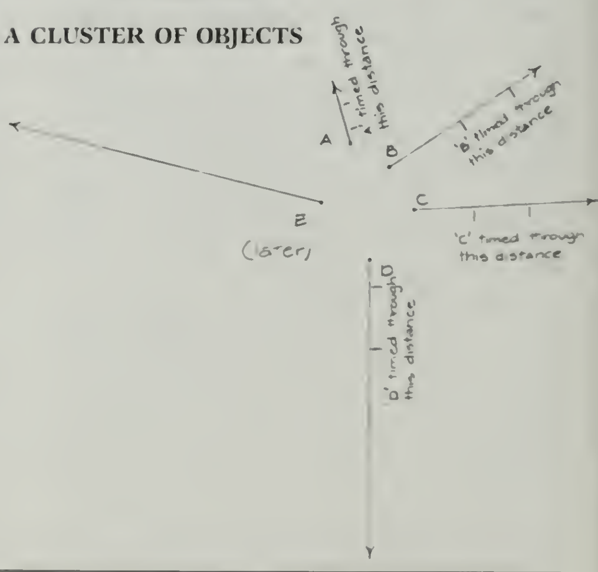
L22 TWO-DIMENSIONAL COLLISIONS. II

L23 INELASTIC TWO-DIMENSIONAL COLLISIONS

L24 SCATTERING OF A CLUSTER OF OBJECTS

L25 EXPLOSION OF A CLUSTER OF OBJECTS





Sample Data

Identification	A	B	C	D	E	Notes
Mass (kg)	0.357	0.539	0.366	1.78	1.02	Given
Projected distance traveled (cm)	1.55	3.05	2.90	3.00	1.25	This will depend upon the size of projection.
Direction of travel	342°	56°	86°	179°	284°	Up is taken as 0°. Direction is measured clockwise.
Time to travel designated distance (sec)	15.0 15.3	17.4 17.0	11.3 11.4	31.7 31.8	10.9 11.8	This is measured time disregarding slow-motion factor.

Sample Calculations

Identification	A	B	C	D	E	Notes
Average time traveled for designated distances	15.2	17.3	11.4	31.8	11.3	$v = \frac{\Delta d}{\Delta t}$
Speed (cm/sec)	0.102	0.176	0.254	0.0945	0.111	
Velocity (cm/sec, degrees)	0.102, 342°	0.176, 056°	0.256, 080°	0.0945, 179°	0.111, 284°	
Momentum	3.60, 342°	9.49, 056°	9.42, 086°	16.8, 179°	11.3, 284°	$p = mv$

From a comparison of expected and observed values for the momentum of ball E, it is clear that momentum is not conserved within the system of the five balls. There is too much predicted momentum. This may lead the student to realize that something is carrying away momentum from the system in a direction opposite to ball E. Looking at the film again, the student may note that the waste matter from the powder charge may be the unmeasured recipient of the required momentum.

An advanced student may wish to study the chemical potential energy stored within the powder charge. However, as this can only be done in arbitrary units, it may have little significant meaning.

An interested student might want to repeat this experiment in order to improve the technique of measuring. One idea might be to time the balls from the explosion to when they are last seen. Furthermore, greater care might be taken to be more accurate in recording the direction of ball E in the smoke.

An error of 2° and at least 1 sec was involved in the above measurements of balls A, B, C, and D. An error of 3° and 2 sec was involved in measuring the motion of ball E.

L26 AND L27 FINDING THE SPEED OF A RIFLE BULLET: METHOD I AND METHOD II

I. INTRODUCTION

These films are not meant to provide a precise laboratory exercise. While it is possible to obtain representative values for the muzzle velocities, the main intention is to bring a few conservation principles into the context of a real experimental problem, *the ballistic pendulum.*

The bullet's speed is calculated from the log's speed after impact by use of conservation of momentum. In Method I, the latter speed is determined directly. In Method II, on the other hand the log's speed after impact must also be calculated. The student can measure only the log's full height of rise during its swing away from impact. To relate the two quantities, we invoke conservation of the sum of the log's kinetic energy and its gravitational potential energy during the swing.

Finally, one can (in each film) compare the kinetic energy of the bullet before impact with that of the log (with the bullet embedded in it) after impact, and ask the student: How was energy conserved here?

II. GENERAL DESCRIPTION

In Method I, there is a slow-motion sequence that permits timing the motion of the log just after the bullet strikes. The circular path of the log has a very large radius and the film sequence is a close-up of the log for a field of view of about 30 cm × 40 cm. Hence, the motion of the log can be considered uniform along a horizontal straight line. The student must convert distance as well as time measurements taken from the projected image of the slow-motion scene to "actual distance" and "real time." The information necessary for this conversion appears in the film. Now the student can calculate the bullet's speed by invoking momentum conservation, as follows

$$mu = (M + m)v, \qquad (1)$$

m = mass of the bullet
M = mass of the log
v = speed of the log plus bullet after impact
u = speed of the bullet

The values of M and m are given in each film.

In Method II, the measurements are simpler; they involve distances only. No measurements of time are required. The film contains a slow-motion sequence showing the log close up, as it goes through its full pendulum swing after impact. The student can measure the vertical height of rise, h, of the log. That is to say, h is the distance from the log's lowest initial position before impact to the highest point of its swing, which is readily identifiable because it reverses its direction of motion at that point.

Since the log's kinetic energy just after impact (at the start of its swing) must equal the gravitational potential energy required to raise it vertically through a distance h (at the end of its swing), then

$$(\tfrac{1}{2})mv^2 = ma_g h \qquad (2)$$

Therefore, $v^2 = 2a_g h$

v = speed of log (plus bullet) just after impact

h = height of vertical rise of log during swing

a_g = acceleration of gravity

Having calculated v from Equation (2), u can be calculated by use of Equation (1).

Method I

In the close-up slow-motion sequence that is intended for taking data, the student must make two horizontal marks on the paper sheet taped to the wall while the log is at rest in the image. These marks must span the vertical dimension of the rod that is given in another portion of the film to correspond to 15 cm "actual distance." By later measuring the distance between these marks, conversion of distances measured on the paper to actual laboratory distances becomes possible by scaling.

A strip of white adhesive is taped to the log. Either of its vertical edges can be used as a reference line for timing the log's horizontal motion after impact. Two vertical lines are drawn on the paper as timing bars. The distance between them is measured, and converted by scaling. The motion is timed by stopwatch.

"Film time" can be converted to "real time" if the slow-motion factor is known. This factor is the ratio

$$\frac{\text{frames/second taken by the camera}}{\text{frames/second delivered by the projector}}$$

The quantity in the numerator is given in the film.

The quantity in the denominator must be measured, since the manufacturer of the projector does not guarantee this rate to within less than 10%. The total number of frames is printed on the cartridge into which the film is looped. There is a single black frame with a large white circle in the "black stretch" between the end and start of the film loop that is visible on the screen as a brief flash. Thus, the student can time the length of the loop.

When we took the indicated measurements of this film loop, we found that our loop of 3,849 frames ran 207.3 sec: a projection rate of 18.57 frames/sec. The slow-motion factor was therefore

$$\frac{2{,}850}{18.57} = 153.4$$

Times measured on the film are converted to absolute time by dividing by the slow-motion factor.

Furthermore, we found the speed of log plus bullet after impact was v = 106 cm/sec and the speed of bullet was $u = 6.43 \times 10^4$ cm/sec ÷ 2 or 3.22×10^4 cm/sec.

The kinetic energy of the bullet before impact was 2,890 J. The kinetic energy of the log (plus bullet) after impact, on the other hand, was only 2.50 J! Most of the initial kinetic energy supply is dissipated in tearing wood and producing heat in this inelastic collision.

Method II

In the close-up, slow-motion sequence that is intended for measurement, the student again marks off the vertical dimension H of the log while it is at rest to serve as a scaling reference. The actual value of H is given in the film as 9.0 cm.

There are two horizontal strips of adhesive taped to the log. Any horizontal edge of these strips can now serve to mark off the initial position of the log and the final position at full swing to determine h.

Our measurements (on the film) yielded h = 5.33 cm, v = 102. cm/sec, and $u = 5.8 \times 10^4$ cm/sec.

The kinetic energy of the bullet before impact was 1,200 J. The kinetic energy of the log with the bullet embedded in it was only 4.2 J.

L28 RECOIL

The film is valuable form both qualitative and quantitative aspects by illustrating the "real life" recoil of an actual cannon and a laboratory cannon suspended from strings. Relative measurements of momentum can be made to test the conservation laws.

With the high-speed camera, a delay is observed between fuse ignition and the emergence of the bullet from the cannon barrel. During this delay the bullet travels through the barrel from its initial position to the end of the muzzle.

To travel a distance of 20 cm on our paper, the projectile required 2.95 sec. The bullet's mass is 3.50 g and its relative momentum 3.5 (20/2.95) g cm/sec. Momentum conservation in one dimension predicts

$$m_p v_p + m_c v_c = 0$$

Thus, the velocity of the cannon v_c should be given by

$$v_c = -\frac{m_p v_p}{m}$$

$$\text{where } \frac{m_p}{m_c} = \frac{1}{100}$$

$$\text{and } v_p = \frac{20}{2.95} \text{ cm/sec}$$

The predicted velocity of the cannon is 0.06 cm/sec or 75 sec for 5 cm with an error in timing of 10% for the bullet and a subsequent error of 10% in the prediction. Our data gave an experimental value of 67.5 sec for the cannon to move 5.0 cm in the opposite direction, which is within the margin of error. One might expect a lower value since the powder charge has some momentum. Also, the projector manufacturer guarantees no less than 10% error of uniformity in projection rate.

The kinetic energy of the bullet is $\frac{1}{2}m_B v_B^2$, or, with our data, $\frac{1}{2}(3.5)(20/2.95)^2 = 80.5$ g·cm²/sec², while the cannon has a kinetic energy of $\frac{1}{2}$ (350) $(5/65.7)^2$ or 9.6 g·cm²/sec². The kinetic energy of the bullet is not equal to that of the cannon; nor would we expect it to be, for this is not an elastic collision but an explosion with energy lost both to the powder charge and through frictional losses within the barrel. Kinetic energy is not conserved.

L29 COLLIDING FREIGHT CARS

The test of coupling strength was made by the Uplands Railway Laboratory for Canadian Pacific Railroad. The test engineer's report for the trial shown in the film's slow-motion sequence gives the peak coupling force as 4,784,850 N; hammer car's velocity after impact is 1.3 m/sec.

An alternative method of finding v_1, involving less accuracy but easier to understand, is to measure the time for the hammer car to come to rest after the collision. Then the initial velocity is just twice the average velocity.

Measurements from the film (in arbitrary units) gave 286 units for total momentum before collision, 280 units after collision. Kinetic energy of the system decreased from 390 units to 167 units.

L30 DYNAMICS OF A BILLIARD BALL

The film has value even if used only qualitatively to illustrate conservation of momentum in a "real-life" situation. Measurements can be made and interpreted at two levels of difficulty.

1. Students should have no difficulty with straightforward conservation of linear momentum, as outlined in the *Handbook*. For best results the velocities after impact should be measured over short distances to avoid complications due to friction. The cue ball's forward linear velocity is negligible just after collision, but this ball does gain forward speed as its rotational speed decreases due to friction. In a typical measurement, balls were timed as their leading edges moved forward a distance equal to one radius. The measured speeds (hence also the measured momenta) agreed within 1%.

2. The following analysis is given primarily for teacher background. The balls rotate as well as translate, so we must consider both translational and rotational momentum. The force of friction between the ball and the table surface affects the motion of a ball whenever there is slipping (that is, a relative motion between the ball's lower surface and the table). A basic assumption is that the coefficient of sliding friction (μ) is the same for each ball and is independent of the speed of slipping. We use Newton's second law for translation ($F = ma$) and for rotation ($\tau = I\alpha$) where τ = torque, I = moment of inertia ($= \frac{2}{5}mr$ for a sphere rotating about an axis through its center), and α is the angular acceleration. When a ball is rolling without slipping, its linear velocity v and its angular velocity ω are related by the equation $v = r\omega$.

At the moment of impact, the only force on each ball is that due to the other ball, acting along the line of centers. Because these forces have no lever arms, they can cause no changes in angular velocity at the moment of impact. The cue ball, which was rolling, must continue to spin with the same ω, and the target ball, which had no initial ω, must start to slide with no rotation. These conditions do not persist, however, because a frictional torque acts on each ball while its lower surface is slipping on the table. *Time of spinning of cue ball.* A frictional force $\mu m a_g$ acting toward the right on the bottom surface of the cue ball does two things: It causes the ball's rotational velocity to decrease, and it causes the ball's translational velocity to increase in the forward direction. After a time t_1 the velocity of the ball's lower surface equals the forward velocity of the ball:

$$r(\omega_1 + \alpha t_1) = 0 + a t_1$$

$$\text{where}\quad r\left(\frac{v}{r} - \frac{\mu m a_g r}{{}^2\!/_5 m r^2}\, t_1\right) = 0 + \frac{\mu m a_g}{m}\, t_1$$

$$\text{then}\quad t_1 = \frac{2v}{7\ m a_g}$$

After this time has elapsed, the cue ball continues to roll without slipping.

Time for target ball to slide. While all this is going on, the target ball starts to slide with velocity v_2. Friction acting to the left causes the ball's translational speed to decrease, and it causes the ball's rotational speed to increase from zero up to some value. The ball starts to roll without slipping after a time t_2 when the linear velocity of the ball's surface (due to rotation) becomes equal to the ball's forward translational velocity:

$$r(0 + \alpha t_2) = v_2 + a t_2$$

$$r\left(0 + \frac{\mu m a_g r}{{}^2\!/_5 m r^2}\, t_2\right) = v_2 - \frac{\mu m a_g}{m}\, t_2$$

$$\text{where}\quad t_2 = \frac{2v_2}{7\ a_g}$$

Now we can explain the strange behavior of the balls. From the law of conservation of linear momentum, $v = v_2$, hence t_1 and t_2 are equal. The changeover to rolling without slipping occurs at the same time for both balls; the cue ball seems to "know" what the target ball is doing.

Conservation of angular momentum. Change in angular momentum equals (torque) × (time). While the cue ball slides, it loses angular momentum $(\mu m a_g r)(t_1)$. While the target ball slides, it gains angular momentum $(\mu m a_g r)(t_2)$. We have seen that $t_1 = t_2$; hence, there is no net change in angular momentum. Since the momentum of inertia is the same for each ball, this means that $\omega = \omega_1 + \omega_2$ where ω is the initial angular velocity of the cue ball, and ω_1 and ω_2 are the angular velocities measured after both balls are rolling without slipping. A typical measurement from the film confirms this to within about 6%.

Coefficient of friction. The time for slipping was found to be

$$t_1 = \frac{2v_2}{7}\mu a_g$$

from which μ can be found if t_1 and v_2 are measured in real-time and real-distance units. (The slow-motion factor is 167 and each ball has a diameter 5.24 cm.)

Perhaps it is easier to work with distances than with times. The distance for the target ball to slide is

$$d = v_2 t_2 + \tfrac{1}{2} t^2$$

$$d = v_2\left(\frac{2v_2}{7\,ma_g}\right) + \tfrac{1}{2}\left(-\frac{\mu m a_g}{m}\right)\left(\frac{2v_2}{7\,ma_g}\right)^2$$

which simplifies to $d = \dfrac{12v_2^2}{49\,ma_g}$.

Thus, μ can be found by measuring v_2 and d. Calculated values of μ are about 0.33. This agrees with a direct measurement of about 0.3 for the coefficient of sliding friction (not shown in the film).

L31 A METHOD OF MEASURING ENERGY: NAILS DRIVEN INTO WOOD

As the nail penetrates deeply into the wood, the force of friction increases somewhat, so the penetration is less than would be expected on the basis of the first few blows. Therefore, the graph is curved downward, as shown in the *Handbook*. For many purposes this effect can be ignored, and the energy of the object striking the nail assumed to be directly proportional to the depth of penetration.

L32 GRAVITATIONAL POTENTIAL ENERGY

In testing and defining gravitational potential energy, we use the graphs from *L31* that relate kinetic energy to nail penetration. Since there is no loss of energy as the bodies fall, we can say that the "potential energy" of the object is the same as the energy just before impact and then measure the nail penetration to find that energy:

nail penetration → energy → potential energy

Further, $\tfrac{1}{2}mv^2 = ma_g h$, where m is the mass, v its velocity just before impact, h the initial height above the zero position ("the nail top"), and a_g the acceleration of gravity.

In the first sequence, we measure the nail position before and after collision to find the penetration and plot this versus weight ma_g. As in *L31*, you may expect the graph to bend downward slightly. (See Fig. 1.)

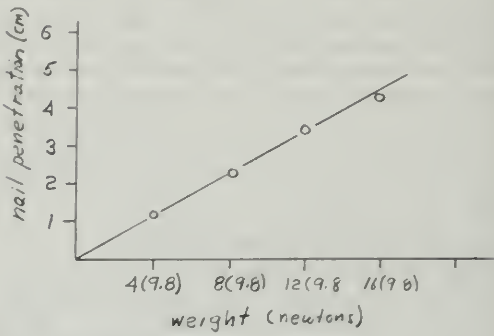


Fig. 1 Weight (newtons)

There is a direct relation between weight and penetration, therefore between weight and energy and between weight and gravitational potential energy. The height above zero position has been held constant at 2 m.

In the second sequence, the mass is the same while the initial height, h, is varied from 1 m to 4 m above the zero position. To graph this data, one might measure values of h from the screen or use the given data as we have done.

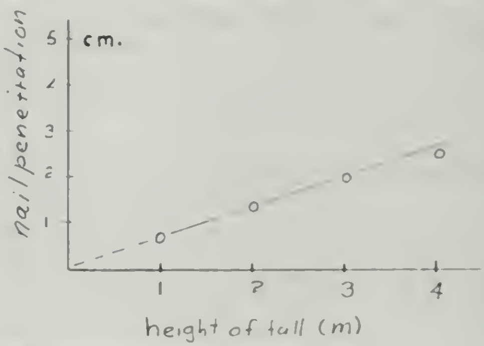


Fig. 2 Height of a fall (m)

The Film Loop does not give a starting measurement in the second sequence to establish the initial position of the nail so we have used the differences between the data to extrapolate an initial position.

From Fig. 2, we can conclude that there is a direct relation between penetration and distance of fall, which implies a direct relation between gravitational potential energy and distance of fall. We can conclude that gravitational potential energy is directly proportional to the product of weight and initial height

$$PE \propto ma_g h \text{ or } PE = K(ma_g h)$$

where K is a constant whose value depends on our chosen units.

The slow-motion factors (33 for the first sequence and 10 for the second) are given. The more ambitious student may wish to use these to determine relative velocities and the relations of v and h or energy and velocity.

L33 KINETIC ENERGY

L31 suggests that nail penetration is a convenient method for studying the energy of an object that hits the nail. In *L33*, we examine the energy in its kinetic form to find a relation between kinetic energy and velocity.

By following the *Handbook*, the students should have no trouble in timing the passage of the objects across the reference lines; they must be consistent in using the same edges for all objects.

Measurements from the film give the following data:

Event	Time (t)	$\frac{1}{t}$	Nail Penetration		$\frac{1}{t^2}$
0	0		0	6.6	
1	2.14	0.466	0.7 units	5.9	0.217
2	1.35	0.741	1.4 units	5.2	0.550
3	0.98	1.010	2.2 units	4.4	1.02
4	0.80	1.25	3.1 units	3.5	1.56
5	0.71	1.41	4.3 units	2.3	1.98

Since velocity is distance/time and the distance is constant, the velocity in each case is inversely proportional to the time. That is $v \propto 1/t$. The student should plot two graphs: kinetic energy *KE* (nail penetration) versus v^2 (or $(1/t)^2$), and another of *KE* versus v $(1/t)$.

The two plots for our data are given below. We can conclude that there is a direct relationship between *KE* and v^2. However, the plot of *KE* versus v is a parabola. (Fig. 4.)

The interested student may wish to determine the relative mass of the last unmarked object by measuring its penetration and velocity and then establishing ratio between these results and those obtained in the first part. Our data would suggest that:

$$t = 1.2 \text{ and } v^2 = 0.7.$$

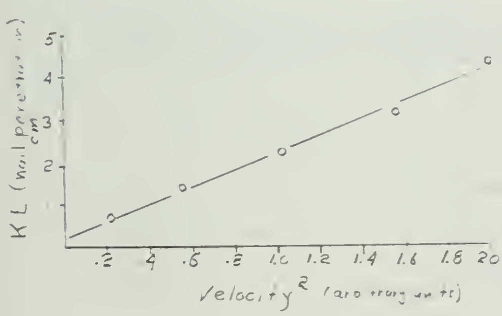


Fig. 3

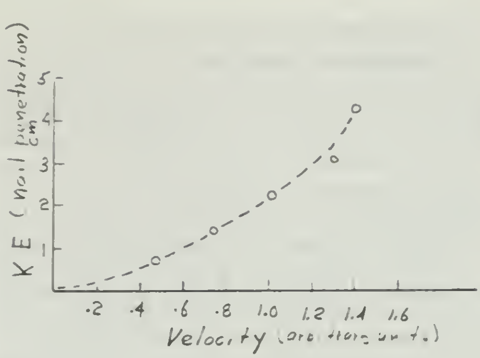


Fig. 4

Interpolating from Fig. 3, the penetration should be 1.6 units if the objects were alike in mass. The filmed sequence suggests that the nail penetration is 5 units so we may conclude that object 6 has a mass three times that used in the first sequence.

L34 CONSERVATION OF ENERGY: POLE VAULT

The film can be used qualitatively. The intermittent freeze-frame sequences are long enough so that the teacher can mention the different forms into which the total energy has been transformed while they are actually "happening." Quantitative measurements are good to about 10%. It is best to concentrate on comparing energy at position 1 with that at position 3 and to leave the more difficult check at position 2 for students who enjoy this type of measurement. Even if no measurements are made at all, you can tell the class that the energy is divided approximately as follows:

1. initial kinetic energy	1300 joules
2a. kinetic energy	450 joules
2b. gravitational potential energy	650 joules
2c. elastic potential energy	300 joules
	1400 joules
3. final gravitational potential energy	1800 joules
less muscular work	−400 joules
	1400 joules

In position 2, each of the three forms of energy is a significant portion of the total.

The intermittent freeze-frame technique is used because even the slow-motion action is too fast for accurate timing of a 1-m displacement. By this new technique, a student can measure the speed to within a few percent by counting, say, 20 frames (1 frame out of 20 is only a 5% error).

Some fine points can be raised for class discussion: (1) The gravitational potential energy of the pole itself is only about 10 J in position 3, and can be neglected. (2) To be precise, we should calculate the work done in bending the pole.

L35 CONSERVATION OF ENERGY: AIRCRAFT TAKEOFF

Air resistance does depend on speed and therefore does decrease somewhat as the plane's speed decreases. Make clear to the student that we are using an approximation when we ignore this effect. The approximation is justified by the large mass (inertia) of the plane. For instance, a typical measurement shows that at the upper level the speed was only two-thirds the value at ground level. This means that the force of air resistance at the upper level is only two-thirds the value at ground level. If, as intended, the engine supplies a constant forward force, a small net forward force builds up when the plane reaches the upper level. But the <u>acceleration</u> caused by this unbalanced force is small because of the large mass of the plane: $a = F/m$. It is this sluggishness of response to changes in air resistance that allows us to make the simple energy analysis outlined for student measurements.

If this seems unreasonable, reflect on the fact that the plane will, if it flies long enough at the upper level, regain its original ground speed when it again reaches terminal velocity; but there is no sign of such an increase during the time the plane remains visible in the film. The plane's inertia is simply too large to allow a rapid change of speed due to such a small unbalanced force.

During preparation of the film, five trials were carefully analyzed. The trial selected for reproduction gave results approximately as follows (height in meters, energies in units of 10^4 joules):

h	KE	PE	E_{total}
0	43	0	43
24	29.5	13	42.5
46	20	24.5	44.5
			E_{av} 43.3

As the table shows, the total energy remained constant to within about 4%.

L36 REVERSIBILITY OF TIME

The film concentrates on two types of sports that depend upon a substantive lack of friction: figure skating and pool. Although the expert in each sport may easily detect the direction of time through minor losses of energy or by a "feeling" for timing and position, the average teacher and student may be momentarily at a loss as to the direction of time.

It is the loss of energy and "timing" that often provides clues to the direction of time. An example of the loss of energy is clear in the next to last sequence where a billiard ball is struck and then slows down to a stop. The reverse motion probably is a reversal of the film. Timing is involved in the analysis of the motion of three balls, one cue ball and two others. From experience, we may "feel" that it is unlikely for two balls to hit the cue so precisely, and we are apt to believe that it was the cue ball that "did" the hitting.

In the more complicated event in which the cue splits the set, it appears that time can only flow in one direction. The balls lose their kinetic energy and come to a halt; it is statistically unlikely that they will then come together. The chances of the resulting random energy in the air, the table, and the internal structure of the billiard balls ever returning to its original form is statistically remote. Yet over short periods of time it seems as if the reversibility of Newtonian laws does hold. It seems that conservation of momentum and energy are "invariant" to within experimental error. When these short periods are added together the resulting continual losses of energy suggest that time is not reversible.

L37 SUPERPOSITION

The amplitudes of component waves are intentionally varied somewhat irregularly while "setting up" a superposition. This is to remind the student that a human operator is really causing these changes on the face of an oscilloscope. Be sure that the student understands that these are <u>not</u> animations.

Among the less familiar aspects of superposition is the fact that when two sinusoidal waves having the same wavelength but different phases and amplitudes are combined, the resultant is again a simple sinusoidal wave of the same wavelength but with an intermediate phase. The phenomenon of beats can also be seen to result from the superposition of two waves having slightly different wavelengths.

L38 STANDING WAVES ON A STRING

The film loops on standing waves (*L30*, *L40*, and *L41*) are designed to emphasize the underlying features common to all standing waves. The source is at the left (tuning fork, loudspeaker, or dipole antenna); a reflector is at the right (wooden rod piston, or aluminum mirror). At the end of *L41*, the three types of standing waves are compared in one composite picture in which the wavelengths are the same and the distances between nodes (given by $\frac{1}{2}\lambda$) are the same. The wave speeds and the frequencies differ by as much as a factor of 10^6.

L39 STANDING WAVES IN A GAS

In symbols, where L is the length of the tube,

$$L = (n + \tfrac{1}{2})(\tfrac{1}{2}\lambda)$$

$$\lambda = \frac{2L}{(n + \frac{1}{2})}$$

Since

$$F = \frac{v}{\lambda}, \quad \text{then } F = \frac{v}{2L}(n + \tfrac{1}{2})$$

and

$$\frac{F}{(n + \frac{1}{2})} = \frac{v}{2L} = \text{constant}$$

For this pipe, $f/(n + \frac{1}{2})$ is about 151 vib/sec, from which v can be found to be 348 m/sec if L is given as 1.15 m.

L40 VIBRATIONS OF A WIRE

The wire was actually a standard brass welding rod 2.4 mm in diameter. A short horizontal right-angle bend near the clamped end of the rod was essential to allow that point to serve as a node without undue restraint of vertical vibrations at neighboring points.

A surplus radar magnet was placed near an antinode of the wire's vibration. The magnetic force is perpendicular both to the current and to the magnetic field.

An audio frequency source of high current and low voltage was needed. An audio oscillator fed a 20-W hi-fi amplifier whose output was matched to the wire by a surplus power transformer used in a reverse or "stepdown" connection. The "high-voltage" (plate) winding was connected to the amplifier's 16-ohm output, and the "filament" winding was connected to the vibrating wire. Audio currents of several amperes passed through the wire.

For the wire, the observed frequencies of the first four modes were 8, 24, 48, 78 vib/sec.

An example will make clear the camera techniques used. For the "time exposure" or blurred shot, the camera speed was 3 frames/sec and the shutter was set at a full opening of 200° (out of 360°). Each frame thus was exposed for $(200/360)(\frac{1}{3}) = 1/5.4$ sec. During this time the wire had a chance to make enough vibrations to give the desired blurred effect, simulating what the eye sees. To obtain the "slow-motion" sequence for, say, the 48 vib/sec mode, the camera speed was set at 45 frames/sec, and the shutter closed to 20°. Each exposure was therefore $(20/360)(1/45) = 1/810$ sec, which was short enough to freeze the wire's motion. The strobe rates was $48 - 45 = 3$/sec as photographed. This becomes about 1/sec when projected in the classroom at 18 frames/sec.

The circular wire was actually clamped at two points very close together, which served as binding posts for the current. For the circular wire, the observed frequencies of the first four modes were 10, 24, 55, and 101 vib/sec.

In discussing the Bohr atom from the point of view of de Broglie waves (Unit 5), a familiar argument is that in the nth energy state there are n wavelengths in a complete circle of radius r. Then, since $\lambda = h/mv$, we have $n(h/mv) = 2\pi r$, whence $mvr = nh/2\pi$. This is Bohr's quantum condition for angular momentum. But the analogy is not as powerful as it seems. The Heisenberg uncertainty principle prevents us from knowing simultaneously both the angular momentum and the direction of the normal to the plane of an orbit. Therefore, the planetary model of an electron's plane orbit is not a valid one, although it is useful in many cases as a first step. The film of a vibrating circular wire can certainly be used to show the student how a simple mechanical system with circular symmetry has a discrete behavior. By analogy, this makes plausible the argument that a simple atom might behave in a similar discrete fashion.

L41 VIBRATIONS OF A RUBBER HOSE,

L42 VIBRATIONS OF A DRUM, AND

L43 VIBRATIONS OF A METAL PLATE

I. INTRODUCTION

This is a set of three qualitative demonstration films. No work notes for the students are provided. The subjects supplement the study of waves. They may be shown in class by the teacher, or viewed by students individually, after the concept of the standing wave has been covered in Chapter 12.

Film Loops make use of the concept of standing waves and extend it. *L41* should be shown first. All the loops demonstrate the following ideas:

1. The vibrations of bodies can be explained in terms of standing waves.

Suppose you have something (an elastic body) capable of a certain type of vibration. Then you will find that:

2. The body can vibrate in more than one *mode* of this type of vibration. Each mode corresponds to a fixed, but different, frequency of vibration.

Moreover, the films, especially *L41*, are so constructed that they suggest the following fact:

3. In principle, the number of possible modes of this type of vibration is infinite.

In each film, we drive the body at continuously increasing frequency (with a motor in *L41*, with a loudspeaker in *L42* and *L43*). When the driving frequency passes through one of the fixed frequencies of vibration of which the body is capable, something happens. This event is shown in detail and illustrates the concept of

4. resonance.

II. GENERAL DESCRIPTION

L41 Vibrations of a Rubber Hose

Unit 3 presents the concept "standing wave" in connection with one-dimensional transverse waves, such as are found in stretched strings.

Whenever two identical transverse traveling sine waves pass over the string in opposite directions the superposed wave pattern appears to be "standing." To put it in another way, the string is *vibrating*. The vibration occurs in *loops*. A loop is exactly one-half wavelength long. If the string vibrates in more than one loop, neighboring loops vibrate in opposite phases. Loops are separated by points on the string that do not move at all, called *nodes*. Successive nodes are separated by one-half the wavelength of the moving wave.

The rubber hose is driven by a variable speed dc motor connected through an eccentric linkage to point A at the bottom of the hose. The motor shakes point A in a sideways oscillatory manner, but the amplitude of this motion is so small that point A can be treated as a node when considering the waves in the hose. Motor speed is controlled by a Variac.

The film opens with a scene in which the hose is stretched to produce tension. The value T of this tension, together with the mass μ, per unit length of the hose, determine the wave speed v:

$$v = T/\mu$$

It follows that the wavelength λ is restricted by the length of the hose and by the fact that end points A and B must be nodes. The wavelength λ can only take on the discrete values

$$\lambda = 2L, L, \frac{2L}{3}, \frac{2L}{4}, \frac{2L}{5}, \ldots, \text{or}$$

$$\lambda_n = \frac{2L}{n}, n = 1, 2, 3, \ldots.$$

If f is the corresponding frequency, and for sine waves $\lambda f = v$,

$$f_n = \frac{v}{2L}, n = 1, 2, 3, \ldots.$$

The overtones are all integer multiples of the fundamental frequency $f_1 = v/2L$. In our hose, $f_1 = 2$ cycles/sec.

The above numerical details need not and probably should not be presented by you in class. It would be better to keep the discussion on a qualitative level.

The main sequence of the film records what happens after the motor is turned on and as its speed is continuously increased from zero. As the speed approaches 2 rps (not given in the film), the amplitude steadily rises (*resonance*). Motor speed continues to increase, a transition to the second harmonic takes place, and so on.

The film shows the first 15 transverse modes of the rubber hose.

L42 Vibrations of a Drum

The vibrating body is now a circular rubber membrane under tension. (The wave speed in this case is the square root of the ratio of surface tension to mass per unit area.)

Here we are dealing with two-dimensional waves that pass radially inward or outward as well as "angularly" around the circle. The standing waves are now not the simple sinusoidal loops we saw in *L41*. *L42* was made primarily to show what two-dimensional standing waves might look like qualitatively.

The "drum" is also capable of transverse vibration in an infinity of modes. The characteristic frequencies are now not integer multiples of each other. In the model drum we used, they were 50, 152, 258, and 373 cycles/sec for the first four symmetric modes shown, and 100 and 205 cycles/sec for the two antisymmetric modes.

The drum head was photographed, in some of the sequences, with a variable-speed, motion-picture camera. For each of the six modes, the speed of the camera was slowly varied from just below to just above the characteristic frequency of the mode in question, while keeping the drum in steady resonant vibration. The effect is stroboscopic. The vibration appears slowed down, revealing the shape of the membrane for each mode.

Fortunately for the viewer of this film, the film is silent. The loudspeaker driving the drum was running at 30 W root-mean-square power.

L43 Vibrations of a Metal Plate

A square aluminum plate is clamped tight at its center. A loudspeaker drives the plate from below at increasing frequency.

This system resonates for many different frequencies. The amplitudes of the two-dimensional, standing-wave patterns in the plate are too small to make them visible.

Sand is sprinkled on the plate. It vibrates furiously. When the output frequency of the loudspeaker reaches resonance for one of the plate's modes, the sand collects along lines, curves in a symmetric pattern, and is quiescent there. These lines and curves are the geometrical loci of the nodes of the two-dimensional standing waves in the plate (*nodal lines*). The patterns of nodal lines are called the *Chladni figures*.

Equipment Notes

TURNTABLE OSCILLATOR

A vertical rod is attached to a rotating phonograph turntable. This rod extends up through a long slot cut into a rectangular platform. The platform is constrained to move in a direction parallel to the slot and in a horizontal plane. As the turntable rotates, the platform moves with simple harmonic motion (SHM). This combination of turntable and platform is referred to as a turntable oscillator (Fig. 1). Various phenomena illustrated with the turntable oscillator are described below.

Fig. 1

Sine Curves

In Fig. 1, the turntable oscillator is shown in operation with a pen attached to the platform. A sine curve, drawn by the pen, can be seen on the strip chart recorder on the left. Figure 2 is a reproduction of the trace, which displays the SHM as a function of time.

Fig. 2

In order to emphasize oscillations and periodic functions in a simple way, students should be encouraged to produce a few hand-drawn traces on moving strip charts. The strip chart recorder has a plate with two slots mounted above the moving paper (see Fig. 3). The student, using a sharp pencil or felt-tipped pen, should make periodic movements back and forth.

Fig. 3

Figure 4 is a sketch of six such hand-drawn traces. Students were here asked to produce traces of (a) a sine curve, (b) a square wave, (c) a sawtooth, (d) an exponential relaxation curve, (e) rectified half waves, and (f) rectified full waves. It can be seen that quite respectable-looking traces can be produced in this way, and it seems likely that efforts to produce the traces will improve understanding of the kinematics of such motion in physical systems. Ask students to give actual examples of each motion they are graphing.

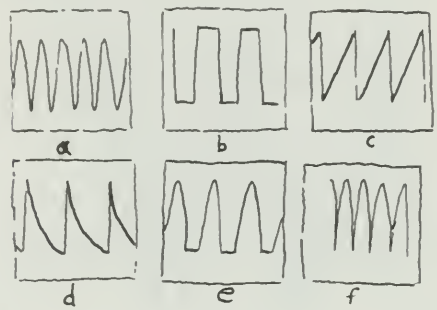


Fig. 4

Harmonic Synthesis

The superposition of two waves can be demonstrated qualitatively and quantitatively with two turntable oscillators arranged so that the reciprocating motions are parallel to each other (see Fig. 5).

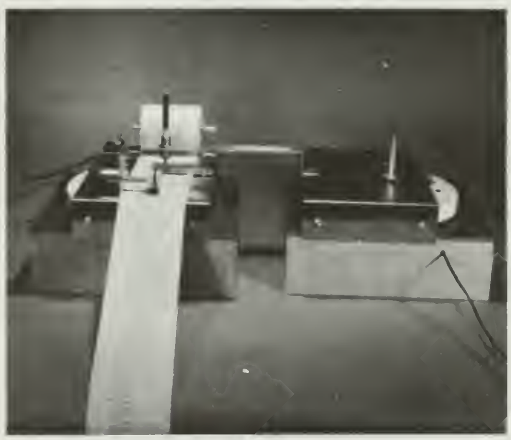


Fig. 5

A pen mounted on one oscillator leaves a trace on a strip chart recorder mounted on the second oscillator. The paper of the recorder moves perpendicularly to the direction of oscillation. If the frequency of one oscillator is *multiple* of the other, the resulting trace illustrates the element of *harmonic synthesis;* that is, the production of complex periodic functions by the addition of two or more frequencies. The traces produced in this way are shown in Fig. 6 through Fig. 9.

The amplitude of the oscillation is increased by moving the vertical peg toward the rim of the turntable. Adjust the amplitude so that either trace alone gives only half-paper width.

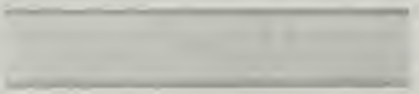


Fig. 6

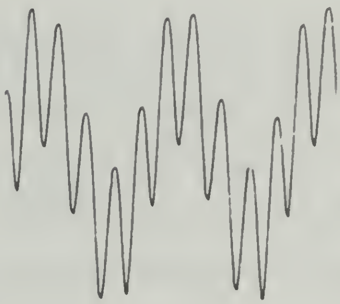


Fig. 7

Fig. 8

Fig. 9

The frequency of oscillation depends upon the rotational speed of the turntables. The turntable speed selector provides the "coarse tuning" at 16, 33, 45, and 78 rev/min. Fine tuning of one turntable can be accomplished by using a Variac or powerstat voltage control. Always *reduce* the speed by *lowering* the voltage. Voltages in excess of 120 V may damage the phonograph motor. (CAUTION: Do not use the transistorized speed controls now available for drills, and other ac-dc motors, as they may be damaged when used with phono motor.) The speed can also be adjusted (slowed) by mechanically loading the motor, i.e., by adding weights to the platform. Weights should be placed in pairs symmetrically on the two sides of the platform. The phase relationship can be altered by adjusting the positions of the two recorders before switching on, and then turning them both on simultaneously.

ACTIVITIES

Some student activities associated with the coupled oscillators follow:

1. Produce superposed traces of two sine curves of different frequency ratios, amplitudes, and phases.
2. Attempt to *analyze* superposed traces in order to identify the components.
3. Compare the sum of two sine curves with the original curves: The ordinates of the original curves should be first added arithmetically, point for point, and the resulting "theoretical" curve can then be compared with the one obtained by actual mechanical addition of two sinusoidal motions.
4. Apply the skills learned from the above to the analysis of oscilloscope traces of simple sound waveforms from musical instruments, tuning forks, combined output of two audio oscillators, etc.

Beats

The outputs of two coupled oscillators produce noticeable beats if the frequencies f_1 and f_2 are nearly the same. Beats are commonly demonstrated by simultaneously sounding two tuning forks of slightly different frequencies. Beats can easily be produced on a piano or organ by playing two adjacent notes (a black key and a white key) simultaneously.

The superposition of two waves to produce beats can be demonstrated quantitatively by coupling the outputs of the two turntable oscillators as described in the previous section. Set each oscillator for equal amplitude, and the "coarse tuning" controls for the same frequency. The amplitude of *each* oscillator should not be greater than one-half the paper width.

Figure 10 shows the trace with, first, only oscillator #1 running at f_1 and, then, only oscillator #2 running at f_1. The beats are produced on the strip chart recorder when both oscillators are operated simultaneously.

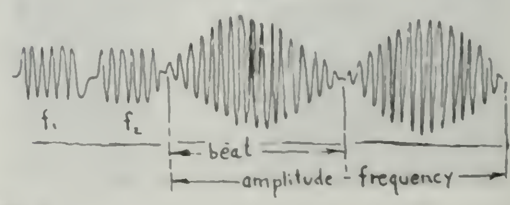


Fig. 10

When two functions have the form $y_1 = \sin a$ and $y_2 = \sin b$ and these functions are added, the result is:

$$y = y_1 + y_2 = \sin a + \sin b$$
$$= 2 \cos\left(\frac{a-b}{2}\right) \sin\left(\frac{a+b}{2}\right) \quad (1)$$

If $a = 2\pi f_1 t$ and $b = 2\pi f_2 t$ and $f_1 > f_2$, Equation (1) becomes

$$y = [2 \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t] \sin 2\pi \left(\frac{f_1 + f_2}{2}\right) t \quad (2)$$

You now see that y is a periodic function with an average frequency, f_{av} given by

$$f_{av} = \frac{f_1 + f_2}{2} \quad (3)$$

Simultaneously, the amplitude of y varies in time with the lower frequency:

$$f_{amp} = \frac{f_1 - f_2}{2} \text{ (the amplitude frequency)} \quad (4)$$

One complete cycle at this frequency is marked "amplitude frequency" in Fig. 10.

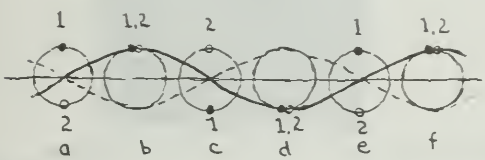


Fig. 11

With two oscillators operating at slightly different frequencies, one gains on the other. Assume that they start at the same time, but 180° out of phase (Fig. 11); their outputs add to zero (a null). As #1 oscillator overtakes #2, they come into phase and their outputs add as in (b). As #1 continues to gain on #2, they again go out of phase [the null at (c)]. The two are again in phase at (d) *but now in the opposite phase;* and again out of phase at (e); and so on. Since the positions at (a) and (e) are identical, one complete amplitude cycle has elapsed. The beat frequency cycle is between consecutive nulls or maxima and occurs twice during each amplitude cycle (marked "amplitude frequency" in Fig. 10).

Spatial Frequency and Wave Number

From the trace on a strip chart recorder one can measure the "spatial frequency" (i.e., the number of oscillations *per centimeter* on the chart. The symbol "υ" (Greek letter "nu") is used for spatial frequency; the units are cm^{-1}. The spatial frequency is quite analogous to the familar time frequency f, i.e., the number of oscillations per second, which is measured in sec^{-1}. Just as time frequency is the reciprocal of period ($f = 1/T$), so spatial frequency is the reciprocal of wavelength $\upsilon = 1/\lambda$. Incidentally, the term *wave number*, used by spectroscopists is also the reciprocal of wavelength.

If the tape moves through the recorder at a uniform rate, $\upsilon \propto f$. If v is constant then

$$f = \frac{v}{\lambda} \propto \upsilon \quad (5)$$

Since you already know that y is a periodic function with a time frequency f, Equation (5) shows that y must also be a periodic function with the *spatial* frequency υ_{av}. Equations (3) and (4) can now be written in terms of spatial frequency:

$$\upsilon_{av} = \frac{\upsilon_1 + \upsilon_2}{2} \quad (6)$$

(the average frequency)

and

$$\upsilon_{amp} = \frac{\upsilon_1 - \upsilon_2}{2} \quad (7)$$

(the amplitude frequency)

From Equations (6) and (7), division gives the number of oscillations in every two beats:

$$\frac{\upsilon_{av}}{\upsilon_{amp}} = \frac{\upsilon_1 + \upsilon_2}{\upsilon_1 - \upsilon_2} \quad (8)$$

These three relationships can be verified with records similar to the one shown in Fig. 12.

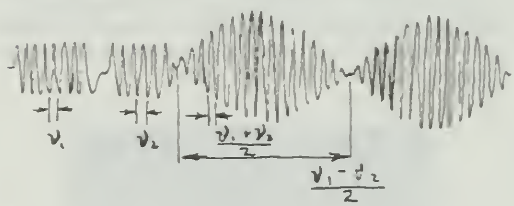


Fig. 12

Lissajous Figures

Lissajous figures can be produced by coupling two oscillators with the output of one perpendicular to the output of the other (see Fig. 13). Demonstrate Lissajous figures first with two turntable oscillators, and then with an audio oscillator and an oscilloscope. Set the oscilloscope sweep control to

Fig. 13

"line" position and adjust the sweep width to about one-half the screen diameter. Connect the audio oscillator to the vertical input, and adjust the amplitude of the signal until the signal is about one-half screen diameter. Stational Lissajous figures will appear on the oscilloscope screen when the oscillator frequency is exactly a multiple, or submultiple, of the line frequency. If the phase changes slightly, the shape of the figure is altered (see Fig. 14).

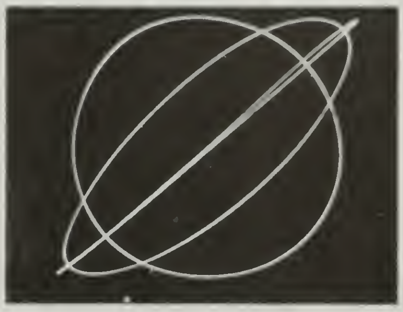


Fig. 14 Same frequency; phase difference changing from zero (straight line) to 90° (circle).

If the frequencies of the oscillators are proportional to whole numbers, the trace closes upon itself and may be repeated again and again. Figure 15 is a reproduction of actual traces produced by a pair of turntable oscillators.

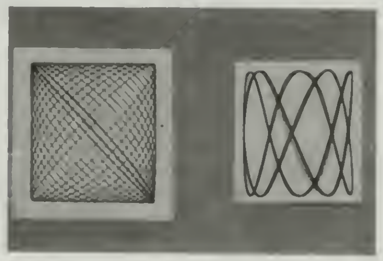


Fig. 15 Left picture shows almost equal frequencies, drifting in phase. Right picture shows frequency 3 to 1, drifting in phase.

Some of the traces show that a given frequency ratio can produce a variety of figures if the relative amplitude and starting phase are changed.

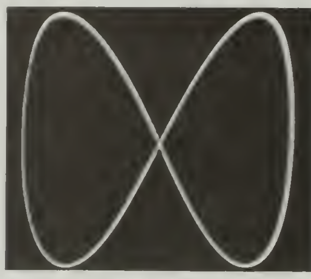


Fig. 16 Frequency 2 to 1 with zero phase difference.

Fig. 17 Frequency 2 to 1 with 90° phase difference.

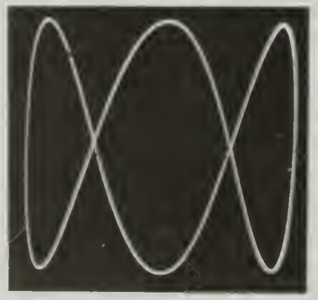


Fig. 18 Frequency 3 to 1 with 90° phase difference.

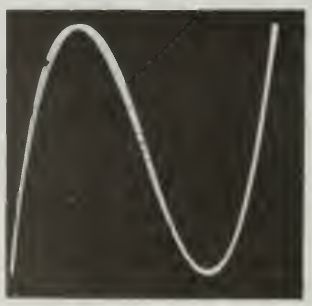
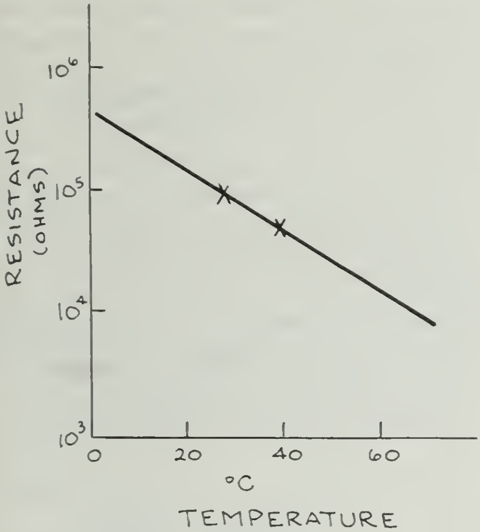
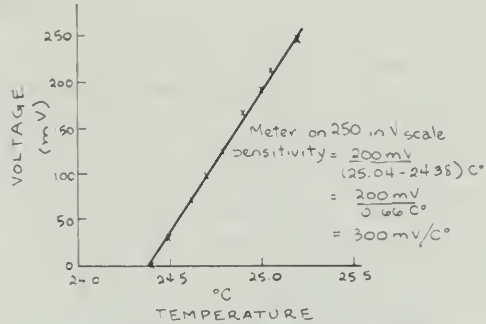
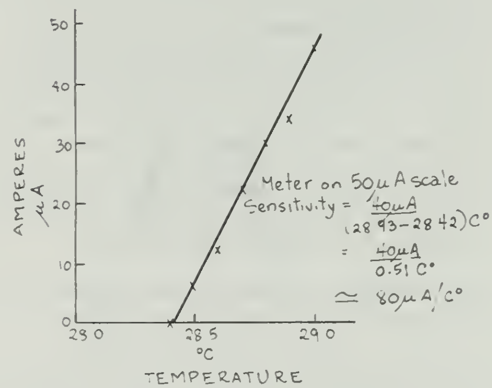


Fig. 19 Frequency 3 to 1 with zero phase difference

A typical thermistor has a resistance of about 100 K at 25°C, and its temperature coefficient is approximately −5%/°C [i.e, resistance at 26°C is 0.95×100 K; at 27°C it is $0.95^2 \times 100$ K, and at $(25 + t)$°C it is $0.95^t \times 100$ K]. A plot of resistance R versus temperature on semilog paper is a straight line:



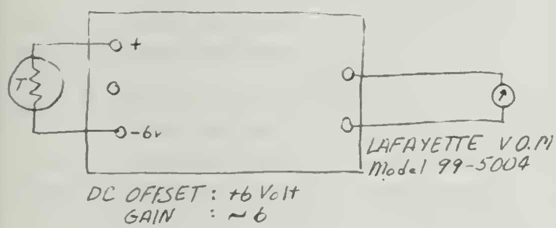




Sample Calibration Curves for Thermistor–Amplifier Combination

The graphs are intended as examples only the slope of the line (sensitivity) will vary with the gain setting and the intercept will depend on the actual thermistor used, the gain, and dc offset settings.

Note that over small temperature ranges the response is linear.



To measure small changes at other temperatures, adjust the gain and dc offset as necessary to get a near-zero reading on a sensitive scale.

To measure temperatures over a larger range it is simplest to measure the thermistor's resistance directly in ohms with a volt-ohm-milliammeter (see also *Experiment 3–10:* "Thermometers and Temperature"). The resistance versus temperature (°C) plot is not linear over a wider range (see above).

Suggested Solutions to Study Guide Problems

CHAPTER 9

2. He called this statement an "uncontestable axiom" because in all cases where he had made careful measurements he could find no deviation from it. Recall Newton's "Rule of Reasoning III" quoted in Chapter 8. If mass were not conserved in all reactions it would have been a remarkable coincidence if Lavoisier had happened to test only reactions where it was.

3. (a) Yes; an increase of 2×10^3 parts in 6×10^{21} (less than a millionth of a millionth of a millionth!) falls far below the limits of accuracy of any experiment.
   (b) The solar system would be sufficiently large if the statement given about the source of meteoric dust is accepted. (Assume the solar system to extend out to 1.6×10^{13} km to include comets.)

4. The purpose of this question is to force students to examine their own thinking with regard to the durability of scientific statements. (If experiments are limited to ordinary chemical reactions, the answer would be "no," based on consideration of mass–energy transformation. Most students will not be aware of this, however, before studying Chapter 20.)

5. No; a difference in weight (pull of gravity) is not the same as a difference in mass (inertia).

6. Place the snake within a container equipped with a device for igniting the pill by remote control (for example, an electric spark). Seal the container and determine its mass before and after ignition.

7. (a) Mass of liquid remaining = sum of initial masses minus mass of precipitate = (19.4 + 100 + 33.1 + 100) − 32.3 = 220.2 g.
   (b) The solids going into reaction weighed
   19.4 g + 33.1 g = 52.5 g total
   The yellow precipitate weighed 32.3 g dry and the white precipitate weighed 20.2 g for a total of 52.5 g of solid. No mass has been added by or lost from the water.

8. (a) The total mass is 60 g on the earth and on the moon.
   (b) Mass is an attribute of material. Weight is a description of the gravitational attraction of a large body (earth or moon) on a smaller mass. Mass does not change, but weight depends on location.
   (c) The statement reports nothing about masses.

9. (a) For an isolated system, there will be no change in the total momentum.
   (b) The momenta of the discs are.
   A. 40 kg·m/sec west
   B. 150 kg·m/sec north
   C. 20 kg·m/sec east
   D. 20 kg·m/sec east
   (c) The momenta east and west cancel. After the collision the total mass is 25 kg. Since $m_1 v_1 = m_2 v_2$,

$$v_2 = \frac{150 \text{ kg·m/sec north}}{25 \text{ kg}}$$
$$= 6 \text{ m/sec north}$$

   (d) Because momentum is a vector quantity momenta in one direction can cancel those in another direction, as for discs A = C + D in part (b).

10. (a) Given: $m_A = 10^5$ kg $\quad m_B = 1.5 \times 10^5$
$v_A = 2.0$ m/sec $\quad m_B v_B = 0 \quad v_A' = v_B'$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$m_A v_A + m_B 0 = (m_A + m_B) v_A'$$

   (b) $$v_A' = \frac{m_A v_A}{m_A + m_B}$$

   (c) $$v_A' = \frac{10^5 \times 2}{10^5 + 1.5 \times 10^5} = \frac{2 \times 10^5}{2.5 \times 10^5}$$
$$= 0.8 \text{ m/sec}$$

11. According to *Webster's Third International Dictionary*, momentum is
   (1) a property of a moving body that determines the length of time required to bring it to rest when under the action of a constant force or moment
   (2) moment
   (3) (a) the force of motion acquired by a moving body as a result of the continuance of its motion
   (b) something held to resemble such force of motion of a moving body.

   None of these statements resemble the technical definition of mass × velocity but definition (1) above is tied to Sec. 9.4 discussion. $F\Delta t = \Delta mv$.

12. To have equal impact the momenta of the cannon ball and the light particle would have to be equal.

mass of light particle × (3×10^8)
= 10 × 100 kg·m/sec

$$\text{mass of light particle} = \frac{10 \times 10^2}{3 \times 10^8}$$
$$= 3.3 \times 10^{-6} \text{ kg}$$

13. (a) The carts exert forces on each other.
(b) Yes, to the system as a whole assuming there is no friction between carts and track.
(c) Take a stroboscopic picture of the collision. Note that there is no well-defined instant of collision and that the carts do not have uniform speeds before or after the collision. So momentum comparisons will have to be made for corresponding time intervals or when the carts are so far apart that the repulsive force is very small.

14. The closed system must include the ball and the earth; as the ball rises the earth moves away in the opposite direction, as the ball falls the earth "falls" toward the ball. Of course, the mass of the earth is so much greater than the mass of the ball that the earth's speed in both cases is too small to detect.

15. Yes. Corollary III states: "The quantity of motion which is obtained by taking the sum of the motions directed towards the same parts, and the difference of those that are directed to contrary parts, suffers no change from the action of bodies among themselves." Definition II states: "The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly."

16. (a) Since $\Delta\vec{p} = \vec{F}\Delta t$,

$$\Delta t = \frac{\Delta\vec{p}}{\Delta\vec{F}} = \frac{80 \text{ kg·m/sec}}{20 \text{ N}}$$
$$= 4 \text{ sec}$$

(b) $v_f = (v_i + \Delta v) = (v_i + a\Delta t)$

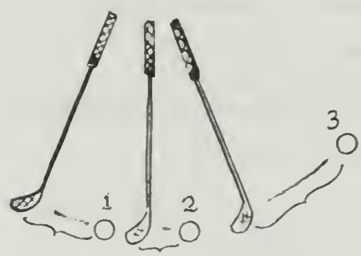
$$= \frac{50 \text{ kg·m/sec}}{5 \text{ kg}} + \frac{10 \text{ N} \times 5 \text{ sec}}{5 \text{ kg}}$$
$$= 20 \text{ m/sec}$$
$$\vec{p}_f = \vec{p}_i + \Delta\vec{p}$$
$$= 50 \text{ kg·m/sec} + (5 \text{ sec} \times 10 \text{ N})$$
$$= 100 \text{ kg·m/sec}$$
$\vec{p}_f = mv_f$, therefore
$$v_f = \frac{p_f}{m}$$
$$= \frac{100 \text{ kg·m/sec}}{5 \text{ kg}}$$
$$= 20 \text{ m/sec}$$

(c) Some problems are easier to solve with the momenta formula, but they are not more basic.

17. (a) Both ocean liners and planes have large momenta: undergoing a significant change in direction implies a velocity (and momentum) change. Such a change requires a force F operating for a time Δt. Water and air do not have a firm enough consistency to allow for a sizeable F: hence Δt must be large.
(b) The argument is similar to that above for planes, unless large rocket blasts are also reported.

18. Initial momentum $= 60 \text{ kg} \times 20 \text{ m/sec}$
$$= 1.2 \times 10^3 \text{ kg·m/sec}$$
$$F\,\Delta t = \Delta \text{ momentum}$$
$$F = \frac{\Delta \text{ momentum}}{\Delta t}$$
$$= \frac{1.2 \times 10^3 - 0}{3}$$
$$= 4 \times 10^2 \text{ N}$$
$$\text{acceleration rate} = \frac{\Delta v}{\Delta t} = \frac{0 - 20}{3}$$
$$= -6.6 \text{ m/sec}^2$$
$$d = v_i t + \tfrac{1}{2} at^2 = 30.3 \text{ m}$$
Her momentum is imparted to the earth.

19. Estimate the scale of the photograph, assuming the golf clubs to be 1 m long. The sketch represents a trace from the photograph on page 27 of Unit 1. Three successive relative positions of club and ball are shown; the first before contact, the second and third after contact.



(a) The distance between ball positions 2 and 3 is about the same as the length of the club, hence the speed of the ball after impact is

$$\frac{1 \text{ m}}{0.01 \text{ sec}} = 100 \text{ m/sec}$$

(b) momentum $= 0.046 \text{ kg} \times 100 \text{ m/sec}$
$$= 4.6 \text{ kg·m/sec}$$

(c) In the interval between the first and second strobe flash (0.01 sec) the club must move to the ball and the ball away from the club. Thus, duration of impact must be less than about ¼ of 0.01 sec or 0.003 sec.

(d) $F = \dfrac{\Delta\text{momentum}}{\Delta t} = \dfrac{4.6}{0.003} = 1.5 \times 10^3$ N (at least)

20. Yes; when the statement is made that $m\Delta v = \Delta mv$.

21.
$$F_A = F_{AB} + F_{AC}$$
$$F_B = F_{BA} + F_{BC}$$
$$F_C = F_{CA} + F_{CB}$$

Since the interaction time Δt is the same for all, the changes in momentum will be as follows.

$$\Delta p_A = F_A \Delta t = F_{AB} \Delta t + F_{AC} \Delta t$$
$$\Delta p_B = F_B \Delta t = F_{BA} \Delta t + F_{BC} \Delta t$$
$$\Delta p_C = F_C \Delta t = F_{CA} \Delta t + F_{CB} \Delta t$$

$$\begin{aligned}\text{Total momentum change} &= \Delta p_A + \Delta p_B + \Delta p_C \\ &= (F_{AB} + F_{BA} + F_{AC} + F_{CA} \\ &\quad + F_{BC} + F_{CB}) \Delta t \\ &= 0 \text{ since } F_{AB} = -F_{BA} \text{ etc.}\end{aligned}$$

22. (a) $F \Delta t = \Delta mv = m(v_0 - v)$

$$\Delta t = \frac{m(v_0 - v)}{F}$$

(b) Exhaust momentum would be equal to momentum change of capsule: $m(v_0 - v)$.

(c) Mass of fuel $\times\ v_e = m(v_0 - v)$

$$\text{mass of fuel} = \frac{m(v_0 - v)}{v_e}$$

23. (a) By the conservation of momentum principle, $\Delta(m_A\vec{v}_A) + \Delta(m_B\vec{v}_B) = 0$, assuming m_A and m_B to be constant.

Then, $m_A \Delta v_A = -m_B \Delta v_B$

and $$\frac{\Delta v_A}{-\Delta v_B} = \frac{m_B}{m_A}$$

(b) Let A be the pellet, B the bowling ball: then

$m_B > m_A$ and $\frac{m_B}{m_A} > 1$

$\Delta v_A = -\frac{m_B}{m_A} \Delta v_B'$ hence $\Delta v_A > \Delta v_B$

(c) $$v_A' = \frac{m_A - m_B}{m_A + m_B} v_A$$

If $m_B > m_A$, we can neglect the contribution of m_A to both the numerator and denominator above, so that v_A' becomes $\frac{-m_B}{m_B} v_A = -v_A$

That is, the speed is the same but with reverse direction

24. $m_C\vec{v}_C + m_S\vec{v}_S = m_C\vec{v}_C' + m_S\vec{v}_S'$

(a) $\vec{v}_C = \vec{v}_S = 0$

(b) Total momentum is 0 since both bodies are at rest.

(c) Total momentum is 0 by the law of the conservation of momentum.

(d) *Magnitudes* of $m_C\vec{v}_C'$ and $m_S\vec{v}_S'$ are equal since their sum is equal to 0

(e) $$0 = m_C\vec{v}_C' + m_S\vec{v}_S'$$

$$m_C\vec{v}_C' = -m_S\vec{v}_S'$$

$$\frac{\vec{v}_C'}{\vec{v}_S'} = -\frac{m_S}{m_C} = -\frac{10 \text{ kg}}{1000 \text{ kg}} = -\frac{1}{100}$$

$$\vec{v}_C' = -\frac{1}{100} \times 1000 \text{ m/sec}$$
$$= -10 \text{ m/sec}$$

25. (a) Δ momentum $= F \Delta t\ (35\text{-}28) \times 10^6 \times 1.5 \times 10^2 = 10.5 \times 10^8$ kg·m/sec.

(b) The amount of fuel that was expended.

26. In cases (a), (b), and (d) Δt is lengthened, thereby decreasing F. In case (c), Δt is shortened making F large.

27. (a) No. Momentum is conserved. The center of gravity of the system remains fixed. As the ball swings forward the cart moves backward and vice versa when the ball swings back.

(b) The cart would continue to move forward with the slight oscillation described in (a).

(c) The cart would continue to move backward with the slight oscillation described in (a).

28. (a) The car on the left was traveling faster.

(b) The speed of one car (*or* the masses of the cars), the distance they slid after the collision, and the retarding frictional force of the ground.

(c) The frictional force between the ground and the cars is constant.

29. Assume the person is fixed to the earth. While the bullet is in flight, person, target, and earth collectively have the same backward momentum as the bullet forward. When the bullet hits the target, the speeds of all the components become zero. Momentum is conserved in (a), (b), and (c). In all three cases the <u>total</u> momentum = 0.

30. (a) 0.8 × mass of ball (toward the cushion)

(b) 0.8 × mass of ball (away from cushion)

(c) Δ momentum $= 2 \times 0.8 \times m = 1.6\ m$

(d) No, for the system that contains only the ball

Yes, for the system includes the ball and the earth. Then the compensating Δ momentum is supplied by the cushion–table–earth system

31. (a) As he leaps into the air the asteroid will recoil in the opposite direction.

(b) The asteroid will always move in the opposite direction to his motion, so it will spin

32. (a) The total kinetic energy of an isolated system involving only elastic collisions is constant

(b) The total kinetic energy does not change

(c) $$\begin{aligned}(KE)_1 &= \tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2v_2^2 \\ &= \tfrac{1}{2} \times 5 \text{ kg}(4 \text{ m/sec})^2 \\ &\quad + \tfrac{1}{2} \times 10 \text{ kg}(3 \text{ m/sec})^2 \\ &= 85 \text{ kg·m/sec}^2 \\ &= 85 \text{ J}\end{aligned}$$

(d) After elastic rebound, the total kinetic energy is not changed

33.

Object	m (kg)	v (m/sec)	mv (kg·m/sec)	$\frac{1}{2}mv^2$ (kg·m²/sec²)
baseball	0.14	30.0	**4.2**	**63**
hockey puck	**0.17**	50.0	8.55	**214**
super ball	0.050	1.5	**0.075**	**0.056**
light car	1460	**49.6**	**7.25×10^4**	1.79×10^6
mosquito	**5×10^{-5}**	**0.4**	2.0×10^{-5}	4.0×10^{-6}
football player	**100**	**5.0**	**500**	**12.5×10^2**

Note: Answers to be supplied by students are in bold type

34. (a) The total mass is 4 g + 6 g + 8 g = 18 g. The total momentum is (4 × 20) g·cm/sec north + (6 × 3) g·cm/sec east + (8 × 10) g·cm/sec south which totals 18 g·cm/sec east. The total kinetic energy is

$$KE = \tfrac{1}{2} \times 4(20)^2 + \tfrac{1}{2} \times 6(3)^2 + \tfrac{1}{2} \times 8(10)^2 = 1{,}227\ \text{J}$$

(b)

	Mass	Momentum	Kinetic Energy
1. open system elastic collisions	18 g	unknown	unknown
2. open system inelastic collisions	18 g	unknown	unknown
3. closed system elastic collisions	18 g	18 g·cm/sec east	1,227 J
4. closed system inelastic collisions	18 g	18 g·cm/sec east	1,227 J

35. Given: $m_A = 3\,m_B$ $\quad v_A = -v_B$

$v_A{}' = 0$ $\quad v_B{}' = -2\,v_B$

To show conservation of momentum:

$$3m(-v_B) + m\,v_B = 3m(0) + m(-2v_B)$$
$$-2m\,v_B = -2m\,v_B$$

To show conservation of kinetic energy:

$$\tfrac{1}{2}(3m)\,v_B^2 + \tfrac{1}{2}m\,v_B^2 = \tfrac{1}{2}(3\,m)(0)^2 + \tfrac{1}{2}\,m\,(2\,v_B)^2$$
$$4(\tfrac{1}{2}m\,v_B^2) = \tfrac{1}{2}m\,4v_B^2$$
$$2m\,v_B^2 = 2m\,v_B^2$$

36. $m_A = m \qquad m_B = 3m \qquad v_A{}' = ?$

$v_A = v \qquad v_B = 0 \qquad v_B{}' = ?$

Conservation of momentum:

$$mv = mv_A{}' + 3mv_B{}'$$
$$v = v_A{}' + 3v_B{}'$$
$$v_A = v - 3v_B{}'$$

Since the collision is head-on the vector aspect of this equation will reduce to only the possibility of different directions for the v's that show up algebraically as a ± sign.

Conservation of kinetic energy:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_A{}'^2 + \tfrac{1}{2}3mv_B{}'^2$$
$$v^2 = v_A{}'^2 + 3v_B{}'^2$$

Substituting for $v_A{}'$ from the result above

$$v^2 = v^2 - 6vv_B{}' + 9v_B{}'^2 + 3v_B{}'^2$$
$$6vv_B{}' = 12v_B{}'^2$$
$$v_B{}' = \frac{v}{2} \text{ assuming } v_B{}' \neq 0$$
$$v_A{}' = v - 3v_B{}' = v - 3\frac{v}{2} = -\frac{v}{2}$$

Thus, the small ball rebounds with one-half its original speed and the larger ball moves in the original direction with one-half the original speed of the small ball.

CHAPTER 10

2. No work is done since the direction of the force is perpendicular to the direction of motion. (Students may argue that he does get tired.)

3. (a) The speeds in the new reference frame are $(v_1 - u)$ and $(v_2 - u)$.

(b) No. In the new reference frame, they are $KE_1{}' = \tfrac{1}{2}m(v_1 - u)^2$ and $KE_2{}' = \tfrac{1}{2}m(v_2 - u)^2$

(c) No.

$$\Delta KE' = KE_2{}' - KE_1{}'$$
$$= \tfrac{1}{2}m(v_2 - u)^2 - \tfrac{1}{2}m(v_1 - u)^2$$
$$= \tfrac{1}{2}m(v_2^2 - 2uv_2 + u^2 - v_1^2 + 2uv_1 - u^2)$$
$$= [\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2] + (muv_1 - muv_2)$$

This is *not* the same as before when

$$\Delta KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

Note the additional terms: $muv_1 - muv_2$

(d) No. $W' = F' \times d'$

Since $F' = F$, and $d' = d - ut$,

$$W' = Fd - Fut$$
$$= Fd - u(Ft)$$
$$= Fd - u(m\Delta v)$$
$$= Fd - um(v_2 - v_1)$$

This is *not* the same as before when $u = Fd$

(e) Yes. $W' = Fd - um(v_2 - v_1)$
$$= \Delta KE - um(v_2 - v_1)$$
$$= [\tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}] + [umv_1 - umv_2]$$
which is the same as $\Delta KE'$.

(f) Only (iii). The relationship $Fd = \Delta(\tfrac{1}{2}mv^2)$

(g) One would think that something an object "has" would be a property of the object itself, and not be dependent on anything outside of the object, as, for example, the reference frame from which it is measured.

4. $KE = \tfrac{1}{2}mv^2$
$$= \tfrac{1}{2}(9.1 \times 10^{-31})(2 \times 10^8)$$
$$= 1.8 \times 10^{-14}\ J$$
$$N = \frac{1J}{1.8 \times 10^{-14}\ J\ \text{per electron}}$$
$$= 5.5 \times 10^{13}\ \text{electrons}$$

5. The final KE is 80 J. The initial KE is
$$(KE)_1 = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2} \times 5\ kg(4\ m/sec)^2$$
$$= 40\ J$$

Therefore, 40 J must also have been added by the 40-N force.

$$\Delta(KE) = Fd$$
$$d = \frac{\Delta_r(KE)}{F}$$
$$= \frac{40\ J}{4\ N}$$
$$= 10\ m$$

6. Estimates may have a wide range.

	m (kg)	v (m/sec)	v^2	$KE = \frac{mv^2}{2}$ (joules)
(a)	0.15	30	900	67.5
(b)	10^5	300	9×10^4	4.5×10^9
(c)	75	10	10^2	3.75×10^3
(d)	6×10^{24}	3×10^4	9×10^8	2.7×10^{33}

7. (a) $a = \frac{F}{m} = \frac{400}{200} = 2\ m/sec^2$
$$d = \frac{at^2}{2}$$
$$t^2 = \frac{2d}{a} = \frac{2 \times 900}{2} = 900$$
$$t = 30\ sec$$
$$v = at = 2 \times 30 = 60\ m/sec$$

(b) work done $= Fd = 400 \times 900$
$$= 36 \times 10^4\ J$$
$$= \Delta KE = \tfrac{1}{2}200v^2 = 10^2v^2$$
$$v^2 = 36 \times 10^2;\ v = 60\ m/sec$$
The result is the same as in (a).

8. (a) $\Delta KE = KE_{final} - KE_{initial}$
$$= 0 - \tfrac{1}{2}mv_1^{\,2}$$
$$= 0 - \tfrac{1}{2}(0.002)(300)^2$$
$$= -\tfrac{1}{2} \times 2 \times 10^{-3} \times (3 \times 10^2)^2$$
$$= -90\ J$$

(b) Work done by tree $= -\Delta KE$ of bullet $=$ 90 J

(c) $W = F \times d$
$$F = \frac{W}{d} = \frac{90\ J}{0.05\ m} = 18 \times 10^2$$

9. $W = F \times d = \Delta KE$ of ball
$$= \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 4.6 \times 10^{-2} \times 100^2$$
$$= 2.3 \times 10^2\ J$$

The ball does the same amount of work on the club in the opposite direction.

10. mass of penny $= 3\ g = 3 \times 10^{-3}\ kg$
thickness $= 1.5\ mm = 1.5 \times 10^{-3}\ m$

The top penny is 49 thicknesses or 74×10^{-3} m above the bottom one.

(a) PE of top penny $= ma_gd = (3 \times 10^{-3}) \times 9.8 \times (7.4 \times 10^{-2})$
$$= 2.2 \times 10^{-3}\ \text{Joules}$$

(b) average height of pile $=$
$$25 \times 1.5 \times 10^{-3} = 37 \times 10^{-3}\ m$$
total PE $=$ no. of pennies $\times$ mass of each $\times\ a_g \times$ average height
$$= 50 \times (3 \times 10^{-3}) \times 9.8 \times (37 \times 10^{-3})$$
$$= 5.4 \times 10^{-2}\ J$$

11. (a) $d = \frac{1\ J}{5\ N} = 0.2\ m$

(b) Energy required $=$ work done lifting the plane to cruising altitude
$=$ weight of plane $\times$ height
$= 7 \times 10^5\ N \times 10^4\ m$
$= 7 \times 10^9\ J$

12. 1. Provide a standard mass raised to the proper height. The proper height can be determined for a 1-kg mass as follows:

$$PE = ma_gd = 1\ J$$
$$d = \frac{1\ J}{1\ kg \times 9.8} = 10.2 \times 10^{-2}\ m$$
$$= 10.2\ cm$$

Note: This height depends on value of a_g at the particular location.

2. Provide a standard mass traveling with the proper speed. For 1 kg, the speed can be determined as follows:

$$KE = \tfrac{1}{2}mv^2 = 1\ \text{J}$$

$$v^2 = \frac{2 \times 1\ \text{J}}{1\ \text{kg}} = 2$$

$$v = 1.4\ \text{m/sec}$$

Note: This method has the disadvantage that the energy cannot be stored.

3. Provide a compressed spring on which 1 J of work has been done.

Note: This method requires a knowledge of the behavior of springs, including the fact that they can lose their elasticity through aging.

13. (a) The stored energy can be measured as a change in potential energy, PE = mgh, where h is the stretch Δx of the rubber band. (b) Δ (PE) = $-\Delta$(KE) (c) As the weights bob up and down, the total energy of the system is distributed in changing fractions between kinetic energy, gravitational potential energy, and elastic (stretch) energy.

14. (a) Force exerted on earth by stone
= force exerted on stone by earth
= 1 kg × 9.8 m/sec²
= 9.8 N
mass of earth = 6 × 10^{24} kg

$$\text{acceleration of earth} = \frac{F}{m} = \frac{9.8\ \text{N}}{6 \times 10^{24}\ \text{kg}} = 1.6 \times 10^{-24}\ \text{m/sec}^2$$

$$d = \tfrac{1}{2}at^2$$

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 1}{1.6 \times 10^{-24}}} = \sqrt{1.25 \times 10^{24}} = 1.1 \times 10^{12}\ \text{sec}$$

There are about 3 × 10^7 sec in 1 yr so this would be about 3 × 10^4 yr.

(b) $t = \sqrt{\dfrac{2d}{a_g}} = \sqrt{\dfrac{2}{9.8}} = \sqrt{0.2} = 0.45$ sec needed for the stone to fall 1 m. During this time the earth, with an acceleration of 1.6 × 10^{-24} m/sec², will move 1.6 × 10^{-25} m since

$$d = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times (1.6 \times 10^{-24}) \times 0.2 = 1.6 \times 10^{-25}\ \text{m}$$

(c) The gravitational potential energy is assigned to the earth–rock system because it describes the PE of the rock relative to the earth. PE always has a frame of reference "potential to something."

15. The PE stored in the ball when it is initially pulled aside is converted into KE during the swing, which is then dissipated on impact in KE of parts of the wall and work done in breaking structural bonds. (Energy is also converted to other forms, such as heat and noise.)

16. (a) If the boulder could somehow fall all the way to the center of the earth, the greatest decrease in PE of the isolated boulder–earth system would be experienced.
(b) No. The system possesses gravitational potential energy with respect to the sun, for example.
(c) No. The usefulness of the concept of potential energy lies in considering *changes* in it.

Convenient zero-levels would be
(a) the lowest point in the pendulum swing
(b) the lowest point on the tracks
(c) the midpoint of the oscillation
(d) The perihelion distance. One may consider the mass of the sun to be concentrated at its center. This would not be a suitable zero-level because the gravitational force $F \propto \dfrac{1}{R^2}$, and approaches infinity when R gets very small.

Another possibility is to make the zero PE level correspond to $R \to \infty$ (infinity). This is a natural one if we

$$\text{consider } \Delta PE = \int_{R_0}^{R} F\,dR$$

$$= \text{constant} \int_{R_0}^{R} \frac{dR}{R^2}$$

$$= -\text{constant}\ \frac{1}{R}\Big|_{R_0}^{R}$$

$$= \text{constant}\left(\frac{1}{R_0} - \frac{1}{R}\right)$$

where R_0 is designated as the R for which PE = 0 and must be ∞. Of course, now all PE values will be negative, but we really are only concerned with *changes* in PE. Furthermore, this would tell you how much work you must do to free the planet from the sun.

17. $KE = PE = \dfrac{mv^2}{2} = ma_g h$

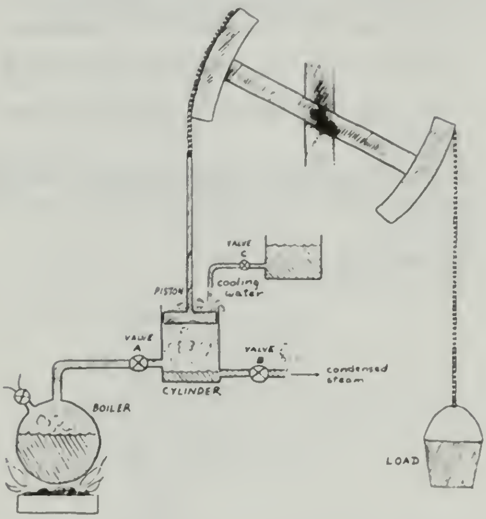
According to the hint: $ma_g \leq \dfrac{mv^2}{R} = \dfrac{2KE}{R} = \dfrac{2ma_g h}{R}$

$$1 \leq \frac{2h}{R} \text{ or } h \leq \frac{R}{2}$$

18. The comet speeds up as it approaches the sun and slows down as it recedes. That is, KE is greatest when it is closest to the sun smallest when it is farthest away. Conversely, PE is at a minimum when the comet is closest and increases as the separation increases. The total amount of energy remains the same.

19. A primitive lever system operates as follows: When the piston reaches the top, pin 2 strikes three levers, two of which are directly linked to the valves B and C, thus opening them, and one which is linked through a pivot to A, thus closing A. On the downstroke, pin 2 releases valve C, which is closed by a spring, but leaves A and B open. At the bottom of the stroke, pin 1 strikes the levers to A and B, opening A, closing B.

Cams driven by a cranked wheel would be smoother. They would have to be shaped to keep the valves open for the appropriate duration.



20. Power $= \dfrac{\text{work done}}{\text{time required}}$

$= \dfrac{\text{force} \times \text{distance}}{\text{time}} = Fv$

21. (a) $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times (75 \times 10^6) \times 16^2$
$= 90 \times 10^8$ J

(b) power = force × speed = 174×10^6 W

force $= \dfrac{174 \times 10^6 \text{ W}}{16 \text{ m/sec}} = 11 \times 10^6$ N

(water drag)

This force would have to do an amount of work equal to the KE of the ship to bring it to rest.

FD = KE

$d = \dfrac{KE}{F} = \dfrac{96 \times 10^8 \text{ J}}{11 \times 10^6}$

$= 88 \times 10^2$ m

(c) $F = \dfrac{KE}{d} = \dfrac{96 \times 10^8 \text{ J}}{2 \times 10^3} = 48 \times 10^5$ N

(d) 1. increase
2. increase
3. decrease

(e) to change direction of ship

22. Various methods could be suggested, such as:
1. For (a) and (b) the ideas of **21** could be adapted to the bicycle or motorcycle traveling at constant speed on a level road.
2. (a), (b), and (c) could be supported so that a pulley attached to the rotating part may lift a weight at the end of a rope. Divide the work done by the time required.
3. A brake horsepower test consists of a brake band pressed against a pulley on the motor shaft so that the difference in tension on the two sides of the band and the speed at which the pulley slips over the band gives the power.

23. (a) Power is work per unit time, P = work/sec, measured in watts. The work done is then

work = 140 W × 500 sec
$= 7 \times 10^4$ J

But work is also equal to force × distance. Therefore,

$d = \dfrac{7 \times 10^4 \text{ J}}{70 \text{ kg} \times 9.8 \text{ m/sec}^2}$

= 102 m

(b) A human body at rest produces about 80 kcal/hr. Therefore,

$P = \dfrac{(4{,}184 \text{ J/kcal})(80 \text{ kcal/hr})}{3{,}600 \text{ sec/hr}}$

= 93 W

24. Both engines receive 100 J of heat.

Engine A: $W = Fd$
= 5 N × 10 m
= 50 J output

efficiency $= \dfrac{50}{100}$
= 50%

power $= \dfrac{50 \text{ J}}{10 \text{ sec}}$
= 5 W

Engine B: W = 2 N × 20 m
= 40 J

efficiency $= \dfrac{40}{100}$
= 40%

power $= \dfrac{40 \text{ J}}{5 \text{ sec}}$
= 8 W

25. (a) Newcomen's engine could be run at will, not dependent on the presence of wind. In addition it delivered slightly greater horsepower.
(b) Watt's engine required less fuel per horsepower.

26. Probably the duty was more important because of fuel costs.

27. Student answers might include the following:

Desirable	Undesirable
(a) 1. Steam engine	
power for mills	need for wood, then coal
railroads	smoke and smog
steamships	crowded tenements
electric generators	loss of workers' independence
profits and wealth	exploitation of natural resources
international trade	waste ashes
2. Gasoline engine	smog
cheap, easy transportation	accidents
jobs	insurance
oil industry	dependence on imported oil
(b) Nuclear power	
another source of electricity	potential massive hazards
low cost (?)	disposal of radioactive wastes
reliable (?)	high cost of construction
clean air	delays in construction
power for ships	centralized power sources
jobs	

28. (a) The maximum efficiency of an engine is described by the change in temperature in degrees Kelvin divided by the initial temperature. On *Text* page 295, the efficiency is given as

$$\text{efficiency} = \frac{(T_2 - T_1)}{T_1} = 1 - \frac{T_2}{T_1}$$

For an engine operating on ocean water between 15 °C and 5 °C, the maximum efficiency is

$$\text{efficiency} = 1 - \frac{278°}{288°} = 3.5\%$$

(b) If the engine operates at 3.5% efficiency, the rate at which water must be pumped through the engine to yield 1 MW of mechanical energy can be found as follows: One metric ton of water cooled 1°C produces 4 MJ (MW sec) of energy. Therefore, 0.1 metric ton cooled 10°C (15° − 5°) would produce 4 MW of energy and 0.025 metric ton (0.1/4) would produce 1 MW. If the efficiency of the engine is 3.5%, (0.025/0.035) = 0.71 metric ton/sec of water would be required to produce 1 MW of energy.

29. The coefficient of performance, described on *Text* page 296 as $T_2/(T_1 - T_2)$, applies to the air conditioner and is always greater than 1.

(a) With an outside temperature T_1 of 40°C (313°K) and an inside temperature T_2 of 21°C (294°K), the coefficient of performance is

$$\frac{T_2}{T_1 - T_2} = \frac{294}{313 - 294} = 15.5$$

(b) If the outside temperature T_1 is raised 10°C to 50°C (323°K) and the inside temperature T_2 is lowered 10°C to 11°C (284°K), the coefficient of performance is

$$\frac{284}{323 - 284} = 7.3$$

The coefficient of performance is decreased by 8.2.

(c) If the outside temperature T_1 is raised by another 10°C to 60°C (333°K), the coefficient of performance becomes

$$\frac{284}{333 - 284} = 5.8$$

30. No. The distances are in a ratio of approximately 1 : 2 : 3; for a given weight, the ratios of costs are not 1 : 2 : 3; for a given distance the costs are not proportional to the weights.

31. Clausius would argue in the sequence: c, a, e, b, d, f.

32. An ideal engine has no heat loss, is therefore reversible, and has a maximum efficiency of 100%.

33. The efficiency of a completely reversible ideal engine must be 100%. For this to occur, the temperature of the cold side must be absolute zero (0°K).

34. (a) 1 kcal = 4,184 J. To raise the temperature of water ½°C requires 2.5 kcal or 10.46×10^3 J. Each descent of the 1-kg weight corresponds to 1 × 9.8 × 1 J. Hence, more than 1,000 such descents would be required.

(b) 1. use less water
2. use a larger mass
3. use a longer distance
4. use a liquid of lower specific heat (for example, mercury).

(Note: Method 4 has not been discussed in this text but some students may be aware of this possibility.)

35. Consider 1 kg of water going over the falls. In a drop of 50 m, 1 × 9.8 × 50 = 490 J will be generated, which is equivalent to $\frac{490}{4,180}$ kcal or ⅛ kcal available to warm the 1 kg of water. Thus, the temperature would be raised by about ⅛°C. The answer would be the same regardless of the amount of water since, although more water would provide more joules, there would be a corresponding increase in the number of Calories required to heat the larger mass of water.

36. The efficiency of a power plant is given by

$$\text{efficiency} = \frac{T_1 - T_2}{T_1}$$

where T is in degrees Kelvin. For the nuclear plant,

$$\text{efficiency} = \frac{600 - 300}{300} = 0.50$$

For the fossil fuel plant,

$$\text{efficiency} = \frac{750 - 300}{300} = 0.60$$

(a) To produce 1 MW of electrical power at these efficiencies, the heat dumped into the environment is the total power produced minus the useful output. The total power is the output divided by the efficiency. Then

$$\text{heat loss} = \frac{\text{output}}{\text{efficiency}} - \text{output} = \text{output}\left(\frac{1}{\text{efficiency}} - 1\right)$$

For the nuclear plant, the heat loss is

$$1\ \text{MW}\left(\frac{1}{0.5} - 1\right) = 1\ \text{MW}$$

For the fossil fuel plant, the heat loss is

$$1\ \text{MW}\left(\frac{1}{0.6} - 1\right) = 0.67\ \text{MW}$$

(b) The rate of flow of cooling water to create an exit flow at 303 °K is given by

$$\text{rate (tons/sec)} = (T_1 - 303) \times \frac{\text{heat loss (MW)}}{4.2\ \text{MJ/sec (per ton per °C)}}$$

For the nuclear plant,

$$\text{rate} = (600 - 303)\left(\frac{1\ \text{MJ/sec}}{4.2\ \text{MJ/sec}}\right) = 70\ \text{tons/sec}$$

For the fossil fuel plant,

$$\text{rate} = (750 - 303)\left(\frac{0.67\ \text{MJ/sec}}{4.2\ \text{MJ/sec}}\right) = 71.5\ \text{tons/sec}$$

37. Digging requires 400 Cal/hr or 200 Cal for 0.5 hr. With 20% efficiency, 1,000 Cal must be supplied. Hamburger supplies 4,000 Cal/kg so 0.25 kg will be required.

38. One-half kilogram of animal fat yields 300 Cal $= 4.3 \times 10^3$ Cal; 22.5 kg of animal fat would yield 21.5×10^3 Cal. If food intake is cut by 1,000 Cal/day, 21.5 days will be required to lose 22.5 kg.

39. (a) Fewer joules of energy are needed to move smaller masses. Note that the ratio of weights (3 : 2) is about equal to the ratio of Calorie requirements.

(b) Fewer Calories are needed to maintain body temperature in a warm climate.

(c) There is a higher percentage of children in the Indian population whose masses would be significantly less than the adult 49.5 kg and who consequently would have lower Calorie needs for mechanical work. (On the other hand, they would have greater intake needs for growth; it is not clear how these would compare.)

40. Our consumer needs of food, fuel, electricity, gasoline, etc., can all be expressed in joules or Calories. With our dollars we are thus buying energy. However, the cost per joule (or Calorie) is not the same for all the various forms of energy and it would not be practical to adopt the dollar as an energy unit.

41. The upward momentum of the rocket equals the downward momentum of the exhaust gases. The chemical potential energy of the fuel is converted into kinetic energy of the rocket (heat, light, and sound energy) plus the gravitational potential energy that the rocket acquires.

42. The statement is basically true. The chemical energy eventually becomes heat energy since at the end of the trip the car is brought to rest, hence having no final KE. (However, if the stopping place is at a higher elevation than the starting place, gravitational PE will be provided from the original chemical energy of fuel.) Any energy radiated as light or sound will eventually warm some absorber.

43. When a hot and a cold body are placed in contact, the entropy change is

$$\Delta S = \frac{H(T_1 - T_2)}{T_1 T_2}$$

44. (a) By firing its retrorockets the space capsule can reduce its speed and will then be pulled closer to the central body about which it is orbiting. For stable orbits, centripetal force equals gravitational force

$$\frac{mv^2}{R} = G\frac{mm_1}{R^2}$$

After the speed is reduced from that needed in the higher orbit, the capsule will be accelerated as it spirals inward. When it reaches the desired lower altitude its speed and direction must again be adjusted by suitable firing of its rockets.

(b) In the lower orbit the required speed will be greater than in the higher orbit, since $v^2R = Gm_1$ = constant. So KE in the lower orbit is *greater* than KE in the higher.
(c) PE will be less when it is closer to the central body.
(d) Less: Δ total $E = \Delta PE + \Delta KE = -2\,\Delta KE + \Delta KE = -\Delta KE$
(e) The difference was dissipated in rocket firings.

45. (a) i. Total energy of system is decreased.
ii. The heat absorbed by the system is less than heat given off by the system.
iii. More work is done by the system than on the system.

(b) i. All three are negative; the man does work and gives off body heat.
ii. All three are negative: the battery does work turning over the motor, some heat is given off, and the internal chemical potential energy is decreased.
iii. ΔH is negative since heat and light are given off.
ΔW is positive (in this case the work is *electrical*).
ΔE is slightly positive (pos. $\Delta W >$ neg. ΔH).
iv. ΔW is positive; ΔH is negative, but now an equilibrium exists, so $\Delta E = 0$.
v. All three are negative: work is done moving the parts; heat and light are given off.
vi. (a) When first starting: ΔW positive; ΔH negative, but greater than ΔW, so ΔE is negative.
(b) After stabilizing: ΔW positive; ΔH negative, but now $-\Delta W = \Delta W$, so $\Delta E = 0$.

46. (a) sun's energy $\begin{cases} \rightarrow \text{evaporation of water} \rightarrow \\ \text{rainfall} \rightarrow \text{plants} \rightarrow \text{fuel (coal, oil, gas)} \end{cases}$

$\rightarrow$ river $\left.\right\}$ $\begin{matrix} \rightarrow \text{electric generator} \rightarrow \text{stove heat} \\ \rightarrow \text{water heat} \end{matrix}$

(b) sun's energy $\rightarrow$ plants $\rightarrow$ fuel $\rightarrow$ $\begin{cases} \text{gravitational} \\ \text{KE} \end{cases}$

PE $\}$ $\rightarrow$ heat of friction

(c) sun's energy $\rightarrow$ water evaporation and condensation $\rightarrow$ wind's mechanical energy of the pump $\rightarrow$ KE (and possibly PE) of water

47. (a) Since $H_1' - H_1 = h$, and energy is conserved, then $H_2' - H_2 = h$ also.
(b) The total entropy change ΔS, defined as $\Delta S = \Delta H/T$, is composed of the entropy change on the hot side ΔS_1 and of that on the cold side ΔS_2 ($\Delta S = \Delta S_1 + \Delta S_2$). But

$$\Delta S_1 = -\frac{H_1' - H_1}{T_1} \text{ and } \Delta S_2 = \frac{H_2' - H_2}{T_2}$$

Since $H_1' - H_1 = h$ and $H_2' - H_2 = h$, we have

$$\Delta S = \frac{h}{T_2} - \frac{h}{T_1}$$

(c) Because T_2 on the cool side is less than T_1 on the hot side, h/T_2 is greater than h/T_1 and ΔS must be positive.

48. The entropy change of the melting ice will be $\Delta S = \Delta H/T = 3.4\text{ MJ}/273\ ^\circ\text{K} = 12.5\text{ kJ}/^\circ\text{K}$. The entropy change of the warming water will be equal but negative: $-12.5\text{ kJ}/^\circ\text{K}$. Since no heat is lost, the entropy of the ice–water universe does not change.

CHAPTER 11

2. Conducting political polls, TV ratings, gambling casino games.

3. Several concepts accepted in present-day kinetic theory are contained in the quotation by Lucretius, such as continual random motion of atoms that may involve elastic collisions. On the other hand, according to Lucretius, some collisions were not elastic; all atoms were identical and different substances were formed by their becoming entangled; and sunlight was atomic in nature.

4. A distribution is a mathematical description stating that the items have a certain characteristic in common. Statistical distributions are more likely to agree with experimental results when the number of items is large.

5. (a) There is a most probable (or average) speed for the molecules.
(b) The higher the temperature the larger the average speed.
(c) No molecules can have negative speeds, so there is a definite cut-off at zero, but no such sharp limit on the curves at high speeds.

6. No. In the transmission of sound the forward and back motion is superimposed on the normal molecular motion.

7. Clausius gave the particles of the simple theory

appreciable size. This meant that frequent collisions would occur among them. He could then explain the slowness of diffusion.

8. (a) The volume of oil equals the area in contact with the water times the thickness of the layer: $V = A \times h$

$$h = \frac{V}{A} = \frac{10^{-6}\ \text{m}^3}{10^3\ \text{m}^2} = 10^{-9}\ \text{m}$$

$(1\ \text{cm}^3 = 10^{-6}\ \text{m}^3)$

(b) The thickness of the layer is 10^{-9} m since the layer is assumed to be one molecule thick.

9. (a) Total volume equals the volume of one molecule times the number of molecules: $V = v \times N$

$$N = \frac{V}{v} = \frac{10^{-6}\ \text{m}^3}{10^{-27}\ \text{m}^3} = 10^{21}$$

(b) Density $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{Nm}{V}$ so there would be 0.001 as many molecules in any volume of gas as compared with the same volume of liquid. One cubic centimeter of a gas will contain 10^{18} molecules. This is in agreement with the *large* numbers mentioned in the text.

10. A lift pump will not operate at all on the moon since the moon has no atmosphere.

11. If the air had constant density, 1,000 × 10.5 m (about 10 km) would balance the 10.5-m column of water. Actually, the density decreases with altitude and the atmosphere has no sharp cut-off point. We have evidence of its existence at more than 100 km above the earth's surface.

12. (a) $P_2 = 100\ \text{N/m}^2 \times 2 \times \frac{1}{3}$
$= 66\ \text{N/m}^2$

(b) $T_2 = T_1(P_2/P_1)(D_1/D_2)$
$= 100\ °\text{C} \times 2 \times \frac{1}{2}$
$= 100\ °\text{C}$

13. $P = \dfrac{F}{A} = \dfrac{\text{weight}}{\text{area of base}}$;

1 atm is about 10 N/cm² (10 Pa)

For a 529-N person standing on:

1. Two shoes, each about 25 cm long and 8 cm wide

$$P = \frac{529}{2 \times 25 \times 8} = 1.3\ \text{N/cm}^2 = \text{about } \frac{1}{7}\ \text{atm}$$

2. Two skis, each about 200 cm long and 8 cm wide

$$P = \frac{529}{2 \times 200 \times 8} = 0.17\ \text{N/cm}^2 = \text{about } \frac{1}{60}\ \text{atm}$$

3. Two skates, each about 30 cm long and 0.3 cm wide

$$P = \frac{529}{2 \times 30 \times 0.3} = 29.4\ \text{N/cm}^2 = \text{about 3 atm}$$

14. Starting with Boyle's law as $P = kD$ and substituting $D = \dfrac{M}{V}$, gives $P = \dfrac{kM}{V}$ or $PV = kM$ = constant for a given mass of gas M.

$$P = kD\,(t + 273°)$$

15. The ideal gas law is $P = kDT$, where

$P \propto D$ for constant T
$P \propto T$ for constant D
$D \propto 1/T$ for constant P

The ideal gas law does not apply to very dense gases or to any gas at a phase transition to a liquid or solid.

16. 1. If the temperature does not change, $(t + 273°)$ = constant or $P \propto D$, which is Boyle's law.
2. If the pressure is kept constant,

$$P = kD\,(t + 273°) = \frac{kM}{V}(t + 273°) \text{ and}$$

$$V = \frac{kM}{P}(t + 273°).$$

Comparing the volumes at two different temperatures,

$$V_2 - V_1 = \frac{kM}{P}[t_2 + 273 - (t_1 + 273)]$$
$$= \frac{kM}{P}(t_2 - t_1)$$

or change in volume is proportional to change in temperature, which is Gay-Lussac's law.

17. The ideal gas law $P = kDT$ becomes $P \propto T$ for a constant mass of air confined in the volume of the tire. (This volume is assumed not to change appreciably.) A specific ratio of $\dfrac{\Delta P}{\Delta T}$ (such as the 12-kPa drop per 10° C cited) would have to be associated with a particular amount of air trapped in the volume available. If the ideal gas law is applied to this situation, it would predict a total pressure of 34 N/cm² (24 N/cm² on the gauge) for a temperature of 4°C. (Remember that absolute zero corresponds to −273° C.) This is a fairly high pressure for most passenger car use.

Next consider the two statements made and convert the data given to a form in which it can easily be checked for consistency.

Gauge pressure (N/cm²)	Temp. (°C)	Actual pressure (N/cm²)	Absolute temp. (related to °C)	P/T
16	26	26	299	0.087
13	−1	23	272	0.085
16	15	26	288	0.090
12	−29	20	244	0.082

Note that for the first set of data the internal agreement is off by about 4% but the second set agree exactly.

The statements as phrased in the quotation suggest two additional comments:

1. The two sets of data cannot be for the same amount of air since in one case 16 N/cm^2 corresponds to 26 °C and in the other to 15 °C.
2. The first case describes a 27°C temperature drop and a pressure drop of 3 N/cm^2 in agreement with the rule stated at the outset. The second case describes a 44° drop in temperature (15° to −29°) and only a 4 N/cm^2 drop in pressure.

The purpose of this question is to encourage students to read such statements critically and to recognize correlations between everyday concerns (such as adequate tire pressure) and physics formulas.

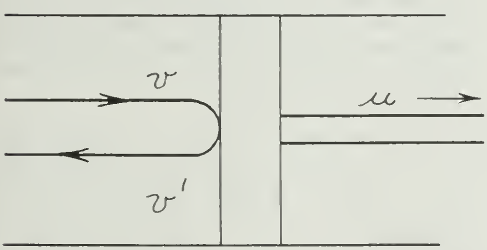
18. A *working model* copies the real thing on a reduced scale; a *theoretical model* idealizes the actual situation to facilitate the mathematical analysis.

19. Speeds relative to *laboratory:*
particle before collision = v
particle after collision = v'
piston at time of collision = u

Speeds relative to the *piston:*
speed of the particle before collision
$= v - u$
(particle and piston headed in same direction)
speed of the particle after collision
$= v' + u$
(particle and piston now headed in opposite directions)

These speeds before and after collision are equal to each other, according to the Galilean relativity principle, so

$$v' + u = v - u$$
$$v' = v - 2u$$



Thus, the speed (relative to the laboratory) after collision v' is smaller than the original speed v by twice the speed of the piston u.

20. The temperature would not change since there is no work done.

21. Pressure, mass, volume, temperature, viscosity, rate of diffusion, color, and odor all might be suggested.

22. The relations among pressure, volume, and temperature; the rate of diffusion. In addition, the sizes and speeds of the particles can be estimated and temperature could be given a mechanical interpretation.

23. Much of the material in the can is in the gaseous phase under pressure. The kinetic theory relates an increase in temperature to an increase in speed of the particles and hence an increase in pressure. The can may burst!

24. Work is done on the gas by the piston. By the first law of thermodynamics, the internal energy of the gas must increase and so the temperature rises. From the kinetic theory, a molecule striking the moving piston has its speed increased. This results in a higher average kinetic energy for all the particles, which corresponds to a higher temperature. The air molecules eventually cool to the temperature of their surroundings by means of successive collisions with the particles of the walls around them. The extra energy is not destroyed but is eventually shared by so many particles that no lasting temperature rise is detected.

25. (a) *Perfectly* insulating would mean that *no* heat could escape, so the gas would have to stay hot. Energy transferred to the container walls by colliding molecules would be eventually transferred back to other molecules.
(b) The chemical energy released in the burning of the fuel in a gas stove (or the electrical energy converted into heat in an electric stove) first speeds up the metal particles of the kettle, which then pass on extra energy to the water, raising its temperature until boiling occurs. At the boiling temperature, the liquid molecules have sufficient energy to escape as a gas.

26. Lucretius and Newton seem to be in basic agreement. However, even though both speak of explaining everything in terms of particles in motion, Newton's idea is primarily deterministic (the motions of the particles should be determinable from laws), while Lucretius' idea seems to be more statistical (only the chance conjunction of large numbers of particles, a conjunction which is slightly probable).

27. The three statements of the law are: (1) Heat will not flow by itself from a cold body to a hot one. (2) It is impossible to fully convert a given amount of heat into work. (3) The entropy of an isolated system tends to increase (it cannot decrease). To show that statements (1) and (2) are equivalent requires two steps: *Step A.* Any violation of (2) is also a violation of (1). *Step B.* Any violation of (1) is also a violation of (2).

Step A. Imagine an engine that can fully convert heat into work [statement (2)]. Then imagine a reversible engine that acts as a refrigerator, and put the two engines together. The net effect of the two engines is to transfer heat from a cold body to a hot body without doing any work. This violates statement (1).

Step B. Imagine an engine that violates statement (1); that is, it causes heat to flow from a cold to a hot body with no other changes taking place. Then take a heat engine that works between the same two temperatures as the first engine. Run both engines at once. Because the resulting engine picks up the heat that is dumped by the reversible engine and returns it to the hot reservoir, the net effect is to convert a given amount of heat entirely into work. This violates statement (2).

28. It would not work. Energy would have to be expended to decrease the temperature of the water. The only time you can get work done at the expense of heat energy is when you have a reservoir of heat at a higher temperature than the surroundings.

29. When the ball is released, the initial potential energy is

$$\begin{aligned} PE &= mgh \\ &= (0.1 \text{ kg})(10 \text{ m/sec}^2)(1 \text{ m}) \\ &= 1 \text{ J} \end{aligned}$$

The PE is converted to KE; therefore, just before the first bounce, KE = 1 J. During the first bounce, some KE is converted to heat. The remainder brings the ball to a lesser height, after which the ball continues to bounce. When the ball finally stops bouncing, its KE = 0, PE = 0, and all its energy has been turned into heat: $Q = 1 \text{ J } (2.4 \times 10^{-4} \text{ Cal})$.

The entropy change equals the amount of heat divided by the temperature, so at 300°K,

$$\begin{aligned} \Delta S &= Q/T \\ &= (1 \text{ J}) / (300°\text{K}) \\ &= 3.3 \times 10^{-3} \text{ J/°K} \end{aligned}$$

It does not matter how much heat goes to the ball assuming everything is nearly at 300°K. The entropy change of the universe is the sum of all the entropy changes:

$$\begin{aligned} \Delta S_{\text{universe}} &= \Delta S_{\text{ball}} + \Delta S_{\text{table}} + \cdots \\ &= Q_{\text{ball}}/T_{\text{ball}} + Q_{\text{table}}/T_{\text{table}} + \cdots \end{aligned}$$

And since $T_{\text{ball}} = T_{\text{table}} = 300°\text{K}$,

$$\Delta S_{\text{universe}} = (Q_{\text{ball}} + Q_{\text{table}})/300°\text{K}$$

30. Eventually, all temperatures will become equalized above the absolute zero. Energy cannot be destroyed.

31. *If* there were such a being, he could get around the second law of thermodynamics.

Maxwell's assumptions:

1. a finite being who knows the paths and velocities of all the particles
2. the ability to open and close a mass-less slide.

Both assumptions are impossible to realize in any laboratory.

32. The paradox lies in what is meant by "long enough." The chances of return to a former state are very slight on account of the large number of molecules. The second law, on the other hand, takes an overall point of view and says it never happens.

33. (a) Newton used a mathematical time. His laws are equations of the first degree (linear) in time. Reverse the sign of the time and the process runs backwards. Every purely mechanical process is reversible.
(b) When heat and temperature are introduced, the law of the increase of entropy with time gives time a definite direction and processes no longer are reversible.

34. The melting of ice is an irreversible process because the ordered arrangement of molecules in the ice crystals is lost, and therefore entropy increases.

35. It would seem as though time were running backwards.

36. The assumptions involved can be formulated as follows:

1. The long-range history of the world is cyclic (ancient belief).
2. The universe has a definite quantity of energy.
3. There is a finite number of molecules.
4. There is a finite number of possible arrangements of the molecules.
5. Time is infinite.
6. All possible combinations of molecules have at least some probability.
7. The kinetic theory fully and adequately describes nature.

Students may differ widely in what they believe to be "true" in a discussion such as this. It should be a good intellectual experience for them to examine *why* they accept or reject statements such as these.

37. In discussing this problem as a physicist, one must take an anthropomorphic or materialistic view of religion. Population increase precludes a cyclical process for individuals. The number of individuals, although numbered in terms of billions, is infinitesimal compared with billions of billions of molecules. You probably contain within your body a few molecules of Julius Caesar!

It is dangerous to extrapolate ideas that arise in one context to a different area or to give them universal validity, but even vague analogies hold an undeniable fascination.

38. Newtonian mechanics could not explain the apparent irreversibility of macroscopic proc-

esses, as well as the processes involved within the molecules and the atoms that compose them.

39. Advantages:
They correlate a number of disparate facts.
They suggest new experiments.
They lead to new results.
Their inadequacies suggest the form that newer and more perfect models must take.

Disadvantages:
They are invariably idealizations, and only approximations to the real world.
They tend to channel thinking (create a paradigm), which is difficult to relinquish, even when it becomes obvious that the model is inadequate.

40. The pressure on the liquid at any depth is proportional to the weight of the water above. Because the pressure is exerted in all directions (sideways on the tank as well as vertically) the lower section of the tank must be stronger to offset the greater outward pressure.

41. (a) The pressure per unit area at the bottom of the cube is P (at top) $+ DgL$.
(b) The force on the top of the cube is the pressure per unit area multiplied by area, or PL^2 directed downward.
(c) The force on the bottom of the cube is $(P + DgL) \times L^2$, directed upward.
(d) The net force on the cube is the difference between the upward force on the bottom and the downward force on the top, which is

$$[(P + DgL) \times L^2] - PL^2 = DgL^3$$

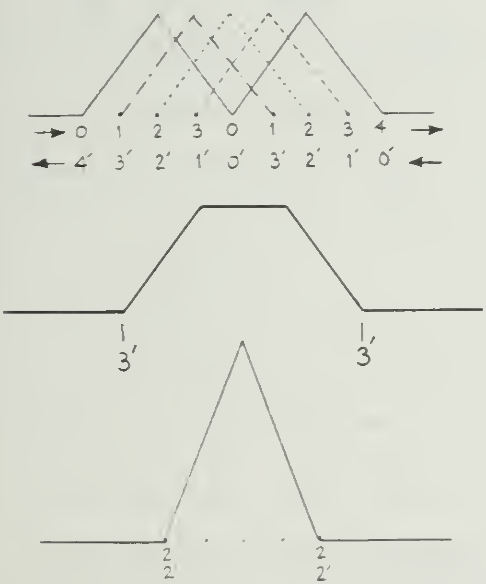
(e) The net force (DgL^3) equals the weight of the fluid displaced, which has a weight of Dg per unit volume for the total volume L^3 of the cube.

CHAPTER 12

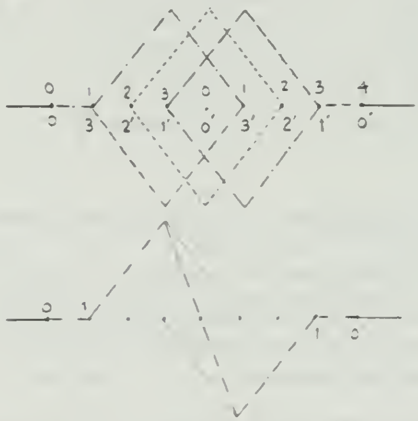
2. If you put your hand at one end of a long metal rod and someone taps the other end sharply, you will feel the vibration almost immediately.

3. If you swing your arms as you walk, the following parts will be in phase: right arm and left leg, left arm and right leg; ½ cycle out of phase: right and left arms, right and left legs. There do not seem to be any generally agreed upon parts that are ¼ cycle out of phase.

4. Use the principle of superposition. The shape at the end of the first and third intervals is the same, a truncated triangle. At the central time a triangle of twice the height would result.



5. The wave shapes at the end of the first and third intervals are reflections of each other. Waves cancel at the central time.

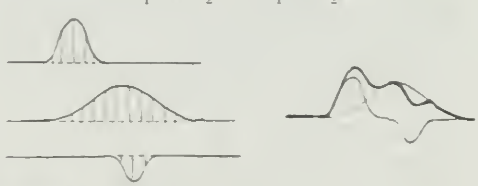


6. Exactly the same shape, but with displacement below the line of the undisturbed rope.

7. No. Kinetic energy does not obey the superposition principle for two reasons. First, the mass of the medium in motion may change as two waves superpose. Second, although the velocities superpose and $v_1 + v_2$ is the velocity of the superposed wave, the squares of the speeds do not superpose. That is,

$$v_1^2 + v_2^2 \neq (v_1 + v_2)^2$$

8.

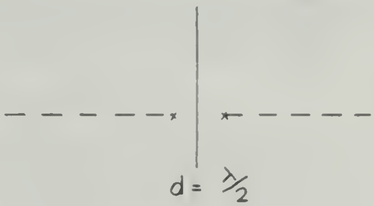


F(beats) = $F_2 - F_1$

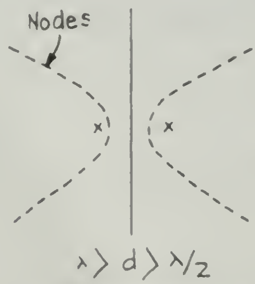
9. They would be surfaces in three dimensions formed by the intersections of a series of hemispheres analogous to the semicircles in the figure on page 366. Nodal surfaces would require the intersection of a *crest* hemisphere with a *trough* hemisphere. (This assumes that the speakers are emitting the same uniform tone.)

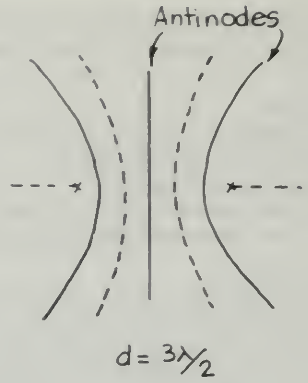
10. The intensity would gradually decrease, until it reached zero at N_1. Then it would gradually increase again and reach a maximum at point A_1. Then the cycle could begin all over again.

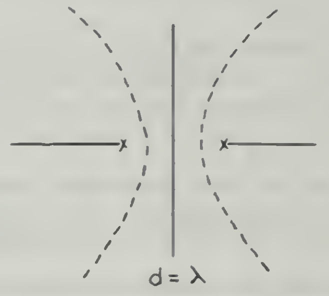
11. The perpendicular bisector of the line joining the sources is always an antinode because the path difference to each source is zero. For a separation less than λ, this is the only antinodal line. For a separation equal to λ, other antinodal lines are the continuations of the line of sources beyond the line joining them. On them, the path difference is just λ.



For separation of sources less than $\frac{\lambda}{2}$, there are no nodal lines. For a separation equal to $\frac{\lambda}{2}$, the nodal line is the same as the additional antinodal line for a separation λ. Along these lines the path difference is $\frac{\lambda}{2}$. For a separation equal to λ, there are two nodal lines of the usual form that meet the line between sources at points equidistant from source and midpoint. The series of sketches in which dotted lines are nodes and solid lines antinodes show how nodal lines are born as the distance between sources is increased.







12. If P is the nth node, then the path difference $PS_1 - PS_2 = S_1M$ must equal $n(\frac{1}{2}\lambda)$. Triangle S_1S_2M can be considered similar to triangle POQ, under the conditions spelled out on page 369. Then $\frac{S_1M}{S_1S_2} = \frac{QP}{OP}$ if OP is much larger than QP. But, OP is practically equal to OQ. Substituting we get

$$\frac{n(\frac{1}{2}\lambda)}{d} = \frac{x_n}{l} \text{ or } \frac{n\lambda}{2} = \frac{dx_n}{l}$$

13. No. You would have to wait for the reflected wave to return. If the wave were appreciably damped, you would have to wait until a final steady amplitude were reached. Then the power supplied would equal that dissipated.

14. (a) The speed of sound v is given by

$$v = \lambda f$$
$$= 256/\text{sec} \times 1.34 \text{ m}$$
$$= 343 \text{ m/sec}$$

(b) $\lambda = \dfrac{v}{f}$

$$= \frac{1{,}500 \text{ m/sec}}{256/\text{sec}}$$
$$= 5.86 \text{ m}$$

(c) $T = \dfrac{1}{f}$

$$= \frac{1}{256/\text{sec}}$$
$$= 3.90 \times 10^{-3} \text{ sec everywhere}$$

15. With the full length, L, the fundamental frequency is obtained, such that $L = \frac{1}{2}\lambda$.

(a) For the musical fourth, $\dfrac{L}{L_4} = \dfrac{4}{3}$, and $L_4 = \dfrac{3}{4}L$.

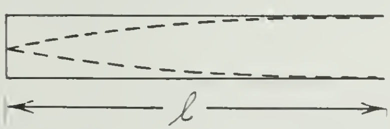
(b) For the musical fifth, $\dfrac{L}{L_5} = \dfrac{3}{2}$, and $L_5 = \dfrac{2}{3}L$.

(c) For the octave, $\dfrac{L}{L_8} = \dfrac{2}{1}$, and $L8 = \dfrac{1}{2}L$.

16. With 8 cycles showing in 10 cm of tube, each cycle occupies 8/(10 cm) = 0.8/cm = k.

$$f = vk$$
$$= 100 \text{ cm/sec} \times 0.8/\text{cm}$$
$$= 80/\text{sec}$$
$$= 80 \text{ Hz}$$

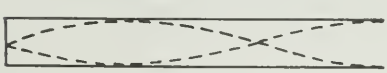
17. (a) The shortest distance from a node to an antinode is $\frac{1}{4}\lambda$. So $\frac{1}{4}\lambda = L$, and $\lambda = 4L$.

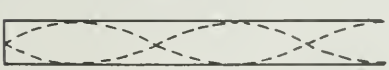


(b) L must be equal to $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, . . . etc.

$$\therefore L = (2n + 1) \cdot \tfrac{1}{4}\lambda$$

and $\lambda = \dfrac{4L}{2n + 1}$, $n = 0, 1, 2, \ldots$

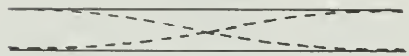


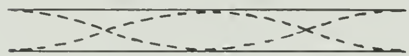


(c) A tube open at both ends must have antinodes at both ends. The fundamental will be produced when $L = \dfrac{\lambda}{2}$ or $\lambda = 2L$. Other possible λ's will be those corresponding to additional half wavelengths.

That is, $L = n\dfrac{\lambda}{2} + \frac{1}{2}\lambda = \dfrac{n + 1}{2}\lambda$

$$\lambda = \frac{2L}{n + 1}, \; n = 0, 1, 2, 3, \ldots$$





Note: Remind the students that the sketches are not intended to imply that the sound waves are transverse; they are merely a convenient way of showing the locations of nodes and antinodes.

18. The symmetrical nodal surfaces would be *spherical* surfaces. Since the outer surface is free to move, outward at least, it would be analogous to a *free end*. The center of the sphere would be a *fixed end*. So there would have to be a node at the center and an antinode at the surface. Let r = radius of the sphere. Then, fundamental mode would require that $r = \frac{1}{4}\lambda$ and $\lambda = 4r$. Other modes would be $r = \frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, . . . etc.

Some practically oriented students may suggest designing an experiment with a real "blob of jello," which of course involves the problem of suspension and raises the question of whether "half a blob" resting on a flat surface would be a satisfactory approximation.

Other imaginative students may speculate about possible *asymmetrical* nodal surfaces.

19. There will be a maximum of disturbance along this line fed by diffracted waves from both ends. Long ocean swells are diffracted more than short choppy waves. The amplitude along this line may be quite large. The energy developed there could cause destruction of boats, docks, etc.

20. Frequencies of sound from the cheerleader's voice might vary between 100 and 1,000 Hz with wavelengths from 3 m to 30 cm. None are small compared with the opening of the megaphone. You can hear the megaphone well off the axis. The quality of the sound will be altered because the higher frequencies will be diffracted less.

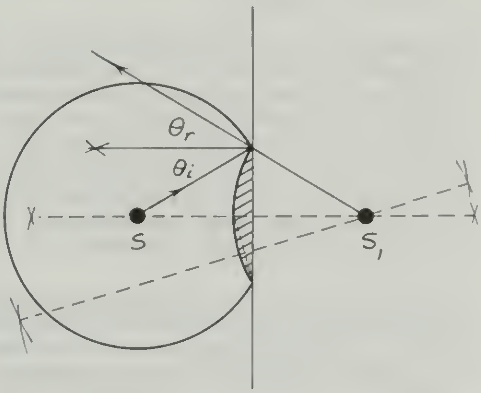
21. The narrower the slit is, the closer it approaches a point on the incident wavefront, which, by Huygens' principle, acts like the source of a spherical wave.

22. The wavelength of light is too small to produce observable diffraction and interference effects with objects the size of fences and houses.

23. For diffraction patterns from double slits,

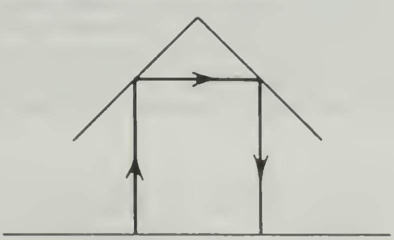
$$\lambda = \frac{dx}{nl}$$
$$= \frac{(2.5 \times 10^{-3}\ \text{cm}) \times 10\ \text{cm}}{1 \times 400\ \text{cm}}$$
$$= 6.25 \times 10^{-5}\ \text{cm}$$

24. Assuming the reflected wave front to be circular, the location of the center of that circle can be determined from the intersection of the perpendicular bisectors of any two chords. Once this is done, attention can be focused on a particular point of reflection. Draw the incident ray from S and the reflected ray *apparently* from S'. Erect the perpendicular to the surface at the point of reflection to show that $\theta_r = \theta_i$.



25. The wave will return as a straight-line wave that goes in just the direction opposite to that along which it started.

You can convince yourself of this by considering a typical ray as follows:



26. The distance from the center of the circle to the point P is about $\frac{r}{2}$. This is just a cross-section of a spherical mirror, for which we get approximate focusing at $\frac{r}{2}$.

Note that this result is obtained only for rays close to the axis (form small θ_i and θ_r). If the rays are farther from the axis, they do not converge to a point and "spherical aberration" results.

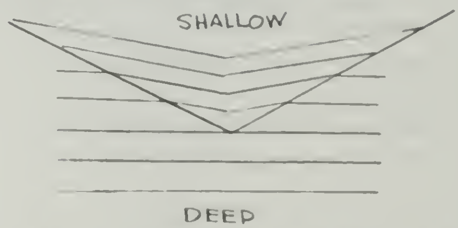
27. Choosing $k = \frac{1}{10}$ makes the calculation of y for a given x especially simple. If one plots the graph $y = \frac{1}{10}x^2$ for $x = 0, \pm 1, \pm 2, \pm 3, \pm 4$, and ± 5, the reflected rays will cut the y axis between $y = 2$ and $y = 3$.

28. It can be shown by methods of analytic geometry that a focal point will occur that is equal to $\frac{1}{4k}$. Using simple geometric methods, the students may not get this answer precisely; but as seen above for $k = \frac{1}{10}$, the focal length was close to 2.5.

29. The wave front will steepen as it moves forward more slowly. On a sloping beach the depth changes rapidly and the wave front becomes so steep it "topples over" or breaks. The propagation speed in deep water is greater than it is in shallow water. Thus, the left side of the sloping water level continues to catch up with the shallower right side, making the slope steeper and steeper. Finally, the higher part of the wave will have a greater speed than the lower part while the slope is exceedingly steep. Then the upper part will just "spill over" and result in a breaker.

30. The speed of a wave does not vary with the frequency. The speed of a wave in a medium depends on the stiffness and density of the medium.

31. The wave splits into two straight-line waves that are inclined toward each other and are propagated more slowly.



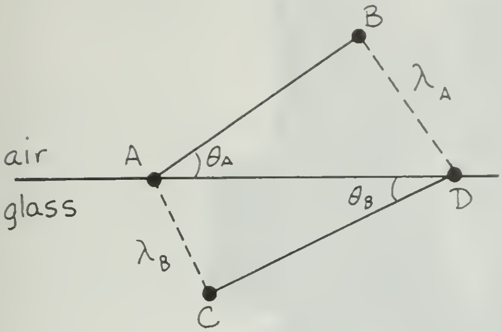
32. (a) Angle of incidence is angle BAD (it is *equal* to the angle of incidence).
(b) Angle of refraction is angle CDA.
(c) Wavelength in air is BD.
(d) Wavelength in glass is AC.
(e) Since point B moved to D and point A moved to C in one period (T),

$$\left.\begin{aligned} v_A &= \frac{\lambda_A}{T} \\ v_B &= \frac{\lambda_B}{T} \end{aligned}\right\} \quad \frac{v_A}{v_B} = \frac{\lambda_A}{\lambda_B}$$

(f) $\sin \theta_A = \dfrac{\lambda_A}{A_D}$

$\sin \theta_B = \dfrac{\lambda_B}{A_D}$

$$\left.\begin{array}{l}\sin \theta_A = \dfrac{\lambda_A}{A_D} \\ \sin \theta_B = \dfrac{\lambda_B}{A_D}\end{array}\right\} \dfrac{\sin \theta_A}{\sin \theta_B} = \dfrac{\lambda_A}{\lambda_B}$$



33. Given: $\theta_i = 45°$ $\quad v_d = 0.35$ m/sec
$\theta_r = 30°$ $\quad f = 10$ Hz
(same for both deep and shallow water)

$$\lambda_d = \frac{v_d}{f} = \frac{0.35}{10} = 0.035 \text{ m} = 3.5 \text{ cm}$$

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\lambda_d}{\lambda_s}$$

$$\lambda_s = \frac{\sin \theta_r}{\sin \theta_i} \cdot \lambda_d$$

$$= \frac{\sin 30°}{\sin 45°} \cdot \lambda_d$$

$$= \frac{0.5}{0.7} \times 3.5 = 2.5 \text{ cm}$$

34. $v = \lambda f$; therefore

$$v = f\frac{dx}{nl}$$

$$= \frac{10/\text{sec} \times 3 \text{ cm} \times 10}{3 \times 40 \text{ cm}}$$

$$= 2.5 \text{ cm/sec}$$

35. The refraction depends upon the ratio of the relative speeds in the two media. If the media are interchanged, so are the speeds and so are the angles of incidence and refraction. See question 32, (e) and (f).

36. (a) The least audible sound above no background noise is 10^{-16} W/cm^2. The sound from the mosquito is spread over the surface of a sphere 1 m in radius. $A = 4\pi r^2$ $= 1.27 \times 10^5$ cm^2. The power of the mosquito is thus 1.27×10^{-11} W.

(b) It would take $100/1.27 \times 10^{-11} = 8 \times 10^{12}$ mosquitoes.

(c) $\dfrac{\text{power}}{\text{cm}^2}$ at 10 m $= \dfrac{100 \text{ W}}{4\pi(10^3)^2 \text{ cm}^2}$

$$= 8 \times 10^{-6} \frac{\text{W}}{\text{cm}^2}$$

Relative intensity: $\dfrac{8 \times 10^{-6}}{10^{-16}}$ = about 10^{11}, which is somewhat louder than a subway train.

37. The speed of sound in sea water is known. The time interval between the sending of a signal and the reception of its echo enables one to compute twice the depth. The process can be made automatic using magnetostriction oscillators and cathode-ray tube display.

38. The speed of sound waves in air is 337.5 m/sec, in sea water it is 1,467 m/sec and in steel it is 4,800 m/sec. The relation among frequency, speed, and wavelength is $v = \lambda f$, where v is the speed, λ the wavelength, and f the frequency. Solving for λ we obtain $\lambda = v/f$. Thus, for air $\lambda = 337.5/1000 = 0.338$ m; for sea water $\lambda = 1.47$ m; and for steel $\lambda = 4.8$ m.

At the higher frequency the wavelengths are one-tenth of these, namely 0.0338 m, 1.47 m, and 0.48 m, respectively.

Place the sound source at one end of a closed, well-padded box about 2 m long. At the other end cut two vertical slits about 20 cm long and 1.25 cm wide. Examine the region outside this end of the box with a microphone connected to an amplifier. The output of the amplifier is fed into a cathode-ray oscilloscope. Movement of the microphone will show interference fringes.

39. Assume that the statement "the wavelength is small compared to the dimensions of the object" means that $\lambda \leq 1/10$ width of object. For a moth 1 cm long, λ must be no larger than 0.1 cm (10^{-3} m).

$$f = \frac{v}{\lambda} = \frac{300 \text{ m/sec}}{10^{-3} \text{ m}} = 3 \times 10^5 \text{ Hz}$$

For a wire 0.12 mm (or 1.2×10^{-4} m) λ must be no larger than 1.2×10^{-5} m.

$$f = \frac{300 \text{ m/sec}}{1.2 \times 10^{-5}} = 2.5 \times 10^7 \text{ Hz}$$

Light and Electromagnetism

Organization of Instruction

THE MULTI-MEDIA SCHEDULE

Day 1
Lab Stations: Properties of Light

1. Reflection. Locate image in plane mirror by ray tracing.
2. Refraction. Observe refracted light beam as suggested in *E4–1.*
3. Diffraction. Use double slit on film as suggested in *E4–2.*
4. Polarized light. Note Chapter 13 activity.
5. Poisson's spot. Note Chapter 13 activity.
6. Dispersion. Observe dispersion through prism.

Day 2
Teacher presentation: Interference

Explain $\lambda = (x/l)\,d$, where x represents the distance between two consecutive nodal lines. For an able class it is worthwhile to derive this expression. Describe the laboratory procedure for *E4–2.*

Day 3
E4–2: Young's Experiment. Students spend entire period completing *E4–2,* including writing it up.

Day 4
Film: "Speed of Light" PSSC #0203 (21 min)

Teacher presentation: History of Measurement of Velocity of Light. Include Röemer's method, Fizeau's toothed wheel, and Michelson's method.

Day 5
Small-group discussion: Is light a particle or a wave?

Students discuss dual nature of light. After 15 min in small groups, assemble class and allow them to compare arguments. It is not necessary to form definite conclusions, since this point is discussed again in Unit 5.

Day 6
Small group problem-solving session
Students work in small groups on the assigned *Study Guide* problems. Some students can work alone if they prefer. The teacher moves from group to group giving assistance.

Day 7
Teacher demonstration: Static Electricity
Procedures will vary with the equipment available, but some possible demonstrations are:

1. charging by inducation and by contact
2. conductors and insulators
3. electroscopes
4. storing charges (capacitance)
5. point discharges

Day 8
E4–4: Coulomb's Law

An alternate or supplement to this experiment is the film Coulomb s Law PSSC #0403 30 min

Day 9

Teacher lecture Current Electricity

Use demonstrations to supplement general discussion of currents conductors semiconductors circuits potential difference etc

Day 10

Student activity day

Students work on one of the following activities

1 *T31* *E* field inside conducting spheres
2 *D45* the electrophorous
3 Detecting electric fields activity
4 Voltaic pile activity
5 An 11c battery activity
6 *E4–7* and *E4–8*: two students Electron Beam Tube

Day 11

Student presentations

Students present what they have discovered during their activities on Day 10.

Days 12 and 13

Lab Stations Current Balance

Refer to *E4–5* and *E4–6* in the *Handbook*. Students do two of the following each day

1 *E4–5* a	3 *E4–5* c	5 *E4–6* b
2 *E4–5* b	4 *E4–6* a	6 *E4–6* c

Day 14

Teacher demonstration lecture

Current Balance
Electric Fields

By questioning the class discuss thoroughly the forces acting upon a current-carrying conductor. Exploit *E4–5* and *E4–6* as much as possible

During the last half of the period demonstrate and explain gravitational electric and magnetic fields

Day 15

Conduct a class discussion concerning the history of electricity and magnetism Include Gilbert Oersted Coulomb Ampere etc Carefully point out the problem of making a field produce a current in a conductor This discussion will give the students a preview of electromagnetic induction which is the subject of Chapter 15

Day 16

Discuss Faraday the son of a blacksmith perhaps stressing his mathematical naivety and his deep intuition about fields Demonstrate and explain electromagnetic induction.

Day 17

Lab Stations Electrical Devices

Students can move from station to station spending 15 min at each device. It is not necessary that each student go to each station Students can be encouraged to return to devices that they missed at some later time. Some suggested devices are

1 Faraday disc dynamo
2 Generator jump rope
3 Simple meters and motors
4 Simple motor generator demonstrations
5 "Lapis polaris magnes"

Day 18

Organize research groups for the study of electricity and society

Days 19 and 20

Library research Electricity and society

Several *Study Guide* questions might present appropriate topics. Other topics might include:

1 Was the effect of society upon pure science during the development of electricity and magnetism the same as during the development of the laws of thermodynamics?

2 Do scientists have an obligation to do electrical research that will help rather than eventually harm society?

Day 22

Teacher presentation and class discussion: Maxwell s contribution to science

Without actually using Maxwell s equations you can point out that his ideas were more abstract than Faraday s lines of force notions. Stress the predictions of Maxwell s theory.

Day 23

Lab Stations Electromagnetic Waves

1 *L45* Standing Electromagnetic Waves
2 *T34* The Electromagnetic Spectrum
3. *E4–9* A Turntable oscillators
4 *E4–9* B Resonant circuits
5 *E4–9* C Elementary properties of microwaves

Day 24

Film Electromagnetic Waves PSSC #0415 33 min

Common behavior of electromagnetic radiation over a wide range of wavelengths is demonstrated. Follow the film by a class discussion.

Day 25

Class discussion Existence of the ether and the Michelson–Morley experiment.

Day 26

Small group problem-solving session

Day 27

Teacher presentation: The Electromagnetic Theory

Using *T34*, make certain that the students appreciate the examples of electromagnetic waves seen in the laboratory namely light radio and microwaves

Days 28–30

One method of evaluation is to review test and then discuss the test Devote a day to each activity Another method of evaluation is through student–teacher conferences over a period of three days Evaluation can be based upon laboratory reports essays poems equipment design sets of *Study Guide* answers etc

Unit 4 SAMPLE MULTI-MEDIA SYSTEMS APPROACH

Note: This is just one path of many that a teacher may take through Unit 4.

Day	Activity	Assignment
1	Lab stations: Properties of light	Write up observations Text: 13.1–13.3
2	Teacher presentation: Interference	Text: 13.4–13.8
3	E 4-2 Young's Experiment: the wavelength of light	Handbook: E 4-2 Write up experiment
4	P.S.S.C. Film: *Speed of Light* Class discussion	
5	Small-group discussion: Is light a particle or a wave?	Selected Study Guide questions
6	Small-group problem solving	Text: 14.1–14.3
7	Teacher demonstration lecture: Static electricity	Handbook: E 4-4 Coulomb's Law
8	E 4-4 Coulomb's Law	Write up lab
9	Teacher lecture: Current electricity	Handbook: Survey Ch. 14
10	Student activity day: Choose one	Text: 14.5–14.10
11	Student presentations from day 10	Text: 14.11–14.13
12	Lab stations: Current balance E 4-5	Write up lab
13	Lab stations: Current balance E 4-6	Write up lab
14	Teacher demonstration lecture: Review current balance Electric fields	Review Chapters 13 & 14
15	Discuss: History of electricity Quiz	Text: 15.1–15.4
16	Teacher demonstration lecture: Life of Faraday and electromagnetic induction	Text 15.5–15.8
17	Lab stations: Electrical devices	Write up observations of electrical devices
18	Organize research groups	
19	Library research: Electricity and society	
20	(Library research: Electricity and society, continued)	Prepare presentations
21	Student presentations	Text: 16.1–16.4
22	Teacher presentation: Maxwell's contribution to science	Handbook: Survey Ch. 16 experiments and activities
23	Lab stations: Electromagnetic waves	Write up observations
24	P.S.S.C. Film: *Electromagnetic Waves*	Text: 16.5–16.6
25	Class discussion: Existence of an ether and the Michelson–Morley experiment	Selected Study Guide questions
26	Small-group problem solving	
27	Teacher presentation: The electromagnetic theory	
28	Review or other evaluation	
29	Test or other evaluation	
30	Discuss test or other evaluation	

Unit 4 SUGGESTED SCHEDULE BLOCKS AND TIMETABLE

Each block represents one day of classroom activity and implies approximately a 50-min period. The words in each block indicate only the basic material under consideration or the main activity of the day. The suggested homework (listed above each block) refers mainly to the **Text** and **Handbook**, but is not meant to preclude the use of other learning resources

CHAPTER 13 LIGHT

	Write up lab. Text: 13.1–13.3	Text: 13.4–13.8	HB E 4-2 Write up exp.	
Lab stations: Properties of light (See day 1.)	Teacher presentation: Interference	E 4-2 Young's Experiment	P.S.S.C. Film: *Speed of Light*	Small-group discussion: Is light a particle or a wave?

CHAPTER 14 ELECTRIC AND MAGNETIC FIELDS

Selected SG questions	Text: 14.1–14.3	HB 4-4	Write up exp.	HB Survey Ch. 14
Small-group problem solving	Teacher demonstration lecture: Static electricity	Lab: E 4-4 Coulomb's Law	Teacher lecture: Current electricity	Student activity day
Text: 14.4–14.10	Text: 14.11–14.13	Write up lab	Write up lab	Review Ch. 13 & 14
Student presentations	Lab stations: Current balance E 4-5 or E 4-6	Lab E 4-7 or E 4-8 Current Balance and or Electron Beam Tube (See days 12–14)	Teacher demonstration lecture: Fields	Discuss history of electricity Quiz

CHAPTER 15 FARADAY AND THE ELECTRICAL AGE

Text: 15.1–15.4	Text: 15.5–15.8		Research	Research
Teacher demonstration lecture: Faraday	Lab stations: electrical devices (See day 17.)	Organize research groups	Library research: Electricity and society	Library research: Electricity and society

CHAPTER 16 ELECTROMAGNETIC RADIATION

Prepare presentation	Text: 16.1–16.4	Handbook: Survey Ch. 16	Write up lab	Text: 16.5–16.6
Student presentation	Teacher presentation: Maxwell	Lab stations: Electromagnetic waves (See day 23.)	P.S.S.C. Film: *Electromagnetic Waves*	Class discussion: Ether
Selected SG questions				
Small-group problem solving	Teacher presentation: Electromagnetic theory	Review	Test	Discuss test

CHAPTER 13 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
13.1 Introduction					T30 The Speed of light		13.1
13.2 Propagation of light	2 3 6 7	4	5		F 30 Speed of light R 4 Velocity of light		13.2
13.3 Reflection and refraction	11 13	9 10 12 14	8	E 4-1 Refraction of a light beam	F 31 Introduction to optics R 20 Lenses and Optical Instruments	Make an ice lens	13.3
13.4 Interference and diffraction	16	17 18 19	15	E 4-2 Young's experiment	R 6 Eye and Camera	Thin-film interference Photographing diffraction patterns Poisson's spot	13.4
13.5 Color	20 21 22				R 7 The Laser—What It Is and Does	Color	13.5
13.6 Why is the sky blue?							13.6
13.7 Polarization	23 24				R 5 Popular Applications of Polarized Light	Polarized light	13.7
13.8 The ether							13.8

CHAPTER 14 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
14.1 Introduction				E 4-3 Electric forces I		An isolated north magnetic pole	14.1
14.2 The curious properties of lodestone and amber					T 31 E field inside conducting sphere		14.2
14.3 Electric charges and electric forces	2 3 5	4		D 47 Electrostatic demonstrations	F 32 Coulomb's law (P.S.S.C.)	Detecting electric fields	14.3
14.4 Forces and fields	7 8 11	10	6 9	E 4-4 Electric forces. II: Coulomb's law			14.4
14.5 The smallest charge	13	12 14	15				14.5
14.6 Early research on electric charges				D 48 The electrophorus		Voltaic pile An 11¢ battery	14.6
14.7 Electric currents	16						14.7
14.8 Electric potential difference	18 21	17 19 20	22 23				14.8
14.9 Electric potential difference and current	25 26 27 28 29	24			R 8 A Simple Electric Circuit: Ohm's Law		14.9
14.10 Electric potential difference and power	33	30 32	31		T 33 Forces between current carriers		14.10
14.11 Currents act on magnets	34			D 49 Currents and forces E 4-5 Forces and currents			14.11
14.12 Currents act on currents			36	D 50 Currents, magnets and forces E 4-6 Currents, magnets, and forces	T 32 Magnetic Fields and Moving Charges T 33 Electrons in a Uniform Magnetic Field (P.S.S.C.) R 17 Radiation Belts around the Earth		14.12
14.13 Magnetic fields and moving charges	40	35 38	37 39	E 4-7 and E 4-8 Electron beam tube	R 9 The Electronic Revolution	Additional activities using the electron beam tube	14.13

CHAPTER 15 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
15.1 The problem: getting energy from one place to another	2 3				R 2 On the Method of Theoretical Physics	Physics collage	15.1
15.2 Faraday's early work on electricity and lines of force				D 51 Electric fields			15.2
15.3 The discovery of electromagnetic induction	4				R 15 Relationship of Electricity and Magnetism		15.3
15.4 Generating electricity by the use of magnetic fields: the dynamo	5 6 7 8				T 32 Magnetic fields and moving charges	Faraday disc dynamo Bicycle generator Generator jump rope	15.4
15.5 The electric motor		9 10			T 32 Magnetic fields and moving charges	Simple meters and motors Simple motor–generator demonstration	15.5
15.6 The electric light bulb	11	12 13 14 15			R 10 The Invention of the Electric Light		15.6
15.7 Ac versus dc and the Niagara Falls power plant	17 20 21 22	16 19	18		R 12 Future of Direct Current Power Transmission		15.7
15.8 Electricity and society	23 24 25				R 9 The Electronic Revolution R 11 High Fidelity R 18 A Mirror for the Brain R 1 Letters from Thomas Jefferson, June 1799	"Lapis Polaris Magnes"	15.8

CHAPTER 16 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
16.1 Introduction	2					Science stamps	16.1
16.2 Maxwell's formulation of the principles of electromagnetism	3				R 14 On the Induction of Electric Currents		16.2
16.3 The propagation of electromagnetic waves	4 5			D 52 Microwaves	L 44 Standing electromagnetic waves F 34 Electromagnetic waves (PSSC) F 16 Electromagnetic field		16.3
16.4 Hertz's experiment	6 7	8		E 4-9 Waves and communication		Microwave transmission systems	16.4
16.5 The electromagnetic spectrum	9 10 11 12 13 14 15 18 19 23	16 17 20 22	21		T 34 The electromagnetic spectrum		16.5
16.6 What about the ether now?	24 25	26 27 28 29			R 13 James Clerk Maxwell Part II R 19 Scientific Imagination	Bell Telephone Science Kits Good reading	16.6

Background and Development

OVERVIEW OF UNIT 4

Chapters 12 through 16 should be considered an integrated sequence covering selected aspects of light, waves, electricity, and magnetism. The primary goal of the sequence is to reach a qualitative understanding of electromagnetic waves (Chapter 16) based on the concept of electromagnetic induction (Chapter 15) and merge it with the wave description of light (Chapter 13). A secondary purpose is to provide the basic physics needed to understand the elements of electrical technology, in order to make contact with an area of applied science that has important social consequences (Chapter 15). A third goal is to present some information about the interaction of electric charges with each other and with magnetic fields (Chapter 14), not only as a prerequisite to understanding electromagnetic induction and light but also for use in Units 5 and 6 in connection with experimental atomic and nuclear physics. While Chapters 12, 13, and 14 cover a certain amount of fairly standard material on waves, light, and electricity, a comprehensive treatment of these subjects is not a major goal of the course, and the temptation to delve *more* deeply into the topic of electric circuits should be resisted (at this time).

There will be some advantage to having the students refer to Unit 3 *Text* and *Handbook*. Such properties of pure transverse waves as polarization and interference need to be transferred to the study of light. For example, students can be reminded of interference by referring to a picture of it in the Unit 3 materials.

CHAPTER 13 / LIGHT

13.1 | INTRODUCTION: WHAT IS LIGHT?

The *Text* suggests two alternative ways of describing light: the scientific, quantitative method and the artistic, emotional approach. Clearly a physics textbook must concentrate on the former, but there is nothing wrong with taking advantage of the natural fascination of the latter to arouse interest in the subject. Don't lose the opportunity to present demonstrations and discussions of color effects with lighting, painting, and photography. Check the latest issues of *Scientific American* and photography magazines for news of recent theories of color perception, and other topics.

Optics is a large subject, of which only a small part is covered in this course. We have omitted almost all of *geometrical optics* on the grounds that both particle and wave models for light give equivalent predictions so that these phenomena cannot be used to distinguish between them. Instead, we concentrate on *physical optics* (interference, diffraction, polarization) and look for phenomena that can be used to test theories of light. If you want to spend extra time on lenses and mirrors, be sure that there is some payoff from the viewpoint of added student interest and motivation, since geometrical optics is not needed for anything else in this course.

13.2 | PROPAGATION OF LIGHT

If there is a *camera obscura* available locally, be sure students have a chance to see it.

Although Römer did not in fact measure the actual value of the velocity of light, his work is of importance in showing that the speed of light is finite. Huygens, two years later in 1678, used Römer's data together with the newly measured earth–sun distance to compute the value often attributed to Römer.

A discussion of the significance of the work of these men and the measurements by Bradley in 1728 by an entirely different method is found in the short article on Römer on page 292 of this *Resource Book*.

The subsequent history of determinations of the speed of light is reported in *Michelson and the Speed of Light*, by Bernard Jaffe, Doubleday Paperback, 1960.

In Chapter 16, we note that Hertz's experimental determination that electromagnetic waves have the same speed as light was one piece of evidence for the hypothesis that light waves are a form of electromagnetic waves. In Chapter 20, we again encounter the speed of light as a maximum speed in relativity theory.

13.3 | REFLECTION AND REFRACTION

In this section we look for properties of light that can be used to distinguish between wave and particle models. Note that we have in mind only one particular particle model, the rather special one proposed by Newton, which is based on assumptions about repulsive and attractive forces exerted by the medium on the particle.

At this point, the discussion of angles of reflection of waves (Chapter 12) should be reviewed. Note that we do not need Snell's law or the concept of index of refraction in our treatment on a qualitative level. However, if you do plan to treat electromagnetic waves more quantitatively using Maxwell's equations, you could introduce the index of refraction here. You can then derive Maxwell's relation between index of refraction and electric and magnetic properties of the medium, $i = \sqrt{\varepsilon\mu}$. This is a neat connection between optics and electromagnetism, and its experimental verification by the "theoretical" physicist Ludwig Boltzmann was one of the first triumphs of Maxwell's theory.

The point to make here, however, does not concern the index of refraction per se but rather the fact that both wave and particle models give the same relation: sine of angle of incidence = a constant times sine of angle of refraction. For the particle model this can be (and should be) derived directly from conservation of momentum, assuming that the "repulsive power" of the medium that the particle enters acts in a direction normal to the boundary. The component of momentum parallel to the boundary must therefore be conserved; and the component normal to the boundary is reversed, just as in an elastic collision of a gas molecule with the wall of its container. (In fact, in kinetic theory this is called "specular reflection" of the molecule.)

Be sure to point out why the light particle would be expected to speed up if it does enter the medium: The "attractive power" now takes over and accelerates it. This is where the peculiarity of Newton's model comes in; don't leave the impression that all particle models would make this prediction. We don't want to convince students so firmly of the validity of the wave theory that they will feel duped when they get to Unit 5!

13.4 | INTERFERENCE AND DIFFRACTION

The theory of the double-slit interference experiment, presented in Sec. 12.6 should be carefully reviewed at this point, so that Young's method for calculating the wavelength of light can be understood. Point out that, because of the very small wavelength of visible light compared to that of sound, interference and diffraction can be observed only in special circumstances. This is why light ordinarily travels in straight lines and casts sharp shadows, whereas sound travels around corners.

The attack on Young by Henry Brougham (referred to briefly in the margin of *Text* page 405) has some historical significance. Brougham was a politician, essayist, and amateur scientist who had earlier published papers on light that Young had criticized. *The Edinburgh Review*, founded at the beginning of the nineteenth century, published articles on literature, religion, politics, and science for a nonspecialist but intellectual audience (perhaps the equivalent of today's *New York Review of Books*). Its authors, writing anonymously, often expressed their opinions intemperately. Brougham was a frequent contributor on scientific subjects; his review of Young's papers, referred to here, seems to have persuaded most British scientists to ignore the wave theory of light for about 20 years. This was partly because of the strong respect for anything Newton had said (or was thought to have said) and partly because British science in the early nineteenth century was mostly empirical and antitheoretical in nature. (Another example of this is the Royal Society's rejection of Herapath's kinetic theory in 1820; see Unit 3, page 326.) Whittaker has the following note on Brougham's character in his book *A History of the Theories of Aether & Electricity, Vol. I*, page 102:

'Strange fellow,' wrote Macaulay, when half a century afterwards he found himself sitting besides Brougham in the House of Lords, 'his powers gone: his spite immortal.'

The statement in the *Text* that "Poisson announced that he had refuted the wave theory" is not supported directly by historical evidence, but it is a plausible reconstruction based on Poisson's known opposition to the wave theory and his persistent attempts to disprove it by any method possible.

Colors of thin films

In discussing Young you may wish to mention his explanation of the colors of thin films. The following is quoted from *Foundations of Modern Physical Science*, by Holton and Roller, p. 555:

Young showed that interference of light waves can account for the color patterns observed by Newton when white light incident on a thin film is reflected to the eye. Examples familiar to everyone are the colors of soap bubbles and of films of oil on a water surface, which are explained as follows. The thin film reflects some light from the front surface and some light from the back surface, and these two beams arrive together at the retina. Now suppose that the difference of distance traveled from the two surfaces to the eye is effectively half a wavelength of red light, or, using modern values, about $\frac{1}{2} \times (7 \times 10^{-5})$ cm; then the *red* component of light will not be seen because the "crests" of the wave coming from one surface are canceled by the "troughs" of the wave coming from the other, and vice versa.

On the other hand, the difference of $\frac{1}{2} \times (7 \times 10^{-5})$ cm corresponds to a *full* wavelength of *blue* light, and therefore the blue component of the reflected white light from the same region where red light has destructive interference will appear to be fully reinforced. For different film thicknesses, different components of light have destructive and constructive interferences, and so different hues can be observed to predominate over the whole surface.

13.5 | COLOR

As was mentioned previously, the subject of color can be used to link physics with other interests of students. Within the structure of this course, the main purpose of talking about color is to introduce the concept of light waves of different wavelengths.

The influence of Newton's theory of color on eighteenth-century poetry is discussed, with many more examples, in Nicolson's book, *Newton Demands the Muse* (Princeton University Press, 1946).

Goethe's attack on Newton's theory of light is another aspect of the antimechanistic Romantic movement that was already mentioned in Sec. 10.9.

13.6 | WHY IS THE SKY BLUE?

New units of length appropriate to atomic dimensions (the Ångstrom: 10^{-10} m and the nanometer: 10^{-9} m) are introduced in this section.

The purpose of this section is to illustrate how physics can answer a question people ask about the world. All one really needs to explain the wavelength-dependence of scattering is a general principle that was mentioned previously in connection with the difference between propagation of light and of sound; the wave doesn't interact much with solid objects unless they are roughly the same size as its wavelength.

13.7 | POLARIZATION

This section introduces the idea that light can be polarized, a fact that is explicable if light waves are transverse rather than longitudinal; hence (as pointed out in the next section), the need for the ether to be solid. Otherwise, polarization as such has little relevance to the unit; the paragraphs on Polaroid serve only to make a connection with the students' common experience.

Review Sec. 12.2 and try to make it plausible that only solids can transmit transverse waves. In fact, the lack of transmission of transverse waves from earthquakes is one evidence that the earth's core is liquid. Substances like Polaroid contain long molecules that absorb waves polarized in the direction along the molecule. In this case, only waves polarized at right angles to the orientation of the molecules will get through. Thus, the usual "picket-fence" analogy for transmission of polarized waves is exactly 90° wrong!

13.8 | THE ETHER

The nineteenth century belief in the elastic ether apparently required a solid medium that could propagate transverse waves; this looks to us more like a suspension of disbelief. But perhaps it's no less plausible than the theories we now accept. One explanation offered was that a substance could act like a fluid for slow deformations but like a solid for rapid motions, such as light. In fact, there are many substances which do act this way. The "silly putty" compound sold in toy stores is an example: It can flow, bounce, or shatter, depending on the magnitude of the applied stress.

Students should leave this chapter with the impression that the wave model explains the properties of light rather well, yet one should not think of the waves as motions of a real physical medium, such as the water waves studied in Chapter 12. The question should be raised, but remain open: How can there be waves without something to wave?

As a further illustration of the interconnection of different areas of physics, you might mention that the acceptance of the wave theory of light after 1820 was one of the factors leading to rejection of caloric theory and the eventual triumph of the law of conservation of energy and thermodynamics. This happened because in the early nineteenth century many scientists were interested in *radiant* heat. They found that it had all the qualitative properties of light: interference, polarization, etc. Hence it was believed that light and heat are essentially the same thing.

If you accept a wave theory of light you must also accept a wave theory of heat; that is, heat involves vibrations of the ether although these are of a longer wavelength than those of visible light. The wave theory of heat provided a transition from caloric theory to thermodynamics in the period from 1830 to 1845, since "caloric" could be identified with "ether." The only, and apparently trivial, change required was from heat as an amount of ether-caloric to heat as motion of the ether. Discussion of radiant heat and its similarity to light would help the students prepare for Chapter 16.

CHAPTER 14 / ELECTRIC AND MAGNETIC FIELDS

SUMMARY OF DEMONSTRATIONS AND EXPERIMENTS USING THE CURRENT BALANCE

In this series of two demonstrations and two experiments used in Chapter 14, students investigate the forces that can occur between two currents or between a current and a magnet. In both experiments (*E4–5* and *E4–6*), students work in three different groups, each group examining a different aspect of the current–current or current–field interaction. Each experiment is preceded by a demonstration.

The *Equipment Notes* section of this *Resource Book* gives a full description of the design and operation of the instrument. Refer to it for any details not given in the *Experiment Notes* and *Demonstration Notes*.

The sequence of demonstrations and experiments is:

1. *D49:* Currents and Forces

   Demonstration of forces between currents as a function of spatial orientation.
2. *E4–5:* Forces on Currents

   (a) Brief qualitative investigation of the interaction between a current and a magnet.

(b) For the simple case of two parallel current-carrying wires, three groups of students work independently to find

$$F \propto I,\ F \propto l,\ \text{or}\ F \propto 1/d$$

These results are combined to give

$$F = k\,\frac{I_1 I_2 l}{d}$$

The value of the constant k can be determined if time allows.

3. *D50:* Currents, Magnets, and Forces
   Demonstrates in more detail that there is a force between currents and magnets and establishes the directions of current, magnetic field, and force.
4. *E4–6:* Currents, Magnets, and Forces
   Two groups of students find that

$$F \propto I \text{ and } F \propto l$$

which are combined to give

$$F = BIl$$

A third group measures a particular B (the vertical component of the earth's magnetic field).

14.1 | INTRODUCTION

It is essential to recognize the unusual nature of the treatment of electricity and magnetism in this course before attempting to teach this chapter. Though it is a long chapter, it covers many topics that occupy several chapters in traditional texts. In fact, electricity and magnetism is one of the subjects that is underemphasized by *Project Physics*. We have tried to cut the material to the bare minimum needed to understand electromagnetic wave theory and some of the later experiments in atomic physics. (The same technical background also can be used in explaining motors and generators in Chapter 15.) Among the subjects not covered in this chapter are: the interactions of magnetic poles with each other, mathematical form of the electrostatic potential of a point charge, properties of various kinds of circuits. We suggest that you do not try adding these topics the first time through; see how long it takes to get through the core of the *Project Physics* Course before expanding it.

14.2 | THE CURIOUS PROPERTIES OF LODESTONE AND AMBER: GILBERT'S DE MAGNETE

This section can be treated as a reading assignment, providing the historical background for the transition from magic to science in electricity and magnetism. Some students may be interested in reading more of Gilbert's book. It's available in a Dover paperback and should be in your library.

14.3 | ELECTRIC CHARGES AND ELECTRIC FORCES

We assume that students have already learned basic electrostatics in earlier grades, or that you will demonstrate it in the laboratory. The section is not intended to teach the subject to beginners, but only to serve as a review and reference. The only things that should be really new here are the terminology for electric units and the idea of a mathematical analogy between different physical phenomena, illustrated by the fact that both electric and gravitational forces cancel out on an object inside a hollow sphere. This is an important way in which mathematics helps to unravel the complexities of nature. We come to it again in Chapter 16, with Maxwell's theory of electromagnetism (analogy between fluid flow and lines of force).

Note that electrostatic induction is treated here only because it will be needed to understand Maxwell's "displacement current" in Chapter 16.

14.4 | FORCES AND FIELDS

The field concept is deceptively simple at first sight, and conceptual difficulties frequently come later. The reason for introducing the idea at such an elementary level is that we want to proceed rapidly to the rather abstract notion of a moving pattern of electric and magnetic fields in space, without reference to any sources. It's hard to believe, even today, that "empty" space can contain forces and energies disconnected from any kind of "matter."

Watch out for the "self-interaction" problem, which some students may stumble on. If a field produced by a point charge is an independent entity existing in space, exerting force on any charge, does that mean it also exerts a force on the charge that produced it?

Another trap: what about Newton's third law? How can a charge react back on the field that exerted a force on it? Such paradoxes result from accepting the field concept too quickly and uncritically.

14.5 | THE SMALLEST CHARGE

We depart from historical sequence (the Millikan experiment will be covered more fully in Unit 5) in order to show how one can actually use an electric field in an important application. You should be prepared to explain how a uniform field can actually be produced (two parallel plates), but there is no need to spend much time on this point. The significant thing to mention here is that, with these elementary ideas about electric charges and forces, we can already start thinking about exploring the atomic world. We will, however, need more information from other areas of physics and chemistry to interpret the results.

14.6 | EARLY RESEARCH ON ELECTRIC CHARGES

Treat this section mainly as a reading assignment, but point out the connection with the law of conservation of charge.

Franklin believed that the two types of charges were not really different. He concluded that a positively charged body had an excess of electrical fire and that a negatively charged body had a deficit of it. This implies that electric charge is not created or destroyed. The modern conservation of charge principle contains some of Franklin's basic ideas.

14.7 | ELECTRIC CURRENTS

Volta's discovery opened a new era in science and technology by making electric currents readily available. The impact on physics can be seen in the remaining sections of this chapter, and the impact on technology in Chapter 15. Note also the use of currents to discover new chemical elements (especially by Humphry Davy in England). As was mentioned in Chapter 10, the discovery of interconnections of various kinds of natural forces (starting with Volta) in the first part of the nineteenth century was one of the historical factors leading to the formulation of the general law of conservation of energy.

14.8 | ELECTRIC POTENTIAL DIFFERENCE

The concept of electric potential difference is part of the essential core of this course, and should be carefully discussed. It serves to make the connection between electric currents and the energy concept (Unit 3) and will be used later in discussing electric power transmission and acceleration of particles.

14.9 | ELECTRICAL POTENTIAL DIFFERENCE AND CURRENT

The following may serve as a roughly correct explanation of Ohm's law: If opposite charges are produced on the ends of a piece of conducting material, an electric field is set up in the material and a potential difference appears across the piece. Loose charges in the material will drift in response to the field in such a direction as to neutralize the unbalanced charge, that is, to cancel out the force field and the potential difference. Left alone, the material would reach a steady state of zero field and uniform potential throughout. But if a potential difference across the material is continually maintained, say, by connecting a battery to the ends of the material, then the drift of loose charges will continue. The charges will accelerate in the field during the time between collisions with molecules of the material. If we assume that after each such collision the charge is scattered in a random direction, then the average component of velocity that it will acquire in the direction of the field will be the product of its acceleration and the average time during which it is accelerated:

$$v_{\text{drift}} = at_{\text{av}} = \frac{F}{m} t_{\text{av}}$$

As long as the force acting on the charge, that is, the potential difference V, is small enough so that the drift velocity is small compared to the average thermal speed of the charge, we can assume that the average time between collisions is roughly independent of V. Then the current (rate of flow of charges past a given point) will be proportional to v_{drift} and hence proportional to F, or to V:

$$I \propto v_{\text{drift}} \propto V$$

which is Ohm's law.

14.10 | ELECTRIC POTENTIAL DIFFERENCE AND POWER

This section provides a good opportunity to show the value of mechanical concepts (such as power and work, as applied in new contexts.

14.11 | CURRENTS ACT ON MAGNETS

Here is Oersted's own account of the history of his discovery, written in the third person as part of an article on "Thermoelectricity" for Brewster's *Edinburgh Encyclopedia* (1830) Vol. 18, pp. 573–589. (In the American edition of 1832, this article is in Vol. 17, pp. 715–732.)

Electromagnetism itself was discovered in the year 1820, by Professor *Hans Christian Oersted*, of the university of Copenhagen. Throughout his literary career, he adhered to the opinion that the magnetical effects are produced by the same powers as the electrical. He was not so much led to this by the reasons commonly alleged for this opinion, as by the philosophical principle, that all phenomena are produced by the same original power. In a treatise upon the chemical law of nature published in Germany in 1812, under the title *Ansichten der chemischen Naturgesetze*, and translated into French under the title of *Recherches sur l'identité des forces électriques et chymiques*, 1813, he endeavoured to establish a general chemical theory, in harmony with this principle. In this work, he proved that not only chemical affinities, but also heat and light are produced by the same two powers, which probably might be only two different forms of one primordial power. He stated also, that the magnetical effects were produced by the same powers; but he was well aware, that nothing in the whole work was less satisfactory, than the reasons he alleged for this. His researches upon this subject were still fruitless, until the year 1820. In the winter of 1819–20, he delivered a course of lectures upon electricity, galvanism, and magnetism, before an audience that had been previously acquainted with the principles of natural philosophy. In composing the lecture, in which he was to treat of the analogy between magnetism and electricity, he conjectured, that if it were possible to produce any magnetical effect by electricity, this could not be in the direction of the current, since this had been so often tried in vain, but that it must be produced by a lateral action. This was strictly connected with his other ideas; for he did not consider the transmission of electricity through a conductor as a uniform stream, but as a succession of interruptions and re-establishments of equilibrium, in such a manner that the electrical powers in the current were not in quite equilibrium, but in a state of continual conflict. As the luminous and heating effect of the electrical current goes out in all directions from a conductor, which transmits a great quantity of electricity ; so he thought it possible that the magnetical effect could likewise eradiate. The observations above recorded, of magnetical effects produced by lightning, in steel needles not immediately struck, confirmed him in his opinion. He was nevertheless far from expecting a great magnetical effect of the galvanical pile; and still he supposed that a power, sufficient to make the conducting wire glowing, might be required. The plan of the first experiment was to make the current of a little galvanic trough apparatus, commonly used in his lectures, pass through a very thin platina wire, which was placed over a compass covered with glass. The preparations for the experiments were made, but some accident having hindered him from trying it before the lecture, he intended to defer it to another opportunity; yet during the lecture, the probability of its success appeared stronger, so that he made the first experiment in the presence of the audience. The magnetical needle, though included in a box, was disturbed; but as the effect was very feeble, and must, before its law was discovered, seem very irregular, the experiment made no strong impression on the audience. It may appear strange, that the discoverer made no further experiments upon the subject during three months; he himself finds it difficult enough to conceive it; but the extreme feebleness and seeming confusion of the phenomena in the first experiment, the remembrance of the numerous errors committed upon this subject by earlier philosophers, and particularly by his friend *Ritter*, the claim such a matter has to be treated with earnest attention, may have determined him to delay his researches to a more convenient time. In the month of July 1820, he again resumed the experiment, making use of a much more considerable galvanical apparatus. The success was now evident, yet the effects were still feeble in the first repetitions of the experiment, because he employed only very thin wires, supposing that the magnetical effect would not take place, when heat and light were not produced by the galvanical current; but he soon found that conductors of a greater diameter give much more effect; and he then discovered, by continued experiments during a few days, the fundamental law of electromagnetism, viz. *that the magnetical effect of the electrical current has a circular motion round it.*

14.12 | CURRENTS ACT ON CURRENTS

We have intentionally avoided stating just what the law of force between currents is (when it is attractive, when it is repulsive, and how it depends on distance) so that students can "discover" it for themselves in the lab. There is no loss of continuity thereby as far as the *Text* is concerned, since we never need the details of this force law later on.

Physics textbooks often state or imply that Ampère discovered the correct law of force between current elements. Actually, the law that is now accepted and attributed to Ampère is somewhat different from the one Ampère himself proposed, although it is equivalent when one integrates over the entire current. The difference is significant in the historical perspective of this section: Ampère himself thought that the force must act along the line between the current elements. For a detailed discussion of this point, see *Early Electrodynamics*, by R.A.R. Tricker, Pergamon Press. This book includes selections from Ampère's own papers. A much shorter mathematical discussion may be found in Whittaker's *A History of the Theories of Aether and Electricity*, Vol. 1, pp. 85–88. Note especially Heaviside's remarks quoted on p. 88.

The numerical factor of 2×10^{-7} in the definition of the ampere may cause some puzzlement. Although it is possible to cook up a pseudo-rational justification for it, it is probably better to state flatly that it is arbitrary, having merely the purpose of giving a convenient-sized unit for currents. (But of course the derived unit for charge, the coulomb, is not at all convenient.)

14.13 | MAGNETIC FIELDS AND MOVING CHARGES

This topic deserves careful detailed discussion. You should familiarize yourself with the general expression for the "Lorentz force" with fields and velocities at arbitrary angles, but for the purposes of this course the special case of right angles should be sufficient. Also, look ahead to the applications of this force law in Chapters 15 (electromagnetic induction) and 16 (E and B vectors at right angles in electromagnetic waves) and in Chapters 18 (Thomson q/m experiment) and Chapter 22 (isotope separation).

CHAPTER 15 / FARADAY AND THE ELECTRIC AGE

This chapter is intended as a change of pace. Very little new physics is introduced, but the applications of physics to technology, and hence to social problems, are stressed. Class time should be spent mainly on (1) explaining the principle of electromagnetic induction and the operation of simple generators and motors, and (2) discussing the social impact of electrical technology. The last section, on electricity and society, should stimulate some debate on the value of technology. Encourage students to amplify or to criticize both the optimistic and pessimistic views presented in the text.

A great deal of additional reading is available on the two central topics of the chapter. Perhaps each of several students can be encouraged to report on a different article or chapter. Do not overlook other physics textbooks for more information on electrical technology. Be sure to consider seriously material that relates electrical technology to current concern with our deteriorating environment.

For example, energy from burning coal and from nuclear fission are both used to run electrical generating plants. Both pollute the atmosphere and both heat the water in rivers and lakes needed for cooling purposes. The relative demerits of these two sources of power ought to be thought about by all students, since within the next decade or two electric power production and all the problems it already creates are expected to increase greatly.

It is impossible to create a comprehensive list of resources. Books and articles on the environment, in particular, are so plentiful that it is more practical for each teacher or student to survey what is available locally and prepare a personal list.

Series and parallel circuits are not discussed quantitatively in the *Text*. However, you may wish to present a brief summary of the ways in which resistances function in the two types of circuits. A few problems on series and parallel circuits are included in the *Study Guide*. The only unusual result is encountered when resistances are added in parallel. In this case, $I \propto 1/R$. The total resistance is

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$$

The numerical characteristics of series and parallel circuits are not significant in the development of the *Project Physics* Course. Note that the problems on ac house circuits are solved as though Ohm's law applied to ac circuits as well as to dc circuits.

CHAPTER 16 / ELECTROMAGNETIC RADIATION

16.1 | INTRODUCTION

The main idea to emphasize in this introductory section is Faraday's suggestion that light might be a traveling disturbance in magnetic and electric lines of force. Maxwell's development and mathematical refinement of Faraday's idea is described in the next section.

16.2 | MAXWELL'S FORMULATION OF THE PRINCIPLES OF ELECTROMAGNETISM

The idea of a displacement current in an insulator is introduced to make plausible Maxwell's contention that a changing electric field has a changing magnetic field associated with it, regardless of whether the changing electric field is in a conductor, insulator, or even in free space. The concept of the displacement current serves as a crutch; little time should be spent on it.

The fact that Maxwell's <u>mechanical</u> model suggested effects that hadn't been yet observed <u>is</u> important. However, the details of how the model "worked" are of little significance.

The two important principles encompassed by Maxwell's theory (the production of a magnetic field by a changing electric field and the production of an electric field by a changing magnetic field) will likely be much clearer in the minds of your students if you spend 5–10 min of class time discussing the illustrative diagrams bordering *Text* page 503

16.3 | THE PROPAGATION OF ELECTROMAGNETIC WAVES

The description given of how electromagnetic waves propagate through space by the reciprocal induction of electric and magnetic fields is handicapped by the limitations inherent in the English language. It is extremely difficult to phrase a description that successfully avoids leaving the impression that there is a time difference in each induction. For example, when we say that a changing electric field "produces" a magnetic field, we wrongly suggest that one precedes the other, when in fact they exist simultaneously. (Draw the attention of the students to the relevant marginal note on *Text* page 504.)

The remarkable similarity between the speed of electromagnetic waves as calculated by Maxwell and the known value of the speed of light should be emphasized. The similarity could have been a coincidence, but Maxwell believed otherwise. His subsequent mathematical efforts resulted in an elegant and comprehensive theory of electromagnetism, but a theory that remained to be proven.

16.4 | HERTZ'S EXPERIMENTS

The experiments of Hertz provide a classic example of the testing of the predictions made on the basis of a theory. Hertz showed that electromagnetic waves of many frequencies could exist, had properties similar to those of light, and had the same speed as light.

To fully understand Hertz's experiments, students would need to know much more about resonance, and, unfortunately, it has not been possible to allocate space to that discussion in the *Text*. If you have a suitable class, it would be time well spent. (See *E4-9*.)

The *Project Physics* microwave equipment is an invaluable tool in developing student appreciation of the "light-like" behavior of electromagnetic waves.

16.5 | THE ELECTROMAGNETIC SPECTRUM

It is worth emphasizing that, although the electromagnetic spectrum covers a range of 10^{25} in frequency (or wavelength), all electromagnetic radiation originates from accelerated electric charges. Be careful to avoid making the general statement that all frequencies of electromagnetic radiation have the same speed, unless you add that this is true only in space. The fact that the speed of radiation in a dispersive medium depends on frequency is of course the reason we can use prisms to obtain the spectrum of white light, and why rainbows are formed. All media except a vacuum are dispersive to electromagnetic radiation. Until recently such was thought not to be the case for sound waves. However, dispersion of sound (ultrasonic) waves is now an active field of experimental and theoretical research. It seems that the effect was too small to be observed before the twentieth century.

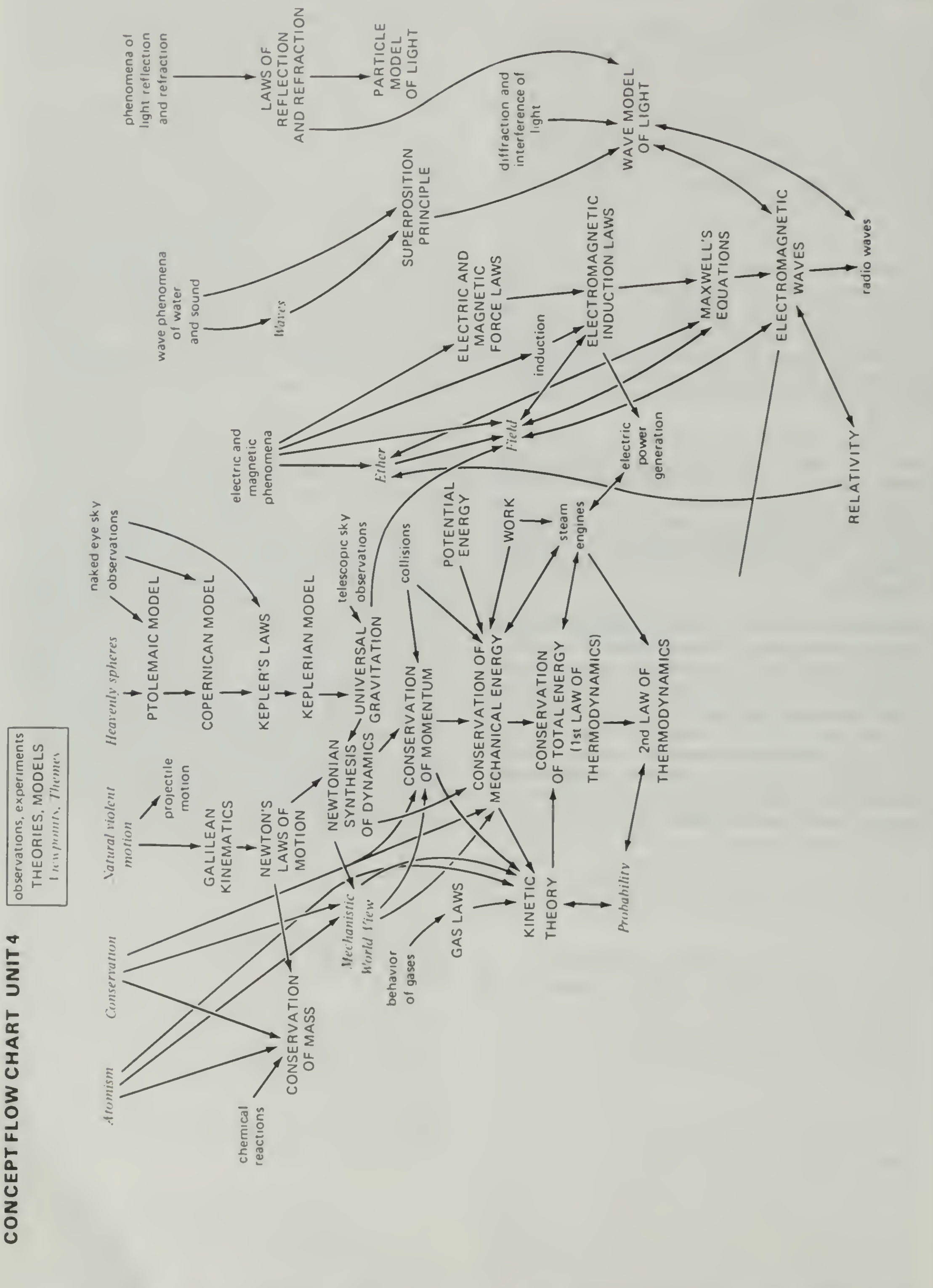
The dispersive effects of interstellar matter have been investigated by studying, at various wavelengths, the times at which eclipsing stars change in brightness. With an accuracy of a few seconds of time, no changes have been found. Thus, over distances of hundreds of light years (one light year is nearly 10^{13} km) the dispersive effects of space are undetectable.

16.6 | WHAT ABOUT THE ETHER NOW?

As indicated in the *Text*, the ether concept returned to nineteenth century physics as a metaphysical necessity for a mechanical interpretation of the wave theory of light and electromagnetism. It is well known that Einstein's 1905 work on special relativity disregarded the ether as superfluous and that this critique of simultaneity was based on the centrality of electrodynamic, rather than mechanical, interpretations of electromagnetic theory. However, many myths have grown around the origins of relativity theory, not least of which is its supposed dependence upon the Michelson–Morley ether-drift experiment of 1887. And so we end this unit with a question in the title that can be picked up again in Secs. 18.5 and 20.1–20.3.

The fact that the luminiferous ether, like the electromagnetic ether, died hard in the controversies over relativity theory early in this century can prove embarrassing to a teacher whose students might discover respectable physicists writing about the ether of space long after 1905. In spite of the null results of Michelson–Morley and other similar experiments (for example, Rayleigh–Brace, Trouton–Noble), the generation of physicists who reached maturity before 1900 were generally extremely reluctant to give up all forms of belief in an ether. Einstein himself after 1920, as well as Michelson, Lorentz, Poincaré, Sir Oliver Lodge, J. J. Thomson, D. C. Miller, and others, seems to have harbored some hope that the vacuum of space might somehow, someday, be filled once again. But the new generation of mathematically proficient professional physicists constituted a majority in favor of fields and particles and likewise felt no need for the ether hypothesis.

In view of recent research on the influence of the Michelson ether-drift experiment on Einstein's special relativity, it is probably best to consider the latter as more of an intuitive leap than an experimentally based conclusion: another example of theory at the vanguard, followed by experimental tests. In the case of the Michelson experiment, only during the decade of the twenties was it definitively repeated, and by then the wave–particle duality and Bohr's complementarity principle, among other developments, had completely changed the presumptions of physics. Students should be encouraged to discover these disjunctions in scientific advance for themselves.



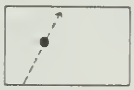
CONCEPT FLOW CHART UNIT 4
observations, experiments
THEORIES, MODELS
Viewpoints, Themes
Atomism
Conservation
Natural violent motion
Heavenly spheres
naked eye sky observations
chemical reactions
CONSERVATION OF MASS
projectile motion
GALILEAN KINEMATICS
NEWTON'S LAWS OF MOTION
NEWTONIAN SYNTHESIS OF DYNAMICS
Mechanistic World View
behavior of gases
GAS LAWS
KINETIC THEORY
Probability
PTOLEMAIC MODEL
COPERNICAN MODEL
KEPLER'S LAWS
KEPLERIAN MODEL
telescopic sky observations
UNIVERSAL GRAVITATION
CONSERVATION OF MOMENTUM
collisions
CONSERVATION OF MECHANICAL ENERGY
POTENTIAL ENERGY
WORK
steam engines
CONSERVATION OF TOTAL ENERGY (1st LAW OF THERMODYNAMICS)
2nd LAW OF THERMODYNAMICS
electric and magnetic phenomena
Ether
Field
electric power generation
RELATIVITY
ELECTRIC AND MAGNETIC FORCE LAWS
induction
ELECTROMAGNETIC INDUCTION LAWS
MAXWELL'S EQUATIONS
ELECTROMAGNETIC WAVES
radio waves
wave phenomena of water and sound
Waves
SUPERPOSITION PRINCIPLE
phenomena of light reflection and refraction
LAWS OF REFLECTION AND REFRACTION
PARTICLE MODEL OF LIGHT
diffraction and interference of light
WAVE MODEL OF LIGHT

Additional Background Articles

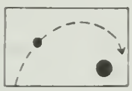
NOTES ON FIELDS

Influence in space
The following two-step explanation is fairly simple to get across and is reinforced in the next section. Perhaps the point can best be made on the chalkboard.

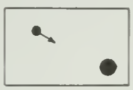
Empty space.



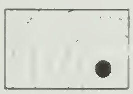
A body moving in empty space has a constant velocity.



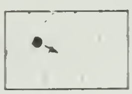
If another body is introduced, the path of the first one is modified.



The modification can be thought of as resulting from a force that the one body exerts directly on the other,



or, the one body can be thought of as producing some kind of effect in the space around it,



and that influence in space exerts a force that modifies the path of the first body.

Test bodies
The expression "force per unit mass" is awkward. It should be made clear that the test body need not be one unit, any more than one must drive for a whole hour in order to go 60 km/hr. It is the *ratio* of force to mass that characterizes the field. If there is more mass to be acted upon, the force will be greater.

The concept of a "test body" is used throughout Chapter 14. A test body must be small in two ways. It must be sufficiently small in mass (or charge or whatever) that it does not appreciably modify the main sources of the field, and so change the field it is trying to explore. It must also be small in size (compared to the scale on which the field is being considered) so that there is essentially one value to the field in the space it occupies. (A striking illustration of this requirement is the supposed fate of a moon that approaches its planet too closely: The gravitational field becomes sufficiently different on near and far parts of the moon to tear the moon apart.)

Does a field really exist?
The independent existence of a field, regardless of whether or not there is something for it to act upon, is as much a philosophical problem as a physical one. The descriptions of phenomena in terms of fields are consistent and useful. Energy can be considered to be stored in a field, and thus field disturbances are accompanied by transfer of energy. But the question "Does a field actually exist in empty spaces or is it just a way of thinking of things?" must be put on the shelf with a number of similar epistemological problems. Suffice it to say that almost all physicists think of fields as if they exist.

Relation of $\vec{g}$ to $\vec{a}_g$
We have tried, with some strain on the conventions of the profession, to use $\vec{a}_g$ for the acceleration produced by gravity. Don't say that the gravitational field $\vec{g}$ is the same as $\vec{a}_g$. There is no need for the relation of the two to be pointed out, and if students don't bring it up it is probably best to let it slip by. The acceleration $\vec{a}_g$ can be measured by distances and times. Using Newton's second law, we would say that the gravitational force on a body is $\vec{F}_g = m_i\vec{a}_g$ where m_i is a measure of the body's inertia. By the definition of gravitational field strength $\vec{g}$, $\vec{F}_g = m_g\vec{g}$, where m_g is a measure of the body's response to gravity. Because the "gravitational mass" of a body is proportional to the "inertial mass," the gravitational acceleration is proportional to the gravitational field strength (as found by experiment to within one part in 10^{11}). Because the same body (the "standard kilogram") is used for defining units of both quantities, and the units are both given the same name "kilogram," $\vec{a}_g$ is actually equal numerically and dimensionally to $\vec{g}$.

In general relativity, the complete equivalence of inertial and gravitational mass is assumed, so that $\vec{a}_g$ and $\vec{g}$ are identical. There is no distinction made between them; acceleration and gravitational force are different ways of viewing the same thing. However, there is an important distinction between $\vec{a}_g$ and $\vec{g}$ in the context of an elementary course. A measured value of $\vec{a}_g$ will be equal to $\vec{g}$ only if a test body is allowed to fall freely. If a body is supported, it is still acted on by $\vec{g}$, but its acceleration is zero. Although it is perfectly legitimate to consider a supported body to have an acceleration component $\vec{a}_g$ (cancelled out by an acceleration component due to the supporting force), that viewpoint will be hard to sell to students.

"Self-action" of a field
Some students may bring up the issue of the effect produced on a body by its own field. There are some very deep problems involved here (see Feynman's Nobel Prize address in *Physics Today*, September 1966). For the purposes of this course, it will be sufficient to appeal to the two-step representation. The effect of one body on another is rep-

resented as a field set up by the first body; that field acts on the second body, and vice versa. The effect of the second body on the first is represented as a field set up by the second; that field acts on the first. If a body does have an effect on itself, then it could be considered to be affected by its own field. In Newtonian mechanics there is no such self-effect (witness Newton's first law), so that a body is properly considered as acted upon only by the combined fields of all other bodies. (By "body" here we mean a minuscule particle. The body of the earth is affected by other parts of the body, so that in that sense it acts on itself.) The essence of this concept can be brought out by asking the students to rewrite

$$F = \frac{GMm}{r^2}$$

in terms of the moon's gravitational field.

Adding fields

If fields are defined in terms of forces and forces are defined in terms of accelerations, then in one sense fields must, by definition, add vectorially as accelerations do. What is at issue, however, is that the effect of one source is the same, whether or not some other source is present and having an effect also; that is, force fields defined in terms of their sources add as vectors.

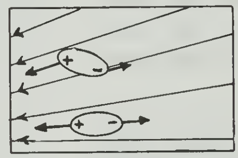
In the case of a field produced by several charged conductors, the presence of one will affect the distribution of charge on the others, so that the field resulting from the presence of all is not the vector sum of the fields that would be produced by each source alone in the absence of the others. The disturbed fields of the disturbed sources do add vectorially, however. The business of mutual disturbance can be very complicated, so that often the configuration of sources required to produce a desired field pattern must be found by trial and error rather than by direct calculation. Film Loop 48, "Thomson Model of the Atom," illustrates changes in the field as more sources are added.

Shielding limitations

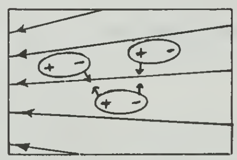
Shielding from external fields is not strictly correct for extremes of field strength or extremes of rate of change of field strength. If an external field were so phenomenally strong as to cause all the loose charge on a conducting shell to move to such locations as to produce a net field of zero inside, then there would be no more shift possible, and any increase in the external field could not be balanced. If the external field is changing at a rate of 10^{20} Hz (that is, X rays), the charges in the conductor could not oscillate rapidly enough to cancel it out inside, and the changing field could penetrate to the interior. For almost closed conducting shells (as in electron-gun electrodes or cyclotron "Dees"), the field is almost zero deep inside. Incidentally, a conducting shell does not shield the space around it from a static charge inside unless the shell is grounded.

Grass seeds

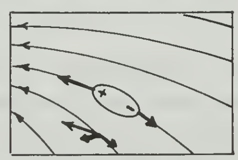
If students specifically ask why grass seeds line up in an electric field, you can explain that the ends of the grass seed become charged oppositely, so that the field exerts opposite forces on the two ends. These forces will tend to turn the grass seed into a position along a line of force. Even when lined up, the force will be greater on the end where the field is stronger, so the grass seed will be pulled into a region of stronger field.



The charged ends of the grass seed attract one another, so that there is a tendency for trains of seeds to form.



Where the lines of force are curved, the vector sum of the forces on the seed will tend to move it into less curved regions of field.



Don't at this time introduce the phenomenon of polarization of an insulator, which involves atoms and charge clouds.

RÖMER

In most textbooks you will find the statement that Römer was the first person to determine the speed of light. You will find various values attributed to him, and most of these values are different even when converted to the same system of units. (The average is about two-thirds the present value.) Anyone who reads Römer's original paper will find that all he did was to estimate the time it takes light to cross a certain fraction of the earth's orbit. He also said that light would take less than 1 sec to cross the diameter of the earth, since he thought the earth's diameter was about 14,400 km (depending on how you interpret the units he used). This means that the speed of light is greater than 14,400 km/sec. But for seventeenth century scientists, the real significance of Römer's work was not the actual value of the speed of light, but the fact that it is finite rather than infinite.

This is analogous to the case of Boyle's work on air pressure. Boyle is known as the discoverer of "Boyle's law," $PV =$ constant; but in fact he wasn't the first to find this relation by experiment. Instead, he did something even more important: He showed that air pressure is finite, and (rather than nature's horror of a vacuum) is responsible for holding up the mercury in a barometer or the water in a suction pump. These two examples, provided by Römer and Boyle, show how different (and how much more interesting) the history of science can become if we try to look at a scientist's work directly and in the context of its own times, rather than judging it on the basis of how it solved problems that we now consider significant.

Although Römer apparently did not have data on the diameter of the planetary orbits from which he could compute the actual speed of light, such data was just becoming available at the time he worked. In 1672, the French astronomers Jean Richer and Jean Dominique Cassini determined the earth–Mars distance by triangulation of Mars. They used a baseline with Paris at one end and Cayenne, on the northern coast of South America, at the other. Since, as we saw in Unit 2, the relative distances of all the planets from the sun, and relative earth–Mars distance at a given time, could be found from the heliocentric theory, all these distances could be found as soon as one of them was known. The Richer–Cassini observations led to an earth–Sun distance of 139 million km. Huygens, in 1678, used this value together with Römer's data on the time interval to compute the speed of light. Huygens' value, 2×10^8 m/sec, was published in his treatise on light in 1690, together with an analysis of Römer's observations. Later scientists seem to have misread this passage and thought that Römer himself had gotten that value.

Another way of determining the velocity of light from astronomical observations was discovered by the British astronomer James Bradley in 1728. He found that certain stars slightly change their positions in the sky during the year in a peculiar way. Although he was originally trying to observe the parallax of such stars, it turned out that this slight movement could not be attributed to parallax, but was due instead to the component of motion of the earth at right angles to the line of sight from the star to earth. During the finite time that it takes for light from the star to go from one end of the telescope to the other, the telescope itself moves because of the motion of the earth, and so the telescope must be tilted slightly to see the star. This effect is known as *stellar aberration,* and its magnitude depends on the ratio of the orbital speed of the earth to the speed of light, as well as on the position of the star relative to the plane of the ecliptic. Since the orbital speed is known, the speed of light can be calculated. The value obtained was within a few percent of the present value.

Bradley's discovery of stellar aberration was historically important for another reason. It was, in a sense, the first direct astronomical evidence for the helicoentric theory! Yet by the time it was found, most scientists had already accepted the heliocentric theory for other reasons (see Unit 2). Here is an interesting example of the relative unimportance of experiments and observations, as compared to convincing theories, in changing a world view.

The subsequent history of determinations of the speed of light is reported in "The Speed of Light," by J. H. Rush, *Scientific American,* August 1955, p. 67.

In Chapter 16 we note that Hertz's experimental determination that electromagnetic waves have the same speed as light was one piece of evidence for the hypothesis that light waves are a form of electromagnetic waves. In Chapter 20 we will again encounter the speed of light as a maximum speed in relativity theory.

THE COST OF AN ELECTRICAL MOTOR IN 1850

In 1821 Michael Faraday exhibited his electromagnetic rotator (described in Sec. 15.3) at the Royal Institution in London. Other electric motors were designed by various scientists in Europe and America. One of them was Joseph Henry's "rocking electromagnet." Henry wrote as follows about his motor:

> I have lately succeeded in producing motion in a little machine by a power, which, I believe, has never before been applied in mechanics—by magnetic attraction and repulsion.
>
> Not much importance, however, is attached to the invention, since the article in its present state can only be considered a philosophical toy; although in the progress of discovery and invention, it is not impossible that the same principle, or some modification of it on a more extended scale, may hereafter be applied to some useful purpose.

Such an oscillating motor is often used in advertising displays in store windows. The reason for Henry's failure to be very enthusiastic about the importance of his invention is that, as long as electric current was only available from batteries, electric motors could not compete with steam engines.

The economics of the situation was summarized in a leading British scientific journal as follows:

> [Notwithstanding] the numerous attempts which have been made to apply electro-magnetism as a power for moving machines ... and the large amount of money which has been expended in the construction of machines, the public are not in

possession of any electro-magnetic machine which is capable of exerting any power economically. . . Estimations made by Messrs. Scoresby and Joule, and the results obtained by Oersted. . . . very nearly agree; and it was stated that one gr. of coal consumed in the furnace of a Cornish [steam] engine lifted 143 lbs. 1 foot high, whereas one gr. of zinc consumed in a battery lifted only 80 lbs. The cost of [one hundred weight] of coal is under 9 pence, and cost of [one hundred weight] of zinc is above 216 pence. Therefore under the most perfect conditions, magnetic power must be nearly 25 times more expensive than steam power. . . . the attention of engineers and experimentalists should be turned at present, not to contriving of perfect machines for applying electromagnetic power, but to the discovery of the most effectual means of disengaging the power itself from the conditions in which it existed stored up in nature.

[*Philosophical Magazine*, 1850.]

Brief Description of Learning Materials

SUMMARY LIST OF UNIT 4 MATERIALS

Experiments

E4–1 Refraction of a Light Beam
E4–2 Young's Experiment: The Wavelength of Light
E4–3 Electric Forces. I
E4–4 Electric Forces. II: Coulomb's Law
E4–5 Forces on Currents
E4–6 Currents, Magnets, and Forces
E4–7 Electron Beam Tube
E4–8 Electron Beam Tube. II
E4–9 Waves and Communication

Demonstrations

D47 Some electrostatic demonstrations
D48 The electrophorous
D49 Currents and forces
D50 Currents, magnets, and forces
D51 Electric fields
D52 Demonstrations and experiments with microwaves

Film Loops

L44 Standing Electromagnetic Waves

Reader Articles

R1 *Letter from Thomas Jefferson, June 1799* by Thomas Jefferson
R2 *On the Method of Theoretical Physics* by Albert Einstein
R3 *Systems, Feedback, Cybernetics* by V. Lawrence Parsegian and others
R4 *Velocity of Light* by A. A. Michelson
R5 *Popular Applications of Polarized Light* by William A. Shurcliff and Stanley S. Ballard
R6 *Eye and Camera* by George Wald
R7 *The Laser—What It Is and Does* by J. M. Carroll
R8 *A Simple Electric Circuit: Ohm's Law* by Albert V. Baez
R9 *The Electronic Revolution* by Arthur C. Clarke
R10 *The Invention of the Electric Light* by Matthew Josephson
R11 *High Fidelity* by Edgar Villchur
R12 *The Future of Direct Current Power Transmission* by N. L. Allen
R13 *James Clerk Maxwell, Part II* by James R. Newman
R14 *On the Induction of Electric Currents* by James Clerk Maxwell
R15 *The Relationship of Electricity and Magnetism* by D. K. C. MacDonald
R16 *The Electromagnetic Field* by Albert Einstein and Leopold Infeld
R17 *Radiation Belts Around the Earth* by James Van Allen
R18 *A Mirror for the Brain* by W. Grey Walter
R19 *Scientific Imagination* by Richard P. Feynman, Robert B. Leighton, and Matthew Sands
R20 *Lenses and Optical Instruments* by Physical Science Study Committee
R21 *Baffled* by Keith Waterhouse

Sound Films (16 mm)

F30 Speed of Light
F31 Introduction to Optics
F32 Coulomb's Law
F33 Electrons in a Uniform Magnetic Field
F34 Electromagnetic Fields

Transparencies

T30 The Speed of Light
T31 *E* Field Inside Conducting Spheres
T32 Magnetic Fields and Moving Charges
T33 Forces between Current Carriers
T34 The Electromagnetic Spectrum

FILM LOOPS

L44 STANDING ELECTROMAGNETIC WAVES

A 435-MHz transmitter feeds an antenna at one end of a metallic trough. When a metallic reflector is positioned at the other end, a standing wave is formed. Many small lamp bulbs at the centers of tuned dipoles make the pattern visible. Polarization is shown. Finally, the standing waves in this and the preceding two loops are displayed simultaneously to emphasize the existence of nodes as a fundamental property of any standing wave.*

SOUND FILMS (16 mm)

F30 SPEED OF LIGHT

21 min. PSSC, William Siebert, MIT, MLA. Outdoors at night Dr. Siebert measures the speed of light in air over a 300-m course using a sparkgap, parabolic mirrors, a photocell, and an oscilloscope. In the laboratory, he compares the speed of light in air and in water using a high-speed rotating mirror.

F31 INTRODUCTION TO OPTICS

Color, 23 min, PSSC, E. P. Little, MLA. Deals with the approximation that light travels in a straight line; shows the four ways in which light can be bent: diffraction, scattering, refraction, and reflection. Refraction is illustrated by underwater photography to show how objects above water appear to a submerged skin diver.

F32 COULOMB'S LAW

30 min, PSSC, Eric Rogers, Princeton, MLA. Demonstrates the inverse-square variation of electric force with distance, and also the fact that electric force is directly proportional to charge. Introduces the demonstration with a thorough discussion of the inverse-square idea. Also tests the inverse-square law by looking for electrical effects inside a charged, hollow sphere.

F33 ELECTRONS IN A UNIFORM MAGNETIC FIELD

11 min, Dorothy Montgomery, Hollins College, MLA. A spherical cathode-ray tube with a low gas atmosphere (Leybold) is used to measure the curvature of the path of electrons in a magnetic field, and, with reference to the Millikan experiment, the mass of the electron is determined. The math involved is worked out with the experiment.

F34 ELECTROMAGNETIC WAVES

33 min, George Wolga, MIT, MLA. Shows why we believe in the unity of the electromagnetic radiation spectrum. Experiment shows that the radiation arises from accelerated charges and consists of transverse waves that can be polarized. Interference (Young's double-slit experiment) is shown in four different regions of the electromagnetic spectrum: X ray, visible light, microwave, and radio.

TRANSPARENCIES

T30 THE SPEED OF LIGHT

Visualizations of the Römer and Michelson methods for determining the speed of light are presented.

T31 $\vec{E}$ FIELD INSIDE CONDUCTING SPHERE

A geometric and electrostatic argument shows in a step-by-step fashion that the electric field strength inside a hollow metallic charge-carrying sphere is zero.

T32 MAGNETIC FIELDS AND MOVING CHARGES

A multiple transparency illustrating forces on moving charged particles in a magnetic and electric field, forces on current carriers, forces on moving conductors, and the principles of the ac and dc generators.

T33 FORCES BETWEEN CURRENT CARRIERS

Explanations for attractive and repulsive forces between parallel current carriers are given in terms of moving charged particles in magnetic fields.

T34 THE ELECTROMAGNETIC SPECTRUM

A double-transparency showing the electromagnetic spectrum with a full-color insert of the visible spectrum, and a number of emission and absorption spectra of the elements.

*Note: A fuller discussion of this *Film Loop* and suggestions for its use will be found in the section of this *Resource Book* entitled "Film Loop Notes."

Demonstration Notes

D47 SOME ELECTROSTATIC DEMONSTRATIONS

In electrostatics there is a great wealth of good demonstrations. We have tried to make a selection on the basis of what is most important, what can be understood, what is likely to be remembered, and what relates to other topics in the course. Since the equipment for giving demonstrations varies so greatly and most of it is well known, we have described a minimum number of specific techniques. We prefer to encourage the teacher's personal enthusiasm for particular demonstrations rather than to prescribe a set of procedures.

When different materials are brought into close contact and then separated, each is likely to show a change in behavior; they may attract or repel other bits of matter. These changes in behavior are called "electrical effects," and a body showing these effects is said to have an "electrical charge." It is important to note that unaided human senses do not respond to the presence of charges. This means that we can detect the presence of charges on an object only by observing the effects that they produce on another object.

DEMONSTRATIONS

There are many ways to produce charges and to show the kind of behavior that is called "electrical effect," such as attracting bits of paper. One specific technique will be useful in a later demonstration, so it should be tried here.

Stick a strip of plastic tape (such as plastic electric tape or transparent tape) to a strip of vinyl; call this unit *A*. Prepare a second similar unit; call it *B*. If the process of preparing the units charges them, they must be discharged before starting the demonstrations. This may be done by running cold water over them and drying them either by waving in air or patting with a towel. The two units neither attract nor repel each other.

When unit *A* is pulled apart, new behavior is observed. The tape attracts unit *B*, the vinyl also attracts *B*, and the tape attracts the vinyl.

Now pull unit *B* apart and observe additional behavior patterns. The two tapes repel each other, and so do the two vinyl strips.

The above demonstrations show that there are three patterns of behavior that the students observe: attraction, repulsion, and no force at all. Be careful at this stage not to let your prejudice as a teacher who "knows the answers" misinterpret the evidence obtained so far. There are three behavior patterns, suggesting three types of charge, *X*, *Y*, and *Z*.

	Type *X*	Type *Y*	Type *Z*
Type *X*	no forces	attraction	attraction
Type *Y*	attraction	repulsion	attraction
Type *Z*	attraction	attraction	repulsion

This is not the time to explain that "Type *X*" is uncharged. After all, Type *X* does show the electric behavior of attracting *Y* and *Z*.

Some properties of electric charge can be shown most easily with an *electroscope*. The explanation of electroscope operation is not simple and is not really relevant. If the description is given at all use terms that are consistent with, but do not actually imply, a fluid theory of electricity. Quantization of charge is a later and not at all obvious discovery although most students will have a vague familiarity with "electrons." It is only required at this point that the student realize that an electroscope works because, when charged, the mutual repulsion of the charges on it causes a light vane to swing out.

An electroscope that is encased on two sides by glass can be made more visible to a class by projection. Almost any small filament light source placed a short distance from the electroscope will project a shadow onto the chalkboard adequately, even in a fully lit room.

Conservation of Charge

If a pie tin or coffee can is placed on top of the electroscope, any charged body lowered into the tin will cause the electroscope vane to deflect. With this "Faraday ice pail" apparatus we can demonstrate a most important aspect of electric charges: The two forms of charge we called *Y* and *Z* tend to cancel each other. (A description of Faraday's "ice pail" experiment can be found in *Foundations of Modern Physical Science*, by Holton and Roller. p. 443.)

Prepare a tape–vinyl unit and produce the two charges by separating the strips. Lower one at a time into the tin to show that they are, in fact charged. Now drop the tape into the tin and leave it there. When you drop the vinyl in (Get it all the way in, but do not touch the tin with your fingers.) the vane will return toward its normal position. If you have moved rapidly enough so that charge leakage has been small, the charges should almost exactly neutralize each other.

Apparently the two kinds of charge tend to cancel each other's effects. Our demonstration suggests, but clearly does not prove, that when creating the two charges by pulling the strips apart you created equal amounts, and that when you put them back into the can, one charge just cancelled the other.

The fact that the tape–vinyl unit was of Type *X* (it did not pick up bits of paper and did not attract another tape–vinyl unit) suggests that Type *X* is not really a third type of charge at all. It can be considered as having equal amounts of Type *Y* and Type *Z* charge; or, in the normal language of physics, Type *X* is merely the condition of being uncharged.

It is at this point in the argument that it is reasonable to talk about two kinds of charge, which we could continue to call Y and Z, but which are called (+) and (−) in physics. Point out that there is no evidence for the existence of any third kind of charge.

At this point, students may recognize a problem. The first column in the table above implies that Type X, the uncharged state, *can* attract (or be attracted by) the (+) and (−) charges, but cannot exhibit repulsion! Let the students ponder this one; do not give them an answer!

Induction

If a charge is brought near an electroscope, the observed deflection cannot be accounted for by saying that the electroscope has been given a charge. Let the students discuss this and guide them to the usefulness of representing various steps with the aid of strip-like cartoons, as found on p. 484 of Holton and Roller. You may not have to tell them that this behavior can be explained in terms of a redistribution of the charges on the electroscope. Let the students develop the thesis that although the electroscope still has as much (+) charge as (−) charge, the forces described by the table could result in a *redistribution* of charges. After the students have worked out a solution in their own words, you can introduce the language of induction.

It may be useful at this stage to introduce the technique of charging an object by induction. If a demonstration is given without any comment from the teacher, it can be a good homework problem for students to try to explain the technique. We do not want to be sidetracked into a lengthy demonstration or discussion of specialized techniques. However, in the experiment on Coulomb's law it will be necessary to charge pithballs by induction.

Electric Charges and "Electricity"

It may be impossible to put students in the frame of mind that existed when no direct connection between static and current electricity was known. Today students are so used to the term "electricity" that they may find it impossible to believe that anyone ever really thought that the two phenomena called by the same name could be different.

Suggest to the students that there is a problem here, but that they already have a possible solution to it. We recognize charges by the effects they produce: repulsion and attraction. Students do not usually think of these same effects as indicators of the presence of electric currents.

When the students were asked to "explain" why an uncharged electroscope deflects when brought near a charge (induction), they accepted the idea that charges could move as they redistributed themselves on the electroscope. Ask the students whether a charge in motion exhibits behavior that we associate with current electricity.

An electroscope can be charged (or discharged) through a neon bulb, such as the bulb used in a blinky, or NE-2. The glow is brief and dim but visible.

Transfer charge from a charged electroscope to a metal plate on an insulating handle. Discharge the electroscope completely by grounding it. Now transfer the charge from the plate back onto the electroscope through the leads of the neon bulb. The bulb will flash. A sensitive meter, about 50 μA, should give just a perceptible kick if used in place of the bulb.

Consider the inverse question: Can a current source be made to exhibit the behavior that we use to detect charges? This effect is difficult to show without having to introduce techniques that distract from the issue at hand. Simply connecting the terminals of a cell to the electroscope will not produce any deflection. A high-voltage supply (giving 500 V or more) should give a small deflection. (Note that a charged comb, which produces deflection in an electroscope, must be a high-voltage source, although a very weak transient current source.)

To use lower voltages it is necessary to add extra capacitance to the system. A metal plate is placed on the top of the electroscope plate and separated from it by a piece of paper. Attach terminals of a 45-V (or more) cell to the two plates. Disconnect the lead to the electroscope, and then pull the upper plate away. The electroscope will show considerable deflection.

The Charge that is "Loose" in Solids

There are other ways than those mentioned for separating charge from neutral matter: using heat, light, chemical reaction, magnetism, or radioactivity. The chemical-reaction effect has already been demonstrated by the battery. Almost invariably, the charge that can be separated is that kind that has been named −, suggesting that in conducting solids the + charges are more or less fixed in place and the − charges are loose and can move around.

Ultraviolet light can knock electric charge out of some metals. Shine a light source with a strong ultraviolet component (carbon arc, mercury arc) on a zinc plate freshly cleaned with sandpaper or steel wool and placed atop the electroscope. If the electroscope has been given a + charge, nothing will happen (because any − charge knocked out will be pulled back immediately). If the electroscope has been give a − charge, the leaves will quickly collapse.

If a metal wire is made quite hot, some electric charge is freed by the agitation. If the heating is done in a vacuum, the charge can drift across to an oppositely charged plate. The 1K3 (1G3, 1B3) vacuum tube is such a setup. Charge knocked out of the hot wire can drift to the cylindrical plate if the plate has a charge opposite to that which came from the wire. Connect the plate terminal on top of the tube to the electroscope and charge the electroscope −. Connect the filament terminals to a 1.5-V battery to heat the wire. Then repeat the experiment with the electroscope charged +. The − charge freed from the wire will be pulled to the plate and balance the + charge on the electroscope, causing the leaves to collapse quickly.

The appearance of charged particles emitted in radioactivity is discussed in a later lab.

Electric Field

Here are two methods that can be used to explore the shape of electric fields:

1. Place electrodes of various shapes into a dish of oil and charge them with a Wimshurst machine or Van de Graaff generator. Grass seeds, hair clippings, lycopodium powder, etc., sprinkled on the oil will polarize and line up along the electric field direction. A transparent tray can be used on the overhead projector.

2. Suspend a charged pithball from a stick by a thin insulating thread, and use it as a roughly quantitative indicator of fields around charged spheres, plates, and wires.

Use a point light source to project the demonstration. The angle between the thread and the vertical gives a rough measure of the forces. Show the uniformity of the field near a large plate, and the $\frac{1}{r}$ dropoff of the field near a long charged wire.

To prevent leakage of charge from the pointed ends of a charged wire, fit the ends with small metal spheres. Even a smooth small blob of solder at the ends of the wire should help.

Plastic strips rubbed with cloth are adequate for charging well-insulated spheres, plates, or wires.

D48 THE ELECTROPHORUS

The electrophorus consists of two parts: a smooth flat slab of wax or other nonconductor, and a circular metal plate, slightly smaller than the wax, with an insulating handle.

1. Charge the wax by rubbing it with fur.
2. Place the metal plate down on it.
3. Touch the top surface of the metal plate.
4. Lift the metal plate off the wax slab by the insulating handle.
5. Show that the metal plate is charged by using it to charge an electroscope, or by discharging it through a neon bulb. On a dry day you may be able to draw a spark from it.
6. Discharge the metal plate completely.

Steps 2–6 can now be repeated over and over again without recharging the wax.

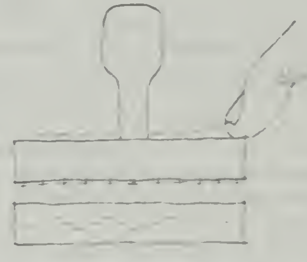
Where does all this charge come from? Why does the wax not lose its charge? Is this a violation of conservation of charge?

To explain the apparent paradox, draw a series of diagrams showing the situation of each of the above six steps:

1. The wax is charged by rubbing.

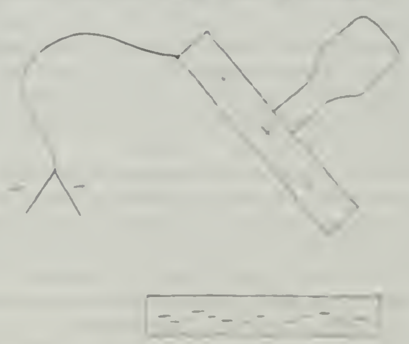
2. The negative charge of the wax attracts the positive charge to the bottom of the metal plate and repels the negative charge to the top. There is separation of charge in the plate, but it still has no net charge.

3. Some negative charge leaves the plate via the demonstrator's finger, hand, and arm, to earth. When the finger is removed, the plate has a net positive charge.

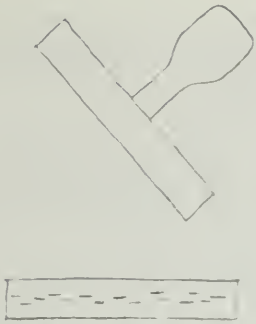


4. Lift the metal plate off the wax slab.

5. Show that the metal plate is charged.



6. No charge remains on the metal plate; all original charge on the wax remains.



This explains why the wax retains its original charge. But it does not explain how we were able to charge the electroscope, or light the neon bulb repeatedly, since the action requires an expenditure of energy every time. Is this a violation of the law of conservation of energy? Or, if not, where does this apparently unlimited energy come from? The answer is that work is done by the demonstrator every time the metal plate (+ charge) is separated from the wax (− charge) in Step 4.

D49 CURRENTS AND FORCES*

(Do before Experiment *4–5*)

The purpose of this demonstration is twofold. It introduces the students to the apparatus used in the experiments, which will save them valuable time. It also points out that the spatial configuration of the two currents (parallel, perpendicular, or antiparallel) has an effect on the forces that result. Among other things, these relationships are an important part of the design of the current balance. An understanding of why only the top wire of the fixed loop is considered depends upon the discussions developed in this demonstration.

Forces exist between two parallel (or antiparallel) currents, but not between currents that are perpendicular.

Equipment:

- Current balance, with the longest loop
- Power supply, 6–8 V dc
- 2 rheostats, 5 Ω, 5 A
- 2 ammeters, 0–5 A dc
- Wire leads
- Ringstand with test-tube clamp
- Hook-up wire with alligator clips
- Paper clips

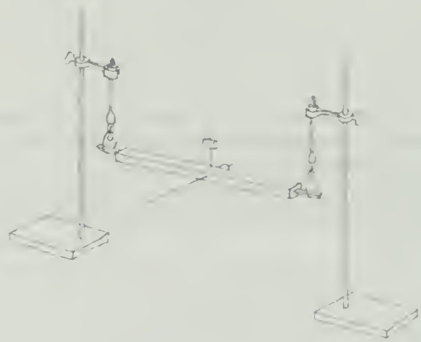
Procedure

1. Turn the frame of the current balance upside down and stand it vertically. This provides a horizontal wire (of 10 turns) near the top of the frame.

2. Support the balance beam, with the longest loop attached, so that it can swing freely with the loop just above the top of the fixed coil. One way to do this is to support the knife edges on paper clips that are in turn held by alligator clips.

*For more details on the operation of the current balance, see *Equipment Notes*.

If you have one of the older versions of the current balance, you can hold the balance support bar in a test tube clamp.



3. Set the balance for minimum sensitivity by sliding the sensitivity clip down as far as it will go. (In this way, the interaction between the loop and the earth's field will not be observed.)

4. Connect the balance loop through the rheostat and ammeter to the power supply. Adjust the current to about 4 A.

Connect the fixed coil in a similar way, and set the current in it to 4 A or so. The values of the currents are not critical. Since students may not have had much experience connecting electric circuits, it is probably best to make these connections in front of the class, not ahead of time. This is the time to explain some of the terms, such as resistance and rheostat, that are used in the *Handbook*. A running commentary or a dialogue between teacher and class on what is being done, and why, will be informative.

The Demonstration

By moving the ring stand or the frame into different relative positions, you can show students the following relationships:

1. If the wires are perpendicular, there is no observable force between them.

2. If the wires are parallel and in the same horizontal plane, there is an observable force, the direction of which (attraction or repulsion) depends upon the relative directions of the currents (parallel or antiparallel).

The same current in the bundles of 10 parallel wires has a 10× greater effect than the same current in the single wire.

3. If the wires are parallel, but the balance loop is directly above the fixed loop, there is no observable thrust.

Be sure to discuss this until students realize that there may be a force, but that this balance does not respond to it. It is a vertical thrust (up or down depending upon whether currents are antiparallel or parallel).

D50 CURRENTS, MAGNETS, AND FORCES

(Do before *E4–6*)

This demonstration prepares students for *E4–6*, "Currents, Magnets, and Forces." A clear demonstration before the experiment will save time and point out some important observations that the students might otherwise overlook. The demonstration shows that there is a force between magnets and currents and establishes the relationship between the directions of current, magnetic field, and force.

Students may have discovered some of this for themselves in the early part of *E4–5*.

Equipment:

- Current balance, with longest loop in place
- Power supply
- Ammeter (0–5 A dc)
- Variac or rheostat, 5 Ω, 5 A
- 2 ceramic magnets on iron yoke
- 2 ceramic ring magnets, if possible (These can be any size, but should have a center of at least 1.25 cm diameter.)
- Wire leads
- Ringstand and test tube clamp

Procedure

Set up the current balance with the longest loop clipped to the balance beam. The current balance frame serves only as a convenient support for the balance beam in this demonstration. There will be no current in the fixed coils. Point this out at the beginning of the demonstration.

Connect the loop to the ammeter and power supply. Either have a rheostat in this circuit, or use a variac on the power supply.

Adjust the current to about 2–4 A. The exact value is not important.

A. CURRENT AND FIELD PERPENDICULAR

Place the magnet pair on the current balance shelf so that the balance loop passes through the field and the field is vertical.

Turn on the current and note the direction of the thrust.

The usual variations are possible. Show the effect of the following charges on the direction of the force:

1. Reverse the current by interchanging the leads to the balance.
2. Invert the magnet pair.
3. Reverse both the field and the current.

All the orientations studied so far will have caused a detectable force on the loop. Now turn the magnet pair through 90° (magnetic field now horizontal), again making sure that the loop passes through the center of the field.

There is no visible displacement of the loop. Is this because there is no force in this orientation; or is it because there is a force, but in such a direction that it doesn't cause the loop to move? Remind students that this balance doesn't respond to vertical forces.

At this stage, if you have not already done so, you must establish directions that describe the orientation of the current, magnets, and force. The directions of the current and of the force are easily defined. If the students have not already been introduced to the idea of magnetic field, this is the time to do so. The important point at this stage is to establish the direction of the field. Using a small compass or iron filings, show that the field between the faces of two ceramic magnets is perpendicular to the faces.

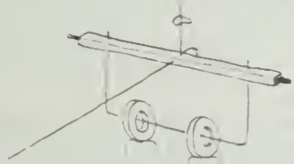
Now you can go back and discuss the relative orientation of current, magnetic field, and force in the different parts of the demonstration. Students should be able to see without too much difficulty that an inference of our demonstration, so far, is that when current and magnetic field are perpendicular to each other, the force on the current is perpendicular to both. If you want, go on to develop a rule for the relative directions of current, field, and thrust (for example, the left-hand rule, where second finger = current, first finger = field and thumb = thrust).

B. CURRENT AND FIELD PARALLEL

With ring magnets you can demonstrate that there is no force on a wire if the current is along the direction of the magnetic field.

Disconnect one end of the magnesium loop, and slide the ring magnets, properly oriented to give a field (unlike poles facing) over the loop. Reconnect the loop.

Use a little putty to support the magnets on the shelf several centimeters apart. The distance is not critical and may depend on the size and strength of the rings. Adjust the balance loop so that the wire hangs freely in the center of the rings.



As current is switched on and off, the only effect on the balance will be a small one, due to the earth's field (which can be avoided by adjusting the sensitivity to a minimum). If any deflection is observed, repeat the test without the magnets in position to show that the observed deflection was, in fact, due to the earth's field.

The inference of this demonstration is that, with current and field parallel, there is no force on the current. (You may again want to show the direction of the field between these two magnets, using a small compass.) If students object that there may still be a force (but a vertical one, and therefore undetected by our balance), invite them to find an orientation of current and field (while still keeping them parallel, of course) that does produce a horizontal force.

C. THE INTERACTION BETWEEN THE EARTH'S FIELD AND THE BALANCE LOOP

Procedure

Set the sensitivity to a maximum. This probably means using two sensitivity clips, one at the top of the vertical rod, one midway (see *Equipment Notes*). Adjust the current in the balance loop to 4 A or so; turn it off. Set the balance to rest, and set the zero mark level with the pointer arm; then turn on the current. The pointer deflects. Show that the deflection depends on the size of the current in the loop. Remind students that there is no current in the fixed coils.

Students already know that there is an interaction between magnets and currents, but there is no magnet near the current now, nor is there another current nearby. This deflection is caused by the earth's magnetic field.

D51 ELECTRIC FIELDS

Mapping of an electric field can be done as follows: Place a small glass tray on the stage of an overhead projector and fill it with slightly salted water. Place the projector microammeter next to it. Connect a 45-V battery to two electrodes in the water, and explore the electric field in the water with two wires connected to the microammeter. Equipotential lines are traced by seeking zero voltage positions. To do this hold one wire motionless in the water and with the second wire hunt for all the points that differ from the first point by zero voltage. Similarly, lines of force are traced by seeking maximum voltage positions. (The latter requires a constant distance between electrodes; a small loop of thread will do.)

This experiment can be done by individual students by putting a piece of white paper at the bottom of the tray and using pencils for electrodes.

Instead of the 45-V source, you can use the audio oscillator–amplifier with small loudspeakers for an audio output.

D52 DEMONSTRATIONS AND EXPERIMENTS WITH MICROWAVES

(For details on operation of the microwave apparatus see *Equipment Notes*. See *E4–9* for application of microwaves to communication.)

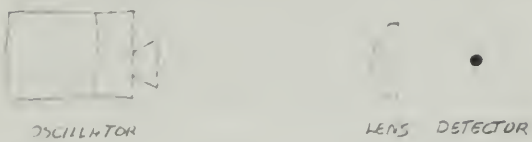
Because most classes will have only one set of microwave equipment, these experiments have not been described in the *Handbook*. Some students at least should be given a chance to use the equipment and do for themselves the experiments described here.

The microwave region is intermediate between radio and light in the electromagnetic spectrum. With a microwave oscillator and detector one can demonstrate the properties of electromagnetic radiation.

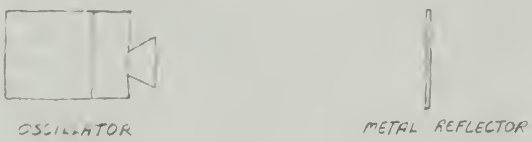
Reflection, Transmission, and Refraction

Material	Reflection	Transmission
Metal	good	none
Wire screen	good	poor
Dry paper	no	yes
Wet paper	some	no
Human hand	some	no
Glass	some	some
Paraffin	some	some

Refraction can be shown by the focusing effect of a semicylindrical lens formed by allowing paraffin wax to solidify in a small juice can.



Produce *standing waves* as shown below. Explore the region between source and reflector. The distance between neighboring nodes (minima) is half a wavelength. From λ and $c = 3 \times 10^8$ m/sec, calculate $f = \frac{c}{\lambda}$. (f is about 8×10^9 Hz.)



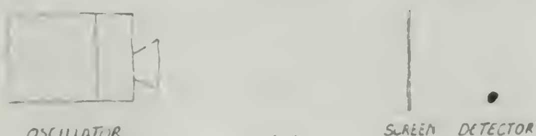
Diffraction

(a) A narrow aluminum screen is provided. Place it 12 cm in front of the source. Explore the field 5 cm behind the screen and at greater distances. Observe the maximum in the center of the "shadow."

(A)

(b) Mask one-half the source with the large screen placed about 12 cm in front of the source. Explore the intensity of the field as the detector is moved parallel to the screen and about 5 cm behind it. You might use the meter to record and plot intensity. You should be able to resolve at least two maxima. If the signal is weak, use the amplifier. (Connect diode to input, meter to output; see *Equipment Notes*).

Note that the intensity at the first maximum is greater than when there is no screen present.



(B)

(c) Use the two large screens to make a single slit about 10 cm in front of the source. The slit should be about 4 cm wide. Explore the region behind the slit.

Two-Source Interference

Place the two-source extension over the horn of the generator. Explore the field about 25 cm in front of the two sources, and plot the positions of maxima and minima. At least three maxima should be picked up on either side of the central one.

This method can be used to calculate wavelength. Use either the formula

$$\lambda = \frac{xd}{l}$$

or the fact that any point on the first nodal line is one-half wavelength farther from one source than the other:

$$S_1N_1 - S_2N_1 = \frac{\lambda}{2}$$

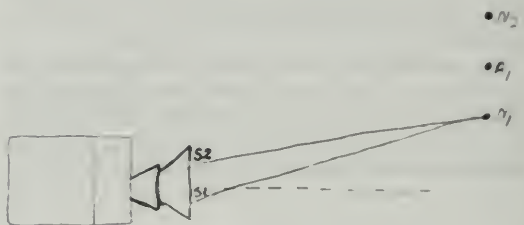
For the first noncentral antinode maximum we have

$$S_1A_1 - S_2A_1 = \frac{2\lambda}{2}$$

and for the next node

$$S_1N_2 - S_2N_2 = \frac{3\lambda}{2}$$

etc.



Lloyd's Mirror

In this variation of the two-source experiment, the source and its own image act as the two sources. It is much easier to demonstrate with microwaves than with light because the greater wavelength makes the adjustments less critical and the interference pattern larger.

Position the source and detector so that intensities of the direct and reflected radiation are about equal. Move the detector toward and away from the mirror, or keep the detector and source fixed and move the mirror.

The variation in total intensity at the detector is analogous to radio "fading." The metal reflector acts the part of the ionosphere (see *E4–9*).

Polarization

It is much easier to demonstrate (and explain) polarization with microwaves than with light.

The radiation from the microwave generator is plane-polarized, with the electric vector vertical. When the detector is held with the antenna vertical, the signal is maximum: The varying vertical electric field causes movement of electrons up and down the antenna. As the antenna is rotated, the effect decreases and disappears when the antenna is horizontal (perpendicular to the field).

For a polarizer use copper strips plated on a piece of board or an equivalent. With the antenna held vertically, place the polarizer between it and the source. Rotate the polarizer about a horizontal axis: The signal is maximum when the copper strips are horizontal and becomes zero when they are vertical.

Don't talk about picket fences, slots in pieces of card, or rope waves at this stage!

Why does the polarizer cut off radiation when the copper strips are parallel to the E field? In this orientation, the varying E field causes a displacement of the electrons in the copper, just as it does in the vertical antenna. It is this displacement, induced by the radiation itself, that causes reflection of the radiation. Because there are free electrons present in metals, they are better reflectors than nonmetals, at light frequencies as well as microwave frequencies.

You can demonstrate that the polarizer is in fact acting as a reflector by placing it at a slight angle and detecting the reflected radiation. When the copper strips are horizontal (perpendicular to the E field), electronic displacement is restricted and reflection is much less.

Not all polarizers work by reflection. An open hand, fingers vertical, acts as a good polarizer too. Here the radiation is partly absorbed, partly reflected. As the hand is rotated the transmitted signal increases and reaches a maximum when the fingers are horizontal.

Most polarizers of visible light ("Polaroid," tourmaline crystals, etc.) evidently work by absorption rather than reflection. These materials have a linear structure, similar in principle to our microwave polarizers, but on a much smaller scale.

For *modulation* of microwaves and application to communication, see *E4–9*.

Experiment Notes

E4–1 REFRACTION OF A LIGHT BEAM

Equipment:

- Millikan apparatus light source
- Telescope tube
- Plastic semicircular dishes
- Polar coordinate graph paper

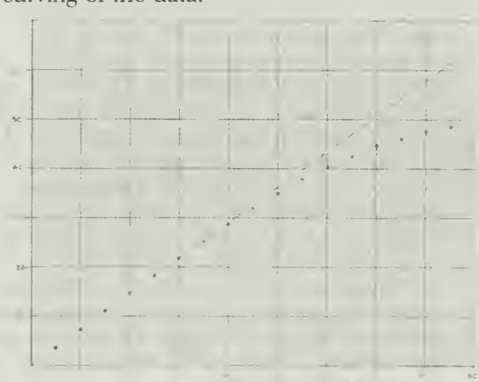
Introduction

This experiment helps the student to understand refraction in more depth. Section 13.3 of the *Text* relates the refraction of light to wave theory and discusses the two conflicting models of light. The laboratory allows the student to see the bending of light in water and to observe the relationship $\frac{\sin \theta_i}{\sin \theta_r} = n$. The strategy is that the details of refraction can be better understood through experiment than by reading the *Text*. However, the presentation of this topic does permit a teacher to skip the experiment and use only the *Text*.

Determinations of the speed of light in water and in air are the crucial experiments that decide between the wave and particle models. The PSSC movie, "The Speed of Light," is a modern version of this experiment. Question 7 of this experiment could lead the students toward an appreciation of this movie.

Answers to questions

1. Light beams entering, passing through, and emerging from a transparent medium follow the same path in either direction.
2. When the beam strikes the curved surface along a radius (that is, when the angle of incidence, θ_i, is 0°), the beam is reflected directly back along the incident beam (angle of reflection is 0°) and hence the incident and reflected beams are indistinguishable. The refracted beam is in the same direction as the incident beam, and hence the angle of refraction, θ_r, is 0°.
3. As the angle of incidence increases, the angle of refraction increases also. (See the table on page 304.)
4. For angles of incidence up to 50°, the two angles seem to be directly proportional; the relationship is linear. However, for angles of incidence greater than 50°, the relationship appears to be more nonlinear, as is indicated by the curving of the data.



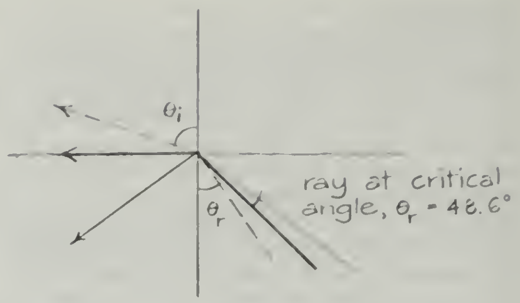
5. Both the wave and particle models predict that the ratio of the sines of the two angles are constant.

6. $\frac{\sin \theta_i}{\sin \theta_r}$ = constant. Students may be interested in knowing that for water the constant has a value of 1.33, and for various kinds of glass the index of refraction ranges from about 1.5 to 1.9.

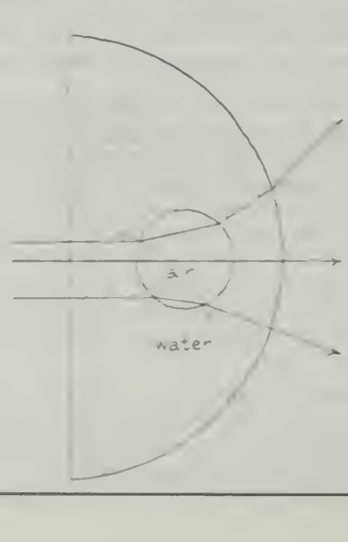
Refraction Angle and Change in Speed Data

θ_i (degrees)	θ_r (degrees)	$\sin \theta_i$	$\sin \theta_r$	$\frac{\sin \theta_i}{\sin \theta_r}$
5	3.7	0.087	0.065	1.33
10	7.5	0.174	0.131	1.33
15	11.2	0.259	0.194	1.33
20	14.9	0.342	0.257	1.33
25	18.6	0.423	0.318	1.33
30	22.1	0.500	0.376	1.33
35	25.5	0.574	0.431	1.33
40	28.9	0.643	0.483	1.33
45	32.1	0.707	0.531	1.33
50	35.2	0.766	0.576	1.33
55	38.0	0.819	0.616	1.33
60	40.6	0.866	0.651	1.33
65	42.9	0.906	0.681	1.33
70	45.0	0.940	0.707	1.33
75	46.6	0.966	0.727	1.33
80	47.7	0.985	0.740	1.33
85	48.5	0.996	0.749	1.33

7. In water the speed of light is $\frac{3.00 \times 10^8}{1.33}$ = 2.33×10^8 m/sec.
8. In the visible portion of the spectrum, violet light is refracted most.
9. This is difficult to do with simple equipment. For red light (of wavelength 6.563×10^{-7} m), the index of refraction is 1.331; for blue light (4.861×10^{-7} m), it is 1.337. These values can best be measured for light emerging from the straight side of the dish at nearly 90° parallel to the straight edge), a condition discussed in the next problem. The velocity of red light (6.563×10^{-7} m) in water is 2.26×10^8 m/sec. For yellow (5.890×10^{-7} m) it is 2.25×10^8 m/sec, and for blue (4.861×10^{-7} m) it is 2.24×10^8 m/sec.



10. Remember that the angle θ_r is the angle inside the water regardless of the direction in which the light beam is traveling. The index of refraction
$n = \sin \theta_i / \sin \theta_r$ or $\sin \theta_r = \sin \theta_i / n$.
When $\theta_i = 90°$, $\sin \theta_i = 1.0$, and $\sin \theta_r = 1/n$.
For water, the angle θ_r required to make $\theta_i = 90°$ is therefore the angle whose sine is $1/n$ or 0.75. This angle is 48.6°. For all angles θ_r greater than this, the beam will be totally reflected inside the water. For water this angle of 48.6° is called the *critical angle*.
11. To predict the path of a light ray through the bubble, draw a perpendicular to the surface of the bubble where the ray hits it (see sketch below). Identify the angle between this perpendicular and the ray. Then draw the refracted ray in such a way that its angle with the perpendicular is greater (since the ray is emerging into air). (The precision made possible by the use of the formula in 10 above is not appropriate to drawing a rough sketch.) At the next intersection with the wall of the bubble draw a perpendicular again, but this time the ray going into the water makes the smaller angle with the perpendicular. Continue similarly for all intersections, noting that a ray that is perpendicular to the air–water surface is not bent at all.



The construction should show that the effect of a bubble of air is to diverge a bundle of rays falling on it, as in the figure on p. 304. On the other hand, if the air bubble is replaced by glass, whose index of refraction is greater than that of water, the parallel rays will be converged.

E4–2 YOUNG'S EXPERIMENT: THE WAVELENGTH OF LIGHT

Equipment:

For each group of at least six students:

- Straight filament light source
- double slit on film
- Pair of telescoping tubes, wooden block, rubber bands
- Black tape
- Transparent tape
- 10× eyepiece and scale
- Meter stick or millimeter ruler
- Red, green, blue filters

Introduction

This is a most important experiment because it supports the wave model of light and serves as an example of how to test wave properties in general as done in the Unit 3 experiments. If students are able to calculate the wavelength of light, they should be convinced that light is a wave rather than a particle. Beware that Unit 5 will stir up the wave–particle controversy when the electron is tested for wave properties.

This experiment can be used before or after *Text* Sec. 13.4, "Interference and Diffraction." If the experiment is completed first, then the students will be better able to appreciate the *Text* discussion. However, the reverse strategy should be just as effective.

The following is a list of possible results from this experiment, from the simplest to the most difficult insights.

1. Dark interference fringes are observed. Light must be a wave.
2. The fringes of blue light are closer together than the fringes of red light. Color is related to wavelength.
3. Since the slit is so small, the wavelength of light is small. The wavelength of light is equal to or less than the distance between the two slits.
4. The wavelength of light is about 10^{-7} m.
5. The wavelength of red light is greater than the wavelength of blue light.
6. The wavelength of red light is about 6.5×10^{-7} m and the wavelength of blue light is about 4.5×10^{-7} m. If the wavelengths of light have been measured, light most certainly must be a wave.
7. Where does $\lambda = \frac{x}{L} d$ come from? (Refer to suggested solution to SG 12.12 in the Unit 3 *Resource Book.*) A laboratory experiment must be something more, in this case, than the verification of the wavelength of a color. A feeling for how small the wavelength is and how the experiment piles up evidence in favor of the wave model of light is of paramount importance. Such a feeling and evidence is a part of the beauty and fun of physics.

Note:

With the geometry used, the symbols l and L, used to express the $\lambda = \frac{xd}{L}$ relationship are virtually the same, and can be used interchangeably. In the *Text*, x is taken as the distance from the central maximum to the first lateral maximum; in the *Handbook*, the meaning of x is extended to be the distance between any two neighboring maxima or minima. It is not advisable to measure a single interval x; precision will be increased by measuring three or four such intervals and deriving their mean value.

Answers to questions

1. As the screen moves farther from the slits, the pattern grows steadily larger much as the picture does on a movie screen as it is moved farther from the projector.
2. Blue light forms a pattern whose lines are closer together than those in the red or yellow patterns. This is a consequence of the fact that waves of blue light are shorter than those of red or yellow.
3. The wavelengths of visible light are too small for nodes and antinodes to be seen.
4. None, so long as the light is traveling through a material in which its speed is constant. This means that it is not passing from a material of one index of refraction into a material of a different index, nor is it passing close to opaque objects around whose edges it will be diffracted. These conditions apply also to sound waves and to water waves.
5. Perhaps the first example that comes to mind is the interference colors produced by a thin film (for instance, by reflection from an oil layer on a puddle), but students probably won't realize this is an interference effect until you tell them. Other examples of this are given in "Suggestions for Some More Experiments."

E4–3 ELECTRIC FORCES. I

Equipment:

- Scotch tape ("magic transparent" type) preferably in a dispenser
- Table stand and horizontal rod
- Variety of plastic strips, rubber rods, cloth, cat's fur, etc.

Introduction

Several important points that are not brought out in the experiment should be mentioned in discussion and (where possible) demonstrated by the instructor.

1. By "generation" of electric charge we mean *separation* of charge. The two rubbed objects have equal and opposite amounts of charge. No new charge is created. This is demonstrated convincingly in *D47*. Indeed, electric charge should be added to the list of conserved quantities that have already been discussed: mass, momentum, and energy.
2. Rubbing or "peeling off" is not the only way to separate electric charge. Charge can also be produced by heating and by shining light on a surface (*D47*). Chemical reactions (separation of charged ions by electrolysis) and radioactive reactions (emission of charged particles) should also be mentioned.

Experimental Technique

On humid days the charged tapes may have to be replaced fairly often, since they will tend to discharge through the air.

The surface of the table should be clean, dry, and nongreasy; it should also be smooth enough that the tape can be peeled off easily. If there are no such tables around, a glass plate or any hard, plastic-covered surface will do equally well.

One should always hold two tapes up with nonsticky sides facing each other, since they might otherwise stick together and alter each other's charge.

Finally, it may often be helpful to fold over about ¼" of tape on one end before pasting it to the table. The nonsticky surface will make a convenient tab with which to pull up the tape.

Answers to questions

1. They attract.
2. They attract.
3. When each of the two tapes is brought near the original test strip, the test strip will attract one and repel the other of the separated pair of strips. (The test strip may need replacement by now, especially on a damp day.)
4. The force between a charged and an uncharged body is always attractive. This occurs because a polarized charge is created by induction in the uncharged body.
5. The reasons that an attractive force always arises between a charged and an uncharged body are explained more fully in the *Text*. For a more difficult puzzle that also depends on induction, demonstrate the electrophorus to students (*see D48*).

E4–4 ELECTRIC FORCES. II: COULOMB'S LAW

Equipment:

For each student team:

- Soda straw
- 2 Coated polyfoam or pith spheres
- 2 Plastic slivers
- Balance support
- 2 Straight pins
- Wire (about 15 cm #30 copper)
- Forceps
- File card or pad, approximately 10 × 15 cm
- Ringstand with test tube clamp
- Plastic strip and cloth for charging spheres
- Ruler

For general use:

- Wire cutters (scissors will do)
- Knife or razor for cutting notches
- Glycerine, mineral oil, or vacuum pump oil, about 10 cc per unit
- Some materials for measuring distances
- Tape—double stick if possible

The "Electrostatics Kits" produced for the PSSC course (for instance, Macalaster Scientific #1001) are a convenient source of many of these items.

Will the experiment work?*

As in all electrostatics, the success of this experiment depends heavily on local conditions. Two factors should be considered:

1. The balance, of necessity, is a sensitive instrument. Students need more manual dexterity for this experiment than for most of the others in the course. Students must work as a smooth team, both to reduce the time required for the experiment *and* to minimize air currents in the laboratory. A simple damping device, a pin moving in an oil bath, is suggested to reduce the tendency of the balance to oscillate excessively.
2. The second condition that determines the probability of success is the rate at which charge leaks from the balls. Under extreme, usually unpredictable, conditions of dampness it will be impossible to carry out this experiment.

Local drying of the atmosphere may be possible in your laboratory.

(a) Leave the steam heat on over the previous night. Success of the experiment may be worth the discomfort of a hot laboratory.

(b) A large aquarium tank from your school's biology department may allow you to perform the experiment as a demonstration-experiment inside a partially enclosed space. Dry this space with heat lamps, but start them well in advance of the period.

Alternatives

There are several methods available for measuring the small forces involved in this experiment. The balance used here does not require indirect geometrical calculations. As the students do the experiment, they have a real and obvious sense that they are measuring forces.

Alternative experiments on the force between charged bodies could be substituted, of course. (For example, PSSC experiment IV-3, "The Force Between Two Charged Spheres"; the equipment is Coulomb's Law Apparatus No. 1002 from Macalas-

*You might have the PSSC film on Coulomb's Law (#0403 available from MLA) on hand, just in case. Running time: 30 min.

ter Scientific Corp.) That experiment, however, involves resolution of forces, a topic that our students have not encountered. The experiment described here involves a simple balancing of forces, a technique that students will use again in the current balance and the electrolysis experiments.

Procedures

You are urged to preassemble the equipment, if at all possible, to save classroom time. Even if students are to assemble the balances, parts should be prepared (plastic slivers cut, balls coated) before the lab period begins. Besides saving time, it may help to reduce the handling of these small bits of equipment. Caution students to touch slivers only by the ends, not by the center portion or the edges, and not to charge the slivers by rubbing them, etc.

Better success has been reported with the use of the milky white polyethylene than with the clear plastic cut from electrostatic charging strips. Some teachers have successfully used plastic "drink sticks," toothpicks, etc.

Slivers should be about 5 cm long, pointed at one end. They must be clean and dry. The other end should fit snugly into a soda straw.

Although polyfoam balls have been used and are usually supplied by manufacturers, some teachers report greater success with pith balls.

It is important that the balls have a heavy coating of graphite in alcohol.

The *Handbook* does not give directions for cutting the slivers or for painting the balls. If you want the students to perform these steps, you will have to give them directions, verbal or dittoed.

If the balls and slivers have been prepared ahead of time, they can be dried under a heat lamp or radiant heater. Do not overheat!

Almost any small plastic container will serve as the base for the balance. It should be about 2–2.5 cm across. Containers for Polaroid Print Coater work well.

The notches must be smooth if the balance is to swing freely.

Stray induced charges in the tabletop can ruin this experiment when the balance is too close to the table. The balance must be far enough above the table so that charging it will not result in a change in the horizontal balance position described in the *Handbook*. How far this must be depends upon the local conditions, but 10 cm should be adequate. A book (or two), a box, or a plastic cup can be used as a base support.

For damping, any slightly viscous liquid will do, such as glycerine, mineral oil, vacuum pump oil, etc. To prevent tipping, the container must be firmly taped to its base support.

A paddle (small piece of cork) on the vertical pin will further increase the damping. Lubricating the bearing with oil may help the balance to swing freely.

A larger charge will generally be obtained when charging is by induction rather than contact. Besides, "scraping" charge onto (or off) the balance ball can disrupt the balance. But students may not have had much information about "charging by induction." So it is not suggested in the *Handbook*.

Measuring distances between the centers of the spheres

The induced charges in a ruler held too close to the charged spheres will significantly influence the experimental results. Students are warned about this in the *Handbook*, but no solution to the problem of good measurement is given. Many methods are possible, and it is hoped that students will come up with good ideas of their own. Here are two techniques that could be suggested.

1. One method is to line up your eye with a sphere and a ruler some 5 cm behind the sphere. Students may not be familiar with parallax and the ways to reduce it. A mirror, handbag-size or a little larger, will do. Stick centimeter tape along the longer edge and stand the mirror in a vertical position about 5 cm behind the balance. When the charged sphere, or the sliver, lines up with its image in the mirror, parallax has been eliminated. This technique is often used with precision meters.
2. Centimeter tape can be stuck to the ringstand post. If the students record the position of the clamp at each step of the experiment, they can reset these positions when the experimental procedure is over and get the distance between centers of the spheres by direct measurement. At this stage it will not matter if the ruler is brought right up to the spheres.

Urge students to work smoothly but quickly. For example, let one record data while the other observes.

If students work from small to large forces (bringing charges closer as the experiment proceeds), they will be following the *Handbook*. If there is appreciable charge leakage during the minute or so required for the experiment, the results of a plot may be ambiguous. Usually, under these circumstances, the student's plot of F against $\frac{1}{d}$ looks almost as linear as one of F against $\frac{1}{d^2}$.

However, if students proceed from large to small forces (moving charges from close up to farther apart), then the ambiguity is likely to be between F versus $\frac{1}{d^2}$ and F versus $\frac{1}{d^3}$! You might ask half the class to reverse the order given in the *Handbook*, but be sure to allow time for discussion so that students can compare results from both methods.

It is important to have the students perform the final part of the force versus distance procedure as described in the *Handbook*. If they have taken measurements while bringing the charges closer together, they must finish by removing one or two hooks and checking the separation required to reestablish balance. This measurement will probably not agree with their earlier data for this number of weights. Of course, if the students have reversed the order and have taken measurements while moving the charges farther apart, they will perform the "check" by adding a hook or two at a time and then comparing the balance position with a previous reading.

Only by actually performing this check will students observe directly just how serious the charge leakage has been.

The *Handbook* does not suggest ways to decide what mathematical relationship best fits the experimental result. Suggestions for plotting were given in the notes on "Behavior of Gases" (*E3–14*), but you may have to suggest that they plot F against $\frac{1}{d}$, F against $\frac{1}{d^2}$, etc.

With regard to question 8, each ball now has one-half the original charge.

Students should find that removing three hooks (one left on) restores the balance better than removing two:

$$F \propto qq \propto q^2$$
$$q' = \tfrac{1}{2}q$$
$$F' \propto q'q' \propto (\tfrac{1}{2}q)(\tfrac{1}{2}q) \propto \tfrac{1}{4}q^2 \propto \tfrac{1}{4}F$$

Students may be able to conclude that $F \propto q_1q_2$, though more experiments are needed to be sure of this.

Answers to questions

1. The balance ball moves downward.
2. When the upper ball is touched by a finger it loses its charge, and the balance ball promptly returns to very nearly its original position.
3. No. Even the rather weak electrical force between the balls is many millions of times stronger than the gravitational force. Although it is hardly a fair comparison, it may be interesting to know that a single proton attracts a single electron with an electrical force 10^{39} times as great as the gravitational force.
4. The charges have been slowly leaking off the balls during the intervening time.
5. There is no way of knowing how much charge is required and no way of providing it accurately even if you did know.
6. If $F \propto \frac{1}{d}$, then all the products, Fd, would have the same value; that is, Fd is constant. Similarly, if $F \propto \frac{1}{d^2}$, then Fd^2 is constant, and so on. To find the best formula of this form, make a series of columns for the values of Fd, Fd^2, Fd^3, etc., and identify the column in which the values are most nearly constant. One (or even two) wildly divergent values are probably the result of experimental error or charge leakage.
7. $F \propto \frac{1}{d^2}$ or $F = \text{constant} \times \frac{1}{d^2}$
8. One-half.
9. $F \propto \frac{q_1q_2}{d_2}$ or $F = \frac{kq_1q_2}{d_2}$
10. (a) The charges on the spheres will attract opposite charges in the ruler, and the resulting unwanted force on the movable ball will contribute to its deflection.

    (b) The value of *d* in the formula is the distance between the charges or, if they are extended over a sphere, it is the distance between the centers of the spheres. In this experiment, *d* is therefore the distance between the centers of the balls over whose surface the charges are uniformly spread. At distances much less than 1 cm, the attraction (or repulsion) of the charges is so strong that charge is displaced toward one side of each ball and is no longer evenly distributed. The distance *d* between the charges is no longer the distance between the centers of the balls.
11. Suspend both balls so that they are free to move.

E4–5 FORCES ON CURRENTS

Equipment:

A. *Force as a function of current*

Current balance, with the longest magnesium loop
Power supply, 6–8 V dc
Wire (about 50 cm #30 copper)
Ammeters, 0–5 A dc (2)
Rheostats, about 5 Ω, 5 A (2)
Ringstand and test tube clamp to hold zero indicator
Hook-up wire or clip leads
Forceps
Ceramic magnets

B. *Force as a function of distance*

Same as Group A, plus:
Small mirror with pressure-sensitive centimeter tape stuck along one edge. This "antiparallax" device is used for measurement of *d*, the distance between wires.

C. *Force as a function of length*

Same as Group B, plus:
Magnesium balance loops (4)

Introduction

Since the *Handbook* tells the students to turn on I_b (current in the balance loop) before they connect the current in the fixed coil (I_f), they may notice that the pointer undergoes a slight deflection that is not explained in the notes. This deflection is due to the interaction of the current in the balance loop with the earth's magnetic field. It may be best to wait until *E4–6*, part (c) before attempting an explanation: meanwhile the students have gained some familiarity with the forces between magnets and currents.

The actual magnitude of the force exerted by the earth's magnetic field depends upon the magnitude of I_b. In the prototype balance, when $I_b = 2.0$ A and $I_f = 0$ A, the force was found to be balanced by the weight of 1 cm of #30 wire on the notch.

The "balance position" is marked with the zero-mark indicator when I_b is on but I_f is not. In this way, the gravitational forces on the balance itself compensate for the earth's magnetic effect. Of course, the zero-mark must be adjusted if I_b is changed. This is one good reason for using a null method of balancing.

The data obtainable with this apparatus are actually quite good. Sample data follow the procedures.

Procedures

A. Group A is expected to report that $F \propto I_f$. Lead the class to accept the argument, from symmetry, that $F \propto I_b$. After all, the fixed loop could have been made to move, and the balance loop could have been held fixed; the experiment would have been the same, only the equipment would have been modified.

Thus, Group A reports $F \propto I_f$ and $F \propto I_b$.

Students may get confused and hold I_f constant, while varying I_b! Since the force on the balance due to the earth's field increases with I_b, the effect is not a constant. A plot of F versus I_b will not be linear, although it will pass through the origin.

B. Group B reports $F \propto \frac{1}{d}$.

The fixed loop consists of a bundle of wires; the location of the "center" is assumed to be the geometric center.

Students are directed to obtain data for values of d from about 0.5 cm to 5.5 or 6.0 cm. Keep in mind that, for $d = 0.5$ cm, an uncertainty in measurement of only ± 0.1 cm represents $\pm 20\%$.

Since the balance is moved between readings, students must recheck the position of the zero mark every time. These readjustments are critical.

Students should plot F against d. The F versus d curve should suggest that the form of the relationship may be $F \propto \frac{1}{d}$. If students next plot F against $\frac{1}{d}$, they see a straight line. For small values of d (large $\frac{1}{d}$) where the uncertainty is large, however, there may be noticeable deviation from linearity.

C. Group C reports $F \propto l$.

In this part of the experiment, the actual distance between the loops and wires is not critical. (It must, of course, be the same for all measurements.) The distance is small enough, however, so that there will be a reasonable force on the *shortest* loop for the currents and distance used. In a trial with the prototype balance, currents of about 4 A were used. The shortest loop, at a distance of 1.5 cm, gave a force equivalent to only 3.5 cm of #30 wire. Since the balance is responsive only to about ± 0.5 cm of #30 wire, the uncertainty in this 3.5-cm value was $\pm 14\%$.

The results of the three groups combine to give:

$$F \propto \frac{I_f I_b l}{d} \text{ or } F = \frac{k' I_1 I_2 l}{d}$$

A possible extension of this series of experiments is the determination of the constant k' (*Q14*). The force F must be converted from centimeters of wire into weight of wire, and the weight of wire must be expressed in newtons. The balance is constructed so that the horizontal part of the loop and the notch on which weights are hung are equidistant from the pivot; that is, the two torques are equal. Therefore, the forces are also equal. (The value of k' is defined as 2×10^{-7} N·m/A^2; it serves to *define* the ampere.)

This constant is related to the constant in Coulomb's law, $F = k\frac{q_1 q_2}{d_2}$, and the velocity of light, c:

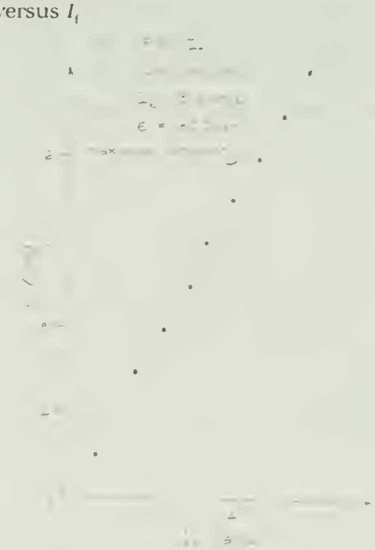
$$c^2 = \frac{2k}{k'}$$

The value of the Coulomb's law constant $k = 9 \times 10^9$ N·m^2/C^2 was given in Sec. 14.3.

Students will probably be intrigued to find this familiar number ($c = 3.0 \times 10^8$ m/sec) cropping up in an apparently unrelated field. If you do bring out this point, we suggest that rather than try to explain it you let it stand as the first clue that there is a connection between electricity, magnetism, and light. This connection is brought out in the discussion of Maxwell's work in Chapter 16.

Sample Data Obtained Using Prototype Current Balance

A. F versus I_f



	F	I_f
d = 1.75–0.25	2 cm #30 wire	0.6 amps
= 1.50 cm	6	1.7
I_b = 3.0 A	8	2.3
maximum	10	2.9
sensitivity	12	3.3
	14	3.9
	16	4.5
	18	5.1
	20	5.6?

B. *F* versus *d*

Use mirror and plastic centimeter tape, for parallax-free scale.

Wire radius = 0.1 cm

Fixed loop diameter = 0.3 cm, so r = 0.15 cm

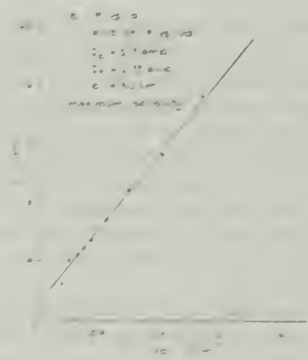
Measure position of inner edge of balance loop and add radius

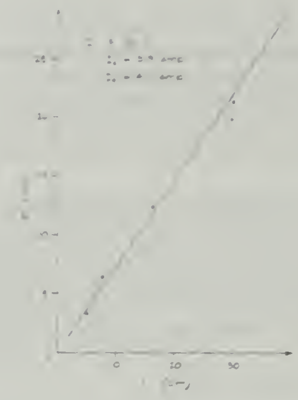
distance = $d + 0.1 - 0.15 = (d - 0.05)$ cm

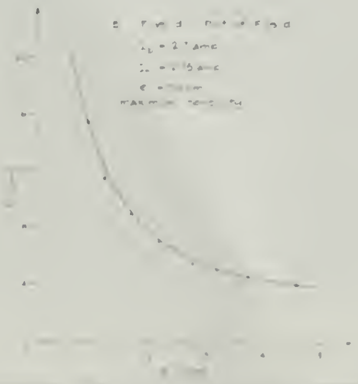
Add F = 20 cm, set d = 0.65, adjusted currents: Set at maximum sensitivity I_f = 2.75 A, I_b = 2.7 A.

distance = d − 0.05	$\frac{1}{d}$	F	$\frac{1}{F}$
0.6 cm	1.7	20	0.05
0.9	1.1	15.5	0.065
1.2	0.83	11.5	0.087
1.65	0.60	9	0.11
2.15	0.46	7	0.14
2.75	0.36	5.5	0.18
3.25	0.31	5	0.20
3.8	0.26	4.5	9.225
4.7	0.21	4	0.25
6.25	0.16	2.5	0.40

Note that in this experiment the zero-mark was set with no current in the balance loop. This accounts for the finite intercept on the *F* axis, which represents the force on the balance loop current due to the earth's magnetic field. If the zero-mark had been set with the current to be used in the experiment passing through the balance loop, then the *F* versus $\frac{1}{d}$ plot would pass through the origin.







C. *F* versus *l*

Set at intermediate distance 1.55 cm between inner edges.

Added F = 20 cm l = 29.7 cm

Adjusted currents: I_f = 4.1 A and I_b = 3.9 A

l = 16.1 cm

Adjust (a) loop parallel to fixed wire and horizontal

(b) sensitivity

(c) balance point

Adjust current

l = 7.8 cm F = 6.5 cm

l = 4.8 cm F = 3.5 cm

Plot: good except for first, long loop.

1. Reset; found loop not quite level with fixed loop. It was about 2 mm high, ∴ low torque ∴ low force.
2. Made new loop! Was now horizontal. Got F = 21.5 cm for l = 30.0 cm. A little better: within reasonable limits

Answers to questions

1. The magnet must be as close as possible to the movable rod with the axis of the magnetic field vertical. The direction in which the rod swings depends upon the relationship between the direction of the magnetic field and the direction of the current through the balance loop

2. Currents flowing in the same direction attract each other; those in the opposite direction repel.
3. $F \propto I_f$
4. $F \propto I_b$
5. $F \propto I_f I_b$
6. Begin by finding the actual weight in grams of the wire hung on the notch. Then multiply the weight in grams by 0.0098 N/g or $F = K\ I_f I_b$, where K is a constant of proportionality, to find the equivalent force in newtons. (1 N = 102 g weight.) This force on the notch is the same as the horizontal force on the conducting wires, since the notch is the same distance from the fulcrum as the horizontal part of the loop. (For most students, it is a waste of time to actually make such calculations.)
7. $F \propto \frac{1}{d}$
8. By Newton's third law it must also be F.
9. See 6
10. $F \propto l$
11. See 6
12. By Newton's third law it must also be F.
13. Since the effects were each measured and confirmed independently, it is reasonable to assume that the independence is real. To answer critics who argue that $F \propto I_f I_b$ only for the particular accidental choices of d and l, suggest that they repeat the confirmation of $F \propto I_f I_b$ using different values of d and l. In the end, the assumption is only as good as the experimental evidence.
14. One can write $F = \frac{K' I_1 I_2 l}{d}$. To predict F quantitatively in newtons from measured experimental values, one must also know that $k' = 2 \times 10^{-7}$ N·m/A^2.

E4–6 CURRENTS, MAGNETS, AND FORCES

Equipment:

A. *Force as a function of current*

Current balance, with longest loop
Power supply, 6–8 V dc
Variac or rheostat, 5 Ω, 5 A
Ammeter 0–5 A dc
Ceramic magnets, iron yoke (2)
Wire leads
Wire (50 cm #30 copper)
Forceps

B. *Force variation with length of magnetic field*

Same as Group A plus:
2 additional iron yokes, enough ceramic magnets so that a total of 3 matched pairs can be found. If *more* than 3 pairs are possible, so much the better.

C. *Strength of earth's magnetic field*

Same as Group A but no magnets! It is possible that students will prefer a thinner wire than the #30 copper. They can then use the longer length. For example, 5 cm of #37 copper weighs about the same as 1 cm of #30

Introduction

If students seem initially to be in trouble, a likely cause is that they have set their magnets on the yoke in such a way that like, instead of unlike, poles are facing.

In parts (a) and (b), students are not told to compensate for the earth's interaction with current. This is because the vertical component of the earth's field is very small compared with the field between a pair of the ceramic magnets supplied (about 0.8×10^{-4} W/m^2, as compared to about 300×10^{-4} W/m^2). Students who raise the question can very quickly check this for themselves.

The forces here are much greater than in *E4–5*, "Currents and Forces." Students who finish parts (a) and/or (b) quickly could be encouraged to go on to part (c): quantitative measurement of the vertical component of the earth's field.

Part (c) is probably the most demanding of the current balance experiments and should be assigned to the more able students. The forces to be measured are small, and a calculation is required.

Students measure (in units of weight, or number of centimeters of wire) the force needed to restore balance with a current in the balance loop alone. No current is in the fixed wires, and no other magnets are nearby. They should work at maximum sensitivity. Let them repeat the measurements for several values of I_b.

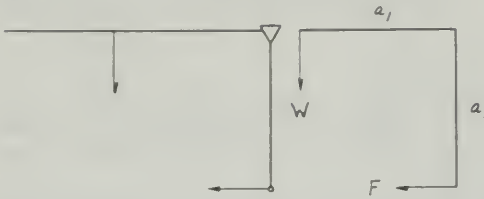
Someone may ask whether the orientation of the loop with respect to the earth's field is important. Students should be able to answer this from the previous demonstration. If there is time, let them try to detect an effect. The point is that the interaction between the vertical component of the earth's field and the horizontal current will always give the same horizontal force, perpendicular to the current. The *horizontal* component of the earth's field will interact with the horizontal current (unless the two are parallel) to produce a *vertical* force. But the current balance doesn't respond to vertical forces. See also "Orientation of Current Balance Unimportant" in the *Equipment Notes* in this *Resource Book*.

Students must convert the force from centimeters of wire into newtons, either by weighing the wire, or by looking up the mass per unit length of the wire used (#30 copper: 0.45 g/m; 1 g = 10^{-3} kg = $10^{-3} \times 9.8$ N). Although students in the *Project Physics* course have not studied levers or torques, their experience with seesaws or their work in elementary science courses should enable them to see that the system in the figure will balance when

$$Fa_2 = Wa_1$$

$$F = W\frac{a_1}{a_2}$$

(From measurement they will probably find that $a_1 = a_2$.)



This is the force on the current. Students are asked to find the force on a wire 1 m long when the current is 1 A. This is the magnetic field (B) in MKSA units:

$$B = \frac{F}{Il}$$

$$B = \frac{W\frac{a_1}{a_2}}{Il}$$

Conclusions

Students doing part (a) are expected to report that $F \propto I$. Students doing part (b) should report that $F \propto l$. These two statements can be combined to give $F \propto Il$. What other factors affect the force? Both groups will realize that if they had used "stronger" magnets they would have found greater forces, for the same values of I and l. So in this case we are not looking for a proportionality constant but for a factor that describes or measures the strength of the magnetic field.

We define the magnetic field strength B by $F = BIl$. B is therefore equal to $\frac{F}{Il}$; it is the force, in newtons, on a conductor 1 m long, carrying a current of 1 A.*

In part (c) students measured the force on the loop due to the vertical component of the earth's magnetic field. To calculate B_{vert} in standard units they must use $B = \frac{F}{Il}$. (In Massachusetts, B_{vert} is about 0.7×10^{-4} N/A·m.)

By comparing the two equations that were obtained in the two current balance experiments,

$$F = k'\frac{I_1 I_2 l}{d} \text{ and } F = BIl$$

We can see that the magnetic field at a distance d from a straight wire carrying a current I is $\frac{k'I}{d}$.

Sample results

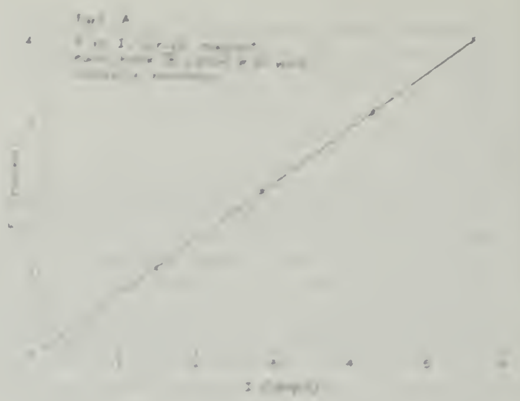
Part (a) F versus I

single magnet: each hook is 2.5 cm #20 wire

sensitivity: minimum

Conclusion: $F \propto I$

*Note that 1 N/A·m = 1 W/m² = 10^4 G



Part (b). Made three magnets: each is a pair of ceramic magnets on an iron yoke.

Used 2.5-cm long hooks of #20 copper wire as weights.

3 A in balance loop; no current in fixed coil.

How does F depend on l?

I. compare magnets, one at a time:
- #1, 2 × 2.5 cm #20 wire to balance
- #2, 2 × 2.5 cm
- #3 2 × 2.5 cm

II. two magnets:
- #1 and #2, 4 × 2.5 cm #20 wire to balance
- #1 and #3, 4 × 2.5 cm
- #2 and #3, 4 × 2.5 cm

III. all three magnets together:
- #1, #2 and #3, 6 × 2.5 cm #20 wire to balance

Conclusion: F varies linearly with the number of magnets interacting with current. Since magnets are far from each other and all have the same strength, $F \propto l$.

Part (c). Effect of earth's field: current in balance loop only.

$I_b = 4.0$ A

$l = 30$ cm (longest loop)

$F = 2$ cm of #30 wire

Wire has mass of 0.45 g/m.

$\therefore$ 2 cm weight $0.45 \times 0.02 = 0.009$ g

$W = mg = 0.009 \times 10^{-3} \times 9.8$ N
$= 8.8 \times 10^{-5}$ N

Lever arm lengths are equal.

$\therefore F = 8.8 \times 10^{-5}$ N

$F = BIl$

$\therefore B = \frac{8.8 \times 10^{-5}}{4 \times 0.3}$ mks units
$= 0.73 \times 10^{-4}$ W/m²
$(= 0.73$ G$)$

Answers to questions

1. $F \propto I_b$ or $F = kI_b$, where k is a constant of proportionality.
2. By Newton's third law it should. To observe that the force on the magnet was equal and opposite in direction one would have to support the magnet so that it could swing freely while the bal-

ance loop is held rigid then calibrate the displacement of the magnet against the force on it.
3. A weaker magnet would exert a weaker force and a stronger magnet a stronger force. Thus, in the equation $F = kl$, the constant k would be smaller or larger.
4. $F \propto l$.
5. To find the weight of the wire in grams multiply the length of the wire by the grams per unit length. Then convert the weight in grams to newtons by multiplying grams by 0.0098 N/g.
6. Student answer.
7. The force exerted by the earth's field will be about 0.8×10^{-4} N/A·m. That of the magnet with a pair of magnet pole pieces is about 300×10^{-4} N/A·m.
8. Redesign the balance arm so that the balance wire is free to move vertically instead of horizontally.

E4–7 ELECTRON BEAM TUBE. I

Equipment.

For assembly:

Electron beam tube kit
Silicone rubber sealant
Wire cutters

At operating station:

Vacuum pump
Power supplies:
(a) about 6 V, 5 A with 5 Ω, 5 A rheostat and ammeter (for filament)
(b) about 100 V dc for anode, with higher and/or lower voltage taps for deflection plate
Hook-up wire
Magnets and yoke (from current balance)

Introduction

The most common cause of failure in this experiment is a poor vacuum pump. A carefully constructed tube used with a moderately good rotary vacuum pump (giving a pressure of 40 microns of mercury or less) should give a fairly well-defined visible beam a few centimeters long that can easily be deflected in electric and magnetic fields.

Condensed vapors (for example, water) in the pump oil cause lower pumping speeds, poorer ultimate vacuum, and may lead to corrosion of the pump. If your pump has not had adequate maintenance, change the oil. Then "run in" the new oil by pumping on a closed system for a little while.

Since you will probably only have one "operating station" (vacuum pump, power supplies, etc.), students will have to test their tubes one at a time. The first student who gets a tube to work could demonstrate it to the rest of the class, so that all students will get a chance to see at least one tube in operation.

You might want to mention the cathode-ray tube and television picture tube. The sensitive coating on the inside of the screen glows where it is struck by the electron beam. As the beam is moved by electric or magnetic fields, the bright spot on the screen moves. A uniform field will be seen to be important again in the Millikan experiment in Unit 5.

The assembly instructions should be self-explanatory; the only tool needed is a pair of wire cutters. It usually takes a student 30–40 min to assemble the tube. You may find it possible to have students do the assembly at home, but they should probably bring their assembled tubes to school before sealing them. It is important that the tube be undisturbed while the sealant is drying.

A spare filament is provided in each kit. If the filament does burn out, the tube can be remade using the spare. It is also possible, using one of the spare leads, to mount two filaments in one tube as shown below. Then, if one filament burns out, the tube does not need to be taken apart to mount another.

However, if this is attempted, one must be very careful about the alignment of the filaments with the hole in the anode cap.

If you need more support tubes, get very thin aluminum tubing at a hobby shop.

If you do not get a visible glow, do not be afraid to increase the filament current to 5 A or more; a tube with a burnt-out filament is no worse than a tube that does not function for some other reason.

After about 10 min of operation, the beam may become less intense since the coating on the filament apparently deteriorates.

Electrostatic deflection

With anode plate and deflecting plate at the same potential (that is, connected to the same battery terminal), the beam should be equidistant from both plates. If it is not, this is almost certainly because the filament and anode hole are not lined up properly, giving a skewed beam. Try the effect of changing the potential of the deflecting plate about 50 V above or below anode potential. Do this at various anode potentials. The lower the anode potential, the slower the electrons in the beam and the easier they are to deflect.

If batteries are used to supply the anode and deflecting potentials, they should be big ones, such as Burgess No. 2308 45-V "B" batteries. Alternatively, use the Linco Power Supply (7100) or Macalaster (MSC 2105). Always include a rheostat and ammeter in the filament circuit.

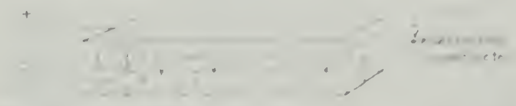
Magnetic deflection

Use the two magnets and yoke from the current balance kit to provide a (fairly) uniform magnetic field. The electron beam is bent into a more or less circular arc, in a plane perpendicular to the magnetic field. The force on the moving particles is perpendicular to their direction of motion and to the field. Reverse the field and determine what happens to the force on the particles.

Electric and magnetic fields must be perpendicular to each other for the two effects to cancel.

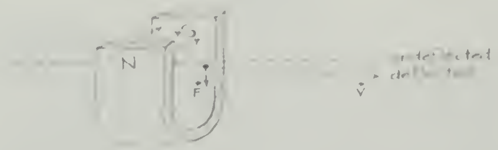
Answers to questions

1.

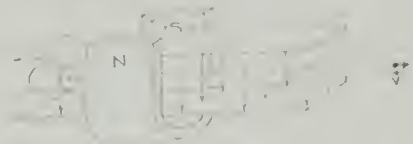


The electric field points toward the negative plate (the direction of force on a positive test charge). The beam is deflected in the opposite direction (toward the + plate), so its charge is negative.

2.



3.



4. $$\frac{B}{E} = \frac{\dfrac{\cancel{N}}{A{\cdot}m}}{\dfrac{\cancel{N}}{C}} = \frac{A{\cdot}m}{C} = \frac{\dfrac{\cancel{C}}{sec} \times m}{\cancel{C}} = \frac{m}{sec}$$

5. Student answer.
6. For a voltage of 50 V to 100 V across the deflecting plates and a space of 2 cm (0.02 m) between them

$$E = \frac{V}{d} = \frac{50\ V}{0.02\ m} = 250\ V/m \text{ up to } 750\ V/m$$

7. Student answer.

E4–8 ELECTRON BEAM TUBE. II

Equipment:

- As for *E4–7*
- Strong magnets or electromagnets
- Heavy gauge copper wire #18
- Ammeters, volt meters
- Variac
- Oscilloscope
- Thin aluminum sheet

This is an extension of the work in *E4–7*. Possibly only a few students especially interested in electrical phenomena will wish to carry through the activities described in the *Handbook*. Variations on those suggestions are to be desired. Any students doing this experiment may wish to present a demonstration to the remainder of the class.

E4–9 WAVES AND COMMUNICATION

Equipment:

A. *Turntable Oscillators*

- Pair of turntable oscillators
- Felt-tipped pen
- Recording paper
- Masking tape for adjusting fit of pen in slot
- Ruler for measuring traces

B. *Resonant Circuits*

- Pair of resonant (coil capacitor) circuits
- Amplifier
- Oscillator
- Cathode-ray oscilloscope
- Loudspeaker
- Hook-up wire

C. *Microwaves*

- Microwave generator
- Amplifier power-supply units (2)
- Diode detector
- 100 μA meter
- Oscillator
- Microphone
- Loudspeaker

Introduction

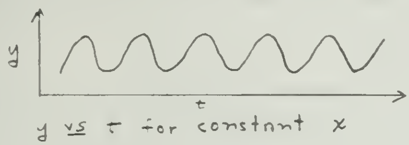
This "experiment" has been designed differently from the others in the *Project Physics* Course. It is really a series of demonstrations using turntable oscillators, tuned circuits and microwave equipment, to be followed by student experimentation with the same equipment. It should tie together much of the material on waves and electromagnetism in Units 3 and 4, and indicate its relevance to communications.

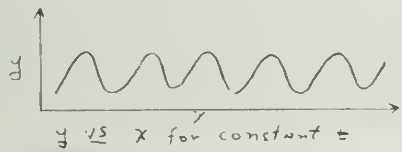
The *Handbook* does not give much detailed instruction about the operation of the equipment; it is assumed that the students are fairly familiar with it from the demonstrations.

You may not have time to do all these demonstrations in class, as part of this experiment. In any case, students may want to repeat some of the demonstrations for themselves before going on to the new investigation described in the *Handbook*.

A. Turntable Oscillators

Waves are generated by an oscillator: ripple-tank waves by an oscillating object at the surface; sound waves by an oscillating diaphragm, string, or air column; radio waves by oscillating electric charge, etc. The back-and-forth motion of the pen attached to a turntable oscillator provides a simple way of seeing how the oscillation is related to the wave. (And, incidentally, it is also an illustration of the relationships between circular motion and simple harmonic motion and sinusoidal waves.) As a wave passes a point in space, some quantity (height of water, pressure of air, electric field) at that point varies with time. On the turntable oscillators, the variation of the pen's position with time is recorded by a paper chart moving under it at uniform speed. In addition to providing a record of the variation with time of the disturbance at a given point, the trace is also a "snapshot" of the wave at a particular instant that shows how amplitude varies with position.



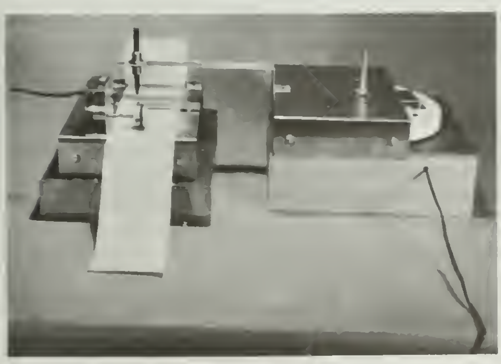


Demonstration of amplitude modulation

Set up a pair of turntable oscillators so that the pen attached to oscillator A writes on the strip-chart recorder ("drag-strip") mounted on oscillator B.

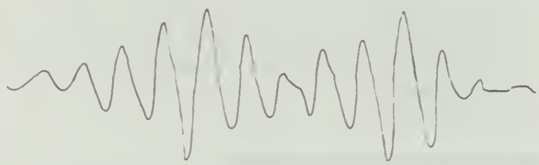
Turn on oscillator A, and if necessary adjust the equipment so that the pen stays on the paper. Now turn on the chart recorder to get a record of the pen's position as a function of time. Slow the effect of changing the frequency of the oscillator by changing the turntable speed.

Demonstrate oscillations of different amplitude by changing the position of the vertical post on the turntable.



The simplest way of using a wave phenomenon to communicate is just to turn the signal on and off according to some prearranged code. A buzzer or light beam can be turned on and off to give a series of long and short "dots and dashes" as in Morse code. Turning the signal abruptly on and off is a primitive form of amplitude modulation. A more sophisticated form of amplitude modulation in which the strength of the signal is varied continuously makes it possible to convey more information.

The amplitude ("strength") of the signal made by the turntable oscillator depends on the distance of the vertical post from the center of the turntable. It is possible, though not easy, to change its position, and thus produce amplitude modulation while the oscillator is working. Set the turntable to its lowest speed (16 rpm). You won't be able to make a very neat trace this way, but it will give an idea of what an amplitude-modulated wave looks like.



The same effect can be produced by adding together the output of two oscillators. (The demonstration of beats was probably done in Unit 3: for details see this *Resource Book*.) Both oscillators are switched on and are oscillating at slightly different frequencies. The resulting trace will be an amplitude-modulated sine curve. Beats and amplitude modulation are mathematically equivalent: *beats* are the result of the addition of two sinusoidal waves.

$$A = A_1 + A_2 = \cos 2\pi f_1 t + \cos 2\pi f_2 t \qquad (1)$$

$$= 2 \cos 2\pi \frac{f_1 - f_2}{2} t \cos 2\pi \frac{f_1 + f_2}{2} t$$

If the two frequencies f_1 and f_2 are nearly equal, the difference $f_1 - f_2$ will be small. The beat frequency is

$$\frac{f_1 - f_2}{2}$$

In amplitude modulation the amplitude of a high-frequency oscillation ($\cos 2\pi f_c t$, the "carrier") s varied at a much lower rate (signal frequency). In the simplest case, the signal is also sinusoidal ($\cos 2\pi f_s t$), and so the result is:

$$A = \cos 2\pi f_s t \cos 2\pi f_c t \qquad (2)$$

Equations (1) and (2) have the same form: the carrier frequency f_c is analogous to the *mean* frequency

$$\frac{f_1 + f_2}{2}$$

and the signal frequency f_s to the beat frequency

$$\frac{f_1 - f_2}{2}$$

Before attempting this demonstration, turn both turntables by hand to make sure that the pen will not leave the recording paper when the amplitude of the resultant trace reaches its maximum value.

You can vary the frequency of the modulation by changing slightly the frequency difference between the two oscillators. This is most easily done by reducing slightly the voltage to one with a "Variac" or "Powerstat" in the line, or by loading down one of the platforms.

Not all waves are sinusoidal

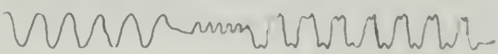
Although simple harmonic oscillators and sinusoidal waves are rather common, they are by no means the only ones possible. You can use an oscilloscope to demonstrate that the sound waves produced by different musical instruments playing the same note can have quite widely different shapes (see this *Resource Book* for Unit 1). But any complex wave form can be produced by adding simple sinusoidal waves (Fourier synthesis). This can be demonstrated with the turntable oscillators.

Turn both oscillators on, but at different frequencies (for example, one at 16 and the other at 45 rpm, or 33 and 78 rpm, etc.). The amplitudes can be varied as well as the frequencies of the two oscillations. The resulting trace will be a complex but regular pattern that repeats itself periodically.

A particularly interesting case is the "square wave." The sinusoidal components that combine to form a square wave are

$$\sin \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots$$

The first two terms can be added using a pair of turntable oscillators. Some care is needed to get a frequency ratio of 1:3, an amplitude ratio of 1:1/3, and the two oscillators in phase.



$\sin x + \frac{1}{3}\sin 3x$

Relevance to communication

The connection between amplitude modulation and communication will become more apparent in the next two demonstrations. It is briefly this: A radio station emits high-frequency electromagnetic radiation at some particular frequency, say 1,000 kHz. The electrical circuits at the station maintain a steady oscillation at this frequency. (Different stations, different frequencies, of course.)

Someone in the studio plays or sings a note, say middle C, into a microphone. The characteristic pressure fluctuations caused by the voice (in this case at 256 Hz) are transformed into electrical signals at the same frequency, which vary the amplitude of the 1,000 kHz oscillation. So a 1,000 kHz oscillation modulated at 256 Hz is transmitted.

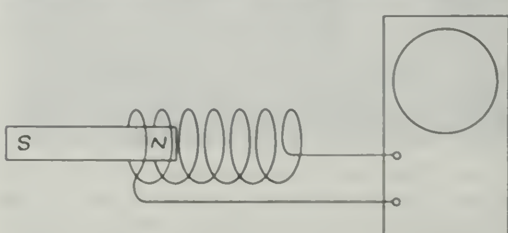
At the receiver (tuned to pick up oscillations at 1,000 kHz), the process is reversed and the 256-Hz signal is recovered, fed into a loudspeaker that sets up pressure fluctuations at 256 Hz, which we hear as middle C.

B. Tuned Circuits

With two circuits each consisting of a coil (about 5×10^{-4} H) and a variable capacitor (10×10^{-12} to 365×10^{-12} F) you can demonstrate many of the phenomena related to "wireless" communication. Use the Fahnstock clips to disconnect any part of the circuit not needed in a particular demonstration. (For example, take the capacitor out of the circuit for the first two demonstrations.)

1. *Changing Magnetic Field Produces Electric Field*

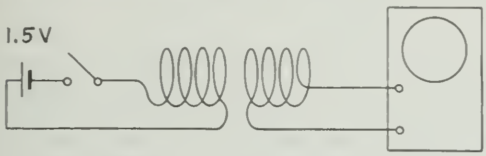
(a) Connect one coil directly across the oscilloscope. Set the oscilloscope gain to maximum. Move a magnet in and out of the coil. The oscilloscope beam shows a deflection only when the magnet is moving: the faster the movement the greater the deflection. Almost any magnet will do, but if you use a powerful one, keep it far enough from the scope to prevent it from affecting the beam directly.



Instead of the oscilloscope, you can, of course, use a galvanometer to show the induced current.

or the *Project Physics* amplifier and projection microammeter.

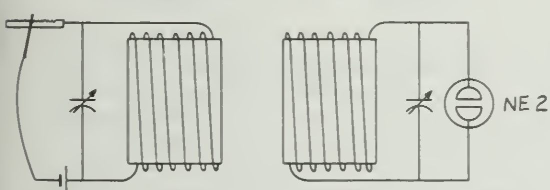
(b) Use the second coil to produce the magnetic field: The oscilloscope beam remains undeflected for either no current or steady current in the left-hand circuit (steady magnetic field). But if the magnetic field changes, owing to "make" or "break" of the left-hand circuit, a current is induced in the right-hand circuit, and the oscilloscope beam is deflected. Try to open and close the left-hand circuit as cleanly as possible: Don't scrape the saw blade in this demonstration.



You may wish to explore the effects of changing the position of one of the coils, adding iron cores, screens of various materials between the two coils, etc., on the voltage induced in the right-hand circuit (as shown on the oscilloscope) when the left-hand circuit is opened and closed.

2. *Resonance in Electric Circuits*

Connect the circuit as shown below.



When contact is broken in the left-hand circuit, oscillations are set up. (A simple exposition of the oscillatory flow of charge in an *LC* circuit is given in many texts; the usual mechanical analogies, such as mass-on-spring, pendulum, etc., are helpful.) Because of resistive and radiative loss of energy, the oscillations die away quite quickly, and so it is necessary to make and break contact repeatedly by scraping the contact wire on the saw blade.

If the coil of the second circuit is placed close to the first coil with axes aligned as shown in the diagram, the oscillating magnetic field of the first will induce an oscillating current in the second. But only if the natural frequencies of the two circuits (each given by $f = \frac{1}{2\pi\sqrt{LC}}$) are the same will large oscillations build up in the second one. Set the capacitor in the first circuit ("transmitter") to a particular capacitance C_1. Vary the capacitance C_2 of the capacitor in the second ("receiver") circuit while scraping the contact wire. When $C_1 = C_2$ electrical oscillations build up in the "receiving" circuit and the neon bulb glows. The demonstration probably won't work if you touch the bare part of the copper scraper wire; pick it up by the insulated part only.

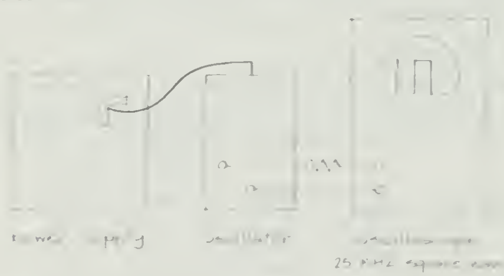
The neon bulb requires about 70 V to make it glow and yet it can glow with only a 1.5 V battery in the "transmitting" circuit. This is because the electric field induced depends on the *rate of change* of magnetic field. In this case the voltage induced in the "receiver" depends on the rate of change of current in the "transmitting" circuit.

Some mention of other resonant systems should be made, for example, a child on a swing, a driven pendulum, which, analogous to our "receiver" circuit, can only build up big oscillations if energy is fed to them at their own natural frequency.

The values of *L* and *C* are chosen so that the frequency range of these oscillations is in the broadcast range (550 to 1,500 Hz). This can be shown by picking up the signal from the "transmitting" oscillator on a regular or transistor radio set. A radio has a frequency sensitive circuit that is tuned in just the same way as our primitive "receiving circuit." Turn on the radio and tune it to some frequency near the low frequency end at a point where no broadcasting station is picked up. The radio should be several meters from the "transmitter," volume control turned up. Scrape the "transmitter" contact and vary the capacitor until a loud scraping sound is heared in the radio. The electrical energy of the oscillations in the demonstration "receiving" circuit caused the neon bulb to glow: In the radio some of the energy is used to control the circuits responsible for audible sound. With the radio tuned to a different frequency, another setting of the variable capacitor in the "transmitter" is required. Or set the "transmitter" and tune the radio to the "transmitter" frequency. Because our primitive *LC* circuit has a fairly wide resonance maximum (low *Q*) some noise will be heard even if the radio or "transmitter" is somewhat "mistuned."

3. *Further Investigation of the Oscillations in a Resonant Circuit*

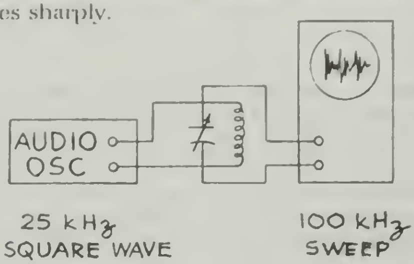
The oscillations themselves can be demonstrated as follows:



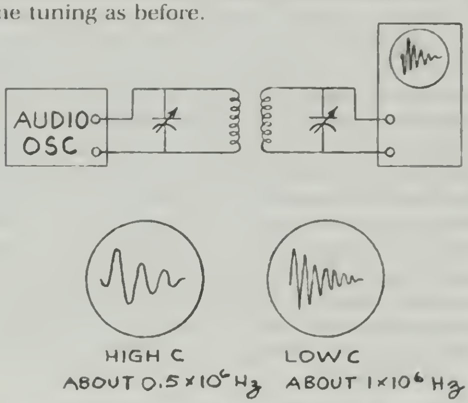
Instead of scraping to provide repeated make-and-break contact, use a signal generator (audio oscillator) set to "square wave" to provide a succession of regular pulses. First connect the signal generator directly to an oscilloscope and show the wave form it produces.

Now connect the signal generator across the coil and capacitor of the "transmitting" circuit. Connect the oscilloscope across the coil and capacitor to show the electrical oscillations in this circuit.

Notice how the oscillations decay while the voltage supplied by the square wave generator is steady and are reestablished every time the voltage changes sharply.



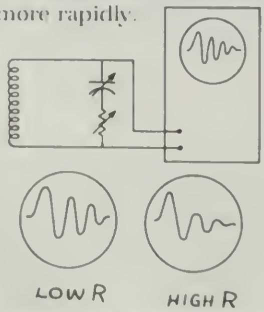
Now bring the "receiving" circuit near, transfer the oscilloscope leads to it, and tune the circuit to resonance. Note that by connecting the signal generator to one circuit and the oscilloscope to the other, we have added different impedances to each. This is why the same capacitor settings in each circuit may no longer necessarily give the same tuning as before.



Electrical oscillations are observed in the second circuit. As the resonant frequency is changed (both capacitors varied), the number of oscillations in the resonant circuit per period of the square wave signal will vary.

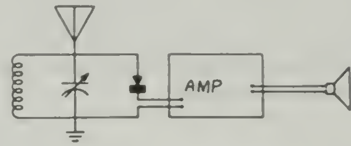
4. *Damping*

Add a 0 to 1 kΩ variable resistor to the receiver circuit. As the resistance is increased, the oscillations decay more rapidly.

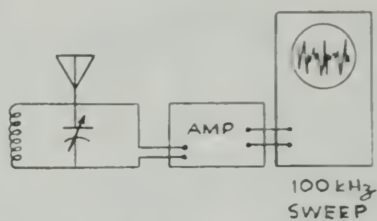


5. *Use of Tuned LC Circuit as Radio Receiver*

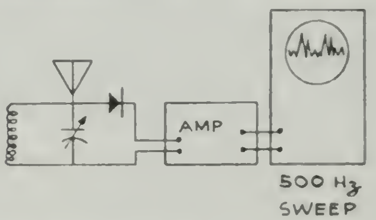
We showed in one of the earlier demonstrations in this sequence that the signal generated by the transmitting circuit can be picked up by a radio. In this demonstration we do the reverse: We use the *LC* circuit to pick up the signal broadcast by a local radio transmitter.



Connect an antenna (a long piece of hook-up wire, preferably insulated, outside the building) to the circuit. Connect the receiver in series with the diode (rectifier) to the input of the amplifier, and connect a loudspeaker to the amplifier output. Grounding the circuit will probably help. Vary the capacitor setting to tune the circuit to the frequency of a local transmitter. If you are near a powerful station and/or have a long enough antenna, you can dispense with the amplifier and use earphones.



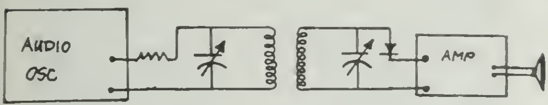
You can use the oscilloscope to show the function of the diode. Connect the oscilloscope directly across the capacitor (no diode) to see the radio frequency carrier wave. Now add the diode in series and reduce the sweep rate to see the audio frequency signal.



For a very effective demonstration let students "see" the signal in this way while they are listening to it on a loudspeaker.

The diode in this circuit performs just the same function as the diode in the microwave probe. You can bring out this point by substituting the microwave diode in the *LC* circuit for the one provided. (But note that you cannot do the reverse, the diode from the *LC* circuit will not rectify at microwave frequencies.)

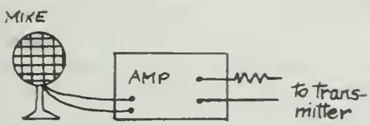
We have seen that the "transmitting" *LC* circuit can broadcast noise that is picked up on a transistor radio, and that the "receiving circuit" can pick up local radio broadcasts. As a final step we ought to be able to transmit a "signal" from one of our circuits to the other. The setup below should do it.



The two coils must be quite close to each other. Vary the frequency setting of the audio oscillation (in the 300–5,000 Hz range) and listen to the tone change in the loudspeaker. Investigate the effects of separating the two coils, placing a metal sheet between them, etc.

Evidently our "transmitter" is a pretty weak one, for the receiver must be very close to it to pick up the "broadcast." In fact, this is really a demonstration of induction rather than radiation.

Instead of using the audio oscillator to provide the signal, try a microphone and amplifier. Reduce the value of r_1 or remove the resistor altogether.

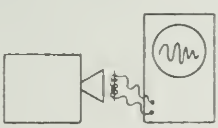


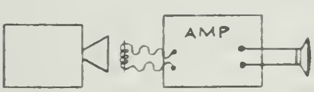
C. Microwaves*

With a microwave oscillator and detector, one can demonstrate all the properties of electromagnetic radiation, often more easily than with light because of the longer wavelength of the microwaves. *D52* covers reflection, transmission, refraction, standing waves, diffraction, interference, and polarization.

Modulation and communication with microwaves

The older microwave units use line frequency ac to supply the plate voltage. This means that the microwave signal is modulated at 60 Hz. This can be shown by connecting the diode detector to an oscilloscope or by connecting the detector to an amplifier and the amplifier output to a loudspeaker. The oscilloscope will show a half-wave 60-Hz signal and the loudspeaker will oscillate at 60 Hz.





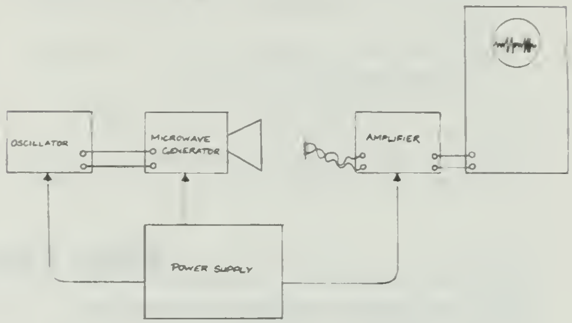
*See *Equipment Notes*.

Be sure that students realize that this 60-Hz signal is the modulation frequency, not the carrier frequency. Show some beat patterns produced by the turntable oscillators to remind students of the difference between carrier and signal frequency. For microwaves the carrier frequency is

$$f = \frac{c}{\lambda} \simeq \frac{3 \times 10^8}{3 \times 10^{-2}} = 10^{10}\ \text{Hz}$$

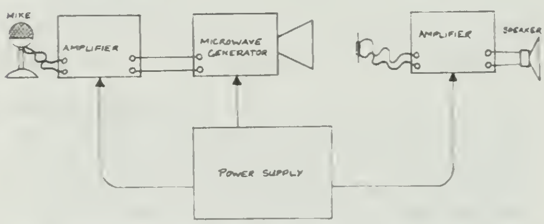
This frequency is unfortunately much too high to be displayed on an oscilloscope.

On the newer microwave units, the modulation frequency can be varied.



With the setup shown above students can see the rectified, modulated signal on the oscilloscope, and hear it on the loudspeaker as the setting of the oscillators is varied in the audible range.

It is also possible to replace the oscillator with a microphone (or even a 45-Ω speaker) and use the amplified signal from the microphone to modulate the microwaves. This demonstration of radio is most effective if you use the setup shown below that avoids leads from the detector back to the source.



Answers to questions

1. See Fig. 4-37 in the *Handbook*.
2. Student answer.
3. If both waves have zero amplitude at the same time (that is, they are in phase), the result will be a sine wave whose amplitude is the sum of the two separate amplitudes. If the waves are not in phase, the resulting pattern will be more complicated: but so long as the separate waves have the same wavelength, the pattern will repeat itself with each wavelength. The pattern will depend on the phase difference and on the separate amplitudes.

4. Metal and wet paper block the signals, but none of the other suggested materials do.
5. Radio signals can penetrate the wood and stone of which most houses are built but cannot penetrate the metal chassis of an automobile.
6. Student answer.
7. There will be a maximum whenever the two paths differ in length by a whole number of wavelengths. There will be a minimum where the paths differ by a whole number of wavelengths plus one-half wavelength.
8. There will be a semicircular pattern of waves intersected approximately radially by lines of nodes.
9. The nodes will probably be about 2.5 cm apart. If so, the wavelength is 5.0 cm.
10. $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/sec}}{0.050 \text{ m}} = 6.0 \times 10^9\text{/sec}$
11. Radio signals with the frequency of sound waves, say 100 Hz, will have a wavelength of $\frac{3 \times 10^8 \text{ m/sec}}{100\text{/sec}} = 3 \times 10^6$ m or 3,000 km. For an antenna to radiate efficiently its length should be about one-half (or one-quarter) wavelength, which would require in this case an absurdly long antenna. Radio waves, on the other hand, have wavelengths in the order of meters.
12. In a television tube, the beam intensity is varied by changing the amplitude of the electromagnetic waves forming the TV signal. If the intensity of the beam in the TV tube is to vary 300 times during a single line scan then at least 300 cycles of TV signal must be received during the time of the scan, which takes 1/525 of 1/30 of a second. Hence, each wave of the TV signal must have a duration smaller than $\frac{1 \text{ sec}}{30 \times 525 \times 300}$ or about 1/5,000,000 sec and therefore a frequency of 5,000,000/sec or 5,000 kHz. Ordinary radio needs only 12 kHz to transmit intelligible sound.

Film Loop Notes

L44 STANDING ELECTROMAGNETIC WAVES

The intensity would go down inversely as the square of the distance if the radiating dipole were in free space. Because of the cavity, the observed decrease of intensity is not as rapid as this.

Although the energy distribution inside the cavity is far from uniform in space, the total power (almost 20 W) supplied by the transmitter is constant in time. The students might expect that when the standing wave exists in the cavity all or most of the reflected energy would be reabsorbed by the transmitter, reducing the net radiation required. This is true, but the meter reads the forward power, not the net power. Some of the power of the transmitter is used to balance losses of energy. (The cavity is not closed on all sides.) Also, some electromagnetic radiation is reemitted in all directions by the receiving dipoles in which electrons are flowing.

Equipment Notes

LIGHT SOURCE

E4–1 calls for a light source. It is suggested that you use the light source that comes with the Millikan Apparatus. The Damon Universal Power Supply can provide electricity for the light source at the jacks marked 0–5 V/5 A, or at the jacks marked 6 V/5 A, ground. If the 0–5 V/5 A jacks are used, the switch directly above these jacks should be in the normal position and the knob below the jacks should be turned clockwise to increase the intensity of the light source. More than one light source may be used from this power supply by gang stacking in either set of jacks. However, if several sources are used, be careful not to exceed the maximum current specified.

CURRENT BALANCE

Assembly

Assembly of the current balance should be clear from the instructions that accompany it.

Parts list

Each current balance package should contain.
- Vertical pegboard with fixed coils
- Balance beam with short vertical rod and long horizontal pointer
- Magnesium loops (4, approx. 30, 16, 8, and 4 cm)
- "Counterweight" cylinder
- Sensitivity clips (2)
- Zero-mark indicator
- #30 Copper wire
- Iron yoke
- Ceramic magnets (4)

Accessories

Ringstand and clamp
Pressure-sensitive centimeter tape

Design
This current balance measures the force between two straight, parallel, horizontal currents; or between one horizontal current and a vertical magnetic field.

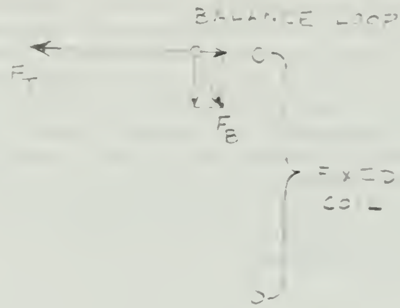
One of the conductors is a fixed rectangular coil mounted vertically on a pegboard frame. There are two such fixed coils on this frame. One consists of a single blue wire, the other is a 10-turn coil of copper magnet wire. For most demonstrations, the 10-turn coil is used, effectively increasing the current in the fixed coil by a factor of 10. The current in the fixed coil will be referred to as I_f.

A balance rests on the frame and consists of a pointer (with notch), counterweight cylinder, sensitivity adjustment clip, and four interchangeable ⊔ or ⎽⊔⎽ shaped magnesium wires that will be referred to as loops. The current to the balance loop (I_b) passes through the knife edges. The knife edges and the plates on which they rest are silver-plated.

The magnesium alloy loops are very light. Students may remember some exciting experiments with magnesium foil or powder from their chemistry classes; and although the loops are actually quite difficult to ignite, the *Handbook* purposely makes no mention of magnesium. This alloy is quite brittle: Once bent, it is almost impossible to change or adjust the bend without breaking it.

A horizontal force on the balance loop makes it swing out of the vertical position. A vertical force has no result except to increase or decrease the effective weight of the loop.

The horizontal part of the balance loop is parallel to the top edge of the fixed wires and must lie in the same horizontal plane. With current in both loops, the thrust on the balance loop is then horizontal. If it is a repulsion force, it can be balanced by hanging weights over the notch on the pointer. If the balance swings the wrong way (that is, if there is an attractive force), one must reverse the direction of one of the currents, of course. This null method is used for all measurements (as in *E4–4*, "Electric Forces, II: Coulomb's Law").



The fact that the loop responds to horizontal forces only is important in the design and operation of the current balance. The magnetic field due to the current in a straight wire varies inversely with distance from the wire ($B = \frac{1}{d}$, $\underline{\text{not}}$ $\frac{1}{d^2}$). The field at the loop due to the current in the bottom wire(s) of the fixed coil is therefore not negligibly small. But because the loop is almost directly above the bottom wires, the force between them will also be in a nearly vertical direction. The *horizontal component* of this force, which is all the loop responds to, is therefore small. If the top wire is 3 cm and the bottom wire 30 cm away from the loop, the total force due to the current in the bottom wire (F_B) will be $\frac{1}{10}$ the force due to the current in the top wire (F_T). But the *horizontal component of* F_B will be about $\frac{1}{100} \times F_T$.

The weights
As in the Coulomb's law experiment, students make their own set of weights by cutting lengths of wire. #30 copper wire cut into lengths of 1 cm, 2 cm, 5 cm, and 10 cm will give a suitable range. Students will need several pieces of each length.

The sensitivity
Move the clip up the vertical rod to increase sensitivity. This raises the center of gravity. You will probably need to add a second clip to get maximum sensitivity (for instance to demonstrate and measure the effect of the earth's field on the current). However, if the center of gravity is too high, the balance is unstable and flops to either side. When the loops are changed, the center of mass of the balance system is moved, and it will be necessary to adjust the sensitivity clip.

Proper adjustment results in a slow oscillation (period of 3 to 5 sec) about the zero, or balance, position. When set in this way, the balance should respond measurably to a "weight" of 0.5 cm of #30 copper wire at the notch in the pointer arm.

Zero adjustment

Use the counterweight cylinder on the short horizontal arm to set the loop vertical and the pointer arm horizontal.

The zero position of the pointer is important, since the null method requires that the balance be returned to this same zero. A small indicator with a notch is provided. You may have to add the index mark (as shown in the sketch below). The indicator should be clamped to a ringstand. Use of a test-tube clamp for this purpose makes it possible to slide the indicator easily for close final adjustments. The pointer on the balance should be inside the notch of the indicator to prevent wide oscillations in the balance.

Measurement of distance between conductors

In some of the experiments, it is necessary to measure the distance between the loop and the fixed coil. The *Handbook* explains how to eliminate parallax when making this measurement by using a mirror and scale (centimeter tape stuck onto the mirror surface).

Effect of the earth's magnetic field

The balance responds to horizontal forces on the loop. Since the earth's magnetic field generally has a vertical component, it will exert a measurable horizontal force on the balance when there is a current in the loop. This force cannot be eliminated by reorienting the system.

The force due to the earth's field is small, but it is possible for students to measure it [part (c) of *E4–6*]. In fact, the value obtained for the vertical component of the earth's field can be used in a later lecture-experiment for a $\frac{ge}{m}$ determination.

As long as the current in the balance loop remains constant, the effect of the earth's magnetic field on it will be constant and can be easily compensated. The simplest technique is just to set the zero mark to the position of the pointer when there is current in the balance loop, but none in the fixed coil. Of course, if the current in the balance loop is changed, or if the balance is moved, the zero mark must be reset. Students are instructed to do this in *E4–5* and *E4–6*.

There are, however, at least two other possible ways to compensate:

(a) A small ceramic magnet can be moved near the balance loop. If located carefully, the force due to its field will just neutralize that due to the earth's field. Once located, this compensating magnet must not be moved. Any changes in I_b will require a readjustment of the compensating magnet, of course. Imbalance between the earth's and the ceramic magnet's fields can be quickly noted. Flick I_b on and off a few times fairly rapidly (about 5 sec apart for a balance whose period of oscillation is 5 sec). If oscillation builds up in the pointer, readjust the position of the ceramic magnet.

(b) Alongside the fixed wires of the current balance is one single blue wire. A "countercurrent" in this wire can be made to balance the earth's effect on I_b. Setting this current requires a power supply, ammeter, and either a rheostat or a Variac. The blue wire is connected, and the direction of the required current found. Adjust this countercurrent until the pointer is pulled back to its zero position. In a prototype balance, with $I_b = 2.0$ A, it was found that a countercurrent of 5.5 A was needed when the two loops were 1.0 cm apart. Any changes in I_b or distance between the blue wire and the loop will require a change in the countercurrent, of course.

Orientation of the current balance unimportant

For any orientation (N–S, E–W) of the balance, the *vertical component* of the earth's field will give rise to a *horizontal force* (perpendicular to the direction of the current).

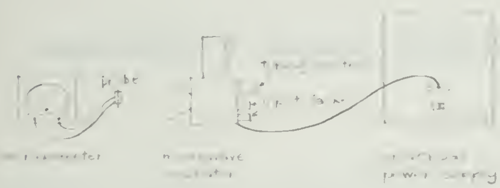
The *horizontal component* of the earth's field will interact with the *vertical* currents in the sides of the loop and give rise to *horizontal forces*. But since the currents are oppositely directed (up and down), the forces will be oppositely directed too.

In particular, if the balance is set up along an E–W line, these forces will be "in" or "out." If the balance is set up along a N–S line, the two forces will form a torque tending to twist the loop, but there will still be no net horizontal force.

The microwave equipment utilizes a reflex klystron, which, when operated in its proper modes, will generate electromagnetic radiation in the microwave band of frequencies. The kit includes a tube and integral cavity, an antenna horn, a microwave diode, reflectors, polarizing screen and double slit adapter.

The oscillator tube is a type 6116 electrically tunable reflex klystron. The tube is mounted on its tuned waveguide and has a horn for better radiation. The unit is designed to operate from the Damon Universal Power Supply and derives all of its necessary power from this unit. Several microwave oscillators can be operated from the power supply simultaneously by gang-stacking the plug-in bars.

The detector is a silicon diode, the leads of which have been cut to form a one-half wave antenna. The diode is supplied mounted at the end of a phenolic tube in order to form a convenient probe.



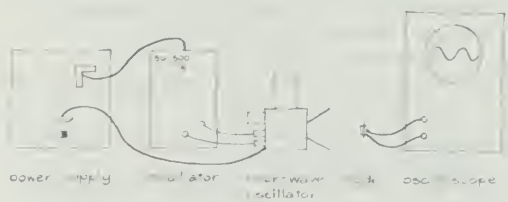
To operate the microwave oscillator, make the connections to the power supply as shown above. Turn the power supply on and wait a few minutes for the tube to warm up. Use the diode and the 100 μA meter as a detector. Hold the diode vertically in front of the horn and while in this position adjust the repeller voltage control to obtain a maximum meter deflection. If the meter goes off scale, move the diode farther away from the horn. Again adjust the tuning control at the rear of the microwave oscillator for maximum deflection while the diode is still in the vertical position.

The microwave is polarized with the electric vector vertical (parallel to the short edge of the cavity). The cavity has been tuned by the manufacturer. The detector must be correctly oriented with the antenna leads parallel to the electric vector, that is, vertical. The signal decreases in strength as the antenna is rotated. With the antenna horizontal, the signal strength is almost zero.

Modulation of Microwave Oscillation

On older models, the microwave oscillation is modulated at line frequency (60 Hz). On the newer models, the 60-Hz internal modulation has been minimized, resulting in a relatively pure carrier wave. The microwave radiation may, therefore, remain unmodulated or be externally modulated at any chosen frequency. A modulation input (MOD INPUT) terminal has been provided on the microwave unit for this purpose.

To modulate the microwave radiation, apply an audio-frequency signal to the MOD INPUT jack. This signal should have about 1 V peak-to-peak voltage: The output of a Damon oscillator unit is ideal. If higher gain is required, use the amplifier with the oscilloscope as shown below. The detector may be connected directly to an oscilloscope so that students can see the demodulated signal.



Alternatively, use an amplifier and loudspeaker as described below.

Readout Devices

If the microwave radiation is unmodulated, you must use a dc meter (or dc oscilloscope) as a readout device. If it is modulated, any of the following devices can be used.

(a) *Meter.* A 100 μA meter is sensitive enough if the detector is not too far from the source. It can be used for both modulated and unmodulated radiation and one can make quantitative measurements.

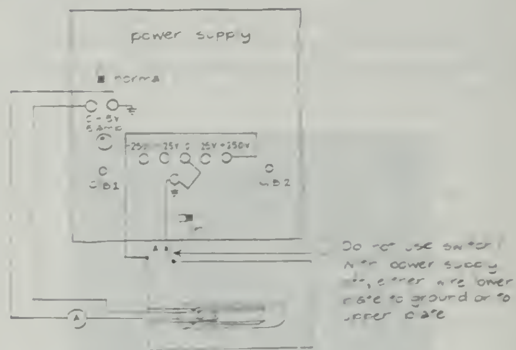
(b) *Oscilloscope.* (Unless you have a dc scope, you can use this for modulated radiation only.) Connect the detector between the vertical input of the oscilloscope and ground. Set the horizontal sweep to 5–10 × the modulation frequency. The rectified envelope of the modulated microwave envelope is clearly visible on the screen. Quantitative or semi-quantitative measurements may be made from the screen, depending on the sensitivity and calibration of the oscilloscope.

(c) *Amplifier and Loudspeaker.* (Can be used with microwave radiation that is modulated in the audio range only.) The position of the nodes in a microwave interference pattern can be measured by listening for the nulls as you probe the field. For this experiment, replace the oscilloscope with a loudspeaker as shown at the right.

ELECTRON BEAM TUBE

Suggestions for operating the electron beam tube from the Damon Universal Power Supply are as follows:

1. CAUTION. You are working with high voltages.
2. Before hooking up any wires, attach the tube to a good vacuum pump and evacuate it.
3. Turn the Damon Universal Power Supply off.
4. Connect the filament of the tube to a 0–5 A ammeter and turn the 0–5 V control knob counterclockwise.
5. Connect the upper plate (attached to the cap) to the 250 V high-voltage jack (red).
6. Connect the high-voltage jack marked "o" to the ground jack (black) below it and slightly to the left. This will make the cap and one plate positive with respect to the filament.
7. Now you have a choice.
   (a) Connect the lower plate (not attached to the cap) to the upper plate,
   (b) OR connect the lower plate (not attached to the cap) to the ground.
   DO NOT DO (a) OR (b) WITH THE POWER SUPPLY ON



Note that in situation (a) the beam will come out of the hole in the cap and spread out. As a result of situation (b) the beam will come out of the hole in the cap and bend away from the grounded plate.

8. Turn on the power supply and turn off the lights.

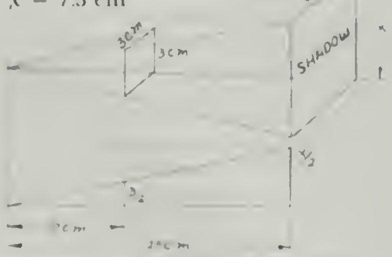
Suggested Solutions To Study Guide Problems

CHAPTER 13

2. $$\frac{\frac{x}{2}}{25\text{ cm}} = \frac{\frac{3}{2}\text{ cm}}{10\text{ cm}}$$

$$x = \frac{25 \times 3\text{ cm}}{10}$$

$$x = 7.5\text{ cm}$$



3. Differences:
   1. The light diffraction pattern was photographed by allowing light coming through the slit to strike photographic film in a camera pointed toward the slit.
   2. The water wave diffraction pattern was photographed with light coming from the surface of the water into a camera that seems to be mounted so as to point perpendicular to the direction of wave propagation.

   Similarities:
   1. Diffraction effects become more pronounced as the slit is narrowed.
   2. The separation between nodes increases as the width of the slit decreases.

4. (a) Galileo was unsuccessful because the distance he used was far too short to detect any

delay in arrival of the light signal. If the total distance had been even 3 km, light would travel that distance in a time of

$$\frac{3 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/sec}} = 10^{-5} \text{ sec}$$

This is much too small a time delay to detect by his method.

(b) There would have been no way to alter the experiment enough to make it successful without making it a different experiment altogether. Even if the distance were increased to the circumference of the earth, the time required would still be only about 0.125 sec, much too small to be detected by Galileo's method.

(c) Suppose the longest time that light might have taken in getting from one observer to the other without the observers detecting the delay was 0.5 sec for the round trip of approximately 3 km. We could then conclude that the speed of light cannot be less than 6 km/sec. Thus, Galileo's experiment did show a lower limit for the speed of light.

(d) Celestial observations can involve very large distances and therefore the corresponding time difference resulting from the finite speed of light can be easily detected and measured accurately.

5. One light year = speed of light in meters per second × number of seconds in one year.
Number of seconds in one year

$$= 60 \frac{\text{sec}}{\text{min}} \times 60 \frac{\text{min}}{\text{hr}} \times 24 \frac{\text{hr}}{\text{day}} \times 365 \text{ days}$$

Therefore, one light year

$$= 3.16 \times 10^7 \text{ sec}$$

$$= 3.0 \times 10^8 \frac{\text{m}}{\text{sec}} \times 3.16 \times 10^7 \text{ sec}$$

$$= 9.5 \times 10^{15} \text{ m}$$

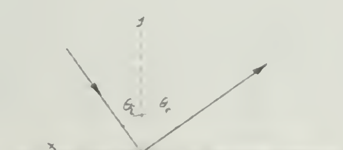
6. Since $d = vt$,

$$d = 3 \times 10^8 \text{ m/sec } (10 \text{ m})(60 \text{ sec/min})$$
$$= 1.8 \times 10^{11} \text{ m}$$

7. To travel 4.3 light years at the speed of light requires, by definition, 4.3 years. At only 1/1 000 of the speed of light, the one-way trip would take 4,300 years.

A spaceship traveling at 1/1,000 the speed of light would have a speed of 3×10^5 m/sec which is 30 times the speed of a capsule traveling at 10 km/sec (10^4 m/sec).

8. (a)



(b) Kinetic energy of a particle = $\frac{1}{2}mv^2$.

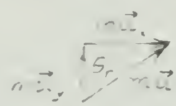
$$m\vec{v}_x = m\vec{v} \sin \theta$$
$$m\vec{v}_y = m\vec{v} \cos \theta$$

(c) Since no work is done on the particle, kinetic energy is conserved. The x-component of the momentum is also conserved since no force acted in that direction; however, the y-component of momentum is not conserved.

(d) Since energy is conserved, kinetic energy of incident particle = kinetic energy of reflected particle: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2$. Therefore, $v = u$. (Strictly speaking, $v = \pm u$, but the negative sign has no significance here.)

(e)

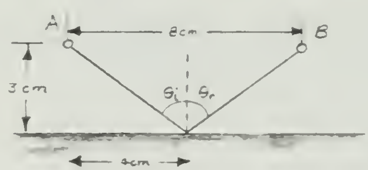
$$m\vec{u}_x = m\vec{u} \sin \theta_r$$
$$m\vec{u}_y = m\vec{u} \cos \theta_r$$



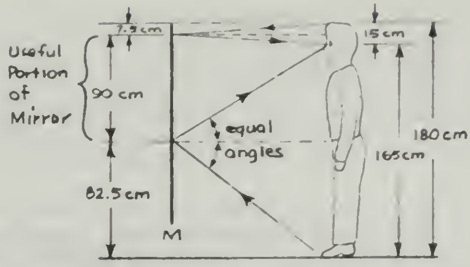
(f) Since the x-components of the momentum are equal before and after collision,

$$m\vec{v}_x = m\vec{u}_x$$
$$m\vec{v} \sin \theta_i = m\vec{u} \sin \theta_r$$
$$\text{But } u = v$$
$$\therefore \sin \theta_i = \sin \theta_r$$
$$\text{and } \theta_i = \theta_r$$

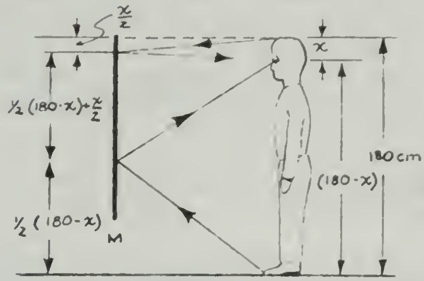
9. The shortest path from A to B via the mirror is the path shown below (each half of which, by Pythagoras' theorem, must be 5 cm). Any other path is longer than 10 cm in total. Light follows a path such that the angles of incidence and reflection are equal. Only the 10-cm path shown has this property. We see then that the light follows the path from A to M to B that requires the least time. Light always travels by least-time paths.



10. To calculate the height of mirror required, we need to consider the path taken by light rays from the man's feet to the mirror and then to his eyes. Also consider light traveling from the top of his head to the mirror and then to his eyes. Assume his eyes are 15 cm from the top of his head. Any reasonable distance from the mirror is satisfactory. The solution of the problem depends on the equality of the angles of incidence and reflection, which is independent of the distance from the mirror.



The distance from his eyes to the top of his head is not important either, as can be seen from the diagram below.

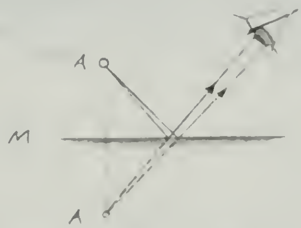


The effective mirror length is

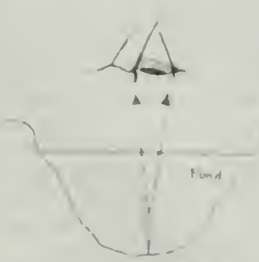
$$\tfrac{1}{2}(180 - x) + \frac{x}{2} = 90 - \frac{x}{2} + \frac{x}{2} = 90 \text{ cm}$$

11. If the reflecting surfaces of visible objects changed so that they completely absorbed any light falling on them, they would no longer be visible. The only things visible under such conditions would be actual sources of light. The world would not in fact "appear" at all; only light sources would "appear."

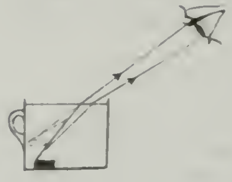
12. (a)



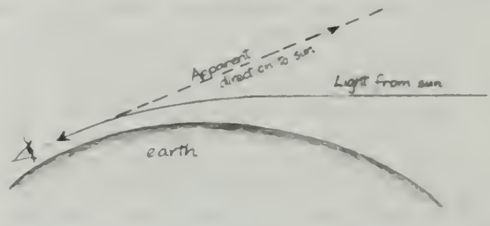
(b)



(c)

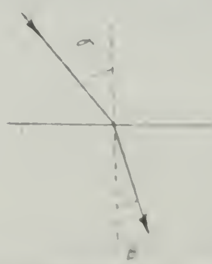


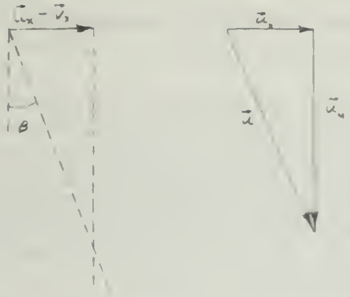
13. (a) Light entering the atmosphere at a near-grazing angle is shown refracted into the atmosphere in a slight curve. Thus, the light received at sunset or sunrise did not come in a "straight line" from the sun, since the sun is still "below" the horizon.



(b) At sunset the sun would seem to be flattened in the vertical direction since light from the bottom edge would be more strongly refracted than the light from the top edge. Furthermore, the delay in actual sunset and the advance in the actual sunrise would be increased. One might even speculate that they could occur quite close together in time but still in opposite directions.

14. We can represent the velocity of the incident particle by a vector $\vec{v}$ at angle α to the y-axis and the velocity of the refracted particle by a vector $\vec{u}$ at angle β to the y-axis. These two vectors must have the same x-component because no force acts in the x direction, but the y component of $\vec{u}$ will be greater than the y component of $\vec{v}$ because of the action of the accelerating force. Therefore, we proceed by drawing $\vec{u}_x$ equal to $\vec{v}_x$. Then from the tail of vector $\vec{u}_x$ we draw a line at angle β to the vertical allowing it to intersect a vertical line dropped down from the head of $\vec{u}_x$. We have thus mechanically constructed vectors representing $\vec{u}_x$ and $\vec{u}$: $\vec{u}$ is thus shown to be greater than $\vec{v}$.



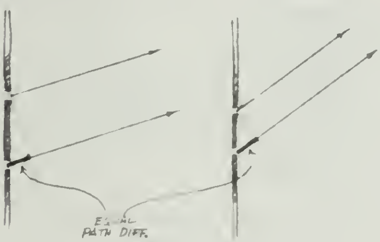


15. (a) Destructive interference occurs if the path difference = $(m + \frac{1}{2})\lambda$, where $m = 0, 1, 2, 3, \ldots$

(b) The separation of the fringes is greater for red light than for blue light because the wavelength, λ, and hence the corresponding path difference is greater for red than blue.

(c) Increasing the distance to the screen results in increased separation of the fringes. (Actually, the distance to the screen is directly proportional to the separation.)

(d) The separation of the fringes becomes greater as the slits are moved closer together.



(e) Narrowing each slit allows less light to pass through, which will make the overall pattern fainter. The diffraction effect will be more pronounced and the pattern will exhibit a greater number of fringes. The spacing between fringes still depends on the distance between slits, not the slit width.

16. Since $f = \dfrac{c}{\lambda}$, for violet:

$$f = \frac{3 \times 10^8 \text{ m/sec}}{400 \times 10^{-9} \text{ m}} = 7.5 \times 10^{14} \text{ sec (Hz)}$$

for red:

$$f = \frac{3 \times 10^8 \text{ m/sec}}{700 \times 10^{-9} \text{ m}} = 4.2 \times 10^{14} \text{ sec (Hz)}$$

17. Since $\dfrac{dx}{l} = n\lambda$,

$$d = \frac{nl\lambda}{x} = \frac{(1)(15 \text{ m})(550 \times 10^{-9} \text{ m})}{0.05 \text{ m}} = 1.6 \times 10^{-4} \text{ m}$$

18. $$x = \frac{nl\lambda}{d} = \frac{(1)(1 \text{ m})(10^{-10} \text{ m})}{10^{-7} \text{ m}} = 10^{-3} \text{ m}$$

19. Diffraction occurs around the disc. Since all points along the center of the shadow are equidistant from the light source and the edge of the disc, constructive interference occurs between the diffracted waves. Hence, there is a bright spot on the screen in the center of the shadow.

20. Artificial light sources do not emit light with a spectrum identical to that of the sunlight that reaches us. The various wavelengths are not present in the same amounts in both. The color of clothing depends on the relative intensities of the various colors it reflects, and this will, of course, depend on the amount of each color present in the light that strikes the clothing.

21. One experienced teacher answers as follows:

(a) As Poetry: "Here I am not competent to give a sure answer; my answer will be akin to an educated guess. I like them both, as poetry, but I think the poem quoted in the problem is somewhat better. The 1727 poem seems a little strained in some of the similes and descriptive adjectives. The 1728 poem seems to flow a little more naturally; the imagery is, to me, more beautiful."

(b) As Physics: "I think the physics in the later poem, *Spring* quoted in the problem, is inferior. I see nothing wrong with the physics in *To the Memory*. . . . The physics in *Spring*, however, is not all correct. First, the bright, primary rainbow is violet on the bottom and red on the top, and so the violet does not fade into the sky. Perhaps more important is the image which Thomson attempts to create of the cloud acting as a prism *between* the sun and the observer. It is not; when we see the rainbow, the sun is *behind* us. Also, the rainbow is not caused by simple refraction, as in a prism, but by a combination of refraction and total internal reflection; and so it is not a good analogy to breaking up light by a prism in the way Newton did."

22. In general, frequency $= \dfrac{\text{speed}}{\text{wavelength}}$

$$\text{For green light, } f = \frac{3 \times 10^8 \text{ m/sec}}{5 \times 10^{-7} \text{ m}} = 6 \times 10^{14} \text{ Hz}$$

The AM broadcast band runs from about 550 to 1,550 kHz, or from 5.5×10^5 to 15.5×10^5 Hz. Thus, the frequency of green light is about 10^9 times as great.

The FM broadcast band runs from 88 to 108 MHz, or from 8.8×10^7 to 10.8×10^7 Hz. The frequency of green light is roughly 10^7 times these values.

23. Suppose the windows of apartment A had polarizing sheets over the glass with the polarizing axis vertical, and apartment B had the sheets oriented with the axis horizontal. Then both apartments would receive light from the courtyard and the sky above, but light would not be able to travel from apartment A into apartment B or vice versa.

24. Suppose every vehicle had polarizing sheets over the windshield and headlights, with the polarizing axis oriented at a 45° angle from the lower left to the upper right as viewed by the driver. Consider light from headlights of an approaching car: In passing through the polarizing sheet over the headlights, it becomes polarized in a plane that is at a 90° angle to the polarizing axis on the windshield of the other car. Hence, the light would be blocked and the drivers of both cars would view the highway without being blinded by glare. Due to imperfections in the polarizing sheets and the action of dust on headlights and windshields the drivers *would* be able to see the other car's headlights, but only dimly.

CHAPTER 14

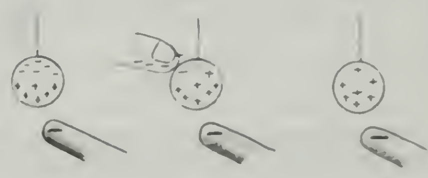
2. (a) The distance must be tripled.
   (b) The distance must be halved.
   (c) The distance must not be changed.

3.
$$F_{el} = \frac{kq_A q_B}{R^2}$$
$$R^2 = \frac{kq_A q_B}{F_{el}} = 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{(\text{C})^2} \times \frac{1 \text{ C}^2}{1\text{N}}$$
$$R^2 = 9 \times 10^9 \text{ m}^2 = 90 \times 10^8 \text{ m}^2$$
Therefore, $R = 9.5 \times 10^4$ m or 95 km.

4. The separation of charge occurs in such a way that one side of the object will be positive and the other negative. The difference in distance between these two concentrations of charge and the charging body results in a net force of attraction.

5. The ball does now have a net charge. One way to test whether it has or not would be to see if any charged objects repel it; repulsion can only occur between two charged objects.

   Negative charges are repelled into the finger when it touches the ball, leaving the ball positively charged.



6. (a) $g = \dfrac{GM}{r^2}$

$$= \frac{(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2})(7.3 \times 10^{22} \text{ kg})}{(1.74 \times 10^6 \text{ m})^2}$$
$$= 1.6 \text{ N/kg}$$

(b) To calculate the mass of the star:
$$M = \text{density} \times \text{volume}$$
$$= 10^{22} \text{ kg/m}^3 \times \frac{4}{3}\pi(1.5 \times 10^6 \text{ m})^3$$
$$= 14.1 \times 10^{40} \text{ kg}$$
$$g = \frac{GM}{r^2}$$
$$= \frac{(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2})(14.1 \times 10^{40} \text{ kg})}{(1.5 \times 10^6 \text{ m})^2}$$
$$= 4.2 \times 10^{18} \text{ N/kg}$$

(c) Assume a uniform density D for the planet, so that the mass inside a sphere of radius r is

$$M_r = \frac{4}{3}\pi r^3 D$$

At a distance r from the center, the effect of all matter outside a sphere of radius r would cancel out. The field due to the matter inside r is

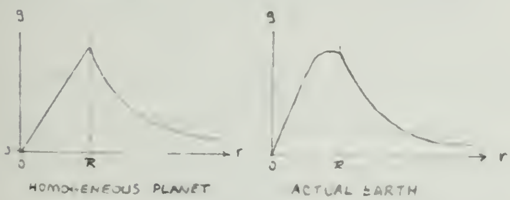
$$g_r = \frac{G \cdot M_r}{r^2}$$

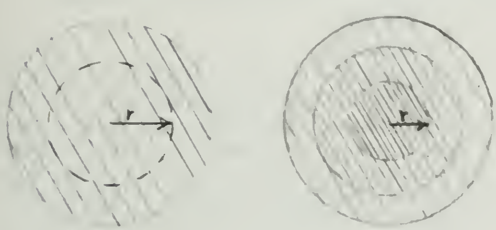
$$= \frac{G \cdot \frac{4}{3}\pi r^3 D}{r^2}$$

$$= \left(\frac{4}{3}\pi G D\right) r$$

Thus, a gravitational field inside a homogeneous planet would increase in direct proportion to the distance from the center.

Since the planet earth is actually several times more dense in the middle than near the surface, moving closer to the dense core overcompensates the amount of material left outside, and the field strength increases for some distance into the earth.

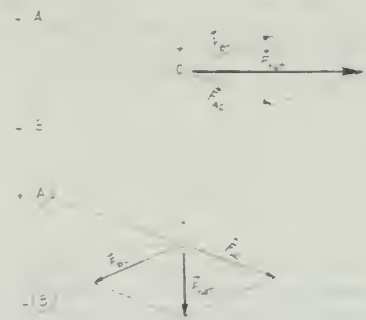




The case for the actual earth is rather complicated. However, some students might enjoy taking average values for core and mantle, making separate graphs for the fields due to each, and adding them to make a rough graph of total field against r.

7. An equal but oppositely directed force must be experienced by the field. Since the field is the connection between the charged particle and the source of the field, the source of the field also experiences the reaction force.

8.



9. (a) $F = \frac{k(Q_1 Q_2)}{R^2}$, $Q_1 = 4 \times 10^{-2}$ C,

$$q_e = 1.6 \times 10^{-19}\text{ C}$$

For $Q_2 = 3q_e$:

$$F = \frac{(9 \times 10^9\ \text{N·m}^2/\text{C}^2)(4 \times 10^{-2}\ \text{C})(4.8 \times 10^{-19}\ \text{C})}{(2.5\ \text{m})^2}$$

$$= 2.76 \times 10^{-11}\ \text{N east}$$

For $Q_2 = 6q_e$:

$$F = 5.5 \times 10^{-11}\ \text{N east}$$

For $Q_2 = 10q_e$:

$$F = 9.2 \times 10^{-11}\ \text{N east}$$

For $Q_2 = 34q_e$:

$$F = 3.1 \times 10^{-10}\ \text{N east}$$

(b) $E = k\frac{Q_1}{R^2}$

$$= \frac{(9 \times 10^9\ \text{N·m}^2/\text{C}^2)(4 \times 10^{-2}\ \text{C})}{(2.5\ \text{m})^2}$$

$$= 5.76 \times 10^7\ \text{N/C}$$

The forces are the same as in part (a).

(c) The field concept specifies the field at any point that will interact with a charge at that point. See part (b).

10. (a) The formula for the electric field strength of a spherically symmetrical charge is $E = kQ/r^2$. In the MKS system of units that uses volts and coulombs, the value of k is about 10^{10}. The radius of the earth is about 10^7 m. If the field strength at the surface is 10^2 N/C, then

$$10^2 = \frac{10^{10}\,Q}{(10^7)^2}$$

$$Q = \frac{10^{14} \times 10^2}{10^{10}}$$

$$= 10^6\ \text{C}$$

(b) The formula for the surface area of a sphere is $4\pi r^2$. For the earth, this is roughly $10 \times (10^7\ \text{m})^2 = 10^{15}\ \text{m}^2$. If 10^6 C of charge is distributed over 10^{15} m^2, the average charge density would be 10^{-9} C/m^2. Common static charges on combs, people, etc., are something like 10^{-9} C. So 10^{-9} C/m^2 is a fairly small static charge.

11.

12. (a) Since the droplet (or plastic sphere) has received a net charge, some other part of the system (perhaps the small opening through which the droplet was forced) must have received an equal and opposite charge.

(b) Air friction is a help because it makes a small charged body stop moving if the electric force and gravitational force on the body are balanced. When the body stops moving the air friction becomes zero and the only forces then acting are electric and gravitational, which are then known to be equal.

13. Call n the number of electrons required for 1 C of charge. Then

$$n \times 1.6 \times 10^{-19}\ \text{C/electron} = 1\ \text{C}$$

$$n = \frac{1}{1.6 \times 10^{-19}}\ \text{electrons}$$
$$= 6.25 \times 10^{18}\ \text{electrons}$$

14. $$\frac{F_{el}}{F_{grav}} = \frac{k\dfrac{q^2}{R^2}}{G\dfrac{m^2}{R^2}} = \frac{kq^2}{Gm^2} = \frac{k}{G}\left(\frac{q}{m}\right)^2$$

Therefore

$$\frac{F_{el}}{F_{grav}} = \frac{9 \times 10^9\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}}{6.7 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}\left(\frac{1.6 \times 10^{-19}\ \text{C}}{10^{-30}\ \text{kg}}\right)^2$$

$$= 3.4 \times 10^{42}$$

15. (a) $$\frac{mv^2}{R} = \frac{kq^2}{R^2}$$

Thus, $$mv^2 = \frac{kq^2}{R}$$

and the $$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{kq^2}{R}$$

(b) $$KE = \frac{1}{2}\frac{kq^2}{R}$$

$$= ½ \times 9 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \times \frac{(1.6 \times 10^{-19}\ \text{C})^2}{10^{-10}\ \text{m}}$$

$$= 4.5 \times 10^9 \times 2.56 \times 10^{-38} \times 10^{10}\ \text{N}\cdot\text{m}$$

$$= 11.6 \times 10^{-19}\ \text{J}$$

$$= 1.2 \times 10^{-18}\ \text{J}$$

(to two significant figures)

(c) $½mv^2 = KE$

Thus, $$v = \sqrt{\frac{2KE}{m}}$$

$$v = \sqrt{\frac{2 \times 1.2 \times 10^{-18}\ \text{N}\cdot\text{m}}{10^{-30}\ \text{kg}}}$$

$$= \sqrt{2.4 \times 10^{12}\ \frac{\text{m}^2}{\text{sec}^2}}$$

$$v = 1.5 \times 10^6\ \text{m/sec}$$

Note: $$\frac{\text{N}\cdot\text{m}}{\text{kg}} = \frac{\dfrac{\text{kg}\cdot\text{m}}{\text{sec}^2}\cdot\text{m}}{\text{kg}} = \frac{\text{m}^2}{\text{sec}^2}$$

16. A metal comb will not acquire a net charge because it is a good conductor and any charge is "grounded" by the person holding the comb. That is, any separation of charge occurring between the comb and the person's hair is immediately redistributed. However, if the comb were insulated from the hand by some material, then it *could* acquire a net charge.

17. The potential difference would be

$$\frac{6 \times 10^{-4}\ \text{J}}{2 \times 10^{-5}\ \text{C}} = 30\ \text{J/C or } 30\ \text{V}$$

18. The electric potential energy must be *the same* everywhere in the region or the electric field must be *zero* everywhere in the region.

19. One volt is one $\dfrac{\text{joule}}{\text{coulomb}}$ or, since a joule is a newton-meter, one $\dfrac{\text{newton-meter}}{\text{coulomb}}$. Therefore,

$$\text{one}\ \frac{\text{volt}}{\text{meter}}\ \text{is one}\ \frac{\dfrac{\text{newton-meter}}{\text{coulomb}}}{\text{meter}} = \text{one}\ \frac{\text{newton}}{\text{coulomb}}$$

The greater the electric field, the greater will be the electric force on a charged particle and so the greater will be the work done on it in moving through some distance. Thus, the greater the electric field, the greater the change of potential with distance. The rate of change of potential with distance (volts per meter) is a measure of the electric field strength (newtons per coulomb).

Another approach would be

$$\text{work} = qEd$$

$$\text{potential difference} = \frac{\text{work}}{q} = \frac{qEd}{q} = Ed$$

$$\text{So } E = \frac{\text{potential difference}}{d}$$

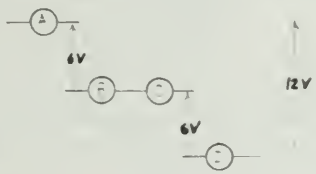
20. A potential difference of 30,000 V across a distance of 1 cm implies an electric field strength in the gap of

$$\frac{30{,}000\ \text{V}}{0.01\ \text{m}} = 3{,}000{,}000\ \text{V/m}$$

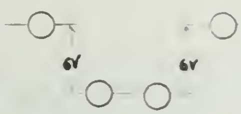
In most cases, the field wouldn't be uniform and so this would be an average value. The air between *pointed* electrodes 1 cm apart can break down when the potential difference between them is only about 10,000 V, because the field is so intense near the points.

21. A potential difference of 10,000 V across a distance of 1 mm implies an electric field in the gap of $\frac{10{,}000\ \text{V}}{0.001\ \text{m}} = 10{,}000{,}000$ V/m. Even in the small gap region the field wouldn't be uniform, so this value is only an average.

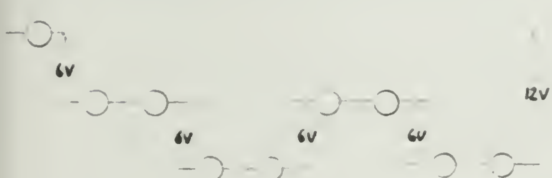
22. (a) The potential at A is 12 V higher than the potential at D.



(b) The potential of the terminal at the left is the same as the potential of the terminal at the right. Both are 6 V above the middle terminals.



(c) The potential of the terminal at the left is 12 V higher than the potential of the terminal at the right.



23. (a) kinetic energy gained = 100 eV or 100 eV $\times\ 1.6 \times 10^{-19}$ J/eV = 1.6×10^{-17} J

(b) $\frac{1}{2}mv^2 = KE$

$$v = \sqrt{\frac{2KE}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-17}\ \text{J}}{10^{-30}\ \text{kg}}}$$

$$= \sqrt{32 \times 10^{12}\ \frac{\text{m}^2}{\text{sec}^2}}$$

$$= 5.7 \times 10^6\ \text{m/sec}$$

24. (a) $I = \frac{E}{R}$

$$= \frac{12\ \text{V}}{3\ \Omega}$$

$$= 4\ \text{A}$$

When the voltage is doubled,

$$I = \frac{24\ \text{V}}{3\ \Omega}$$

$$= 8\ \text{A}$$

(b) Voltage describes a potential *difference* between two points and is measured *across* the circuit. Current describes the flow of electric charge *through* the circuit.

25. $R = \frac{E}{I}$

$$= \frac{100\ \text{V}}{4\ \text{A}}$$

$$= 25\ \Omega$$

When the current is cut in half,

$$E = IR$$

$$= 2\ \text{A} \times 25\ \Omega$$

$$= 50\ \text{V}$$

If Ohm's law does not apply, nothing is known about the relation of current to voltage.

26. $P = IE$

$$= 3\ \text{A} \times 50\ \text{V}$$

$$= 150\ \text{W}$$

$$P = I^2R$$

$$R = \frac{P}{I^2}$$

$$= \frac{150\ \text{W}}{(3\ \text{A})^2}$$

$$= 16.7\ \Omega$$

27. The initial power use (before the cut) is:

600 W (bulbs)
200 W (TV)
5,000 W (dryer)
25 W (radio)
5,825 W (total)

After the 5% voltage reduction, the power used is

$$5{,}825\ \text{W} \times 0.95 = 5{,}534\ \text{W}$$

You are not being cheated because you pay for power use in watts.

28. The maximum power through the circuit breaker before it will open is

$$P = IE$$
$$= 10\ A \times 120\ V$$
$$= 1{,}200\ W$$

With the stereo drawing 500 W, 700 W remain available. Therefore,

$$\frac{700\ W}{150\ W/\text{lamp}} = 4 \text{ lamps of } 150\ W \text{ each}$$

29. Voltage	Current	Resistance
(a) 2 V	**$I = V/R$ = 4 A**	0.5 Ω
(b) 10 V	2 A	**$R = V/I$ = 5 Ω**
(c) **$V = I/R$ = 15 V**	3 A	5 Ω

Note: Answers in bold type are to be supplied by the student

30. (a) If thunder clouds are roughly 1,000 m high, then a field strength of about 10^4 V/m under the cloud would imply an earth–cloud potential difference of roughly 10^7 V.
(b) A charge of 50 C transferred across a potential difference of 10^7 V (that is, 10^7 J/C), would release roughly 5×10^8 J of energy as heat, light, and sound.

31. The rate at which the kinetic energy of the charges in the beam is being increased is $4 \times 10^6\ V \times 4 \times 10^4\ A = 1.6 \times 10^{11}$ W! The pulse lasts for only 3×10^{-8} sec, so the energy of a pulse is $1.6 \times 10^{11}\ W \times 3 \times 10^{-8}\ \text{sec} = 4.8 \times 10^3$ J. It isn't clear from the advertisement what accuracy is intended for the figures, nor is it clear whether they are average or peak values. So, the expression "5,000 J" is not unreasonable, but certainly it seems that the company hasn't sold itself short.

32. Power $= VI = 20 \times 10^3\ V \times 10^{-3}\ A = 20\ W$

33. $P = VI$, or $P = I^2R$. Using either of these relationships, the power in each part of question 29 is
(a) 8 W
(b) 20 W
(c) 45 W

34. Since the current is in a horizontal wire, its circular magnetic field at the same horizontal level is vertical. Thus, the compass needle would tend to turn in a vertical plane, which its suspension was not designed to allow. (The compass *would* have responded if it had been held over or under the wire, providing that the current in this situation does not happen to be in an east–west direction. Why?)

35. (a) to the north
(b) 1 A to the north

36. (a) As stated in Sec. 14.12: "One ampere is the amount of current in each of two long, straight parallel wires, set 1 m apart, that causes a force of exactly 2×10^{-7} N to act on each meter of each wire."
(b) To find the total force,

$$F = 2 \text{ wires} \times \frac{(5A)(8A)(3m)(3m)(2 \times 10^{-7}\ \text{N·m·A}^2)}{0.5\ m}$$
$$= 28.8 \times 10^{-5}\ N$$

Since the total force on the two wires is 28.8×10^{-5} N, the force on each wire is $28.8 \times 10^{-5}\ N \div 2 = 14.4 \times 10^{-5}$ N. Since each wire is 3 m long, the force on each meter of each wire is $14.4 \times 10^{-5}\ N \div 3 = 4.8 \times 10^{-5}$ N.

37. (a) $qvB = \dfrac{mv^2}{R}$; $R = \dfrac{mv}{qB}$; so $R \propto mv$
(b) Since $q/m = v/RB$, you would need to know the speed v, the magnetic field strength B, and the radius R of the particle's orbit.

38. (a) The period T is given by the circumference divided by the speed:

$$T = \frac{2\pi R}{v} = \frac{2\pi\left(\dfrac{mv}{qB}\right)}{v} = \frac{2\pi m}{qB}$$

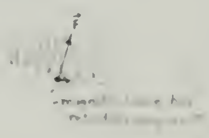
Note that T does not depend on v.
(b) The magnetic force Bev is a centripetal force, so we can write

$$Bev = \frac{mv^2}{r}$$

Solving this equation for r, we get

$$r = \frac{mv}{qB}$$

Obviously, r decreases as B increases.
(c) The force on the particle is perpendicular to both its velocity and the field. If the field lines converge, there will be a component of force in a direction away from the region of convergence. For certain combinations of v, m, q, and B, the particle will "reflect" and reverse direction along the field.



39. The magnetic field of the earth is directed from south to north. Using the right-hand rule you will find that positively charged bodies will be

deflected toward the east. They will appear to be coming "out of the west" and a directional detector would have an appreciably higher count rate if directed somewhat toward the west.

40. The students may wish to pursue this topic in their own locality by looking into such questions as:

1. What are the main air pollutants present? What are the causes of these pollutants?
2. Which of these could be successfully treated with an electrostatic precipitator? Which could not?
3. Are there any such devices currently in use in nearby plants?
5. How expensive is one of these to install? to operate?

CHAPTER 15

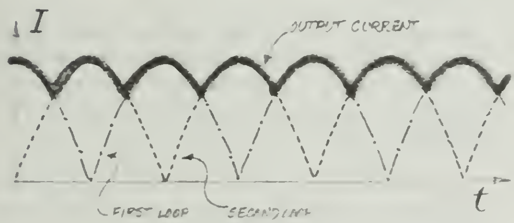
2. The main sources of energy were:
   (a) humans
   (b) the energy of work animals such as oxen, horses, etc.
   (c) the energy of moving water
   (d) the chemical energy of coal, wood, oil, gas, and other fuels, which could be changed to heat energy by combustion and then to mechanical energy by steam or by internal combustion engines

   The energy was transported by such things as trains, wagons, trucks; humans often walked. Oil and gas flowed in pipes to a place where it could be used; the energy of moving water was transported in flumes, etc.

3. Yes. Newton's third law of motion implies a reaction. To detect this force, suspend a loop of current-carrying wire that is free to turn in the vicinity of the magnetic needle. (One type of sensitive current-indicating meter uses such a mechanism; it is called the D'Arsonval galvanometer.)

4. In all cases except (d).

5.



6. (a) If you use the *right*-hand rule, you will predict that *positive* charges tend to move along *b* toward you. If you use the *left*-hand rule, you will predict that *negative* charges (free electrons) flow along segment *b* away from you.
   (b) The additional force is directed *upward*, opposite to the direction of motion of *b*.
   (c) The additional force is directed *downward*, opposite to the direction of motion of *a*.

   Again there is a reaction against what caused the motion of the wire in the first place. (There is a general principle, "Lenz's law," which states that induced motion of a charge is always in such a direction as to cause a reaction against whatever initially produced the motion.)

7. There is no magnetic opposition to the turning of the coil unless a current is being generated, and of course, there must be a complete circuit before there can be a current in the coil of the generator. The appliance serves to complete the circuit.

   When the current is being generated, the opposing magnetic force causes a torque (tuning effect) that opposes the applied torque. So work is required to rotate the coil; mechanical energy is changed into electrical energy.

8. The magnet falling *outside* the loop of wire reaches the ground first. The magnet passing through the loop induces a current in the loop. The induced current has a magnetic field that opposes the motion of the magnet (see answer to question 6). So when the magnet is above the loop, the force on it is repulsive, reducing its acceleration. When the magnet is below the loop, the force is attractive, again reducing the acceleration.

9. Using the hand rule twice shows that the additional force is *opposite* to the original direction of the charges. Thus, the current decreases. This decrease is due to the "back voltage" developed by the induced motion of the wire across the field.

10. The more slowly the motor goes, the greater is the current. As the motor speeds up, the "back voltage" across the coils increases, reducing the current in the coils.

11. (a) the series circuit
    (b) the series circuit
    (c) the parallel circuit
    (d) When resistances are added in parallel, the total effective resistance decreases and the total current and power increase.

12. In series, each resistor carries 5 A. In series, the total resistance is 8 Ω and the total current is 1.5 A. For each resistor, the current is 1.5 A at 6 V. In parallel, the total resistance is 2 Ω and the current is 6 A. For each resistor, the voltage is 6 V and the current is 4 A.

13. In series, the total resistance is 15 Ω. The total current is

$$I = \frac{50\ V}{15\ \Omega}$$
$$= 3.3\ A$$

The voltage across the 5 Ω resistor is 16.7 V at 3.3 A. The voltage across the 10 Ω resistor is 33.3 V and the current is 3.3 A. In parallel, the total voltage is 50 V and the total current is 15 A. The voltage is 50 V across both resistors. The current through the 5 Ω resistor is 10 A and through the 10 Ω resistor is 5 A.

14. (a) Each of the dozen lamps dissipates 10 W, so the entire set dissipates 120 W. The electric power input is

$$P = IV$$
$$120\ W = I \times 120\ V$$
$$I = 1\ A$$

(b) We know the power dissipation in each loop is I^2R, and the current in each bulb is 1 A, so R must be 10 Ω.

(c) If a 10-Ω lamp were connected directly across the 120-V line, the current would be

$$I = \frac{V}{R} = \frac{120\ V}{10\ \Omega} = 12\ A$$

The electric power going to the bulb would be

$$I^2R = 1{,}440\ W$$

The lamps would burn out very quickly, probably before the fuse in the wall circuit burned out. (Note that the total current for a very short time would be 144 A.)

15. (a) The electric power input is $P = IV$, so, 10 W $= I \times 120$ V. Thus $I = \frac{1}{12}$ A.

(b) The power dissipated by each lamp is I^2R, so 10 W $= (\frac{1}{12})^2R$, and $R = 1{,}440$ Ω.

The total current is the same as in the circuit in question 14, 1 A.

16. (a) $P = VI$

$$I = \frac{P}{V} = \frac{6\ W}{6\ V} = 1\ A$$

(b) The power loss in the connecting wires $=$ I^2R_w $(1\ A)^2 \times \frac{1}{5}\ \Omega = \frac{1}{5}\ W$

(c) $$I = \frac{6\ W}{12\ V} = \frac{1}{2}\ A$$

$$P = (\tfrac{1}{2}\ A)^2 \times \tfrac{1}{5}\ \Omega = \tfrac{1}{20}\ W$$

(d) For the 6-V system, the total resistance would be 6.2 Ω. Thus, the current would be $\frac{6.0\ V}{6.2\ \Omega} = 0.97$ (rounded to two significant figures.) The power loss would then be $(0.97\ A)^2 \times \frac{1}{5}\ \Omega$, or 0.19 W. The power used by the lamp would be $(0.97\ A)^2 \times 6\ \Omega = 5.6$ W.

For the 12-V system, the total resistance would be 24.20 Ω. Thus, the current would be $\frac{12\ V}{24.2\ \Omega} = 0.50$ A. Therefore, the power loss will then be nearly 0.05 W, and the power used by the lamp would be nearly 6 W.

17. The output power is assumed equal to the input power. So,

$$I_S V_S = I_p V_s$$
and $I_S \times 6\ V = \frac{1}{4}\ A \times 120\ V$

Thus, $I_S = 5$ A

18. The voltage ratio is the same as the turn ratio, that is,

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

where N_p and N_s represent the numbers of turns on the primary and secondary coils, respectively. Assuming 100% efficiency, the output power is equal to the input power:

$$I_S V_S = I_p V_p$$
$$\frac{I_S}{I_p} = \frac{V_p}{V_S}$$

Thus, equating both expressions for $\frac{V_p}{V_s}$,

$$\frac{I_S}{I_p} = \frac{N_p}{N_S}$$

19. The changing current and magnetic field of the primary coil, induces a changing magnetic field in the transformer core, which in turn induces a changing current and magnetic field in the secondary coil.

20. The low-voltage coil needs the thicker wire, because the current is greater in the low-voltage coil than in the high-voltage coil.

21. To produce a current in you, there must be a potential difference across you. Contact between a "live" power line and the car would create a large voltage between the car (and its contents) and the ground, but this would not in itself be dangerous. Leaping from the car could be safe; but if you were to touch the ground at the same time as you were touching the car. . . .

22. Some of the factors were: the unexpected daytime uses for electric power (in motors, street railways, elevators, sewing machines, etc.) and the fact that new industries needing constant power were attracted to a site at the falls.

23. Student report.

24. The students may draw on Sec. 15.8 and add some ideas of their own.

25. The efficiency of electric power plants is limited by the second law of thermodynamics. Modern power plants can achieve about 38–40% efficiency for fossil fuel plants (maximum 60%) and 30% for nuclear plants (maximum 50%).

CHAPTER 16

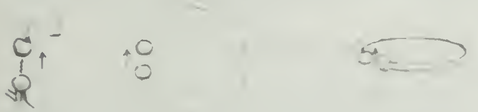
2. Oersted was inspired by a conviction, not supported by the existing evidence, that all physical phenomena are different forms of the same basic force.

3. No, a displacement current is only caused by a *changing* electric field. A steady electric field will hold + and − charges at a *constant* displacement, but a displacement current requires a shifting (a changing displacement) of charge.

4. An electromagnetic wave is initiated by an accelerating charge. It propagates itself by the mutual induction of electric and magnetic fields.

5. (a) the height of the water surface
   (b) the pressure or density of the air
   (c) the electric field strength and the magnetic field strength

6. Hertz could show they were polarized by rotating the detector ring: The sparks would be strongest when the two ends of the ring were in the same plane as the gap in the secondary winding of the induction coil.



7. Hertz showed that the waves had about the same speed as light and that they could be similarly reflected, diffracted, refracted, and focused.

8. (a) Other theories accounted for the same observations.
   (b) The concept of the displacement current seemed mysterious to many scientists.
   (c) Most scientists were unaccustomed to the field concept.
   (d) Prior to Hertz's work, no *new* property of electromagnetism had been discovered using a prediction from Maxwell's theory.

9. (a) Energy must be supplied to a source of electromagnetic radiation.
   (b) When waves are absorbed, the absorber is heated. The kind of waves that we experience in everyday life diffract (spread out behind obstacles); they do not travel in straight lines in all circumstances. Particles, on the other hand, do travel in straight lines. Since energy transfer seemed, for a long time, to be always in straight lines in the case of light, it was reasonable to favor a particle theory.

10. Solve the equation $f\lambda = c$ for the wavelength λ:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{60 \text{ Hz}} \text{ (alternating current)}$$
$$= 5 \times 10^6 \text{ m}$$
$$= 5{,}000 \text{ km (approx. } \tfrac{1}{8} \text{ the earth's circumference)!}$$

For standard AM radio frequencies, λ is between

$$\frac{3 \times 10^8 \text{ m/sec}}{500 \times 10^3 \text{ Hz}} = 600 \text{ m}$$

and

$$\frac{3 \times 10^8 \text{ m/sec}}{1{,}500 \times 10^3 \text{ Hz}} = 200 \text{ m}$$

For AM "Citizen Band" frequency, λ is around

$$\frac{3 \times 10^8 \text{ m/sec}}{27 \times 10^6 \text{ Hz}} = 11 \text{ m}$$

11. Wavelengths of "short" radio waves range from roughly 10 m to 100 m. They are short relative to the commercial AM broadcast band.

12. Wherever charged particles are accelerated, radio waves can result, and static can be produced in AM radios. Some other common sources are: sparking between brushes and commutator in motors; spark plugs in an automobile; radio emissions from the sun, especially during times of sunspot activity; radio emissions (21 cm) from many large hydrogen clouds in the galaxy; cyclotrons, and other particle accelerators; diathermy machines.

13. Newspapers and magazines print on their own paper. Radio and TV stations are broadcasting in a medium common to all, where regulation

and cooperation are required to prevent overlapping and interference.

14. (a) The approximate frequencies and wavelengths of the radiations and (b) their behavior at a 2-m space between buildings are as follows:

	(a)		(b)
	f (Hz)	λ (m)	
TV/FM	10^8	1	little diffraction
red light	10^{14}	10^{-6}	sharp shadow
infrared	10^{13}	10^{-5}	sharp shadow
60-Hz wires	10^2	10^6	great diffraction

15. The radiation that could escape from the surface into space would be FM and TV broadcasts, and visible light. AM radio wouldn't get through. Orbital satellites and radiation from nuclear explosions would be additional sources of information.

16. There are two reasons for the greater reception distances of radio waves than of waves used for TV and FM broadcasting. First, radio waves are trapped between the ionosphere and the earth, whereas the waves used for TV and FM pass through the ionosphere into space. Second, the longer wavelength of radio waves causes them to diffract more readily around obstacles such as mountains.

17. Kepler's third law states that $T^2 = kR^3$. We can find the constant k that applies for earth satellites by using the period T and radius a of orbit of the moon, the only natural satellite of the earth.

    From Unit 2, T of moon = 27.3 days and R = 384,000 km. For this problem the arithmetic is made easier if we call this distance 1 unit. Then,

$$K = \frac{T^2}{R^3} = \frac{(27.3\ \text{days})^2}{(1\ \text{unit})^3} = 743\ \text{days}^2/\text{unit}^3$$

Thus, for a satellite of period 1 day,

$$R^3 = \frac{T^2}{k}$$

$$R = \sqrt[3]{\frac{T^2}{k}}$$

$$R = \sqrt[3]{\frac{(1\ \text{day})^2}{743\ \text{day}^2/\text{unit}^3}} = \sqrt[3]{1.35 \times 10^{-3}\ \text{unit}^3}$$

$$R = 1.11 \times 10^{-1}\ \text{unit} = 0.111\ \text{unit}$$

$$\text{or } R = 0.111\ (384{,}000\ \text{km}) = 42{,}624\ \text{km}$$

18. TV waves are reflected well by a large conducting object like an airplane, and the reflected wave interferes with the direct wave at the TV antenna. As the distance of the plane changes, the phase difference of reflected and direct wave changes, so that the superposed wave amplitude at the antenna goes through maxima and minima.

19. The time required for the radar signal to travel from the earth to the moon and back to the earth will be the ratio of the total distance traveled divided by the speed of the signal. The distance to the moon (Actually we should use the radius of the orbit, but we will neglect the relatively small difference between the radius and the distances between the surfaces of the earth and moon.) is given as 3.84×10^5 km.

$$\text{Thus, the time} = \frac{2 \times 3.84 \times 10^5\ \text{km}}{3 \times 10^5\ \text{km/sec}}$$

$$= 2.56\ \text{sec (or 2.6 sec to two-figure accuracy)}$$

20. The trees and clouds appear bright in the photo because they absorb invisible light and reradiate energy of a longer wavelength (infrared). The sky appears black because infrared (due to its relatively long wavelength) is scattered very little by the earth's atmosphere. Also, because the air is cool it *emits* very little infrared radiation.

21. An argument can be made that natural selection favored organisms whose eyes were sensitive to the light that was most plentiful. Which wavelengths are most plentiful depends on the sun's output and the transparency of the earth's atmosphere. (For the earth, both of these factors peak at roughly the same wavelengths; it isn't clear whether such a nice match is necessary for the development of seeing beings.)

22. The thermometer will receive ultraviolet radiation beyond one end of the visible spectrum and infrared radiation beyond the opposite end.

23. An interesting discussion topic.

24. A principal reason for loss of support of the ether concept was that it wasn't used in the mathematical description of electromagnetism.

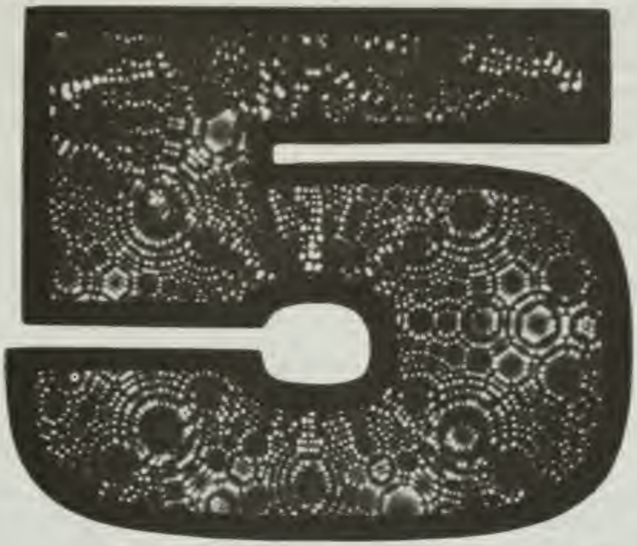
25. Other examples: inertial motion, circular orbits, the mobility of the earth, the remoteness of the stars, equal acceleration of all objects in free fall; some students may have heard of relativistic mass increase, time dilation, wave nature of matter, Heisenberg uncertainty principle.

26. Discussion.

27. The body of the cat is analogous to mechanical models of the ether; the grin represents the mathematical description of the electromagnetic field.

28. Essay.

29. Essay.



Models of the Atom

Organization of Instruction

THE MULTI-MEDIA SCHEDULE

Day 1
Lab stations: Law of Multiple Proportions

1. mechanical models of chemical compounds (See CHEM study materials.)
2. other mechanical models
3. *L45*, "Production of Sodium by Electrolysis"
4. Pass current through very weak H_2SO_4 solution. Measure ratio of volumes of gases produced.
5. Dalton's Puzzle Activity (See *Handbook*.)
6. "Black Box" Atoms (See *Handbook*, page 231 or CHEM Study Experiment.)

Day 2
Film: Definite and Multiple Proportions PSSC #0110 (30 min)

This film is used to tie together ideas introduced on Day 1 and hence does not require elaborate follow-up discussion.

The rest of the class period is spent solving *Study Guide* problems. See end of Chapter 17 for examples.

Day 3
E5–1: Electrolysis

Teacher or students gather data in advance. Experiment is shown qualitatively during class and calculations are made from previous data

Day 5
Lecture demonstration of cathode rays and Thomson $q_e m$ experiment. Treatment will vary with equipment available. A good ending for the class might be *L46* on the Thomson atom.

Day 6
E5–3: Measurement of Elementary Charge

Have Millikan apparatus set up in advance. Students spend first 15 to 20 min with apparatus. Since adequate data is difficult to obtain in a short time, data from previous experiments or from the *Resource Book* may be given out.

Day 7
Film: Millikan Experiment PSSC #0404

Show the first 15 min of the film up to the point where charge is changed by X-ray bombardment. Stop the film at that point and discuss briefly.

Teacher presentation: Introduction to Photoelectric Effect

Day 8
E5–4: Photoelectric Effect

Set up three or four stations using *Project Physics* amplifiers and photoelectric tubes. Half the class spends half the period gathering data.

Set up three or four stations using an electroscope and zinc plate. Charge the electroscope and shine light on the plate. Measure the rate of discharge. (A similar experiment is written up in PSSC.) Half the class spends half the hour on this, then rotates to *Project Physics* apparatus.

Day 11
E5–5: Spectroscopy
Students observe and make notes describing spectra from

1. incandescent lamp
2. neon lamp
3. hydrogen capillary tube
4. helium capillary tube
5. sodium vapor in flame
6. fluorescent tube

Day 12
Spectra continued
Students are issued photographs of spectra and asked to calculate wavelengths and identify elements. Photographs may be taken in advance by any students who are interested.

Days 13–16
Teacher presentations:

1. Thomson model of the atom
2. Rutherford's experiment
3. Rutherford's model of the atom

Days 17–19
Small-group library research on topics related to Unit 5.
Some examples:

1. special theory of relativity
2. De Broglie waves
3. wave–particle dualism
4. Compton effect
5. uses of spectra in astronomy

Days 19–21
Small groups prepare and present research material to class.
Encourage students to effectively communicate ideas that they have researched. Dramatizations, reading, and use of audio-visual aids will add interest and allow all students to participate.

Unit 5 SAMPLE MULTI-MEDIA SYSTEMS APPROACH

Note: This is just one path of many that a teacher may take through Unit 5.

1 Lab stations: Law of Multiple Proportions	2 P.S.S.C. film *Law of Definite and Multiple Proportions*	3 Demonstration: E 5-1: Electrolysis	4 CHEM Study film: *Chemical Families*
Text: prologue, 17.1–17.2	Text: 17.2	Text: 17.3	Text: 17.3
5 Lecture–demonstration: Cathode-Ray Tubes	6 E 5-3: The Measurement of Elementary Charge	7 P.S.S.C. film: *Millikan Experiment*	8 E 5-4: Photoelectric Effect
Text: 18.1–18.3		Text: 18.4–18.5	Text: 18.6
9 Discussion: Photoelectric Effect	10 Teacher presentation: X rays	11 E 5-5: Spectroscopy	12 Demonstration–experiment: Spectra
Text: 19.1–19.3		Text: 19.4–19.7	Text: 19.8–19.9
13 Teacher presentation: Models of the Atom	14 P.S.S.C. Film (40 min): *Rutherford Atom*	15 Teacher presentation: Bohr Model of Hydrogen Atom	16 Teacher presentation: Where Bohr Fails. . . .
Text: 20.1–20.2	Text: 20.3–20.4		Text: 20.5–20.6
17 Small-group research	18 Small-group research continued	19 Groups prepare presentations	20 Class presentations of special topic
Special Learning	Special Learning	Special Learning	Special Reading
21 Presentations continued	22 Review Unit	23 Unit Test	

Unit 5 SUGGESTED SCHEDULE BLOCKS AND TIMETABLE

Each block represents one day of classroom activity and implies approximately a 50-min period. The words in each block indicate only the basic material under consideration or the main activity of the day. The suggested homework (listed above each block) refers mainly to the **Text** and **Handbook**, but is not meant to preclude the use of other learning resources.

CHAPTER 17 A SUMMARY OF SOME IDEAS FROM CHEMISTRY

Text: Prologue, 17.1–17.2	Text: 17.3	Text: 17.3 and HB: E 5-1		Review
Chemical properties and the periodic table	Synthesis of electricity and matter	Lab E 5-1: Electrolysis Effects	Post lab and/or problem seminar	Test

CHAPTER 18 ELECTRONS AND QUANTA

Text: 18.1–18.2	Text: 18.3 and HB: E 5-3		Text: 18.4–18.5 and HB: E 5-4	Text: 18.6–18.7
Discovery of the electron	Lab E 5-3: Measurement of Elementary Charge	Post lab and/or problem seminar	Lab E 5-4: Photoelectric Effect	X rays Atomic models

CHAPTER 19 THE RUTHERFORD–BOHR MODEL OF THE ATOM

Review	Text: 19.1 and HB: E 5-5	Text: 19.2	Text: 19.3–19.4	Text: 19.5–19.8
Test	Lab E 5-5: Spectroscopy	Post lab discussion: Balmer relation	Rutherford's model	Bohr theory

CHAPTER 20 SOME IDEAS FROM MODERN PHYSICAL THEORIES

Text: 19.9–19.11	Review	Text: 20.1	Text: 20.2	Text: 20.3–20.4
Shortcomings and prelude to a new theory	Test	Mass–energy equivalence	Particle behavior of waves	Wave behavior or particles

Text: 20.5–20.6	Review			
Quantum mechanics	Chapter test	Unit review	Unit test	Discuss unit test

CHAPTER 17 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
Prologue					R 4 Ideas and Theories		Prologue
17.1 Elements, atoms, and compounds	2 3	4 5 6 7			F 35 Definite and multiple proportions F 36 Elements, compounds, and mixtures R 1 Failure and Success		17.1
17.2 Electricity and chemistry	12 13	9 10 11	8	D 53 Electrolysis of water E 5-1 Electrolysis			17.2
17.3 The periodic table			14	D 53 Electrolysis of water E 5-1 Electrolysis	R 21 Looking for a New Law T 35 Periodic table F 37 Counting electrical charges in motion L 45 Production of sodium by electrolysis	Periodic table Electrolysis of water Single-electrode plating	17.3

CHAPTER 18 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
18.1 The idea of atomic structure					R 17 The Sentinel		18.1
18.2 Cathode rays		2		D 54 Charge-to-mass ratio for cathode rays E 5-2 The charge-to-mass ratio for an electron		Cathode rays in a Crookes tube Measuring q/m for the electron	18.2
18.3 The measurement of the charge of the electron: Millikan's experiment	3			E 5-3 The measurement of elementary charge	F 38 Millikan experiment		18.3
18.4 The photoelectric effect	4			D 55 Photoelectric effect E 5-4 Photoelectric effect	T 36 Photoelectric experiment	Lighting an electric lamp with a match	18.4
18.5 Einstein's theory of the photoelectric effect	5 6 7 9	8 11	10 16		T 37 Photoelectric equation F 39 Photoelectric effect R 5 Einstein R 19 Space Travel: problems of Physics and Engineering	Writings by or about Einstein	18.5
18.6 X rays	12	13 14					18.6
18.7 Electrons, quanta, and the atom	15				L 46 Thomson model of the atom		18.7

CHAPTER 19 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
19.1 Spectra of gases		2		D 56 Blackbody radiation E 5-5 Spectroscopy			19.1
19.2 Regularities in the hydrogen spectrum	3	4	5			Modeling atoms with magnets Another simulation of the Rutherford atom	19.2
19.3 Rutherford's nuclear model of the atom	6 7				T 38 Alpha scattering L 47 Rutherford scattering F 40 The structure of atoms F 41 Rutherford atom		19.3
19.4 Nuclear charge and size	9						19.4
19.5 The Bohr theory: the postulates		10	11		R 11 The Teacher and the Bohr Theory of the Atom		19.5
19.6 The size of the hydrogen atom			8		T 39 Energy levels: Bohr theory F 42 A new reality		19.6
19.7 Other consequences of the Bohr model					F 43 Franck–Hertz experiment	Measuring ionization: a quantum effect	19.7
19.8 The Bohr theory: the spectral series of hydrogen	12	13			T 35 Periodic table		19.8
19.9 Stationary states of atoms: The Franck–Hertz experiment	15	14		D 57 Absorption	R 19 The Sea Captain's Box	"Black box" atoms	19.9
19.10 The periodic table of the elements	17 18	16					19.10
19.11 The inadequacy of the Bohr theory and the state of atomic theory in the early 1920's		19 20 21	22				19.11

CHAPTER 20 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
20.1 Some results of relativity theory	5	2 3 4 6 7			R 6 Mr. Tompkins and Simultaneity R 7 Mathematics and Relativity R 8 Parable of the Surveyors R 9 Outside and Inside the Elevator		20.1
20.2 Particle-like behavior of radiation	9 10	8			F 44 Interference of photons		20.2
20.3 Wave-like behavior of particles	11 12 13				F 45 Matter waves	Standing waves on a band-saw blade Turntable oscillator patterns Standing waves in a wire ring	20.3
20.4 Mathematics versus visualizable atoms				D 58 Ionization potential	F 46 Light: wave and quantum theories R 12 The New Landscape of Science R 14 Dirac and Born R 15 I Am This Whole World: Erwin Schrodinger		20.4
20.5 The uncertainty principle	14	15 16			R 13 The Evolution of the Physicist's Picture of Nature		20.5
20.6 Probability Interpretation		17 19 20 21	18 22 23 24		R 16 The Fundamental Idea of Wave Mechanics		20.6
Epilogue					R 10 Einstein and Some Civilized Discontents		Epilogue

Background and Development

OVERVIEW OF UNIT 5

Evidence that supports an atomic theory of matter is presented in the first half of this unit. There is no single experiment upon which atomic theory is based. Rather, a number of experimental data, like the interlocking pieces of a puzzle, provide the basis for the theory.

PROLOGUE

One of the oldest, yet one of the most exciting current problems of physics concerns the nature of matter. As long ago as the fifth century B.C. it was suggested that material things are made up of small, indivisible particles; yet in the early part of the twentieth century reputable physicists could still challenge the validity of the atomic theory of matter. This merger of a rich history with current episodes of an incomplete story brings intrigue and excitement to the study of matter.

The Prologue to Unit 5 gives a brief sketch of the early theories of matter. All of these theories share the search for an explanation of the multitude of macroscopic changes in terms of a microscopic "stuff." The earliest thinkers argued whether the basic "stuff" was atomistic or continuous. Was there one basic "stuff" or were there several? Aristotle provided an answer that satisfied scholars for 2,000 years.

The early theories of matter all interpreted macroscopic change as the consequence of some kinds of transformations at the microscopic (invisible) level. Thus, several ingredients of modern atomic theory were present in these early theories of matter. Chief among these ingredients are (a) the forerunner of our concept of the elements and (b) the interpretation of change as a consequence of transformations among the "elements."

With these notions forming a vital part of our view of matter, the stage was set for some very penetrating questions. Many of the techniques developed by the alchemists were available as a means for carrying out experiments. These ideas, these questions, and these techniques, in the hands of Boyle, Lavoisier, and Dalton, were the beginning of modern chemistry.

Chapter 17 has been shortened in this edition of the *Text*. Many teachers reported that their students had studied chemistry and that the historical review of Dalton's work, while interesting, added little to their knowledge. The chapter has been completely rewritten to present tersely those aspects of chemistry that contributed significantly to the development of the physical concepts of atoms.

The internal structure of the atom is the subject of Chapters 18 and 19. The experiments of Thomson, Millikan, and Rutherford, along with Einstein's quantum interpretation of the photoelectric effect, set the stage for the Bohr model of the atom. The Bohr model was successful in correlating much of the data that had accumulated by 1913.

Chapter 20 surveys the quantum theory of matter, which followed the failure of the Bohr theory.

CHAPTER 17 / THE CHEMICAL BASIS OF THE ATOMIC THEORY

17.1 | ELEMENTS, ATOMS, AND COMPOUNDS

The historical background for this section is given in the Prologue to Unit 5. It should be pointed out that the work of the eighteenth century chemists provided important evidence for the atomic theory. Two accomplishments of these scientists were essential to the work of Dalton. First was the establishment of the concept of "element." This concept, which has its roots in antiquity, was brought close to its modern form by Lavoisier in 1789. Second was the establishment of quantitative methods as the approach to chemical problems. This latter point was particularly significant.

The postulates of Dalton's atomic theory reflect his confidence in the validity of the law of conservation of mass. As Dalton wrote,

> No new creation or destruction of matter is within the reach of chemical agency. We might as well attempt to introduce a new planet into the solar system, or to annihilate one already in existence as to create or destroy a particle of hydrogen.

Perhaps the most important contribution of Dalton was his emphasis on the weights of atoms. Again to quote Dalton:

> In all chemical investigations it has justly been considered an important object to ascertain the relative weights of the simples which constitute a compound.

("Simples" as used by Dalton is equivalent to elements.)

The conclusions about relative atomic masses are drawn in an indirect manner. The indirectness follows from the fact that several types of experimental results must be utilized to arrive at the relative atomic masses.

Dalton's theory provided an explanation for the conservation of mass and the law of definite proportions. Yet many questions were left unanswered. Chief among these was the question "What makes atoms unite with each other?" The idea of affinity was introduced in an attempt to "explain" why nitrogen will react with more hydrogen than will lithium. Unfortunately the idea of affinity explained nothing. It is equivalent to saying that wood burns because it is combustible.

The concept of combining capacity slowly developed as an attempt to understand and therefore to predict the way in which atoms would combine. By an analysis of previous experience one could predict the outcome of a reaction between two elements; however, one still could not understand why atoms united. Understanding did not come until the atom took on an internal structure

A note of caution: The word *valence* came to mean different things to different chemists; consequently, it has been abandoned in some modern chemistry texts. However even where the noun form is rejected the adjective form still finds broad usage. The bonding electrons are called *valence electrons* and these electrons are found in *valence orbitals*. A frequently used near-synonym of valence is *oxidation number*.

17.2 | ELECTRICITY AND CHEMISTRY

Two developments stimulated the discovery of new elements. The first was the precise definition of an element framed by Lavoisier. This definition suggested new experiments, for example the investigation of gases, which earlier had all been considered to be the same element. The second was the development of new physical techniques that aided in the discovery of new elements. Electrolysis and spectroscopy (Sec. 19.1) were experimental techniques that assisted in the quest for new elements. The periodic table, an empirical relationship found to exist among the elements suggested the properties of unknown elements and hence hastened their discovery (Sec. 17.3).

Elements were found that possessed very similar properties and classifications were made. Here is an example of the fact that classification is a very important kind of scientific activity. It represents the first step in bringing order out of chaos. Frequently the development of a theory is preceded by the classification of facts. Two such theories to be considered later, deal with atomic spectra and radioactive series. A number of broad classifications can be made by even the casual observer: organic versus inorganic, metal versus nonmetal solid versus liquid versus gaseous dense versus porous, etc. Such relationships found among the elements set the stage for the next development.

17.3 | THE PERIODIC TABLE

The accumulation of data concerning the properties of the elements and the discovery of relationships among the elements gave rise to a more general relationship based on the atomic weights of the elements. In Mendeleev's own words

The law of periodicity was a direct outcome of the stock of generalizations and established facts which had accumulated by the end of the decade 1860–1870: it is an embodiment of those data in a more or less systematic expression.

The periodic table is the basis for the periodic law that states that when the elements are arranged (ordered) according to their atomic masses a periodicity of their properties results. At about the same time that Mendeleev published his version of the periodic table, Meyer in Germany, enunciated the periodic law while in England Newlands noted the repetition of properties when the elements are arranged in order of atomic weights. Why then is Mendeleev credited with the discovery?

The reason is that Mendeleev did much more than produce an arrangement of the elements. Generally, the periodic law is regarded as growing out of the periodic table; however, as far as Mendeleev was concerned, perhaps the opposite was true. It appears that he was convinced of the validity of the periodic law and his arrangement of the periodic table merely conforms to it. Thus, titanium had to belong to the same family as silicon. This represents a departure from the schemes proposed by his contemporaries.

Furthermore, the placement of titanium under silicon left a vacancy under aluminum. This provided Mendeleev with an opportunity to use the periodic law to deduce the properties of an unknown element. This clinched it! Given the alternatives, the scientific community will always choose a theory that not only correlates data, but also predicts new results. Again to quote Mendeleev:

The confirmation of a law is only possible by deducing consequences from it, such as could not possibly be foreseen without it and by verifying these consequences by experiment.

In 1894, Sir William Ramsay wrote to Lord Rayleigh: "Has it occurred to you that there is room for gaseous elements at the end of the first column of the periodic table?" Prior to this letter Ramsay had been working with atmospheric nitrogen. He found that a small fraction of residual gas was left after the nitrogen had been absorbed by hot magnesium. A spectroscopic analysis of the gas showed it to be a hitherto unknown constituent of air. The gas was argon. Thus began a series of discoveries that resulted in a major modification of the Mendeleevian version of the periodic table.

Helium had been "known" since 1868. In 1868 helium was observed spectroscopically as a constituent of the sun's chromosphere It went unexplained until Ramsay in 1894 extracted small amounts of gas from uranium ore and discovered terrestrial helium. With helium (atomic weight 4.0) and argon (atomic weight 40) discovered the periodic table was again left with a vacancy. In 1897, Ramsay speaking in Toronto stated:

There should therefore be an undiscovered element between helium and argon with an atomic weight 16 units higher than that of helium and 20 units lower than that of argon . . . and pushing this analogy further still, it is to be expected that this element should be as indifferent to union with other elements as the two allied elements.

Thus, neon was predicted!

The modern periodic table is one of the most useful devices to the chemist. In experienced hands, it can lead to new discoveries of many kinds. Haber used the table in the development of his high-pressure catalytic synthesis of ammonia, an important industrial process. A more recent example is the discovery of the freons, which are important gases for use in refrigerators and air conditioning units. In a matter of a few hours after determining the desirable qualities of these refrigerants, Thomas Midgley, Jr. and two associates deduced from the relationships of the periodic table that the substance CCl_2F_2, a freon, should have the desired properties of being stable, nontoxic, nonflammable, noncorrosive, etc. This discovery led to the development of a large industry.

By 1900, a fundamental basis for the periodic law was needed. At this time, the periodic table was an empirical device. As an empirical device it was very useful, just as Kepler's laws were useful for calculation purposes. Until the inverse-square law was postulated by Newton, however, Kepler's laws remained a mystery. Likewise, until the internal structure of the atom was studied, the periodic table remained a mystery.

Here we see another great synthesis: two subjects, previously thought to be unrelated, are beginning to coalesce. Some of the other syntheses that have occurred are terrestrial physics and celestial physics, electricity and magnetism, electromagnetism and light, heat and matter, and space and time (Chapter 20). Syntheses always lead to a deeper understanding of phenomena. Here the synthesis is between electricity and matter; a link is established between them.

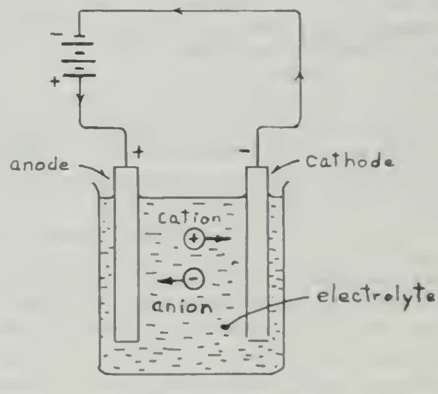
All substances seem to fall into one of two categories: those that allow an electric current to pass through them with ease and those that do not. The former we call conductors and the latter insulators. Conductors can be gaseous, liquid, or solid. Ordinarily, gases are poor conductors; however, when they are subjected to a high potential or to certain kinds of radiation, they become highly conductive. The solid conductors are the metals. (An important class of materials, such as silicon and germanium, also conducts electricity, but with difficulty. Such materials are called semiconductors.) Liquids that are electrical conductors are called electrolytes.

Among the first investigators of the interrelationships between electricity and matter were Humphrey Davy and J. J. Berzelius. Growing out of their work was one of the first attempts to systematize chemical behavior. Berzelius developed the *dualistic theory*, which was to form the basis for chemical combination. He assumed that chemical and electrical attraction were essentially the same. Atoms were believed to be polar. Chemical combination was the result of the interaction of the polar atoms. This theory, advanced in 1812, was important until the advent or organic chemistry in the 1830's. Then the dualistic theory became a handicap rather than a help.

Earlier it was established that chemical changes are brought about by electricity. This was conclusively demonstrated by Davy. Later Faraday, who started his scientific career as an assistant to Davy, established quantitatively the amount of chemical change caused by a given quantity of electricity.

One of Faraday's important contributions was the devleopment of the descriptive terms that are now universally used. In 1833, together with William Whewell, Faraday devised the terminology that enables one to describe the mechanism of electrolysis. The conductor, solution, or molten salt, is the *electrolyte*.* The conductors by which the positive current enters or leaves the electrolyte are the electrodes; the positive current enters the positive electrode, the *anode*, and leaves the negative electrode, the *cathode*. The charge particles that move toward the anode are called the *anions* and those that move toward the cathode are called *cations*.

In the process of electrolysis, interesting energetics are involved. The battery converts chemical energy into electrical energy; that is, a potential difference is established between the terminals, making the anode positive with respect to the cathode. Thus, positive particles are attracted to the cathode and negative particles to the anode. Where does the electrical energy go? It is dissipated as heat in the electrolyte and in the external circuitry.



The Process of Electrolysis

Quite early in his research, Faraday believed that a quantitative relationship could be established between chemical change and quantity of electricity. His efforts to discover this relationship were frustrated by the fact that secondary reactions

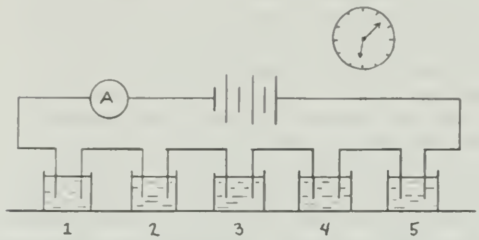
*All italicized words were devised by Faraday and Whewell.

often occurred to complicate the results. However after he developed the *volta-electrometer*, since 1902 called the *coulometer*, his work proceeded more smoothly. The coulometer served as a standard. When it was placed in series with other electrolytic cells, the amount of decomposition occurring in them could be compared to the amount of hydrogen liberated.

A typical experimental arrangement for verifying Faraday's laws of electrolysis is shown in the diagram. Five beakers are placed in series with a battery and an ammeter. A timing device is also necessary. If the experiment is conducted in such a way that the product of current and time equals 96,540 C, data such as shown in Table 1 will result. The relative masses have the same ratios as those determined from chemical analysis. Thus there is a quantitative connection between chemical change and amount of electricity.

This type of experiment greatly assisted the chemists with one of the major problems of their day: atomic weight values. Under explicit conditions, the amount of an element liberated at an electrode is called the equivalent mass. In Faraday's own words:

I have proposed to call the numbers representing the proportions in which they are evolved electrochemical equivalents.



A coulometer: Beaker 1: water with H_2SO_4
Beaker 2: HCl solution
Beaker 3: molten NaCl
Beaker 4: molten $MgCl_2$
Beaker 5: molten $AlCl_3$

Thus in Table 1, the equivalent masses of H, Na, Mg, and Al are 1, 23, 12, and 9. The equivalent masses of these elements determine how much of them will combine with other elements.

TABLE 1

Beaker	Element	Anode Weight Released (grams)	Element	Cathode Weight Released (grams)	Atomic Weight	Total Charge Passed (coulombs)
1	O_2	8.00	H	1.008	1.008	96,540
2	Cl	35.5	H	1.008	1.008	96,540
3	Cl	35.5	Na	23.0	23.0	96,540
4	Cl	35.5	Mg	12.16	24.32	96,540
5	Cl	35.5	Al	8.99	26.98	96,540

There is a profound implication of the laws of electrolysis. This implication was eloquently expressed by Helmholtz in 1881 in his Faraday Lecture delivered at the Royal Institution:

Now the most startling result of Faraday's law is perhaps this: if we accept the hypothesis that the elementary substances are composed of atoms, we cannot avoid concluding that electricity also, positive as well as negative, is divided into elementary portions which behave like atoms of electricity.

As we shall see in the next chapter, it was just 16 years after "atoms of electricity" were proposed by Helmholtz that the electron was discovered.

Thus, we have gone a full circle. We have used electricity to gain insight into the nature of matter and we have succeeded. However, our success was greater than we anticipated, for in the process we gained insight into the nature of electricity. The stage is set for the next part of our story in which the basic structure of the atom is explored.

CHAPTER 18 / ELECTRONS AND QUANTA

18.1 | THE IDEA OF ATOMIC STRUCTURE

Historically, one might say that physics began with Galileo and chemistry with Lavoisier and Dalton. It is important to stress at this point the intermingling of chemistry and physics. The concept of element postulated by Lavoisier became inseparable from Prout's law (Chapter 23). Prout's hypothesis (1815) of basic building-up units of matter found acceptance in latter-day physics and chemistry. Mendeleev's periodic table revealed a need for basic "structure" on which atoms might be built. However, a lack of any direct experimental evidence pointing toward a structure for atoms prevented further speculation. No reasons could be advanced at the time for the periodicity of elements with similar properties. It was left for later experimenters like Becquerel and others finally to find the way, and not until the theory of atomic structure was advanced by Bohr did the periodic table find complete acceptance and experimental support in the microscopic domain. See *From Atomos to Atom*, by Andrew G. van Melsen, a Harper Torchbook.

18.2 | CATHODE RAYS

The pioneer work of Geissler and Plücker led to subsequent discoveries connected with cathode rays. However, one must not forget to give due credit to Crookes. Crookes' new interest in vacuums led him to study the Geissler tubes. He perfected them for more efficient study of the radiation, and they have been called "Crookes' tubes" ever since.

Crookes represented the results dramatically and systematically. He showed that cathode rays travel in straight lines and can cast shadows. He also showed that the radiation can turn a small wheel when it strikes one side and that radiation can be deflected by a magnet. He was convinced, therefore, that he was dealing with charged particles and not electromagnetic radiation. Crookes spoke of these charged particles as a fourth state of matter, or an ultra-gas, as far beyond the ordinary gas in rarefaction and intangibility as an ordinary gas is beyond a liquid.

Crookes on several occasions nearly stumbled onto great discoveries that were eventually made by others. (More than once he fogged photographic plates during the operation of his tube, though his plates were contained in their boxes. However, he missed the connection and it was Roentgen who later, using Crookes' tube, discovered X rays.)

The aurora borealis phenomenon has been the subject of speculation ever since the time of Benjamin Franklin, who attributed the phenomenon to electricity. That it was due to electric rays from the sun was suggested in 1872 by A. B. Donati of Florence. Eugen Goldstein of Berlin held that they were cathode rays from the sun. Kristian Birkeland (1867–1917) of Christiania (now Oslo), adopting this view, constructed a miniature model of the earth (terrela) and exposed it to cathode rays in a vacuum tube. Later when this terrela was magnetized, it possessed an illumination concentrated upon a sprial path about the poles and a thin luminous ring about the equator. The mathematical theory of these phenomena was elaborated by Carl Stormer.

During the last decade of the nineteenth century, the scientific world was divided over the nature of cathode rays. The English school favored Crookes' theory that cathode rays consisted of tiny negatively charged particles. The German investigators were unanimously opposed. The German school was of the opinion that cathode rays were ether waves similar to the electromagnetic (radio) waves discovered by Hertz in 1887. This view was strengthened by Hertz's discovery that cathode rays were small enough to penetrate gold leaf.

If cathode rays really consisted of negatively charged particles, they would be deflected both by an electric field (electrical force) and by a magnetic field. Hertz was unable to detect such an effect in an electric field, no matter how he tried to perform this experiment. Thomson decided to repeat Hertz's experiment and also to remove what he thought was a discrepancy in earlier procedures.

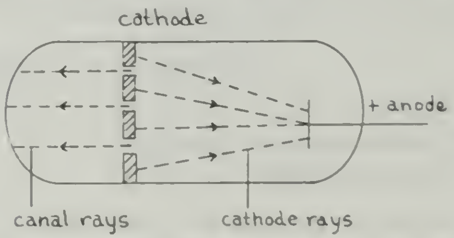
When Thomson tried to deflect cathode rays by passing them between electrically charged plates within the vacuum tube, he also obtained no result. Although he observed a slight flicker when the electric field was first turned on, he discovered that he could not obtain a permanent deflection no matter how strong he made his electric field.

After much thought Thomson finally theorized that the cathode rays were converting the particles of gas into charged ions upon collision, and these charged ions were then immediately attracted to the oppositely charged plate. Thus, the plates were neutralized by the ionized gas particles almost instantaneously and they could no longer produce an electric field.

The remedy was to have the gas in the discharge tube at a very low pressure. When this was done, Thomson was able to detect a beam of cathode rays being bent in the electric field. He obtained rays with the fantastically high velocity of 256,000 km/sec in subsequent experiments. His conclusions were then incontestable.

Cathode rays were first noticed in 1855. It may be helpful to mention here that in 1866 Goldstein discovered a new kind of radiation in the Crookes tube that he called "canal rays." These are discussed in some detail in Unit 6. If the cathode is made of a perforated metal plate (that is, one with holes in it), then, in addition to the cathode rays traveling from the cathode to the anode, a stream of rays behind the cathode is also observed. When the charges of these rays were found by deflection in electric and magnetic fields, they were discovered to be positive and integral multiples of elementary charges. If q_e = the elementary charge, then only integral multiples of q_e appear; that is, nq_e, where $n = 1, 2, 3, \ldots$

The masses of these positive rays depend on the nature of the substances in the tube. When hydrogen is used in the tube, the multiple of integral charge is always $n = 1$. This was, in fact, one of the first instances of the direct evidence of singly charged positive ions of hydrogen. In a sense, it was an "alter ego" of the electron.



The positive rays appear only when there is enough gas in the tube so that ionization can be produced by the collisions due to the cathode rays.

It may be significant at this point to mention positive rays so that later concepts of a positive center and negative orbit for a neutral atom in the Bohr theory may find easy acceptance.

18.3 | THE MEASUREMENT OF THE CHARGE OF THE ELECTRON: MILLIKAN'S EXPERIMENT

Among the several successful attempts made to measure the charge on an electron, those of J. S. Townsend, J. J. Thomson, H. A. Wilson, and R. A. Millikan are prominent. Several of these scientists are familiar today as Nobel Prize winners. Although Millikan's method was probably the most accurate and the simplest, he was not the only one who carried out experiments on the charged particles. From his measurements, Millikan demonstrated what had been previously surmised by Benjamin Franklin and by more recent physicists, namely, that electricity has a corpuscular structure.

As early as 1899, it was shown by John S. Townsend of Oxford that the positive or negative charge carried by an ion in a gas was equal to the charge carried by the hydrogen ion in the electrolysis of water. Millikan showed conclusively that electricity consists of equal units, that the electrtic charge of each single ion is always a multiple of this unit, and that this unit of charge is not merely a statistical mean, as the atomic weights have been shown to be.

For reasons of historical interest, consider Thomson's measurement of the electronic charge. He had already performed experiments yielding q_e/m values for the cathode rays, as explained in the *Text*. Though the results were consistent enough, Thomson was by no means certain that the difference between the q_e/m value for cathode rays and the q_e/m value for hydrogen ions in electrolysis was entirely due to the enormous difference in the relative masses of the hydrogen ion and the "corpuscle" (electron). Indeed, the charge on the negative electron could be several times that of the hydrogen ion and still leave the latter very much heavier than the former.

The only way to resolve this was to determine q independently. Thomson's method was described in a paper published in *The Philosophical Magazine*, December 1898. He used ions in a gas that acted like nuclei for condensation from supersaturated water vapor, thus producing a cloud of water droplets. One can determine the charge carried by each of them, and hence by the original ions in the following way.

By measuring the downward velocity of the cloud falling under gravity, Thomson calculated the radius of each droplet. This was done by using Stoke's formula for the terminal velocity of a sphere falling through a viscous fluid:

$$v = \frac{2(\rho - \sigma)ga^2}{9\mu}$$

μ = coefficient of viscosity of air
ρ = density of a drop
σ = density of air
g = acceleration due to gravity
a = radius of drop
v = velocity of drop

Next, the cloud was made to move under the influence of an electric field and the ions fell on a plate connected to a condenser. The rate at which the potential on the condenser changed was measured. Since the number of ions originally present could be calculated by knowing the amount of water that was condensed on all the ions, the charge on a single ion could be calculated. The results showed large variations, and the method was obviously inapplicable to the problem of determining the charge carried by individual corpuscles.

Contrast this with the simplicity and accuracy of Millikan's method. The smallest charge that Millikan measured had the value 1.6×10^{-19} C. All other electric charges were multiples of this charge. More recently discovered positrons have a charge equal in magnitude to that of electrons but opposite in sign. Since then, particles of various masses have been discovered, but so far all have charges that are integral multiples of the electronic charge. Likewise, recently discovered particles called *mesons* of masses 273 times that of the electron also have just one electronic charge.

There is no direct correlation between the mass of a particle and its charge. The electronic charge has been found on some of the lightest as well as some of the heaviest particles in existence, for example, a positron and a proton.

Millikan's method has now been outdated by visual methods using bubble chambers, etc., where the tracks of particles bent in magnetic fields can be actually observed and direct inferences can be drawn.

18.4 | THE PHOTOELECTRIC EFFECT

The photoelectric effect was discovered by Hertz in the course of his work designed to show experimentally that Maxwell's prediction of electromagnetic waves was correct. In particular, the wave theory of light had been incorporated in Maxwell's electromagnetic theory, and thus all observable phenomena were explained in terms of it. The example given below serves to illustrate why the wave theory in its electromagnetic form was incapable of giving the correct order of magnitude for the time taken in ejecting electrons.

Derivation of time delay required by wave model

In a properly lit classroom, there are at least 10 foot-candles of illumination at the laboratory tables. This is equivalent to about 1,500 ergs/sec/cm^2 of radiant energy.

Suppose that these 1,500 ergs/sec fall on 1 cm^2 of zinc, causing it to emit photoelectrons.

In zinc, the atoms are 2.5×10^{-8} cm apart. Thus, in the top layer of zinc atoms (1 cm^2) there are:

$$\frac{1}{(2.5 \times 10^{-8})^2} = 1.6 \times 10^{15} \text{ atoms/cm}^2$$

If the light consists of waves that penetrate 10 layers of atoms, its energy will be distributed over

$10 \times (1.6 \times 10^{15}) = 1.6 \times 10^{16}$ atoms. Now suppose that each atom absorbs an equal share of the 1,500 ergs/sec that are available. (Notice how our wave model differs from our particle model interpretation on this point.)

This is:

$$\frac{1.5 \times 10^{3} \text{ ergs/sec}}{1.6 \times 10^{16} \text{ atoms}}$$
$$= 9.4 \times 10^{-14} \text{ ergs/sec/atom}$$

The work function of zinc is about 4.9×10^{-12} ergs.

To acquire this much energy at the rate of 9.4×10^{-14} ergs/sec, an atom of zinc would have to "save up" for

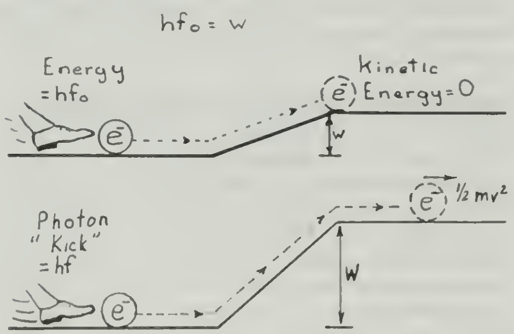
$$\frac{4.9 \times 10^{-12} \text{ ergs}}{9.4 \times 10^{-14} \text{ ergs/sec}} = 52 \text{ sec}$$

before it had enough energy to emit a photoelectron.

The work functions of some other materials yield times twice as long under similar conditions.

18.5 | EINSTEIN'S THEORY OF THE PHOTOELECTRIC EFFECT

Einstein's theory of the photoelectric process can be suitably explained in terms of the illustrative analogy shown in the following diagram. The illustration explains clearly the notion of work function and threshold frequency.



The kick hf_0 is just sufficient to overcome the potential height W, so that the electron at the top of the hill has zero kinetic energy. It is obvious that any energy less than hf_0 (which is just enough to send the electron up the potential hill) will fail to eject the electron.

A quantum "kick" of energy hf sends the electron up a height W and provides some additional kinetic energy as well. This explains the character of W as potential energy.

18.6 | X RAYS

X rays are similar to γ rays; however, the two have a different origin. X rays derive from atomic electrons, where γ rays originate in the atomic nucleus. Gamma rays have discrete energies whereas X rays from a conventional X-ray tube have a continuous energy spectrum over a considerable range of wavelengths. The "peak voltage" is used to characterize both the voltage at which the tube is operated and the maximum energy of the resulting X rays.

Supplementary X-ray Information

Frequency (Hz)	Wavelength (cm)
10^{16} to 3×10^{20}	3×10^{-6} to 10^{-10}

The frequencies of γ rays and X rays overlap at about 10^{18} Hz, where the lower limit of γ ray frequencies lies. Radiation of wavelength 3×10^{-6} cm could be produced by a voltage of 100 V across an X-ray tube, and would be called "soft" X rays. Wavelengths of the order of 3×10^{-9} cm are obtainable from tubes using 100,000 V.

Identical radiation, called γ rays, is also obtainable from nuclei. When electrons are decelerated, thus producing very high-frequency (or very small-wavelength) radiation, it is difficult to decide whether to characterize the radiation as X ray or γ ray. Electrons with energies up to a billion electron volts can produce such high-frequency radiation. Actually, the term X rays is reserved for lower-frequency radiation produced by the X-ray tube.

The decision to discuss γ rays at this point is left to the discretion and inquiry of the teacher and student, respectively. Since they are very similar types of radiation, there may be an advantage in a brief comparison.

Brief Note on Roentgen

Wilhelm Konrad Roentgen received the Nobel Prize in 1901. It is said that he objected strongly to calling his X rays by the name "Roentgen rays." He believed that scientific discoveries belonged to all people and that they should not in any way be hampered by patents. In fact, he chose to donate his Nobel Prize to the University of Wurzburg. Roentgen died on February 10, 1923, in his seventy-eighth year. Ironically, he died of cancer, a disease that often responds to treatment by X rays.

18.7 | ELECTRONS, QUANTA, AND THE ATOM

Directly after the discovery of cathode rays (electrons) and the indirect evidence for positively charged matter, speculations were made about the nature of neutral atoms. A number of theories of atomic structure were proposed in the period 1897 to 1907. The most prominent of all these theories was that advanced by J. J. Thomson. His theory attempted to meet the following requirements.

(a) The atom had to be a stable configuration of positive and negative charges.

(b) The atomic theory had to offer some explanation for the details of atomic spectra.

(c) The theory had to account for the chemical difference and resemblances of elements.

Thomson's interest in the relation between chemical properties and atomic structure may well have derived, in part at least, from the indifferent attitude of chemists to his discovery of the electron.

Thomson's model had to deal with a collection of positive and negative charges in equal number, where the charges obeyed the inverse-square law of attraction and repulsion. These charges would be required to settle in a position of stable equilibrium, without falling into one another. Oscillations of these charges about this stable position would result in the emission of line spectra. The problem was not an easy one, and was eventually explained differently. (See Sec. 19.5.)

J. J. Thomson proposed his theory in 1904. His model had stable distributions of rings of different numbers of electrons rotating within a sphere of positive electricity. He made an attempt to correlate his stable configurations to chemical properties as seen in Mendeleev's table. It may be worth mentioning at this point that this picture of the atom was not due to Thomson alone, and was, in effect, based on an idea previously suggested by Kelvin. Kelvin's theory was published in *The Philosophical Magazine*, Vol. 3, p. 257, 1902, entitled "Aepinus Atomised," in which Kelvin developed a theory of electricity based on the properties of electrons. J. J. Thomson's paper was first seen in 1904 in *The Philosophical Magazine*, Vol. 7, p. 237, entitled: "On the structure of the atom: an Investigation of the stability and periods of oscillation of a number of corpuscles arranged at equal intervals around the circumference of a circle; with application to the results of the theory of atomic structure."

In his mathematical treatment, Thomson considered corpuscles (electrons) at rest within the positive sphere, and also corpuscles in angular motion about the center of the sphere. He arrived at various configurations for various angular velocities and states of rest. The fundamental idea of his theory was that the atom consists of a number of corpuscles moving about in a sphere of uniform positive electricity. This theory raises at least three questions:

(1) How do the corpuscles arrange themselves in the sphere?
(2) What properties does this structure confer on the atom?
(3) How can this model explain the theory of atomic structure?

The details of Thomson's model are mathematical and much too complicated for class discussion. The model survived at the time because it seemed to offer more possibilities for further development than did other theories. A quotation from Thomson's original paper is as follows:

> The analytical and geometrical difficulties of the problem of the distribution of the corpuscles when they are arranged in shells are much greater than when they are arranged in rings, and I have not, as yet succeeded in getting a general solution.

CHAPTER 19 / THE RUTHERFORD–BOHR MODEL OF THE ATOM

19.1 | SPECTRA OF GASES

During the latter part of the nineteenth and early part of the twentieth century, two lines of interest were converging. The first was the interest in the nature of matter. With the discovery of the electron and radioactivity, new questions about the internal structure of the atom were being asked. The Thomson model of the atom was one attempt to answer some of these questions.

A second line of interest was the study of spectra. By the beginning of this century, a vast amount of spectroscopic data had accumulated. It was known that each element possessed a unique spectrum. Certainly, any proposed theory of atomic structure had to account for the origin and characteristics of spectra.

Since no experimental technique has contributed more to our understanding of the intrinsic structure of atoms and molecules than spectral analysis, some further comments on the technique will be given in the *Additional Background Articles*.

19.2 | REGULARITIES IN THE HYDROGEN SPECTRUM

In the Balmer formula, we once again encounter an empirical relationship. It is important to see the role played by such relationships. First, an empirical formula should never be identified with an explanation, or theory. Second, while empirical formulas do not provide understanding, they simplify and clarify what the theory must explain. Newton's inverse-square law very quickly took on significance when the known laws of Kepler could be deduced from it. Likewise, Bohr's model of the atom was enhanced when Balmer's formula could be deduced from it.

19.3 | RUTHERFORD'S NUCLEAR MODEL OF THE ATOM

During the year 1908, Rutherford and his associates, Geiger and Marsden, initiated experiments on the scattering of α particles by a thin metallic foil. In 1909, they observed to their surprise that α particles could be scattered through a large angle (>90°). The scattering experiments were completed in 1909; however, Rutherford pondered on their significance for a long time. Early in 1911, Geiger relates that

> One day Rutherford, obviously in the best of spirits came into my room and told me that he now knew what the atom looked like and how to explain the large deflections of α particles. On the very same

day I began an experiment to test the relations expected by Rutherford between the number of scattered particles and the angle of scattering.

In order to understand the drama associated with this discovery, it would be effective to recall the then current thinking on atomic structure. In the Thomson model of the atom, the mass of the atom was distributed uniformly through the volume of a sphere. With such an atom, only small-angle deflections of the α particle should be observed. An analogy can serve to show this quite convincingly. Think of the Thomson atom as a marshmallow. A bullet ("α particles") incident upon layers of such "atoms" would suffer little deflection. Since the Thomson model of the atom was the most popular with physicists, they were indeed surprised when α particles were observed coming backwards.

When Rutherford informed Geiger that he knew what the atom looked like and could explain the scattering results, what was the basis for experimental verification? In the first place, Rutherford envisioned a new model of the atom—the nuclear model: a massive, positively charged nucleus surrounded by planetary electrons. (Actually, an earlier nuclear model had been proposed by H. Nagaoka, a Japanese physicist. Nagaoka's model derived its inspiration from the planet Saturn. He envisioned electrons traveling in rings about a massive center forming a miniature Saturn-like system.) In its totality, however, the atom is mostly empty space. Now instead of the gold foil being thought of as layers of marshmallow-like atoms, it becomes an array of widely spaced massive nuclei.

Rutherford scattering is further discussed in the *Additional Background Articles*, page 364.

A very interesting biography of Rutherford has been written by E. N. Andrade. it is entitled *Rutherford and the Nature of the Atom* and appears in the *Science Study Series.*

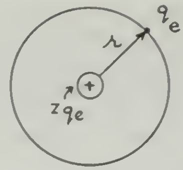
19.4 NUCLEAR CHARGE AND SIZE

Rutherford's scattering experiments were the beginning of an experimental technique that has been one of the most fruitful in producing information about the nucleus. Even today, scattering experiments are a widely used technique. By 1912 a rather precise model of the atom had emerged. The scattering experiments had indicated that:

(1) the mass of the atom was concentrated in a very small volume relative to the atomic volume. This was indicated by the fact that most α particles suffered no deflection whatsoever in passing through the foil.

(2) the concentration of mass carried a positive charge. This was suggested by the fact that of those α particles scattered, a very large fraction were scattered through small angles.

(3) the size of the nucleus was on the order of 10^{-14} m in diameter. This can be deduced by calculating the distance of closest approach of the α particle to the nucleus. The calculation is made by assuming that all the initial kinetic energy of the α particle is converted into electrostatic potential energy at the distance of closest approach. Thus knowing the masses of the α particle and the scattering nucleus and knowing the initial kinetic energy of the α particle, the nuclear diameter can be deduced.



The magnitude of the nuclear charge was found to be equal to the product of the atomic number and the electronic charge. Thus, the arrangement of the elements in the periodic table in terms of atomic number could now be related to atomic structure.

Moseley, who was killed during World War I, provided convincing evidence for the importance of the atomic number. He systematically followed up a discovery made by Bragg in 1913 that the heavier elements, when strongly excited, exhibit characteristic lines lying in the X-ray region of the spectrum. These X-ray spectra are quite simple. Each element gave peaks having a slightly different wavelength. There was such regularity that Moseley was able to express the results in the form of an empirical formula similar to the empirical formula of Balmer. This formula may be written

$$f = 2.48 \times 10^{15} (Z - 1)^2$$

where f is the frequency in cycles per second and Z is the atomic number. This equation led to the discovery of new elements. For example, when the known elements were arranged according to increasing frequency of their X-ray lines, a gap existed at $Z = 43$, indicating the existence of an element (now called technetium) then unknown.

In spite of its usefulness, the nuclear model was not without its difficulties. As a review of some of the mechanics learned in Unit 1, analyze the motion of a planetary electron. The centripetal force maintaining the electron in its circular orbit about the nucleus is supplied by the electrostatic attraction between electron and nucleus. Thus, we can write

electrostatic force = centripetal force

or

$$9 \times 10^9 \frac{Z^2 q_e^2}{r^2} = \frac{mv^2}{r}$$

Classically, an accelerated charge produces electromagnetic waves. Since an electron moving in a circular orbit is constantly accelerating, it should radiate. The expected frequency of the electromagnetic waves is just the frequency of the electron's motion about the nucleus. What is the frequency of the electron's motion?

frequency = number revolutions/second

$$f = \frac{v}{2\pi r}$$

From the relation above

$$\frac{v}{r} = \left(9 \times 10^9 \frac{Zq^3_e}{mr^3}\right)^{1/2}$$

so

$$f = \left(\frac{1}{2\pi}\right)\left(9 \times 10^9 \frac{Zq^2_e}{mr^3}\right)^{1/2}$$

If one assumes $Z = 1$ and $r = 0.5$ Å one finds that $f = 7 \times 10^{15}$ Hz.

Thus, the atom should be emitting ultraviolet radiation! If it does, it loses energy. If it loses energy, r gets smaller, the electron makes more trips around the nucleus per second making the frequency of emitted radiation even higher. On this basis the atom should collapse in less than 10^{-8} sec! The question is, how do we account for the stability of the atom?

19.5 | THE BOHR THEORY: THE POSTULATES

Before discussing the Bohr theory of the atom it might be wise to list the questions to be explained by any atomic model.

1. combining capacity: What determines the ability of an atom to combine with other atoms?
2. periodic law: What is the basis of the family relationships?
3. periodic table: Can any model give insight into the ordering of elements as they are in the periodic table?
4. electrolysis laws: Would an understanding of combining capacity provide an understanding of the laws of electrolysis?
5. scattering data: How can a nuclear model be stable?
6. spectra of elements: What is the origin of spectra?

All of the consequences of the Bohr theory can be logically deduced from his basic postulates. An alternative way of stating his postulates follows.

Postulate 1

The electron can exist only in certain stable circular orbits in which the electron obeys the laws of mechanics. (Here the word "stable" means that the electron does not lose energy by radiating.) When the electron is in a stable orbit the atom is said to be in a stationary (stable) state.

Note: This postulate avoids the difficulty discussed in Sec. 19.4 that the atom should be unstable. With this postulate, the atom is stable *by definition!*

Postulate 2

An atom can undergo a transition from one stationary state to another stationary state and in so doing emits or absorbs radiation of frequency

$$f = \frac{(\text{higher energy}) - (\text{lower energy})}{h}$$

where h is Planck's constant

Note: Such discrete energy changes would appear as a line spectra, where each line represents a specific energy change.

Postulate 3

The stationary states of an atom are those for which the angular momentum (mvr) of the atom is an integral multiple of $h/2\pi$.

19.6 | THE SIZE OF THE HYDROGEN ATOM

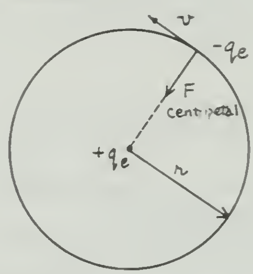
Consequences of Postuate 1:

coulomb force = centripetal force

$$C\frac{q^2_e}{r^2} = \frac{mv^2}{r}$$

or, $$Cq^2_e = mv^2r$$

Here the unknowns are v and r.



Invoke Postulate 3

$$mvr = n\frac{h}{2\pi}$$

With these two equations the unknowns can be determined. *From Postulate 3,*

$$v = \frac{nh}{2\pi mr}$$

and thus,

$$Cq^2_e = m\frac{n^2h^2}{4\pi^2m^2r^2}r = \frac{n^2h^2}{4\pi^2mr}$$

Solving for r_n:

$$r_n = \frac{n^2h^2}{4\pi^2mCq^2_e}$$

In this equation all the factors on the right are known so r can be computed. At this point we have a checkpoint for the theory. The diameter of hydrogen was known from kinetic data to be on the order of 1 Å. The above equation gives a value for the radius of 0.529 Å. This is a very encouraging result.

Defining r_1 as the value of r when the integer n equals 1, we can write

$$r_1 = \frac{h^2}{4\pi^2 mCq^2_e}$$

The radius for an arbitrary orbit is $r_1 = n^2 r_1$. This result means that only orbits with certain radii are permitted. Since $r_1 = 0.529$ Å, we have

$$r_2 = 4\,(0.529\text{ Å}) = 2.12\text{ Å}$$

$$r_3 = 9\,(0.529\text{ Å}) = 4.76\text{ Å}$$

etc. In this model, no other intermediate orbits, such as $r = 3.5$ Å, can exist.

19.7 OTHER CONSEQUENCES OF THE BOHR MODEL

Now the velocity of the electron can be determined.

$$\begin{aligned} v &= \frac{nh}{2\pi mr} \\ &= \frac{nh}{2\pi m} \times \frac{4\pi^2 mCq^2_e}{n^2h^2} \\ &= \frac{2\pi Cq^2_e}{nh} \end{aligned}$$

or,

$$v_1 = \frac{2\pi Cq^2_e}{h} \text{ and } v_n = \frac{1}{n}\, v_1$$

Again the factors on the right-hand side are known so that v can be computed.

With the velocity known, the energy can be determined. The total energy is the sum of the kinetic energy and the electrostatic potential energy, or

$$E = \tfrac{1}{2}mv^2 + \left(-C\,\frac{q^2_e}{r}\right)$$

Substituting our derived expression for r and v, we can write:

$$\begin{aligned} E &= \tfrac{1}{2}m\,\frac{4\pi^2C^2q^4_e}{n^2h^2} - Cq^2_e\,\frac{4\pi^2mCq^2_e}{n^2h^2} \\ &= \frac{2\pi^2mC^2q^4_e}{n^2h^2} - \frac{4\pi^2mC^2q^4_e}{n^2h^2} \\ &= -\frac{2\pi^2mC^2q^4_e}{n^2h^2} \end{aligned}$$

Or we can write $E_n = \frac{1}{n^2}\,E_1$, where E_1 can be computed from the known quantities to be equal to -13.6 eV. This was another experimental checkpoint of the Bohr theory, for the energy needed to remove the electron from the hydrogen atom was known to be approximately 13.6 eV. Note that the atom can exist only in certain energy states, namely,

$$E_1 = -13.6\text{ eV}$$

$$E_2 = -\tfrac{1}{4}(13.6\text{ eV}) = -3.40\text{ eV}$$

etc. In terms of this model there is no energy of -10.0 eV.

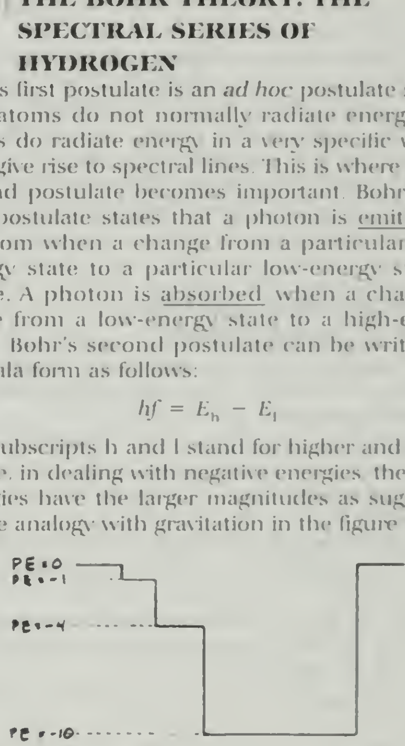
A gravitational analogy might be instructive at this point in the development. A gravitational "well" is pictured below. At "ground level" the potential energy is by definition zero. A stone can exist in stable positions (stationary states) at certain potential energies below the ground level that is defined as zero. Thus, energy must be absorbed for the stone to undergo a transition from the -10 level to the -4 level.

19.8 THE BOHR THEORY: THE SPECTRAL SERIES OF HYDROGEN

Bohr's first postulate is an *ad hoc* postulate stating that atoms do not normally radiate energy. Yet, atoms do radiate energy in a very specific way as they give rise to spectral lines. This is where Bohr's second postulate becomes important. Bohr's second postulate states that a photon is emitted by an atom when a change from a particular high-energy state to a particular low-energy state is made. A photon is absorbed when a change is made from a low-energy state to a high-energy state. Bohr's second postulate can be written in formula form as follows:

$$hf = E_h - E_l$$

The subscripts h and l stand for higher and lower where, in dealing with negative energies, the lower energies have the larger magnitudes as suggested by the analogy with gravitation in the figure below.



Write Bohr's second postulate in terms of the results of the last section:

$$\begin{aligned} hf &= E_h - E_l \\ &= \frac{E_1}{n^2_h} - \frac{E_1}{n^2_l} \\ &E_1\left(\frac{1}{n^2_h} - \frac{1}{n^2_l}\right) \end{aligned}$$

or

$$hf = -\left[\frac{2\pi^2mC^2q^4_e}{h^2}\right]\left(\frac{1}{n^2_h} - \frac{1}{n^2_l}\right)$$

All the factors in the brackets are known constants. The appearance of the integers in the parentheses is reminiscent of the Balmer formula. Balmer's formula was, however, written in terms of wavelength rather than frequency. When the last equation is rewritten in terms of wavelength it can be compared to the Balmer formula. (Note the change of algebraic sign.)

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{2\pi^2 mC^2 q^4_e}{ch^3}\left(\frac{1}{n^2_1} - \frac{1}{n^2_h}\right)$$

where the signs have been changed to make the first term positive.

The Balmer formula was written in terms of an empirical constant (the Rydberg constant) whose value is 109,677.59 cm^{-1}. The question is, what is the value of the fraction in the derived formula above, which is made up of known constants (the fraction in brackets)? Does it equal 109 677.58 cm^{-1}? It does! This was indeed a triumph for the Bohr theory.

The Balmer series of spectral lines occurs when $n = 2$ and $n_h = 3, 4 \ldots$. Thus, we can derive the Balmer formula in its entirety. In deriving it we understand it. Now the origin of spectral lines can be explained and understood. The mechanism at the atomic level responsible for the production of spectral lines is known. One of the main goals of an atomic model has been reached.

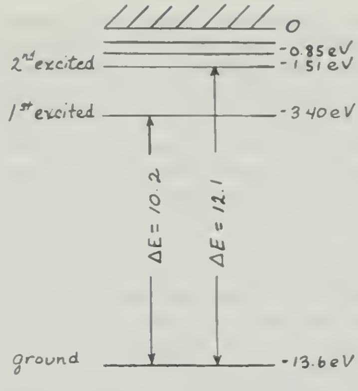
19.9 STATIONARY STATES OF ATOMS: THE FRANCK–HERTZ EXPERIMENT

The Bohr theory predicts that the energy of an atomic electron is quantized according to the relation $E_n = E_1/n^2$. A direct proof of the existence of discrete energy states in atoms and a confirmation of Bohr's view of the origin of emission and absorption spectra is provided by the experiment of Franck and Hertz.

The experiment can be understood in terms of energy principles. The energy states of the hydrogen atom are shown in the energy-level diagram that follows. The first excited state is 10.2 eV above the ground state and the second exicted state is 12.1 eV above the ground state. According to the Bohr theory of the hydrogen atom, its energies are precisely defined.

Franck and Hertz studied the collisions between electrons and heavy monatomic atoms. However, since we have already solved for the energies of the hydrogen atom, we shall use it as an example. (Hydrogen is diatomic and the Franck–Hertz experiment can only be done with difficulty when hydrogen is used.) The energy of the bombarding electrons can be controlled by controlling the potential through which the electron "falls."

Establish a potential difference of 5 V across the tube. With a potential difference of 5 V an electron can obtain a kinetic energy of 5 eV. If a 5-eV electron collides with a hydrogen atom, an elastic collision occurs; that is, the electron has essentially as much energy after the collision as before the collision.



The same would be true if the electron had any energy of less than 10.2 eV.

However, when the electron has an energy of 10.2 eV, a new result appears. The electron no longer collides elastically, but inelastically; that is, it loses energy in the collision. In fact, it loses all its energy! Now 10.2 eV is just the energy difference between the first excited state and the ground state.

An atom can also gain energy by photon absorption and lose energy by photon emission. In fact, in the Franck–Hertz experiment, after the atom gains energy by collisional excitation, it loses energy by photon emission. The emitted photon is the signal that excitation by electron collision has occurred.

When the electron energy reaches 12.1 eV, again the electron loses all its energy in the collision. The difference between the second excited state and the ground state is 12.1 eV!

Thus, the Franck–Hertz experiment is a demonstration that excitation of atoms by collision is governed by the Bohr quantization of energy.

19.10 THE PERIODIC TABLE OF THE ELEMENTS

The Bohr theory provided a model of the atom that can be correlated with the periodic table and the periodic law. In the Bohr model, electrons move in well-defined orbits. The chemical and physical properties of an element depend upon the arrangement of the electrons about the nucleus. To account for the periodic table and the periodic law, we must be able to determine the arrangement of electrons in the atom and show that the chemical and physical properties follow from this arrangement. This could not be done in a rigorous fashion until after the advent of quantum mechanics, yet the Bohr model was suggestive of the coming solution.

The electrons in an atom can be regarded as grouped into shells and subshells. Each shell and subshell has a fixed capacity for electrons; that is, no more than a certain number of electrons can be accommodated. The chemical and physical properties are related to the relative "emptiness" or "fullness" of the shells. For example, the inert gases have completely full shells. Thus, full shells can be associated with stability.

In Bohr's periodic table pictured on *Text* page 594, the inert gases form the turning points in the progression of the elements. The elements just before the inert gases are short one electron of having a full shell. These elements, short one electron, are the halogens. Thus, the halogens are found to be prone to react chemically with elements from which an electron can be captured, hence filling their shells to capacity. Likewise, the elements just after the inert gases, the alkali family, have one electron in excess of a full shell. Thus, the alkali metals are prone to react chemically with elements to which an electron can be given, hence leaving them with a filled shell. One could predict that the halogens and the alkali metals are ideally suited to react with each other.

In this manner, the periodic table and periodic law are explained. Thus, the Bohr model was instrumental in reaching another of the goals set up for any atomic model.

19.11 | THE INADEQUACY OF THE BOHR THEORY AND THE STATE OF ATOMIC THEORY IN THE EARLY 1920's

The Bohr model has been eminently successful. The goals established earlier have been reached. In review, the nuclear model of the atom was rendered stable by one major *ad hoc* postulate. The origin and mechanism of spectral lines was explained by the theory. The periodic law was given a basis in atomic structure.

In addition to these obvious successes, there was a more subtle one: namely, the Bohr theory left its indelible mark on physics. Bohr's emission and absorption of photons between stationary states remains predominant in the minds of spectroscopists. Bohr's model set the stage for further work. His quantization of angular momentum and energy was the beginning of a vital part of quantum mechanics.

Yet, for all its successes, it did not survive. Many questions that the theory was unable even to begin to answer concerned the intensities of spectral lines and the effects of a magnetic field on an atomic spectrum. To quote a contemporary physicist:

> The instant relief which Bohr's theory provided in sorting out the hopeless muddle of spectroscopic data, down to the wavelengths of the X rays, at first overshadowed all other considerations. Then, in the following years the strange emptiness of this ingenious and successful model began to impress itself on the minds of the physicists.

One of the principal reasons that the Bohr theory did not survive was that it represents a hybrid between classical and quantum ideas. It was not until new quantum ideas replaced classical ideas that these questions were answered.

CHAPTER 20 / SOME IDEAS FROM MODERN PHYSICAL THEORIES

20.1 | SOME RESULTS OF RELATIVITY THEORY

The theory of relativity brought about a revolution in the thinking of scientist and nonscientist alike. Because of its revolutionary nature, it has been regarded as an abstract theory, extremely difficult to understand. This is not the case. The theory of relativity is not abstract (at least not the special theory). The difficulty in understanding relativity occurs because some of our most basic concepts concerning space and time have to be reexamined and modified.

In Units 1 and 2, we saw the difficulty people had in accepting new modes of explanation. The idea that an object in violent motion tends to remain in motion was completely foreign to the Aristotelian natural philosopher. Another idea that was difficult to assimilate was the earth's daily rotation and annual revolution about the sun as assumed in the Copernican system. The concepts of inertia and axial rotation were only slowly accepted into the mainstream of our thought. Likewise, some time will be required for the concept of relativity to become a natural part of our thinking processes.

In Newtonian physics, physical phenomena are described in terms of position coordinates and momenta. When the description of a physical system is complete, it is sometimes desirable to express the state of the system in terms of a reference frame moving relative to the original reference frame. The results of transforming our description of events from one reference frame to another, and the form of physical law in aribtrary reference frames, is the concern of relativity theory.

The measurement of the speed of light has been of interest to scientists since the time of Galileo. Galileo's attempts to measure the speed of light led him to believe that the propagation of light is instantaneous. As he wrote in *Two New Sciences*:

> Everyday experience shows that the propagation of light is instantaneous; for when we see a piece of artillery fired at a great distance, the flash reaches our eyes without lapse of time; but the sound reaches the ear only after a noticeable interval.

The first successful experiment to measure the speed of light was concluded in 1675 by a Danish astronomer, Olaf Romer. From a study of the periods of Jupiter's satellites. Römer concluded that the speed of light was finite. Later experimenters using Römer's method concluded that the speed was in the neighborhood of 320,000 km/sec.

A great synthesis occurred when Maxwell showed that electromagnetic waves should propagate at the speed of light. This result suggested that light was electromagnetic in nature. However, Maxwell's equations gave the speed of light as a constant. Questions soon arose about the frame of reference to which Maxwell's value referred. The Michelson–Morley experiment was an attempt to answer these questions by isolating some absolute frame of reference. However, the Michelson–Morley experiment failed to do so.

Much confusion followed the failure to establish an absolute frame of reference and the way was cleared for Einstein's ideas. His special theory of relativity rests on two postulates: (1) The speed of light in space is the same for all observers regardless of their velocities with respect to the light source. (2) The laws of physics are the same when stated in terms of either of two reference systems moving at constant velocity relative to each other. Hence, the absolute frame of reference sought by the Michelson–Morley experiment does not exist.

The first and second postulate together form a basis from which many deductions can be made. Among these deductions are the equivalence of mass and energy and the velocity dependence of mass, length, and time.

The theory of relativity forced a reexamination of some fundamental physical concepts. The twentieth century has seen two such periods of reassessment. The second period of reexamination took place after the development of the quantum theory. These two periods of introspection have taught physicists to be more critical of common-sense notions that tend to dominate their thinking. (One might recall the common-sense appeal of the geocentric system.) Werner Heisenberg, one of the chief architects of quantum theory, has emphasized this point. Speaking of relativity, he says:

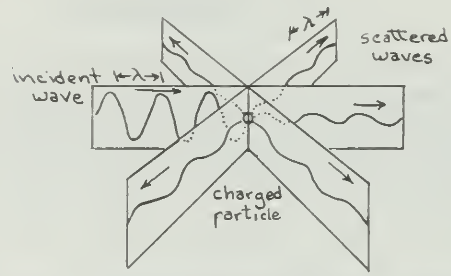
It was the first time that scientists learned how cautious they had to be in applying the concepts of daily life to the refined experience of modern experimental science. . . . This warning later proved extremely useful in the development of modern physics, and it would certainly have been still more difficult to understand quantum theory had not the success of the theory of relativity warned the physicists against the uncritical use of concepts taken from daily life or from classical physics.

20.2 PARTICLE-LIKE BEHAVIOR OF RADIATION

In Chapter 18, we studied the photoelectric effect in which the theoretical treatment assumed a particle nature of light. In the photoelectric effect, a photon loses all of its energy in ejecting a bound electron. The initial energy of the photon appears as the binding energy (work function) plus the kinetic energy of the photoelectron.

A second example in which light must be treated as corpuscular is known as the Compton effect. In this example, the photon collides with an electron and is scattered. Unlike the previous example, the photon carries some energy away from the collision. This scattering experiment clearly distinguishes between the wave and particle models of light.

When light (treated as a wave) is incident upon a charged particle, it is scattered in all directions, as the next figure shows. The incident wave sets the charge into oscillation; the oscillating charge in turn radiates electromagnetic radiant energy in all directions. However, as the figure shows, there is no change in wavelength. (There is a change in the amplitude since the energy of the incident wave is being spread in all directions.)



Early scattering experiments showed, however, that the scattered radiation was less penetrating and seemed to have a longer wavelength than did the incident radiation. This observation contradicts the prediction based on the wave theory of light. However, when light is considered as photons (that is, particles), one can predict that the scattered radiation should have a longer wavelength than the initial radiation, as is observed.

If we now endow our light particle with all the properties of a particle, namely, energy and momentum, we can derive the change in the wavelength of a scattered photon. The result of this derivation is as follows:

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0 c}(1 - \cos\phi)$$

where $\Delta\lambda$ is the change in wavelength, λ_0 is the wavelength of the incident photon, λ is the wavelength of the scattered photon, and ϕ is the angle through which the photon is scattered.

The derivation of this formula is very tedious algebraically; however, the argument in terms of physical concepts is very simple and is based upon the conservation laws studied in Unit 3. Let the frequency of the incident photons be f_0 and the frequency of the scattered photons be f. Then, if energy is conserved, we must have

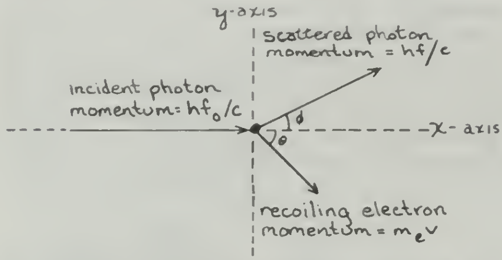
$$hf_0 = hf + \tfrac{1}{2}m_0v^2$$

where m_0 is the rest mass of the electron. If momentum, m_0v (a vector quantity), is also conserved, we have two equations representing two components. Thus, if momentum is conserved, we may write:

$$x\text{-component: } \frac{hf_0}{c} = \frac{hf}{c}\cos\phi + m_0v\cos\theta$$

$$y\text{-component: } 0 = \frac{hf}{c}\sin\phi - m_0v\sin\theta$$

where the angles are illustrated in the figure below.



For a particular scattering angle ϕ, we have three equations and three unknowns: f, v, and θ. We can solve these equations simultaneously for any of the unknowns. For comparison with experiment however, we are interested in the shift in the wavelength. Remembering that $\lambda_0 = c/f_0$ and $\lambda = c/f$, we can rewrite the three equations in terms of $\Delta\lambda$. The result is the same as that given above and in the *Text*.

With this equation, we can solve for $\Delta\lambda$ for any scattering angle ϕ. When we insert the known values for h, m_0, and c, we get:

$$\Delta\lambda = (0.0242)(1 - \cos\phi)\ \text{Å}$$

When $\phi = 90°$, $\cos\phi = 0$, and $\Delta\lambda = 0.0242$ Å. When visible light with a wavelength on the order of 5,000 Å is used, the percentage of change in the wavelength of the scattered light is too small to detect. However, when X rays with a wavelength of approximately 1 Å are used, the percentage of change in the wavelength of the scattered X rays is observable. This change in the wavelength is known as the Compton effect.

20.3 | WAVE-LIKE BEHAVIOR OF PARTICLES

Apparently everything in the physical world can be classified into one of two categories, matter and radiation, a symmetry that impressed Louis de Broglie. He was also impressed by the dual nature of radiation. These considerations led de Broglie to question the particle nature of matter. Why, he asked, should not the same dualism that characterizes the basic quanta of radiation also characterize the basic quanta of matter? That is, should particles also have wave characteristics? He developed his ideas and presented them in the form of a doctoral dissertation.

As we have stated them, de Broglie's ideas sound highly speculative. However, de Broglie did much more. First, he gave a definitive relation equating the wave properties with the particle properties (*Text* page 608). Second, he was struck by the fact that the stability condition for atomic orbits, namely the quantization of angular momentum, introduced integers. With his postulate, he was able to deduce the quantum condition of Bohr studied in Chapter 19. Thus, the wave nature of matter gave a basis for understanding the Bohr atom; that is, those orbits are allowed that represent an integral number of wavelengths.

Of course, the impressive result came when the de Broglie hypothesis was verified experimentally. To demonstrate wave properties, one does a diffraction experiment that crucially depends on the wave nature. Davisson and Germer showed that electrons could be diffracted and thus demonstrated the wave nature of electrons.

20.4 | MATHEMATICAL VERSUS VISUALIZABLE ATOMS

The first step in understanding the internal structure of the atom was provided by the work of Rutherford and Bohr, discussed in Chapter 19. It might be argued that this was the most decisive step. Yet, while Bohr's theory was stimulating, it lacked the breadth to accommodate all the facts and could not be structured to answer the vital questions. It became obvious that a new, more fundamental approach was needed. Bohr himself was one of the leaders in the quest for deeper understanding.

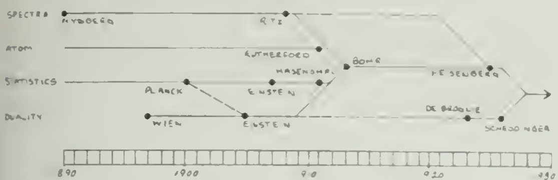
We have seen that the first big breakthrough came with the de Broglie hypothesis. On the basis of this hypothesis, Schrodinger developed a mathematical formalism that came to be known as wave mechanics. The mathematical representation of waves has deep roots in classical physics. Water waves, sound waves, and radio waves are significantly different. The first is a transverse mechanical wave, the second a longitudinal mechanical wave, and the third an electromagnetic wave. However, in spite of their differences, the same type of mathematical equation will describe all three. Schrödinger, viewing the electron as a wave, wrote an equation that is very similar in mathematical form to the equations that describe water waves, sound waves, and light waves. His equation is written below. (This equation is not necessarily to be shown to the students; however, there may be some students whose curiosity has been aroused and who would like to see this famous equation.)

$$-\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) - \left(\frac{e^2}{\sqrt{x^2 + y^2 + z^2}}\right)\psi = E\psi$$

With the Schrodinger equation, properties of the hydrogen atom that include not only the results of Bohr, but many more besides can be computed.

Heisenberg developed an alternate approach to quantum mechanics. His approach grew more directly from the experimental data of atomic spectra. The two approaches, Schrödinger's and Heisenberg's, were ultimately shown by Dirac to be equivalent.

The various lines of investigation that converged to form the impetus for the quantum mechanical revolution are illustrated below.



A sketch indicating the different roads (and main contributors) that have led to our present understanding of quantum mechanics: spectral analysis, atomic structure, statistical mechanics, and wave–particle duality of both light and matter. (Adapted from F. Hund, *Physics Today,* August 1966, Vol. 19, p. 25.)

20.5 | THE UNCERTAINTY PRINCIPLE

One of the consequences of the wave–particle dualism is the uncertainty principle, first enunciated in 1927 by Heisenberg. This principle sets a fundamental limit on the ultimate precision with which we can simultaneously know both the position of a particle and its momentum.

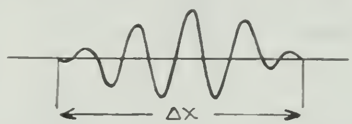
In Newtonian mechanics, even as modified by special relativity, the position and momentum relative to a given frame of reference can be both exactly and simultaneously defined. Then, with Newton's laws of motion, all future motions of the particle can be accurately determined. This result formed the basis for a philosophy of mechanistic determinism. One of the most ardent disciples of this philosophy was Laplace, a French mathematician. He articulated the essence of this philosophy as follows:

An intellect which at any given moment knew all the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit its data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom: for such an intellect nothing could be uncertain; and the future just like the past would be present before its eyes.

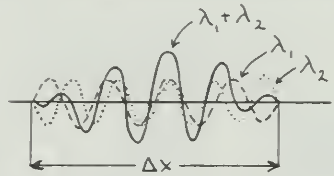
This statement represents quite a contrast with current physical thought. The conclusion of current physical thought is that we are unable to know the present exactly, hence we cannot know the future exactly. For example, if we attempt to measure precisely the position of an electron, we lose all knowledge of its momentum. The very act of measurement changes the position or motion of the particle (as the example on page 609 of the *Text* suggests), with the result that its future positions cannot be precisely predicted.

The uncertainty principle is also a consequence of the de Broglie hypothesis. This hypothesis relates the wavelength of a particle to its momentum. Therefore if we know the momentum exactly, we also know the wavelength exactly. But any wave with an exact wavelength (such as a sine wave) is a continuous wave having infinite extent. If, then, a particle is represented by a wave having infinite extent, its position is completely undefined. We conclude, therefore, that precise momentum (exact wavelength) implies an undefined position.

A localized wave, in contrast to an infinite wave, can be built up from a superposition of sine waves. The figure below shows such a localized wave. A particle, represented by such a wave packet, can be located within an uncertainty of Δx. The synthesis of such a wave packet from the two sine waves of wavelengths λ_1 and λ_2 is shown below.



The question now is, what is the wavelength (momentum) of the particle? The superposition of two waves of different wavelengths is required to give the particle some localizability. Thus, in localizing



the particle we have lost our precise knowledge of its momentum (wavelength). And so it goes. We purchase knowledge of the one at the price of uncertainty of the other: complete knowledge at the price of complete ignorance.

Further discussion of the uncertainty principle appears in the January 1958 *Scientific American* in an article by George Gamow, "The Principle of Uncertainty."

20.6 | PROBABILITY INTERPRETATION

In Born's interpretation, the square of the amplitude of the de Broglie wave is related to the probability of finding a particle. A particle is most likely to be observed in regions of large amplitude.

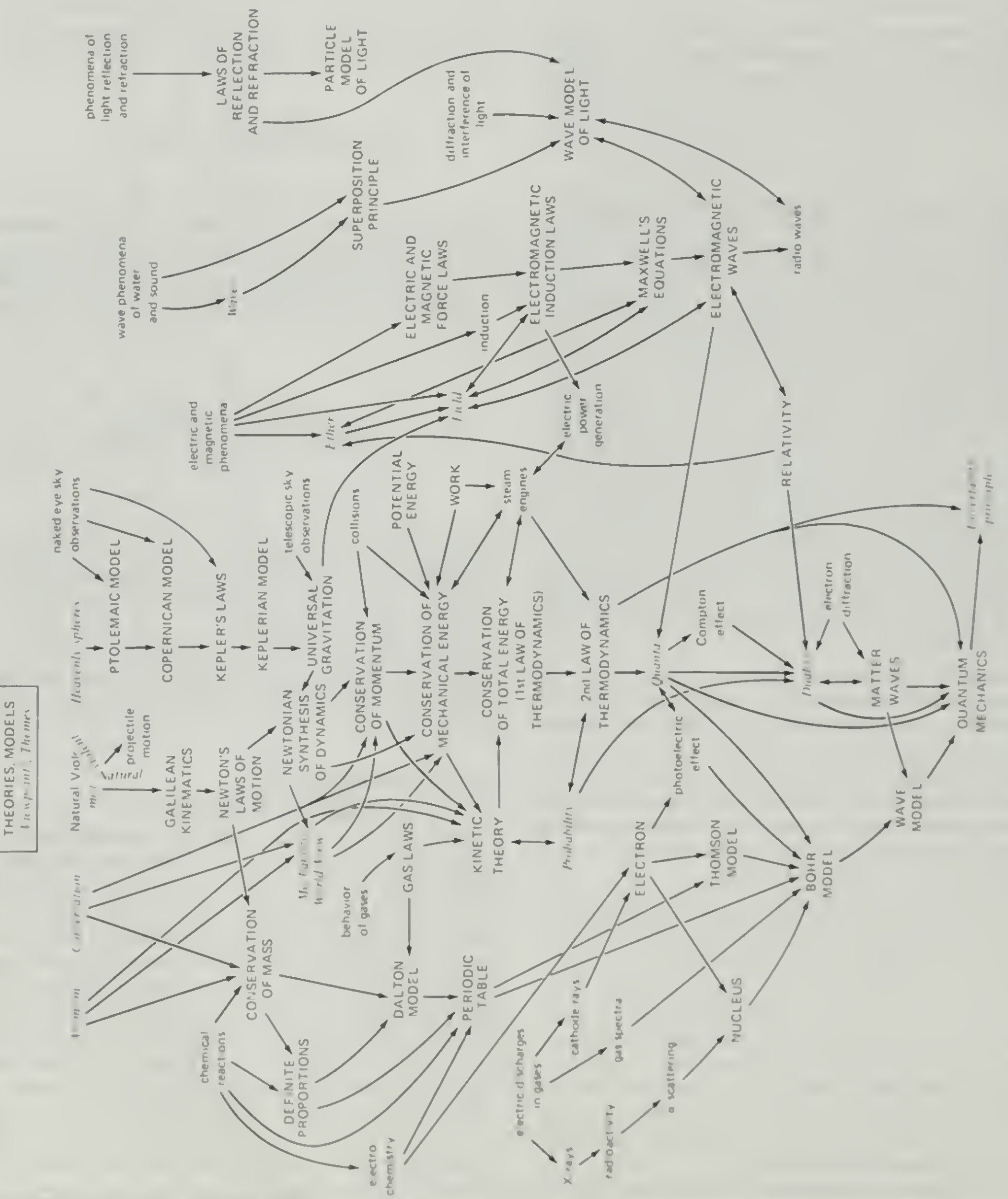
How does one find these amplitudes? They are found by solving the Schrödinger equation. As was pointed out in Sec. 20.4, Schrödinger's equation describes the motions of electrons. We now see that Schrödinger's equation gives us an amplitude, the square of which is a probability. Thus, we can compute where an electron is most likely to be.

These considerations have forced physicists and chemists to change their conception of the atom. The Bohr model of the hydrogen atom pictured the electron traveling in prescribed "planetary" orbits about the nucleus. Such a view is inconsistent with the intrinsic nature of the electron, that is, its dual nature. In fact, an electron having an energy of -13.6 eV can be found anywhere. One is most likely, however, to find the -13.6-eV electron at a distance of 0.529 Å from the nucleus, which corresponds to the first Bohr orbit. Thus, that which is most probable on the submicroscopic level corresponds to that which is observed on the macroscopic level.

This relation between the microscopic and the macroscopic is reminiscent of the kinetic theory of gases. In kinetic theory, the average value of dynamic variables on the molecular level is related to observable parameters. Our ability to compute the values of these observables applies only to those systems made up of large numbers of particles.

In a similar fashion, we are able to compute the values of observables in quantum theory because we are dealing with extremely large numbers of particles. With large numbers of particles, that which is highly probable for one overwhelmingly determines the macroscopic behavior of the system of particles.

CONCEPT FLOW CHART UNIT 5



Additional Background Articles

THE DETERMINATION OF RELATIVE ATOMIC MASSES

There are three approaches that can be taken to determine relative atomic masses.

Chemical reaction method
The relative masses of elements entering into a reaction can be determined. In this way, for example, it can be demonstrated that in the formation of water the mass of oxygen is 7.94 times the mass of hydrogen. If one now assumes, as did Dalton, that water consists of one oxygen atom and one hydrogen atom, then one can conclude that the mass of one oxygen atom is 7.94 times the mass of one hydrogen atom.

Electrolysis method
A second line of evidence comes from electrolysis experiments. The decomposition of water by electrolysis produces 1.008 g of hydrogen and 8.00 g of oxygen. Again, the mass of oxygen is 7.94 times the mass of hydrogen.

Gas density method
In the discussion of the kinetic theory of gases in Chapter 11, we saw a result of a calculation by Loschmidt. He calculated the number of molecules (N) in a cubic meter of gas at 0° C and normal atmospheric pressure. The result is

$$N = 2.687 \times 10^{25}$$

We can use this number together with gas density measurements to determine relative atomic masses. The density of hydrogen is:

$$\frac{N_H \times \text{mass of one hydrogen gas particle}}{1\ m^3}$$

Likewise, the density of oxygen under the same conditions is:

$$\frac{N_o \times \text{mass of one oxygen gas particle}}{1\ m^3}$$

But, by Loschmidt's work, $N_H = N_o$, so the ratio of densities is:

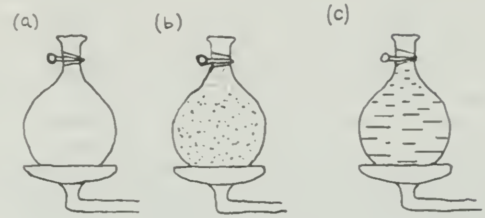
$$\frac{\text{density of hydrogen}}{\text{density of oxygen}}$$

$$= \frac{N_o \times \text{mass of one oxygen gas particle}}{N_H \times \text{mass of one hydrogen particle}}$$

$$= \frac{\text{mass of one oxygen gas particle}}{\text{mass of one hydrogen gas particle}}$$

Hence, a knowledge of densities can provide relative atomic masses.

Gas densities can be meausred by weighing a known volume of gas. This can be done in a few simple steps. First an evacuated bulb is weighed. It is then filled with gas under measured conditions of temperature and pressure and weighed again. The difference in weight gives the mass of the gas.



The bulk is then filled with water and weighed again. The difference between this mass and that of the evacuated bulb is the mass of the water. Since the density of water is known, the volume of the water, which is the same as the volume of the bulb, is determined. This volume is also the volume of the gas. The ratio of the mass of the gas to the volume of the bulb (or gas) gives the density of the gas. The densities of a number of gaseous elements at room temperature are listed in Table 1 below.

TABLE 1
Gas Densities of Some Elements at 298° K and 760 mm pressure

	Density (g/m^3)	Density relative to hydrogen density
Hydrogen	82.5	1
Nitrogen	1,146	13.89
Oxygen	1,309	15.87
Chlorine	2,900	35.2

$$\text{The ratio } \frac{\text{density of oxygen}}{\text{density of hydrogen}} = \frac{1.309}{82.5} = 15.87$$

Therefore, from the gas density data, it is concluded that the mass of one oxygen particle is 15.87 times the mass of one hydrogen particle. If we compare this value with the value obtained by the chemical reaction method and the electrolysis method, we see that our new value is twice as large. Two explanations could account for this discrepancy. Either a gas particle of oxygen contains twice as many atoms as a gas particle of hydrogen, *or* water contains twice as many hydrogen as oxygen atoms. In the first case, the relative atomic masses of oxygen to hydrogen would be 8:1; in the second case 16:1.

If we calculate other relative densities (density of element/density of hydrogen), we obtain the values in the last column of Table 1.

We now have two sets of relative densities for some elements: one set obtained from electrolysis and chemical reaction meausrements, the other from gas densities. To establish the relative masses of atoms, we must have some reliable way of assigning formulas to elements and compounds. Such a way was proposed in 1858 by S. Cannizzaro who called attention to a regular pattern developed from data like those shown in Table 2.

The second column of Table 2 shows values of gas densities measured at 100° C and 1 atm pressure. In the third column are values of the fraction of the weight of each substance due to the presence in it of hydrogen. This fraction can be determined by measuring the amount of water obtained when each of the indicated substances reacts with oxygen. On multiplying together the values for a given gas in the second and third columns we get the corresponding value in the fourth column. These values represent the numbers of grams of hydrogen per liter of each compound. That is:

$$\text{density}\left[\frac{\text{g compound}}{\text{liter}}\right] \times \begin{matrix}\text{weight}\\ \text{fraction}\\ \text{hydrogen}\end{matrix}\left[\frac{\text{g hydrogen}}{\text{g compound}}\right] = \frac{\text{g hydrogen}}{\text{liter}}$$

In the fifth column we show that the weights of hydrogen per liter of the various compounds stand to each other in the ratio of small whole numbers. What interpretation shall we place upon this strikingly simple relationship?

Loschmidt's number is independent of the gas under investigation. It implies that under specified conditions of pressure and temperature, a cubic meter (or any volume) contains the same number of molecules. It then follows that the masses of hydrogen listed in the fourth column of Table 2 calculated for equal volumes of the various substances, also represent the masses of hydrogen contained in equal numbers of molecules of the various substances. The atomic-molecular theory then clarifies why these masses should stand in the ratio of small whole numbers. A given hydrogen-containing substance may contain one atom of hydrogen per molecule (or 2 or 3 or 4 or more atoms of hydrogen per molecule). If we denote by y the number of molecules of substance present the numbers of atoms of hydrogen present in a unit volume of each substance must be $1y$, $2y$, $3y$, $4y$, etc. But, in that case, the number of grams of hydrogen present must stand in the corresponding small whole number ratio 1:2:3:4. . . . which is precisely the relationship expressed in the fifth column of Table 2. This explanation suggests the partial formulas given in the sixth column of Table 2.

This argument leads us to the somewhat surprising conclusion (see Table 2) that the gaseous particle of hydrogen is not a single atom but rather, a pair of atoms joined in an H_2 molecule. But, we are still unable to assign the complete formula of any compound. There is, however, no real reason for being discouraged: we can easily reach our goal by preparing a series of tables like that given below for hydrogen. Table 3 is such a table for oxygen.

As before, all entries in the fourth column are integral multiples of a minimum value, in this case 0.52. As before, the gaseous element itself is seen to consist not of individual atoms, but rather of diatomic molecules (O_2). We can assign partial formulas as indicated in the last column of the table.

Putting together the findings in both our tables we see that we have obtained the complete formula for water: H_2O. Once we have established such a formula, accurate atomic masses can be obtained from measurements of chemical combining masses. These are obtained more easily than accurate values of gas densities.

As noted earlier, 1 g of water contains

0.1119 g hydrogen
0.8881 g oxygen

Given that the formula of water is H_2O, we see that 0.8881 g is the mass of oxygen containing only half as many atoms of oxygen as there are atoms of hydrogen in 0.1119 g of that element. The masses containing equal numbers of the respective atoms would be 2 × 0.8881 g oxygen and 0.1119 g hydrogen, and the relative masses of individual atoms of the two species are given by the ratio of these two numbers: $(2 \times 0.8881)/0.1119 = 1.7762/0.1119 = 15.87/1.000 = 16.00/1.008$.

TABLE 2
Cannizzarro Method for Arriving at Formulas of Hydrogen Compounds

1	2	3	4	5	6
Substance	Gas density in grams per liter, at 1 atm and 100° C	Fraction by weight of hydrogen in substance	Grams of hydrogen in 1 L of compound	Values (from column 4) expressed as multiples of 0.033	Partial formulas
Hydrogen chloride	1.19	0.0276	0.033	1	$H_1Cl?$
Chloroform	3.90	0.00844	0.033	1	$C?H_1Cl?$
Hydrogen	0.066	1.00	0.066	2	H_2
Water vapor	0.59	0.112	0.066	2	$H_2O?$
Ammonia	0.556	0.178	0.099	3	$N?H_3$
Methane	0.525	0.251	0.132	4	$C?H_4$
Ethyl chloride	2.11	0.0781	0.165	5	$C?H_5Cl?$
Ethane	0.982	0.201	0.197	6	$C?H_6$

TABLE 3
The Formulas of Some Oxygen Compounds

1 Substance	2 Gas density in grams per liter, at 1 atm and 100° C	3 Fraction by weight of oxygen in substance	4 Grams of oxygen in 1 L of compound	5 Multiples of 0.52 g per liter	6 Partial formulas
Nitric oxide	0.98	0.533	0.52	1	$N?O_1$
Water vapor	0.59	0.888	0.52	1	$H?O_1$
Carbon monoxide	0.915	0.571	0.52	1	$C?O$.
Carbon dioxide	1.44	0.727	1.04	2	$C?O_2$
Oxygen	1.04	1.00	1.04	2	O_2
Sulfur trioxide	2.61	0.600	1.57	3	$S?O_3$

We have studied two tabulations and have obtained the ratio of two atomic masses. Our procedure can be generalized to other elements. The elements themselves need not be gaseous (as are hydrogen and oxygen); all that is necessary is that the element in question forms a considerable number of compounds that are either gaseous or readily volatile. In that case, we can with confidence assign to the element the minimum mass that corresponds to unit volume of a substance containing in its gaseous particle just one atom of the element in question. Given this minimum mass and/or the assignments of partial formulas it makes possible we can then proceed by the three methods noted above, to assign the atomic mass of the element. In this way atomic masses can be assigned to a substantial fraction of the known elements. There are additional procedures which we need not discuss, that make it possible to find the atomic masses of all the known elements.

SPECTROSCOPY

BACKGROUND AND SCOPE OF THE TECHNIQUE

Almost everyone has seen at least one spectrum: a rainbow. The band of colors (arranged from red to violet) is the visible portion of the sun's spectrum. The sun's spectrum can also be viewed by means of a spectroscope. In 1666 when he was 23, Sir Isaac Newton constructed the first spectroscope.

Spectroscopy is that branch of physics and chemistry that studies the absorption and emission of electromagnetic radiation by matter. Radiation of all wavelengths can be used. Matter in all physical states (gaseous liquid and solid) can be studied. A range of information can be obtained depending upon the wavelength of electromagnetic radiation used. The table on page 571 of the *Text* gives a sample of information obtainable from spectral analysis.

INSTRUMENTATION

The basic instrument consists of three parts: a radiation source, a dispersive device and a detector.

Radiation source

The source employed depends upon the problem being studied. There is no universal source. For example, gaseous discharges or arcs give rise to visible and ultraviolet radiation tungsten lamps to visible, glowing filaments to infrared and klystrons to microwave radiation. A wide variety of sources is used for different purposes.

Dispersive device

Two principal dispersive devices are used: a prism and a diffraction grating. Each of these will disperse, or spread out, radiation into a spectrum, thus giving intensity information as a function of wavelength. The difference between an expensive and cheap instrument is often the quality of the dispersive device. The more detail one wishes to resolve, the better the dispersive device should be. Thus for example, to resolve the sodium doublet (see page 570 of *Text*), the lines of which differ in wavelength by only 5.970 Å a good instrument is required.

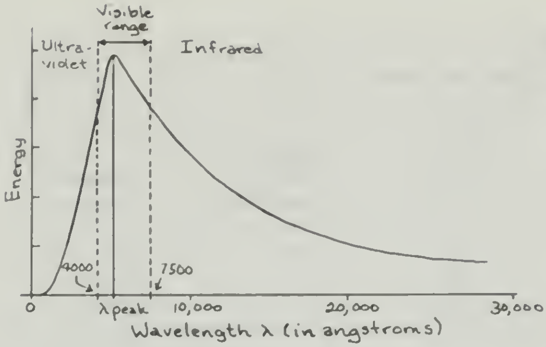
Detector

The type of detector used depends upon the radiation source being used. For work in the ultraviolet and visible regions photographic film is useful. Thermocouples are used to detect infrared radiation, while crystals are used in the detection of microwaves. In the latter two cases, electrical signals produced at the thermocouple or crystal must be amplified electronically. Often these amplified signals are then recorded on a stripchart recorder or viewed on an oscilloscope.

SPECTRA TYPES

Continuous spectra

Continuous spectra include all wavelengths between certain limits (for example a rainbow). A continuous spectrum results from heating a solid, liquid or gas under high pressure (like the sun) until it glows brightly. Such a glowing object makes a good source of radiation for spectroscopic work. In the next figure, it is seen that an object heated to 6 000°K radiates with a maximum intensity in the visible region. (The tungsten filament of a photoflood lamp is approximately 6,000°K when in use.)



Discrete spectra

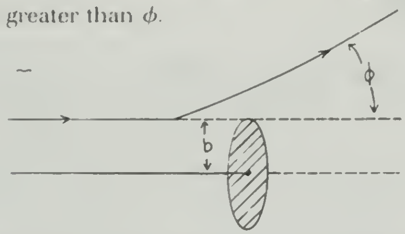
These are spectra in which only certain wavelengths appear. Line spectra and band spectra are discrete spectra.

(a) *absorption spectra:* When light is passed through a medium that abosrbs radiation of specific wavelengths, the spectrum of the transmitted light is known as an absorption spectrum.

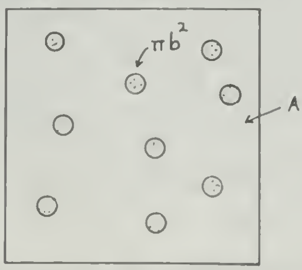
(b) *emission spectra:* These spectra are formed when gases under low pressure radiate.

RUTHERFORD SCATTERING

In Sec. 19.3 of the *Text* it is explained that scattering results when the alpha particle is incident along a line close to the nucleus. Looking at the figure below, we can say that any α particle whose undeflected path touches the circumference of the circle will be scattered through an angle ϕ. Furthermore, any α particle whose undeflected path interesects the circle (or radius b) itself will be deflected by an angle greater than ϕ. Thus, the α particle must strike the area πb^2 to be scattered by an angle greater than ϕ.



Now we can ask, what is the probability that an α particle will be scattered by an angle greater than ϕ? Each nucleus presents a target of size πb^2. The probability of an α particle hitting one of the targets is proportional to the total target area: that is, the total shaded area in the figure below.



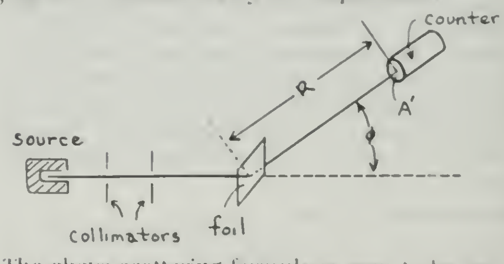
If there are n nuclei per unit volume, then the total shaded area is $n\pi b^2 tA$, where t is the thickness of the foil and A is the total foil area. We are assuming that the thickness of the foil is small enough that target areas do not overlap: in other words, we are considering that only single scatterings occur. The probability of scattering through an angle greater than ϕ is simply the ratio of the total target area to the total foil area: that is, $n\pi b^2 tA/A$ or $n\pi b^2 t$. Thus, if we have N_i incident α particles, the number scattered through an angle greater than ϕ (N_s) is given by $N_s = N_i n\pi b^2 t$, or the fraction scattered is $N_s/N_i = n\pi b^2 t$.

All factors in the above equation, with the exception of b, can be determined experimentally. With the aid of the calculus, a relationship between b and ϕ can be derived. When this is done one gets the following:

$$\frac{N_s}{N_i} = \frac{(9 \times 10^9)^2 Z^2 q^4_e ntA}{4R^2K^2 \sin^4 \dfrac{\phi}{2}}$$

where Z = the atomic number of the scattering nuclei
q_e = the electronic charge
A = the area of the counter window
R = the distance from the foil to the counter
K = the kinetic energy of the α particle

With this equation, which is essentially the relation given to Geiger by Rutherford, experimental tests on the nuclear model could be made. It was found that for both gold and silver foils there was agreement between theory and experiment.

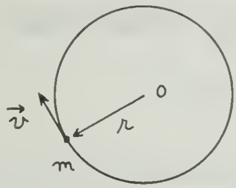


The above scattering formula is seen to be proportional to Q^2 and inversely proportional to the square of the α particle's kinetic energy (or $v\alpha^4$) and to $\sin^4\phi/2$. It has been stated that Rutherford did not derive the above scattering formula. According to George Gamow, Rutherford's mathematical skill was not high and therefore the famous Rutherford formula for α particle scattering was derived for him by a young mathematician, R. H. Fowler.

For a complete derivation of the Rutherford scattering law see *Introduction to Atomic and Nuclear Physics* (Fourth Edition), Henry Semat, Holt, Rinehart and Winston, 1963, Appendix VII, page 598.

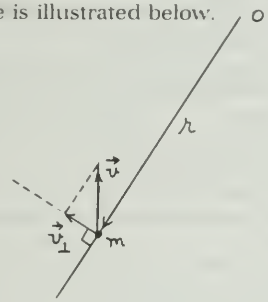
ANGULAR MOMENTUM

The angular momentum of a particle is always defined with respect to a point. Sometimes this point is called the center of rotation. In the case of circular motion it is particularly simple; that is, the point is the center of the circle. In circular motion, it is also simple because the velocity, v, is always perpendicular to the radius, r. In this case, the angular momentum can be represented as mvr, where m is the mass. The situation is pictured below.

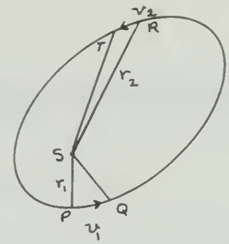


Angular momentum of mass m with respect to center O is mvr.

In the more general case, the angular momentum is written as mv_1r, where v_1 indicates the component of v that is perpendicular to r, the distance from the point of reference to the object. This more general case is illustrated below.



As a point of interest, angular momentum is conserved for a system just as is linear momentum. In fact, Kepler's second law, the law of equal areas, is an expression of the conservation of angular momentum. This can be shown as follows:



Kepler's second law states that for equal transit times between PQ and RT, area A_1 = PQS, area A_2 = RTS; and by Kepler's law, $A_1 = A_2$. When transit time between PQ and RT is small, arc $\overline{PQ}$ = chord $\overline{PQ}$. Now

$$A_1 = \tfrac{1}{2} r_1 (\overline{PQ})_\perp$$

where $(PQ)_\perp$ is the component of $(\overline{PQ})$ perpendicular to r_1. Likewise, $A_2 = \frac{1}{2} r_2 (\overline{RT})_\perp$.

But $\overline{PQ} \propto v_1$ and also $\overline{PQ} \propto mv_1$; thus, $(PQ)_\perp \propto (mv_1)_\perp$. Substituting, we get $A_1 \propto \frac{1}{2} r_1 (mv_1)_\perp$. In like manner, $A_2 \propto \frac{1}{2} r_2 (mv_2)_\perp$. Since $A_1 = A_2$, proportionality constants will cancel and

$$mv_{1\perp}\, r_1 = mv_{2\perp}\, r_2$$

which says that the product, mv_1r (the angular momentum), is a constant; that is, angular momentum is conserved.

NAGAOKA'S THEORY OF THE "SATURNIAN" ATOM

Excerpted from Conn and Turner,
The Evolution of the Nuclear Atom
(London: Iliffe Books Ltd., 1965, pp. 111–118)

In the same volumes of the *Philosophical Magazine* in which this [Thomson's] paper appeared, another theory of atomic structure was propounded by H. Nagaoka. This was the theory of the 'Saturnian' atom and was the precursor of the nucleus theory so brilliantly developed by Rutherford and Bohr. Nagaoka's theory derived its inspiration from the mathematical analysis of the stability of the system of rings surrounding the planet Saturn, by James Clerk Maxwell who, in 1856, was awarded the Adam's Prize for an essay, entitled 'On the Stability of the Motion of Saturn's Rings'. ... In this essay, Maxwell discusses the stability first of a solid ring of matter and then of a ring consisting of a number of separate particles. In both cases the rings were presumed to be rotating around, and to be attracted inversely as the square of their distances from, a massive central body. He concluded that although the system containing a solid ring would be unstable, a massive central body surrounded by a ring of separate satellites would form a stable system if the angular velocity of the ring were sufficiently high. . . .

Such a stable structure of separate particles rotating in a series of concentric rings round a massive central body which attracts the satellite particles with a force inversely proportional to the square of their radii of rotation was suggested as a possible model of the atom by H. Nagaoka in 1903 in a paper read before the Physico-Mathematical Society of Tokyo. This was published in the *Phil. Mag.*, Vol. 7, p. 445, 1904, and was entitled 'Kinetics of a System of Particles illustrating the Line and Band Spectrum and the Phenomena of Radioactivity'. . . .

In order to account for the characteristic frequency lines of the band spectrum Nagaoka supposed that the rings of electrons in the 'Saturnian' atom would vibrate and that these vibrations would give rise to radiation....

"There are various problems which will possibly be capable of being attacked on the hypothesis of a Saturnian system, such as chemical affinity and valency, electrolysis and many other subjects connected with atoms and molecules. The rough calculation and rather unpolished exposition of various phenomena above sketched may serve as a hint to a more complete solution of atomic structure."

Brief Descriptions of Learning Materials

SUMMARY LIST OF UNIT 5 MATERIALS

Experiments
E5–1 Electrolysis
E5–2 The Charge-to-Mass Ratio for an Electron
E5–3 The Measurement of Elementary Charge
E5–4 The Photoelectric Effect
E5–5 Spectroscopy

Demonstrations
D53 Electrolysis of water
D54 Charge-to-mass ratio for cathode rays
D55 Photoelectric effect
D56 Blackbody radiation
D57 Absorption
D58 Ionization potential

Film Loops
L45 Production of Sodium by Electrolysis
L46 Thomson Model of the Atom
L47 Rutherford Scattering

Reader Articles
R1 *Failure and Success* by Charles Percy Snow
R2 *The Clock Paradox in Relativity* by C. G. Darwin
R3 *The Island of Research* by Ernest Harburg
R4 *Ideas and Theories* by V. Guillemin
R5 *Einstein* by Leopold Infeld
R6 *Mr. Tompkins and Simultaneity* by George Gamow
R7 *Mathematics and Relativity* by Eric M. Rogers
R8 *Parable of the Surveyors* by Edwin F. Taylor and John Archibald Wheeler
R9 *Outside and Inside the Elevator* by Albert Einstein and Leopold Infeld
R10 *Einstein and Some Civilized Discontents* by Martin Klein
R11 *The Teacher and the Bohr Theory of the Atom* by Charles Percy Snow
R12 *The New Landscape of Science* by Banesh Hoffmann
R13 *The Evolution of the Physicist's Picture of Nature* by Paul A. M. Dirac
R14 *Dirac and Born* by Leopold Infeld
R15 *I Am This Whole World: Erwin Schrödinger* by Jeremy Bernstein
R16 *The Fundamental Idea of Wave Mechanics* by Erwin Schrödinger
R17 *The Sentinel* by Arthur C. Clarke
R18 *The Sea-Captain's Box* by John L. Synge
R19 *Space Travel: Problems of Physics and Engineering* by Project Physics Staff
R20 *Looking for a New Law* by Richard P. Feynman
R21 *A Portfolio of Computer-made Drawings* by Darel Eschbach, Jr.

Sound Films (16 mm)
F35 Definite and Multiple Proportions
F36 Elements, Compounds, and Mixtures
F37 Counting Electrical Charges in Motion
F38 Millikan Experiment
F39 Photoelectric Effect
F40 The Structure of Atoms
F41 Rutherford Atom
F42 A New Reality
F43 Franck–Hertz Experiment
F44 Interference of Photons
F45 Matter Waves
F46 Light: Wave and Quantum Theories

Transparencies
T35 Periodic Table
T36 Photoelectric Experiment
T37 Photoelectric Equation
T38 Alpha Scattering
T39 Energy Levels: Bohr Theory

FILM LOOPS

L45 PRODUCTION OF SODIUM BY ELECTROLYSIS

Davy's classical experiment in which molten NaOH is electrolyzed to form metallic sodium.

L46 THOMSON MODEL OF THE ATOM

Small magnets floating on the surface of water are aligned into various patterns by a radial magnetic field. The apparatus, a model of a model, was described by Thomson.

L47 RUTHERFORD SCATTERING

A computer-animated film, in which projectiles are fired toward a nucleus that exerts an inverse-square repulsive force.

Note: A fuller discussion of each *Film Loop* and suggestions for its use will be found in the section of this *Resource Book* entitled "Film Loop Notes."

SOUND FILMS (16 mm)

F35 DEFINITE AND MULTIPLE PROPORTIONS

30 min, Modern Learning Aids.

Here is the evidence on which Dalton based his conviction that matter came in natural units (atoms). The chemical laws of definite proportions are demonstrated by electrolysis and recombination of water; and of multiple proportions by the quantitative decomposition of N_2O, NO, and NO_2.

F36 ELEMENTS, COMPOUNDS, AND MIXTURES

Color, 33 min, Modern Learning Aids.

A discussion of the difference between elements, compounds, and mixtures, showing how a mixture can be separated by physical means. Demonstrates how a compound can be made and then taken apart by chemical methods, with identification of components by means of their physical properties, such as melting point, boiling point, solubility, color, etc.

F37 COUNTING ELECTRICAL CHARGES IN MOTION

22 min, Modern Learning Aids.

This film shows how an electrolysis experiment enables us to count the number of elementary charges passing through an electric circuit in a given time and thus calibrate an ammeter. Demonstrates the random nature of motion of elementary charges, with a current of only a few charges per second.

F38 MILLIKAN EXPERIMENT

30 min, Modern Learning Aids.

Simplified Millikan experiment described in the *Text* is photographed through the microscope. Standard spheres are substituted for oil drops. An analysis of the charge related to the velocity of the sphere across the field of view of the microscope emphasizes the evidence that charge comes in natural units that are all alike. Numerous changes of charge are shown, produced by X rays, with the measurements clearly seen by the audience.

F39 PHOTOELECTRIC EFFECT

Color, 28 min, Modern Learning Aids.

Qualitative demonstrations of the photoelectric effect are shown using the sun and a carbon arc as sources. A quantitative experiment is performed measuring the kinetic energy of the photoelectrons emitted from a potassium surface. The data are interpreted in a careful analysis.

F40 THE STRUCTURE OF ATOMS

B&W, 12.5 min, McGraw-Hill (code 612010); color, 12.5 min (code 612022).

This film provides the experimental evidence for our basic concepts concerning the structure of the atom. An experiment similar to Rutherford's historic α-particle scattering demonstrations shows that atoms have dense, positively charged nuclei. Another fundamental experiment shows the charge on the electron and the ratio of charge to mass.

F41 RUTHERFORD ATOM

40 min, Modern Learning Aids.

A cloud chamber and gold foil in a simple α-particle scattering experiment to illustrate the historic Rutherford experiment that led to the nuclear model of the atom. Behavior of α particles clarified by use of large-scale models illustrating the nuclear atom and Coulomb scattering.

F42 A NEW REALITY

Color, 51 min, produced by Statens Filmcentral and Laterna Films, Denmark, and by OECD, and sponsored by the Carlsberg Foundation in association with the International Council for Educational Films, International Film Bureau Inc.

Traces the discovery of the structure of the atom and emphasizes the work of the Danish physicist Niels Bohr. The story begins at the Institute for Theoretical Physics in Denmark, where experts from all parts of the world study and experiment with the atom. They devised means of visualizing the submicroscopic structure of molecules and by using advanced electronic equipment, gained an understanding of the character of the atom. We see how one element can be converted to another by atomic bombardment, which changes the number of protons in the nucleus. Other demonstrations using light waves establish color measurement in terms of energy. Also illustrated are proofs that the electron components of the atom are both particles and wave energies. The modern concept of the atom is basically that determined by Niels Bohr, and its implications reach into the realms of biology, psychology, and philosophy.

F43 FRANCK–HERTZ EXPERIMENT
30 min, Modern Learning Aids.

A stream of electrons is accelerated through mercury vapor, and it is shown that the kinetic energy of the electrons is transferred to the mercury atoms only in discrete packets of energy. The association of the quantum of energy with a line in the spectrum of mercury is established. The experiment retraced in the film was one of the earliest indications of the existence of internal energy states within the atom.

F44 INTERFERENCE OF PHOTONS
13 min, Modern Learning Aids.

An experiment in which light exhibits both particle and wave characteristics. A very dim light source, a double slit, and a photomultiplier are used in such a way that less than one photon (on the average) is in the apparatus at any given time. Characteristic interference pattern is pointed out by many individual photons hitting at places consistent with the interference pattern. Implications of this are discussed.

F45 MATTER WAVES
28 min, Modern Learning Aids. The film presents a modern version of the original experiment that showed the wave behavior of the electron. The student sees electron diffraction patterns on a fluorescent screen. The patterns are understandable in terms of wave behavior. Alan Holden presents an optical analogue showing almost identical patterns. The electron diffraction experiments of G. P. Thomson are described by Holden, who also presents brief evidence for the wave behavior of other particles, such as neutrons and helium atoms.

F46 LIGHT: WAVE AND QUANTUM THEORIES
B&W and color, 13.5 min, Coronet Films. This film clearly and simply demonstrates the accepted theory of light as consisting of both a wave motion and of discrete bundles, (quanta) of energy. Young's double-slit experiment is performed to show the wave character of light, while the photoelectric effect indicates that light consists of energy quanta. The Compton effect and other major experiments associated with the present theory are shown in laboratory demonstrations and in animation.

TRANSPARENCIES

T35 PERIODIC TABLE
The modern long form of the periodic table is presented with various overlays highlighting chemical families and other pertinent groupings.

T36 PHOTOELECTRIC EXPERIMENT
Schematic drawings of a photoelectric tube and circuit show the procedure for measuring the stopping voltage.

T37 PHOTOELECTRIC EQUATION
A sliding mask on a plot of maximum kinetic energy of photoelectrons versus frequency of photons permits the derivation of Einstein's photoelectric equation.

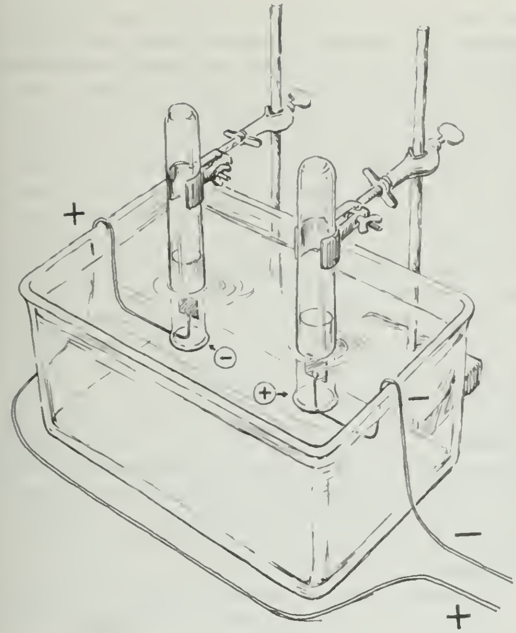
T38 ALPHA SCATTERING
A two-part transparency showing a diagrammatic sketch of the Rutherford scattering experiment and potential hill diagrams for the Thomson and Rutherford atomic models.

T39 ENERGY LEVELS: BOHR THEORY
A two-part transparency showing the Bohr orbits and energy levels for hydrogen. Illustrates production of Lyman and Paschen series in general and Balmer series in detail.

Demonstration Notes

D53 ELECTROLYSIS OF WATER
Set up the apparatus as shown in the next figure. The electrodes are clamped alongside the inverted test tubes and connected either to a 6-V battery or power supply. Any gas that forms on either of the platinum-plated spiral electrode tips will be collected when it rises and displaces water from the corresponding test tube. Since this reaction is very slow with pure water, it is necessary to add about 5–15 mL of 6 normal (dilute) sulfuric acid to the water. The 6 normal solution can be prepared by adding 1 volume of concentrated sulfuric acid to 5 volumes of water and stirring with a glass stirring rod. Caution: *Always pour the acid into the water.*

A full test tube of hydrogen can be collected in about 20 min. Mark the level of oxygen in the sec-



ond tube; test both gases to demonstrate that hydrogen and oxygen have been produced. Then fill each test tube with water from a graduated cylinder to the former gas level to measure the volume of each gas produced. Calculate the ratio of volumes.

The two gases can also be collected in one test tube and ignited. *Caution: There will be a violent reaction, from which it can be concluded that the gases readily combine.* It is not possible to conclude that water was formed as a result of this reaction, since the test tube is wet before ignition.

The platinum electrodes with connecting wires are available from most apparatus supply houses.

D54 CHARGE-TO-MASS RATIO FOR CATHODE RAYS

A simple demonstration of the deflection of the beam of a cathode-ray tube in a magnetic field can bring together parts of Units 4 and 5, and can give quite a good value for the ratio q_e/m. In Unit 4 (*E4–7,* "Electron Beam Tube. I") students saw, qualitatively, the deflection of the cathode rays in the magnetic field of a pair of permanent magnets. In *E4–6,* "Currents, Magnets, and Forces," they learned how to measure magnetic fields, and one group of students used the current balance to measure the vertical component of the earth's magnetic field. This field will be used to deflect an electron beam, and its measured value used to calculate q_e/m. This ratio in itself is probably not of great interest to students, but together with q_e found in the Millikan-type experiment (*E5–3,* "The Measurement of Elementary Charge"), it enables us to estimate m, the mass of the electron.

Equipment:

- Cathode-ray tube, such as the 5-cm 902A
- Power supply for CRT (for 902A, 6.3 V, 1 A for heater, about 200 V dc for anode)
- (Permanent magnet)
- (Sticky centimeter tape)

Procedure

Connect the CRT to power supply. The simplest wiring diagram for the 902A is shown in Fig. 1. Use of fixed resistors will give an adequately, but not perfectly, focused beam. To improve focus and control brightness, replace the 1.5 megohm (meg) resistor by a potentiometer (about 1 meg) and a resistor (about 0.5 meg) in series and adjust the potentiometer for best focus. [A more complete schematic, suggested by the manufacturers, is given at the end of this note (Fig. 5).]

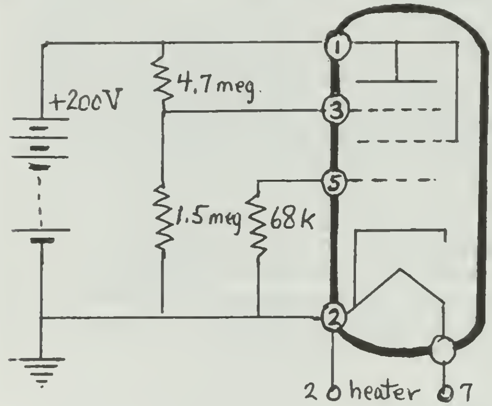


Fig. 1 Schematic for CRT 902A.

It is important to keep the anode voltage as low as possible. With the 902A, a spot is obtained with only 200 V, but other tubes may require more voltage. In a typical cathode-ray oscilloscope, the accelerating (anode) potential is more than 1,000 V. This produces a beam of faster electrons that are correspondingly less deflected in the external magnetic field, whereas a beam of 200-V electrons is appreciably deflected in the earth's field.

First demonstrate that the beam is affected by an applied magnetic field by bringing a small permanent magnet near the tube and showing the deflection of the spot.

Electric deflection can be shown by applying a potential difference of a few volts (for instance, from a 6-V battery) between the pairs of deflection plates. Note that one of the x-plates and one of the y-plates are connected internally to anode 2. Terminals 4 and 6 are connected to the other x- and y-plates. With the wiring suggested here (Fig. 1) these temrinals will be "hot" (about 200 V above ground).

The significance of this first qualitative part of the demonstration is discussed on *Text* pages 542–544. The fact that the cathode ray is deflected

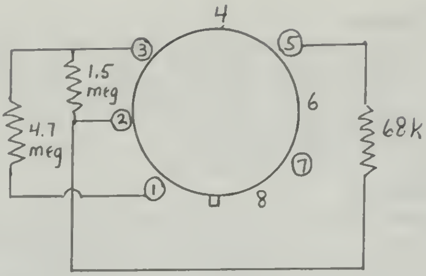


Fig. 2 Bottom view of the tube socket (medium shell octal 8-pin) with resistors. Ground pin no. 2, connect 6.3 V between pins 2 and 7, and 200 V to pin 1.

by magnetic (and electric) fields suggests that it consists of charged particles. That the beam remains narrow and well-defined suggests that all the particles are identical; if they were not, we would expect some to be deflected more than others, which would cause the beam to spread out. J. J. Thomson went further than this and showed that the deflection was independent of the cathode material and of the residual gas in the tube. To get the effect of the vertical component of the field, measure the total displacement of the spot (how much it shifts when the tube is moved from the N–S to the S–N orientation) and divide by 2. A question for students: In what direction must the beam be oriented to be undeflected by the earth's field?

To make any quantitative interpretations of the deflection, we must be able to measure the magnetic field. The field due to a permanent magnet is unsatisfactory for this purpose because it is nonuniform. Fortunately the vertical component of the earth's magnetic field is strong enough to deflect the beam appreciably; it is certainly uniform over the length of the tube; and it can be measured with the *Project Physics* current balance (E4–6, part C).

Set up the tube horizontally along a N–S line. Mark the position of the spot on the screen. Turn the tube through 180°, so that the electron beam is still horizontal but is in the opposite direction (S–N). Mark the spot again and measure the horizontal displacement from its first position.* (A strip of sticky centimeter tape stuck on the tube face is useful.) Ignore any vertical displacement, which at most should be small. Use a voltmeter to record the anode potential.

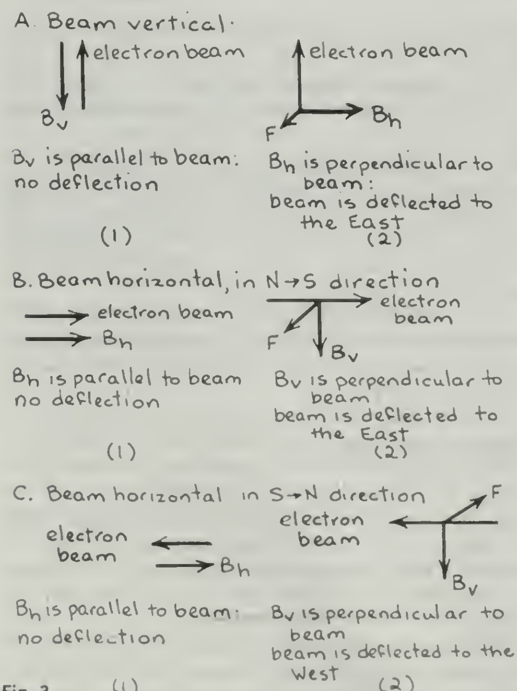


Fig. 3

Theory

Electrons are emitted from the hot cathode and are accelerated by a positive potential on the anode. After passing through a hole in the anode they move with constant velocity till they strike the luminescent screen.

The kinetic energy gained by an electron due to acceleration through a potential difference of V volts is

$$\tfrac{1}{2}mv^2 = Vq_e$$

where q_e is the charge of an electron.

After the electrons have passed through the anode hole, they can be deflected by electric and magnetic fields. The magnitude of the force (F) on an electron moving with velocity (v) in a magnetic field (B) is

$$F = Bq_ev$$

The force is perpendicular to the directions of both v and B. In this case, v is horizontal and B vertical; thus, F will be horizontal.

When a particle is moving with constant velocity v and is acted on by a force perpendicular to that velocity, the particle will move in a circle (see Unit 1). The centripetal force needed to keep a particle of mass m moving in a circle of radius R is mv^2/R. But we already know that

$$F = Bq_ev$$

$$\frac{mv^2}{R} = Bq_ev$$

$$\frac{q_e}{m} = \frac{v}{BR}$$

Substituting for v from equation (1) above

$$\frac{q_e}{m} = \frac{(2Vq_e)^{1/2}}{m^{1/2}BR}$$

$$\frac{q_e}{m} = \frac{2V}{B^2R^2}$$

*Notice that the deflection is not symmetrical about the spot's position when the tube is vertical. This is because with the tube vertical the beam is deflected (to the east) by the horizontal component of the earth's field B_h (see Fig. 3A(2)).

To find the radius of curvature of the electron beam, R, apply Pythagoras' theorem to triangle OPQ (see Fig. 4):

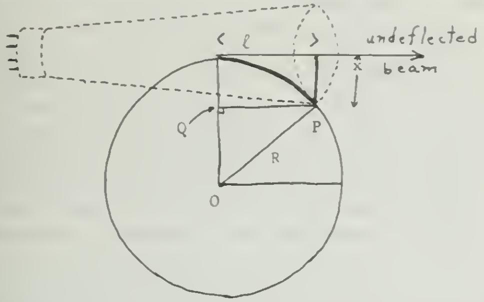


Fig. 4 l is the length of the tube from anode to screen; x is the displacement of the spot on the screen (shown vertical in the figure); R is the radius of curvature of the circle into which the electron beam is bent.

$$(R - x)^2 + l^2 = R^2$$
$$R^2 - 2Rx + x^2 + l^2 = R^2$$
$$2Rx = x^2 + l^2$$

if $\quad x < l, x^2 << l^2$

and $\quad 2Rx \cong l^2$

$$R \cong \frac{l^2}{2x}$$

(Compare the geometry of the pendulum in the *Handbook*, Unit 3, page 123.)

The distance l for the 902A tube is 10 cm.

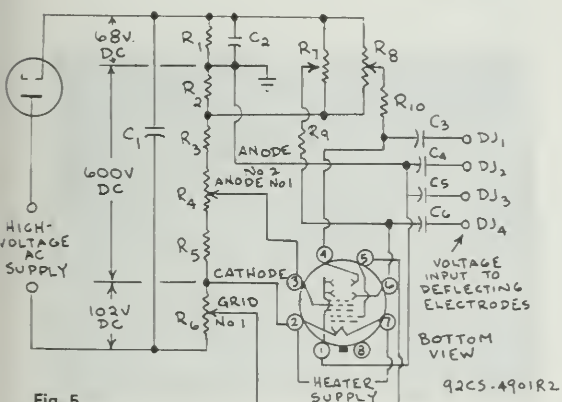


Fig. 5

C1: 0.1 µF
C2: 1.0 µF
C3 C4 C5 C6: 0.05-µF blocking capacitors*
R1 R2: 1 meg
R3: 1.3 meg

*When the cathode is grounded, capacitors should have a high voltage rating; when anode No. 2 is grounded, they may have low voltage rating. For dc amplifier service, deflecting electrodes should be connected directly to amplifier output. In this service, it is usually preferable to remove deflecting-electrode circuit resistors to minimize loading effect on the amplifier. In order to minimize spot defocusing, it is essential that anode No 2 be returned to a point in the amplifier system that will give the lowest possible potential difference between anode No. 2 and deflecting electrodes.

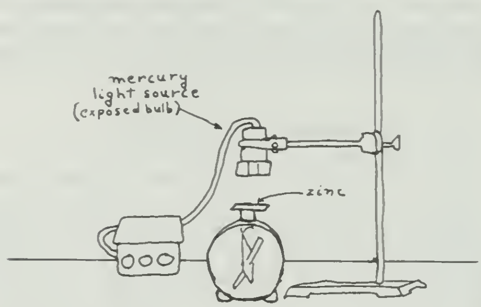
R4: 1-meg potentiometer
R5: 0.3 meg
R6: 0.5-meg potentiometer
R7 R8: 2-meg potentiometers
R9 R10: 2 meg

D55 PHOTOELECTRIC EFFECT

A simple electroscope demonstration introduces the photoelectric effect vividly in a qualitative way and displays features not shown at all in the *Handbook.*

The necessary equipment consists of:

1. a simple electroscope whose electrode can be surmounted or replaced by 10 cm × 10 cm pieces of zinc and copper and possibly lead and iron
2. a source of ultraviolet light such as a small mercury vapor sterilizing lamp to illuminate the metal plates
3. a sheet of ordinary glass to hold between the lamp and the metal plates
4. plastic strips or other materials for giving the electroscope positive and negative charges
5. a piece of sandpaper or steel wool for cleaning the metal plates.



Procedure

Each metal plate should be scrubbed hard with sandpaper or steel wool to remove any traces of oxide. After a few hours, the cleaning must be done again, and the plate wiped with a clean cloth to remove any traces of sand or steel wool whose sharp points allow rapid leakage of charge.

Mount the zinc plate on the electroscope as shown in the diagram, and give it a negative charge. (Charging by induction often gives a much larger charge than charging by contact. If the day is humid, it may be necessary to dry the charging materials, for example, over a bright lamp bulb or a radiator.)

Watch the charged electroscope for a few moments to make sure its charge is not leaking at a visible rate. Then illuminate the plate with light from the mercury lamp.

The electroscope should discharge rather rapidly. A large class can see this best if the electroscope shadow is projected on the chalkboard by a bright light 1 m away.

To show that the photoelectric effect is related to chemical activity and therefore to the ease with which a metal loses electrons, replace the zinc sheet by metals of lower chemical activity, such as copper, iron, and lead, cut to the same size.

To show that the effect is not caused by visible light:

(a) replace the ultraviolet lamp by an ordinary incandescent lamp. However bright, it will not drive electrons from these metals; though, of course, it will do so from more active elements, such as cesium and lithium, which are used to coat the emitters of photoelectric cells; and

(b) shield the metal plate from the ultraviolet radiation with the glass plate. Ultraviolet does not penetrate glass. The electroscope's discharge immediately stops.

None of these qualitative effects is shown in the student laboratory experiment, the purpose of which is to show how the energy of the emitted electrons depends on the frequency of the light and so to demonstrate the inadequacy of the wave theory of light for photoelectric phenomena.

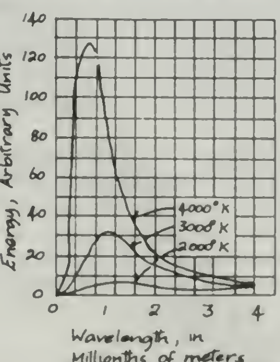
It is worth noting that the photoelectric effect was discovered originally by Heinrich Hertz in 1887 in the course of research that gave massive support to Maxwell's wave model of light. He found that ultraviolet light shining on the terminals of a spark gap facilitated the formation of sparks.

D56 BLACKBODY RADIATION

As the brightness of an ordinary 150- or 200-W incandescent lamp is varied, its color changes markedly. Students can see this easily through their pocket spectroscopes as the brightness of an incandescent lamp is varied by means of a variable transformer. Do this in a partially darkened room. Warn students to ignore the immense change in brightness and to concentrate on the changing distribution of *color*.

First only red is faintly visible. Then orange, yellow, etc. are added until the bright light is almost (but not quite) white.

This demonstration leads easily to a description (not a derivation) of the radiation curves shown below. (The intensity is plotted against wavelength for each temperature.) It leads also to Planck's derivation of their formula in 1900. Note the use of these curves for finding the surface temperatures of the source of any continuous spectrum: molten steel or a distant star.



Stefan's law, $E = kT^4$, and Wien's law, $\lambda_{max} = \frac{C}{T}$, could perhaps also be mentioned at this point. (E is the total energy radiation by a blackbody; T is the absolute temperature; λ_{max} is wavelength of most intense emission; and c and k are constants.) There are not many fourth-power laws in physics.

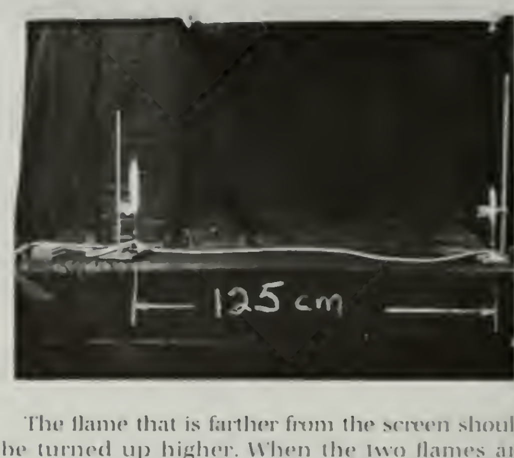
The important point to raise here is that Planck's successful explanation of the continuous spectrum energy distribution curve required for the first time the assumption that radiant energy was emitted in chunks, or "quanta," of energy *hf*. This was the origin of the quantum theory.

It is worth noting, too, that the next significant use of the quantum idea was by Albert Einstein five years later to explain the emission of photoelectrons. His photoelectric equation is described in *E5–4*, which the students have probably already performed.

D57 ABSORPTION

This demonstration shows the absorption of light by excited molecules. It should be done in a dark room.

Set up two Bunsen burners and a white screen as shown in the photo. The distances are approximate only.



The flame that is farther from the screen should be turned up higher. When the two flames are burning steadily, with luminous flames (air supply shut off), the farther one will cast a shadow of the nearest burner on the screen.

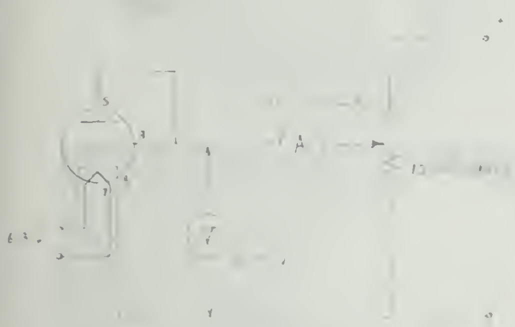
Now open the air holes of both burners and introduce some sodium into each flame. Look carefully at the screen and you will see that the flame of the nearer burner also casts a shadow. Some of the light emitted by excited Na atoms in the farther flame is absorbed by Na atoms in the nearer flame. Adjust the brightness and distances to get maximum effect. Try not to set up air currents that will disturb the flames and make the shadows flicker excessively.

To emphasize the point that it is the same monochromatic light emitted in one flame and absorbed in the other that causes the shadow, remove the sodium from the farther flame and cut off the air to make it luminous again. The nearer flame casts no shadow now because nearly all the light emitted by the farther flame passes through it without being absorbed. But the flame is still bright enough to cast a shadow of the burner tube. If you remove the sodium from the nearer flame the effect will be the same: no absorption and therefore no shadow.

D58 IONIZATION POTENTIAL

The existence of a discrete ionization potential for each element confirms the existence of stationary states, postulated by Bohr and mathematically derived by Schrödinger. The ionization potential of argon can be demonstrated rather simply using a thyratron 884 tube.

Connect the tube as shown in the next diagram. Notice that since the grid and the plate are connected together, the tube is essentially a diode.



A power supply with a low internal resistance, capable of delivering a high current, should be used. The variable resistance is an ordinary 5,000-Ω potentiometer.

If the 5,000-Ω potentiometer is not available, the voltage can be varied instead by running the power supply from a variable transformer. Of course, the filament of the 884 tube in this case requires an independent 6.3-V supply.

To operate the circuit, vary the voltage between the cathode and the grid plate while monitoring the current with the ammeter. When the tube is operating properly, the grid is positive with respect to the cathode and electrons thermally emitted from the cathode are accelerated toward the grid. Along the way, the electrons may strike argon atoms with which the tube is filled at low pressure. At low voltages the collisions are elastic, and the argon atoms are not altered. At any potential V the electrons have a kinetic energy

$$\frac{mv^2}{2} = Vq_e$$

which they may transfer to the argon atoms by collision.

As the grid plate is made increasingly positive, the electrons finally acquire enough energy to ionize argon atoms with which they collide. The cathode–grid potential difference at which this occurs is called the *ionization potential*, V_i. Thus $V_i q_e$ is the minimum energy sufficient to ionize the argon atoms: that is, to remove an electron.

Experimentally you recognize this voltage by the rapid increase in the ammeter reading and by the sudden glow of light that appears in the center of the tube at the same moment.

The sudden onset of light in the argon and the sharp increase in anode current occur simultaneously as the critical potential V_i is exceeded. This is strong evidence that the argon gas is in an entirely different condition. That this is a condition of ionization seems fairly clear. It is also reasonable to assume that V_i is nearly equal to the ionization potential of the argon.

The ionization potential for argon is 15.7 V. At this point, the kinetic energy of the ionizing electrons is

$$\frac{mv^2}{2} = V_i q_e = 15.7 \times (1.60 \times 10^{-19})$$
$$= 2.51 \times 10^{-18}\ \mathrm{J}$$

The implication of this demonstration is that electrons are bound to argon atoms by a definite binding energy that is being measured by the energy $V_i q_e$ needed to remove them altogether from the argon atoms.

Since the Rutherford atom model has already been discussed, it is worth pointing out that the glow is caused by the emission of quanta as argon atoms recapture their lost electrons.

Students may wish to plot grid voltage (reading of voltmeter) against plate current (reading of ammeter) in order to see on a graph the abrupt increase of current.

There is a small current even at voltages far below the ionization potential, since, of course, electrons will make their way across the tube to the grid plate so long as it is positive.

There is a similarity between this demonstration and the Franck–Hertz experiment. In that experiment, the various excitation energies of mercury vapor were measured instead of the ionization potential of argon. It may be worth referring to the more detailed description of the experiment in Sec. 19.9.

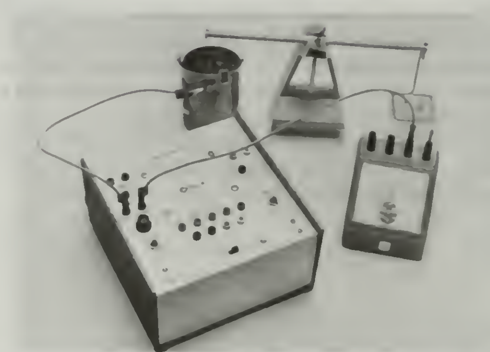
Experiment Notes

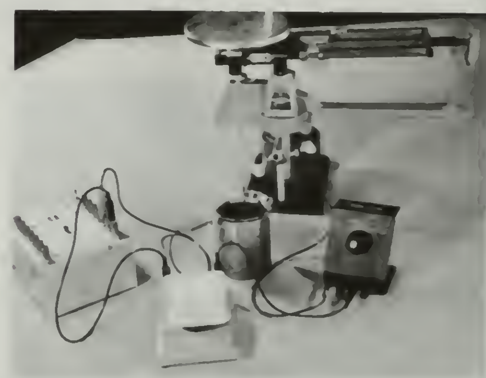
E5–1 ELECTROLYSIS, CALCULATING MASS AND VOLUME OF AN ATOM

Equipment:

- Beaker, 500 or 600 mL
- Copper sheet
- Balance, equal arm or triple beam
- Saturated solution of copper sulfate in distilled water, with two or three drops of concentrated sulfuric acid
- Power supply, 6-V 5-A dc
- Ammeter, 0–5 or 0–10 A dc
- Rheostat or variable autotransformer
- Fine copper wire and clips
- Hook-up wire
- Stopwatch (optional)

Although not an ideal arrangement, there is an argument in favor of doing this experiment after *E5–3*, "The Measurement of Elementary Charge." Students will then know the value of the charge on the electron ($q_e = -1.6 \times 10^{-19}$ C) and they can use the measurement of the mass of metal deposited by the passage of a known quantity of charge to calculate such things as the mass and volume of a single metal atom. If, in the experiments, we followed the strict historical sequence of the *Text* (Faraday's work on electrolysis preceded Millikan's oil-drop experiment by about 80 years.), students would use electrolysis to determine the faraday (F). Although this is an important quantity, its significance would emerge only after doing a series of experiments with many different elements and finding that the same quantity of electricity (F = 96,540 C) will deposit, or release, the gram equivalent weight of any element.





Procedure

An unusual and very convenient feature of this experiment is that the cathode is supported in the electrolyte from the balance beam. Thus, the cathode need not be removed from the cell for weighing, and we have eliminated the risky step of drying it after the experiment.* Either equal-arm or triple-beam balances can be used.

Another refinement is to control the current by means of an autotransformer ("Variac" or "Powerstat"), which provides current control over a wider range than the more conventional rheostat in series with the cell. Of course, a rheostat can still be used if more convenient and should be connected in series with the output of the power supply.

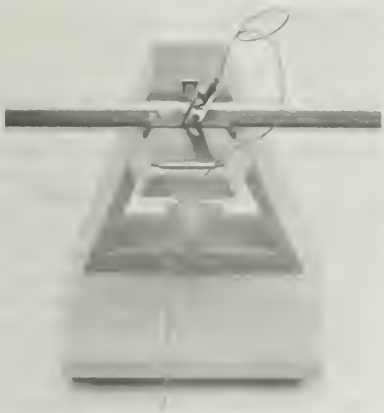
If an ordinary power supply is not available, one can be made from a bell transformer with a rectifier in series with its output. The ammeter in the circuit will read the average of the resulting pulsating direct current, which gives correct results.

Notice that the electrical connection to the cathode must be made through the wire by which it hangs from the balance beam. The knife edge and its seat are not electrically conducting, so they must be bypassed as indicated. Even if the pivots of the balance are made of metal, you cannot pass a 5-A current through them.

The anode connection is made in any convenient manner.

Care must be taken to see that neither solder nor any battery clips touch the copper sulfate electrolyte, since some foreign metal will dissolve and by replacing copper atoms in the electrolysis process will drastically alter the results. It is for this reason that both electrodes must have protruding tabs for electrical connections.

*If you still have trouble due to some of the deposited copper dropping off the cathode, try reversing the current and have students measure the loss in weight of the anode.



Since the rate of deposit of copper will not be constant if the cathode surface is dirty or impure it is a good idea to form a deposit of pure copper on it by a preliminary run of 10–15 min. During this time, the controls can also be adjusted to obtain the desired current. Explain to the students why they can start from any state of the electrode not just the unplated electrode at $t = 0$.

Answers to questions

1. $Q = I \times t$. A typical answer: a current of 5 A for 10 min would be $5 \times 10 \times 60 = 3{,}000$ C. This would give a true mass increase of 1.02 g. This is a theoretical value. In practice the mass increase is usually 3%–8% lower than theoretical.
2. Number of electrons $= \dfrac{Q}{1.6 \times 10^{-19}}$

For the example given above

$$\frac{Q}{1.6 \times 10^{-19}} = \frac{3{,}000}{1.6 \times 10^{-19}}$$
$$= 1.9 \times 10^{22} \text{ electrons}$$

3. Two electrons:
$$Cu^{++} + 2e^{-} \rightarrow Cu$$

4. Number of copper atoms deposited
$$= \frac{\text{number of electrons}}{2}$$
$$= 0.95 \times 10^{22} \text{ atoms (in this example)}$$
5. Mass of each atom
$$= \frac{\text{mass deposited}}{\text{number of atoms}}$$
$$= \frac{1.02 \text{ g}}{0.95 \times 10^{22}} = 1.07 \times 10^{-22} \text{ g}$$
6. Number of atoms in a penny
$$= \frac{\text{mass of penny}}{\text{mass of each atom}}$$
$$= \frac{3 \text{ g}}{1.07 \times 10^{-22} \text{ g/atom}} = 2.8 \times 10^{22} \text{ atoms}$$
7. Volume occupied by copper atom
$$= \frac{\text{volume of penny}}{\text{no. of atoms in penny}}$$
$$= \frac{0.3 \text{ cm}^3}{2.8 \times 10^{22}}$$
$$= 1.07 \times 10^{-23} \text{ cm}^3$$

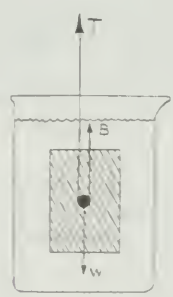
Derivation of the Buoyancy Correction Factor

In this experiment, the cathode whose mass is m and whose density is ρ_c has a volume $V = m/\rho_c$. It is submerged in a liquid (the electrolyte) whose density is ρ_e. The three forces on it are shown in the next diagram where T is the upward force exerted by the balance, B is the buoyant force exerted by the liquid, and $W = ma_g$, the weight of the cathode. When the cathod is in equilibrium, $T + B = W$.

Now by Archimedes' principle the buoyant force B is simply the weight of the liquid displaced by the volume $V = ma_g/\rho_c$ of the cathode. Since the liquid's mass is $\rho_e V$,

$$B = \rho_e V_g = \frac{\rho_e m a_g}{\rho_c}$$

Also T is the *apparent* weight of the cathode $m'a_g$ and W is the *true* weight of the cathode ma_g.



Putting these values of B, T, and W into the first equation above, we get

$$m'a_g + \frac{\rho_e m a_g}{\rho_c} = ma_g$$

Dividing both sides by ma_g

$$\frac{m'}{m} + \frac{\rho_e}{\rho_c} = 1$$

or

$$m = \frac{m'}{1 - \frac{\rho_e}{\rho_c}}$$

Of course, the experimenter is not interested in the toal mass m of the cathode, but in the *increase* in mass Δm; however, this is proportional to the increase in apparent mass $\Delta m'$. So

$$\Delta m = \frac{\Delta m'}{1 - \frac{\rho_e}{\rho_c}}$$

Using typical density values, the buoyancy correction is

$$\rho_c = 8.9 \text{ g/cm}^3$$

$$\rho_e = 1.3 \text{ g/cm}^3$$

$$\therefore 1 - \frac{\rho_e}{\rho_c} = 1 - \frac{1.3}{8.9} = 0.85$$

$$\therefore \Delta m = \frac{\Delta m'}{0.85}$$

Discussion

Notice that the idea that 1.6×10^{-19} C is the smallest possible charge rests only on the fact that despite an enormous number of measurements like the foregoing, no smaller charge has been observed. There is no other basis for the idea that this must be the smallest charge.

This may be a good place to point out to the class that any physical quantity that exists in "smallest possible" parts is said to be quantized. We have seen that mass is quantized. We shall presently see that energy is quantized, too. Also note how, on the scale of everyday sizes, mass, charge, and energy do not appear to be quantized. Thus, the everyday world observed with our unaided senses is remote from this aspect of the "real" world. As an example, in an ordinary 110-V, 100-W lamp bulb, 6×10^{18} separate elementary charges enter and leave the bulb each second.

E5–2 THE CHARGE-TO-MASS RATIO FOR AN ELECTRON

Equipment:

- Electron beam tube that gives at least 5 cm visible beam (from *E4–8*)
- Vacuum pump, power supply, hook-up wire as in *E4–8*
- Cardboard tube, about 7.5 cm diameter and 15 cm long
- Copper magnet wire (any size between 18 and 28 gauge can be used)
- Current balance and all accessories (power supply, ammeter, etc.) for calibration of magnet

The experiment described here will give only an order of magnitude result for q_e/m (see sample data), but students can get a lot of satisfaction out of using their own homemade beam tubes.

It is essential to have an electron beam tube that will give a fairly well-defined and clearly visible beam. Procedures are not described in great detail. The experiment is recommended only for more enterprising and resourceful students. For another method see *D54*, "Charge-to-Mass Ratio for Cathode Rays," on page 369 of this *Resource Book*.

Any wire between #18 and #28 gauge could be used. About 12 m are needed for each pair of coils.

After students have discussed and performed the Millikan experiment, they can calculate m (the mass of an electron) by combining the results for q_e/m and q_e.

Answers to questions

1.–2. Student answers.

Sample Results

Accelerating voltage between filament and anode plate: 150 V.

With the deflecting plate also 150 V above filament potential, the beam goes straight up the tube.

With the deflecting plate connected to the ground, a current of 0.94 A in the magnetic deflection coils is needed to straighten out the beam.

With magnetic deflection above, the beam hits the plate 2.5 cm from the hole in a node.

The distance between the anode plate and the deflecting plate equals 1.5 cm.

$$\therefore R = \frac{d^2 + x^2}{2x} = \frac{6.25 + 0.5}{1.5} = 4.5 \text{ cm}$$

$$= 4.5 \times 10^{-2} \text{ m}$$

$$\text{Field } E = \frac{V}{2x} = \frac{150}{1.5 \times 10^{-2}} = 10^4 \text{ V/m}$$

From calibration curve for coils ($I = 0.94$ A), $B = 5.2 \times 10^{-4}$ T.

$$\therefore \frac{q_e}{m} = \frac{2V}{B^2R^2} = \frac{2 \times 150}{(5.2 \times 10^{-4})^2 (4.5 \times 10^{-2})^2}$$

$$= 5.5 \times 10^{11} \text{ C/kg}$$

E5–3 THE MEASUREMENT OF ELEMENTARY CHARGE

Equipment:

- Millikan apparatus, complete
- Suspension of latex spheres in water
- Power supply 6 V, 5 A for light source
- Power supply 200–250 V dc
- Voltmeter 0–250 V dc

Chapters 17 and 18 of the *Text* follow in historical sequence the development of atomic theory from the laws of chemical combination to Thomson's model of the atom as a positive blob embedded with negatively charged electrons. In Chapter 17, Faraday's work on electrolysis is presented as evidence for a connection between electricity and matter. In Chapter 18, the early work on cathode rays is followed by a description of Thomson's determination of q_e/m and Millikan's experiment to determine the value of the charge on the electron.

Our apparatus for the simplified Millikan experiment is one in which the charge on a tiny particle of polystyrene is measured by adjusting the vertical electric field until the particle neither rises nor falls. The electric force upward on the particle is then equal to the gravitational force, mg. The same apparatus can, of course, be used for the more conventional version of the experiment in which the two forces do not balance, and the sphere moves with terminal velocity.

If you plan to do the entire experiment and compute values of charge you should first raise with the students the questions: "Is there a limit to how small an electric charge can be?" and if so, "Does electric charge come in multiples of some basic 'atom' of electricity?"

One should ask students how the attribute of "smallest" can be demonstrated. Perhaps the answer to this question can be left for the experiment to clarify. Or perhaps it will help to illustrate the question by means of the following analogy:

You have a collection of cardboard egg cartons. Each of these essentially weightless containers has concealed in it anywhere between none and a dozen eggs. How do you now find that an "egg" is not endlessly divisible, but comes in multiples of a "smallest possible" part? And how do you find the size (here the weight) of such a part?

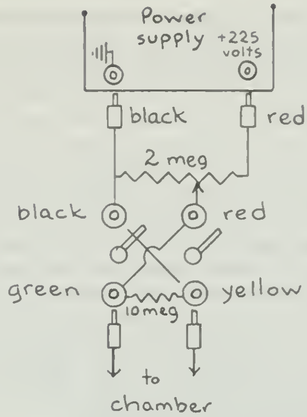
The rather obvious answers to these questions, achieved with the help of a balance, are analogous to the answers to our questions about electric charge achieved with the help of the instrument described below.

Students may ask, "How do they ever manage to make batches in which all the particles have the same size?"

"Seed" particles are added to a mixture of monomer (that is, styrene, if the polymer being made is polystyrene) and catalyst. The seed particles act as nuclei for growth of larger particles of polymer. The rate at which a particle grows depends on its size, and the smaller the particle the faster it grows. The result is that there is a "sharpening" of the size distribution. The "seeds" need not be all the same size, as long as they are small. Of course, there should be no new nucleation once the process has started. Soap is added to prevent the formation of new particles, and to prevent coagulation of particles already formed.

The student instructions assume that the apparatus has been set up and put in working order. For details of how to assemble and adjust the apparatus, refer to the instructions packed with the equipment.

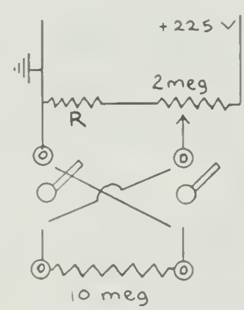
In passing, it is worth noting that the Millikan apparatus can be used for a demonstration of Brownian motion (Unit 3). Draw a little smoke from a just-extinguished match into a medicine dropper, and expel into the chamber through the hole in the wall. Too much smoke will make the whole field of view look gray. If this happens, wait until most of the smoke has settled, or blow some out.



You should see a relatively small number of bright spots against an almost black background. The "jiggle" of these tiny particles is very noticeable. You may want to prepare the ground a little for the Millikan experiment by showing the effect of an electric field on the smoke particles (which are electrically charged). Use the reversing switch and potentiometer to demonstrate the effect of changing field strength and direction.

If students have difficulty seeing the latex particles, let them introduce a little smoke into the chamber. When most of the smoke has settled, students should have no difficulty in seeing the tiny smoke particles, and can adjust the light source and microscope for maximum visibility. Then go back to latex spheres. If none are now visible, the atomizer may be at fault rather than the optics.

The evidence that charge is quantized is less convincing if one works with highly charged particles. For example, the "balancing voltages" for particles carrying 6 and 7 units of charge are 39 and 34 V, respectively. The difference is probably not significant experimentally. On the other hand, the balancing voltages for singly, doubly, and triply charged particles are 234, 117, and 78 V, respectively, and the difference is much clearer.



You can make it impossible for students to work at uselessly low voltages by adding a fixed resistor between the potentiometer and the black input terminal.

If R is 250 Ω the minimum voltage will be about 25 V.

One group of students may not accumulate enough data to give clear evidence of the quantization of charge. If data from the whole class are pooled, the quantization should be more obvious. Before pooling data, try to make sure that there are no systematic errors in the results of any group. The data might be graphed by each lab group before pooling their results. This makes it easier to identify any systematic errors among different groups. It might be well, for example, to check the voltmeters against one another.

Answers to questions

1. Field $E = \frac{V}{d}$ V/m
2. Electric force $F = qE$ N
3. Gravitation force $F = ma_g$ N
4. Some particles appear to move up, others down in the electric field.
5. Some particles are positively, others negatively, charged.
6. The most rapidly moving particles are the more highly charged particles.

7.–12. Student answers.

Some Typical Data

It may be useful to know the density of polystyrene, $p = 1{,}050$ kg/m^3, and the mass of a sphere, $m = \frac{4}{3}\pi r^3 p$. A sphere whose diameter ($2r$) is 1.305 microns (1.305×10^{-6} m), therefore, has a weight $mg = 1.18 \times 10^{-14}$ N.

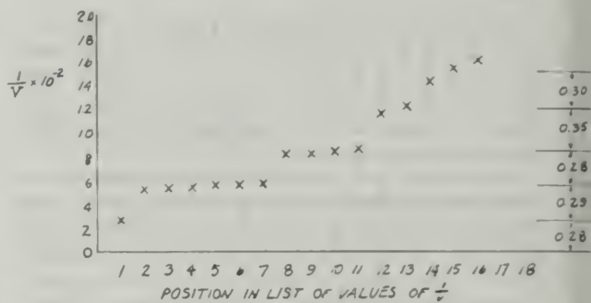
The results given here were obtained with spheres having a diameter of 1.305 microns. The balancing voltages for spheres of 1.099-micron diameter will, of course, be smaller. The values of V will be reduced in the ratio $\left(\frac{1.1}{1.3}\right)^3 = 0.61$, and the values of $\frac{1}{V}$ will be increased by the factor $\frac{1}{0.61} = 1.64$.

Balancing Voltages

V	$\frac{1}{V} \times 10^{-2}$	V	$\frac{1}{V} \times 10^{-2}$
11 volts	9.1 volt^{-1}	60	1.67
80	1.25	68	1.47
115	0.87	170	0.59
120	0.83	360	0.28
85	1.18	175	0.57
35	2.86	65	1.54
112	0.89	170	0.59
175	0.57	170	0.59
175	0.57	120	0.83

Values of $\frac{1}{V}$ Arranged in Increasing Order

$\frac{1}{V} \times 10^{-2}$	Position in list	$\frac{1}{V} \times 10^{-2}$	Position in list
0.28 volt^{-1}	1	0.87	10
0.57	2	0.89	11
0.57	3	1.18	12
0.57	4	1.25	13
0.59	5	1.47	14
0.59	6	1.54	15
0.59	7	1.67	16
0.83	8	2.86	17
0.83	9	9.1	18



Values of $\frac{1}{V} \times 10^{-2}$ plotted against the position of each value on the list.

Remember that charge q is directly proportional to 1/V. The graph could be drawn to show values of q directly instead of 1/V, but this requires more calculations before plotting. Whichever way it is done, the graph of pooled results should show vividly that the charge on each particle is a whole number of "smallest observed" units. If we measure from the graph the size of this "smallest observed" unit, we find in our example

$$ma_g = 1.18 \times 10^{-14}\ \text{N}$$

$$d = 5.0 \times 10^{-3}\ \text{m}$$

$$ma_g d = 5.9 \times 10^{-17}\ \text{J}$$

$$q = \frac{ma_g d}{V} = 5.0 \times 10^{-17} \times 0.28 \times 10^{-2}$$

Therefore, $q = 1.6 \times 10^{-19}$ C

This value of $q = q_e$ is considered to be the elementary unit of charge. The elementary particle carrying it is the electron.

For particles having diameter 1.099 microns the corresponding values are

$$ma_g = 7.3 \times 10^{-15}\ \text{N}$$

$$ma_g d = 3.6 \times 10^{-17}\ \text{J}$$

and the difference in 1/V values between successive steps in the graph should be 0.44×10^{-2} V^{-1}

E5–4 THE PHOTOELECTRIC EFFECT

Equipment:

- Phototube unit
- Amplifier power supply
- Loudspeaker, earphones or CRO (or microammeter)
- Colored filters
- Light source: mercury lamp or fluorescent or incandescent lamp
- Voltmeter, 0–2.5 V dc

Do the qualitative demonstrations (*D55*) using an electroscope before students start the experiment. The demonstration shows that negative charge (electrons) can be driven off from a clean metal plate by light, and that the effect depends on the wavelength of the light and the nature of the metal surface.

T36 can be used to explain the construction of a photocell and the measurements to be made in the experiment.

The central purpose of the student experiment is to give evidence that the wave model of light is inadequate.

Specifically, the experiment shows that the kinetic energy of photoelectrons knocked out of a photosensitive surface depends on the color (and hence the frequency) but not on the intensity of the incident light. According to the wave model the kinetic energy depends on the intensity of the light.

The experiment then goes on to show that the maximum kinetic energy of the electrons is a linear function of the frequency of the incident light. A graph of energy against frequency is a straight line whose slope (as found by precise measurements) is Planck's constant h. Measurements of h in this experiment are within an order of magnitude of the accepted value $h = 6.62 \times 10^{-34}$ J·sec.

Equipment Notes

Since many important details differ from one model of the equipment to another, it is important at this point to heed the manufacturer's instructions packed with each piece of equipment.

Phototube unit. One cannot really see the phototube inside the box, and students may not have seen one before. To prevent the experiment from becoming too mysterious, show them an unmounted phototube before they begin the experiment, and point out the emitting surface and collecting wire.

Amplifier power supply. The case of the phototube unit is connected to the ground terminal of the amplifier. In an experiment like this where small signals are amplified, noise is always likely to be a problem. In general, grounding the case of the phototube unit in this way will reduce noise. But in some instances it may be better to unground the amplifier by using a three-to-two adapter plug to connect it to the line.

Earphone or loudspeaker. At this frequency the human ear can detect ac currents as small as 1 μA in the earphone. Since the maximum gain of the amplifier is probably at least 100, currents of less than 10^{-8} A can be detected. The loudness of hum in the earphone or speaker increases with the current in the photoelectric cell.

A cathode-ray oscilloscope can be used to detect the amplified photocurrent instead of an earphone or loudspeaker. It is important to use shielded connecting wires to avoid extraneous pick-up. Set the 'scope to a sweep rate of a few hundred per second; set vertical gain to maximum.

Light source. Mercury vapor lamps are the best for this experiment (Macalaster #3400, Damon 03-075080-6, or the small 4-W "ozone" lamps made by General Electric, which cannot be run without ballast). These emit the four frequencies listed. Because they emit ultraviolet, students must not look directly at them.

Fluorescent lamps give a continuous spectrum with bright mercury lines superimposed on it, as can be seen easily with a pocket spectroscope. Fluorescent room lighting is adequate if the photocell is directly below the ceiling fixture. A large lens can be used to concentrate more light on the cell.

Fluorescent desk lamps may be unsatisfactory unless you can screen out the large inductive hum they cause in the earphones, which has nothing to do with the amount of light shining on the photocell.

An incandescent light source (such as the light source of the Millikan apparatus) is less satisfactory. It gives a continuous spectrum and therefore filters whose cut-off frequencies are known exactly must be used (see discussion of filters, below). If earphones, loudspeakers, or oscilloscopes are going to be used as detectors, the beam must be "chopped" to give an ac current in the phototube. The 12-slot strobe disc driven by the 300 rpm motor gives a 60-Hz signal that is unsatisfactory because it can easily be confused with line frequency pick-up. You can make a cardboard disc with about 60 teeth that will give a chopping rate of 300/sec when mounted on the 300 rpm motor.

An uninterrupted beam of light from an incandescent lamp (or daylight) can also be used: The photocurrent will be dc and so will the amplified current. Use a dc milliamter or 0–2.5 V dc voltmeter instead of speaker or oscilloscope to detect the amplifier output. Set the dc offset control very carefully so that the meter reading is zero when the phototube is covered. This is certainly the simplest method to use and needs least specialized equipment. It has the advantage that the noise problem is largely eliminated.

Filters. At least three filters to fit over the photoelectric cell windows are necessary. Ideally, these will be yellow, green, and blue filters that isolate the mercury yellow, green, and blue lines. With no filter, the highest frequency effective is either the violet or the ultraviolet mercury line, probably the former. If you use a fluorescent lamp source, you can assume that the dominant frequencies transmitted by the filters are the mercury lines.

Voltmeter. A voltmeter, 0–2 or 0–5 V dc, is used for measuring "stopping voltages." Its use is described more fully in the procedure section below.

Students may notice that if they continue to turn the voltage control knob past the cut-off setting, the signal increases again. This is because some photoelectrons are emitted from the collector wire and, because the collector is now quite negative with respect to the emitter, they are drawn to the emitter. The current is now in the reverse direction, but this cannot be established with the loudspeaker detector (though it can with oscilloscope or dc meter).

If students ask about this, you can either explain the cause and tell them to find the voltage setting for minimum signal, or reduce the reverse current by making a black stripe with narrow tape or grease pencil down the middle of the tube face to prevent light from falling on the collecting wire.

The Wrattan series of filters have sharp cut-off at the high-frequency (short-wavelength) end. By using some of these filters and an incandescent lamp (continuous spectrum), many more values of stopping voltage can be obtained. The filters are made by Kodak and can be obtained through shops that sell photographic supplies.

Wrattan Filter	Cut-off λ
Glass alone	3,200 Å
#1 (poor stability)	3,500 Å
2B	3,900 Å
4 (poor stability)	4,500 Å
8	4,600 Å
12 + 8	5,000 Å
15 + 8	5,100 Å
16	5,200 Å
23A	5,600 Å
26	5,900 Å
29	6,100 Å
70	6,500 Å
97 + 26	6,800 Å

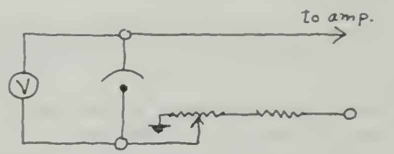
Answers to questions (Part 1)

1. Students will not be able to detect any time delay. The wave model of light requires a delay of about 100 sec (see *Text* page 549); the particle model is consistent with instantaneous emission.
2. Yes, the greater the light intensity, the louder the hum (and we can deduce that more photoelectrons are emitted).
3. No, stopping voltage does not depend on intensity. (But because the signal is weaker for lower light intensities, it may be difficult to find precise cut-off settings at low intensities.)
4. The stopping voltage increases as the frequency of light increases.

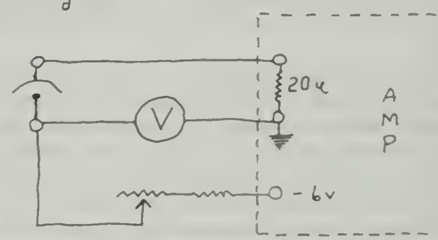
You may want to stop here. For a discussion of the meaning of the results so far, see the discussion notes at the end of these instructions.

The second part of the experiment requires that students make more precise measurements of stopping potential, then graph their data and deduce Planck's constant from it. They should already have read Sec. 18.4 of the *Text* and understood Millikan's graph on *Text* page 550, since this is not material that can be learned best by starting with the experiment.

Voltages measured in this way will not be absolute values. The resistance of the phototube is very high (~ 10 megohms). When a voltmeter of appreciably lower resistance is put across it, the meter draws some current and voltage across the phototube drops. But the relative values for stopping voltage can still be compared and used to plot the vq_e versus f graph.



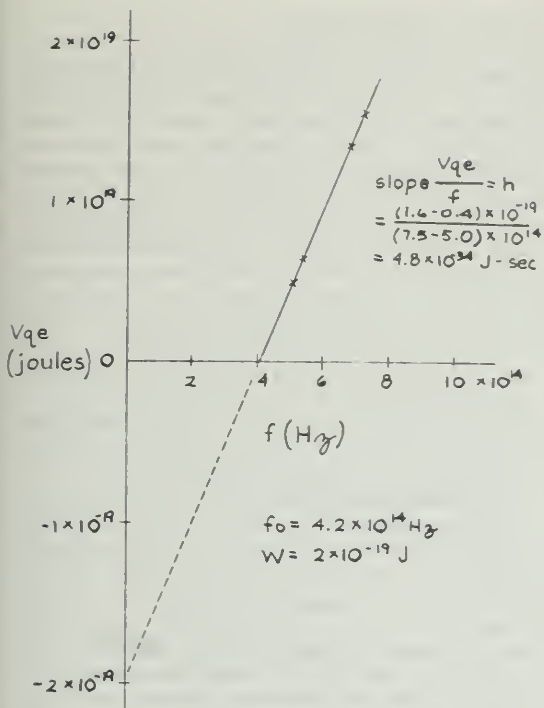
a) Voltage readings are relative only.



b) Absolute voltage measurement.

Alternatively, if the voltmeter is connected between the collector of the phototube and the ground terminal of the amplifier, it will give absolute readings.

The value of h will be approximate for several reasons. For example, a fluorescent lamp gives out a continuous spectrum that contains all visible frequencies at low intensity. Some of these frequencies that pass through the filters will be greater than those of the bright emission lines. Also, the electron-emitting surface is never uniformly clean. Various spots on it have various work functions. (Millikan had to prepare his pure metal surfaces by shaving off the oxide coating in a high vacuum.) The signal is small, and there may be considerable "noise" as well, which makes it difficult to make accurate determinations of stopping voltage. The use of inexpensive voltmeters limits the precision with which you can measure the low values of stopping voltage. Results should, however, be to better than an order of magnitude. A typical plot is given below.



The value of f_0, the threshold frequency, and of W, the work function, varies from metal to metal and depends on the condition of their surfaces. Students' results will depend on the accuracy of their voltage readings.

color	frequency f (Hz)	stopping voltage V (volts)	kinetic energy Vq_e (joules)
yellow	5.2×10^{14}	0.30	0.48×10^{-19}
green	5.5×10^{14}	0.38	0.61×10^{-19}
blue	6.9×10^{14}	0.83	1.33×10^{-19}
violet	7.3×10^{14}	0.96	1.54×10^{-19}

Answers to questions (Part 2)

5. (a) Yes, if it is assumed that the frequency of light is in some way related to the energy it delivers to the photoelectron.
   (b) Yes, since the intensity of a light beam is a measure of the energy it carries. Brighter sunlight, for example, causes a more severe sunburn.
   (c) Yes, since the photoelectron "sees" only a tiny fraction of the advancing wavefront, it may have to save up the incoming energy over a period of time.
6. (a) Same as 5(a).
   (b) If a light beam is a stream of particles, a beam of greater intensity might be explained as a beam that carries more particles per second, none of them any more energetic than those in a low-intensity beam. In this case, the stopping potential would not be related to the intensity of the beam.
   (c) No delay would be expected since photoelectrons would receive the necessary energy to be emitted at the instant they were struck by light particles of sufficient energy.
7. Student answers.
8. No, because there is no result in this experiment that demands a wave theory for its explanation.

Discussion

Transparencies *36* and *37*, which show idealized results of this experiment, and the photon theory explanation can be used after students have finished the experiment.

The important ideas to emphasize in a discussion of this experiment are:

1. The stopping voltage (and hence the maximum energy of photoelectrons) is a linear function of the frequency, and is independent of the intensity of the incident light.
2. The photoelectrons are emitted immediately when light falls on the photoelectric cell.
3. The preceding two statements are both inconsistent with the wave model of light (*Text*, pages 549–550) according to which (a) the maximum energy of the photoelectrons should also depend on the intensity of light, and (b) the photoelectrons should not emerge until several hundred seconds after light strikes the cell.
4. The slope *h* of the graph line turns out to be a quantity already shown to be important by Planck's study of radiation, in which he assumed light came in discrete quanta of energy *hf*.

Two papers give details of some of the finer points of this experiment:

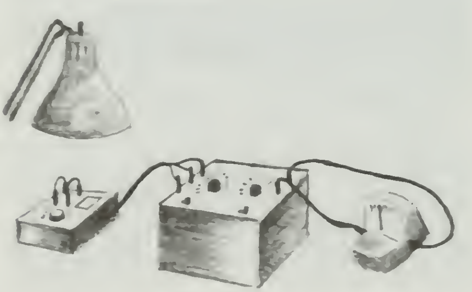
A. Ahlgren, "Inexpensive Apparatus for Studying the Photoelectric Effect and Measuring Planck's Constant," *The Physics Teacher*, October 1963.

H. H. Gottlieb, "Photoelectric Effect Using a Transistorized Electrometer," *The Physics Teacher*, November 1965.

A Simpler Qualitative Setup

You may be bewildered by the number of alternative light sources and detectors that have been suggested for *E5–4*. The simplest setup (and one that requires the least special equipment) uses:

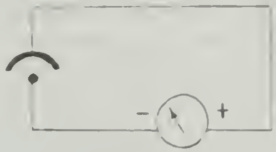
(a) a desk lamp (or other incandescent source)
(b) voltmeter (2.5 V dc) or milliameter to detect amplifier output



There is no noise problem with this arrangement. Disadvantages are:

1. The incandescent lamp has a continuous spectrum that makes it difficult to know what the highest effective frequency passed by the filter is.
2. The dc offset control must be carefully set to make sure that the meter reads zero when there is no light falling on the phototube.

You can show the direction of the photocurrent and its dependence on light intensity very simply with the *Project Physics* phototube unit and microammeter, and a desk lamp. Connect the meter directly across the phototube, using the black jacks on the front of the unit.



A 100-W tungsten bulb about 10 cm from the phototube gave about 5 μA.

E5–5 SPECTROSCOPY

There are several parts to this sequence of observations.

Certainly all students should look at as many spectra as possible (bright line, absorption, and continuous) including a photograph of the Balmer spectrum of hydrogen.

The work becomes more quantitative as it goes along. Some teachers may not want to pursue it to the end, which requires that students calculate wavelengths and from them perhaps Rydberg's constant and some of the energy levels of an excited hydrogen atom.

Equipment

Replica grating and/or "take-home" pocket spectroscopes (Damon 03-075075-X, Macalaster 32582)
Sources of line and continuous spectra, such as the following:

(a) incandescent lamp (see *D56*, Blackbody Radiation)
(b) spectrum tubes of various gases and power supply
(c) flames, including Bunsen burner with various metallic salts added
(d) concentrated solutions of colored salts, dyes, chlorophyll, etc., to show absorption
(e) fluorescent lamp
(f) Balmer tube (atomic hydrogen) with power supply. If you use the Macalster #1300 high-voltage source here, be sure to remove the 6.8-megohm resistor taped to its output. Macalaster #1350 spectrum tube power supply needs no alteration.

Polaroid camera (model 95, 150 or 800; film speed 3000), and tripod

Procedure: qualitative

Have the class observe as many different spectra as possible to establish (a) the existence of bright-line, absorption, and continuous spectra, and (b) the qualitative observation that the red light (longer wavelength) is "bent" more than the violet by a grating. In explaining spectra it is a good idea to contrast emission and absorption spectra first of all and then contrast line and continuous spectra.

Let the students use the pocket spectroscopes outside the classroom to look at illuminated signs, street lights, sky light, moonlight, etc. The Fraunhofer absorption lines of the solar spectrum (described on *Text* page 569) can probably be seen with the pocket spectroscopes only by looking directly at the sun through a dark filter and with a somewhat narrowed slit. Razor blades taped over the slit can narrow it very well. Remind students of the danger of looking at the sun directly.

Ask whether anyone can explain why a fluorescent lamp gives both a continuous spectrum (from coating on the walls of the tube) and the bright-line spectrum of mercury (from mercury vapor in the tube). This is a good point to observe that continuous spectra are emitted by solids (for example, lamp filaments) or highly compressed gases (the body of the sun), while bright-line spectra are emitted by excited gases (for instance, discharge tubes or salt sprinkled into a Bunsen flame). Absorption spectra are formed when light having a continuous spectrum passes through relatively cool gases, liquids, or transparent solids.

Ideally, students should see a hydrogen spectrum against a black background in a darkened room. Probably only three lines (red, blue-green, violet) will be visible, in which case the idea of a regular series (Balmer's) will not be very convincing. For this reason, the second part of the experiment is devoted to photography which will reveal several additional lines in the ultraviolet to which the eye is insensitive.

Discuss these observations to bring out the point that, because the emission of bright lines is evidently an atomic process, an explanation of spectra can reveal a great deal about the structure of atoms. Note in this connection how each element has its own characteristic bright-line spectrum. Note how the simplest atom, hydrogen, seems to produce the simplest spectrum.

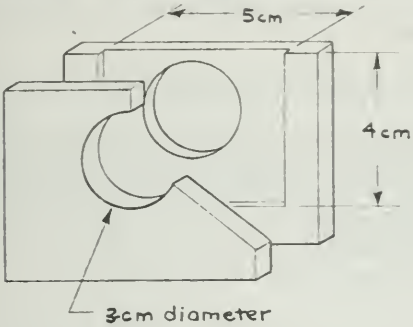
Procedure: quantitative

The hydrogen tube should give a bright red light. Old tubes give bluish light that does not produce a good Balmer spectrum.

Mount the hydrogen spectrum tube vertically against a black background in a darkened room (see photo at bottom right). Set a meter stick horizontally just behind the tube. Secure a grating directly in front of the camera lens, making sure that the grating lines are parallel to the spectrum tube. (You see a spectrum running horizontally.)

Set up the camera on a tripod or other firm support about 1.3 m in front of the tube.

An easy way to attach the diffraction grating to the lens is to use a cardboard holder. This is made with two layers of corrugated cardboard: one with a hole that fits tightly around the lens, the second with a 5-cm × 4-cm slot cut in one half of the corrugation and a 3-cm hole cut in the other half. When the two layers are cemented together, the grating is inserted and the holder can be easily positioned on and removed from the lens. The



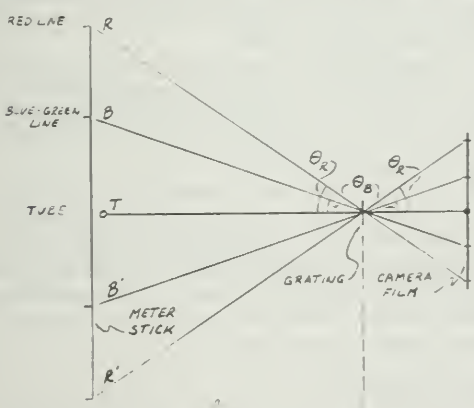
Hole to fit camera lens (5cm diameter for model 002, 3-cm diameter for models 95, 150, 800).

grating must, of course, be oriented so that its lines are parallel to the spectrum tube, that is, vertical.

Exact exposure time will depend on the lighting conditions in your room, and is best found by experiment. You may be able to record both the spectrum lines and the meter stick scale with a single exposure in a darkened, but not dark, room. Or you may have to make a double exposure: one of the spectrum when the tube is on, the second exposure with the tube turned off and the room lights on in order to show the meter stick scale. It is not necessary to remove the grating for the second exposure.

The resulting picture should show the spectrum lines clearly against the meter stick scale.

If you have the spectrum tube directly in front of the camera lens so that the central (undiffracted)



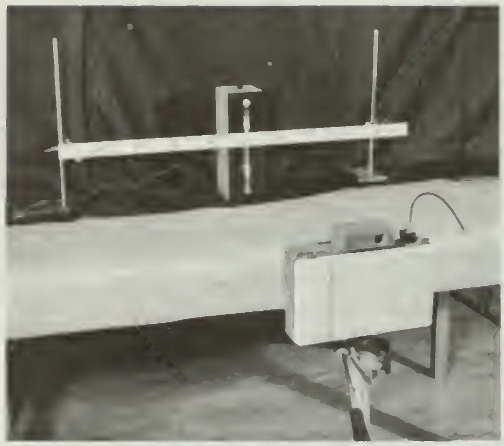
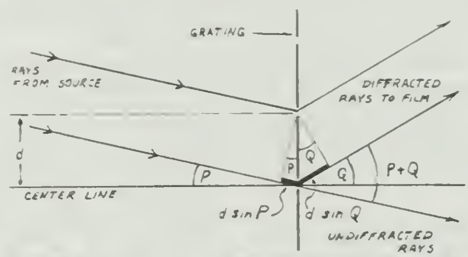
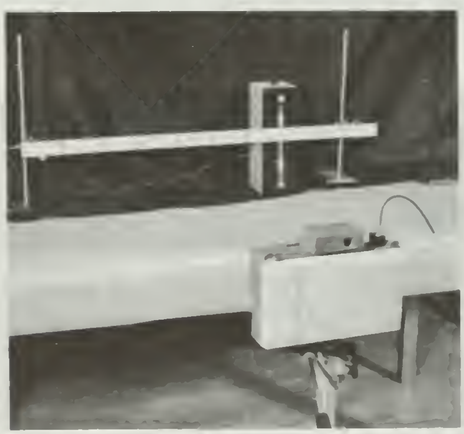


image is in the center of the picture, there may not be enough space on each side of it to show the red line (longest wavelength), which is at the greatest angular distance from the source. This will certainly occur if you use the commonly available gratings with 5 360 lines/cm and any of the cameras suggested here. To get a photograph showing the Hα line, a different setup will be required, as shown in the figure and photo below. This geometry gives a picture with the source near one edge of the photograph and the first-order red line near the other. Compare the camera's field of view with the spectrum seen by the eye through the same grating to find the best orientation for the camera, or use the focusing screen to make sure that the red line will be recorded.





Qualitative observation of the hydrogen spectrum (with or without a photograph) is as far as some teachers will wish to proceed. The important point to be made is the regularity of the spectrum, and it may be better for some students just to look.

To go further requires the presentation of the grating equation ($\lambda = d \sin \theta$) and probably a correction to it to account for the off-axis geometry of the setup just described.

The derivation of the grating formula $\lambda = d \sin \theta$ for normally incident parallel light is probably familiar to you, but for convenience it is reprinted at the end of this note.

The grating space d is presumably known. For the commonly available gratings of 5,360 lines/cm, $d = 1.89 \times 10^{-6}$ m.

The angle θ for each spectrum line is found from the source-grating distance l and its position on the film in accordance with one or the other of the two preceding diagrams.

If the source is in the center of the photograph (light from the source is normally incident on the grating), the geometry of the first diagram applies.

It can be seen from the diagram that for the red line

$$\tan \theta = \frac{BT}{l} = \frac{B'T}{l}$$

and similarly for the other lines. With more accuracy

$$2 \tan \theta = \frac{BB'}{l}, \text{ etc.}$$

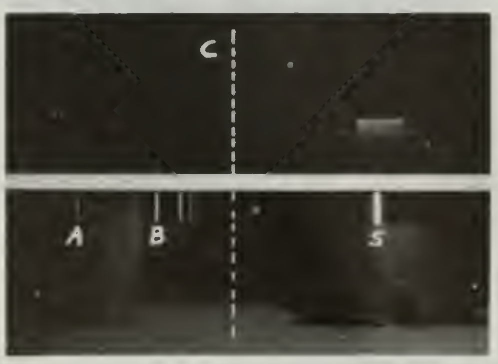
Since BB' is identified from the photograph and l (the source-to-grating distance) is known, the values of $\tan \theta$ can be found for each line.

Tables then give values of θ and also $\sin \theta$ so that values of $\lambda = d \sin \theta$ can now be found.

If the source is not at the center of the photograph, refer to the second diagram. In this arrangement, the beam from the source is no longer incident perpendicular to the grating. Hence, the rays are not diffracted through an angle θ, but through an angle $P + Q$ as shown in the diagram.

The path difference between the two rays shown in the diagram is:

$$d \sin P + d \sin Q$$



The angle P is the same for all spectrum lines. It is the angle between the source S and the center of the field of view C in the labeled photograph shown.

Angle P is easily found since its tangent is the distance SC (from the photograph) divided by the distance l from source to grating:

$$\tan P = \frac{SC}{l}$$

The angle Q is the angle between a given spectral line (A, B, . . .) on the film and the center of the field of view C. Thus

$$\tan Q_A = \frac{AC}{l} \quad \tan Q_B = \frac{BC}{l}, \text{ etc.}$$

The procedure for finding wavelengths is as follows. From the measurements taken from the apparatus and the photograph, find $\tan P$ and $\tan Q$ for a given spectrum line. From tables find $\sin P$ and $\sin Q$. Then

$$\lambda = d \sin P + d \sin Q$$

or

$$\lambda = d (\sin P + \sin Q)$$

Students should be able to get results that agree within a few tens of angstroms. (See Sample Results.)

R_H = 109,678 cm in a vacuum. The value in air differs from this in the last two digits. This will not show up in students' results, of course.

$$E = hf = h\frac{c}{\lambda}$$

$$= 6.6 \times 10^{-34} \times \frac{3.0 \times 10^{8}}{0.66 \times 10^{-6}}$$

$$= 3.0 \times 10^{-19} \text{ J}$$

Discussion of the hydrogen spectrum

Class discussion should bring out the point that the Balmer series is only one of several spectral series of hydrogen and that the lowest energy state of this series is not the lowest possible (ground) state of the atom. It is next to the lowest state. When electrons do fall to the ground state, the quanta emitted have higher energy and lie in the ultraviolet (Lyman series).

Discussion should also make it clear that spectral series of other elements are generally much more complex, but from them it is also possible to work out the related energy level differences of excited valence electrons.

It is probably also worth reminding students that the lines of the Balmer series involve excited atoms but not ionized atoms. If possible, show pictures of the Balmer series in absorption in the spectra of stars. From this kind of evidence we know that hydrogen is by far (~90%) the most prevalent element in the universe as a whole.

Sample Results

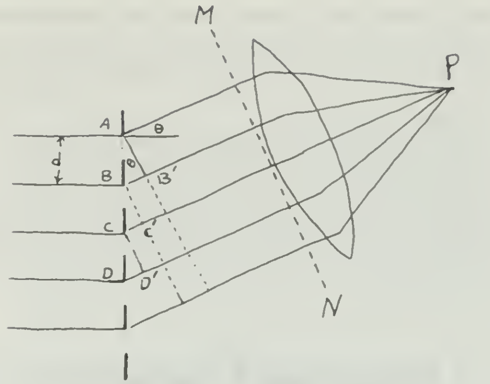
As many as nine lines of the hydrogen Balmer spectrum have been measured by this technique. More typically, students will be able to measure about five lines:

measured value (Å)	accepted value (Å)	% difference
6,350	6,562	3.2
4,830	4,861	0.6
4,430	4,340	2.1
4,050	4,101	1.3
3,920	3,970	1.3
3,820	3,835	0.4

Derivation of the grating formula $\lambda = d \sin \theta$
Consider a parallel beam, normally incident at the grating.

The diffracted rays are brought to a focus by the (eye or camera) lens. There will be reinforcement at the point P if the diffracted rays are in phase in the plane MN. For this to happen, the path difference between rays diffracted at successive openings in the grating must be a whole number of wavelengths. Thus, $BB' = n\lambda$; $CC' = n\lambda$; $DD' = n\lambda$, etc., for P to be bright.

If d is the grating spacing, $BB' = CC' = DD' = \ldots = d \sin \theta$. So point P will be bright if



$$n\lambda = d \sin \theta$$

There is a series of values of $\sin \theta$ corresponding to $n = 1, 2, 3 \ldots$ that will satisfy this equation for a given λ. In this experiment we are only concerned with the first-order spectrum ($n = 1$) for which the formula simplifies to

$$\lambda = d \sin \theta$$

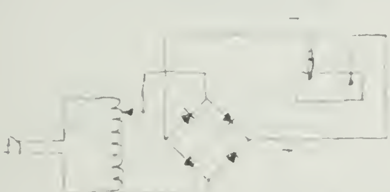
Longer wavelengths are diffracted through larger angles ($\sin \theta \propto \lambda$).

Film Loop Notes

L45 PRODUCTION OF SODIUM BY ELECTROLYSIS

Solid-state rectifiers are used in a bridge circuit. The dc potential difference between electrode and crucible was about 15 V; the current was 18 A. In the film, the rectifier circuit is seen mounted on a copper heat-dissipating plate.

One might expect water vapor in the atmosphere to react with the film of sodium on the surface of the melt. Evidently there is sufficient updraft of heated air near the surface to prevent this from happening.



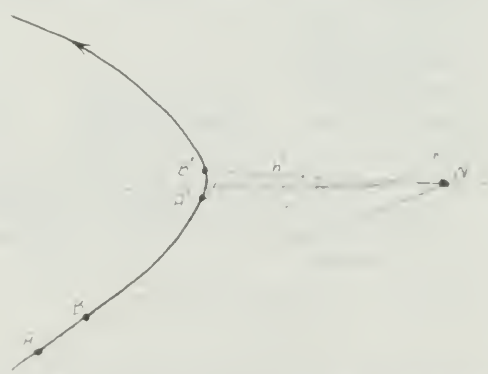
L46 THOMSON MODEL OF THE ATOM

The horizontal component of the field of the electromagnet gave rise to an inward force on the upper poles of the floating magnets; the outward force on the lower poles was weaker since they were farther from the electromagnet. Thus, the electromagnet exerted a net inward force on the ping-pong balls. The model was not exact, since the net force on the magnets was only approximately proportional to the distance away from the center of the pattern.

L47 RUTHERFORD SCATTERING

The law of areas holds because the nuclear repulsion is a central force. This conclusion is true whether or not the force is a Coulomb inverse-square force. (See the notes for *L15*, "Central Forces," and *Text* Sec. 7.2.)

A student may observe that the α particle slows down as it approaches the nucleus. (The displayed points are closer together.) This is easily interpreted. Consider the extreme case when the par-



ticle is aimed head-on at the nucleus, slows down, stops, and then moves out again. In general, for any path, the α particle gains potential energy in coming close to the nucleus, hence its kinetic energy must decrease.

The student may feel that the slowing of the particle conflicts with the law of areas, since in the case of planetary motion the planet speeds up when it is closest to the sun. (But the planet loses potential energy as it "falls" toward the sun!) Have the student draw a path similar to the diagram shown. (It need not be a perfect hyperbola.) The long triangle ANB has altitude h. Now the law of areas can be used to find the distance $A'B'$; it is less than AB because the height h' is greater than h. Thus, the law of areas does not conflict with the slowing down of the α particle as it approaches the nucleus.

You might wish to reconsider *E2–11* with the central force now being repulsive. Students might develop various paths for different initial motions of the intruding particle.

Suggested Solutions to Study Guide Problems

CHAPTER 17

2. The atomic mass of zinc is listed as 65.4 units and that of oxygen as 16.0 units. Therefore, the mass of a combination of 1 zinc atom and 1 oxygen atom is:

$$\underset{\text{(zinc)}}{65.4} + \underset{\text{(oxygen)}}{16.0} = \underset{\text{(zinc oxide)}}{81.4}$$

The percentage of zinc in the combination is the fraction of zinc times 100:

$$\frac{65.4}{81.4} \times 100 = 80.3\% \text{ zinc}$$

Therefore, the percentage of oxygen is

$$(100 - 80.3)\% = 19.7\%.$$

3. As in question 2, the molecular mass for the compound is computed from the atomic masses listed:

$$\underset{\text{(zinc)}}{65.4} + \underset{\text{(chlorine)}}{2\,(35.5)} = \underset{\text{(zinc chloride)}}{136.4}$$

The percentage of zinc is therefore

$$\frac{65.4}{136.4} \times 100 = 47.9\%$$

4. Using an equation similar to that in question 5 we have:

$$\frac{\text{mass of hydrogen}}{\text{mass of nitrogen}} = \frac{\text{mass of H atom}}{\text{mass of N atom}} \times \frac{\text{number of H atoms}}{\text{number of N atoms}}$$

Since the mass of nitrogen obtained was 4.11 g, the mass of hydrogen is $(5.00 - 4.11)$ g = 0.89 g. From the formula NH_3, the ratio of the number of H atoms to the number of N atoms is 3/1. Therefore,

$$\frac{0.89\text{ g}}{4.11\text{ g}} = \frac{3}{1} \times \frac{\text{mass of H atom}}{\text{mass of N atom}}$$

or mass of N atom = 13.9 × mass of H atom.

Using values of the atomic masses from the modern version of the periodic table yields a similar result:

$$\frac{\text{mass of H}}{\text{mass of N}} = \frac{1.01}{14.0} = \frac{1}{13.9}$$

5. Question 4 states that 5.00 g of ammonia is composed of 4.11 g of nitrogen and 0.89 g of hydrogen. This ratio will be the same for any quantity of ammonia. Therefore:

$$\frac{4.11\text{ g nitrogen}}{5.00\text{ g ammonia}} = \frac{x\text{ g nitrogen}}{1\,200\text{ g ammonia}}$$

$$x = 986\text{ g}$$

With this quantity of nitrogen, 214 g of hydrogen will be needed to form 1 200 g (1.2 kg) of ammonia.

6. Using an equation similar to that in question 5, we have:

for NO: $$\frac{\text{mass of N in sample}}{\text{mass of O in sample}} = \frac{1}{1} \times \frac{\text{mass of N atom}}{\text{mass of O atom}}$$

or

$$\frac{\text{mass of N atom}}{\text{mass of O atom}} = \frac{0.47\text{ g}}{(1 - 0.47)\text{ g}} = 0.89$$

for NO_2: $$\frac{\text{mass of N in sample}}{\text{mass of O in sample}} = \frac{1}{2} \times \frac{\text{mass of N atom}}{\text{mass of O atom}}$$

or

$$\frac{\text{mass of N atom}}{\text{mass of O atom}} = 2 \times 0.89 = 1.78$$

$$\text{for } N_2O: \frac{\text{mass of N in sample}}{\text{mass of O in sample}} = \frac{2}{1} \times \frac{\text{mass of N atom}}{\text{mass of O atom}}$$

$$= \frac{1}{2} \times 0.89$$

$$= 0.445$$

These numbers are the ratios of the atomic masses. If oxygen is defined as having an atomic mass of 16.00, then the calculated atomic mass of N would be:

(a) for NO: $16 \times 0.89 = 14.1$.
(b) for NO_2: twice as much or 28.2.
(c) for N_2O: half as much or 7.0.

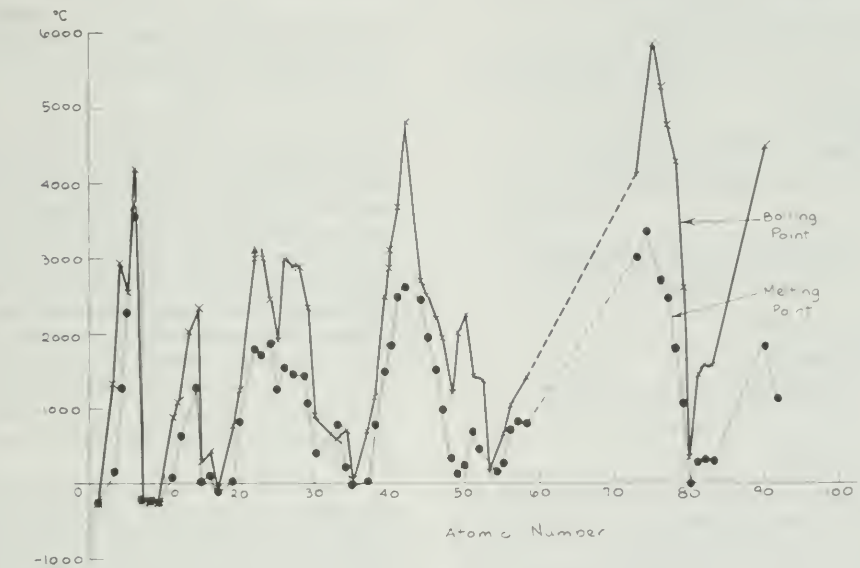
7. If we assign Cl the combining capacity of 1, then the combining capacities of the other elements are:

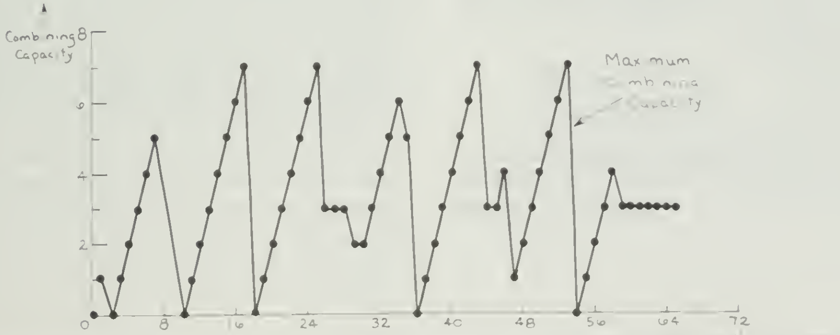
sodium:	1
calcium:	2
aluminum:	3
tin:	4
phosphorus:	5

It should be noted, however that the information given does not preclude the *possibility* of other combining capacities for the same elements.

8. There is no true periodicity, but there are noticeable regularities (see the following graph). For example, every return to zero valence (at atomic numbers 2, 10, 18, 36, and 54) is followed by a steady rise in valence toward a maximum valence of 7 (at atomic numbers such as 17, 25, 41, and 53). We note that the zero valence elements are the inert gases and that they are followed by chemically very active elements Li, Na, K, etc. There thus appears to be a significant relationship between the valence of the atoms of an element and the chemical activity of that element. The more active elements have valences of 1 or 7. The physical significance lies in the electron structure of the atom. This will be discussed in Chapters 19 and 20.

The question of the necessity of this relationship is a philosophic one. There seem to be very few coincidences in nature that have no deeper connection than a chance correlation.





Therefore, physicts tend to believe that for every correlation there is an underlying reason.

9. 8.0 g of oxygen.

Since 96,500 C will produce 1.008 g of hydrogen, a portion of this quantity will produce a smaller quantity of hydrogen. A simple proportion may be written as follows: (Let x represent the number of grams of hydrogen.)

$$\frac{1.008\ \text{g}}{96{,}500\ \text{C}} = \frac{x}{3\ \text{A} \times 3{,}600\ \text{sec}}$$

$$3\ \text{A} \times 3{,}600\ \text{sec} = 10{,}800\ \text{C}$$

$$x = \frac{1.008\ \text{g}}{96{,}500\ \text{C}} \times 10{,}800\ \text{C}$$

$$= 0.113\ \text{g hydrogen}$$

Similarly, a proportion for oxygen is set up:

$$\frac{8.00\ \text{g oxygen}}{96{,}500\ \text{C}} = \frac{x}{10{,}800\ \text{C}}$$

$$x = 0.895\ \text{g oxygen}$$

Note that the ratio of hydrogen to oxygen remains the same:

$$1.008/8.00 = 0.122/0.895 \cong 1/8$$

10. As in question 9, a proportion may be set up:

(a) for 5 min:

$$\frac{32.69\ \text{g zinc}}{96{,}500\ \text{C}} = \frac{x}{0.5\ \text{A} \times 300\ \text{sec}}$$

$$x = 0.05\ \text{g zinc}$$

(b) for 30 min:

$$x = \frac{32.69\ \text{g zinc} \times 0.5\ \text{A} \times 1{,}800\ \text{sec}}{96{,}500\ \text{C}}$$

$$= 0.30\ \text{g zinc}$$

(c) for 120 min:

$$x = \frac{32.69\ \text{g zinc} \times 0.5\ \text{A} \times 7{,}200\ \text{sec}}{96{,}500\ \text{C}}$$

$$= 1.2\ \text{g zinc}$$

11. (a) The table on *Text* page 536 indicates that 96,540 C will produce 35.45 g of chlorine. If $2.0 \times (1.2 \times 10^3\ \text{C})$ (that is, $2.0\ \text{A} \times 1{,}200$ sec) are used, the following proportion will indicate the amount of chlorine produced:

$$\frac{35.45\ \text{g chlorine}}{96{,}540\ \text{C}} = \frac{x}{2.0\ \text{A} \times 1{,}200\ \text{sec}}$$

$$= 0.88\ \text{g chlorine}$$

(b) The table on *Text* page 536 does not indicate the quantity of iodine that would be produced by 1F of charge. This can be found from Faraday's second law of electrolysis. Since iodine has a valence of 1, the amount produced by 96,540 C is 126.9 (atomic mass)/1 (valence). The proportion is written:

$$\frac{126.9\ \text{g iodine}}{96{,}540\ \text{C}} = \frac{x}{2.0\ \text{A} \times 1{,}200\ \text{sec}}$$

$$x = 3.14\ \text{g iodine}$$

(c) The quantity of zinc in part (b) would be identical to that produced in part (a) because the mass liberated is proportional to the amount of charge (Faraday's first law), which is the same in both cases.

(d) The copper spoon to be plated and a piece of silver could be immersed at opposite sides of a container of silver nitrate solution. The spoon and the silver electrode should be connected to a low-voltage source of electric current, with the spoon connected to the negative side of the voltage source. Current will pass through the solution, and silver will be deposited on the spoon.

12. Amount of Cl $= \dfrac{A}{v} = \dfrac{35.45}{1} = 35.45$ g

13. It suggests that the valence indicates the number of "atoms" of electricity associated with each atom of matter.

14. Students have been told of atoms authoritatively since their earliest grades. Most of them accept the notion on faith without ever questioning it. However, this question provides an excellent opportunity for those few "atom skeptics" you may have in your group who feel that atoms really don't exist. If you allow them to go far enough, these nonbelievers will soon be accepting the existence of atoms by means of their own astute reasoning.

The second portion of the question can be employed in demanding clear-cut evidence to reinforce and defend the faith of your "atom believers."

"Atom skeptics" can probably devise a model that gives a rough, first-approximation explanation for the phenomena. But if precision is carried far enough and if their experiments are made sufficiently quantitative, any such model will eventually be seen to be inadequate. Furthermore, even though chemical phenomena usually involve a large number of atoms, the properties of the individual atoms can still be inferred indirectly from the results of the experiments. When many such inferences converge toward the same model, belief that it is the correct model becomes logically very strong.

2. (a) Where the effects of the electric and magnetic fields cancel, we have

$$qE = qvB, \quad \text{or} \quad v = \frac{E}{B}, \text{ and}$$

$$\text{since } E = \frac{V}{d}, \; v = \frac{V}{Bd};$$

$$\text{so} \quad v = \frac{200 \text{ V}}{1.0 \times 10^{-3} \frac{\text{N}}{\text{A} \cdot \text{m}} \times 0.01 \text{ m}}$$

$$= 2.0 \times 10^{7} \text{ m/sec}$$

$$\left(\frac{\text{V} \cdot \text{A}}{\text{N}} = \frac{\frac{\text{N} \cdot \text{m}}{\text{C}} \cdot \frac{\text{C}}{\text{sec}}}{\text{N}} = \frac{\text{m}}{\text{sec}}\right)$$

(b) When the magnetic field acts alone, a circular orbit results, and

$$qvB = \frac{mv^2}{R}, \quad \text{or} \quad \frac{q}{m} = \frac{v}{BR}$$

$$\frac{q}{m} = \frac{2.0 \times 10^{7} \frac{\text{m}}{\text{sec}}}{1.0 \times 10^{-3} \frac{\text{N}}{\text{A} \cdot \text{m}} \times 0.114 \text{ m}}$$

$$= 1.8 \times 10^{11} \text{ C/kg}$$

$$\left(\frac{\text{A} \cdot \text{m}}{\text{N} \cdot \text{sec}} = \frac{\frac{\text{C}}{\text{sec}} \cdot \text{m}}{\frac{\text{kg} \cdot \text{m}}{\text{sec}^2} \cdot \text{sec}} = \frac{\text{C}}{\text{kg}}\right)$$

3. Since 1 A = 1 C/sec past a given point, we need to show how many basic units of charge (electron or proton charges) are equal to 1 C. Call n this number of electrons. Then $n \times$ charge on each electron = 1 C

$$\text{or} \quad n = \frac{1 \text{ C}}{1.6 \times 10^{-19} \text{ C/electron}}$$

$$\text{so} \quad n = 6.25 \times 10^{18} \text{ electrons}$$

4. The final energy = the initial energy. Consider the simple case of a uniform electrostatic field between cathode and anode. Then the force on the electron is q_eE everywhere. The electron leaves the plate with some kinetic energy. It is slowed by the electrostatic field and stops a distance d from the cathode. The initial kinetic energy must equal the work done by the electron so

$$q_eEd = \tfrac{1}{2}mv_I^2$$

Now the electron is pushed back toward the cathode by the field. When it hits, the electron has traveled the same distance d back. The work done on the electron is equal to the kinetic energy it gains, so

$$q_eEd = \tfrac{1}{2}mv_F^2$$

We see that $\tfrac{1}{2}mv_I^2 = q_eEd = \tfrac{1}{2}mv_F^2$. The initial and final kinetic energies are equal.

Similar arguments can be made by considering the potential energy–kientic energy interplay, or by considering an arbitrary force that depends only on the distance from the cathode.

Some may see intuitively that the energies are equal from consideration of the symmetry of the electron path. Symmetry arguments are occasionally misleading though, and should, where possible, be accompanied by an argument like the one above.

5. The light energy is either absorbed by the crystal lattice as a whole, increasing its thermal energy, or is reflected.

6. The work function $W = hf_0$,

$$\text{so} \quad f_0 = \frac{W}{h} = \frac{10^{-18} \text{ J}}{6.6 \times 10^{-34} \text{ J} \cdot \text{sec}}$$

$$= 1.5 \times 10^{15} \text{ Hz}$$

This corresponds to a wavelength given by

$$\lambda = \frac{3 \times 10^{8} \text{ m/sec}}{1.5 \times 10^{15} \text{ Hz}}$$

$$= 2 \times 10^{-7} \text{ m or } 2{,}000 \text{ Å}$$

This wavelength lies in the ultraviolet region of the spectrum.

7. The energy of a photon is given by

$$E = hf; \text{ and, since } f = \frac{c}{\lambda}$$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{sec} \times (3 \times 10^{8} \text{ m/sec})}{5 \times 10^{-7} \text{ m}}$$

$$= 4 \times 10^{-19} \text{ J}$$

For $\lambda = 5 \times 10^{-8}$ m, $E = 4 \times 10^{-18}$ J.

8. Light of threshold frequency has an energy that is just sufficient to free an electron from the metal. The energy associated with that minimum frequency (f_0) is called the work function (W) of the metal, and $W = hf_0$. For copper,

$$W = 6.6 \times 10^{-34} \text{ J} \cdot \text{sec} \times 1.1 \times 10^{15} \text{ Hz}$$

$$= 7.3 \times 10^{-19} \text{ J}$$

When light of greater than threshold frequency is used, the photoelectrons will be emitted with a maximum kinetic energy given by

$$KE_{max} = hf - W$$

$$= 6.6 \times 10^{-34} \text{ J} \cdot \text{sec} \times 1.5 \times 10^{15} \text{ Hz} - 7.3 \times 10^{-19} \text{ J}$$

$$= (9.9 - 7.3) \times 10^{-19} \text{ J}$$

$$= 2.6 \times 10^{-19} \text{ J}$$

[Alternatively, $KE_{max} = nf_0 - hf_0 = h(f - f_0)$, etc.]

Since $1\ eV = 1.6 \times 10^{-19}$ J,

$$KE_{max} = \frac{2.6 \times 10^{-19}\ J}{1.6 \times 10^{-19}\ J/eV} = 1.6\ eV$$

9. The energy of a photon that will cause the emission is given by

$$f_0 = \frac{W}{h} = \frac{2.0\ eV \times 1.6 \times 10^{-19}\ J/eV}{6.6 \times 10^{-34}\ J \cdot sec}$$

$$= 4.8 \times 10^{14}\ Hz$$

10. (a) $f = \dfrac{c}{\lambda}$

$$= \frac{3 \times 10^8\ m/sec}{5 \times 10^{-7}\ m}$$

$$= 6 \times 10^{14}/sec$$

(b) $E = hf$

$= 6.6 \times 10^{-34} \times 6.0 \times 10^{14}$

$= 4.0 \times 10^{-19}$ J

(c) If the atomic diameter = 1 Å, 10^{10} atoms will fit along a 1-m line and in 1 m^2 there will be $10^{10} \times 10^{10}$ atoms, or 10^{20} atoms.

Then in 1 sec, 2.5×10^{20} photons will fall on 10^{20} atoms.

(d) If all photons are absorbed by the surface layer of atoms, each atom will absorb on the average 2.5 photons/sec.

(e) For an average of one photon/atom it would take $\dfrac{1}{2.5}$ the time it takes for 2.5 photons, or 0.4 sec.

(f) The number of photons arriving per atom in 10^{-10} sec will be

$= 2.5$ photons/sec $\times 10^{-10}$ sec

$= 2.5 \times 10^{-10}$ photons

(g) The cathode area is $(0.05\ m)^2$ or 2.5×10^{-3} m^2. Thus, the rate of arrival of photons at the cathode

$$= 2.5 \times 10^{20} \frac{photons}{m^2 \cdot sec} \times 2.5 \times 10^{-3}\ m^2$$

$= 6.3 \times 10^{17}$ photons/sec

By assumption, this yields 6.3×10^{17} electrons/sec. The current is thus

$$= 6.3 \times 10^{17} \frac{electrons}{sec} \times 1.6 \times 10^{-19} \frac{C}{electron}$$

$= 0.1$ C/sec, or 0.1 A

(Note: You may object that the above answers are given to more significant figures than are warranted by the problem as stated. You would be correct. Significant figures are very important when making calculations from data; but the round numbers given in this problem are contrived for the purpose of an exercise, to keep arithmetic from getting in the way of ideas.)

11. By definition, 1 W = 1 J/sec. However, the energy transformed into light is only 5% of this or 0.05 J. For one photon $E = hf$, and for n photons, $E = nhf$. Thus, substituting

$$f = \frac{c}{\lambda},\ E = \frac{nhc}{\lambda} \quad \text{or} \quad n = \frac{E\lambda}{hc};$$

$$n = \frac{0.05\ J \times 5 \times 10^{-7}\ m}{6.6 \times 10^{-34}\ J \cdot sec \times 3 \times 10^8\ \dfrac{m}{sec}}$$

$= 1.3 \times 10^{17}$ photons

12. (a) 96,540 C contain N electrons each of charge 1.6×10^{-19} C. Therefore,

$$N = \frac{96{,}540}{1.6 \times 10^{-19}} = \frac{9.654 \times 10^4}{1.6 \times 10^{-19}}$$

$= 6.0 \times 10^{23}$

(b) Assuming that there are 3.0×10^{23} copper atoms in 31.77 g there will be $\dfrac{8.92}{31.77} \times 3.0 \times 10^{23}$ atoms in 8.92 g. Since 8.92 g occupies 1 cm^3, there will be 84×10^{21} atoms/cm^3.

(c) If there are 84×10^{21} atoms in 1 cm^3 each will have a volume $= \dfrac{1}{84 \times 10^{21}} = \dfrac{1}{0.84 \times 10^{23}} = 1.2 \times 10^{-23}\ cm^3$.

(d) The side of a cube whose volume is 1.2×10^{-23} cm will be $\sqrt[3]{1.2 \times 10^{-23}} = \sqrt[3]{12 \times 10^{-24}} = \sqrt[3]{12} \times 10^{-8} = 2.3 \times 10^{-8}$ cm.

13. (a) $2x = n\lambda$

(b) $2x$ = any odd number of half wavelengths

(c) $\cos\theta = 2d/\lambda$ for first order

14. We are given $hf_{max} = q_eV$, thus

$$f_{max} = \frac{q_eV}{h} = \frac{1.6 \times 10^{-19}\ C \times 5 \times 10^4\ V}{6.6 \times 10^{-34}\ J \cdot sec}$$

$= 1.2 \times 10^{19}$ sec

15. If q_eV represents the energy of the electron in each case and if hf represents the energy of the photon in each case, then the major difference in the form of the equations is that the photoelectric equation contains a work function term that is absent from the X-ray equation.

The similarity is accounted for by the fact that in each case energy is being exchanged between a photon and an electron, and so it is to be expected that terms representing photon and electron energies would appear in both equations. The subscript "max" is appended to the *product* of the reaction, since the energy interchange is not always total, and therefore the product can have less energy than that of the incident "particle."

The absence of a work function in the X-ray equation can easily be explained. Strictly speaking, even the X-ray equation needs a work function term, but it is so small compared to the other terms that it can be omitted without large errors. The work function is about a few electron volts, whereas the other terms are typically on the order of several thousand electron volts. Thus, the error in omitting it is at most about a tenth of a percent.

16. The energy of a photon is given by $E = \frac{hc}{\lambda}$. This energy can at most be as large as the electron energy q_eV. Thus, for minimum wavelength (maximum energy)

$$q_eV = \frac{hc}{\lambda_{min}}, \quad V = \frac{hc}{q_e\lambda_{min}}$$

$$V = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec} \times 3 \times 10^8\ \text{m/sec}}{1.6 \times 10^{-19}\ \text{C} \times 10^{11}\ \text{m}}$$

$$= 1.2 \times 10^5\ \text{J/C}, \quad \text{or}$$

$$V = 1.2 \times 10^5\ \text{V}$$

This corresponds to a maximum energy of 1.2×10^5 eV, or

$$1.6 \times 10^{-19}\ \text{C} \times 1.2 \times 10^5\ \text{J/C} = 1.9 \times 10^{-14}\ \text{J}$$

17. *photon:* a discrete quantity of electromagnetic radiant energy; a "bundle of energy" whose value *hf* is proportional to the frequency of radiation.

quantum: one of the very small increments or quantities into which forms of energy are found to be subdivided.

cathode rays: emanations from the cathode (negative electrode) of a vacuum tube under the influence of an electric field; found to be electrons.

photoelectron: an electron released, generally from a metal, by means of energy absorbed from photons of light (usually ultraviolet) shining on the surface of the material.

photoelectric effect: the release of electrons from matter when exposed to certain frequencies of electromagnetic radiation.

quantum theory: that branch of modern physics based on the concept of the subdivision of electromagnetic energy into discrete quanta.

threshold frequency: the frequency of incident radiation below which the photoelectric effect will not take place.

stopping voltage: the voltage between cathode and anode in a photoelectric tube that will just stop the most energetic electrons emitted from the cathode.

classical physics: the physical theories concerning the nature of the universe and their philosophical implications that were developed prior to the advent of quantum theory.

X rays: electromagnetic radiation of short wavelength produced by electron bombardment of matter.

18. (a) Both models conceive of light as a stream of separate particles.

To explain refraction, Newton's model assumes a force of attraction between light particles and atoms that cause the particles of light to move according to the laws of classical mechanics. The photon model does not.

Newton's model gave no quantitative relationship on which to base precise predictions of experimental results. The photon model did.

Newton's model gave no reason to assume that light particles should give up all their energy when interacting with particles of "gross matter," whereas the photon model did.

(b) As indicated in part (a), Newton's model was qualitative and tied to classical particle mechanics. Newton could not have predicted the slope, intercept, or general form of the energy versus frequency curves. He might have had difficulty explaining the rapid emission of photoelectrons.

CHAPTER 19

2. (a) One way that has been used to tell if Fraunhofer lines are caused by absorption in the earth's atmosphere is to make observations over a range of zenith angles (angles measured between the line of sight and the vertical). As observations are made at larger and larger zenith angles, the light from the sun traverses a greater and greater thickness of the earth's atmosphere. This results in the darkening of absorption lines due to the earth's atmosphere.

When an absorption line is due to some element in both the sun's and earth's atmospheres, the respective contributions can be found by observing reflected sunlight from the planet Mercury when it is moving toward the earth and then when it is moving away; that is, at both quadratures. The sun's contribution to the total line will then be Doppler-shifted to the left and to the right of the earth's contribution. (Mercury is used because of its lack of atmosphere and high speed.)

A more expensive method that can now be used is to place a spectrograph on a satellite orbiting the earth above the earth's atmosphere. Any record made by the satellite of the sun's atmosphere would then involve only solar absorption. The Orbiting Solar Observatory (OSO-4) made such observations over a narrow range of wavelengths (about 1,500 Å, mostly in the ultraviolet).

(b) If light from the moon or planets were found to have absorption lines characteristic of sunlight, this would be strong evidence that the light was from the sun. Additional selective absorption from sunlight would indicate both the extent and composition of any atmosphere the moon and planets might have. For example, methane, which is not present on the sun, is present on Jupiter and thus causes additional absorption lines in sunlight reflected from Jupiter.

That the moon has no atmosphere (less than 10^{16} of earth's) can be determined from its sharp occultation of background stars. Also, a lunar atmosphere would cause scattering of sunlight that would result in the "horns" of the moon appearing slightly larger. All the planets except Mercury (and perhaps Pluto) have atmospheres.

3. The *Text* gives five series, but in theory there are an infinite number: one for each of the infinite number of possible values of n_f. (Most of them comprise "lines" in the radio region of the spectrum.) Only four lines (the first four of the Balmer series) are in the visible region.

4. The empirical formula giving the wavelength of the spectral lines of hydrogen is

$$\frac{1}{\lambda} = R_H \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$$

where $R_H = 1.10 \times 10^{7}$ m

For the Balmer series, $n_f = 2$. Then for $n_i = 8$, the quantitiy in brackets becomes

$$\frac{1}{2^2} - \frac{1}{8^2} \text{ or } \frac{1}{4} - \frac{1}{64} = \frac{15}{64}$$

$$\text{for } n_i = 10, \quad \frac{1}{2^2} - \frac{1}{10^2}, \text{ or } \frac{24}{100}$$

$$\text{for } n_i = 12, \quad \frac{1}{2^2} - \frac{1}{12^2}, \text{ or } \frac{35}{144}$$

Solving the above formula for λ, and evaluating for

$$n_i = 8, \lambda = \frac{1 \text{ m}}{1.10 \times 10^7 \times \frac{15}{64}}$$

$$= 3.88 \times 10^{-7} \text{ m or } 3{,}800 \text{ Å } (388 \text{ nm})$$

$$n_i = 10, \lambda = \frac{1 \text{ m}}{1.10 \times 10^7 \times \frac{24}{100}}$$

$$= 3.79 \times 10^{-7} \text{ m or } 3{,}790 \text{ Å } (379 \text{ nm})$$

$$n_i = 12, \lambda = \frac{1 \text{ m}}{1.10 \times 10^7 \times \frac{35}{144}}$$

$$= 3.74 \times 10^{-7} \text{ m or } 3{,}740 \text{ Å } (374 \text{ nm})$$

In the table on *Text* page 571 where n increases from 3 to 6 and here where n increases from 8 to 12, it is apparent that λ decreases as n increases. Also the increment by which λ decreases as n increases, say, from 6 to 8, 8 to 10, 10 to 12, becomes smaller and smaller, implying that as higher quantum numbers are approached, the difference between energy states approaches zero.

This problem illustrates a very important trend, namely, as high quantum numbers are approached, quantum effects become less apparent. In other words the energy states get closer and closer together until the state defined by quantum number n cannot be distinguished from the state defined by quantum number $n \pm 1$. This fact forms the basis of a principle that relates classical and quantum physics. This principle, first articulated by Bohr, is called the *correspondence principle* and says that in the limit of large quantum numbers, quantum physics merges into classical physics.

5. (a) In the figure on *Text* page 571 the bunching occurs at the high-frequency end of the spectrum. The figure on page 586 shows that the high-frequency transitions involve large quantum numbers. As seen in the preceding problem solution, high values of n lead to nearly identical values of λ, so the bunching is predicted by the formula.

(b) $n_i = \infty$

(c) The Lyman series has $n_f = 1$, so the series limit, where $n_i = \infty$, is

$$\lambda = \frac{1 \text{ m}}{1.10 \times 10^7 \times \left(\frac{1}{1^2}\right)}$$

$$= 0.910 \times 10^{-7} \text{ m or } 910 \text{ Å } (91 \text{ nm})$$

The Balmer series has $n_f = 2$, so the series limit is

$$\lambda = \frac{1 \text{ m}}{1.10 \times 10^7 \times \left(\frac{1}{2^2}\right)}$$

$$= 3.64 \times 10^{-7} \text{ m or } 3{,}640 \text{ Å } (364 \text{ nm})$$

The Paschen series has $n_f = 3$, so the series limit is

$$\lambda = \frac{1 \text{ m}}{1.10 \times 10^7 \times \left(\frac{1}{3^2}\right)}$$

$$= 8.18 \times 10^{-7} \text{ m or } 8{,}180 \text{ Å } (818 \text{ nm})$$

(d) The series limit is 910 Å (91 nm) for the Lyman series. This wavelength corresponds to an energy given by $\dfrac{hc}{\lambda}$:

$$E = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec} \times 3.0 \times 10^{8}\ \text{m/sec}}{0.910 \times 10^{-7}\ \text{m}}$$

$$= 21.8 \times 10^{-19}\ \text{J, or}$$

$$\frac{21.8 \times 10^{-19}\ \text{J}}{1.6 \times 10^{-19}\ \text{J/eV}} = 13.6\ \text{eV}$$

6. The Thomson and Rutherford models of the atom are similar in the following ways: Atoms contain positive and negative charge in equal amounts; nearly all of the mass of the atom is associated with the positive charge; the diameter of the atom is of the order of 10^{-10} m.

The models differ in that in the Thomson atom the positive charge is spread out through the entire volume of the atom; in the Rutherford model the positive charge is concentrated (localized) into a very small volume at the center of the atom. Also, in the Thomson model, the electrons are distributed throughout the positive charge; in the Rutherford model the electrons are separated from the positive charge, being distributed around the positive charge in some undefined way. Lastly, Rutherford's model has much empty space; Thomson's atom is "full" of charged matter.

7. (a) Lenard's model is like those of Thomson and Rutherford in having positive and negative electricity in equal amounts so that the atom as a whole is neutral. It is similar to Rutherford's model in having much empty space. But Lenard's model did not distinguish between the electrons and the positive charge as did the Thomson and Rutherford models. Hence, in Lenard's model the positive charge was not associated with nearly all the mass of the atom.

(b) It would not be possible to account for the backscattering of swift α particles. A neutral dynamide would have to have a mass no greater than that of a hydrogen atom (if we assume that a hydrogen atom consists of one dynamide). Hence a heavier nucleus, such as those used by Rutherford in his scattering experiments, would contain many dynamides, each with much smaller mass than an α particle. A collision between an α particle and a neutral dynamide would be like a billiard ball collision of a moving heavy ball with a lighter, stationary ball. The heavier ball would not be deflected significantly from its forward direction in a single collision. The angular distribution of the scattered α particles would then be in a small angle about the direction of the incident (α particle) beam, with no backward scattering.

(c) No. Since the observed backscattering is inconsistent with Lenard's model, the model is not valid, at least without modification.

8. $v_\alpha = 2 \times 10^{7}$ m/sec $\qquad Z_{Au} = 79$
$m_2 = 7 \times 10^{-27}$ kg $\qquad k = 9.0 \times 10^{9}$
$q_e = 1.6 \times 10^{-19}$

Equating kinetic energy of motion to electrical potential energy,

$$\frac{mv^2}{2} = \frac{kZq_e}{r} \times 2q_e$$

and substituting known values

$$\frac{7 \times 10^{-27} \times (2 \times 10^{7})^2}{2} = \frac{9.0 \times 10^{9} \times 79 \times (1.6 \times 10^{-19})^2 \times 2}{r}$$

and solving for r

$$r = \frac{9 \times 10^{9} \times 316 \times 2.56 \times 10^{-38}}{28 \times 10^{-13}}$$

gives

$$r = 2.6 \times 10^{-14}\ \text{m}$$

This is an upper limit to the radius of the gold nucleus.

9. (a) The 1 Å estimate for the size of atoms is the result of a variety of different kinds of experiments: mean free path in the kinetic theory of gases, monolayer experiments, etc. The evidence for the 10^{-4} Å estimate for the size of the nucleus is mainly the result of collision experiments where the distance of closest approach can be calculated and used to establish an upper limit on the nuclear size (see question 8) and of experiments with "strong interactions," (nuclear forces) which occur only within about 10^{-14} to 10^{-15} m. (See Unit 6.)

(b) $10^{-4}/1$

Certainly the burden of proof is on the author. The nuclear model of the atom is consistent with the scattering experiments; it has served to suggest new experiments; and in general it nicely ties together data pertaining to atomic structure.

Furthermore, the author in question proposes that the atom is a small neutral particle. If this were the case, how would the author account for the scattering forces that cause the observed scattering? In the nuclear model, scattering forces between a positively charged α particle and positively charged nucleus are Coulomb forces. What kind of forces would act between an α particle and a small neutral atom? It is not at all clear what the origin of the required forces would be; hence it would be very difficult to account for the observed α scattering.

10. The magnification is $\frac{10^{-2}\text{ cm}}{1.5 \times 10^{-13}\text{ cm}}$, or 0.67×10^{11}. Thus, the magnified radius of the first Bohr orbit would be the product of the magnification and the actual radius: $0.67 \times 10^{11} \times 5.29 \times 10^{-9}$ cm $= 3.5 \times 10^2$ or 3.5 m

11. $$E = KE + PE$$

But $\frac{mv^2}{r} = \frac{kq^2}{r^2}$

Multiplying this expression for $\frac{mv^2}{r}$ by $\frac{r}{2}$ gives

$$\frac{mv^2}{2} = \tfrac{1}{2}\frac{kq^2}{r} = KE$$

$$\text{So } E = \tfrac{1}{2}\frac{kq^2}{r} + \left(-\frac{kq^2}{r}\right)$$

$$= -\tfrac{1}{2}\frac{kq^2}{r}$$

Now if $r = \frac{n^2h^2}{4\pi^2 mq_e^2 k}$ as given,

$$E = -\tfrac{1}{2}kq^2\left(\frac{4\pi^2 mq_e k}{n^2h^2}\right) = -\frac{2\pi^2 k^2 mq_e^4}{n^2h^2}$$

For the first orbit $n = 1$ and $E = E_1$

$$E_1 = \frac{-2\pi^2k^2mq_e^4}{h^2}$$

Therefore

$$E_n = \frac{1}{n^2}, E_1$$

$$E_1 = \frac{-2\pi^2(9\times10^9)^2\times(9.1\times10^{-31})\times(1.6\times10^{-19})^4}{(6.6\times10^{-34})^2}$$

$$= -2.18 \times 10^{-18}\text{ J}$$

$$= -13.6\text{ eV}$$

$$E_2 = \frac{-13.6}{4} = -3.4\text{ eV}$$

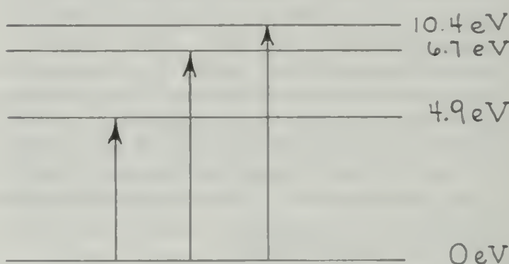
$$E_3 = \frac{-13.6}{9} = -1.5\text{ eV}$$

$$E_4 = \frac{-13.6}{16} = -0.85\text{ eV}$$

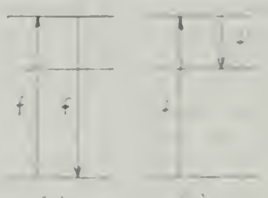
12. The Bohr model can account for the lines of absorption spectra if it is assumed that the orbital electron can absorb a light quantum only if the energy so absorbed raises the electron into another allowed orbit. The absorption of light is then the exact inverse of emission and every absorption line should correspond exactly to an emission line, in agreement with experiment.

13. Atoms in the stationary state corresponding to $n = 5$ can make transitions to states corresponding to $n = 4, 3, 2, 1$, respectively. An atom that is now in the state with $n = 4$ can make transitions to states with $n = 3, 2, 1$, and so on.
   Ten lines are possible:
   four correspond to transitions ending in $n = 1$
   from 2, 3, 4, and 5;
   three correspond to transitions ending in $n = 2$
   from 3, 4, and 5;
   two correspond to transitions ending in $n = 1$
   from 4 and 5;
   one corresponds to transitions ending in $n = 4$
   from 5.

14. The upward direction of the arrows indicates that mercury atoms are absorbing energy and being raised from the ground state to states of higher energy.



15. When a substance is illuminated with ultraviolet light of frequency f, an atom of the substance in its ground state absorbs energy in the amount hf. The atom is raised from its ground state to an excited state; or, to put it another way, an electron is raised from its normal orbit into an outer orbit. The electron can then drop back to some lower orbit between its initial orbit and the one to which it is raised. In other words, it can give up either the energy hf [see (a) below] or an amount of energy hf' [see (b) below]. It is seen that $hf' < hf$ or $f' < f$.



The wavelength is inversely proportional to frequency. Hence, $\lambda' > \lambda$, which is what Stokes found. Ultraviolet light includes frequencies greater than those of visible light. Thus, in fluorescence caused by ultraviolet light, much of the light reradiated is visible; that is, f is ultraviolet while f' is visible.

16. (a) The concept of atoms as expressed by Newton is quite similar to that attributed to Leucippus and Democritus. One difference is that, according to Leucippus and Democritus, atoms are eternal; according to Newton, God created atoms in the beginning. For Leucippus and Democritus, all atoms are of the same kind, but differ in size, shape, and position; Newton's atoms have sizes, shapes, and "such other properties . . . , as most conduced to the End for which He formed Them." There is a theological aspect to Newton's views that is not found in the Greek atomists.

(b) Dalton hypothesized that each *element* (an idea not mentioned by Newton or Leucippus and Democritus) consists of a characteristic kind of identical atoms: The atoms of an element "are perfectly alike in weight and figure, etc." Between Newton and Dalton much progress had been made in the understanding of the concepts of chemical element and chemical compound. Dalton could also, therefore, make an hypothesis concerning the details of the formation of compounds by the atoms of different elements.

(c) In the Rutherford–Bohr model, the atom was no longer "solid," "impenetrable," "uncuttable," or "indivisible." The atom of Rutherford and Bohr consists of a nucleus and electrons and empty space. In view of what was known about radioactivity in 1913, especially since it was known that atoms could emit α particles, Rutherford and Bohr made no detailed hypotheses about the nature of the nucleus. It was already evident that the nucleus (the only place α particles could come from) was not "indivisible."

17. An atom normally has its electrons in the lowest possible energy states. An atom of potassium has $Z = 19$ and thus has 19 electrons situated as follows: (see the chart on *Text* page 593, noting that each circle represents a *pair* of electrons) 2 electrons in K shell, 8 in L, 8 in M; then the one electron remaining would be in the lowest energy level of the N shell, because that level is lower in energy than the five pairs of locations still open in the M shell.

18. Refer to the chart on *Text* page 593. Starting with argon ($Z = 18$), we continue adding electrons in pairs: 2 electrons in the N shell, 10 in M, 6 more in N; now we have a stable arrangement of 8 outer electrons. The element having this number of electrons is krypton, $Z = 36$.

To find the next inert gas after krypton, we continue from $Z = 36$, adding 2 electrons in the O shell, 10 in N, 6 more in the O shell; now we have another stable configuration of 8 electrons in the outer shell. The element having this number of electrons is xenon, $Z = 54$.

19. A glossary of some of the terms that should be defined are:

α particles	optical spectra
Bohr orbit	planetary atom
empirical relation	quantum mechanics
energy-level diagram	radioactive substances
excitation energy	scintillation
ground state	shell
line-absorption spectrum	spectrum
line-emission spectrum	stationary states
nuclear atom	X-ray spectra
nucleus	

20. This is open-ended, but we would like to point out the following: In particle dynamics, collisions can be considered singly, and the collision of a collection of particles is simply the summation of individual collisions. It appears, however, that a different level of analysis is needed for mental phenomena. Thought is made up of the interaction of complex systems, rather than of discrete entities. The model is more like a committee meeting than a game of billiards. Furthermore, the particles of atomism are of a limited number of types: those of the same type being in fact identical. However, brain cells and the system of connections among them are infinitely variable.

21. (a) The statement in the *Text* really deals with all the quantized things the student would know. Additional concepts they wouldn't know are "strangeness" and "baryon number."

(b) Some properties or things outside physics that can be thought of as being quantized are the following:

salary increases in large corporations
consumer prices (in units of \$0.01)
snow (rain, hail, sleet)
formal education (in units of courses)
letter grades (not percent grades, though)
dates ("tomorrow" suddenly becomes "today," etc.)

Note: we do not mention quantities that are intrinsically collections of things.

22. Essay.

2. The mass m at relativistic speed v is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ so } \frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this problem, $m = 1.01\ m_0$,

$$\text{so } \frac{1.01\ m_0}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring both sides,

$$1.02 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{1.02} = 0.98$$

$$\text{Thus,} \quad \frac{v^2}{c^2} = 0.02, \frac{v}{c} = 0.14,$$

$$\begin{aligned} v &= 0.14\ c \\ &= 0.14 \times 3.0 \times 10^8 \text{ m/sec} \\ &= 4.2 \times 10^7 \text{ m/sec} \end{aligned}$$

3. The relativistic mass (m) is related to the rest mass (m_0) by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{9.1 \times 10^{-31}\text{ kg}}{\sqrt{1 - \left(\frac{0.6\ c}{c}\right)^2}}$$

$$= \frac{9.1 \times 10^{-31}\text{ kg}}{\sqrt{1 - 0.36}} = \frac{9.1 \times 10^{-31}\text{ kg}}{0.80}$$

$$= 11.4 \times 10^{-31}\text{ kg}$$

The centripetal force required is mv^2/R, but

$v = 0.60\ c$, so

$$F = \frac{m(0.60\ c)^2}{R}$$

$$= \frac{11.4 \times 10^{-31}\text{ kg} \times (0.60 \times 3 \times 10^8\text{ m/sec})^2}{1.0\text{ m}}$$

$$= 14.4 \times 10^{-31}\ (3.24 \times 10^{16})\text{ kg} \cdot \text{m/sec}^2$$

$$= 3.7 \times 10^{-14}\text{ N}$$

4. (a) By substitution of $v/c = 0.1$, the series becomes $1 + \frac{1}{2}(0.1)^2 + \frac{3}{8}(0.1)^4 + \ldots$

$$= 1 + 0.005 + 0.0000375 + \ldots$$

(b) From (a) only the first two terms may be necessary. The relativistic momentum is then

$$p = m_0\left(1 + \tfrac{1}{2}\frac{v^2}{c^2}\right)v \text{ and since } \frac{v^2}{c^2} < 10^{-10}$$

we can neglect even that term. So for human-sized objects, $p = m_0v$.

(c) Similarly, the relativistic kinetic energy is

$$KE = m_0\left(1 + \tfrac{1}{2}\frac{v^2}{c^2}\right)c^2 - m_0c^2 = \tfrac{1}{2}m_0v^2$$

Here the second term must be retained because the m_0c^2 terms drop out of the equation. Therefore, for human-sized objects, $KE = \frac{1}{2}m_0v^2$.

5. (a) The mass changes that are due to energy changes in chemical reactions are too small to be detected. Support for this statement is given in part (b).

(b) The mass change is related to the energy change as follows:

$$\begin{aligned} \Delta m &= \frac{\Delta E}{c^2} = \frac{10^5\text{ J}}{(3 \times 10^8\text{ m/sec})^2} \\ &= 1.1 \times 10^{-12}\text{ kg} \end{aligned}$$

This is only one billionth of a gram and is, of course, not detectable in chemical reactions.

$$\left(\frac{\text{J}}{\frac{\text{m}^2}{\text{sec}^2}} = \text{N} \cdot \cancel{\text{m}} \cdot \frac{\text{sec}^2}{\text{m}^2} = \frac{\text{kg} \cdot \cancel{\text{m}}}{\cancel{\text{sec}^2}} \cdot \frac{\cancel{\text{sec}^2}}{\cancel{\text{m}}} = \text{kg}\right)$$

6. (a) The kinetic energy of the earth is

$$\begin{aligned} KE &= \tfrac{1}{2}m_0v^2 = \tfrac{1}{2} \times (6.0 \times 10^{24}\text{ kg}) \\ &\quad \times (3 \times 10^4\text{ m/sec})^2 \\ &= 27 \times 10^{32}\text{ J} \end{aligned}$$

(b) The mass equivalent is $\Delta m = \frac{\Delta E}{c^2}$

$$= \frac{27 \times 10^{32}\text{ J}}{(3.0 \times 10^8\text{ m/sec})^2} = 3.0 \times 10^{16}\text{ kg}$$

(c) The percentage increase in mass is then

$$\frac{3.0 \times 10^{16}\text{ kg}}{6.0 \times 10^{24}\text{ kg}} \times 100\% = 5 \times 10^{-7}\%$$

(d) Any measurements of the mass of the earth made *on* the earth will yield the rest mass of the earth, since the observers are at rest with respect to the earth.

7. (a) The relativistic momentum is the product of the relativistic mass and the velocity of the electron:

$$p = \frac{m_0v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{9.1 \times 10^{-31}\text{ kg} \times 0.4 \times 3 \times 10^8\text{ m/sec}}{\sqrt{1 - \left(\frac{0.4\ c}{c}\right)^2}}$$

$$= \frac{9.1 \times 0.4 \times 3 \times 10^{-23}}{0.92}\text{ kg} \cdot \text{m/sec}$$

$$= 1.2 \times 10^{-22}\text{ kg} \cdot \text{m/sec}$$

(b) The Newtonian momentum at that speed:

$$m_0v = 9.1 \times 10^{-31}\text{ kg} \times 0.4 \times 3 \times 10^8\text{ m/sec}$$
$$= 1.1 \times 10^{-22}\text{ kg} \cdot \text{m/sec}$$

(c) The relativistic momentum at $v = 0.8\,c$

$$= \frac{9.1 \times 10^{-31}\text{ kg} \times 0.8 \times 3 \times 10^8\text{ m/sec}}{\sqrt{1 - (0.8)^2}}$$
$$= 3.6 \times 10^{-22}\text{ kg} \cdot \text{m/sec}$$

Therefore, the change in relativistic momentum due to v increasing from $0.4\,c$ to $0.8\,c$ is

$(3.6 - 1.2) \times 10^{-22}$ kg · m/sec, or 2.4×10^{-22} kg · m/sec

(d) Since the Newtonian momentum at a speed of $0.8\,c$ is simply twice its value at $0.4\,c$, the change in Newtonian momentum is

1.1×10^{-22} kg · m/sec

8. The momentum of the photon is given by

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}\text{ J} \cdot \text{sec}}{4 \times 10^{-7}\text{ m}}$$
$$= 1.7 \times 10^{-27}\text{ kg} \cdot \text{m/sec}$$
$$\left(\frac{\text{J} \cdot \text{sec}}{\text{m}} = \frac{\text{kg} \cdot \text{m} \cdot \text{sec}}{\text{sec}^2 \cdot \text{m}} = \text{kg} \cdot \text{m/sec}\right)$$

For an electron to have the above momentum it must have a speed given by

$$v = \frac{p}{m} = \frac{1.7 \times 10^{-27}\text{ kg} \cdot \text{m/sec}}{9.1 \times 10^{-31}\text{ kg}}$$
$$= 1.9 \times 10^3\text{ m/sec}$$

9. Compton theorized that in a collision between a photon and an atom, the law of conservation of momentum should apply. He then calculated how much energy a photon should lose in a collision with an atom if the momentum of the photon is hf/c. Compton concluded that if the photon strikes an entire atom, the change in energy is too small to observe. However, if a photon strikes an electron, the photon should transfer a measurable amount of energy to the electron. In 1923, Compton performed an experiment in which X rays struck electrons. Compton's experiment proved that a photon can be regarded as a particle with momentum as well as energy. It also showed that collisions between photons and electrons obey the law of conservation of momentum and energy.

10. The experiments that led to the acceptance of the wave theory of light involving reflection, refraction, diffraction, and interference were done in the eighteenth and early nineteenth centuries with relatively simple experimental equipment.

The experiments that could be interpreted only in terms of the *particle* aspect were the photoelectric and Compton effects. The electron had to be discovered (Thomson, 1897) and methods developed for making quantitative experiments with electrons and X rays before those effects could be analyzed and interpreted. These things were not done until the end of the nineteenth century and the first quarter of the twentieth century.

11. The momentum of an electron is given by the de Broglie relation:

$$mv = \frac{h}{\lambda};\ \text{thus } v = \frac{h}{m\lambda}$$
$$v = \frac{6.6 \times 10^{-34}\text{ J} \cdot \text{sec}}{10^{-30}\text{ kg} \cdot 10^{-10}\text{ m}} = 6.6 \times 10^6\text{ m/sec}$$
$$\left(\frac{\text{J} \cdot \text{sec}}{\text{kg} \cdot \text{m}} = \frac{\text{N} \cdot \text{m} \cdot \text{sec}}{\text{kg} \cdot \text{m}} = \frac{\text{kg} \cdot \text{m} \cdot \text{sec}}{\frac{\text{sec}^2}{\text{kg}}} = \text{m/sec}\right)$$

12. The de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}\text{ J} \cdot \text{sec}}{(0.2\text{ kg})(1\text{ m/sec})}$$
$$= 3.3 \times 10^{-33}\text{ m}$$

13. By definition, the de Broglie wavelength is given by $\lambda = h/mv$. But $KE = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{2(KE)}{m}}$$

Hence, the momentum $mv = m\sqrt{\frac{2(KE)}{m}}$
$$= \sqrt{2m(KE)}$$

Substituting this for mv in the de Broglie relation,

$$\lambda = \frac{h}{\sqrt{2m(KE)}}$$

When mass and velocity both decrease, λ becomes larger.

14. (a) $\lambda = 2(1\text{ m}) = 2\text{ m}$

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{2\text{ m}}\text{ J} \cdot \text{sec}$$
$$= 3.3 \times 10^{-34}\text{ kg} \cdot \text{m/sec}$$

But $p = mv$, so $v = p/m$

$$v = \frac{3.3 \times 10^{-34}\text{ kg} \cdot \text{m/sec}}{10^{-9}\text{ kg}}$$
$$= 3.3 \times 10^{-25}\text{ m/sec}$$

(b) Similarly, we can solve for v as above <u>or</u> in one step as follows:

$$v = \frac{h}{\lambda m} = \frac{6.6 \times 10^{-34}\text{ J} \cdot \text{sec}}{2 \times 10^{-1}\text{ m} \times 6.6 \times 10^{-26}\text{ kg}}$$
$$= 5 \times 10^{-8}\text{ m/sec}$$

$$(c)\ v = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec}}{2 \times 10^{-6}\ \text{m} \times 10^{-22}\ \text{kg}}$$
$$= 3.3 \times 10^{-6}\ \text{m/sec}$$

$$(d)\ v = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec}}{2 \times 10^{-10}\ \text{m} \times 10^{-30}\ \text{kg}}$$
$$= 3.3 \times 10^{6}\ \text{m/sec}$$

(In general, as the size of the particle and its containing "box" decreases, the least speed the particle could have increases!)

15. You would be unable to learn about the following:
   (1) objects outside your throwing range;
   (2) objects from which the ball would not bounce (sponge, etc.);
   (3) fluids like air;
   (4) objects appreciably smaller than the ball.

   Although you might learn of the existence of small objects, you couldn't learn the details of their shape. (Similarly, if you are to "see" detail with electromagnetic waves, the wavelength must not be much smaller than the dimensions of the detail.)

16. The uncertainty principle states

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$
$$\geq \frac{6.0 \times 10^{-34}}{2\pi}\ \text{J} \cdot \text{sec}$$
$$\geq 1 \times 10^{-34}\ \text{J} \cdot \text{sec}$$

The uncertainty in momentum is the product of the mass, 1×10^{-2} kg, and the uncertainty in speed, 3 cm/sec or 3×10^{-2} m/sec.

That is, $\Delta p = 3 \times 10^{-4}\ \frac{\text{kg} \cdot \text{m}}{\text{sec}}$

Thus, $\Delta x \geq \frac{1 \times 10^{-34}\ \text{J} \cdot \text{sec}}{3 \times 10^{-4}\ \text{kg} \cdot \text{m/sec}}$

$$\Delta x \geq 3 \times 10^{-31}\ \text{m}$$

(This result indicates that uncertainty effects are completely negligible for objects of "normal" size and speed.)

17. The essence of the quantum theory is that energy exists in bundles of size proportional to the frequency. If the constant of proportionality were zero, the bundles would have no energy and there could be no quantum effect. Light would become entirely a wave phenomenon and there would be no photoelectric effect or Compton effect.

18. This is an open-ended question, but the following comments are relevant.

   The uncertainty principle expresses limitations on our ability to obtain information about certain detailed and particular phenomena on the atomic scale. At this time, the application of the uncertainty principle to problems such as free will represents an extrapolation of physical theory that is unjustified in light of the absence of experimental facts.

   The claims of these philosophers and physicists represent in part a reaction against the rigid determinism represented by Laplace's view expressed in question 22.

   The facts do not support a dogmatic stand on either side of this issue.

19. In one sense, this statement is acceptable. If a scientific theory correctly predicts experimental results, it is doing its job. In another sense, however, this statement falls short of those ends desired by most physicists. That is, in addition to a formalism (mathematical symbols and processes), a good theory has models in terms of which the physicist can "visualize" the implications of the theory. Such "visualizations" or "picturizations" can often lead to deeper understanding.

20. The idea of complementarity has been used by Bohr and other physicists to describe the use of the wave and particle models in atomic physics. The idea is another way of expressing human limitations in the study of atomic phenomena. Bohr has argued that complementarity must be used in other connections than atomic physics, but there has been small success, if any. So far the only uses have been in analogies, or in highly general statements. Some examples follow.

   Bohr's biological example: The vital aspect and the physiochemical aspect of living beings. The complete description (by means of physics and chemistry) of a living being would require an analysis so extreme in its various parts that it would inevitably lead to the death of the subject studied. The study of the vital functions would have to ignore in a large measure the details of the physical and chemical processes taking place in tissues and cells. Neither study by itself would give a complete description of the behavior of living beings.

   Bohr's psychological example: We speak of living beings acting by instinct or with the use of reason. "Instinct" and "reason" seem to be mutually contradictory but complementary aspects of behavior.

   An example of the use of the idea of complementarity in a more limited (perhaps trivial) way: An account of what happens on a TV screen in terms of pictures of people and things is in a sense complementary to the description in terms of electron beams scanning the screen.

21. The latter interpretation is the correct one. Quantum theory really applies to all phenomena, but its results are indistinguishable from the predictions of classical physics when the

objects under study are large and or slow-moving. The most important piece of evidence to support this view is that quantum theory does indeed predict correct results even for large objects and slow-moving ones. Thus it is not true that quantum theory is *only* for the phenomena of the atomic world. Also, one of the assumptions that has exerted great influence on the endeavors of physicists throughout the centuries is that there is a basic unity in nature, and that therefore one should not assume one set of laws for the atomic world and another set for the everyday world unless experimental results require it. So far they have not done so.

22. (a) No. The precision with which the position and momentum can be determined is limited by the uncertainty principle. Thus, since the state of the particles of the universe is uncertain at any time, one cannot know the future with absolute certainty.

    Of course, what a superior intelligence could or could not do is not within the scope of scientific discussion.

    (b) Relativity is deterministic only in the sense that it expresses the exact bookkeeping that must be done whenever there is a change in the mass or total energy in any defined system, whether a single particle or a collection of particles. However, it is not really "deterministic" in the sense used by Laplace, since relativity theory by itself does not address itself to causes for those changes, and is compatible with probability theory.

    (c) Quantum theory simply eliminates the concept of determinism on the atomic level and replaces it by a theory of probability.

23. The passage from Lucretius is remarkably prophetic of modern views. Though we regard the random motion of atoms and molecules as rather closely determined just as Lucretius did, the uncertainty principle does not grant complete determinism. Electrons, being of far smaller mass, are much more susceptible than atoms to "that ever-so-slight swerve," which in Heisenberg's principle is equated with "uncertainty."

    Whether free will follows as a consequence of the unpredictability of either Lucretius' "atoms" or our electrons is a highly debatable point. One's actions may be just as firmly determined by the "slight atomic swerve" as they were thought to be under the old determinism: those actions are simply made less predictable, but no less determined.

24. In the Escher drawing, one can focus on the objects (fish birds) as objects or as background. (The birds become the sea, lose their identity, and define the fish; and vice-versa.) From an esthetic standpoint, one of the merits of this drawing is that it forces us to pay attention to the interdependent identity of discrete objects and their continuous background. However, this or any other such analogy must be of limited scope, since the purpose of a scientific idea such as the dual nature of light is to explain or predict, whereas that of a drawing is usually to evoke an emotional response.

The Nucleus

Organization of Instruction

DETAILS OF THE MULTI-MEDIA SCHEDULE

Day 1
Teacher introduction to Unit 6
Stress that:

1. there are particles of matter smaller than atoms;
2. new techniques and ideas are required in order to study what one cannot see;

Film: Discovery of Radioactivity (color), International Film Bureau

Small-group discussion of film. Provide guide questions.

Day 2
Lab stations: Detection of Radioactivity

1. Cloud chamber, *Project Physics* type. Provide sources so that students can compare tracks of α and β particles.
2. Geiger counter, *Project Physics* or other type. Provide α and β sources.
3. Electroscope. Observe discharge rate of electroscope with and without presence of radioactive sources.
4. 3-D viewer and pictures from bubble chamber.
5. Spinthariscope. Many designs are available for purchase or construction; all require time for eyes to become dark-adapted.
6. Photographic plate. See *D59* for details.

Day 3
Library Day: give students an opportunity to pick an area for an individual study.
Some possible topics:

- Accelerators (or a specific accelerator, for example, Brookhaven)
- Detection devices (or specific device, for example, bubble chamber)
- Types of research with isotopes (medicine, biology)
- Political issues of nuclear science (detection of tests, control of power, financing of research)
- Engineering applications of nuclear power
- The future of the nuclear age (nuclear power, cheap power)
- New particles (Quarks, Ω^-)
- "The eight-fold way"
- Radiation safety

Day 4
Lab Stations: Behavior of Nuclear Particles
Use same stations as for Day 2, but emphasize the behavioral characteristics of particles.

Investigate absorption, magnetic deflection, scattering, and inverse-square law of radiation intensity.) Be careful about the inverse-square law because air is a good absorber of radiation.)

Day 5

Library Day

Help students find information concerning topics related to Unit 6 concepts and consistent with their ability and interests. In general, a student should (a) find a topic of interest, (b) read about it, (c) tell the rest of the class about it on Days 20–22. The contract approach may be used.

Days 6, 7, 8: Experiments on radioactivity

Students do one experiment each day.

E6-1 Random events

E6-2 Range of α and β particles

E6-3 Half-life. I

Day 9

Summary

Teacher leads class in summarizing experiments and results.

Day 10

Problem-solving Day

Select appropriate Study Guide questions on displacement rules, half-life, decay constant, etc.

Day 11

Student Activity

Students continue to read and plan for contract. Give students latitude in the way topics can be described to class. For example, a student who studies reactors may wish to make a model to show; a student who studies political implications of nuclear power may wish to write an essay to read: and students studying the financing of nuclear research may want to dramatize a make-believe request in front of the class.

Day 12

Lab Stations: Models and Applications

1. *D61* Model of mass spectrograph. Drop steel balls of various masses past a strong magnet and note where they land. Compare principle to mass spectrograph.
2. Model of nuclear scattering. Mount magnet under glass tray. Use magnetic discs and beads to show Rutherford scattering. Other models are possible.
3. Model of chain reaction. "Mouse-trap-and-cork" model is ideal. A set of dominoes arranged in a pyramid also illustrates the reaction.
4. *L48* (Collisions with an object of unknown mass)
5. Dice model of decay. Twenty-sided and eight-sided dice are used to show decay rates. See *E6-3*.
6. Model cyclotron. Place a marble in the center of a flat board (approximately 60 cm × 60 cm). Tilt the board back and forth to cause the marble to roll faster and faster in a circular path. Compare the principle to that of the cyclotron.

Days 13, 14, 15

Ditto a contract form for students to fill in. The contract should contain a bibliography of material studied and a description of the way it will be presented to the class. You may want to have the students specify what grade they will receive for successful completion of the contract terms. This technique seems to appeal especially to slower students. Help students write *reasonable* contracts.

Day 16

Lecture presentation on fission

Show quantity of ΔE from Δm. Avoid homework assignments at this time to allow students to concentrate on projects.

Day 17

Fission (continued)

Local power companies often have information on nuclear power reactors that provides an extension of this topic. A good summary film (10 min) is *Principles of Nuclear Fission* (McGraw-Hill).

Day 18

Lecture presentation on fusion.

Show quantitative relationships of ΔE and Δm.

Day 19

Continue discussion from Day 18.

Days 20, 21, 22

Student Report Days

Students give presentations, demonstrations, and dramatizations to the class.

Day 23

Review for Unit 6 Test

A quiz could be set up by a student or students as their contract project. All the class can participate as judges, scorers, timekeepers, and team members.

Day 24

Unit 6 Test. Other methods of evaluation may also be used.

Unit 6 SAMPLE MULTI-MEDIA SYSTEMS APPROACH

This is just one path of many that a teacher may take through Unit 6.

DAILY PLAN

1 Teacher introduction to Unit 6 Unit 6 Prologue	2 Lab stations: Detection of radioactivity Text: 21.1–21.3	3 Library day: Students look for research topics Text: 21.4–21.5	4 Lab stations: Behavior of nuclear particles Text: 22.1–22.2 Reader: "Nature of Alpha Particles," Rutherford
Text: 22.3–22.4 5 Library day: Continue research	6 Experiments on radioactivity Text: 23.1–23.3	7 Experiments cont'd. Write up experiments	8 Experiments cont'd. Write up experiments
Write up experiments 9 Follow-up discussion of experiments	10 Problem-solving day Problems: Chapters 21 and 22	11 Reading and planning Finish problems	12 Lab stations: models and applications Reader: "Success," Fermi
Text: 23.4–23.6 13 Discuss contracts: Students draw up contracts	14 Students work on contracts and research; teacher assists Text: 23.6–23.7	15 Continue work on contracts Text: 24.1–24.4	16 Teacher presentation: fission Text: 24.5–24.8
17 Fission reactors	18 Teacher presentation: (fusion quantitative)	19 Continue fusion	20 Student reports from research
21 Student reports continued	22 Student reports continued	23 Review of Unit 6 optional: quiz Reader: "Calling all Stars" Szilard	24 Unit 6 Test Unit 6 Epilogue
Review			

Unit 6 SUGGESTED SCHEDULE BLOCKS AND TIMETABLE

Each block represents one day of classroom activity and implies approximately a 50-min period. The words in each block indicate only the basic material under consideration or the main activity of the day. The suggested homework (listed above each block) refers mainly to the *Text* and *Handbook,* but is not meant to preclude the use of other learning resources.

CHAPTER 21 RADIOACTIVITY

Text 21.1–21.3 HB: E6-1	Text: 21.4–21.5	HB: E6-2	Text: 21.6–21.7	Text: 21.8 HB: E6-3
Lab E6-1: Random Events	Postlab and or problem seminar	Lab E6-2: Range of α and β Particles	Postlab and or problem seminar	Lab E6-3: Half-Life. I

CHAPTER 22 ISOTOPES

Review		Text: 22.1–22.3	Text: 22.4–22.5	Text 22.6–22.8
Postlab and or problem seminar	Test	Isotopes	Mass spectrograph separation	Nuclides

CHAPTER 23 PROBING THE NUCLEUS

Review	Text: 23.1–23.3	Text: 23.4	Text: 23.5–23.6	Text: 23.7–23.9
Test	Proton–electron hypothesis	Chadwick and problem seminar	Proton–electron hypothesis versus Proton–neutron theory	Nuclear reactions

CHAPTER 24 NUCLEAR ENERGY: NUCLEAR FORCES

Review	Text: 24.1–24.3	Text: 24.4–24.5	Text: 24.6–24.7	Text: 24.8–24.10
Test	Nuclear binding	Fission and problem seminar	Fission (cont'd.)	Fission and problem seminar
Text: 24.11–24.12	Text: 24.13 HB: E6-5	Review		
Models Discuss Unit Test	Radioactive tracers	Chapter Test	Unit Review	Unit Test

CHAPTER 21 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
21.1 Becquerel's discovery				E 6-1 Random events D 59 Mineral autoradiograph			21.1
21.2 Other radioactive elements are discovered		2		D 60 Naturally occurring radioactivity	F 47 Discovery of radioactivity		21.2
21.3 The penetrating power of the radiation: α, β, and γ rays	3			E 6-2 Range of α and β particles	L 6-1 Radioactive decay	Ionization by radioactivity	21.3
21.4 The charge and mass of α, β, and γ rays	5	4	6		R 1 Snow, "Rutherford"		21.4
21.5 The identity of α rays: Rutherford's "mousetrap"					T 40 Separation of α, β, γ rays	Magnetic deflection of β rays	21.5
21.6 Radioactive transformation	7				T 41 Rutherford's α particle "mousetrap" R 2 Rutherford and Royds, "The Nature of the Alpha Particle"		21.6
21.7 Radioactive decay series			8		F 48 U-238 Radioactivity series T 42 Radioactive disintegration series		21.7
21.8 Decay rate and half-life	13 15	9 12 14 10	11	E 6-3 Half-life. I E 6-4 Half-life. II	F 49 Random events T 43 Radioactive decay curve	Exponential decay in concentration	21.8

CHAPTER 22 RESOURCE CHART

Text	Study Guide E	Study Guide M	Study Guide H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities
22.1 The concept of isotopes	2				F 50 Long time intervals	22.1
22.2 Transformation rules		3			T 44 Radioactive displacement rules	22.2
22.3 Direct evidence for isotopes of lead					L6-2 Thomson's positive-ray parabolas	22.3
22.4 Positive rays					F 51 Isotopes T 45 Mass spectrograph L6-3 Aston's mass spectrograph	22.4
22.5 Separating isotopes		4	5		T 46 Chart of the nuclides	22.5
22.6 Summary of useful notation for nuclides: nuclear reactions	6	7 8 9				22.6
22.7 The stable isotopes of the elements and their relative abundances			10			22.7
22.8 Atomic masses	11 12	13				22.8

CHAPTER 23 RESOURCE CHART

Text	Study Guide E	M	H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities	
23.1 The problem of the structure of the atomic nucleus		2					23.1
23.2 The proton–electron hypothesis of nuclear structure	3						23.2
23.3 The discovery of artificial transmutation	4				R 5 Yagoda, "The Tracks of Nuclear Particles" R 6 O'Neill, "The Spark Chamber"		23.3
23.4 The discovery of the neutron	5 6	7			L 48 Collisions with an unknown object R 3 Chadwick, "Some Personal Notes on the Search for the Neutron"	Neutron detection Problem analogue (Chadwick's problem)	23.4
23.5 The proton–neutron theory of the composition of atomic nuclei	9 10						23.5
23.6 The neutrino					R 12 Pierls, "Models of the Nucleus" R 17 Gardner, "The Fall of Parity" R 18 Gardner, "Can Time Go Backward?"		23.6
23.7 The need for particle accelerators			8		R 4 Chamberlain, *et al.*, "Antiprotons"		23.7
23.8 Nuclear reactions	12	11			F 52 The linear accelerator R 7 Lawrence, "The Evolution of the Cyclotron" R 9 Judd and MacKenzie, "The Cyclotron as Seen by. . ." R 10 Bernstein, "CERN"		23.8
23.9 Artificially induced radioactivity	13	14	15		T 47 Nuclear equations L6-4 Nuclear reactions F 53 Positron–electron annihilation		23.9

CHAPTER 24 RESOURCE CHART

Text	Study Guide E	M	H	Experiments and Demonstrations	Transparencies (T), Loops (L), Films (F), Reader (R)	Student Activities
24.1 Conservation of energy in nuclear reactions					R 16 Ford, "Conservation Laws" R 23 Infeld: "One Scientist and His View of Science"	24.1
24.2 The energy of nuclear binding						24.2
24.3 Nuclear binding energy and stability	2 3				T 48 Binding energy curves	24.3
24.4 The mass-energy balance in nuclear reactions		4 5 7	6			24.4
24.5 Nuclear fission: discovery		8 9 10				24.5
24.6 Nuclear fission: controlling chain reactions		11 12			F 54 Principles of nuclear fission L 6-5 Critical size R 14 Fermi, "Success"	Two models of a chain reaction 24.6
24.7 Nuclear fission: large-scale energy release and some of its consequences					R 15 Weinberg and Young, "The Nuclear Energy Revolution" R 19 Franck, *et al.*, "A Report to the Secretary of War" R 20 Weisskopf, "The Privilege of Being a Physicist" R 21 Szilard, "Calling All Stars"	24.7
24.8 Nuclear fusion						24.8
24.9 Fusion reaction in stars	13 14	16 17 18	15		R 13 Lapp, "Power from the Stars" R 24 Feynman, "The Development of the Space-Time View of Quantum Electrodynamics	24.9
24 10 The strength of nuclear forces					R 25 Feynman, "Physics and Mathematics"	24 10
24.11 The liquid drop nuclear model		19 20			R 12 Peierls, "Models of the Nucleus"	24.11
24.12 The shell model						24 12
24.13 Biological and medical applications of nuclear physics		21		E 6 5 Radioactive tracers	R 22 Brown, "Tasks for a World Without War"	Peaceful uses of radioactivity 24 13

Background and Development

OVERVIEW OF UNIT 6

The main purpose of this unit is to trace the development of our ideas of the constitution and structure of the atomic nucleus. The story line is simpler than that of Unit 5 and extends over a much shorter time interval: from 1896 to the present. The material has been chosen to accomplish these additional purposes: (1) to emphasize important concepts from earlier units (for example, the use and importance of the principles of conservation of energy and momentum, and the motion of charged particles in electric and magnetic fields); and (2) to relate physics to practical applications and to social and economic problems. The material of this unit is more amenable to these purposes than was the material of Unit 5.

The story of the atomic nucleus begins with the discovery of radioactivity. The phenomena of radioactivity furnished information about atomic transformations that were later shown to occur in the nucleus. Radioactive materials supplied the first projectiles (α particles) that made possible the discovery of the atomic nucleus, artificial transmutation, and nuclear reactions. The investigation of radioactivity also led to the concept of isotopes. Hence, the study of radioactivity opened several roads that led to the nucleus and its properties. Chapter 21 deals, therefore, with the discovery of radioactivity, the phenomena of radioactivity, and the theory of radioactive transformations.

The discussion in Chapter 21 leads, in a direct way, to the discovery of isotopes, treated in Chapter 22. The quantitative investigation of isotopes by means of the mass spectrometer made possible the measurement of the masses of individual atoms. Such measurements are based on the motion of charged particles in electric and magnetic fields.

The large amount of data obtained on isotopic masses and their natural abundance made possible theories concerning the composition of the nucleus. These theories are treated in Chapter 23. This chapter provides a fine example of the interplay between theory and experiment. The first hypothesis of the constitution of the nucleus (the proton–electron hypothesis) was unsuccessful. Further experimentation led to the discovery of artificial transmutation, which led, in turn, to the discovery of the neutron. The proton–neutron hypothesis of nuclear constitution was then possible.

The discovery of artificial transmutation and nuclear reactions opened up the field of radiochemistry, and led to the invention of charged-particle accelerators. The use of these machines resulted in the accumulation of an enormous amount of information, analogous to the development of chemistry in the 19th century. The phenomena studied in Chapter 23 are closely dependent on the principles of conservation of energy and momentum (the neutron and neutrino).

The ideas developed in Chapter 22 and 23 make possible the quantitative study of the energy balance in nuclear reactions and lead to the concept of nuclear binding energy, the first subjects studied in Chapter 4. These topics are intimately connected with nuclear fission and fusion reactions and their vast release of energy. Hence, nuclear fission is studied in some detail, along with its practical applications and industrial, economic, and political consequences. The study of energy release in these nuclear reactions focuses attention on the forces holding the nucleus together, and on models of nuclear structure.

Nuclear physics is far from complete, and the unit ends with hints of the problems and possibilities that remain.

CHAPTER 21 / RADIOACTIVITY

SUMMARY OF CHAPTER 21

In Unit 5 we discussed a number of experimental results that led to a model for the atom. In particular, we looked at the results of spectroscopic measurements. For many years an enormous amount of spectral data was taken for many different gases; Balmer's empirical relation gave this data an order that had to be explained by postulated models. We accepted Bohr's postulates because they led to a model for the hydrogen atom from which predictions checked with the known experimental data. The Rydberg constant (originally found purely experimentally) could now actually be calculated.

This approach (constructing a model to explain the experimental data) can be used for the nucleus as well as for the atom. In this unit we shall discuss some of the experimental results of nuclear physics just as we discussed experimental results of atomic physics in Unit 5. The problems are, however, much more difficult.

While the size of the atom is of the order of 10^{-10} m, the nucleus has a diameter of the order of only 10^{-14} m. Moreover, the energies involved on the atomic scale are relatively low: the ionization potential of hydrogen is -13.6 eV and the work functions of most materials for the photoelectric effect are only a few electron volts. Even X-ray energies go up to only a few thousand electron volts. All of these energies can be produced without too much difficulty in the laboratory. But the energies of particles inside the nucleus are several million electron volts or higher. The production of such energies is very difficult, and requires machines that appeared only gradually. As a result of these and other difficulties, we are not yet in a position to postulate a nuclear model that will explain all of our experimental results.

Although the practical applications of nuclear physics have increased at a rapid rate during the last 20 years, our understanding of the detailed structure of the nucleus and the role of the so-called fundamental particles is far from complete. We therefore present some of the experimental facts of nuclear physics with a few possible explanations. But the end of the story has yet to be written.

In this unit, we apply a number of concepts discussed in earlier units; some of these are really threads throughout the study of physics. The α and β particles follow the dynamics discussed in Unit 1. Conservation of energy (and mass) as well as conservation of momentum (Unit 3) are used throughout this discussion. Forces on charged particles in magnetic fields (Units 4 and 5) once again play an important role. The students should be aware that they now apply principles already studied to the nucleus; there are relatively few new concepts in this unit.

21.1 | BECQUEREL'S DISCOVERY

The story of Becquerel's discovery is an exciting one, and probably little teacher explanation will be required. It would be useful to point out examples of fluorescence, such as fluorescent lights, the face of a TV tube, etc. It is also useful to emphasize that phosphorescence is similar to fluorescence, the difference being that phosphorescence involves a time delay between the absorption of radiation and the subsequent re-emission. The word phosphorescence is unfortunately also used to describe the light emitted by small marine organisms (bioluminescence). It is worthwhile mentioning to the students that there are two different uses of the term.

21.2 | OTHER RADIOACTIVE ELEMENTS ARE DISCOVERED

Here is an opportunity to give your students a feeling for: (1) the tremendous progress in the technology of laboratory equipment, and (2) the ability of research to exist and succeed under conditions of minimum technology.

You could point out to your students that the Curies worked under conditions that would repel the average graduate student in science today. Their "laboratory" was just a shed with no heat, and their equipment was far less elaborate than that found in most junior high school science laboratories today! Nevertheless, they were able to carry out experiments of high sophistication and to make fairly exact measurements. Since they were chemists, their approach to the job of finding the source of the mysterious Becquerel rays was a chemical one. They used the technique of reducing radium ore as far as possible, until they arrived at the nonreducible substance radium. You might indicate at this point that the energies involved in the chemical processes of reduction were very, very small, compared to the energies inherent in the radioactive atoms themselves.

Since the major quantitative instrument used by the Curies was essentially a rather sensitive electroscope, you might refer back to Unit 4 to remind your students that electroscopes can be used as measuring instruments. You might ask: Is it just sheer luck that the amount of ionization produced by a "Becquerel ray" is directly proportional to the deflection of the charged electroscope leaves?

During this early work on both X rays and radioactive substances, knowledge of the biological effects of radiation was completely lacking. Pierre Curie carried samples of radium in glass vials in his pocket. After his death, his skin was found to have burn marks in that area. Until the invention of better tubes, notably the Coolidge tube, X-ray photos were made with very long exposures without any thought about possible damage to cell structure. Even as late as the 1940's and early 1950's, there were X-ray machines (fluoroscopes) in shoe stores, and children could have "fun" by irradiating their feet!

You could recommend the famous biography of Marie Curie to interested students; most libraries have it. Often, the movie based on this book is shown on television. Public television stations in your area might also show the series based on Marie Curie's life. Your science class might be interested in knowing that despite her unique and international fame, the French Academy of Sciences refused by one vote to elect her; an interesting comment on the attitude toward women as scientists at that time. (You might ask your class to find out if that attitude has undergone any change during the last 70 years.)

A pertinent article is: "The Early Years of Radioactivity," by G. E. M. Jauncey, in the *American Journal of Physics*, vol. *14* (1946), pp. 226–241.

21.3 | THE PENETRATING POWER OF THE RADIATION: α, β, AND γ RAYS

If your students inquire further about the meaning of the word "range," when applied to the distance traveled by radioactive emanations, you might explain that the less penetrating (charged) particles give up their energy gradually. For example, an α particle in air loses about 35 eV for each ion pair formed, until all its kinetic energy is gone. It then captures two electrons to become a helium atom. (You might ask your class how many ion pairs will be formed in air by a 6-MeV α particle.) The energy of a β particle is absorbed in the same way; however, β particles have a much wider range of energies than do α articles or γ rays.

Gamma rays, on the other hand, do not ionize air molecules and so do not lose their energy gradually. A γ ray photon is removed from the beam in a single event: by photoelectron absorption, electron scattering, or pair production.

21.4 | THE CHARGE AND MASS OF α, β, AND γ RAYS

Students will be much aided in their understanding of Rutherford's ingenious "mousetrap" by a brief teacher-led discussion based on Transparency *41*.

21.5 | THE IDENTITY OF α RAYS: RUTHERFORD'S "MOUSETRAP"

It is worth mentioning to your students that the separation of α, β, and γ rays illustrated on page 640 is, of necessity, greatly exaggerated. (Students doing SG question 3 will discover that if α and β particles enter a given magnetic field with the same speed, the ratio of their radii of curvature will be 7,350:1.) Note that this ratio suggests that α particles have a much larger momentum than β particles. The other possibility was that α particles are deflected much less than β particles because they have a much smaller charge, but independent measures of the charge indicated that such was not the case. Students will realize that the greater momentum of the α particle could be due to either a greater mass or to a greater speed. The greater momentum was found to be primarily due to greater mass.

21.6 | RADIOACTIVE TRANSFORMATIONS

Though it is correctly pointed out in this section that the establishment of α and β rays as particles emerging from atoms broke down the earlier idea that atoms were "indivisible," you might want to point out that the discovery of cathode rays was also a step in this direction. Thomson's empirical measurement of e/m for the electron provided the basis for an atom model made of two separate parts. However, you might then ask: What is the difference between the electrons in cathode rays and the electrons that are β rays, in terms of their places in the model of an atom?

The discovery that radium released heat as part of its radioactivity has had technological consequences. You can acquire additional information about how the heat energy resulting from a nuclear event can be transformed to do useful work in two publications: *Power from Radioisotopes*, and *Direct Conversion of Energy*. Obtain these by writing to the Department of Energy.

It should be emphasized that the proposal of Rutherford and Soddy that there was a *transmutation* of elements in the radioactive series was a very bold step, and that their idea was an exciting breakthrough. If the loss of an α or β particle from an atom resulted in a *different* atom, that difference had to lie in a change in the *nucleus*. Recall the nuclear atom model proposed by Rutherford: from where else could an α particle emerge?

21.7 | RADIOACTIVE DECAY SERIES

The uranium–radium decay series on page 645 can be most effectively discussed by referring to *T42*. (Although the term *half-life* is not defined until the next section, it would be useful to briefly mention its meaning in class and to point out the tremendous range of half-lives listed in the table.) Discourage students from attempting to learn the decay sequence in detail. It is important to understand the kinds of transformations that take place, but specific examples are for illustrative purposes only.

21.8 | DECAY RATE AND HALF-LIFE

The radioactivity decay curve framing page 646 is a natural focus for class discussion of this section. If students understand what is meant by half-life, they will have little difficulty understanding the shape of the decay curve. They should be led to realize that the units of time on the horizontal axis are "half-lives"; that is, T represents the half-life of a radioactive substance. It is important to emphasize that knowing that 50% of a sample will decay during its half-life does not imply that we have any way of predicting which atoms of the sample will decay during any given half-life. Of course, if the sample size is extremely small, large fluctuations from the predicted decay are likely.

Note that the mathematics of the decay has been set aside on a separate page (page 649) and is appropriate reading only for those students who are particularly mathematically inclined.

CHAPTER 22 / ISOTOPES

22.1 | THE CONCEPT OF ISOTOPES

It might motivate your students to realize that with this chapter they will begin to understand a physical phenomenon that for hundreds of years was the dream of so many. Ancient, medieval, and Renaissance alchemists spent their lives hopelessly searching for the "philosopher's stone," the unique substance that would enable them to change one metal into another (preferably iron, lead, or mercury into silver or gold). Radioactivity and the concept of the nuclear atom, in this sense, are the "philosopher's stone" of modern physical science for they provide the key to understanding how the atoms of one chemical element can be transmuted into the atoms of another. Perhaps one or two interested students would like to look into the history of alchemy. A fascinating book on this subject is *Prelude to Chemistry; An Outline of Alchemy* by John Read. MIT Press paperback.

22.2 | TRANSFORMATION RULES

The terms "displacement rules" was considered misleading and was not used. "Transformation rules" is a far more appropriate term because it is descriptive of the two processes involved.

One question you might ask (if your students don't) is: "Why do the radioactive series elements only lose mass by emitting α particles?" For example, in the gradual change from U^{238} to Pb^{206}, the nucleus loses 32 atomic mass units (and 10 units of charge). Why isn't a nuclear fragment of this size thrown out all at once, instead of a gradual loss of mass and charge by α and β emission?

Ask your students to calculate the ratio of mass to charge (A/Z) for such a fragment, and then ask them to search through the periodic table or a table of the nuclides if you have one, to see if such a nucleus exists. If they keep testing the ratio A/Z, they will not find a nucleus with ratio 3.2 in existence. Ask them to consider the possible emission of other particles, like ${}_1H^2$ or ${}_3Li^6$, instead of α particles. Would these particles provide the required change of mass and charge?

Your students may come to the conclusion that since α particles are the only significant massive particles emitted by nuclei in the radioactive series, such particles must exist in the nucleus as entities.

As a final check, point out that the earth's crust is made up mostly of such stable nuclei as ${}_6C^{12}$, ${}_8O^{16}$, and ${}_{14}Si^{28}$. Can these nuclei each be divided into a whole number of α particles?

22.3 | DIRECT EVIDENCE FOR ISOTOPES OF LEAD

This section provides a good opportunity to emphasize the property of the isotopes of an element that enables us to say that they are in fact varieties of the same element. What is it about the four isotopes of lead that makes them chemically identical? Of course, it is the atomic number 82 that they have in common. The atomic number determines the electron configuration of the atom and hence determines all the chemical properties and most of the physical properties.

That three of the four naturally occurring isotopes of lead are end products of three different decay series is very fortunate. We can determine both the initial composition and age of rock samples from the relative abundance of the lead isotopes in the rocks.

22.4 | POSITIVE RAYS

The term "canal rays" is often used to refer to positive rays; however, because it is a misleading term it was not used in this unit.

22.5 | SEPARATING ISOTOPES

Encourage students to study the captions and diagram of the mass spectrograph on page 663. They will likely be pleased that their knowledge of a few simple principles enables them to understand the operation of this ingenious device.

Whereas the preceding section dealt with the *theory* of the mass spectrographic determination of the mass of ions and the separation of isotopes, this section recounts Thomson's *experiment* with the mass spectrograph in which he discovered that neon has two isotopes.

To understand the work of Aston in separating isotopes by gaseous diffusion, students probably need to be reminded that the average kinetic energies of different types of molecules in a mixture of gases are the same if the temperature of the mixture is uniform. Thus, the lighter molecules must, on the average, move faster, and hence will diffuse through a porous wall more quickly.

22.6 | SUMMARY OF A USEFUL NOTATION FOR NUCLIDES: NUCLEAR REACTIONS

Although the latest convention for symbolizing nuclides is to write both the subscript Z and superscript A to the left of the element symbol, due to printing considerations, this convention was not followed in this unit. To separate the superscript and subscript adequately puts them too close to the lines of type above and below.

The proton and neutron do not enter the story until Chapter 23, but the chart on page 667 has its axes labeled A-Z versus Z, and Number of Neutrons versus Number of Protons. This was done to make the chart a more useful reference when reading the next chapter. Note that the first open square (at $A - Z = 1$, $Z = 0$) represents a free neutron.

Point out to students that the large arrow in the diagram on page 666 is a "process" arrow; that is, it represents a process by which, in this case, U^{238} gives off an α particle and becomes Th^{234}. The large arrow does *not* represent a velocity. The small arrows shown represent the relative velocities of the reaction products.

22.7 | THE STABLE ISOTOPES OF THE ELEMENTS AND THEIR RELATIVE ABUNDANCES

It is interesting to note that if an element with an odd atomic number has two isotopes, the atomic masses of the isotopes will also be odd numbers. For example, the atomic number of chlorine is 17 and it has two isotopes Cl^{35} and Cl^{37}. The isotopes are *stable*.

But, for heavy elements, if an isotope has an odd Z but an *even* mass number A, it is radioactive! For example, potassium has an atomic number of 19 and has an isotope K^{40} that is radioactive.

If you bring the above generalizations to the attention of the students, stimulating questions might arise as to the reason for these regularities in behavior. What is it about the nucleus that allows the "oddness" or "evenness" of its composition to determine its stability? You might drop a hint here about the shell model of the nucleus, a model that students will read about in Sec. 24.12.

A point relating to stability that should eventually be brought to the attention of students is the way the plot of the nuclides on page 667 curves up (away from an imaginary 45° line that would represent equal numbers of protons and neutrons in nuclei). This curve implies that the masses of nuclei increase faster than their charges. At higher Z numbers, the greater positive charge of the nucleus is in effect "diluted" by uncharged matter (neutrons) in such a way that the nucleus is not forced apart by electrostatic repulsion. (This coulomb repulsion is very large due to the extremely small distance between protons in the nucleus.) The effect of the greater proportion of uncharged matter is to give rise to very strong short-range attractive forces that hold the nucleus together.

Another interesting point arises from the chart on page 667. If you imagine a best-fit "stability line" drawn through the black squares, the unstable nuclides above that line have excessive negative charge (too few protons) for stability and hence undergo $\beta-$ decay to become stable; whereas those unstable nuclides below the stability line have excessive positive charge (too few neutrons) for stability and hence undergo $\beta+$ decay. In other words, the heavier isotopes of a given element emit electrons, the lighter isotopes of the element emit positrons. (In addition to positron emission as a way of reducing positive charge, there are other processes such as electron capture and α decay that accomplish the same result.)

22.8 | ATOMIC MASSES

This brief section presents Aston's whole-number rule, which suggests so strongly that nuclei consist of different numbers of identical pieces each of which has a mass of 1 amu.

CHAPTER 23 / PROBING THE NUCLEUS

23.1 | THE PROBLEM OF THE STRUCTURE OF THE ATOMIC NUCLEUS

At the beginning of this chapter, it might be helpful for students to summarize the information they would have had at their disposal at this point in history (about 1925) relative to the structure of the nucleus. This might be tabulated on the board. Items such as Rutherford scattering, half-life phenomena, particle energies, knowledge about the electron, isotopic mass variation, etc., might be included. Students should be asked what each piece of evidence might imply about nuclear structure. They should be reminded that the neutron had not been discovered at this time, and also that Heisenberg did not formulate the uncertainty principle until the 1930's.

A second provoking question to think about throughout the chapter is: What is charge? There is presently no simple answer to this question, and it gives one practice in spinning out hypotheses. An interested student might be asked to look into the present state of inquiry into the nature of charge.

23.2 | THE PROTON–ELECTRON HYPOTHESIS OF NUCLEAR STRUCTURE

It might not be entirely wrong to say that the whole point of Chapter 23 is the development of answers to the questions: What is the purpose of a model? On what criteria do models succeed or fail? How real is a model? How much do models allow the scientist to understand or explain nature; that is, what are the limitations of models?

These should be the kinds of questions that have been coming up in class during the entire course. The aware student ought to realize that, as the investigation of natural phenomena has passed from the macroscopic world to the microscopic one, the models have become increasingly complex. This makes for an interesting contrast: as we proceed from the behavior of large and complex masses of matter to that of simpler masses, the models go from simpler to more complex.

The proton–electron theory of the nucleus is an interesting example of how a model inevitably depends upon the empirical evidence available at the time. Thus, the proton–electron model works very well, if all you know is that certain heavy nuclei are emitting either α or β particles.

Note that at the end of this section, no reason is given for the failure of the proton–electron model. Yet, some perceptive student may want to know upon what criteria the model failed. The fact is that the criteria are rather sophisticated. One was the discovery that the nucleus had an angular momentum, called *spin*. This property could be measured by spectroscopic analysis (very high resolution of hyperfine structure). The result of such analysis is an ability to measure a quantity called the *magnetic moment* of the proton and electron. It turns out that the magnetic moment of the electron is much larger than the magnetic moments of different nuclei. So, one must ask the question, if there are electrons in the nucleus, why isn't the magnetic moment of the whole nucleus greater than that of the electron?

23.3 | THE DISCOVERY OF ARTIFICIAL TRANSMUTATION

One question related to the Rutherford observation of artificial transmutation that you might use to stimulate class discussion is: Why are the chances of a collision between an α particle and the nitrogen nucleus so small (one for every 10^6 α particles passing through the nitrogen gas)?

In terms of the kinetic theory (Unit 3), it seems that the probability of capture should be much greater. However, remind your students that kinetic theory does not take into account (1) the extreme smallness of nuclear diameters, or (2) the effect of the electrical fields of the nuclei.

The nitrogen atom has a radius of about 10^{-10} m; however, the nucleus is far smaller, with a radius of about 3.6×10^{-15} m. Imagine a target with a diameter of 300 m, whose bull's-eye is only 1 cm across! To make things worse, imagine standing a few thousand meters away and shooting at this target blindfolded. What are the chances of hitting the bull's-eye? Suppose all the targets were moving around randomly in space at the same time, what would happen to the probability?

As for the effect of the electric field, the N nucleus has a charge of +14 elementary charge units, while the α particle has a charge of +2. Ask your students to calculate, by Coulomb's law, the repelling force between two such charged particles at a distance equal to the sum of the radii of both particles (the radius of the α particle is about 2.4×10^{-15} m). They should be surprised at the result. In fact, they have a right to ask, how is it that an α particle ever manages to get inside a nitrogen nucleus? (The answer, of course, is outside the realm of classical physics; only a quantum mechanical analysis can offer an explanation.)

You can relate Wilson's invention to the Millikan oil drop experiment (Unit 5) by pointing out that Wilson first used his cloud chamber to calculate the charge on the electron. He created the water cloud between two parallel metal plates. First, he observed the gravitational fall of the top surface of the cloud; then he created a uniform electric field between the plates and observed the fall under the influence of both gravitational and electric fields. By comparing the two rates of fall, Wilson calculated a charge of 1.0×10^{-19} C, about two-thirds of the accepted value today. Ask your students what kind of error could be caused by using a water cloud (evaporation), and why the technique of using single droplets of oil was a better one.

Fifteen years later, Wilson realized that his cloud chamber could be used to observe the tracks of particles from radioactive disintegration. For this work (together with Arthur Compton), he received the Nobel Prize.

23.4 | THE DISCOVERY OF THE NEUTRON

This section explains the evidence for the neutron in a fairly clear and detailed fashion, and your students should be able to follow the arguments without much difficulty. Remind them that in this chapter, they are seeing the application of Newton's idea of the universality of the laws of nature. The principles of conservation of momentum and energy came from observations in the macroscopic world, but here they are being applied to the motion of tiny, invisible particles moving with high speeds. Nevertheless, the predictions that are made on the basis of the validity of these conservation principles are verifiable.

23.5 | THE PROTON–NEUTRON THEORY OF THE COMPOSITION OF ATOMIC NUCLEI

This brief section provides an excellent example of how models change in physics. In the long run, the validity of the model depends only upon how nature behaves. Scientists must accommodate the model to the behavior; not the other way around. You might ask your students to contrast this model change with the attempts to change the solar system model in early astronomy.

If your students are curious, the empirical evidence for the existence of neutrinos was discovered by Reines and Cowan at Los Alamos in a famous nuclear pile experiment. They made a very large neutron counter and placed it near one of the atomic piles at the Savannah River Project. The reaction they were looking for was this proposed one:

proton + neutrino = neutron + positron.

Since the heavy pile shielding kept all other particles except neutrinos from coming through, the appearance of neutrons and positrons in the counter showed that the above predicted reaction was indeed taking place.

23.6 | THE NEUTRINO

If your students want more information about the neutrino, there is an excellent article in the January 1956 *Scientific American* by Philip Morrison, "The Neutrino."

It might interest your students to know that though Pauli suggested the existence of another particle, the name "neutrino" was coined by Enrico Fermi. (It means "little neutral one" in Italian.)

For better students, who might want to know just how the principle of conservation of energy is violated without emission of the neutrino, there is a lively explanation of the neutrino in *The Atom and Its Nucleus*, by George Gamow.

Point out that a free neutron tends to disintegrate fairly quickly into a proton and a β particle. However, if we write the nuclear equation as:

$$_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0$$

the masses on each side of the equation do not balance. The rest mass of the neutron turns out to be 0.00084 amu larger than the combined rest masses of proton and electron. This rest mass difference represents an energy of 0.78 MeV, which should be the enargy of the emerging electrons. However, very few electrons emerge with as much energy as this. Therefore, Pauli suggested the existence of another energetic particle to make up the difference in energy.

How is the conservation of momentum principle violated by the nuclear equation above? If the neutron does break up into only a proton and electron, the two particles should recoil from each other at an angle of 180°; that is, they should fly diametrically apart, relative to their centers of mass. But observation of the tracks showed that they did not seem to obey the law of conservation of momentum. The angle of separation suggested that another particle must be part of the interaction.

In the case of β emission, the missing particle is an antineutrino. (The neutrino and antineutrino are similar particles. The former is associated with positron emission; the latter, with electron emission.)

Tell your students to conceptualize the neutrino as a kind of photon with zero rest mass and with a velocity equal to that of light (c). But the neutrino behaves quite differently from a light photon. For example, it does not cause a photoelectric effect. In fact, the neutrino rarely interacts with other particles; this is why it has such tremendous penetrating power. Many neutrinos probably pass through the entire earth without interacting with any other particles! The chances of a neutrino interacting with the atoms of your body are only about 1 in 10^{13}.

Neutrinos are produced in the upper atmosphere by cosmic ray bombardment. They also come from many stars, and some astronomers think that, as supernovas degenerate into even more incredibly dense stars (black holes and neutron stars), vast numbers of neutrinos are emitted.

23.7 | THE NEED FOR PARTICLE ACCELERATORS

It is interesting to point out the remarkable contrast between the size of the particles that are accelerated and the size of accelerating machines (shown on pages 690 and 691). Similarly, the scale of detection devices, such as the Brookhaven bubble chamber assembly shown on page 694, is worthy of mention. Another thing to contrast with the size of that immense bubble chamber assembly is Glaser's tiny bubble chamber, also shown on page 694.

Students who might be particularly interested in elementary particles should be encouraged to look through the *Project Physics* supplemental unit "Elementary Particles," and to study it in detail if time permits.

A good paperback book on accelerators is *Accelerators: Machines of Nuclear Physics*, by Wilson and Littauer, Doubleday-Anchor, paperback

23.8 | NUCLEAR REACTIONS

The purpose of this section is to emphasize that a change in nuclear charge means a change in place in the periodic table, the result of a transmutation.

Many examples of nuclear reactions can be found to supplement the few examples in the unit. But it might be more interesting and informative for your students to be aware of the Bohr theory of the compound nucleus proposed in 1936 to explain such reactions. Niels Bohr made two assumptions about the order of events in a nuclear reaction:

(1) the particle that strikes the nucleus is absorbed into the nucleus to form a compound nucleus;

(2) the compound nucleus is unstable and disintegrates by ejecting a particle or γ photon, with a new nucleus formed as product.

Bohr also assumed that the compound nucleus depends upon its own energy state and its angular momentum (and has nothing to do with the way the nucleus is formed). So, in terms of the Bohr theory, the first reaction on page 693 would look like this:

$$_{1}H^{2} + {}_{80}Hg^{199} \rightarrow [{}_{81}Tl^{201}] \rightarrow {}_{79}Au^{197} + {}_{2}He^{4}$$

Have your students rewrite each of the nuclear equations in this and in the following section in terms of the compound nucleus theory.

Bohr assumed that the energy of the entering particle was shared by all the other nucleons in the capturing nucleus; he called this newly available nergy the *excitation energy*. If this energy is large enough, it may provide the means for one nucleon or a combination (like the α particle) to escape. This amount needed for escape is called the separation energy: about 8 MeV.

Thus, a compound nucleus is formed whenever a nucleus captures a proton, neutron, electron, deuteron, α particle, or even an X-ray photon of high enough energy. In a very short time, the compound nucleus disintegrates to eject a particle and leave a new product nucleus.

23.9 | ARTIFICIALLY INDUCED RADIOACTIVITY

One of the most interesting cases of induced radioactivity is that discovered by E. O. Lawrence. He bombarded rock salt with deuterons of 2 MeV energy and obtained radioactive sodium. The reaction can be written

$$_{11}Na^{23} + {}_{1}H^{2} \rightarrow [{}_{11}Na^{24}] + {}_{1}H^{1}$$

followed by

$$[{}_{11}Na^{24}] \rightarrow {}_{12}Mg^{24} + {}_{-1}e^{0}$$

with a half-life of 15 h. The magnesium nucleus, however, is in an excited state and, in falling to the normal state, emits γ rays. Hence radiosodium emits β and γ rays identical in nature with those from natural radioactive substances. These γ rays are more penetrating than those from radium.

The advent of induced radioactivity creates new possibilities in biological and chemical research. The active atoms are so easily detected that they can be traced through a process or reaction. They serve as marked or "tagged" atoms that can be located by their effect upon a counter. An in-depth discussion of radiation biology, medicine and agriculture is appropriate with Sec. 24.13. Today, there are far more artificially produced radioactive isotopes than natural ones, and there is no essential difference between them.

CHAPTER 24 / NUCLEAR ENERGY; NUCLEAR FORCES

24.1 CONSERVATION OF ENERGY IN NUCLEAR REACTIONS

The statement that nuclear reactions are far more energetic than chemical reactions can be made more vivid by having your students calculate the number of electron volts of energy released in a typical exothermic chemical reaction.

A well-known reaction is the combination of oxygen and hydrogen to form water. The heat of formation for a gram molecular weight of water is -68.4 kcal. (The minus sign means the reaction is exothermic.) The steps your student should follow are:

(1) change kilocalories into joules by using the mechanical equivalent of heat:

$$W = JH;$$

(2) compute the number of joules of energy released per molecule. Remind students that Avogadro's number of molecules comprise a gram molecular weight;

(3) change the number of joules/molecule to electron volts.

The students will discover that the total energy released per molecule of water is about 3 eV. This is considered a fairly energetic chemical reaction!

You can find other values of heats of formation in any chemistry text for similar examples.

24.2 THE ENERGY OF NUCLEAR BINDING

The concept of binding energy (remind your students about the actual definition in terms of "unbinding") brings up the question of nuclear force. When nucleons are held together in a very, very small volume, what kind of force keeps them together? What part is played by the binding energy here?

There is a fine article in the March 1960 *Scientific American* on the subject about which we still know so little: "The Nuclear Force," by Robert E. Marshak.

If any student who has heard of the term "mass defect" asks about it with respect to the subject matter of this section, point out that "mass defect" is the same as "binding energy" except that the former is measured in mass units and the latter in energy units.

It may be worthwhile to point out to your students that the conceptualization and measurement of binding energy is responsible for the decision not to use the proton as a standard (mass = 1 amu) for all atomic masses. For all atoms with A greater than 12, the mass defect is about 0.0085 amu/nucleon. On this scale C^{12} would have a mass = 11.907 amu, and the mass of U^{238} would be 238.20. Thus, there would be a noticeable discrepancy between mass numbers and the actual numbers of nucleons in nuclei. Actually, any atom more massive than B^{11} could have been chosen as a standard in order to make the mass numbers nearly equal to the numbers of nucleons; C^{12} was chosen for the reasons given on page 670.

24.3 NUCLEAR BINDING ENERGY AND STABILITY

The point to emphasize in this section is the way in which the average binding energy per nucleon varies with the mass number A. The figure on page 706 illustrates that variation. This will be seen in later sections to account for the release of energy in nuclear reactions, and to make possible the large-scale energy release in both nuclear fission and fusion.

Since the figure on page 706 wil be referred to in later sections, focus the attention of students on it briefly at this point but don't take time to go into all its implications. Let the implications arise naturally in later sections.

Clarify that it only makes sense to speak of binding energy per particle when the particle is, in fact, bound to other particles. An isolated particle, of course, has no binding energy.

24.4 THE MASS–ENERGY BALANCE IN NUCLEAR REACTIONS

Here is an excellent opportunity to clarify student understanding of the equivalence of mass and energy. The *Text* describes a nuclear reaction in which a proton is captured by a lithium nucleus which then disintegrates into two α particles moving apart at high speed. As shown on page 707, this disintegration results in a loss of rest mass and a gain in kinetic energy. Students may think mass has been converted into energy! But this interpretation is valid only if the student is careful to say rest mass and kinetic energy (and/or γ ray energy). There is no change in total mass or total energy.

Prior to disintegration, the lithium nucleus and proton had a certain total energy, made up of kinetic energy and rest energy: the energy equivalent of their rest mass. After disintegration, the total energy is the same as before, but now consists of more kinetic energy and less rest energy!

As total energy is conserved, so is total mass. Suppose measurements of the mass of the α particles were made as they separated at high speeds after their formation. Such a mass measurement at high speed could be made by measuring the initial curvatures of the α particle tracks in a cloud chamber in a magnetic field. The kinetic energy of the α particles contributes to their relativistic mass, and the total mass so measured would be identical to the total mass of the proton and lithium nucleus prior to the reaction!

However, if we stop the α particles and again measure their masses, we will be measuring their total rest mass, which we will find to be smaller than before by an amount we call the mass defect of the reaction. Note that mass and energy are still conserved, because the kinetic energy and accompanying mass increase are transferred by collision and ionization from molecule to molecule of the air through which the α particles travel, resulting in thermal motion of the air molecules.

24.5 | NUCLEAR FISSION: DISCOVERY

This section of the course should be exciting for most students; they all know about atomic weapons by now. What is important, then, is that they understand (1) exactly what fission is, (2) how it takes place, and (3) what the consequences are. The students ought also to understand how the phenomenon of fission affects nuclear theory; that is, the making of nuclear models.

You might emphasize the effect of accident (of a kind) in the case of the Fermi attempts to create transuranium elements. The discovery of reaction products that "shouldn't have been there" (such as Ba^{139} and La^{140}) turned the direction of research toward the discovery of fission by Hahn and Strassmann, and its explanation by Lise Meitner and O. R. Frisch.

It may interest your students to know that Lise Meitner shared the Enrico Fermi Award in 1966 with Hahn and Strassmann and was the first woman to receive this award.

24.6 | NUCLEAR FISSION: CONTROLLING CHAIN REACTIONS

Unit 6 discusses some of the conditions for neutron capture in terms of neutron energy, that is, "slow" or "fast" neutrons are mentioned. This whole subject area is rather complex and the treatment should be sufficient; however, there is no reason why interested students should not be encouraged to find out more on their own if they so desire.

Some good references for student reading are: *The Neutron Story*, by Donald Hughes (a Doubleday-Anchor paperback); *Scientific American*, August 1965, "Nuclear Fission," by R. B. Leachman; and the *American Journal of Physics*, January 1964 (vol. 32, no. 1), "A Study of the Discovery of Fission," by Esther Sparberg, and "Discovery of Nuclear Fission," by Hans Graetzer.

24.7 | NUCLEAR FISSION: LARGE-SCALE ENERGY RELEASE AND SOME OF ITS CONSEQUENCES

One of the questions you might bring up in class is: If we assume that at the time of the formation of the earth there was as much U^{235} created as U^{238}, how can the present-day ratios of 99.28% U^{238} to 0.72% U^{235} be explained? (Hint: Assume the earth is about 5.6 billion years old. The half-life of U^{238} $= 4.5 \times 10^9$ y; that of $U^{235} = 7 \times 10^8$ yrs.)

A quick but interesting approximation of the magnitude of nuclear energies can be worked out quite easily in class. The fission of one U^{235} nucleus liberates about 200 MeV of energy. Have your students convert this quantity to joules. Suppose you had an Avogadro's number of U^{235} nuclei. How much energy would be liberated? Five times this amount is about 1 kg of U^{235}. When they have calculated the equivalent of this amount of energy liberated by the fission of 1 kg, point out that this is more than is liberated by exploding 20,000 tons of TNT. The difference between an exploding bomb and a nuclear reactor, of course, is simply the time taken to release the total energy.

Your students could write to the Department of Energy in Washington, or to private corporations like Westinghouse and General Electric, for more information about the peaceful uses of atomic power. Most of these agencies have education and information sections for the dissemination of such information free of charge. Here is a good opportunity to relate what is being learned in the classroom to the realities of world politics and economics.

The accident at Three Mile Island, Pennsylvania, in March 1979 brought into public discussion the potentials of nuclear power and nuclear disasters. In retrospect, it appears that the difficulties which did not become a major disaster, resulted from a combination of human error in responding to warning signals and inadequacies in the engineering design of the power plant. At the time, conflicting media reports led to public confusion. However, the safety devices did hold. Had the human response been quicker, much of the internal damage to the plant that did occur might have been avoided.

Because many of your students will have little or no knowledge of the Three Mile Island affair, you might recommend that they review the daily reports and consider the subsequent analyses in newspapers and news magazines of that time. Perhaps three major points might be made: (1) No device is used without risk. One risk may be more apparent than the risk resulting from another device; long-term consequences must also be taken into consideration. Basically, the question becomes: What is a tolerable risk to be accepted by society? (2) The quantitative aspects of a risk are of utmost importance. Many writers and the public at large seem unable to realize that some radioactivity is present in our bodies already (from cosmic rays, etc.) and that even doubling that amount might be of little consequence. Also, not only the level, but also the atomic carrier is important. For example, radioactive argon has less effect on humans than radioactive calcium or strontium, which are absorbed by bones. (3) The engineering design of any device or power plant involves compromises between "complete safety" at impossible costs, and a reasonable "safety factor" that takes into account forseeable conditions. (Recall the Tacoma Narrows Bridge, which collapsed while oscillating in a high wind.)

The thoughtful assessment of the physical and social consequences of accidents, pollution, etc., requires an increasingly well-informed public. A discussion of the advantages and disadvantages of using alternate sources of electrical energy could be a culmination of the course. There is no "right answer" in choosing among the possible sources of power. However, reasonable arguments backed by data and social consciousness should be encouraged.

24.8 | NUCLEAR FUSION

Though this section describes to some extent the problems inherent in controlling a fusion reaction, you might pose the questions a little more precisely for your students. For example, if a magnetic field is used to contain the deuterons and tritium nuclei, what will the paths of these particles be in the field? How does this "contain" the charged particles? How could particles "escape," and what would happen if they did?

In this way, you can relate the technology of thermonuclear reactions to fundamental principles already studied in previous units on electricity, magnetism, and kinetic theory.

A nice lead-in to Sec. 24.9 would be the intriguing question: How can thermonuclear reactions be taking place in the sun and stars without a special mechanism of confinement? Of course, the answer involves the gravitational fields of such massive bodies.

A discussion of thermonuclear energy always brings up the threat of the H-bomb and its promise of total destruction. Here is a good opportunity to "integrate" with the social science course in your school. There are certainly many books and articles in print on this subject, and discussions of the problem are still continuing on an international scale. Journals like *The Bulletin of Atomic Scientists* and *Daedalus* have many articles on the subject. Try to find time for an informal seminar class on the question of atoms for war or peace.

24.9 | FUSION REACTIONS IN STARS

The discussion applies only to "normal," or "main-sequence" stars, of which our sun is one. Special types of stars, such as white dwarfs, red giants, variables, and supernovae, are more complicated and less well understood. Hydrogen is by far the most abundant element in those parts of the universe that we can explore; thus a very large fraction of all the nuclei present will be protons. For interested students, first assign SG question 15, then summarize the two nuclear reactions that are thought to occur as follows:

(1) Carbon–nitrogen cycle

$C^{12} + H^1 \rightarrow N^{12} + \gamma$
$N^{13} \rightarrow C^{13} + e^+ + \text{neutrino}$
$C^{13} + H^1 \rightarrow N^{14} + \gamma$
$N^{14} + H^1 \rightarrow O^{15} + \gamma$
$O^{15} \rightarrow N^{15} + e^+ + \text{neutrino}$
$N^{15} + H^1 \rightarrow C^{12} + He^4$

(2) Proton–proton cycle

$H^1 + H^1 \rightarrow H^2 + e^+ + \text{neutrino}$
$H^2 + H^1 \rightarrow He^3 + \gamma$
$He^3 + He^3 \rightarrow He^4 + 2H^1$

Note that for the third reaction in the proton–proton cycle to occur, the second reaction must occur twice.

The net effect of both cycles is to form a strongly bound α particle from four protons. The carbon cycle is more appropriate to stars that are more luminous than our sun and whose central temperatures are higher. The proton–proton cycle is more important to stars whose central temperatures and luminosities are lower than that of our sun. Both reactions take place in the sun, but the proton–proton cycle predominates.

There may seem to be a discrepancy between two passages in this section. Reference is made to "a hot plasma at 10^8 degrees" and the statement is also made that the sun's interior "has been estimated to be 10 to 20 million degrees." These are not necessarily inconsistent, however; the two cases are different. The first refers to experiments in the laboratory under the conditions available there. The second refers to the explosion of a hydrogen bomb under conditions that must be quite different (and are not publicly available). One probable difference is that of the pressure of the materials that interact in fusion. For example, in the first case the gas pressure may be expected to be lower than in the second case. This must affect the temperature at which fusion might occur. In any case, the difference between the two temperatures mentioned is effectively quite small in controlling the rate of so complex a process as fusion under "exotic" conditions.

24.10 | THE STRENGTH OF NUCLEAR FORCES

The nuclear force is the third type of force your students will have studied; the previous two being gravitational and electrostatic forces. (Magnetic force is not similarly classed because it results from the relative motion of charged particles.)

In the case of nuclear force, there seems to be need for a description that does not seem to have the same kind of logical basis as the first two. How can the tremendous repelling coulomb force between protons (that certainly must overcome any gravitational force) be rendered inoperative? What kind of cohesive power works here? It ought to be evident to your students that anything said about "nuclear force" must indeed be quite speculative.

There is a fairly easy way to demonstrate the techniques of speculation in nuclear theory by developing the argument for the particle called the *meson*.

You begin with the rather weird assumption that within the space of the nucleus one nucleon might eject a very small particle that would be absorbed by a neighboring nucleon. The new particle would have an amount of energy that can be designated by Einstein's equation: $E = mc^2$, where m is the mass of the particle. This new particle, then, could move through the range of "nuclear force"; that is, across a very small distance inside the nucleus. Measurements show that this range is about 1.5×10^{-15} m.

If we call this distance s, then the time it takes the particle to move through s at the speed of light is simply s/c. Now remind your students about the Heisenberg uncertainty principle (Unit 5, Sec. 20.5), and indicate that at such a short distance and at so great a speed, this principle must come into play. Then, the energy multiplied by the time interval must be of the order of Planck's constant; that is:

$$\Delta mc^2 \, \Delta t \approx \frac{h}{2\pi}$$

Ask your students to compare the units of $\Delta mc^2 \, \Delta t$ with those of $\Delta p \, \Delta x$ (the more familiar statement of the principle); they will find that the units are exactly the same. Now, since Δt is the same as s/c, we can write:

$$\Delta mc^2 s/c \approx \frac{h}{2\pi}$$

or

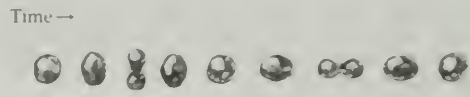
$$ms \approx \frac{h}{2\pi c}$$

Now, your students can fill in the values for h (6.62×10^{-34}), π (3.14), and c (3×10^8). Have them calculate the value of ms in kilogram-meters. If they divide this value by $s = 1.5 \times 10^{-15}$ m, they will have found the mass of the theoretical particle (it should come out to about 2×10^{-28} kg). Have them compare this mass with that of the proton and electron.

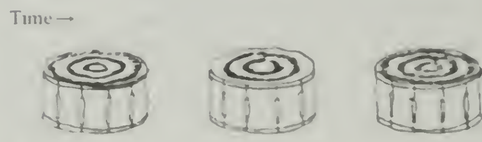
After this calculation is finished, you can point out to your students that they have just worked out a sequence of logical thought similar to that done by Yukawa in order to predict the existence of mesons. In spite of the seemingly illogical nature of the assumptions, the tracks of mesons were first identified in 1938 by Anderson in cloud-chamber photos of cosmic-ray events. More than four different kinds of mesons have been identified. Emphasize the fantastic nature of making a prediction that turns out to be so exact (Anderson's first measurements showed the new particle to have a mass equal to about 200 electron masses), while the "underpinnings" of the prediction are shaky! This is the way in which much of the prediction in nuclear theory has been going along.

24.11 | THE LIQUID-DROP NUCLEAR MODEL

The liquid-drop model, with the ability to account for the variation in nuclear binding energy with mass number and for the process of nuclear fission, depends upon the assumption that interactions occur only between adjacent nucleons. We are all familiar with the characteristic oscillations of a stretched string (Unit 3, page 370), a taut membrane (Unit 3, page 372), etc. Less familiar perhaps are the characteristic oscillations of a liquid drop. It might be well to review briefly just how a liquid drop oscillates with time when disturbed. The students must understand this before they can appreciate the application to fission via the model. A sketch similar to the following might help



Vibrating liquid drop



Vibrating drum

Vibrating string

With this model it is easy to see that once a nucleus is set into oscillation the electrostatic forces of repulsion (positive charges of the protons) between the two halves may drive it into a complete break. This is the fission process.

The question "Why doesn't a nucleus separate into three lighter nuclei?" is answered by considering the nucleus to act like a liquid drop in which it seems unlikely that an oscillation would take place in such a mode as to break the drop into three pieces.

The shock that sets a heavy nucleus into oscillation can be delivered by any kind of particle. $_{92}U^{238}$ needs a very large disturbance while $_{92}U^{235}$ can undergo fission by the addition of a neutron with zero kinetic energy. There has been considerable study of photon-induced fission; but for chain reaction, however, the important particle is the neutron.

24.12 | THE SHELL MODEL

There is a good article for the teacher on the two models by Aage Bohr (winner of the 1975 Nobel Prize in Physics and son of Niels Bohr) in the November 1957 *American Journal of Physics*, "On the Structure of Atomic Nuclei" (vol. 25 no. 8).

Experimental and theoretical results regarding so-called "elementary" particles are changing so fast that even current journals may be out of date before they are printed. However, two general review articles may be useful. In the July 1974 issue of *Scientific American* (vol. 231, No. 1) Steven Weinberg reviewed the status of knowledge about elementary particles in an article entitled "Unified Theories of Elementary-Particle Interaction." In the January 1980 *Scientific American* (Vol. 242, No. 1) Robert R. Wilson discussed "The Next Generation of Particle Accelerators." Since high-energy machines are essential to the production of particles with increasingly heavy mass, the two papers are complementary.

Wilson reports that energies of about 500 billion electron volts (500 GeV; 10^9 eV = 1 GeV) have been achieved. Energies of 1,000 GeV (1 TeV) are expected during the 1980's at Fermilab in Batavia, Illinois, and at a machine being completed in the U.S.S.R. (The new unit for 1,000 GeV is called a tera-electron volt: 10^{12} eV = 1 TeV.)

As shown in Table 1, Weinberg described the four natural forces: gravitational, electromagnetic, "strong" interaction, and "weak" interaction. The latter two forces appear only in nuclear interactions. Both "strong" and "weak" nuclear forces have very small ranges: less than 10^{-14} cm. The range of the force appears to be inversely related to the mass of the particle exchanged. Since gravitational (in hypothetical units called gravitons) and electromagnetic forces (photons) involve essentially massless particles, they have infinite range. However, the "strong" and "weak" forces do involve particles with mass and have very short ranges.

Table 2, also taken from Weinberg's paper, reports the grouping of particles into photons, leptons (which are massless or of low mass), and heavier hadrons (which are involved in "strong" interactions). With both experimental and theoretical work proceeding rapidly, any such table is soon incomplete. However, mathematical models are revealing some common characteristics of the "weak" and the electromagnetic interactions, which may reduce the apparent complexity of the data.

Table 1. The four types of particle interaction that are believed to account for all physical phenomena. ("Range" indicates the distance beyond which the interaction ceases to operate. "Strength" characterizes the strength of the force under conditions typical of current observations.)*

	Gravitational	Electro-magnetic	Strong	Weak
RANGE	∞	∞	10^{-13}–10^{-14} cm	$<<10^{-14}$ cm
EXAMPLES	astronomical forces	atomic forces	nuclear forces	nuclear beta decay
STRENGTH (NATURAL UNITS)	$G_{NEWTON} = 5.9 \times 10^{-39}$	$e^2 = \frac{1}{137}$	$g^2 \approx 1$	$G_{FERMI} = 1.02 \times 10$ [illegible]
PARTICLES ACTED UPON	everything	charged particles	hadrons	hadrons leptons
PARTICLES EXCHANGED	gravitons	photons	hadrons	?

*From "Unified Theories of Elementary-Particle Interaction" by Steven Weinberg.

Table 2. A partial list of observed elementary particles with lifetimes greater than 10^{-20} sec. A symbol with a bar above it indicates an antiparticle. (In the case of photons, neutral pions, and eta mesons, the particle is its own antiparticle.) The charges shown are in units of *e*, the charge on the electron ($e = 1.602 \times 10^{-19}$ C). The masses are given in energy units (1 MeV = 1.783×10^{-27} g).*

		PARTICLE	SYMBOL	CHARGE	MASS (10^6 eV)	LIFETIME (sec)
		Photon	γ	**0**	**0**	∞
LEPTONS		neutrino	$\nu_e\ \bar{\nu}_e$	0	0	∞
LEPTONS		neutrino	$\nu_\mu\ \bar{\nu}_\mu$	0	0	∞
LEPTONS		electron	e^-	$\pm e$	0.511	∞
LEPTONS		muon	μ^-	$\pm e$	105.66	2.199×10^{-6}
HADRONS	MESONS	pion	$\pi^\pm$	$\pm e$	139.57	2.602×10^{-8}
HADRONS	MESONS	pion	π^0	0	134.97	0.84×10^{-16}
HADRONS	MESONS	kaon	$K^\pm$	$\pm e$	493.71	1.237×10^{-8}
HADRONS	MESONS	kaon	K^0	0	497.71	0.882×10^{-10}
HADRONS	MESONS	eta	η	0	548.8	2.50×10^{-17}
HADRONS	BARYONS	proton	$p\ \bar{p}$	$\pm e$	938.259	∞
HADRONS	BARYONS	neutron	$n\ \bar{n}$	0	939.553	918
HADRONS	BARYONS	lambda hyperon	$\Lambda\ \bar{\Lambda}$	0	1,115.59	2.521×10^{-10}
HADRONS	BARYONS	sigma hyperon	$\Sigma^+\ \overline{\Sigma^+}$	$\pm e$	1,189.42	8.00×10^{-11}
HADRONS	BARYONS	sigma hyperon	$\Sigma^0\ \overline{\Sigma^0}$	0	1,192.48	$<10^{-14}$
HADRONS	BARYONS	sigma hyperon	$\Sigma^-\ \overline{\Sigma^-}$	$\pm e$	1,197.34	1.484×10^{-10}
HADRONS	BARYONS	cascade hyperon	$\Xi^0\ \overline{\Xi^0}$	0	1,314.7	2.98×10^{-10}
HADRONS	BARYONS	cascade hyperon	$\Xi^-\ \overline{\Xi^-}$	$\pm e$	1,321.3	1.672×10^{-10}
HADRONS	BARYONS	omega hyperon	$\Omega^-\ \overline{\Omega^-}$	$\pm e$	1,672	1.3×10^{-10}

*From "Unified Theories of Elementary-Particle Interaction" by Steven Weinberg. Copyright © 1974 by Scientific American, Inc. All rights reserved.

24.13 | BIOLOGICAL AND MEDICAL APPLICATIONS OF NUCLEAR PHYSICS

The ramifications of this final section are many, and if time permits, you may want to encourage your students to find out more about the use of radioactive tracers. They can write to the Department of Energy, or to various industries; local doctors or hospitals can probably furnish information on the part played by radioactive isotopes in fighting malignant diseases. Another interesting feature is the general effect of radiation on animal and plant communities.

Students who can handle mathematics fairly easily can be asked to investigate the logic of using tracers in agriculture. What is the advantage of using P^{32} (half-life = 14 days) to find out how rapidly a plant takes up phosphate fertilizer?

What the students ought to discover is that only a very tiny amount of radioactive phosphate is needed: so little that it could not be detected with the finest analytical balance. Thus, analysis becomes a much simpler task, depending on the counting rate of the detection equipment and the half-life of the tracer atoms.

Brief Descriptions of Learning Materials

SUMMARY LIST OF UNIT 6 MATERIALS

Experiments
E6–1 Random Events
E6–2 Range of α and β Particles
E6–3 Half-Life. I
E6–4 Half-Life. II
E6–5 Radioactive Tracers
E6–6 Measuring the Energy of β Radiation

Demonstrations
D59 Mineral Autoradiograph
D60 Naturally Occurring Radioactivity
D61 Mass Spectrograph
D62 Aston Analogue

Film Loops
L48 Collisions with an Object of Unknown Mass

READER ARTICLES
R1 *Rutherford*
by Charles P. Snow
R2 *The Nature of the Alpha Particle*
by Ernest Rutherford and T. Royds
R3 *Some Personal Notes on the Search for the Neutron*
by Sir James Chadwick
R4 *Antiprotons*
by Owen Chamberlain, Emilio Segré, Clyde E. Wiegand, and Thomas J. Ypsilantis
R5 *The Tracks of Nuclear Particles*
by Herman Yogoda
R6 *The Spark Chamber*
by Gerard K. O Neill
R7 *The Evolution of the Cyclotron*
by Ernest O. Lawrence
R8 *Particle Accelerators*
by Robert K. Wilson
R9 *The Cyclotron as Seen by . . .*
by David C. Judd and Ronald G. MacKenzie
R10 *CERN*
by Jeremy Bernstein
R11 *The World of New Atoms and of Ionizing Radiations*
by V. Lawrence Parsegian *et al.*
R12 *The Atomic Nucleus*
by Rudolf E. Peieres
R13 *Power from the Stars*
by Ralph E. Lapp
R14 *Success*
by Laura Fermi
R15 *The Nuclear Energy Revolution*
by Alvin M. Weinberg and Gale Young
R16 *Conservation Laws*
by Kenneth W. Ford
R17 *The Fall of Parity*
by Martin Gardner
R18 *Can Time Go Backward?*
by Martin Gardner
R19 *A Report to the Secretary of War*
by James Franck, Donald J. Hughes J. I. Nickson, Eugene Rabinowitch Glenn T. Seaborg, Joyce C. Stearns and Leo Szilard
R20 *The Privilege of Being a Physicist*
by Victor F. Weisskopf
R21 *Calling All Stars*
by Leo Szilard
R22 *Tasks for a World without War*
by Harrison Brown
R23 *One Scientist and His View of Science*
by Leopold Infeld
R24 *The Development of the Space–Time View of Quantum Electrodynamics*
by Richard P. Feynman
R25 *The Relation of Mathematics to Physics*
by Richard P. Feynman
R26 *Where Do We Go from Here?*
by Arthur E. Ruark

Sound Films (16 mm)
F47 Discovery of Radioactivity
F48 U-238 Radioactive Series
F49 Random Events
F50 Long Time Intervals
F51 Isotopes
F52 The Linear Accelerator
F53 Positron–Electron Annihilation
F54 Principles of Nuclear Fission

Transparencies
T40 Separation of α. β γ Rays
T41 Rutherford s Particle "Mousetrap"
T42 Radioactive Disintegration Series
T43 Radioactive Decay Curve
T44 Radioactive Displacement Rules
T45 Mass Spectrograph
T46 Chart of the Nuclides
T47 Nuclear Equations
T48 Binding Energy Curves

Quantitative measurements can be made with film loops marked (Lab), but these loops can also be used qualitatively.

L48 COLLISIONS WITH AN OBJECT OF UNKNOWN MASS

Elastic collisions between balls of appropriate relative masses illustrate Chadwick's discovery of the neutron (Lab).

Note: A fuller discussion of this *Film Loop* and suggestions for its use will be found in the section of this *Resource Book* entitled Film Loop Notes.

NonProject Physics Loops

L6-1 RADIOACTIVE DECAY

The assembly of a scintillation detector is shown. Samples of Cu-64 and Mn-56 are placed in position. Gamma ray spectra are displayed. Radioactive decay of Cu-64 (half-life: 12.84 hr) and Mn-56 (half-life: 2.56 hr).

Distributed by the Ealing Corporation, Cambridge, Mass. (8-mm loop, 4 min, 55 sec)

L6-2 THOMSON'S POSITIVE RAY PARABOLAS

Mostly animation. After showing the original apparatus, a beam of positive ions (Ne-20) passes through the cathode and strikes the screen. The beam is deflected by electric plates producing a vertical line on the screen. The magnetic field produces a horizontal line. When both fields are applied and the magnetic field is changed, a parabola is produced.

Distributed by Encyclopaedia Britannica Films Inc., 1150 Wilmette Avenue, Wilmette, Ill. (8-mm loop, color, 3 min, 25 sec, No. 20210)

L6-3 ASTON'S MASS SPECTROGRAPH

The film opens with a shot of Aston's original mass spectrograph, introduces electric plates, passes a deflected beam through a magnetic field, and also introduces a photographic plate.

Distributed by Encyclopaedia Britannica Films Inc. (8-mm loop, color, 2 min, 20 sec)

L6-4 NUCLEAR REACTIONS: CHAIN REACTION AND CONTROLLED CHAIN

Animated. Shows various stages of a chain reaction and of a controlled chain reaction around a U-235 nucleus cluster.

Distributed by Encyclopaedia Britannica films, Inc. (8-mm loop, color, 2 min, 35 sec, No. 20206)

L6-5 CRITICAL SIZE (NUCLEAR REACTIONS: CRITICAL SIZE)

Animated. Demonstrates the importance of critical mass in the creation of a nuclear reaction similar to that taking place in an atomic explosion.

Distributed by Encyclopaedia Britannica Films Inc. (8-mm loop, color, 2 min, 10 sec, No. 20207)

16-mm SOUND FILMS

F47 DISCOVERY OF RADIOACTIVITY

Color, 15 min, International Film Bureau.

Presents an historical survey of progressive developments leading to the present knowledge of radioactivity. Includes the discovery work of Roentgen, Becquerel, Curie, Elster, Geitel, and Rutherford.

F48 U-238 RADIOACTIVE SERIES

B & W, McGraw-Hill.

The film traces the various stages in the decay of U-238 to stable lead. Alpha emission and the statistical nature of the process are emphasized. A brief mention is made of other radioactive series.

F49 RANDOM EVENTS

B & W, 31 min, Modern Learning Aids.

This film shows how the overall effect of a very large number of random (unpredictable) events can be very predictable. Several unusual games can be played to bring out the statistical nature of this probability. The predictable nature of radioactive decay is explained in terms of what is shown.

F50 LONG TIME INTERVALS

25 min, Modern Learning Aids.

A discussion of the significance of long time intervals with a detailed description of radioactive dating arriving at an estimate for the age of the earth.

F51 ISOTOPES

B & W or Color, 15 min, McGraw-Hill.

This film shows uranium being separated into two isotopes: U-238 and U-235. It explains how J. J. Thomson first demonstrated the existence of isotopes and how Aston developed the first mass spectrometer. It then shows two methods of separating isotopes and concludes by illustrating the uses of radioisotopes.

F52 THE LINEAR ACCELERATOR

B & W, 12 min, McGraw-Hill.

This film introduces the theory of nuclear transmutations and the production of hard X rays with laboratory accelerated particles. It shows the development and techniques from the original Cockcroft and Walton experiments up to the most recent traveling-wave linear accelerator, the design and underlying theory of which are described in detail.

F53 POSITRON–ELECTRON ANNIHILATION
B & W, 27 min. Educational Services, Inc.

Using brief demonstrations and emphasizing conservation of energy, the film "proves" the annihilation and shows the two 0.5-MeV γ rays moving in opposite directions.

F54 PRINCIPLES OF NUCLEAR FISSION
Color, 10 min, McGraw-Hill.

After considering the historic and the modern conceptions of the structure of the atom, the film shows diagrammatically the relation of the basic particles: electrons, protons, and neutrons. It describes in detail how bombarding neutrons cause fission in U-235 atoms and the production of chain reactions. The film then deals with the graphite nuclear reactor, showing methods of controlling the reaction in a nuclear reactor and relating this to the production of electricity.

Also see the notes on the 16-mm sound films, "People and Particles," "Electron Synchrotron," and "The World of Enrico Fermi" in Unit 1 of this *Resource Book*.

TRANSPARENCIES

T40 SEPARATION OF α, β, γ RAYS
Observed deflections of the emanations from a radioactive source are shown in the presence of a magnetic field.

T41 RUTHERFORD'S PARTICLE "MOUSETRAP"
A simplified detail of the apparatus used by Rutherford and Royds in 1909 to show that the α particle is a doubly ionized helium atom is presented along with spectra observed during the experiment.

T42 RADIOACTIVE DISINTEGRATION SERIES
The radioactive series uranium, radium, thorium, actinium, and neptunium are presented for completion by the teacher and students.

T43 RADIOACTIVE DECAY CURVE
A number of overlays displaying sample data for a radioactive element and its accumulating "daughter" atoms leads to the half-life concept.

T44 RADIOACTIVE DISPLACEMENT RULES
Three types of radioactive decay (α, $\beta-$, $\beta+$) are presented in a visualized "before-and-after" sequence as well as in generalized and specific equation form.

T45 MASS SPECTROGRAPH
A schematic diagram of the mass spectrograph with its velocity selector and mass-determining sections are shown.

T46 CHART OF THE NUCLIDES
A chart of the stable and unstable radioactive isotopes is presented with other pertinent nuclear information.

T47 NUCLEAR EQUATIONS
Visualizations and equations for important nuclear reactions (the first artificial transmutation, the discovery of the neutron, and the mass–energy relation) are presented.

T48 BINDING ENERGY CURVES
Two plots, one of the total binding energy in thousands of electron volts versus the number of nucleons, and another of the average binding energy per nucleon versus number of nucleons, are presented.

Demonstration Notes

D59 MINERAL AUTORADIOGRAPH
Autoradiographs played an important role in early discoveries of radioactivity. For example, in 1895, during the set of experiments in which Roentgen discovered X rays, he developed a photographic plate that had been accidentally exposed while lying near an apparatus that emitted X rays. Later, while attempting to expand one part of Roentgen's work, Becquerel discovered natural radioactivity when he developed a photographic plate that had been exposed to the phosphorescent substance potassium-uranyl sulfate, under conditions in which Becquerel only expected a weak image. Autoradiographs are still used for work with tracers and can provide some quick and useful evidence of radiation.

This demonstration of the "Becquerel effect" is designed as an introduction to radiation and should be used as the first piece of evidence of spontaneous radiation and its effects. The demonstration might even be done before the first reading in the *Text* is assigned. The techniques used here are also used later in a tracer experiment on plant growth.

Equipment
Polaroid sheet film, and film roller or camera back for developing sheet film. (Better results are obtained using a thick emulsion X-ray film, but this requires darkroom supplies for development.) We encourage those who have developed films to do this experiment by wet-method using a darkroom, so that students will see the procedure at least once.

Kit of six samples of materials, three of which are radioactive. It is also desirable to have a watch with a radium fluorescent dial.

Procedure
The specimen samples should be placed on the film and left for over 48 hr. A watch, if it is a strong source, needs less than 12 hr exposure. A 50-μc source of Ti^{204} will produce a noticeable mark in 15 min. After exposure, develop the film, and raise questions about the cause of the exposed areas and how the exposing rays passed through the paper wrapper covering the film.

D60 NATURALLY OCCURRING RADIOACTIVITY

It will probably come as a surprise to most students that even very "ordinary" things can be radioactive. Radioactive matter does not have to be artificially made, nor is uranium the only naturally occurring radioactive element.

Natural potasium contains three isotopes: K^{39} and K^{41}, which are both stable, and K^{40}, which decays by $\beta-$ emission to stable Ca^{40}. The approximate natural abundances of the three isotopes are K^{39}, 93.1%; K^{40}, 0.01%; K^{41}, 6.9%. The half-life of K^{40} is 1.3×10^{9} yr. A straightforward calculation (based on the natural abundance, half-life, and Avogadro's number) shows that there should be about 10^{6} disintegrations per minute in 1 kg of naturally occurring potassium chloride. Of course, not all the emitted β particles will ever be counted, but the calculation certainly indicates that with suitable geometry a count significantly above background should be obtained. The β radiation has an energy of 1.32 MeV and can be counted with the *Project Physics* GM setup.

Make sure that the KCl is as close to the window of the tube as possible. A count of 40 to 60 per min (two to three times background) should be obtained. It is not necessary actually to count to appreciate the difference.

Connect the counter system to an amplifier and speaker that will let the whole class hear or see the pulses. Demonstrate the background level before and after the KCl count.

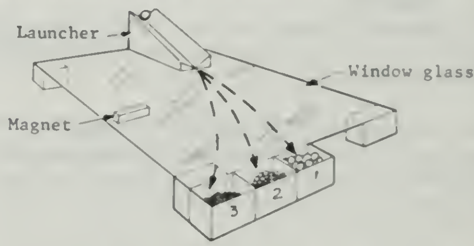
It is a good idea to use "Analytically Pure" KCl, to bring home the point that the radioactivity is not an impurity, but that all naturally occurring potassium contains a small amount of the radioactive isotope.

Discussion points

1. The material contains less than 0.01% of radioactive material. (The total amount is actually about half this amount because KCl is roughly half potassium, half chlorine, and the chlorine is not radioactive.) It has a half-life of about 10^{9} yr. The count rate is about one disintegration per second. These numbers give us a feeling for the number of atoms present in the sample or alternatively for the size of an atom. In 10^{9} yr the rate will be down to about one every 2 sec, and taking the "average" decay rate to be about one per sec, $1 \times 60 \times 60 \times 24 \times 365 \times 10^{9}$ atoms will have decayed. This number (of the order of 10^{18}) is about half of 0.01% of the total number of atoms present and close enough to the Geiger tube so that the β particles they emit reach it.

2. Ask students with an interest in chemistry these questions: If potassium salts always contain K^{40}, and K^{40} decays to Ca^{40}, how can any potassium compound ever be obtained free of calcium? And if a calcium free sample were obtained by chemical separation, would it remain calcium free? How quickly would it become contaminated? (About 0.0025% Ca^{40} after 10^{9} y!)

D61 MASS SPECTROGRAPH

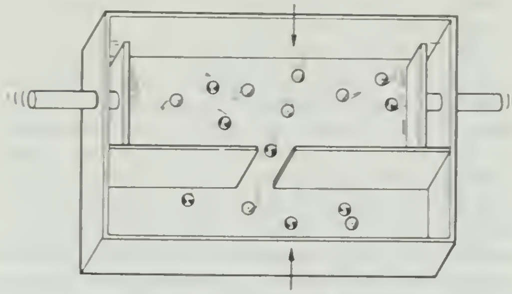


To construct the model of the mass spectrograph mount a piece of window glass horizontally and place a strong magnet under it, close to the glass. For the inclined "launcher," fold several index cards and attach them to the glass with rubber cement. When ball bearings of various sizes are launched from the incline to roll across the glass along a path that passes near the magnet, the larger bearings are deflected less than the smaller ones. If boxes are placed at the edge of the glass, as shown in the diagram, each will catch bearings of a different given size (or range of sizes). Adjustments can be made by changing the position of the magnet. The bearings can be launched in any order; however, if the launching rate is too rapid the fields induced in the bearings will interfere with the paths of consecutive bearings

If some students wish to build such a model its inadequacies should be thoroughly discussed. Does the model have a velocity selector? No, but a single calculation shows that bearing velocities at the bottom of the ramp are constant: $mgh = \frac{1}{2}mv^2$, $v = \sqrt{2gh}$. Is the deflecting force similar to that in the mass spectrograph? No since the qvB force in the mass spectrograph exists in discrete multiples of the *charge* on the particle, and as a function of v; while here it is a function of the ferromagnetic mass of the bearing and independent of v.

D62 ASTON ANALOGUE

An analogue of Aston's porous plug isotope separation might be constructed by interested students as follows: Hand operated pistons at the sides of a box generate fairly random motion of the contained marbles and steel bearings (of the same size). A hole in one of the other sides of the box allows the "molecules" to escape and should favor the lighter, faster-moving marbles. (This analogue has never been constructed and may not work. If it does not, discuss why not.)



Experiment Notes

E6-1 RANDOM EVENTS

Many phenomena occur in a completely random way. The rolling of dice and the breakdown of unstable atomic nuclei are two convenient examples for laboratory study.

This group of experiments should heighten students' appreciation for three important ideas:

1. As well as the variation in data introduced by difficulties in the *measuring process* (for example, scale interpolation, as discussed briefly in *Experiment 1-8:* "Newton's Second Law"), the observed *quantity itself* (for example, number of radioactive nuclei disintegrating per minute) may vary from observation to observation.
2. Although the outcome of single events of this kind (roll of a die or count of radioactive breakdowns in a minute) is unpredictable, it is quite possible to get useful information about such randomly occurring events provided enough events are observed.
3. This useful information follows from the properties of the regular pattern of distribution that evolves as the number of random events grows large. The larger the number of observations, the more precise become the deductions and predictions that can be made.

Divide students into four groups so that each student does only one of the following four experiments.

Equipment

Part (a) Radioactive Analogues For this experiment students will need a large piece of graph paper, preferably about 40 × 50 cm with heavy grid marks at intervals of 2–3 cm. (This grid can easily be drawn on stiff paper by a student.) As analogues to radioactive atoms that disintegrate randomly, the students are given thin washers (not discs) of two sizes.

A "hit" or radioactive disintegration occurs when a cross of the heavy grid marks shows through the hole in the washer. The large holes include an area of 0.50 cm². If the heavy grid crosses are spaced at 2-cm intervals, an average of 12.6 washers would "hit" for each trial of 100 washers. The standard deviation (the range from the mean that would include 2/3 of the trials) will be near 3.5; that is 2/3 of the trials are expected to have between 9 and 16 hits. Enough of the washers are supplied so that several groups can simultaneously do experiments with 100 washers in each set. As the notes to the students suggest, several students of a group can count in various areas of the grid.

One hundred washers with a smaller hole are also supplied. These can be used by one group to investigate the behavior of another nucleon that has a slower rate of disintegration. Because the area of the hole is about 0.124 cm², about 3 "hits" per 100 washers should result.

Variations: Several variations can be made as analogues to other aspects of radioactive decay.

(1) If each washer that "hits" is removed, and the number of hits per trial is recorded in sequence, the results will be a "decay curve." Although results will vary between runs with washers having large holes 50 hits, leaving 50 on the grid will occur in about 8 trials (the half-life) and 75 hits (leaving 25 on the grid) will occur in about 16 trials (twice the half-life).

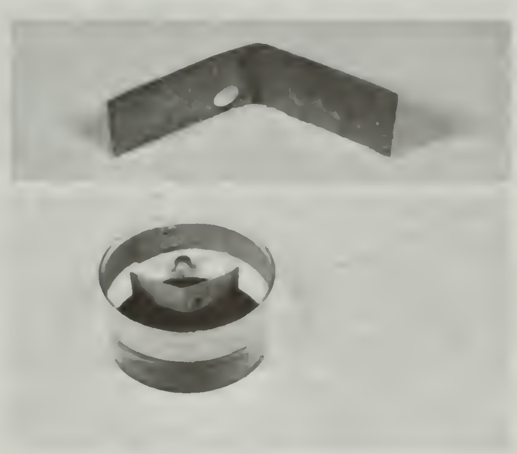
(2) Since most disintegrations result in daughter elements that are themselves radioactive but have different half-lives, a small-hole washer can be substituted for each large-hole washer that "hits." If a small-hole washer "hits," it should be removed or

replaced by a solid slug (an inert end product). After only 7 to 8 trials, nearly half of the washers will be those with small holes, and these will disappear very slowly. A graph of the number of "hits" against the sequence of trials will show an initial rapid decline in "hits" and then a flattening out. Such a curve occurs for mixtures of radioactive nuclides of differing half-lives.

The standard deviation of a distribution and the standard error of a mean are introduced to acquaint students with the definitions and terminology used to describe the spread of results around a statistical mean. In the exercise, only a rough approximation to the numerical values is sought; the idea is more important than the number.

Part (b) Dice Before class, put a large spot on one side of each of the 20-sided dice with the marking pen provided. Provide a container about the size of a shoe box for shaking the dice. It may be necessary to muffle the sound of shaken dice to prevent interference with the students who are listening to Geiger counter "clicks."

Part (c) Cloud chamber Prepare the cloud chamber by placing the α source inside and putting a small barrier nearby as shown in the following photos. This can be a piece of cardboard with a small hole in it, folded to form a V so it will stand on edge. The hole should be to the side of the needle. Relatively few particles emerge parallel to the needle from its end. The size of the hole must be determined by trial; for ease in counting, tracks should appear beyond the hole in the barrier at not more than about one per second.



Moisten the felt or paper ring in the chamber with alcohol and place the chamber on a slab of dry ice. If dry ice is not available, aim the nozzle of a CO_2 fire extinguisher at a pad of cotton or other insulator and discharge the extinguisher at it briefly. The resulting CO_2 "snow" should operate the chamber for several minutes before it must be replenished. Make sure your fire extinguisher does contain CO_2 (and not foam, for instance!) before trying this.

Typical data table for a cloud chamber

number of tracks observed in 1 min (n)	number of times observed (frequency) (f)	total number of events observed ($n \times f$)
0		0
1		0
2		0
3		0
4	|	4
5	||	10
6	|	6
7	~~||||~~	35
8	||||	32
9	~~||||~~ ||||	81
10	~~||||~~ ~~||||~~	70
11	|||	33
12	||||	48
13	||||	52
14	||||	56
15	|	15
16		0
17	|	17
18	||	36
19		0
20	|	20
21		0
22		0
23		0
24		0
25	|	25
	50	540

Mean, $\bar{n} = \| \frac{540}{50} \cong \|$

To observe, direct a fairly bright beam of light horizontally across the chamber. A flashlight, small spotlight, the light source from Millikan apparatus, or a slide projector will do.

If *E6-2* ("Range of α and β Particles") will be done later, you can have students measure the source-to-hole distance and the area of the hole. They will then have all the data they need for the first half of the range experiment.

The counts of α particles in the diffusion cloud chamber may be adversely affected by several factors.

1. It is difficult to maintain a constant temperature gradient in the cloud chamber for any length of time. The dry ice continually disappears and the alcohol evaporates from the blotter and may trickle down the sides of the chamber. The light source tends to warm the chamber. Due to all these slow changes, the conditions for observing tracks probably deteriorate during the experiment.
2. The static electric charge on the plastic container slowly changes, altering the clarity (fuzziness) of the tracks unless you rub the cover occasionally with a clean dry cloth.
3. If a flashlight is used, its intensity slowly decreases, also decreasing the probability of seeing all the tracks in a given region.

Part (d) Geiger tube Determine the mean background count rate for your Geiger tube before class. Then choose a time interval for the experiment such that students will observe from 5 to 10 "clicks" per time interval when they are obtaining their data. In order to get a reasonable amount of data in one class period, you may need to increase the count rate by placing a radiation source at such a distance from the tube that 5–10 counts are recorded in about 15 sec. On the other hand, the background rate may already be as high as this in some locations.

Obtaining the data

Assign students to teams according to the number of sets of apparatus you have.

If possible, have the recorder for each team display the data on the chalkboard or a large sheet of paper using the kind of table suggested in the *Handbook*. The entire class can then watch the developing regularity of the patterns of numbers.

Analyzing the data

Stress the point that there is no "correct" histogram. But the larger the number of observations, the more closely the results approach the pattern predicted by probability theory.

Notice that the sample histogram is lopsided. This will always occur when the mean value (11 in this case) is small. Moreover, when the mean is as small as this, the "mean ± $\sqrt{\text{mean}}$" range will enclose more nearly three-quarters of all the observations as the number of data grows very large.

The fractions of all observations included in "mean ± $\sqrt{\text{mean}}$" range for various mean values are given in the following table. This table refers to the theoretically predicted distribution, not to actual results based on a limited sample.

In the table it can be seen that the fraction of observations included in the "mean ± $\sqrt{\text{mean}}$" range decreases toward two-thirds as the mean value grows large.

$\overline{n}$	Fraction within $\overline{n} \pm \sqrt{\overline{n}}$
4	0.8166
9	0.7600
16	0.7413
25	0.7295
36	0.7217
100	0.7080

The fact that the fractions that students report vary from these values should not cause concern since the tabulated fractions are approached only for very large samples. Students doing the present experiment gather so little data, even in several class periods, that the fraction of their results in the range, "mean ± $\sqrt{\text{mean}}$," will fluctuate rather sharply above and below the "two-thirds" figure mentioned. And as pointed out in the *Handbook*, the range "mean ± $\sqrt{\text{mean}}$" is difficult to interpret when $\sqrt{\text{mean}}$ is not a whole number. So for this experiment the precision implied by the table is quite irrelevant.

Strictly speaking, from the fact that the value of a single count = mean ± $\sqrt{\text{mean}}$ (with ⅔ probability), it follows that mean = count ± $\sqrt{\text{mean}}$ (with ⅔ probability). But if one is trying to estimate the mean, this formula is not much help since $\sqrt{\text{mean}}$ is obviously unknown. The expression mean = count ± $\sqrt{\text{count}}$ given in the *Handbook* is used instead.

Students who are interested in probability theory will want to explore this matter further. Several excellent books on probability are available, including *Lady Luck*, by Warren Weaver, *Facts from Figures*, by M. J. Moroney; and *Mathematics for the Million*, by Lancelot Hogben. These books explain the various kinds of distributions to be expected from observations of random behavior. The distribution that applies to this experiment on random events is called the *Poisson* distribution. Stated in formal terms, the probability of observing n events per unit time when the mean of a great number of observations is $\overline{n}$ is given by

$$P(n) = \frac{\overline{n}^n e^{-\overline{n}}}{n!}$$

It has been found by experiment that measurements of radioactive decay fit this distribution very well.

The theoretical distribution for 1,000 observations given in the *Handbook* was obtained by setting $\overline{n}$ = 6 and letting n take the values 0, 1, 2 ... in the above expression.

Answers to questions

1.–2. Student answers
3. 1/20, which means that, on the average, 6 marked faces will appear.
4. 160 + 160 + 140 = 460 trials.
5. 0.460
6. 6 ± 1 includes 460 or almost half the observations. 6 ± 2 includes about 700 or a little over ⅔ of the observations.
7. About 700 trials or 0.700 of the total.
8–9. Student Answers
10. In 1,000 min you would expect 10,000 ± 100 counts. Dividing by 100 gives 10 ± 0.1 counts/min, which is a "⅔ range" of 1% of the average value.

11. As the size of the sample increases, the predictability of random events (like fires) improves. The number of fires predicted will depend also on such variables as rainfall, population density, character of the neighborhood (industrial, residential, etc.), and time of year.

E6-2 RANGE OF α AND β PARTICLES

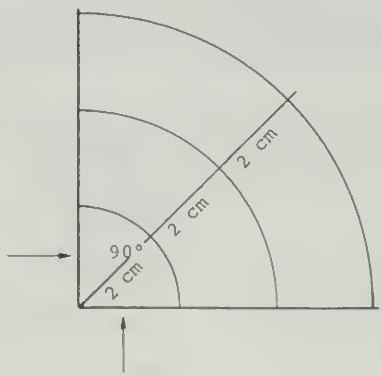
One important idea to emphasize is that different kinds of radiation, though invisible to the eye, have observably different properties and require different instruments for their measurement.

Another idea is the manner in which α and β particles lose their energy. Their loss of energy by the ionization of matter is one of the significant reasons for the effect radiation has on people and is one of their useful properties as tools for studying the structure of atoms.

There is nothing in this experiment that requires the students to have studied the properties of α and β rays. Hence this may be treated, if desired, as a "discovery" exercise.

If those students who did the random events experiment measured the size of the hole through which α particles were counted and also its distance from the source, they are almost ready to do the first half of the present experiment as an exercise in calculation. They already have their data except for a measurement of the range of the α particles.

In addition to the equipment listed in the *Handbook*, you or the students need to make a distance scale on the bottom of the cloud chamber by scratching the black paint on the underside in the following pattern:



Alpha particles

Students may observe α tracks of energies as high as 6 MeV, although they should notice that the length of tracks does not vary much. In general, α particle energies from naturally radioactive sources vary from 4 to 8 MeV; but for a particular source, they cluster closely around one principal energy. One source often supplied with cloud chambers is radium DEF, which emits α particles of 5.3 MeV.

The following typical calculations involving α particles are based on the data tabulated in *E6-1* ("Random Events"). It was shown there that the mean count was 11 α particles/min emerging through the small hole.

From the geometry of the cloud chamber arrangement as described in the *Handbook* (Fig. 6-9) we can find the total number of α particles (true count) by proportion.

$$\frac{\text{true count}}{11/\text{min}} = \frac{4 \times 3.14 \times 2.5 \times 2.5\ \text{cm}^2}{0.5\ \text{cm}^2}$$

$$\text{true count} = 1{,}727/\text{min}$$

These particles are observed to have a range of about 3.8 cm and hence (from *Handbook*, Fig. 6-8) an energy of 5.3 MeV each.

The total energy of all 1,727 particles/min will therefore be

$$\frac{1{,}727}{\text{min}} \times 5.3\ \text{MeV} = \frac{9{,}150\ \text{MeV}}{\text{min}}$$

Converting this to joules

$$\frac{9{,}150\ \text{MeV}}{\text{min}} \times 1.6 \times 10^{-13}\ \frac{\text{J}}{\text{MeV}} = 1.46 \times 10^{-9}\ \frac{\text{J}}{\text{min}}$$

or to calories

$$1.46 \times 10^{-9}\ \frac{\text{J}}{\text{min}} \times \frac{1\ \text{cal}}{4.18\ \text{J}} = 0.349 \times 10^{-9}\ \frac{\text{cal}}{\text{min}}$$

To generate the necessary 100 cal would require

$$\frac{100\ \text{cal}}{0.349 \times 10^{-9}\ \text{cal/min}} \simeq 5.5 \times 10^{5}\ \text{yr}$$

which is why calorimeters are not used as radiation detectors, and why very much "hotter" sources are in nuclear power plants.

As a final note, it is worth observing that the luminous paint on pre-World War II watch dials (and on some new ones too) emits α particles of a wide variety of ranges from the accumulated decay products of the original radium.

Beta particles

The curve of β particles counted by the Geiger tube against the thickness of absorber looks approximately like an exponentially decreasing one. If all the β particles could penetrate the same thickness of absorber, the curve would be flat up to the given thickness, then drop sharply off to the background count. Curves made with different absorbers should have the same form, but with differing half-value thicknesses; that is, the thickness of absorber required to reduce the count rate to just half what it was when no absorbers were between the source and the Geiger tube.

If the curve is plotted on semilog paper with counting rate along the logarithmic axis, the graph is nearly a straight line for high counting rates and thin absorbers. Then as the absorber thickness increases, the rate of decline slows, which means that the straight line curves to become more nearly horizontal. It is as if additional layers of absorber were decreasingly successful in absorbing additional β particles.

The explanation for this effect is quite interesting. In Unit 4 it was explained that an accelerating electric charge must radiate energy. Beta particles are electric charges and they are being accelerated (negatively) in the absorbing material. The electromagnetic radiation they emit, called "bremsstrahlung," is being detected by the Geiger counter and added to the counts of the β particles. The thicker the absorber, the larger is the fraction of counts due to bremsstrahlung.

Answers to questions

Answers all involve the properties of the particular sources used.

E6-3 HALF-LIFE. I

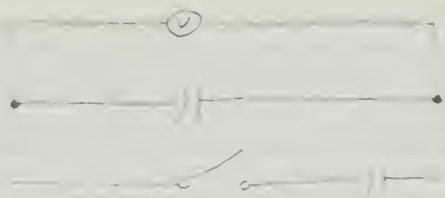
Exponential growth and decay occur in so many natural processes that it is important for students to study examples in the laboratory. Four kinds of exponential decay are described in this experiment; students may be able to think of other examples to investigate on their own. The results of students' observations are examined graphically, and knowledge of the mathematics of exponents and logarithms is not assumed. Teachers may choose to have students use semilog paper for their graphs or to plot logarithmic values if this seems appropriate.

Preparation

Part A. Random discs By use of "chips" like washers thrown on large graph paper, a radioactivity analogue is created. "Hits," equivalent to radioactive disintegrations, occur when selected points on the graph paper are seen through the central holes of the washers. Washers producing "hits" are removed to simulate radioactive decay.

Part B. Twenty-sided dice (optional: eight-sided dice) As in *E6-1* ("Random Events"), a shoebox is convenient for shaking the dice. You can save time by having the dice marked before class. The *Handbook* suggests marking two sides of each die. This means that twice as many marked sides show after each shake, and the experiment proceeds more rapidly. One side will already be marked from *E6-1*. Mark a second side in another color so that experiments requiring a one-in-twenty probability can still be done.

Part C. Electric circuit Refer to the circuit diagram (top). The capacitor suggested by *Project Physics* has a capacitance of 600 μF. A high value of capacitance is necessary to give a reasonably long time constant. Closing the switch causes the battery to charge the capacitor; when the switch is opened, the capacitor discharges through the voltmeter. The difference in potential V across the capacitor is a function of the time t that has elapsed after the switch is opened, the capacitance C, and the resistance R of the voltmeter:



$$V = V_0 e^{-\frac{t}{RC}}$$

where V_0 is the initial value of V, at $t = 0$.

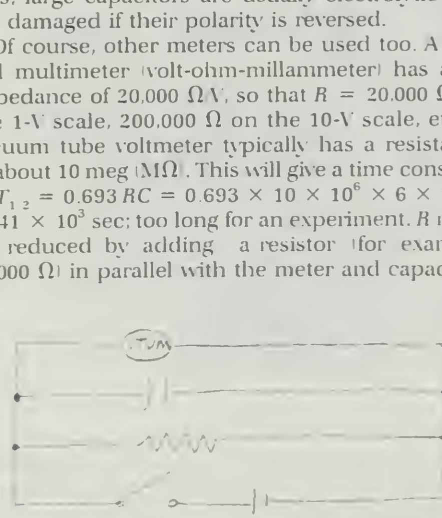
The quantity RC is called the *time constant* of the circuit. When R is in ohms and C in farads, the dimensions of the product are seconds. The quantity $\frac{1}{RC}$ in this expression is analogous to the quantity λ, the radioactive decay constant. The "half-life" of the circuit is

$$T_{1/2} = \frac{0.693}{\lambda} = 0.693\ RC$$

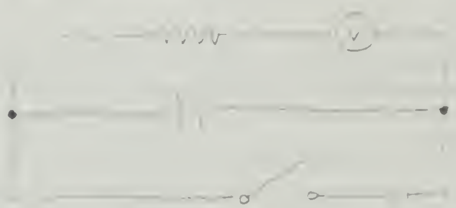
It will not be necessary to go through this calculation with students. However, the information given here will enable you to select suitable values for R and C. For example, a 2.5-V dc meter would typically have a resistance of 10,000 Ω/V; that is, a resistance of $25 \times 10^3\ \Omega$. With $C = 6 \times 10^{-3}$ F, this gives $RC = 150$ sec and $T_{1/2} = 104$ sec.

A simple way to charge the capacitor to 2.5 V (or any other desired voltage below 6 V) is to use the *Project Physics* amplifier power-supply unit. Simply set the dc offset control to the required voltage and connect the capacitor across the output. Do not exceed the maximum voltage indicated on the capacitor, and be sure to observe the polarity markings; large capacitors are usually electrolytic and are damaged if their polarity is reversed.

Of course, other meters can be used too. A typical multimeter (volt-ohm-millammeter) has a dc impedance of 20,000 Ω/V, so that $R = 20{,}000\ \Omega$ on the 1-V scale, 200,000 Ω on the 10-V scale, etc. A vacuum tube voltmeter typically has a resistance of about 10 meg (MΩ). This will give a time constant of $T_{1/2} = 0.693\,RC = 0.693 \times 10 \times 10^6 \times 6 \times 10^{-3} = 41 \times 10^3$ sec; too long for an experiment. R must be reduced by adding a resistor (for example 10,000 Ω) in parallel with the meter and capacitor.



On the other hand, if you have to use a meter with a low resistance, you will need to put additional resistance in series with the meter. For example, if the meter resistance is 1,000 Ω and C = 6000 μF, RC = 6 sec, which is much too short a time. Putting an additional 9,000 Ω in series with the meter makes R = 10,000 Ω and RC = 60 sec. The voltmeter reading will now be only a fraction (in this instance one-tenth) of the total voltage across the capacitor, but it will still vary with time in the same way. To get a nearly full-scale voltmeter reading when the capacitor is charged, the voltage across the capacitor must be 10 times the full-scale reading of the meter.



It may be more convenient (and certainly the analogy to radioactivity is better served) if you replace the voltmeter in *Handbook* Figure 2 (a) and (b) by a microammeter in *series* with the resistor and have students plot currents against time. The character of the resulting decay curve is essentially the same as before.

*Part D. Short-lived radioisotope** This experiment requires quite a bit of advance preparation by the instructor. However, students should find working with a short-lived isotope a rewarding experiment, well worth a little extra effort. The advantages of this particular method are:

1. The starting material, thorium nitrate, can be purchased from chemical suppliers at any time and stored indefinitely.
2. The isotope to be used has a half-life of only 3 min, so students can observe its decay over several half-lives in one class period.
3. The procedure is relatively safe since the isotope decays quickly to a stable end product.

The experiment is based on the fact that a thorium compound contains not only thorium atoms, which are naturally radioactive with a half-life of over 10^{10} yr, but also a series of daughter atoms, as explained in the *Text*. The isotope used in this experiment, thallium-208, is separated from the mixture by selective adsorption when the dissolved salts are poured over a layer of ammonium phosphomolybdate on filter paper.

The decay of thallium (Tl) may be written

$$ {}_{81}\mathrm{Tl}^{208} \xrightarrow{\beta} {}_{82}\mathrm{Pb}^{208} $$

*The method for separating thallium-208 from thorium has been adapted from an article by John Amend in *The Science Teacher*, May 1966.

The materials to prepare in advance are:

(a) Dilute nitric acid. Dilute 25 mL of concentrated nitric acid to 200 mL with distilled or deionized water and store in a flask or bottle.

(b) Thorium nitrate solution. For each series of counts to be taken, dissolve 6 g of thorium nitrate in 12 mL of dilute nitric acid. In other words, if you expect to have the experiment performed five times in a day, dissolve 30 g of thorium nitrate in 60 mL of dilute nitric acid. However, after the solution has been poured over the adsorbing material to remove the thallium-208, the decay of other members of the thorium series very quickly replaces the thallium in the filtrate. In theory, you should be able to continue using the same solution indefinitely, simply waiting 10–15 min between successive filtrations.

(c) Ammonium phosphomolybdate. This is supplied as a yellow powder. Place it in a bottle, add distilled water, and shake vigorously. The powder is insoluble and will form a slurry.

Procedure

Part A. Random discs The materials used for this experiment are the same as those used for *E6-1*. Now, however, the washers that score "hits" are removed on each trial. A plot of the number of hits against the trials will create the decay curve. Students can observe the diminishing number of washers on the graph and anticipate that the number of hits must gradually diminish. When the results from several sets of parallel trials are pooled, the mean will be more stable than for any one series of trials. Enough trials should be made to reduce the number of hits per trial to one-fourth the number at the start of the activity. This number, which is equivalent to twice the "half-life," can be compared to the observed half-life when the hits have decreased to one-half their initial number.

By substituting a smaller washer for each one that shows a hit and is withdrawn from the board, the model simulates one nucleon disintegrating into another that is itself radioactive, but with a different half-life.

Part B. Twenty-sided dice Students are asked to shake the dice and remove those with a marked side on top. Since the dice have two marked sides, the probability that a marked side will appear on top is 1 in 10. This is analogous to a radioactive decay process with λ = 0.10 per observation. The half-life is

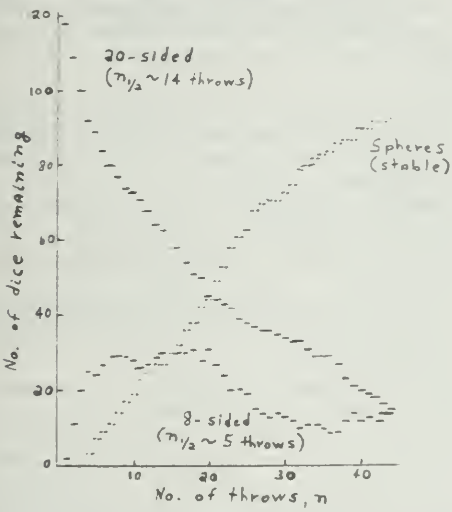
$$ T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.10} = 6.9 \text{ shakes} $$

After 7 shakes, the dice should "decay" from 120 to 60; after 7 more shakes, 30, etc.

The *Handbook* suggests alternatively that students shake the tray 5 times, find the average number of spots showing for the 5 shakes and remove that number. This is time-consuming but may be necessary if students are to be convinced that the number "decaying" is proportional to the number shaken.

The reason for plotting two graphs, one showing the number of dice removed per shake and the other the number remaining, is to show that the curves have the same shape. Averaged over many shakes, or (which amounts to the same thing) if the sample is large enough, the number of dice removed per shake is proportional to the number of dice remaining: $\Delta N \propto N$. This is the key idea of exponential decay processes and the fundamental law of radioactive decay.

A more elaborate version of this experiment, which is analogous to a radioactive decay series, follows. For this experiment the 20-sided dice should have a 1-in-20 probability of decay; ignore the second marked side. Each time a 20-sided die is removed, replace it with an 8-sided die that has one side marked. After each shake, record the number of each kind of die that has a mark on top. Then remove 20-sided dice with marked side up, replace with 8-sided dice, and replace all marked-side-up 8-sided dice with spheres (representing stable atoms). Count the number of 20-sided dice, 8-sided dice, and spheres remaining in the tray after each shake. When the resulting numbers are plotted, the graph will look like the one in the *Text* illustrating the decay of polonium-218.

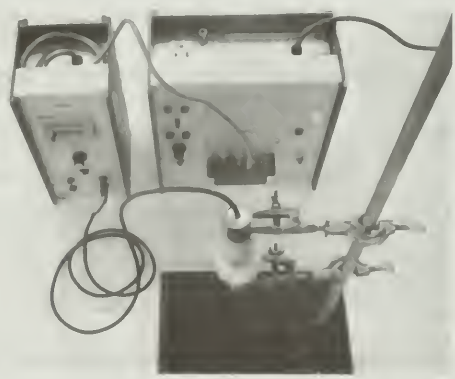


Part C. Electric circuit The amount of emphasis placed on the process involved in this part of the experiment will depend on how much students learned about simple electric circuits in Unit 4. If circuits containing resistors in series and parallel have not been studied previously, it will not be worthwhile to introduce them now. But students can still record voltmeter readings as a function of time and see that the resulting graph is a curve similar to those obtained from radioactivity and dice.

Students can look for a relationship between R and $T_{1/2}$ if they understand how to compute the resistance of resistors in series and parallel, and if the resistance of the voltmeter is known. A plot of $T_{1/2}$ versus R should be a straight line.

Part D. Short-lived radioisotope At the sink where you have set up the filter flask and pump, place the funnel on the filter flask, shake the bottle containing the slurry of ammonium phosphomolybdate, and pour the slurry into the funnel, depositing a layer of the precipitate uniformly over the filter paper to a thickness of about 1 mm. Wash this with distilled water, then pour the thorium nitrate solution slowly, a few drops at a time, over the precipitate. Wash with dilute nitric acid and let the filter pump dry it for a few seconds.

The Geiger tube recommended by *Project Physics* is an end-window tube with a diameter less than that of the funnel. If you are using this tube, it will not be necessary to remove the radioactive sample from the funnel. Instead, wrap a single layer of plastic sandwich wrap around the tube to avoid contaminating it. Then remove the top part of the funnel and insert the Geiger tube carefully into it until the window is very close to the sample but not touching it. Emphasize to students that the window of the Geiger tube is very thin and fragile.



The radioisotope that is adsorbed from the mixture of thorium daughter products is thallium-208, which has a half-life of 3.1 min. If students can count for at least 10 half-minute periods this will include about three half-lives. Again, stress the fact that an exponential curve results when equal <u>fractions</u> decay in equal time intervals.

Discussion
Students should have the opportunity to compare graphs from all four decay experiments. If students have learned about logarithms, have them make a second graph, plotting the logarithm of the quantity that decays as a function of time (or shakes). Alternatively, distribute semilog graph paper, so that students can come to appreciate the convenience of this kind of plotting. Point out how much easier it is to determine the half-life from the straight line resulting from a semilog plot.

If two isotopes with differing half-lives are mixed together, the resulting curve will be hard to interpret. In the thallium adsorption experiment, most of the activity in the sample will be thallium-208, with a half-life of 3.1 min (plus, perhaps, traces of other elements with much longer half-lives). As a result, a plot of the first 10 min (after subtracting background counts) will show a decay curve almost entirely due the the thallium. (The net count rate will level off to a steady value only slightly above zero if the separation has not been quite complete.) A semilog plot would show a very slight curve rather than a straight line.

Students are asked to apply their knowledge of the relationship $T_{1/2} = \dfrac{0.693}{\lambda}$ to find the number of dice in a very large tray, given that 50 marked dice appeared on the first shake, and that 4 shakes reduced this number by one-half. From this, we find that

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4} = 0.173$$

But λ is the fraction that "decay" per shake, so $\lambda N = 50$, where N is the number of dice in the tray. Then $N = \dfrac{50}{0.173} = 290$ approximately.

The same kind of operation is used to find the half-life of a very long-lived element such as uranium-238 ($T_{1/2} = 4.5 \times 10^{9}$ yr) or thorium-232 ($T_{1/2} = 1.39 \times 10^{10}$ yr). The total number of atoms in a sample is determined, then the number of atoms decaying per unit time, $\dfrac{\Delta N}{\Delta t}$, is measured. From this,

$$\lambda = \frac{\Delta N}{N \Delta t}, \text{ and } T = \frac{0.693}{\lambda}.$$

For example, suppose you have a sample containing 0.1 g (1.0×10^{-4} kg) of thorium-232 and observe 600 counts/min, or 10 counts/sec. You estimate from the geometry of the counting arrangement that the Geiger tube is actually counting about 2.5% of the atoms that are decaying. The number of atoms N in the sample is

$$N = 1.0 \times 10^{-4}\ \text{kg} \times \frac{1\ \text{amu}}{1.7 \times 10^{-27}\ \text{kg}} \times \frac{1\ \text{atom}}{232\ \text{amu}}$$

$$= 2.7 \times 10^{20}\ \text{atoms}$$

$$\text{Then } \lambda = \frac{\Delta N}{N \Delta t}$$

$$= \frac{\text{counts}}{\text{sec}} \times \frac{1}{2.5 \times 10^{-2}} \times \frac{1}{2.7 \times 10^{20}\ \text{atoms}}$$

$$= 1.5 \times 10^{-18}/\text{sec}$$

$$\text{and } T_{1/2} = \frac{0.693}{\lambda}$$

$$= \frac{0.693}{1.5 \times 10^{-18}/\text{sec}}$$

$$= 4.6 \times 10^{17}\ \text{sec}$$

$$= 4.6 \times 10^{17}\ \text{sec} \times \frac{1\ \text{yr}}{3.15 \times 10^{7}\ \text{sec}}$$

$$= 1.5 \times 10^{10}\ \text{yr}$$

Answers to questions

1.–4. Student answers.
5. As one curve descends, the other rises. If the two curves are plotted on the same sheet of paper, they can be added together; the resulting sum is a horizontal straight line.
6. 1/10.
7. The answers to all parts of this question should be very roughly the same, and for a very large number of throws, close to 6.9 rolls. See answer to 13 below.
8. Whatever the student answer, it should be the same for all the parts of this question.
9. As the resistance is doubled, the time is doubled, etc.
10. The answers to all parts of this question should be the same and close to 3.10 min, the half-life of thallium-208.
11. $_{81}Tl^{208}$ (thallium) is decaying with the emission of a β particle to $_{82}Pb^{208}$ (lead).
12. No, but if you plot your counts against the *logarithm* of time (or plot time on the log axis of semi-log paper), you will get a straight line if your sample contains only one radioisotope, and a curve if your sample is a mixture.
13. Since $\lambda = 0.100$, $0.100T = 0.693$, and so $T = 6.93$ rolls.

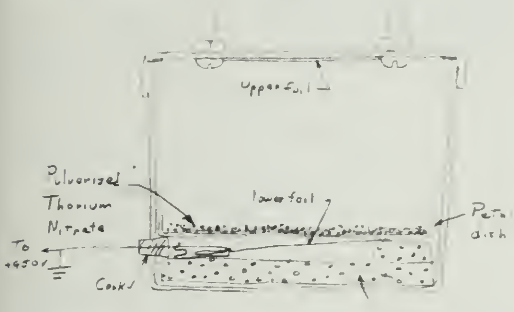
E6-4 HALF-LIFE. II

The sample used in this experiment is easier to prepare than the thallium-208 used in *E6-3*. On the other hand, it has a rather awkward half-life (10.6 hr), which means that counting should be continued over several days. Even if a count is taken early in the morning and late in the afternoon, the plot of count rate against time will have large gaps in it.

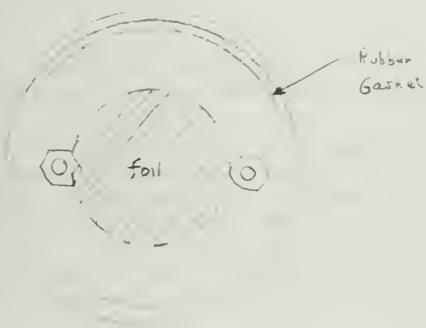
Procedure

Plastic refrigerator jars, ice cream cartons, etc. work well as containers. It is easy to make holes in the top and side with the tip of a hot soldering iron. Use a thin disc of sponge rubber at the bottom of the container and moisten it with 10–20 drops of water. (A damp atmosphere increases the amount of deposit collected on the top plate.) The top foil of aluminum wrap is held in place by the screws; the lower one is held in the alligator clip.

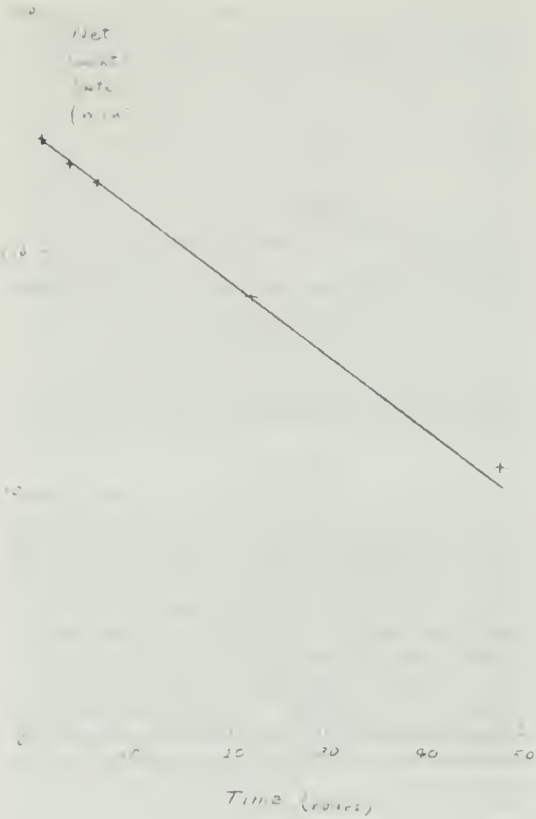
Spread about 50 g of pulverized thorium nitrate in a shallow dish (for instance, a Petri dish). No special power supply is needed for the high voltage. Use the 450-V terminal on the scaler. Make sure that the top foil is at lower potential than the bottom foil.

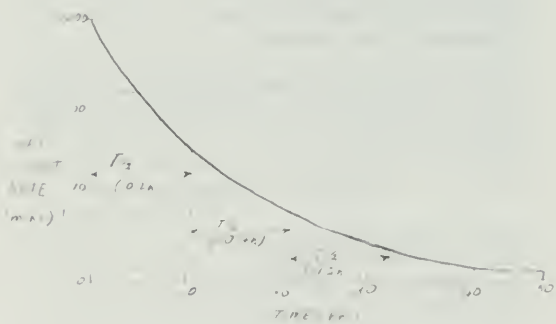


Side view



Top view



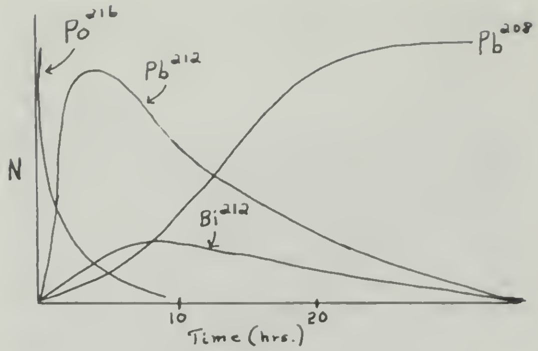


Let the apparatus stand for about 2 days to get a sample of maximum activity. Turn off the high voltage before removing the lid of the container. If you have several Geiger counters, you can cut several samples from each piece of foil; the sample need be only slightly larger than the window of the Geiger tube.

Data

Students will probably report that the sample *does* have a constant half-life (this is much easier to see on a semi-log plot), in spite of the fact that several isotopes with different half-lives are present in the sample. They should report a value of $T_{1/2} \simeq 10.5$ hr. Po^{216} decays very rapidly ($T_{1/2} \sim 0.16$ sec) to Pb^{212}. Pb^{212} has a half-life of 10.6 hr. Although there are three more radioactive daughters before the end of the series (Pb^{208}) is reached, these subsequent isotopes have half-lives much shorter than Pb^{212}. The decay of Pb^{212} is therefore the process that determines the activity of the sample. For a discussion of the activity of a sample containing several members of a decay series, see, for instance, Kaplan, *Nuclear Physics*, Addison-Wesley Publishing Co., 1963.

The student's sketch of isotope concentration versus time should look something like this:



Since $T_{1/2}$ for Pb^{212} is about 10 hr and is about 1 hr for Bi^{212},

$$\frac{\lambda Pb}{\lambda Bi} \simeq \frac{1}{10}.$$

You could make a model of this decay scheme using the multi-faced dice, as in the variation of Part B of *E6-3*. Start with 20-sided dice with one face marked ($\lambda = 1/20$) to represent the Pb^{212}. Replace each "decayed" atom by an 8-faced die with 4 faces marked ($\lambda = 1/2$) to represent the Bi^{212} atom formed. Replace "decayed Bi^{212} atoms" by balls to represent the stable Pb^{208} atoms. (Make sure you use an ink that can be removed! You will need to have fewer faces marked for other experiments and in future years.)

Answers to question 2

Initial count: 2,959 in 10 min

Background: 12 per min

$\therefore$ Net count rate $= 296 - 12 = 284$/min

Not all disintegrations are detected by the counter. Assume that for our geometry about one-quarter are detected. Then

$$\left(\frac{\Delta N}{\Delta t}\right)_o \sim 1{,}000/\text{min}$$

$T_{1/2} = 10.5$ hr (from graph)

$$\therefore \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{10.5 \times 60}\ \text{min}^{-1}$$

$$= 1.10 \times 10^{-3}\ \text{min}^{-1}$$

$$\left(\frac{\Delta N}{\Delta t}\right)_o = \lambda N_o$$

$$\therefore N_o = \frac{1}{\lambda}\left(\frac{\Delta N}{\Delta t}\right)_o \text{ atoms}$$

$$= \frac{1}{1.10 \times 10^{-3}} \times 1{,}000 \text{ atoms}$$

$$\simeq 10^6 \text{ atoms}$$

$$\text{mass of } Pb^{212} = 212 \times 1.7 \times 10^{-27} \times 10^6\ \text{kg}$$
$$= 3.60 \times 10^{-19}\ \text{kg}$$
$$= 3.6 \times 10^{-10}\ \mu\text{g}$$

Answer to discussion

For the α particle,

$$KE = \tfrac{1}{2} mv^2 = 6.8 \text{ MeV}$$
$$1 \text{ eV} = 1.6 \times 10^{-19}\ \text{J}$$
$$\therefore \tfrac{1}{2} mv^2 = 6.8 \times 10^6 \times 1.6 \times 10^{-19}\ \text{J}$$
$$= 1.1 \times 10^{-12}\ \text{J}$$
$$v^2 = \frac{2 \times 1.1 \times 10^{-12}}{m}\ (\text{m/sec})^2$$
$$m = 4 \times 1.7 \times 10^{-27}\ \text{kg}$$
$$= 6.8 \times 10^{-27}\ \text{kg}$$
$$\therefore v^2 = \frac{2 \times 1.1 \times 10^{-12}}{6.8 \times 10^{-27}}\ (\text{m/sec})^2$$
$$v^2 = 3.2 \times 10^{14}\ (\text{m/sec})^2$$
$$v = 1.8 \times 10^7\ \text{m/sec}$$

(This velocity is considerably less than 3×10^8 m/sec, so we were justified in using the nonrelativistic expression $\tfrac{1}{2} mv^2$ for kinetic energy.)

Momentum is conserved at the collision:

$$(MV)_{\text{Po atom}} = (mv)_{\alpha \text{ particle}}$$
$$\therefore (MV)^2 = (mv)^2$$
$$\frac{1}{M}(MV)^2 = \frac{1}{M} mv^2$$
$$\therefore MV^2 = \frac{m}{M}(mv^2)$$
$$\tfrac{1}{2} MV^2 = \frac{m}{M}(\tfrac{1}{2} mv^2)$$

$$\therefore \text{KE of polonium atom} = \frac{m}{M}(\text{KE of } \alpha \text{ particle})$$
$$= \frac{4}{216} \times 6.8 \text{ MeV}$$
$$= 0.12 \text{ MeV}$$

which is very much more than the ionization energy.

A note on safety

The fact that we are dealing with a radioactive *gas* in this experiment may seem to cause an additional safety hazard. A simple calculation shows that there is nothing to worry about.

The amount of radon present is determined by two factors: its rate of formation and its rate of decay.

The rate of decay is about 10^{13} greater than the rate of formation from thorium (10^{10} yr: 1 min), so the radon never builds up a high concentration.

The rate of formation is governed by the decay of the parent, thorium-232:

$$\frac{\Delta N}{\Delta t} = \lambda N.$$

For thorium-232

$$\lambda = \frac{0.693}{1.4 \times 10^{10}\ \text{yr}} = 1.58 \times 10^{-18}\ \text{sec}^{-1}$$

In a 50-g sample there are

$$N = \frac{50 \times 10^{-3}}{232 \times 1.7 \times 10^{-27}} \text{ atoms}$$

$$\therefore \frac{\Delta N}{\Delta t} = \lambda N$$

$$= 1.58 \times 10^{-18} \times 1.25 \times 10^{23} \text{ atoms/sec}$$

$$= 2.0 \times 10^5 \text{ atoms/sec}$$

One microcurie is 3.7×10^4 disintegrations/sec so the activity here is about 5 μc. The decay rate of Rn^{220} is given by

$$\frac{\Delta N}{\Delta t} = \lambda N$$

and

$$\lambda = \frac{0.693}{51.5} \text{sec}^{-1} = 1.34 \times 10^{-2} \text{ sec}^{-1}$$

$$\therefore \frac{\Delta N}{\Delta t} = 1.34 \times 10^{-2} N \text{ sec}^{-1}$$

The equilibrium concentration of radon (N) is found by setting the rate of formation equal to the rate of decay, i.e.

$2.0 \times 10^5 = 1.34 \times 10^{-2} N$

$N = 1.5 \times 10^7$ atoms

The mass of radon

$$= 220 \times 1.7 \times 10^{-27} \times 1.5 \times 10^7 \text{ kg}$$
$$= 5.6 \times 10^{-18} \text{ kg}$$
$$= 5.6 \times 10^{-9} \text{ μg}$$

Disposal of waste

At the end of the experiment the activity of the sample will be insignificant, and it can be safely discarded with the trash.

E6-5 RADIOACTIVE TRACERS

Student responses to a "blank check" experiment such as this one will be varied. You should act as resource person, giving suggestions as to where to find ideas, helping to order isotopes, and seeing that safety precautions are strictly maintained.

As a teacher, you may confront the situation of a student planning an experiment you know will not give a positive result. While negative results are very important in the advancement of science, they may overly discourage a poor student. A confident student, however, might be left alone to pursue such an experiment.

Emphasize to students the possibility of doing a variation of an experiment they read about, beginning with a hypothesis they wish to test, rather than simply repeating an experiment already done.

Students should be encouraged to peruse any literature they can find for possible ideas. *The Physics Teacher, The Biology Teacher, Senior Science, Journal of Chemical Education,* and *Scientific American* occasionally have tracer experiments. A number of useful sources are listed at the end of the student instructions, and others of interest to teachers are listed below.

To do the simple autoradiograph experiment, a radioactive source is needed. One simple possibility is a lump of uranium ore from a mineral supply company such as the Foote Mineral Company, 18 W. Chelten Avenue, Philadelphia, PA 19144; or Wards Natural Science Establishment, Rochester, NY.

Once an experiment has been chosen, you should discuss what safety precautions will be necessary.

Finally, you will need to order the isotopes students will need, as suppliers will not usually ship to minors. Choose the nearest suppliers from the supplier's list, and write well in advance in order to check on the shipping procedures. Then when the experiment is ready, the order can be placed at the advance interval specified by the supplier. For example, many suppliers ship on Friday to be received the following Monday at the specified strength.

References

In addition to the articles listed in the *Handbook*, the following books and articles may be of use to teachers.

Nuclear Science Teaching Aids and Activities, J. Woodburn and E. Obourn. U.S. Office of Education. Available at no cost from Superintendent of Documents, Government Printing Office, Washington, DC 20402.

Radioisotope Experiments for the Chemistry Curriculum, Teacher's manual. See corresponding title in student references.

Radioactive Tracer Research. M.D. Kamen, Holt, Rinehart, and Winston, Publishers, 383 Madison Avenue, New York, NY 10017 (paperback).

Isotopes in Action. D. Harper. Pergamon Publishing Co., Maxwell House, Fairview Park, Elmsford, NY 10573.

"Power from Radioisotopes," "Radioisotopes in Industry," "Radioisotopes and Life Processes," and "Radioisotopes in Medicine," a series of pamphlets for the general reader available at no cost from the Department of Energy, P.O. Box 62, Oak Ridge, TN 37830.

"Tracers," M.D. Kamen, *Scientific American*, February 1949; and "Radioactivity and Time," P.M. Hurley, *Scientific American*, August 1949. Reprints of these two articles are available from W. H. Freeman and Company, 660 Market Street, San Francisco, CA 94104.

Note also the references to articles on radiation safety reprinted elsewhere in this *Resource Book*.

Answers to questions

1. 1 μc is 37,000 disintegrations/sec; $\sqrt{37,000}$ = ±192 counts
2. 1% of the 1μc source is 370 counts/sec; $\sqrt{370}$ = ±19.2 counts
3. Its relatively small size intercepts only a small fraction of the emitted particles and even some traveling in the right direction fail to penetrate the counter wall.

E6-6 MEASURING THE ENERGY OF β RADIATION

The experiment, rather fully described in the *Handbook,* involves a crude but illustrative β-ray spectrometer. As advised in the *Handbook,* exposures of several days are necessary when Polaroid film is used. Do not forget to remove the magnets so that the spot of undeflected electron impacts is recorded. Because of the range of electron energies the image from deflected electrons will be a smear. Also, some spread of image will occur even in the undeflected spot because the electron stream is not highly collimated.

Film Loop Notes

L48 COLLISIONS WITH AN OBJECT OF UNKNOWN MASS

It is intended that this film be used to encourage interest in Chadwick's experiment. Obtaining the numerical value of the unknown mass is not as important as the experience of finding it by an ingenious indirect method.

In the slow-motion scenes, the collisions have been planned to occur near the left of the frame, so that there can be no attempt to measure the incoming ball's velocity v.

The iron balls used for the film were hardened by heat treatment after holes for the suspension strings were drilled. Evidently the treatment was insufficient; the collisions are not very elastic. As shown in the *Handbook,* the coefficient of restitution e need not be known, but it should be the same for the two events. However, in case 1, e is about 0.44, and in case 2, e is about 0.64. Thus, the assumption of constant e is not correct for the actual filmed experiment. The numerical value of m found by the student will therefore be in error. However, this point need not be stressed, since the film's main purpose is to illustrate Chadwick's indirect method of finding the neutron's mass. In Chadwick's neutron experiment, the target nuclei did not store any potential energy, and the collisions were indeed perfectly elastic, with $e = 1$ in each case.

From measured values of V_1 and V_2 and the given values of M_1 and M_2, the unknown mass m turns out to be about 460 g. For most students, this should be the end of the experiment: They have determined an unknown mass by indirect measurements, using a method entirely analogous to Chadwick's historic experiment.

For the teacher's background information, m was actually 449 g, with 1% of the mass M_1 of the smaller target ball. Also, the velocity v_1 *can* be measured from the film (in violation of the spirit of Chadwick's experiment). If m and v_1 are known, we can find v using Eq. (2), and then we can find v_2 using Eq. (3). (It turns out to be negative.) In this way, all the velocities become known, and the coefficients of restitution can be found:

$$e_1 = \frac{(V_1 - v_1)}{v} = 0.44$$

$$e_2 = \frac{(V_2 - v_2)}{v} = 0.64$$

It is this difference between the elastic behavior of the balls that accounts for the lack of precise numerical agreement between the computed mass m and the true value not given in the *Handbook.*

The answers to the questions are:

1. In our experiment $\frac{M_2}{M_1} = 12.2$; in Chadwick's experiment, $\frac{M_2}{M_1} = 14.0$.
2. $e = 1.000$ in Chadwick's experiment.
3. $v = \frac{(M_2 - M_1)(V_1 V_2)}{(1 + e)(M_2 V_2 - M_1 V_1)}$

Suggested Solutions to Study Guide Problems

CHAPTER 21

2. If the Curies had relied only on photographic techniques for detecting radioactivity they would have been unlikely to discover:

   (1) that the intensity of radiation from thorium was directly proportional to the amount of thorium in the sample.

   (2) two other radioactive elements (polonium and radium) in pitchblende.

   The use of the sensitive electrometer invented by Pierre Curie yielded quantitative information that greatly facilitated the above discoveries.

3. (a) The energy of a photon

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34}\ \text{J}\cdot\text{sec} \times 3.0 \times 10^8\ \text{m/sec}}{0.016 \times 10^{-10}\ \text{m}}$$

$$= 1.2 \times 10^{-13}\ \text{J}$$

(b) Since 1 eV = 1.6×10^{-19} J, the energy in (a) is $\frac{1.2 \times 10^{-13}\text{ J}}{1.6 \times 10^{-19}\text{ J/eV}} = 0.75 \times 10^6$ eV or 0.75 MeV

4. (a) The magnetic force is a centripetal force:

$$Bqv = \frac{mv^2}{R}, \text{ so } R = \frac{mv}{Bq}$$

$$R = \frac{9.1 \times 10^{-31}\text{ kg} \times 1.0 \times 10^{7}\text{ m/sec}}{1.0 \times 10^{-3}\text{N/A·m} \times 1.6 \times 10^{-19}\text{ C}}$$
$$= 5.7 \times 10^{-2}\text{ m}$$

(b) As seen in part (a),

$R = \frac{mv}{Bq}$, or $R \sim \frac{m}{q}$ since v and B are constant.

Hence,

$$\frac{R_\alpha}{R_e} = \frac{m_\alpha}{m_e} \cdot \frac{q_e}{q_\alpha} = \frac{(6.7)(10^{-27})}{(9.1)(10^{-31})} \cdot ½ = 3{,}700$$

and

$$R_\alpha = 3{,}700\ (5.7 \times 10^{-2}\text{ m}) = 210\text{ m}$$

(c) The radius of curvature of the path of the α particles is much greater than that of the electrons. That is, the α particles are much less deflected. The ratio of the radii of curvature is
$\frac{R_\alpha}{R_e} = 3{,}700.$

5. The charges in the beam are positive, since they are repelled by the positively charged top plate. The magnetic field is into the page, by the right-hand rule.

6. (a) When the electric and magnetic forces on a charged particle are in balance,

$$qE = Bqv, \text{ so } E = Bv$$

$$E = 1.8 \times 10^{-3}\frac{\text{N}}{\text{A·m}} \times 1.0 \times 10^{7}\frac{\text{m}}{\text{sec}}$$
$$= 1.8 \times 10^{4}\frac{\text{N}}{\text{A·sec}}$$
$$= 1.8 \times 10^{4}\frac{\text{N}}{\text{C}}$$

(b) The electric field strength is the ratio of the voltage to the plate separation:

$$E = \frac{V}{d}, \text{ so } V = Ed$$

$$V = 1.8 \times 10^{4}\frac{\text{N}}{\text{C}} \times 0.10\text{ m}$$
$$= 1.8 \times 10^{3}\frac{\text{J}}{\text{C}}$$
$$= 1.8 \times 10^{3}\text{ V}$$

(c) As can be seen in part (a), the condition for balance does not involve the charge of the particle but only the speed. Thus, the α particles will pass through the crossed fields undeflected, since they have the same speed as the electrons.

7. (a) γ (f) α*
(b) α (g) β
(c) α (h) α
(d) γ (i) α
(e) γ (j) β

8. In the case of Crookes' observation, the uranium compounds and the daughters that had resulted from its decay were separated. Since the daughters have shorter half-lives than the uranium, the activity of the portion containing the daughters would initially be greater.

In the case of Becquerel's observation, the decay of the uranium would gradually lead to a buildup of the daughters in the portion originally containing uranium. Since the daughters have shorter half-lives, the activity of this portion would increase. On the other hand, some of the daughters in the second part would have, in the interval, reached Ra G (stable lead), so a smaller proportion of atoms would be radioactive.

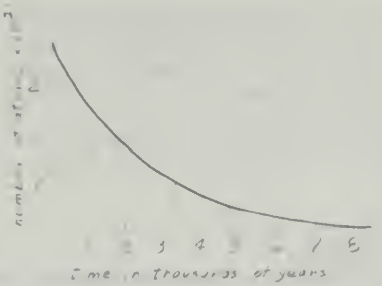
A poor analogy might be balls dropped periodically down a long and irregular flight of stairs. We notice the bounces as the ball goes from one step to another. If the rate at which balls are let loose at the top is slow (the half-life is long), the number of bounces on an initially empty staircase (daughters separated initially) will slowly build up. On the other hand, if the source at the top is cut off (no more uranium), more and more balls will reach the bottom of the staircase without replacement so the total number of bounces will slowly decrease. That only several months are required to approach equilibrium is surprising, since some of the other half-lives are quite long.

9. (a) Since the rate of emission is proportional to the fraction of the sample remaining, one-half of the original number will remain after 25 hr.
(b) Since one-quarter of the original number will remain after 50 hr, three-quarters will have disintegrated.
(c) We have assumed that the radioactive substance did not decay into daughter products that are unstable and contribute to the β-emission. To check this possibility, one would have to separate the daughter products by chemical means and determine whether or not they were radioactive.

10. If 10% of the sample decays in the first 10 yr, 10% of the remaining 90% decays in the *next* 10 yr; 10% of 90% is 9% of the original amount.

*The radius of curvature of the γ is infinite; therefore one could put γ for this answer

11. (a)



(b) 8,000 yr = 5 half-lives

$$N_{8000\ \text{years}} = N_0(\tfrac{1}{2})^5 = 2.66 \times 10^{21} \cdot (\tfrac{1}{32})$$
$$= 0.0831 \times 10^{21}$$
$$= 8.31 \times 10^{19}\ \text{atoms}$$

(c) $N_{4000\ \text{years}} \simeq 0.5 \times 10^{21}$
$= 5.0 \times 10^{20}$ atoms

12. (a) The rate of energy release is 360 W, or 360 J/sec. This corresponds to 17,000 C $\times$ 3.70 $\times$ 10^{10} disintegrations = 6.3 $\times$ 10^{14} disintegrations.

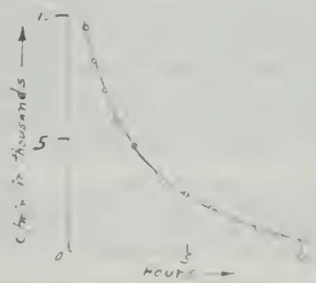
The energy release per disintegration is

$$\frac{360\ \text{J/sec}}{6.3 \times 10^{14}\ \text{disintegrations/sec}}$$
$= 5.7 \times 10^{-13}$ J/disintegration.

(b) After 15 yr, or 3 half-lives, the rate of heat production will be 360 W $\times$ $(\tfrac{1}{2})^3$ = 45 W.

13. The activity is 10 μc, or 10×10^{-6} C, which is 10^{-5} C $\times$ 3.70 $\times$ 10^{10} disintegrations/C/sec = 3.70 $\times$ 10^5 disintegrations/sec.

14. (a)



The half-life, T, can be obtained from a graph of the data (see above curve). For example, it takes about 2.3 hr (138 min) for the counting rate to drop from 8,000 counts/min to 4,000 counts/min. To drop from 4,000 counts/min to 2,000 counts/min requires about 2.3 hr (138 min). To drop from 2,000 counts/min to 1,000 counts/min again requires about 2.3 hr (138 min). Thus, the half-life appears to be constant at about 138 min.

(b) The decay rate can be computed from

$$\lambda T = 0.693$$
$$\lambda = \frac{0.693}{138} = 5.0 \times 10^{-3}/\text{min}$$

Since λ is the fraction of any sample of atoms that decay per minute, a sample of 10^6 atoms will have 5.0×10^{-3}/min $\times$ 10^6 = 5×10^3 atoms decaying each minute.

Yes, the number of atoms decaying per minute for every 10^6 atoms in the sample does remain constant.

15. (a) From the graph, the half-life of thorium X appears to be about 4 days. The graph is based on early experiments; more recent, more precise work has given a half-life of 3.64 days.

(b) With a little imagination, the following connections may be seen:

(1) The curves of the graph are reproduced on the shield of the coat of arms with very little, if any, change.

(2) The Latin inscription ("to see the origin or beginnings, of things") seems to be reinforced by the two figures: one possibly representing the beginnings of the human race the other possibly a medieval scientist. The combination may correspond to progress as related to the work of a scientist in trying to find the reasons and the bases for things.

CHAPTER 22

2. The chemical properties of an element are determined by its electron configuration, which in turn is determined by the atomic number Z. The isotopes of an element have the same Z and hence the same chemical properties.

3. If the apparently new element had chemical properties different from those of any known element, then one could be certain that the element deserved a separate place in the periodic table. If, on the other hand, it was found to have chemical properties identical to those of a known element (although having a slightly different mass than the known element), then it could be regarded as an isotope of the known element.

4. (a) The *net* rate of escape of hydrogen mole-

cules from the surface of the liquid hydrogen depends on two factors: the probability that a molecule leaves the surface and the probability that it reenters the surface because of collisions with another molecule in the vapor phase or with the wall of the container. The escape probability is the same for the light and heavy molecules because it depends on cohesive (chemical) forces, which are the same for the two isotopes. But of the molecules that have left the surface, the lighter ones have greater speeds than the heavier ones and tend to move away from the surface more rapidly. They are, therefore, less likely to return to the surface than the heavier ones, and their *net* rate of escape is greater.

(b) The hydrogen isotopes have the largest ratio of isotopic masses (2:1) and hence the largest ratio of speeds ($1:\sqrt{2}$). Separation of the hydrogen isotopes by evaporation therefore proceeds more rapidly than separation of isotopes of other elements.

5. (a) The magnetic force is the centripetal force, so

$$Bqv = \frac{mv^2}{R}, \text{ thus } R = \frac{mv}{Bq}, \text{ and so } R \propto m.$$

(b) $\dfrac{R_{35}}{R_{37}} = \dfrac{D_{35}}{D_{37}} = \dfrac{m_{35}}{m_{37}}$

$$\frac{D_{35}}{1.000 \text{ m}} = \frac{34.97}{36.97} \text{ (See Table 22-1 on page 669.)}$$

$$D_{35} = 0.946 \times 1.000 \text{ m} = 0.946 \text{ m}$$

Therefore, the separation is

$$D_{37} - D_{35} = 1.000 \text{ m} - 0.946 \text{ m}$$
$$= 0.054 \text{ m or } 5.4 \text{ cm}$$

(c) $D_{208} = \dfrac{208.0}{36.97} \times 1.000 \text{ m} = 5.640 \text{ m}$

(d) $\dfrac{D_{207}}{D_{208}} = \dfrac{m_{207}}{m_{208}}$

$$D_{207} = \frac{207.0}{208.0} \times 1.000 \text{ m}$$
$$= 0.995 \text{ m}$$

Then, $D_{208} - D_{207} = 1.000 \text{ m} - 0.995 \text{ m} = 0.005 \text{ m}$.

6. (a) ${}_{82}Pb^{212} \rightarrow {}_{83}Bi^{212} + {}_{-1}e^0$
(Since the mass number didn't change, the transmutation was due to β decay.)

(b) ${}_{83}Bi^{212} \rightarrow {}_{84}Po^{212} + {}_{-1}e^0$
In a β decay, the atomic number increases by one, but the mass number remains unchanged. The product nucleus here then has $Z = 84$ and $A = 212$. Reference to a list of elements or a periodic table shows that the element with $Z = 84$ is polonium.

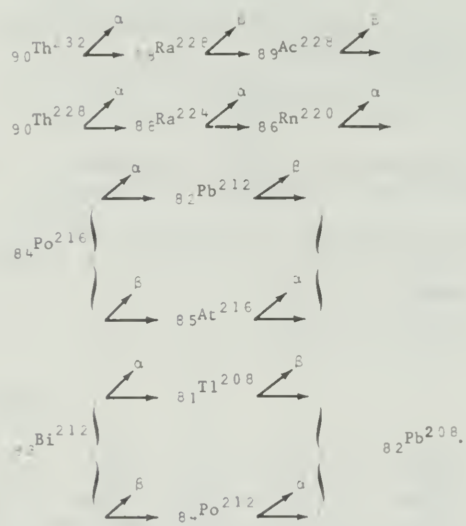
(c) ${}_{84}Po^{212} \rightarrow {}_{82}Pb^{208} + {}_2He^4$
The atomic number of lead is 82; the atomic number and mass number of the initial nucleus must therefore be 84 and 212. As was seen in part (b), the element is polonium.

7.

Actinium Series

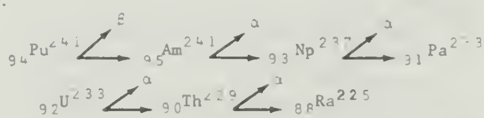
New Symbol	Decay Mode	
${}_{92}U^{235}$	α	$({}_2He^4)$
${}_{90}Th^{231}$	β	$({}_{-1}e^0)$
${}_{91}Pa^{231}$	α	
${}_{89}Ac^{227}$	β	
${}_{90}Th^{227}$	α	
${}_{88}Ra^{223}$	α	
${}_{86}Rn^{219}$	α	
${}_{84}Po^{215}$	α	
${}_{82}Pb^{211}$	β	
${}_{83}Bi^{211}$	α	
${}_{81}Tl^{207}$	β	
${}_{82}Pb^{207}$	stable	

8.



(The decay route through ${}_{82}Pb^{212}$ is much more common than through ${}_{85}At^{216}$. The other case of branching is unusual in that the two decay routes are both common; ${}_{81}Tl^{208}$ is formed 33.7% of the time and ${}_{84}Po^{212}$ is formed 66.7% of the time.)

9.



10. The percentage of the original activity at the time of measurement is $\frac{9.2}{15.3} \times 100\% = 60\%$. Referring to the radioactivity decay curve on page 646, it can be seen that 60% corresponds to a time of approximately $0.7T$ or $0.7 \times 5{,}760$ yr, which is about 4,000 yrs.

An activity of 1.0 β emission per minute per gram of carbon is $\frac{1.0}{15.3} \times 100\% = 6.5\%$ of the original activity. From the curve on page 646, this activity is seen to correspond to a time of nearly $4T$, or 23,000 yr.

The answer can also be obtained algebraically.

$$N = N_0 e^{-\lambda t}, \text{ and } \lambda = \frac{0.693}{T} = \frac{0.693}{5760}$$

$$= 1.2 \times 10^{-4}/\text{yr}$$

$$\frac{N}{N_0} = \frac{9.2}{15.3} = 0.602.$$

Hence, $e^{-\lambda t} = 0.602$, $\lambda t = 0.51$

and $t = \frac{0.51}{(1.2)10^{-4}} = 4200$ yr

For an activity of 1.0 β emission per minute per gram of carbon,

$$\frac{N}{N_0} = \frac{1}{15.3} = 0.0653 = e^{-\lambda t};$$

$$\lambda t = 2.7 \text{ and } t = \frac{2.7}{(1.2)(10^{-4})} = 23{,}000 \text{ yr}$$

11. The average atomic mass is the sum of the products of relative abundance and isotopic mass.

(a) For carbon, the average mass

$$A = 0.9889 \times 12.000 + 0.0111 \times 13.00$$
$$= 11.867 + 0.144$$
$$= 12.011 \text{ amu}$$

(Five significant digits are justified because 0.9889 is accurate to 1 part in 10.000.)

(b) For lithium, the average mass

$$A = 0.0742 \times 6.015 + 0.9258 \times 7.016$$
$$= 0.446 + 6.495$$
$$= 6.941 \text{ amu}$$

(c) For lead, the average mass is

$$A = 0.0148 \times 203.97 + 0.236 \times 205.97$$
$$+ 0.226 \times 206.98 + 0.523 \times 207.98$$
$$= 3.02 + 48.6 + 46.8 + 108.8$$
$$= 207.2 \text{ amu}$$

12. The mass of the α particle can be found by subtracting the mass of two electrons from the mass of the helium atom:

$$m\,\alpha = m\text{He} - 2\,m_e$$
$$= 4.00260 \text{ amu} - 0.0011 \text{ amu}$$
$$= 4.0015 \text{ amu}$$

Note: The actual mass is slightly larger than the amount obtained by simple subtraction because of the transformation of binding energy to mass. The difference is negligible though, because the mass equivalent of the binding energy of an electron is of the order of 10^{-9} amu.

13. (a) ¼ (¾ are still U^{238})
(b) about ½ (initial portion of graph is almost linear)
(c) about 2.25×10^9 yr
(d) Yes; the half-life of any intermediate substance is so much shorter than that of U^{238} that it can be assumed that practically all the U^{238} atoms that did decay are now Pb^{206}.

CHAPTER 23

2. Since the mass number of ${}_{92}U^{235}$ is not exactly divisible by 4 (the mass of an α particle), it is hard to see how a nucleus could be made up of only electrons and α particles. Also, by the uncertainty principle, the electrons could not exist in the nucleus.

3. According to the proton–electron hypothesis, the nucleus represented by ${}_{92}U^{235}$ would be composed of 235 protons; $235 - 92 = 143$ electrons.

4. (a) ${}_5B^{10} + {}_2He^4 \rightarrow {}_6C^{13} + {}_1H^1$
Since mass number is conserved. $4 + 10 = 14$, $14 - 1 = \underline{13}$. Since nuclear charge is conserved. $5 + 2 = 7$, $7 - 1 = \underline{6}$. So $Z = 6$, which is the atomic number of carbon

(b) ${}_{11}Na^{23} + {}_2He^4 \rightarrow {}_{12}Mg^{26} + {}_1H^1$
For mass number: $23 + 4 = 27$, $27 - 1 = \underline{26}$. For nuclear charge: $11 + 2 = 13$. $13 - 1 = \underline{12}$. $Z = 12$ is the atomic number of magnesium.

(c) ${}_{13}Al^{27} + {}_2He^4 \rightarrow {}_{14}Si^{30} + {}_1H^1$
For mass number: $27 + 4 = 31$ $31 - 1 = \underline{30}$. For nuclear charge: $13 + 2 = 15$ $15 - 1 = \underline{14}$. $Z = 14$ is the atomic number of silicon.

(d) ${}_{16}S^{32} + {}_2He^4 \rightarrow {}_{17}Cl^{35} + {}_1H^1$
For mass number: $35 + 1 = 36$ $36 - 4 = \underline{32}$. For nuclear charge: $17 + 1 = 18$ $18 - 2 = \underline{16}$. $Z = 16$ is the atomic number of sulfur.

(e) $_{19}K^{39} + {}_2He^4 \rightarrow {}_{20}Ca^{42} + {}_1H^1$

For mass number: $42 + 1 = 43$, $43 - 4 = \underline{\underline{39}}$. For nuclear charge: $20 + 1 = 21$, $21 - 2 = \underline{\underline{19}}$. $Z = 19$ is the atomic number of potassium.

5. (a) $_3Li^6 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^3$

mass number: $6 + 1 = 7$; $7 - 4 = 3$

nuclear charge: $3 + 1 = 4$; $4 - 2 = 2$

($_2He^3$ is a rare isotope of helium, natural abundance = 0.00013%.)

(b) $_4Be^9 + {}_1H^1 \rightarrow {}_2He^4 + {}_3Li^6$

(c) $_4Be^9 + {}_1H^1 \rightarrow {}_4Be^8 + {}_1H^2$

(d) $_5Be^{11} + {}_2He^4 \rightarrow {}_7N^{14} + {}_0n^1$

6. (a) $Al^{27} + {}_0n^1 \rightarrow Al^{28} + \gamma$

Since Z does not change ($Al \rightarrow Al$) and since mass numbers already balance, only a γ ray (no charge, no mass) can result.

(b) $Al^{27} + {}_1H^2 \rightarrow {}_1H^1 + Al^{28}$

Since $Z_{Al} + 1 = Z_{Al} + 1$, the resulting nucleus must be aluminum. Total mass number on the left side is 29 (27 + 2) = total mass number on right side. So $A_{Al} + 1 = 29$ and we have Al^{28}.

(c) $Al^{27} + {}_1H^1 \rightarrow {}_2He^4 + Mg^{24}$

Since the particle input on the left has charge 1 and the particle output on the right has charge 2, the nucleus on the right has one less positive charge than the nucleus on the left: Move one place toward hydrogen in the periodic table. The nucleus must be magnesium.

Alternatively, $Z_{Al} = 13$ so $13 + 1 = 14$; $14 - 2 = 12$ and $Z_{Mg} = 12$.

For mass number: $27 + 1 = 28$, $28 - 4 = 24$

(d) $Al^{27} + {}_1H^2 \rightarrow {}_2He^4 + Mg^{25}$

By reasoning of (c), the nucleus is magnesium.

Mass number: $27 + 2 = 29$, $29 - 4 = 25$

Reactions (b) and (d) show that when a given nuclide is bombarded by a given particle (Al^{27} and a deuteron) different reactions can occur giving different products.

7. (a) Since nitrogen nuclei are about 14 times more massive than neutrons, if a neutron were to hit the nitrogen nucleus and stop dead in its track, the nitrogen nucleus would have a velocity of only 1/14 the original velocity of the neutron, v_n. Actually, the neutron rebounds somewhat, but its velocity of rebound cannot be greater than its initial velocity, or energy would not be conserved. Thus, the momentum given the nitrogen nucleus must be less than twice the original neutron momentum, and the nitrogen nucleus must have a velocity less than 2(1/14) of the original neutron velocity. We can see then, without calculating the actual velocity of the nitrogen nucleus, that it must have been between 1/14 and 1/7 the original neutron velocity (that is, about an order of magnitude less). But since the neutron and the hydrogen nucleus (proton) have nearly the same mass, a complete transfer of momentum from neutron to proton would give the proton very nearly the same velocity as the original velocity of the neutron. Thus, the nitrogen nucleus will have a velocity about an order of magnitude less than the velocity of the hydrogen nucleus.

(b) In Unit 3, where the conservation laws were studied.

8. From page 686, $\dfrac{v_p}{v_N} = \dfrac{m_N + m_n}{m_p + m_n}$

Substituting for known masses and speeds:

$$\frac{14 + m_n}{1 + m_n} = \frac{3.4 \times 10^9 \text{ cm/sec}}{4.7 \times 10^8 \text{ cm/sec}} = 7.2$$

so

$$m_n + 14 = 7.2\,(m_n + 1)$$
$$6.2\, m_n = 14 - 6.2 = 7.8$$
$$m_n = 1.10 \text{ amu}$$

Difference in mass: $1.16 - 1.10 = 0.06$ amu

Percent difference: $\dfrac{0.06}{1.16} = 5.2\%$

Thus, a difference of measurement of 3% is multiplied in the calculation to a difference of 5.2%.

9.

	A	Z	number of protons	number of neutrons
H^1	1	1	1	0
H^2	2	1	1	1
He^4	4	2	2	2
Li^7	7	3	3	4
C^{13}	13	6	6	7
U^{238}	238	92	92	146
Th^{234}	234	90	90	144
Th^{230}	230	90	90	140
Pb^{214}	214	82	82	132
Pb^{206}	206	82	82	124

10. (a) 78
(b) 79
(c) 80
(d) 80

11. (a) $_{11}Na^{23} + {}_1H^2 \rightarrow {}_1H^1 + {}_{11}Na^{24}$

For a method of solution, see question 4.

(b) $_{11}Na^{23} + {}_0n^1 \rightarrow \gamma + {}_{11}Na^{24}$

(c) $_{12}Mg^{24} + {}_0n^1 \rightarrow {}_1H^1 + {}_{11}Na^{24}$

(d) $_{12}Mg^{26} + {}_1H^2 \rightarrow {}_2He^4 + {}_{11}Na^{24}$

Notice that the nuclide $_{11}Na^{24}$ can be produced in at least four ways. Of course, the reactions noted are not unique. When bombarding $_{11}Na^{23}$ with neutrons, for example, the major product varies with the neutron energy and, even with single-energy neutrons, more than one type of product may result. (The target, after all, contains many more than one nucleus of $_{11}Na^{23}$.)

12. When a target of the aluminum nuclide with mass number 27 is bombarded by neutrons, the neutrons react with the aluminum to produce a magnesium nuclide with mass number 27 and an ejected proton. The magnesium nuclide is radioactive, undergoing β decay, accompanied by emission of a γ ray and an anti-neutrino. The half-life of the artificially radioactive Mg nucleus is 9.5 min.

13. Assume that the tracks originate at the point in the lower center: The law of conservation of momentum requires that the vector sum of the momenta of the two particles be equal to the momentum of the neutral particle that "exploded" at point A.

A similar argument holds if the tracks terminate at A, except that in this case the neutral particle (which leaves no track) goes off in the direction determined by the vector sum of the momenta of the particles that combined at A.

It is possible that (neutral) electromagnetic radiation either originated at A or was annihilated at A. Indeed, something is, or was, present, which could be either a neutral particle or electromagnetic radiation.

14. The existence of artificially radioactive nuclides provided much more data on unstable (radioactive) nuclides. There are only 54 naturally occurring radioactive nuclides; thus, only 54 test cases for theories of nuclear stability. Most of these are heavy nuclides. The manufacture of 1,200 artificial nuclides that span the entire spectrum of mass number and charge made possible far more sensitive tests of theoretical prediction.

15. Essay.

16. See Table 22-1, page 669.

Mass of neutral helium atom = 4.002604 amu

Mass of 4 hydrogen nuclei = 4 (1.00727) = 4.02908 amu

Mass of 2 electrons = 2 (0.000549) = $\underline{0.00110 \text{ amu}}$

4.03018 amu

The mass of a helium atom is less than the sum of the masses by 0.02758 amu. We may conclude that if a helium atom were to be made from 4 hydrogen nuclei and 2 electrons, the "missing" mass might be accounted for by energy considerations.

CHAPTER 24

2. $C^{12} + n \rightarrow C^{13}$

The mass of the neutron added is 1.008665 amu, whereas the mass of C^{13} is only greater than that of C^{12} by 1.003354 amu. The discrepancy in mass (often called the "mass defect") of 0.005311 amu is equivalent to 0.005311 amu × 931 MeV/amu = 4.95 MeV, and represents the binding energy of the neutron.

3. The sum of the individual masses of the 2 protons, 2 neutrons, and 2 electrons comprising the helium atom is 2 × (1.007276 + 1.008665 + 0.000549) amu = 4.03298 amu. Since the atomic mass is only 4.00260 amu, there is a mass difference of 0.03038 amu, which is equivalent to 0.03038 × 931 MeV/amu = 28.3 MeV. Therefore, the average binding energy is 28.3 MeV/4 nucleons = 7.07 MeV/nucleon. (The binding energy of the two electrons was ignored; it is only 13 eV per electron.)

4. By the law of the conservation of momentum, the two α particles will fly off with identical speeds but in opposite directions. Their total KE is calculated on page 707 to be 17.3 MeV; thus they would each have a KE of 8.65 MeV.

5. The total atomic mass of the reactants is 14.003074 amu + 4.002604 amu = 18.005678 amu. The total atomic mass of the products is 16.999134 amu + 1.007825 amu = 18.006959 amu. Since the products have a greater total mass than have the reactants, energy must have been <u>absorbed</u> in the reaction.

The mass difference is 0.001281 amu, which is equivalent to 0.001281 amu × 931 MeV/amu = 1.19 MeV. Thus,

$_7N^{14} + {_2He^4} + 1.19 \text{ MeV} \rightarrow {_8O^{17}} + {_1H^1}$

6. As calculated in the preceding question, 1.19 MeV is required to promote the reaction. If we subtract this from the KE of the α particles, the amount of energy remaining will represent the total KE of the reactants. (We assume that the N^{14} target is stationary.) Thus, 7.68 MeV − 1.19 MeV = 6.49 MeV is the total KE of the reactants. The "recoil" energy of the O^{17} nucleus is then

the difference between 6.49 MeV and 5.93 MeV, or 0.56 MeV.

7. The total atomic mass of the reactants is 14.003074 amu + 2.014102 amu = 16.017176 amu. The total atomic mass of the products is 15.000108 amu + 1.007825 amu = 16.007933 amu. The mass difference (mass defect) is thus 16.017176 amu − 16.007933 amu = 0.009243 amu. The energy liberated is 0.009243 amu × 931 MeV/amu = 8.61 MeV.

8. We consider three successive steps:
1. Addition of a neutron. This changes the total number of nucleons by one, but does not change the charge number.

$${}_{90}\mathrm{Th}^{232} + {}_{0}\mathrm{n}^{1} \rightarrow {}_{90}\mathrm{Th}^{233}$$

(neutron capture)

2. β decay. The total number of nucleons remains the same. A neutron changes to a proton with β decay, that is, the emission of an electron. This increases the charge number by 1.

$${}_{90}\mathrm{Th}^{233} \rightarrow {}_{91}\mathrm{Pa}^{233} + {}_{-1}e^{0}$$

3. Another β decay gives the final transition to ${}_{92}\mathrm{U}^{233}$

$${}_{91}\mathrm{Pa}^{233} \rightarrow {}_{92}\mathrm{U}^{233} + {}_{-1}e^{0}$$

9. From the figure we find approximately:
BE per nucleon for Ba^{141} = 8.4 MeV
BE per nucleon for Kr^{92} = 8.7 MeV
BE per nucleon for U^{235} = 7.6 MeV

The total binding energy is found in each case by multiplying by the total number of nucleons in the nucleus.
Thus, we have
Total BE of Ba^{141} = 8.4 × 141 = 1,180 MeV
Total BE of Kr^{92} = 8.7 × 92 = 800 MeV
BE of products = 1,980 MeV
Total BE of U^{235} = 1,790 MeV

The energy released in the reaction is equal to the difference of the total binding energy of the products and the total binding energy of the incident particles.

energy released = (1,980-1,790) MeV
= 190 MeV
= 200 MeV

10.

U^{235} :	235.04393 amu	La^{139} :	138.9061 amu
n^{1} :	1.00867 amu	Mo^{95} :	94.9057 amu
		$2\mathrm{n}^{1}$:	2.0173 amu
sum =	236.05260 amu	sum =	235.8291 amu
		mass defect =	0.2235 amu

Binding energy = 0.2235 amu × 931 MeV/amu = 208 MeV.

11.

$${}_{36}\mathrm{Kr}^{92} \xrightarrow{{}_{-1}e^{0}} {}_{37}\mathrm{Rb}^{92} \xrightarrow{{}_{-1}e^{0}} {}_{38}\mathrm{Sr}^{92} \xrightarrow{{}_{-1}e^{0}} {}_{39}\mathrm{Y}^{92} \xrightarrow{{}_{-1}e^{0}} {}_{40}\mathrm{Zr}^{92}$$

12. A given mass of fissionable material will lose the least number of neutrons through its surface if it is spherical in shape. Any other shape would mean a larger surface area and hence a larger loss of neutrons.

The neutron loss would be greatest if the material were formed into a thin, flat sheet; the thinner the sheet, the larger the surface area and neutron loss.

13. The high temperatures are required for fusion to commence because the kinetic energies of the protons must be great enough to overcome the electrical repulsion between them.

14. As the vast clouds of hydrogen collapse gravitationally, the gravitational potential energy of the particles is transformed into kinetic energy. Eventually the contraction results in particles of sufficient kinetic energy to initiate fusion reactions.

15. The net result of the proton–proton chain is given as $4\,{}_{1}\mathrm{H}^{1} \rightarrow {}_{2}\mathrm{He}^{4} + 2\,{}_{+1}e^{0}$.
(Note that to be complete, we should also show two neutrinos on the right-hand side of the equation.) Since we are to show that the energy released per cycle is about 27 MeV, we are only interested in two-figure accuracy. Therefore, we can simply deal with atomic masses and ignore the masses of the electrons and positrons. The mass defect is then

4 (1.007825 amu) − 4.002604 amu = 0.028696 amu.

The corresponding energy release = 0.02870 amu × 931 MeV/amu = 26.7 MeV.

16. (a) We can use the Einstein relation $E = \Delta mc^2$ to calculate the rate of the mass loss in the sun.

$$\Delta m = \frac{E}{c^2} = \frac{3.90 \times 10^{26}\ \mathrm{J/sec}}{(3 \times 10^{8}\ \mathrm{m/sec})^2}$$
$$= 0.433 \times 10^{10}\ \mathrm{kg/sec}$$
$$= 4.33 \times 10^{9}\ \mathrm{kg/sec}$$

(b) 3.90×10^{26} J/sec is 3.90×10^{26} W.

This is equivalent to $\dfrac{3.90 \times 10^{26}\ \mathrm{W}}{746\ \mathrm{W/hp}}$

= 5.23×10^{23} horsepower.

17. The completed six steps of the carbon cycle are as follows:

$${}_{6}\mathrm{C}^{12} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{7}\mathrm{N}^{13} + \gamma$$
$${}_{7}\mathrm{N}^{13} \rightarrow {}_{6}\mathrm{C}^{13} + {}_{+1}e^{0} + \nu$$
$${}_{6}\mathrm{C}^{13} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{7}\mathrm{N}^{14} + \gamma$$
$${}_{7}\mathrm{N}^{14} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{8}\mathrm{O}^{15} + \nu$$
$${}_{8}\mathrm{O}^{15} \rightarrow {}_{7}\mathrm{N}^{15} + {}_{+1}e^{0} + \nu$$
$${}_{7}\mathrm{N}^{15} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{6}\mathrm{C}^{12} + {}_{2}\mathrm{He}^{4}$$

(b) Four atoms of $_1H^1$ have been converted into one atom of $_1He^4$, two positrons, γ rays, neutrinos, and 26.7 MeV of energy. The nuclides $_6C^{12}$, $_6C^{13}$, $_7N^{14}$, $_7N^{15}$, and $_8O^{15}$ come out unchanged.

18. The total atomic mass of the reactants is 3.016030 amu + 4.002604 amu = 7.018634 amu. The mass difference is 7.018634 amu − 7.016929 amu = 0.001705 amu. Therefore, the energy released is 0.001705 amu × 931 MeV/amu = 1.59 MeV.

19.

$$\begin{aligned} \text{Mass of } U^{233} &= 233.039498 \text{ amu} \\ \text{mass of } n^1 &= \underline{1.008665} \\ \text{mass of } U^{233} + n^1 &= 234.048163 \\ -\ \text{mass of } U^{234} &= \underline{234.040900} \\ \text{mass defect} &= 0.007263 \text{ amu} \\ \text{excitation energy} &= 0.007263 \text{ amu} \\ &\times 931 \text{ MeV/amu} \\ &= 6.76 \text{ MeV} \end{aligned}$$

But the activation energy of U^{234} is 4.6 MeV. Thus, since the excitation energy is greater than the activation energy, $_{92}U^{233}$ <u>is</u> fissionable with slow neutrons.

20.

$$\begin{aligned} \text{Mass of } Pu^{241} &= 241.056711 \text{ amu} \\ \text{mass of } n^1 &= \underline{1.008665} \\ \text{total mass} &= 242.065376 \\ -\ \text{mass of } Pu^{242} &= \underline{242.058710} \\ \text{mass defect} &= 0.006666 \text{ amu} \\ \text{excitation energy} &= 0.006666 \times 931 \text{ MeV} \\ &= 6.20 \text{ MeV} \end{aligned}$$

Since the activation energy is only 5.0 MeV, Pu^{241} <u>is</u> fissionable with slow neutrons.

21. Atoms of the isotopic tracer $_{15}P^{32}$ with a half-life of 14 days can be incorporated chemically into molecules of a substance from which phosphate groups are transferred to ADP. The newly produced molecules of ATP are radioactive, and the rate at which they are produced can be determined by means of counters.

22. Essay.

Appendix: Radiation Safety

(Based on material supplied by Oak Ridge Associated Universities)

Many schools, including those planning to adopt *Project Physics*, are incorporating some form of radioisotope work into their science curriculum. As a result, questions concerning the potential hazards and control of radiation are bound to be raised by teachers, students, and parents. It is the purpose of this article to deal as briefly as possible with such questions and to provide the information that will be necessary for the safe introduction of radioisotopes into the high school course.

MEASUREMENT OF RADIOACTIVITY

The basis of all measurements of radioactivity is the creation of an ion pair by radiation.

The most common units used for measuring radioactivity and radiation exposure are the *curie*, the *roentgen*, the *rad*, and the *rem*. Each unit indicates a different quantity that is of interest to the scientist.

Curie (ci)

The curie is a unit used to measure the rate at which radioactive material, or a combination of radioactive materials, is giving off nuclear particles. One curie = 37,000,000,000 disintegrations/sec (dps). Since it is the number of disintegrations per second that determines the amount of radiation emitted, the activity of the source is a significant factor. However, the type and energy of the radiation are also important in evaluating the potential hazard, and the curie does not measure this. The curie is not a measure of exposure to radiation damage.

The curie is a very large amount of radioactivity. Historically, the unit was chosen because it was approximately the amount of radiation emitted by 1 g of pure radium. More practical units for laboratory use are the millicurie (mc) or 10^{-3} c, and the microcurie (μc), or 10^{-6} c.

Roentgen (R)

The roentgen is a unit of *exposure dose*. It measures the ionization in air produced by a source of γ or X rays. One roentgen produces 2.58×10^{-4} C of charge (about 1.6×10^{15} ion pairs) per kilogram of dry air. This unit is not applicable to particle radiations such as α, β, and neutrons. Radiation survey instruments are usually calibrated in roentgens per hour or milliroentgens per hour.

Rad

The rad is the unit of absorbed dose. It amounts to 100 ergs (10^{-5} J) of energy imparted to 1 g of irradiated material, by any ionizing radiation.

Rbe

This factor stands for *relative biological effectiveness*. It is a comparison of the biological effectiveness of different types of ionizing radiation.

Rem

The rem was devised to allow for the fact that the same dose in rads delivered by different kinds of radiation does not necessarily produce the same degree of biological effect; some radiations are biologically more effective than others. The rem may be thought of as an abbreviation for *radiation effect, man*. Since various radiations such as α, β, γ rays, and neutrons have different biological effects per rad of absorbed energy, they are assigned values depending on the biological effect being considered. For a given biological effect the number of rems = rads × rbe.

Relationships among radiation units

The relationships among the units of radiation exposure as they apply to the radiation of water and soft tissue are summarized below.

Type of Radiation	R	RAD	REMS
X rays and gamma rays	1	1	1
Beta particles	—	1	1
Fast neutrons	—	1	10
Thermal neutrons	—	1	4–5
Alpha particles	—	1	10

Since the human body is about two-thirds water, this table provides a means for making a rough estimate of the biological effect of simultaneous or consecutive absorption of different kinds of radiation.

RADIATION PROTECTION GUIDELINES

Nuclear radiations are a natural part of the environment, just as are the sunlight and the earth's magnetic field. Humans have always lived in an environment that includes a great deal of natural radiation. In addition, we have created new sources of radiation as we have explored nature and discovered ways to improve our control and use of nature.

Natural radiation comes from both the earth and the sky. Such radioactive minerals as uranium and thorium, and decay products associated with them exist everywhere in the earth. The places where they are mined are simply the locations of extremely large concentrations of these minerals. The radioactive gas radon is present in small amounts in the air we breathe. Cosmic radiation, particles of very high energy that strike the earth's atmosphere from outer space, contributes both directly and indirectly to the amount of radiation to which we are exposed. Cosmic rays contribute directly by striking our bodies, ionizing body materials, and causing radiation damage. They contribute indirectly when cosmic neutrons create radioactive carbon-14 by striking atmospheric nitrogen. C^{14} is then quickly converted into $C^{14}O_2$. The $C^{14}O_2$ is utilized by plants in photosynthesis and the plants are eaten by animals. The radioactive carbon eventually finds its way into our bodies through our food. In fact, the disintegration of C^{14} causes the liberation of about 200,000 β particles per minute in the average adult. K^{40}, which is also in our bodies, liberates approximately 240,000 β particles/min in the average adult. Other radioactive materials in our bodies include radium-226, strontium-90, cesium-137, and iodine-131.

It is true that all radiation is harmful, but it is also true that human beings can be, and in fact are, continuously exposed to radiation at low intensity without any apparent harmful effects. Body tissue is both damaged and destroyed whenever one cuts oneself, bruises oneself, or breaks a bone. But just as the body is able to adjust to this type of an injury and mend the damage, so it is able to adjust to and counteract the harmful effects of small doses of radiation. Similarly, large doses of radiation can cause severe injury and even death just as many other types of injuries can.

Sources of synthetic radiation that contribute to our total exposure to radiation include X rays, radioactive fallout, luminous watch dials, television tubes, radioactive industrial wastes, etc.

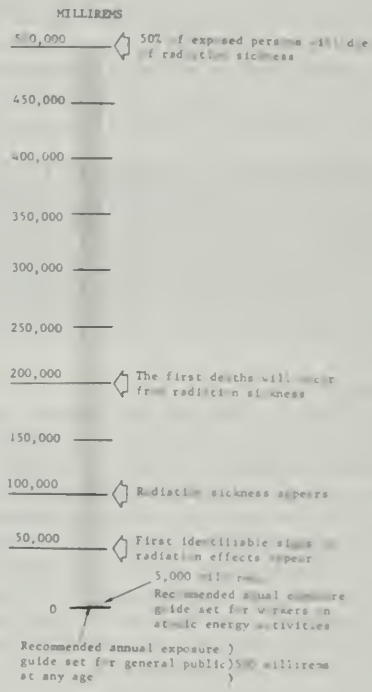
It is estimated that the average annual exposure in the United States amounts to 267 millirems. Natural sources (cosmic radiation, minerals, etc.) contribute 101 millirems and synthetic sources, mostly from medical X rays, make up the remaining 166 millirems. Radioactive fallout accounts for 4 millirems.

One of the questions health physicists are called upon to answer concerns the safe and maximum permissible radiation exposures that a person can receive. To answer a question of this type, it must be understood that the effects of radiation exposure depend on several factors. These include:

1. the amount and rate of radiation exposure.
2. the kind of radiation: whether it is penetrating radiation such as γ rays and X rays, or relatively nonpenetrating such as α particles;
3. the tissue exposed, which depends in turn on the source of the exposure: whether the radiation came from outside the body or from radioactivity inside the body;
4. the kind of radioactive material involved, its radioactive and chemical nature, and its biological path if taken into the body. (See Ref. 2, p. 452.)

Balancing the risks involved in radiation exposure against the benefits to be gained from increased knowledge, the National Council on Radiation Protection and Measurements (NCRP) and the International Commission on Radiological Protection have set limits on what is considered an acceptable exposure for persons occupationally exposed to radiation. These limits are in addition to background, medical, and dental exposures. The primary goal is to keep radiation exposure of the individual well below a level at which adverse effects are likely to be observed during one's lifetime. Another objective is to minimize the incidence of genetic effects.

The chart that follows shows the recommended limits of exposure in relation to biological effects.



Note that the exposure guide set up for workers in atomic energy activities is one-tenth of the level at which the first identifiable signs of radiation effects occur. The exposure guide set for the adult general public is just about twice the average background level due to natural and synthetic sources. It is one-hundredth of the level at which the first identifiable signs appear.

By adhering to the guidelines that have been set, industries engaged in atomic energy activities in the United States have insured that the risk of damage to exposed persons is not greater than the risks normally accepted in other present-day industries.

The following additional limits apply to the high school situation (See Ref. 3, p. 452):

1. Students under 18 years of age who are exposed to radiation during educational activities should not receive whole body exposures exceeding 0.1 rem per year. To provide an additional factor of safety, it is recommended that each experiment be so planned that no individual receives more than 0.01 rem while carrying it out.

It should be emphaiszed that there is no difficulty in performing radiation experiments and demonstrations in conformity with the above recommendations if appropriate safeguards are taken. These appropriate safeguards are discussed in the following sections.

2. Persons under 18 years of age shall not be occupationally exposed to radiation. They shall not be employed or trained in an X-ray department, radioisotope laboratory, or industrial radiation facility.

External hazard control

1. *Quantity and Type of Radiation Source*

Because each radioisotope has its own characteristic mode of decay, equal activities (curies) of different radioisotopes may provide different exposure rates (roentgens). Radiation safety is concerned with limiting exposures, so control deals with limiting the amount of isotope used to the minimum necessary to achieve the desired results and the careful selection of isotopes according to the exposure rate they produce.

2. *Distance*

The easiest means of controlling radiation exposure is to use the fact that radiation intensity follows an inverse square.

3. *Length of Exposure*

The total exposure that a person receives is the product of the exposure rate and the length (time) of exposure:

$$I \times T = \text{total exposure}$$

where I = exposure rate, R/hr

T = length of exposure

Radiation safety requires that the product of length of exposure and exposure rate not exceed the recommended limits. Figure 1 shows this relationship. For high exposure rates, only very short exposure periods are allowed, although very long exposure periods are permitted at very low exposure rates.

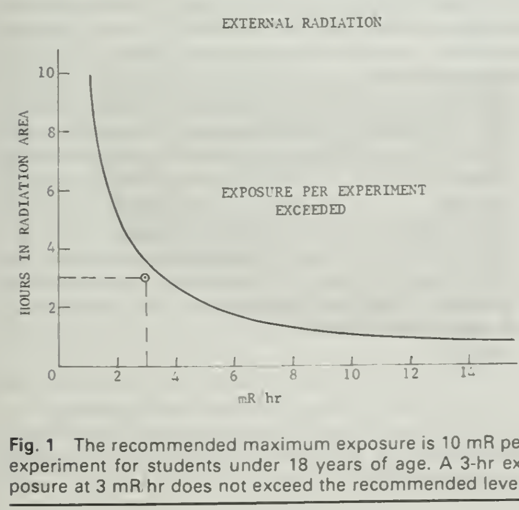


Fig. 1 The recommended maximum exposure is 10 mR per experiment for students under 18 years of age. A 3-hr exposure at 3 mR/hr does not exceed the recommended level.

4. *Shielding*

For sources of radiation either very large in quantity or in size, it is not always possible to achieve the degree of radiation safety required by using the first three factors alone. The use of some type of shielding is required. The type of shielding used depends on the type of radiation to be absorbed.

The rate at which α particles lose energy is so great that even 1 mm of any solid or liquid material will stop them. Hence, α particles are not usually considered an external radiation hazard because their penetration in tissue is only a fraction of a millimeter. As a result, they expend all their energy in the dead (cornified) layer of skin that covers the body.

Beta particles (having less mass and charge and a much greater velocity than α particles) are more penetrating but also have a limited range. Figure 2 shows the range of β particles of various energies in different materials.

Note that while the range of β particles in air may be as much as 1 m, the range in a material such as glass is a fraction of a millimeter. Therefore, the body is easily protected against β and α particles with relatively thin shields.

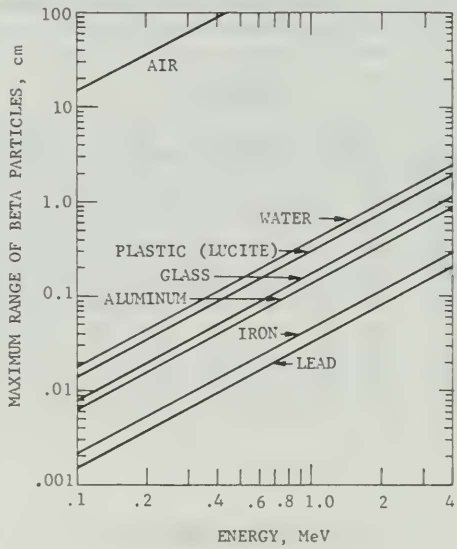


Fig. 2 Range of β particles as a function of energy.

The absorption of γ rays by matter, like the absorption of light, follows a negative exponential law:

$$I = I_0 e^{-\mu x}$$

So in $\dfrac{I}{I_0} \times -\mu x$ or log $\dfrac{I}{I_0} \times -0.434\ \mu x$,

where I_0 = exposure rate at a given point with no shielding; I = exposure rate at the same point, but with a shield of thickness x between the source and the point where intensity is measured; x = thickness of shield; and μ = linear attenuation coefficient.

The relationship between I and x is not linear, but use of the equation will enable one to determine the thickness of lead required to reduce the intensity to the desired level.

Theoretically it is not possible to attenuate γ radiation completely, but the exposure rate can be reduced by any desired factor. A useful concept regarding γ attenuation is the half-value layer (HVL) or the half thickness $x\frac{1}{2}$, which is defined as the thickness of any particular material necessary to reduce the intensity of a beam of X rays or γ rays to one-half its initial value. Similarly the tenth-value layer (TVL) or the tenth thickness $x\frac{1}{10}$ is defined as the thickness of any particular material necesary to reduce the intensity of a beam of X rays or γ rays to one-tenth its initial value. Hence, three tenth-value layers will reduce the dose received from a γ source to $\frac{1}{1000}$ of the initial amount ($\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$). We can see from the table below why lead is so often used as a shield for radioisotopes.

Approximate Half-Value and Tenth-Value Layers for Gamma rays

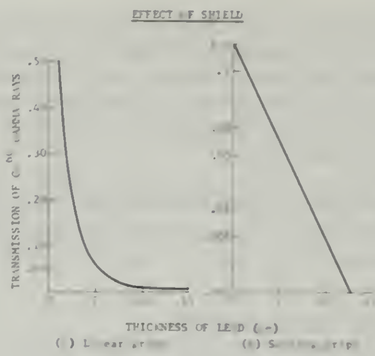
MATERIAL

Gamma energy (MeV)	Water HVL $x\frac{1}{2}$ (cm)	Water TVL $x\frac{1}{10}$ (cm)	Aluminum HVL $x\frac{1}{2}$ (cm)	Aluminum TVL $x\frac{1}{10}$ (cm)	Iron HVL $x\frac{1}{2}$ (cm)	Iron TVL $x\frac{1}{10}$ (cm)	Lead HVL $x\frac{1}{2}$ (cm)	Lead TVL $x\frac{1}{10}$ (cm)
0.20	5.2	17.3	1.9	6.3	0.66	2.20	0.14	0.47
0.50	7.2	24.0	3.0	10.0	1.11	3.70	0.43	1.43
1.0	9.8	32.6	4.1	13.7	1.56	5.20	0.88	2.93
1.5	12.2	40.6	5.1	17.0	1.74	5.80	1.17	3.90
2.0	14.0	46.6	5.9	19.7	2.05	6.83	1.37	4.57
2.5	15.8	52.6	6.7	22.3	2.22	7.40	1.47	4.90
3.0	17.4	58.0	7.2	24.0	2.31	7.70	1.51	5.03
4.0	20.0	66.7	8.4	28.0	2.55	8.50	1.48	4.93
5.0	22.1	73.7	9.4	31.3	2.88	9.60	1.42	4.73

Linear Attenuation Coefficients (cm^{-1})

Shield Material	Gamma Ray Energy 0.1 MeV	0.5 MeV	1.0 MeV
Water	0.17	0.097	0.06
Iron	2.7	0.66	0.55
Lead	62	1.7	0.79

The next figure shows the relationship of I to x for lead. Because of its high value of μ, lead is often used as a shielding material.



Internal Radiation Hazard

Any radioactive substance entering the body is hazardous. Common sense rules can prevent radioisotopes from being ingested. They are:

1. Never place any materials used in the laboratory in the mouth. Pipetting, glass blowing, etc., should be done using indirect methods. (Various inexpensive types of pipettors are available on the market.)
2. Never handle radioactive materials directly. Always use rubber or plastic gloves, tweezers, or some means of indirect contact.
3. Do not eat, smoke, or apply cosmetics in areas where radioisotopes are handled.
4. Never place your hands near your face or mouth while working with radioisotopes.
5. When working with liquid sources, always cover your working area with absorbent material, preferably inside a tray, to retain any spilled liquid.
6. At the end of an experiment check all tools, glassware, etc., with a Geiger counter. Wash your hands carefully.

RADIONUCLIDE SOURCES

The most commonly used naturally occurring radioactive substances with sufficient activity to constitute a possible hazard are radium, polonium, actinium, thorium, and uranium. Of these radium and polonium are particularly significant since they are readily available and are frequently used in quantities sufficient to constitute a potentially serious radiation hazard. Also, uranium and thorium salts can be purchased from chemical supply houses by the case and shipments generally are delivered without any radiation warning signs on the bottles. Although these salts are no real hazard in terms of external radiation, they could be a potential internal radiation hazard and should be treated as such.

Note: Normal uranium and thorium, in soluble form, are more dangerous as chemical poisons inside the body than as sources of radiation since relatively large quantities are required to cause severe radiological damage.

Artifically produced radionuclides are produced either by the fission of heavy elements in a nuclear reactor or by the bombardment of nonradioactive isotopes in high-energy accelerators or nuclear reactors. Over 1,000 radionuclides are known, and of these, about 100 are in common use. Small amounts of certain commonly used nuclides are available without specific license. Acquisition of larger amounts of nuclides requires a specific license from the Department of Energy or state regulatory agency, or from both.

"Generally Licensed" Radioisotopes in the High School Classroom

The quantity of radioisotopes used in the high school classroom is usually limited to very small amounts that are generally licensed by the Department of Energy. A complete listing of these (termed "generally licensed" by the DOE, "exempt" by states, and "license free" by some suppliers) may be obtained from the DOE (see Ref. 7, p. 452). A teacher may purchase any of these radioisotopes without fulfilling any specific licensing requirements provided that he or she does not at any one time possess or use more than a total of 10 such quantities. Although generally licensed quantities may be purchased without the need of any specific license from the DOE, the user is *not* exempt from adhering to the regulations that are concerned with their use; hence, it is recommended that teachers obtain copies of these regulations. (See Ref. 6, p. 452.)

"Generally licensed" quantities of materials in solution *cannot* be added together to obtain a source of a higher activity. If higher activities are desired, proper authorization must be obtained for their acquisition. Hence, if you want a 30 μc source of P^{32}, you cannot buy three 10 μc liquid sources and pour them all together.

The limitation of quantity by the Department of Energy *practically* assures the safety of the persons using or coming in contact with radioisotopes in the high school program. However, the fact that they are radioactive materials and can constitute a safety hazard should always be kept in mind, and the methods of controlling both external and internal hazards should become a concern of the student as well as the teacher.

For any γ-emitting radionuclide, the specific γ ray constant is the exposure rate in roentgens per hour for a point source of 1 mc at a distance of 1 cm. If used as γ sources, β-γ emitters can be enclosed in a sufficient thickness of glass, plastic, or metal to eliminate essentially the β radiation. As stated earlier, both β and γ radiations are capable of producing biological effects.

A "generally licensed" or "exempt" source, when used for a short time (1-2 hr), represents a negligible external hazard. However, if it were to be kept close to the skin for many hours, injurious effects could possibly be produced. Radiation sources should, therefore, not be carried in pockets or handled without proper tools. Adequate precautions should be taken to prevent radioactive materials, *irrespective of amount*, from gaining entry to the body through the mouth, the nose, or the skin (Ref. 3, p. 452).

Just about any laboratory exercise involving the use of radioisotopes to be done at the high school level can be accomplished using "generally licensed" quantities of radioactive materials. A few exceptional cases may arise, but in the vast majority of cases, the "generally licensed" quantities will suffice. As a rule of thumb in choosing radioisotopes to be used in a classroom experiment, choose the source of minimum strength (activity) required to perform the experiment.

Records should be kept of all radioactive materials that have been and are being used in the classroom. The records should contain the name of the isotope, the activity, the specific γ constant, if it is a γ emitter, the half-life, the date of shipment, the storage location, the names of persons using the isotope, and the date of disposal.

Calculation of Radiation Exposure

In an experiment to study the absorption of γ rays by lead sheets, some high school students will be using a new 1 μc Co^{60} plastic-sealed disc source. Since 1 μc is only one-tenth of the maximum amount of Co^{60} that may be purchased as a generally licensed quantity in the form of a sealed source, it may be considered safe for student use under ordinary conditions. However, if you want to determine what dose in rems a student would receive from this source if the student were sitting, on the average, about 20 cm from the source during the experiment, which takes about 30 min, proceed as follows:

$$\text{dose (rem)} = \begin{pmatrix}\text{specific} \\ \gamma \text{ ray} \\ \text{constant}\end{pmatrix} \times \text{present activity of source}$$

$$\times \text{ time of exposure (hr)}$$

$$\times \frac{1}{(\text{distance in cm})^2}$$

$$\times 1 \frac{\text{rem}}{\text{roentgen}}$$

For Co^{60},

$$\text{dose} = 12.9 \frac{\text{roentgen} \cdot \text{cm}^2}{\text{mc} \cdot \text{hr}} \times 1\ \mu c$$

$$\times\ 10^{-3} \frac{\text{mc}}{\mu c} \times 1/2 \text{ hr}$$

$$\times \frac{1}{(20 \text{ cm})^2} \times 1 \frac{\text{rem}}{\text{roentgen}}$$

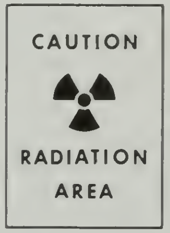
$$\simeq 1.6 \times 10^{-5} \text{ rem}$$

This exposure is well below the 0.01 rem/experiment maximum that was suggested earlier.

Aso, since 0.01 rem/experiment is the recommended maximum, no student should work in an area where a survey instrument reads above 10 mR/hr. (Note: Survey instruments are often available from local Civil Defense authorities.)

Radiation Warning Signs

All radiation sources should be identified by the standard radiation symbol.



The Standard Radiation Symbol

Besides warning of the presence of radioactive material, the warning sign should also indicate the person responsible for the material and how to contact this person.

Storage of Radioactive Materials

All radioactive materials should be stored in a plainly marked and properly shielded area. The area should be secured against the unauthorized removal of radioisotopes from the place of storage. All radioactive materials should be signed for on removal and checked back in on their return so that the whereabouts of the materials is always known. The activity immediately outside the storage area should not be above background.

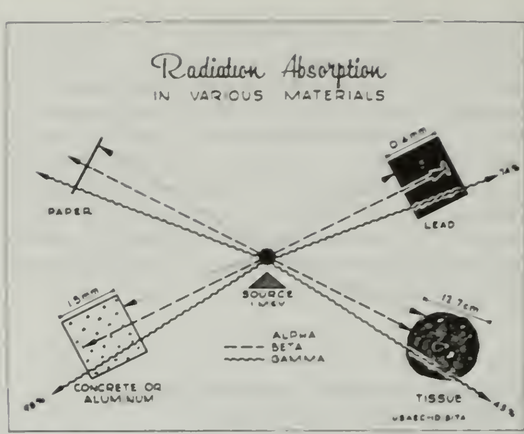
Waste Disposal

The following general rules apply:

1. The Code of Federal Regulations, Title 10, Part 20, or comparable State Regulations should be consulted for detailed information on the proper disposal of quantities of liquid source materials that exceed the quantities that are termed "generally licensed." (See also NCRP Report No. 30.)

2. When work with generally licensed quantities of liquid sources has been completed, they may be disposed of in the sanitary sewer system if flushed away with a considerable amount of extra water.

3. The final disposal of solid radioactive waste may require the use of a licensed commercial disposal service. If there is doubt, a radiological physicist, a health physicist, or the local health agency should be consulted.



4. Solid sources of short-lived isotopes may be stored for decay to possibly 1/10 of the permissible exempt value, and then disposed of, singly, in the ordinary trash. (Ref. 8, p. 452.)

5. Never throw long-lived solid radioactive wastes into the ordinary trash receptacles. You have no control over their final destination.

Some question may arise as to the method of disposal of various uranium and thorium salts that have been placed in solution. The method is probably best illustrated by an example.

Example: In *E6-3* in the *Handbook,* an acidic solution of thorium nitrate is prepared by dissolving 5 g of thorium nitrate in 10 mL of 2 molar nitric acid. During the course of the experiment, the solution is further diluted with another 10 mL portion of 2 molar nitric acid. The end result is that, for each lab group, the instructor is faced with the problem of safely disposing of a solution of 5 g of thorium nitrate dissolved in 20 mL of liquid. How can safe disposal be accomplished?

Procedure:

Thorium nitrate: $Th(NO_3)_4 \cdot H_2O$

Formula weight: 522.146

Grams of thorium in a 5-g sample of thorium nitrate =

$$5 \frac{(232)}{552.146} = 2.1 \text{ g}$$

Activity of the Th^{232} sample:

$= (\text{mass of } Th^{232})(\text{specific activity of } Th^{232})$[1]

$$= (2.1 \text{ g}) \left(1.11 \times 10^{-7} \frac{c}{g}\right) \left(10^6 \frac{\mu c}{c}\right)$$

$= 2.33 \times 10^{-1}\ \mu c$

The radioactivity concentration guide permits a concentration of Th^{232} in water $\leq 5 \times 10^{-5}\ \mu c/\text{mL}$.[2]

Notes [1]The specific activity of various isotopes can be found in the Radiological Health Handbook (See Ref 10 p 452.)
[2]From Appendix B of Title 10 Ch 1 Part 20 of the Code of Federal Regulations

$2.33 \times 10^{-1}\ \mu c/5 \times 10^{-5}\ \frac{\mu c}{mL}$

Dilution factor:

$= 4.66 \times 10^3$ mL

$= 5$ L

Therefore, each sample of 5 g of thorium nitrate in 20 mL of liquid should be flushed down the sewer with about 5 L of water.

SUMMARY

1. Some properties of α, β, and γ radiations:

	Charge	Relative Specific Ionization	Approximate Range: Air	Soft Tissue	Bone
α	+2	2500	few cm	several μ	few μ
β	−1	100	several m	few mm	several μ
γ	0	0	indefinite	indefinite	indefinite

2. The ingestion of any type of radioactive material must be avoided at all costs. High specific activity α sources are the most dangerous.

3. External radiation exposures can be minimized by controlling:

(a) quantity and type of radiation source
(b) length of exposure
(c) distance
(d) shielding

4. Internal radiation exposures can be controlled by reducing the probability of isotopes entering the body by:

(a) inhalation
(b) ingestion
(c) injection
(d) absorption through the skin

5. Experiments should be carefully planned and carried out to minimize both accidents and exposure time.

6. Our bodies have no built-in warning system that tells us when we are in a high radiation area. Therefore, radiation areas must be clearly marked, the strength of the source known, and any hazards clearly understood by both teacher and student.

7. The Department of Energy places limits on the quantity of radioisotopes most teachers can possess or use at any one time. Because of this limitation, injurious doses of radiation could be received by a high school student in a laboratory situation only as a result of gross carelessness, or through ignorance of the presence of radiation.

8. The guidelines for the use of radioisotopes in the high school classroom are set forth in NCRP Report No. 32. These guidelines were adhered to in this article. Reference should be made to this report if questions arise which this article does not answer.

9. Radioactivity should be respected but not feared. Fear implies a lack of knowledge and understanding of the subject. Good instruction on the part of the teacher will win respect for both oneself and the subject.

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PROJECT PHYSICS

TESTS

Unit 1 / Concepts of Motion — TEST A

Directions

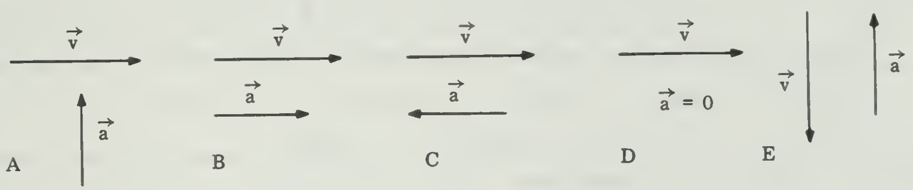
This test consists of 15 multiple-choice questions and six problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants and equations that may be useful in the test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. The arrows show the direction of the velocity and acceleration vectors for a car at five separate instants of time.

Which diagram applies to the car while it is turning a corner?



2. Several cars are racing on an oval track. Which of the following statements is correct for every racing car after it completes exactly one lap of a race?

A. Its acceleration is the same as when it crossed the starting line.
B. Its speed is the same as when it crossed the starting line.
C. Its displacement from the starting line is zero.
D. Its acceleration has not changed since it crossed the starting line.
E. Its velocity has not changed since it crossed the starting line.

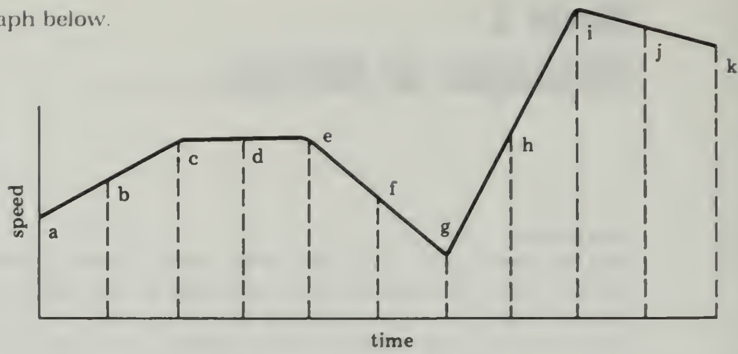
3. ALL EXCEPT ONE of the following require the application of a net force. Which one is the exception?

A. to change an object from a state of rest to a state of motion
B. to maintain an object in motion at a constant velocity
C. to change an object's speed without changing its direction of motion
D. to maintain an object in uniform circular motion
E. to change an object's direction of motion without changing its speed

4. Which one of the following statements correctly describes a satellite orbiting about the earth?

A. The acceleration and velocity of the satellite are in roughly the same direction.
B. There is no force acting on the satellite.
C. The velocity of the satellite is constant.
D. The satellite must fall back to earth when its fuel is gone.
E. The satellite is always accelerating toward the earth.

Questions 5 and 6 refer to the graph below.



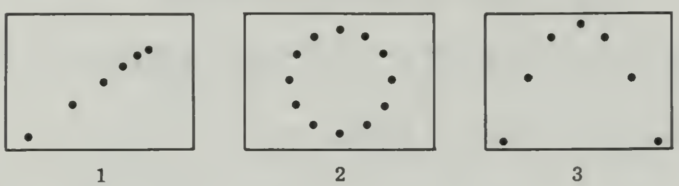
5. The magnitude of the acceleration is greatest in the time interval

A. a to c.
B. c to e.
C. e to g.
D. g to i.
E. i to k.

6. The speed is greatest at the time corresponding to point

A. c B. g C. h D. i E. k

7. The following diagrams represent stroboscopic photographs of a ball. The strobe rate is constant and is the same for all three pictures.



Which of the photographs could be produced with a stationary ball and a moving camera?

A. none B. 1 only C. 2 only D. 1 and 2 only E. 1, 2, and 3

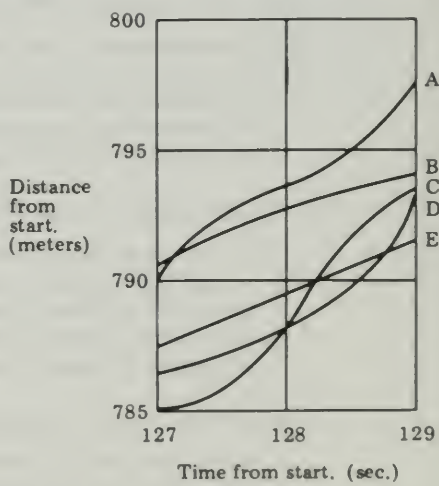
Questions 8 and 9 refer to the graph and the statement below. The graph shows the positions of five sprinters near the end of an 800-m race.

8. The average speed of sprinter C in the time period 127 sec to 129 sec from the start was approximately

A. 4 m/sec.
B. 6.5 m/sec.
C. 8 m/sec.
D. 9 m/sec.
E. 13 m/sec.

9. Which sprinter runs with uniform speed during the time period shown?

A. sprinter A
B. sprinter B
C. sprinter C
D. sprinter D
E. sprinter E



10. A golf ball is hit toward the pin from a point on the same level as the pin and 99 m away. it strikes the ground near the pin. Assuming that air resistance had no effect on the ball's path, what is the best estimate of the location at which it reached the highest point in its path?

A. within 27 m of where it was hit
B. about halfway to the pin
C. approximately two-thirds of the way to the pin
D. almost directly over the pin
E. No estimate is possible. The data are not sufficient for a decision.

Questions 11 and 12 refer to the following, which is a hypothetical report submitted by an astronaut about a space maneuver intended to link two capsules:

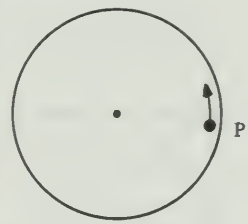
1. "At 5:01:00 A.M. we activated Rocket Z-4 for 10 sec.
2. The thrust gauge showed that the rocket produced a force of 77 N.
3. Accordingly, we estimated a velocity toward the target vehicle of 4 m/sec.
4. We expected to touch the target vehicle in 300 sec, at 5:06:10.
5. Actually, we touched at 5:06:28."

11. Which of the sentences above gives the result of a computation done by the astronaut that involved the use of Newton's second law?

A. 1 only B. 2 only C. 3 only D. 4 only E. 2, 3, and 4 only

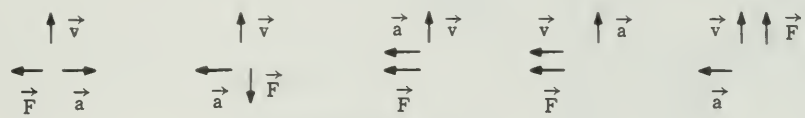
12. Which of the sentences above describes how the astronaut changed conditions to perform the maneuver?

A. 1 only B. 4 only C. 5 only D. 1 and 4 only E. 1 and 5 only



13. A child is riding on a merry-go-round, as shown at the right. When the child is at point P, which set of vectors shows the direction of the velocity $\vec{v}$, the acceleration $\vec{a}$, and the centripetal force $\vec{F}$ acting on the child?

A. set A
B. set B
C. set C
D. set D
E. set E

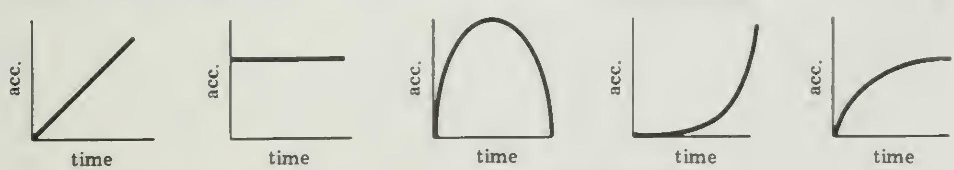


A B C D E

14. A steel ball rolls down an inclined plane. Which graph best represents how the distance traveled changes with time?

A B C D E

15. A propeller rotates at a constant rate. If we consider the two ends of the propeller, which graph best represents how the magnitude of their acceleration changes with time?

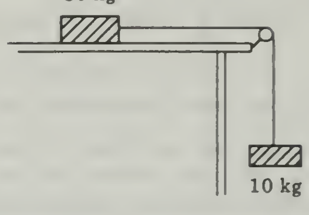


A B C D E

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following four questions.

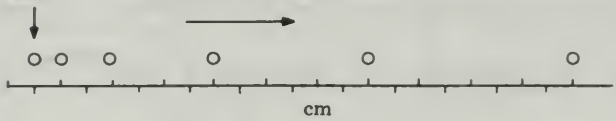


1. A 30-kg block lies on a frictionless table, and is connected to a 10-kg block by a rope passing over a frictionless pulley, as shown in the diagram.

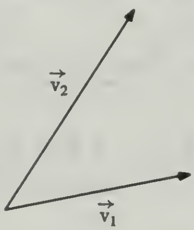
   (a) What is the acceleration of the 10-kg block?
   (b) What is the acceleration of the 30-kg block?

2. A car accelerates away from a stoplight. Use this example of a moving object to explain the difference between average speed and instantaneous speed.

3. The diagram below illustrates the motion of a ball as if it were recorded by a camera whose shutter remained open and whose only source of light was a stroboscopic lamp flashing 10 times/sec. On this diagram, 1.0 cm represents 1.0 cm in the laboratory. What was the acceleration of the ball?



4. Find $\Delta\vec{v}$, where $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.

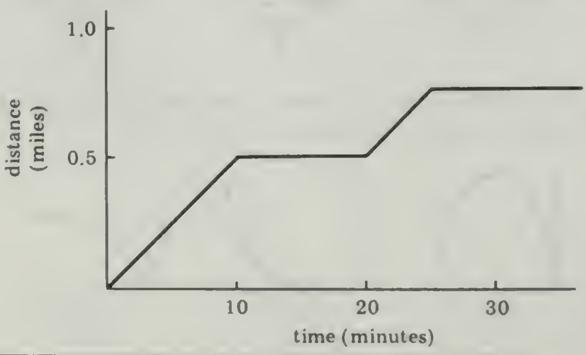


Group Two

Answer ONE of the following two questions.

5. Galileo believed that the proper language with which to describe nature is mathematics. How is our understanding of natural phenomena aided by describing what we observe in mathematical terms?

6. A man goes for a walk. Write a description of his motion that includes numerical values of time, distance, and speed, based on the information contained in the graph below.



Unit 1/ Concepts of Motion

TEST B

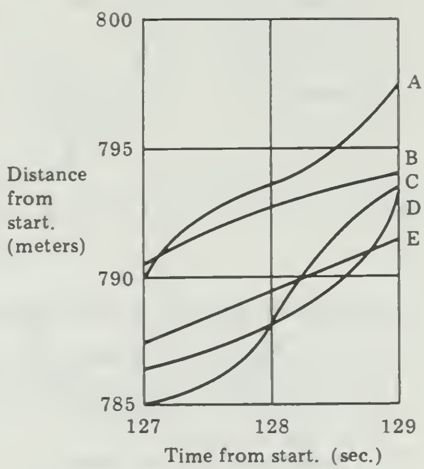
Directions

This test consists of 15 multiple-choice questions and six problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants and equations that may be useful in the test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

Questions 1 and 2 refer to the figure at right, which shows the positions of five runners near the end of an 800-m race.



1. Which sprinter was ahead after exactly 127 sec?

A. sprinter A
B. sprinter B
C. sprinter C
D. sprinter D
E. sprinter E

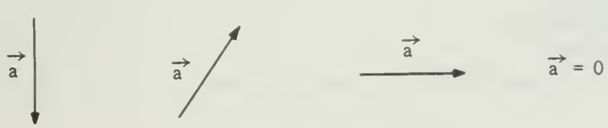
2. With what average speed did E run in the interval 127–129 sec?

A. 1 m/sec
B. 2 m/sec
C. 4 m/sec
D. 5 m/sec
E. 11 m/sec

3. Which of the following four statements describes the motion of a bullet that has been fired by a supersonic jet fighter plane flying parallel to the ground? (Neglect air resistance.)

A. uniform straight-line motion
B. uniformly accelerated straight-line motion
C. circular motion
D. projectile motion

4. Which of the following four diagrams represents the acceleration of a golf ball the instant after it leaves the face of a golf club?



A B C D

5. An ice skater gives a sudden push to a sled that sends it sliding away from her. Consider the following statements. (Assume friction is negligible.)

1. The force exerted on the sled by the skater is equal in magnitude to the force exerted on the skater by the sled.
2. During the push, the acceleration of the skater is equal in magnitude to the acceleration of the sled.
3. The skater will accelerate for the same length of time as the sled.

Which of the statements is true if the skater and the sled have the same mass?

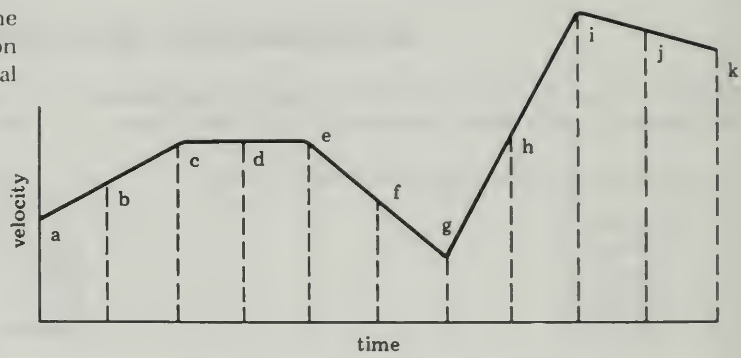
A. 1 only B. 2 only C. 3 only D. 2 and 3 only E. 1, 2, and 3

6. A child is riding on a merry-go-round that is rotating at a constant rate. The child has

A. constant velocity.
B. constant acceleration.
C. constant speed.
D. constant acceleration and speed.
E. constant velocity, acceleration, and speed.

7. In the graph at the right, the magnitude of the acceleration is greatest in the time interval

A. a to c
B. c to e
C. e to g
D. g to i
E. i to k



8. ALL EXCEPT ONE of the following require a net unbalanced force. Which is the exception?

A. to set into motion an object that is initially at rest
B. to maintain an object in a state of constant velocity
C. to maintain an object in a state of uniform circular motion
D. to stop a moving object
E. to change an object's direction of motion while keeping its speed constant

9. The distance d traveled by an object is given by the equation $d = \frac{1}{2}at^2$, when the object

A. is moving in a circle.
B. has a constant velocity.
C. starts from rest and accelerates uniformly.
D. is thrown upward.
E. is thrown downward.

10. This test paper is sitting at rest on your desk. Which of the following statements best describes this situation?

A. There are no forces acting on your paper.
B. Your paper is at rest in any coordinate system.
C. Your paper exerts no force on the desk.
D. There are many forces acting on your paper, but they balance each other.

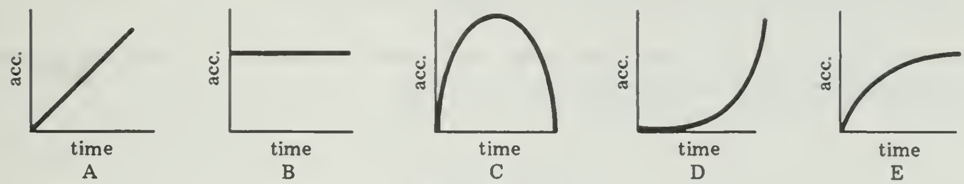
11. A satellite is in orbit around the earth. In the absence of air friction, which of the following statements is necessarily true?

A. The acceleration and velocity of the satellite are in approximately the same direction.
B. There is no force acting on the satellite.
C. The velocity of the satellite is constant.
D. The satellite must fall back to earth when its fuel is gone.
E. The satellite always accelerates towards the earth.

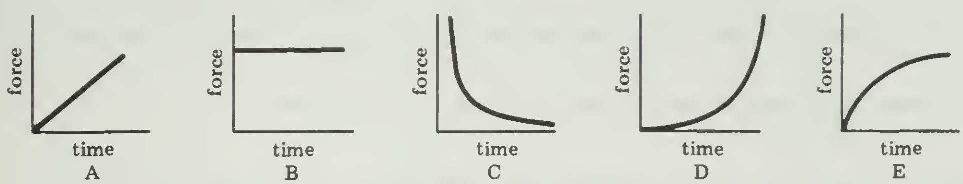
12. If you must choose between two hypotheses, which of the following is the best reason for selecting hypothesis 1 rather than hypothesis 2?

A. Hypothesis 1 is more in agreement with the observed facts.
B. Hypothesis 1 contains more mathematics.
C. Hypothesis 1 is newer.
D. Hypothesis 1 is more easily understood.
E. Several people think hypothesis 1 is more likely to be correct.

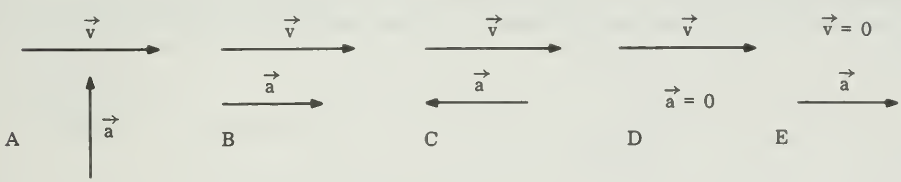
13. A rock is thrown into the air. Which graph represents how the magnitude of its acceleration changes with time while it is in the air? (Neglect air resistance.)



14. A propeller blade rotates at a constant rate. Which graph best represents how the magnitude of the force on one tip of the propeller changes with time?



15. In the diagrams shown below, arrows show the direction of the velocity and acceleration vectors for a car at five separate instants of time. Which diagram represents the car starting from rest?



PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following four questions.

1. Upon observing a rolling object, you obtained the following values for d/t^2, where d is the distance rolled and t the elapsed time.

d/t^2
0.00185
0.00183
0.00192
0.00182
0.00187
0.00182

Do these data justify the conclusion that d/t^2 is a constant? Explain.

2. Consider the motion of a flare dropped from an airplane flying at constant velocity. Describe this motion as seen by observers in the airplane and on the ground.

3. What is the difference between the concepts "weight" and "mass"?

4. An object resting on a level, frictionless surface on the earth is subjected to a horizontal force equal to its weight. What is the magnitude of its acceleration?

Group Two

Answer ONE of the following two questions.

5. Joe and Maria are arguing about uniform acceleration. Joe says that acceleration means "the longer you go, the faster you go." Maria states that acceleration means "the farther you go, the faster you go."

(a) Present their points of view in terms of an equation or equations.
(b) Who is right and why?

6. Galileo approached scientific problems in ways different from Aristotle and the Scholastics. Two of these differences are listed below. Select one and explain why you think it was important to the development of physics.

(a) insistence that experiment and observation must be quantitative, not just qualitative
(b) abstraction from real situations to idealized ones that show the laws of nature in their simplest form

Unit 1/ Concepts of Motion

TEST C

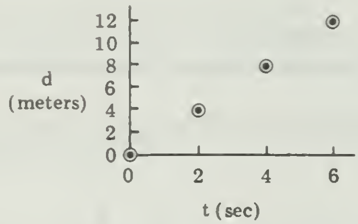
Directions

Answer ALL 40 multiple-choice questions by marking the letter corresponding to the one best answer.

The numerical values of some physical constants and equations that may be useful in the test are given at the end of the tests for this unit.

1. An experiment yielded the data given in the table and graph below.

t (seconds)	d (meters)
0	0
2	4
4	8
6	12



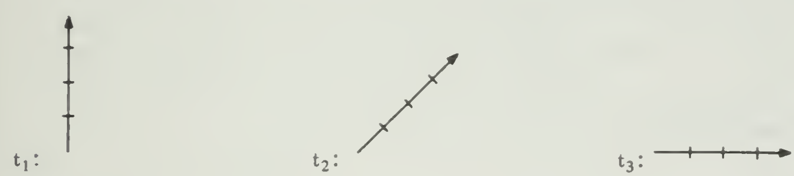
If these data are expressed as an equation, $d = kt$, the value of k is

A. 1 m/sec. B. 1 sec/m. C. 2 m/sec. D. 2 sec/m. E. 0.5 m/sec.

2. Referring to his work, Newton wrote, "If I have seen further than others, it is because I have stood on the shoulders of giants." Who of the following was one of the "giants" whose work on motion immediately preceded Newton's?

A. Fermi B. Galileo C. Simplicio D. Aristotle

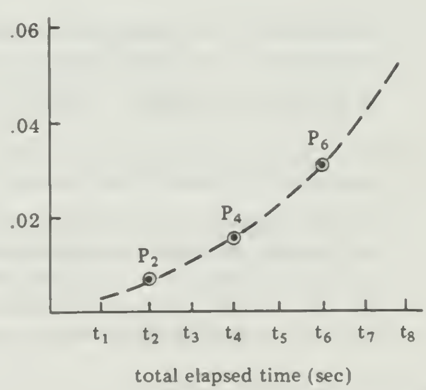
3. The arrows drawn below represent the velocity vectors of a 747 jet at three successive times.



We may conclude that the jet was

A. changing direction.
B. speeding up.
C. slowing down.
D. maintaining a constant velocity.

Questions 4 and 5 refer to the following statement and graph: The graph at the right shows the relationship between the time and the total distance traversed by a glider moving on a nearly frictionless air track. Points P_2, P_4, and P_6 represent the experimental measurements. The dotted curve is a smooth curve drawn through these points.



4. If the values of the total distance traversed at times t_5, t_6, and t_8 are arranged in order of uncertainty with the most *uncertain* value of distance first, the order is

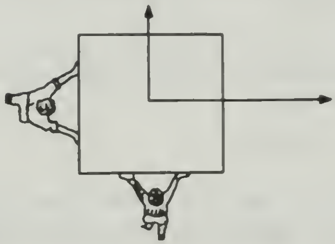
A. t_5, t_6, t_8.
B. t_8, t_6, t_5.
C. t_5, t_8, t_6.
D. t_6, t_5, t_8.
E. t_8, t_5, t_6.

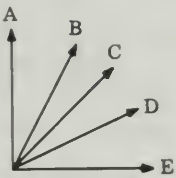
5. The slope of the curve at t_4 represents the

A. total distance traversed.
B. instantaneous speed.
C. acceleration.
D. rates of change of speed.
E. average speed.

6. Two people push on a box resting on a smooth, level floor as indicated in the diagram below. The lengths of the arrows are drawn proportional to the magnitude of the force each person exerts on the box.

In the diagram below, which arrow indicates the direction in which the box will start to move?





7. A satellite is in a circular orbit around a planet. The satellite's period of revolution T and the radius of the orbit R are known. Which of the following equations must you use to compute its acceleration?

A. $d = \frac{1}{2}aT^2$ only

B. $v = \dfrac{2\pi R}{T}$ and $v = aT$

C. $v = \dfrac{2\pi R}{T}$ and $a = \dfrac{v^2}{R}$

D. $d = \frac{1}{2}aT$ and $a = \dfrac{v^2}{R}$

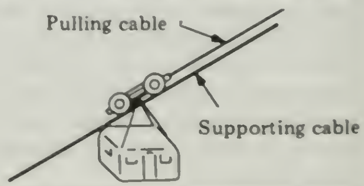
E. $v = aT$ and $a = \dfrac{v^2}{R}$

8. A student pushes a puck on a frictionless horizontal surface with a force of 10 N. The resulting acceleration is 4.0 m/sec². What is the mass of the puck?

A. 0.4 kg B. 2.5 kg C. 4.0 kg D. 10 kg E. 40 kg

9. The diagram at right shows a cable car supported by an overhead cable and pulled uphill by a second cable. Which of the following forces is zero when the cable car moves with constant velocity?

A. new unbalanced force on the car and carriage
B. frictional force on the wheels of the carriage
C. force of gravity on the car and carriage
D. force exerted by supporting cables
E. force exerted by the cable that pulls the car upward



10. In ALL EXCEPT ONE of the following situations, an object is being accelerated. Which one is the exception?

A. The object changes direction without changing speed.
B. The object changes speed without changing direction.
C. The object maintains speed and direction.
D. The object maintains uniform circular motion.
E. The object moves in the trajectory of a projectile.

Questions 11 and 12 refer to the following situation.

During a planned maneuver in space flight, a free-floating astronaut pushes a free-floating instrument package. The mass of the astronaut is greater than that of the instrument package.

11. The force exerted by the astronaut on the instrument package

A. is equal to the force exerted by the package on the astronaut.
B. is greater than the force exerted by the package on the astronaut.
C. is less than the force exerted by the package on the astronaut.
D. is equal to zero.
E. may be greater than, less than, or equal to the force exerted by the package on the astronaut; one cannot tell with the information given here.

12. During the push

A. the magnitude of the acceleration of the astronaut is greater than that of the instrument package.
B. the magnitude of the acceleration of the astronaut is smaller than that of the instrument package.
C. neither astronaut nor instrument package is accelerated.
D. the accelerations of each are equal in magnitude but opposite in direction.
E. the accelerations of each are equal in magnitude and in the same direction.

13. In *Two New Sciences*, Salviati, speaking for Galileo, defines "a motion to be uniformly accelerated, when starting from rest it acquires, during equal time intervals, equal increments of speed." This definition is important because it

A. convinces Simplicio, the spokesman for Aristotelian physics.
B. corresponds closely to the way real objects fall near the surface of the earth.
C. explains the cause of acceleration of falling objects.
D. is correct regardless of the air resistance of falling objects.
E. is the only definition that can be tested by experiment.

14. ALL EXCEPT ONE of the following statements would be operational definitions of 1 sec of time. Which one is the exception?

One second is

A. a little more time than there is between the pulse beats of most people.
B. the shortest unit of time.
C. 1/86,400 of the time it takes the earth to make one rotation about its axis.
D. the length of a time interval a little shorter than it takes a student to answer this question.

Questions 15 and 16 refer to the following statement.

Scientists on the imaginary planet Q have defined a unit of length, the "lar," to be the distance between two mountain peaks on the surface of the planet. The unit of time on the planet Q is called the "tik" and is defined as the average interval between pulse beats of the king.

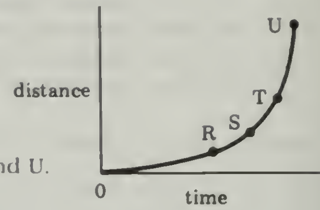
15. What units would express acceleration on planet Q if acceleration were defined as it is on earth?

A. lar/tik B. lar/sec C. lar^2/sec D. tik/lar^2 E. lar/tik^2

16. If the distance between the cities Zytropolis and Elany on planet Q is 20 lars, what would your average speed be if you made the trip in 100 tiks?

A. 0.2 lars/tik B. 0.1 tiks/lar C. 5 tiks/lar D. 5 lars/tik E. 100 tiks/lar

17. The graph at the right represents the distance traveled by an automobile as a function of time. The instantaneous speed at the time corresponding to point S is best approximated by the slope of a straight line drawn between points



A. S and T. B. 0 and S. C. R and S. D. R and T. E. R and U.

18. A subway car is at rest in a subway station. A person sitting in the car flips a dime into the air: the dime hits the floor. Later, when the car is moving over a straight, level section of track at a high, constant speed, the person flips the dime again in exactly the same way. Where does the dime hit the floor?

A. at the same spot on the floor as before
B. ahead of where it hit before
C. behind where it hit before

Questions 19 and 20 refer to the following statements concerning Galileo's work with balls rolling down inclined planes. This work led to the acceptance of the idea that falling objects accelerate uniformly. The quotations are from *Two New Sciences*.

1. If speed during fall increases with time, $\frac{d}{t^2}$ is constant.
2. "We took a piece of wooden scantling, about 12 cubits long, half a cubit wide, and three finger breadths thick. In its top edge we cut a straight channel."
3. "Having raised the scantling in a sloping position by raising one end some one or two cubits above the other, we let the ball roll down the channel."
4. "We always found that the distances traversed were to each other as the squares of the times."
5. Since for a rolling ball $\frac{d}{t^2}$ is constant, $\frac{d}{t_2}$ is constant for a falling ball also.

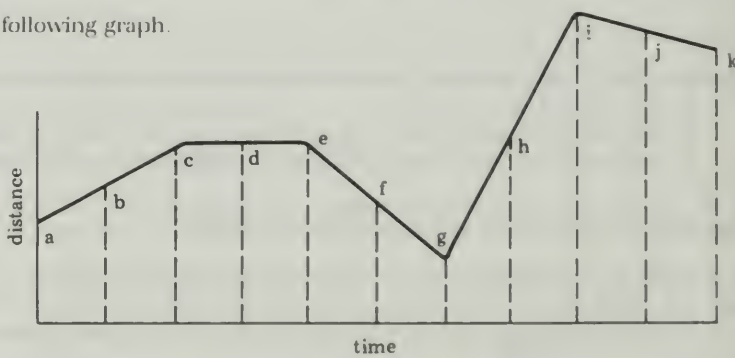
19. Which of the statements are assumptions made by Galileo?

A. 1 only B. 4 only C. 5 only D. 1 and 4 only E. 4 and 5 only

20. Which statement presents experimental results?

A. 1 only B. 4 only C. 5 only D. 2 and 3 only E. 1 and 5 only

Questions 21 and 22 refer to the following graph.



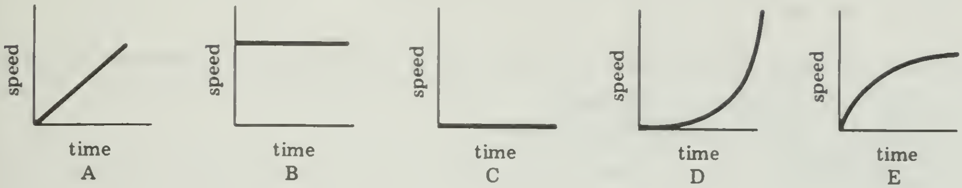
21. The greatest distance is traveled between the times corresponding to points

A. a and c. B. c and e. C. e and g. D. g and i. E. i and k

22. The velocity v is greatest between times corresponding to points

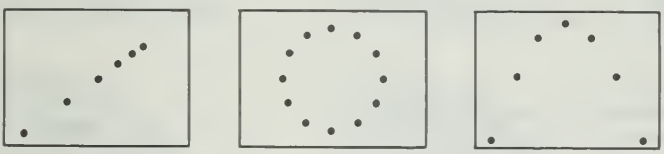
A. a and k. B. c and e. C. e and g. D. g and i. E. i and k.

23. A cart, initially at rest, is pulled with a constant, unbalanced force. Which graph best represents how the speed of the cart changes with time?



A. graph A B. graph B C. graph C D. graph D E. graph E

For questions 24 and 25, use the following figures which represent stroboscopic photographs of a moving ball. The strobe rate is constant and is the same for all three "photographs."



1 2 3

24. Which of the "photographs" could have been produced with the camera in motion and the ball fixed in position?

A. none B. 1 only C. 2 only D. 1 and 2 only E. 1, 2, and 3

25. If the camera is fixed in position, which of the "pictures" show a ball being acted upon by a net unbalanced force?

A. 1 only B. 3 only C. 1 and 3 only D. 2 and 3 only E. 1, 2, and 3

26. Which of the following increases with time if an object moves with uniform velocity?

A. instantaneous velocity B. average velocity C. acceleration D. direction E. displacement

27. A sprinter reaches top speed 3 sec after the start of a race. In those 3 sec, she moves 18 m. Assume that she accelerates uniformly. What is her acceleration?

A. 2 m/sec^2 B. 3 m/sec^2 C. 4 m/sec^2 D. 7 m/sec^2 E. 18 m/sec^2

Questions 28 and 29 refer to the following statement and table.

Main Street in Centerville is crossed by streets called 1st St., 2nd St., 3rd St. . . . 46th St. Blocks between the numbered streets are equally long.

Five cars are traveling along Main Street, and their locations are recorded at 5-min intervals, as shown in the following table:

	10:00 A.M.	*10:05 A.M.*	*10:10 A.M.*	*10:15 A.M.*
car A	25th	30th	35th	40th
car B	30th	25th	15th	10th
car C	1st	2nd	5th	10th
car D	9th	10th	20th	38th
car E	35th	33rd	23rd	20th

28. Which car traveled with the greatest average speed during the period described?

A. car A B. car B C. car C D. car D E. car E

29. Assuming that all cars started from rest at 10:00 A.M., which car could have traveled with uniform acceleration during the entire period described?

A. car A B. car B C. car C D. car D E. car E

Use one of the following statements to describe the motion mentioned in questions 30 and 31:

A. straight-line motion at uniform speed
B. uniformly accelerated straight-line motion
C. circular motion
D. projectile motion

30. The motion of a shirt in a washing machine in the middle of the "spin-dry" cycle

31. The motion of a bicyclist seen from a passing car when each is moving with constant speed on a straight, horizontal road

32. ALL EXCEPT ONE of the following conditions must apply if one is to use the equation $d = \frac{1}{2}at^2$. Which one is the exception?

A. The motion must be free fall.
B. The acceleration must be constant in magnitude.
C. The initial velocity of the body must be zero.
D. Displacement must be measured from the point where motion begins.
E. Acceleration must be constant in direction.

33. An 80-kg fireman slides down a pole in a fire station. His grip on the pole causes a frictional force of 240 N opposing his fall. What is the approximate value of his acceleration toward the floor below?

A. 13 m/sec^2 B. 10 m/sec^2 C. 8 m/sec^2 D. 7 m/sec^2 E. zero

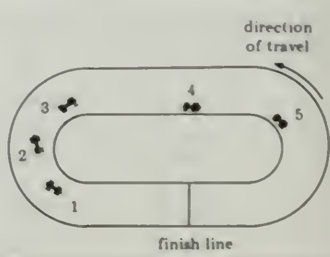
34. Two barrels roll off the deck of a barge and describe identical paths from the edge of the deck to the water. Which of the following conclusions is necessarily true?

A. Both have the same mass.
B. Both have the same weight.
C. Both moved with the same velocity at the instant they fell overboard.
D. Both were pushed with the same force across the deck before they fell overboard.
E. They were chained together.

35. Aristotle's scientific beliefs were different from Galileo's. Which one of the following statements would be in agreement with those of Aristotle?

A. Mathematics has no important place in scientific thought.
B. An object on earth will move at a constant speed if there are no unbalanced forces acting on it.
C. Different objects near the surface of the earth fall freely with the same acceleration.
D. Objects on the earth and heavenly bodies obey the same basic laws of motion.

36. Just before the end of a 25-lap auto race, the five leading cars moving in a counterclockwise direction are in the positions shown in the diagram. Which of the following statements is necessarily true?

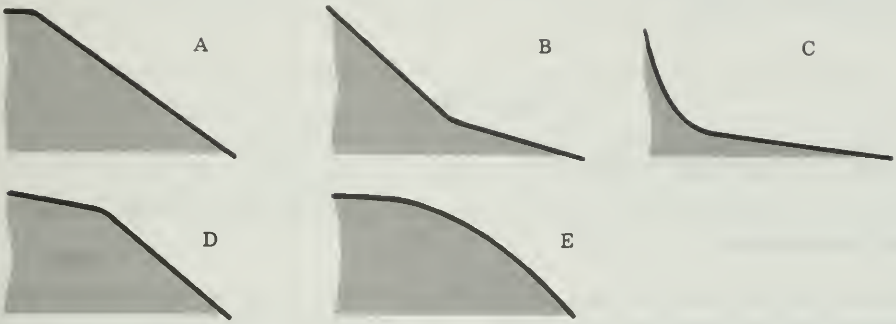


A. Car 5 is traveling with the lowest speed.
B. Car 2 can cross the finish line without changing velocity.
C. Car 4 can cross the finish line without changing speed.
D. Car 1 will finish first.
E. All 5 cars are traveling with the same velocity.

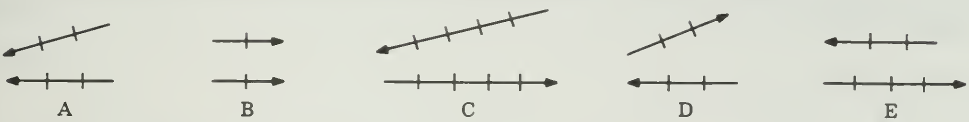
37. Measurements made on a ball rolling down a hill of unknown shape provided the following data:

Time (sec)	*Instantaneous Speed (m/sec)*
0	0
1	6
2	12
3	18
4	20
5	22
6	24

Which of the following diagrams could represent the shape of the hill?

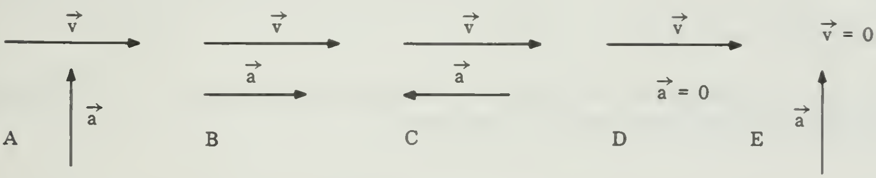


38. Which of the following pairs of vectors has the resultant of largest magnitude?



A. pair A B. pair B C. pair C D. pair D E. pair E

39. The arrows in the diagrams below show the directions of the velocity and acceleration vectors that apply to a car at five separate instants of time.



ALL EXCEPT ONE of the diagrams above show an instant at which the velocity is changing. Which is the exception?

A. diagram A B. diagram B C. diagram C D. diagram D E. diagram E

40. A satellite is in a circular orbit around the earth. Which of the following statements must be true?

1. The speed is constant.
2. The velocity is constant.
3. The period is constant.

A. 1 only B. 2 only C. 1 and 3 only D. 2 and 3 only E. 1, 2, and 3

Unit 1/ Concepts of Motion — TEST D

Directions

This test consists of eight questions in two groups. Answer only FOUR of the five questions in Group One, and only TWO of the three questions in Group Two. Spend about 5 min on each of the questions from Group One, and 10 min on each of the questions from Group Two.

The numerical values of some physical constants and equations that may be useful in the test are given at the end of the tests for this unit.

Group One

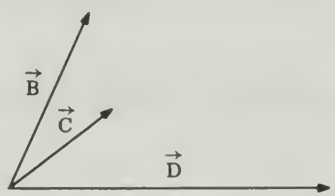
Answer FOUR of the five questions in this group.

1. A satellite is in a circular orbit. Describe the force acting on the satellite, the satellite's acceleration, its velocity, and its speed.

2. At time zero, a boy mounted a bicycle and rode off, accelerating uniformly at 2 m/sec^2 for 3 sec. He continued at constant speed for an additional 5 sec and then stopped abruptly. Plot the boy's motion on a speed-versus-time graph.

3. List three experimental techniques now available for the study of motion that were not available to Galileo in 1632.

4. Find $\vec{A}$, where $\vec{A} = \vec{B} + \vec{C} + \vec{D}$.



5. To an Aristotelian, it seems clear that a force is necessary to maintain uniform motion. Comment on this statement from the point of view of a Newtonian.

Group Two

Answer TWO of the three questions in this group.

6. An airplane is flying horizontally over the ocean at a speed of 200 m/sec and at an altitude of 2 000 m. The pilot drops a flare. (Neglect air resistance.)

   (a) How many seconds after release does the flare hit the water?
   (b) At what distance from point P, directly under the point of release, does the flare strike the water?

7. Acceleration is the rate of change of speed. Instantaneous acceleration is the slope of a speed–time graph at a point. Suppose we call the rate of change of acceleration *surge*.

(a) What is the algebraic expression defining average surge?
(b) What are the units of surge?
(c) How can we calculate instantaneous surge?

8. In *Two New Sciences*, Galileo uses the character Salviati to present his own views concerning free fall. At one point in the discussion Salviati states:

If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. . . . But if this is true, and if a large stone moves with a speed of, say, eight while a smaller moves with a speed of four, then when they are united, the system will move with a speed of less than eight; but the two stones when tied together make a stone larger than that which before moved with a speed of eight. Since the heavier body moves with less speed than the lighter . . . you see how, from the premise that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly.

Salviati based the preceding argument on several assumptions that are not necessarily valid.

(a) State one of these assumptions.
(b) Consider that this assumption is not valid. Propose a more appropriate assumption.
(c) Based on your assumption, what conclusions can be drawn regarding the rate of fall of the two stones that are tied together?

Unit 1

Physical Constant:

Acceleration of gravity on the surface of the earth (approximate value to be used in these tests) $a_g = 10$ m/sec^2

Equations:

$$d = vt \qquad v = at$$

$$d = \tfrac{1}{2} at^2 \qquad d_y = k(d_x)^2$$

$$v = \frac{2\pi R}{T} \qquad a = \frac{v^2}{R}$$

$$\vec{F} = m\vec{a} \qquad F_c = \frac{mv^2}{R}$$

Unit 2 / Motion in the Heavens — TEST A

Directions

This test consists of 15 multiple-choice questions and seven problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants and equations that may be useful in the test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. To ancient observers, the principal difference between the planets and the stars was that the planets appeared

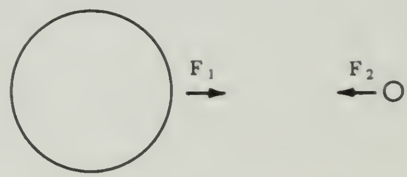
A. brighter.
B. more like the earth.
C. to wander among the other stars.
D. closer to the earth.
E. to travel around the sun.

2. Which of the following statements must be part of any heliocentric theory?

A. The planets revolve around the sun.
B. The sun is a sphere.
C. The earth is a sphere.
D. The planets revolve around the earth.
E. The earth turns on its axis.

3. If F_1 is the magnitude of the force exerted on the sun by the earth and F_2 is the magnitude of the force exerted on the earth by the sun, then

A. F_1 is much greater than F_2.
B. F_1 is slightly greater than F_2.
C. F_1 is equal to F_2.
D. F_1 is slightly less than F_2.
E. F_1 is much less than F_2.



4. Which one of the following men is famous for the decision to abandon Plato's association of heavenly bodies with "uniform motion in perfect circles"?

A. Aristotle
B. Copernicus
C. Kepler
D. Galileo
E. Tycho Brahe

5. Galileo gathered a great deal of evidence that was at odds with the medieval view of the universe. ALL EXCEPT ONE of the following are examples of this evidence. Which is the *exception?*

A. the discovery of a new star in 1604
B. the rough appearance of the moon's surface
C. the motion of four luminous objects around Jupiter
D. the moon-like phases of the planet Venus
E. the wanderings of the planets among the stars

6. Two sacks of marbles are hung 1 m apart. Which of the following would approximately double the gravitational force that one sack of marbles exerts on the other sack?

A. Double the number of marbles in one sack.
B. Double the number of marbles in both sacks.
C. Move them closer, to one-half the separation.
D. Move them farther apart, to twice the separation.
E. Move them farther apart, to four times the separation.

7. ALL EXCEPT ONE of the following statements are acceptable. Which is the *exception?*

A. The earth is moving fastest when closest to the sun.
B. The path of the earth lies in a plane that passes through the sun.
C. A line drawn from the sun to the earth sweeps over the same area from March 21 to March 23 as it does from December 21 to December 23.
D. The sun is at the exact center of the earth's path.

8. Assume that the earth suddenly shrank to one-half its original diameter, but that its mass remained unchanged. Under these circumstances, the weight of a person standing on its surface would be

A. four times as great.
B. twice as great.
C. the same.
D. one-half as great.
E. one-fourth as great.

9. A friend tells you that the earth is fixed in space and that the sun revolves about it. Which one of the following facts contradicts his hypothesis?

A. Each day the sun rises in the east and sets in the west.
B. During the night the stars appear to move.
C. The sun makes one complete trip among the stars in 1 year.
D. Eclipses of the sun sometimes occur.
E. none of the above

10. Which one of the following was an important factor that worked against the acceptance of Copernicus' heliocentric solar system hypothesis in the sixteenth century?

A. When Venus was observed through a telescope, phases were seen.
B. Stellar parallax had never been observed.
C. The calendar failed to keep pace with the seasons.
D. Galileo observed four satellites moving around Jupiter.
E. Venus had never been observed more than 48° from the sun.

11. ALL EXCEPT ONE of the following were among Tycho Brahe's contributions to astronomy. Which one is the *exception?*

A. He established an astronomical observatory.
B. He proposed an inverse square law of attraction for the solar system.
C. He developed a theory of the solar system.
D. He determined the limits of accuracy of his instruments.
E. He made very accurate observations of the positions of the heavenly bodies.

12. Assume that the following measurements were made on three planets revolving about a star. (Planets are listed in order of discovery.)

Planet	*Orbital Period*	*Mass*
Alpha	14 earth years	10 earth masses
Beta	188 earth years	17 earth masses
Gamma	50 earth years	½ earth mass

On the basis of Kepler's laws of planetary motion, these planets could be arranged in order of their increasing distances from the star. If the planets were listed in sequence, starting with the planet nearest the star, the would be arranged

A. Alpha, Beta, Gamma.
B. Beta, Gamma, Alpha.
C. Gamma, Alpha, Beta.
D. Beta, Alpha, Gamma.
E. Alpha, Gamma, Beta.

13. The following men made significant contributions to our present understanding of planetary motion:

1. Copernicus
2. Newton
3. Kepler

If the names of these men were arranged in the order of their contributions starting with the earliest first, they would be

A. 1, 2, 3. B. 2, 3, 1. C. 3, 1, 2. D. 1, 3, 2. E. 2, 1, 3.

14. Time-exposure photographs of stars show arcs of circles. An astronomer who believed the Ptolemaic theory of planetary motion would explain that these arcs result from

A. the rotation of the earth on its axis.
B. stellar parallax.
C. retrograde motion.
D. the inclination of the earth's axis.
E. the rotation of the starry sphere.

15. Which one of the following is evidence that supports the universality of Newton s law of gravitation?

A. Pairs of stars have been observed that move around each other in accordance with Kepler's laws
B. A few stars have moved from the positions given for them in Ptolemy's catalogue of stars.
C. Stellar parallax has been observed for several thousand stars.
D. A calendar reform was needed in the sixteenth century to keep months and seasons in agreement.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions.

1. Indicate briefly the major contributions to astronomy of

   (a) Ptolemy
   (b) Copernicus

2. With the aid of a diagram, briefly describe the experiment used by H. Cavendish to determine the value of G, the constant of universal gravitation in the equation

$$F_{grav} = G\frac{m_1 m_2}{R^2}.$$

3. The orbit of Mercury about the sun has a radius about one-third that of the orbit of the earth about the sun, while the period of Mercury is about 3 months. With the aid of a diagram of a heliocentric (sun-centered) system, explain the retrograde motion of Mercury as seen from the earth.

4. According to the theory of relativity, nature may be validly observed and described from all frames of reference. Why then have astronomers, in describing the motions of planets, preferred a heliocentric rather than a geocentric frame of reference?

5. Describe the daily and annual motion of the sun from a geocentric point of view.

Group Two

Answer ONE of the following two questions.

6. According to Newton, the moon is continually falling toward the earth. In what sense is the word "falling" used here? Explain your answer by relating the motion of the moon with the motion of a freely falling object near the surface of the earth.

7. Given Newton's assumption for the gravitational force due to the sun on a planet: $F \propto 1/R^2$, and given that the distance of fall of a planet is determined by $d = \frac{1}{2}at^2$, derive Kepler's third law in the form

$$\frac{(T_s)^2}{(T_p)^2} = \frac{(R_s)^3}{(R_p)^3}.$$

Unit 2 / Motion in the Heavens — TEST B

Directions

This test consists of 15 multiple-choice questions and seven problem-and-essay questions divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants and equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. In its orbit, the earth travels

A. fastest when it is nearest the sun.
B. fastest at night.
C. fastest around the time of the new moon.
D. with constant speed.
E. with zero speed, since the earth is stationary.

Select answers to questions 2 and 3 from the following list.

A. Ptolemy   B. Kepler   C. Copernicus   D. Tycho Brahe   E. Galileo

2. He tried to relate planetary distances to the five regular geometric solids.

3. He developed a model of the solar system in which the planets revolved around the sun but the earth remained motionless.

4. ALL EXCEPT ONE of the following are characteristics of geocentric models of the solar system. Which one is the *exception?*

A. The earth is at or near the center of the solar system.
B. The stars are at the greatest distance from the earth.
C. The sun moves daily around the earth.
D. The moon's motion is tied to the motion of the sun.
E. The stars move daily around the earth.

5. Kepler's three laws of planetary motion were

A. almost self-evident from Tycho Brahe's data of Mars' orbit.
B. little used in Newton's development of a general law of universal gravitation.
C. developed only after Kepler took imaginative steps from the available data.
D. widely discussed early in the seventeenth century.
E. used by Copernicus in deriving his heliocentric hypothesis.

6. Tycho Brahe's most important contribution to science was

A. the accurate observation of the positions of the stars and planets.
B. the discovery of a new star that changed its brightness.
C. the discovery of elliptical orbits.
D. his theory of planetary motions.

7. Galileo accumulated a great deal of evidence that was inconsistent with the medieval view of the universe. ALL EXCEPT ONE of the following are examples of this evidence. Which is the *exception?*

A. the discovery of a new star in 1604
B. the rough appearance of the moon's surface
C. the motion of four luminous objects around Jupiter
D. the moon-like phases of the planet Venus
E. the wanderings of the planets among the stars

8. A satellite with a television camera is placed in an orbit 38,400 km above the earth so that it remains exactly above the same point on earth at all times, with its camera pointed toward the earth. As seen from the sun the orbit of the satellite is

A. an ellipse with the sun at one focus.
B. an epicycle with its center on the orbit of the sun.
C. an epicycle with its center on the orbit of the earth.
D. a circle with the sun at the center.
E. a parabola constantly accelerated toward the earth.

9. Assume the earth suddenly became one-half its original diameter, but that its mass was unchanged. Under this assumption, the strength of the earth's gravitational pull on the moon would be

A. four times as great.
B. twice as great.
C. the same.
D. one-half as great.
E. one-fourth as great.

10. What is the acceleration due to gravity of a meteor at 2 earth radii from the center of the earth? Assume the acceleration due to gravity at 1 earth radius from the center of the earth to be 10 m/sec^2.

A. 2.5 m/sec^2 B. 5 m/sec^2 C. 7.07 m/sec^2 D. 10 m/sec^2 E. 20 m/sec^2

11. Which one of the following is evidence that supports the universality of Newton's law of gravitation?

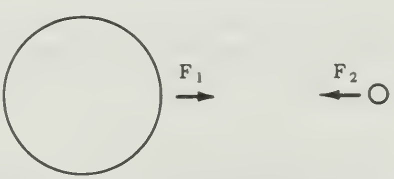
A. A few stars have moved from the positions given for them in Ptolemy's catalogue of stars.
B. Stellar parallax has been observed for several thousand stars.
C. Eclipses of the sun do not occur at every new moon.
D. A calendar reform was needed in the sixteenth century to keep months and seasons in agreement.
E. Pairs of stars have been observed that move around each other in accordance with Kepler's laws.

12. If a theory predicts a result that is contrary to common sense, we should

A. reject the theory since we must rely on common sense.
B. devise an experiment to test the predicted result.
C. disregard common sense because it is of no value in a scientific study.
D. revise the theory to produce a compromise between the theory and common sense.

13. If F_1 is the magnitude of the force exerted on the sun by the earth and F_2 is the magnitude of the force exerted on the earth by the sun, then

A. F_1 is much greater than F_2.
B. F_1 is slightly greater than F_2.
C. F_1 is equal to F_2.
D. F_1 is slightly less than F_2.
E. F_1 is much less than F_2.



14. In describing the motion of a thrown rock, Newtonian physics introduced a premise that was *not* part of Aristotelian physics. Which of the following is the Newtonian premise?

A. A force is needed to change the state of the rock from rest to motion.
B. The rock has no properties that affect its motion.
C. The undisturbed motion of the rock is uniform motion along a straight line.
D. The natural state of the rock is rest.

15. Galileo's discovery of Jupiter's moons provided supporting evidence for

A. Aristotle's solution to Plato's problem.
B. the Copernican theory of planetary motion.
C. the existence of epicycles.
D. the geocentric hypothesis of Ptolemy.
E. the accuracy of Tycho Brahe's observations.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min for each.

1. Describe the annual motion of the sun against the background of the stars as observed from the earth.

2. Describe the role played by stellar parallax in the arguments for and against a helicoentric theory of the universe.

3. Describe the relationship between Newton's law of gravitation $F_{grav} = G\frac{m_1 m_2}{R^2}$ and Galileo's observation that the gravitational acceleration at the surface of the earth is the same for all falling bodies.

4. The scientific theories of Galileo and Copernicus were severely criticized by some of their contemporaries. Were they criticized on scientific grounds? Explain your answer.

5. If two planets A and B have the same mass but the radius of A is twice that of B,

(a) which planet has the greater gravitational pull at its surface?
(b) what is the ratio of the surface gravitational pulls of the two planets?

Group Two

Answer ONE of the two questions in this group. Allow about 10 min.

6. The *Text* indicates that Galileo, Kepler, and Newton each made contributions that helped change the scientific outlook from an Aristotelian to a modern point of view. Pick two of these men and describe briefly their contributions in producing this changed outlook.

7. Two criteria for an adequate theory are:

1. that it be based upon simple assumptions.
2. that it be in agreement with experimental observations.

Consider the sun-centered system of planetary motion published by Copernicus in 1543.

(a) Did it satisfy these criteria?
(b) Did it satisfy them better than the older Ptolemaic (earth-centered) system? Explain your answer.

Unit 2 / Motion in the Heavens — Test C

Directions

Answer ALL 40 multiple-choice questions by marking the letter corresponding to the one best answer.

The numerical values of some physical constants and equations that may be useful in this test are given at the end of the tests for this unit.

1. Which of the following statements must be part of any heliocentric theory?

A. The planets revolve around the sun.
B. The sun is a sphere.
C. The earth is a sphere.
D. The planets revolve around the earth.
E. The earth turns on its axis.

2. Time-exposure photographs of stars show an arc of a circle for each star. An astronomer who believed the Ptolemaic theory of planetary motion would explain that these arcs result from

A. the rotation of the earth on its axis.
B. stellar parallax.
C. retrograde motion.
D. the inclination of the earth's axis.
E. the rotation of the starry sphere.

3. Which of the following was an important factor that worked against the acceptance of Copernicus' heliocentric solar system hypothesis in the sixteenth century?

A. When Venus was observed through a telescope, phases were seen.
B. The calendar failed to keep pace with the seasons.
C. Galileo observed four satellites moving around Jupiter.
D. Venus had never been observed more than 48° from the sun.
E. Stellar parallax had never been observed.

4. Galileo's discovery of Jupiter's moons provided supporting evidence for

A. Aristotle's solution to Plato's problem.
B. the Copernican theory of planetary motion.
C. the existence of epicycles.
D. the geocentric hypothesis of Ptolemy.
E. the accuracy of Tycho Brahe's observations.

5. To ancient astronomers, planets were different from stars because planets

A. moved in circles.
B. were unlike the earth in composition.
C. moved against the background of stars.
D. moved around the sun.
E. were within the sphere of the sun.

6. Which of the following led to a numerical value of the constant G in the equation $F_{grav} = G\frac{m_1 m_2}{R^2}$?

A. calculations of the moon's orbit by Tycho Brahe
B. a laboratory experiment by Cavendish
C. observations of Jupiter's moons by Galileo

D. an experiment with balls rolling down an incline by Galileo
E. calculations of the orbit of Mars by Kepler

7. ALL EXCEPT ONE of the following are objections that were raised against the heliocentric model of the solar system. Which one is the *exception?*

A. It failed to fit the observations.
B. It displaced humanity from its unique position in the center of the universe.
C. It was contrary to common sense, which demonstrates that the earth is motionless.
D. It failed to distinguish between base terrestrial matter and heavenly matter.
E. It conflicted with Aristotelian physics.

8. ALL EXCEPT ONE of the following were among Tycho Brahe's contributions to astronomy. Which one is the *exception?*

A. He established an astronomical observatory.
B. He proposed an inverse square law of attraction for the solar system.
C. He developed a theory of the solar system.
D. He determined the limits of accuracy of his instruments.
E. He made very accurate observations of the positions of the heavenly bodies.

9. Which of the following correctly places the earth, Jupiter, Mars, the moon, and the sun in order of increasing mass?

A. moon, earth, Mars, sun, Jupiter
B. moon, Mars, earth, Jupiter, sun
C. Mars, earth, moon, Jupiter, sun
D. moon, Jupiter, Mars, earth, sun
E. moon, earth, Jupiter, Mars, sun

10. Explorer 7 is a U.S. satellite in an elliptical orbit in which its distance from the center of the earth varies between 6,640 and 8,800 km. Compared to the speed at a distance of 8,800 km its speed at a distance of 6,640 km is

A. greater, in the ratio 6,640 to 1.
B. greater, in the ratio 8,800 to 6,640.
C. the same.
D. smaller, in the ratio 6,640 to 8,800.
E. smaller, in the ratio 1 to 8,800.

11. ALL EXCEPT ONE of the following were arguments used by followers of Copernicus to defend his heliocentric theory. Which one is the *exception?*

A. A rotating earth is no more likely to break up than a much larger rotating celestial sphere.
B. Lenses change what one sees and, therefore, telescopic evidence may be distorted.
C. It is a simpler, more harmonious system and, therefore, more pleasing to the divine architect.
D. Friction drags the atmosphere along with the rotating earth; therefore, clouds and birds are not left behind.
E. The stars are at too great a distance to show a parallax.

Select answers to questions 12 and 13 from the following list.

A. Ptolemy B. Galileo C. Kepler D. Tycho Brahe E. Newton

12. He demonstrated that the observed planetary motions were consistent with general principles that described all motion everywhere in the universe.

13. His major contributions to astronomy were his accurate measurements.

14. Opponents of Copernicus gave many arguments in support of the Ptolemaic system. ALL EXCEPT ONE of the following support the Ptolemaic system. Which one is the *exception?*

A. It predicted positions of the bodies of the solar system with fair accuracy.
B. It explained why the fixed stars showed no stellar parallax.
C. It was in accordance with the ideas of "natural motion" and "natural place."
D. It was based on what they sensed: that the earth is motionless and the sun, planets, and stars are moving.
E. It explained how comets' orbits could come close to the sun.

15. ALL EXCEPT ONE of the following statements are true. Which is the *exception?*

A. The earth moves fastest when it is nearest to the sun.
B. The earth's orbit lies in a plane that passes through the sun.
C. A line drawn from the sun to the earth sweeps over the same area from March 21 to March 23 as it does from December 21 to December 23.
D. The sun is at the exact center of the earth's orbit.
E. The earth's orbit around the sun is an ellipse.

16. ALL EXCEPT ONE of the following are correct statements about a satellite in an elliptical orbit around the earth. Which is the *exception?*

A. One focus of its elliptical orbit is at the center of the earth.
B. Its orbit lies on a plane that passes through the center of the earth.
C. A line drawn from the center of the earth to the satellite sweeps over equal areas in equal time intervals.
D. The satellite moves fastest when it is closest to the earth.
E. There is no net force acting on the satellite.

17. Which one of the following is evidence that supports the universality of Newton's law of gravitation?

A. Pairs of stars have been observed that move around each other in accordance with Kepler's laws.
B. A few stars have moved from the positions given for them in Ptolemy's catalogue of stars.
C. Stellar parallax has been observed for several thousand stars.
D. Eclipses of the sun do not occur at every new moon.
E. A calendar reform was needed in the sixteenth century to keep months and seasons in agreement.

18. If the planets were to be listed in order of increasing mean distance from the sun (i.e., Mercury, Venus . . . Pluto), that order would necessarily be the same as listing the planets in order of increasing

A. period of revolution around the sun.
B. period of rotation around their axes.
C. eccentricity of orbit around the sun.
D. size of the planet.
E. brightness of the planet in the sky.

19. Kepler's three laws of planetary motion were

A. almost self-evident from Tycho Brahe's data of Mars' orbit.
B. little used in Newton's development of a general law of universal gravitation.
C. developed only after Kepler took imaginative steps from the available data.
D. widely discussed early in the sixteenth century.
E. used by Copernicus in deriving his heliocentric hypothesis.

20. Heliocentric models of the solar system explain retrograde motion of planets by means of

A. the differences between the rates of motion of the earth and the planets along their orbits.
B. the daily rotation of the earth.
C. a combination of the sun's motion northward on the ecliptic and the planets' revolution around the sun.
D. changes in the speed of planets as their distance from the sun changes.
E. corrections to the inaccurate measurements of the ancient astronomers.

21. The following men made significant contributions to our present understanding of planetary motion:

1. Copernicus   2. Newton   3. Kepler

If the names of these men were arranged in the order of their contributions starting with the earliest first, they would be

A. 1, 2, 3.
B. 2, 3, 1.
C. 3, 1, 2.
D. 1, 3, 2.
E. 2, 1, 3.

22. What is the acceleration due to gravity of a meteor at 2 earth radii from the center of the earth? Assume the acceleration due to gravity at 1 earth radius from the center of the earth is 10 m/sec^2.

A. 2.5 m/sec^2 B. 5 m/sec^2 C. 7.07 m/sec^2 D. 10 m/sec^2 E. 20 m/sec^2

23. Which of the following *can* be explained by a model of the solar system in which the sun revolves around the earth?

1. Each day the sun rises in the east and sets in the west.
2. During the night the stars appear to move about the north celestial pole.
3. Eclipses of the sun sometimes occur.

A. 1 and 2 only
B. 2 and 3 only
C. 1 and 3 only
D. 1, 2, and 3
E. none of the above

24. A simple geocentric model with uniform motion about the center of circular orbits explains only one of the following observations. Which one is it?

A. The sun moves slower in the summer than in the winter.
B. The moon shows phases with a period of one month.
C. the planets vary in speed at different points in their orbits.
D. The size and duration of Mars' retrograde motion is not the same at successive occurrences.
E. The planets move with different speeds against the stars at different points of their orbits.

25. Two sacks, each containing 10 oranges of equal mass, are hung 4 m apart. Which one of the following would double the gravitational force that one sack of oranges exerts on the other sack?

A. adding 20 oranges of the same mass, 10 to each sack
B. adding 10 oranges of the same mass, 5 to each sack
C. adding 10 oranges of the same mass, to one sack only
D. moving 5 oranges from one sack to another
E. moving 10 oranges from one sack to another

26. Two baseballs are supported at a distance of 4 m from each other. If they were moved, at what distance would the gravitational force between them be approximately one-fourth its previous value?

A. 1 m   B. 2 m   C. 6 m   D. 8 m   E. 16 m

27. Which one of the following posed *no* serious difficulties for the Platonic view of heavenly perfection?

A. the new star discovered in 1572
B. the rough appearance of the moon's surface
C. the four satellites of Jupiter
D. sunspots
E. the observed motion of the stars

28. Which ONE of the following statements was *not* an observation used to support the ancient idea that there is a fundamental difference between the substance of the heavenly bodies and the substance of the earth.

A. Heavenly bodies moved along a regular path, whereas earth-bound objects moved in an irregular fashion.
B. The heavens seemed perfect, whereas the earth did not.
C. The material of the heavenly bodies seemed eternal and unchanging, whereas earthly matter was constantly changing.
D. Events in the heavens could not generally be predicted, whereas those on earth could be.
E. Heavenly bodies seemed to move without forces, whereas a force was required to keep an object in motion on the earth.

29. To Ptolemy ALL EXCEPT ONE of the following statements were part of the solution to Plato's problem of accounting for the motions of the planets. Which one is the *exception*?

A. Planetary paths must be composed of circles or combinations of circles.
B. Planets must move relative to the stars.
C. The sun must stand still.
D. The stars must move in paths composed of circles.
E. The moon must be nearest the earth.

30. A space probe that missed its target went into orbit around the sun at a mean distance 9 times as great as the earth's. On the basis of Kepler's third law, the period of the space probe is approximately

A. 3 years. B. 9 years. C. 27 years. D. 54 years. E. 81 years.

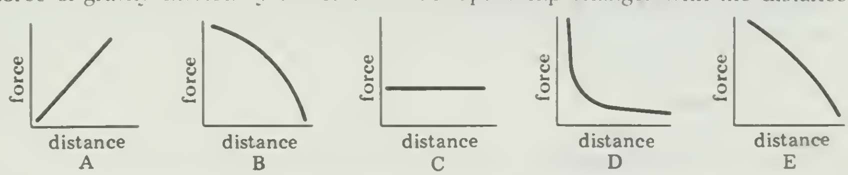
31. If the earth's mass were twice as great as it is, its period of revolution about the sun (assuming it stayed in the same orbit) would be

A. increased by a factor of 4.
B. increased by a factor of 2.
C. hardly changed at all.
D. decreased by a factor of 2.
E. decreased by a factor of 4.

32. A theory predicts a result that is contrary to common sense. We should

A. reject the theory since we must rely on common sense.
B. devise an experiment to test the predicted result.
C. disregard common sense because it is of no value in a scientific study.
D. revise the theory to produce a compromsie between the theory and common sense.
E. revise common sense to agree with the new theory.

33. A spaceship travels at a constant speed directly away from the earth. Which of the following graphs shows how the force of gravity exerted by the earth on the spaceship changes with the distance from the earth?



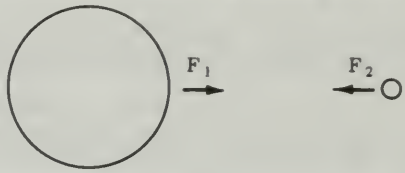
34. Scientists usually publish their findings in journals that are then distributed throughout the world. One reason they do not try to keep their discoveries secret is that

A. few discoveries have any economic or military importance.
B. scientists are not concerned with political or economic affairs.
C. someone would soon give the secret away anyway
D. scientists want to let others know what they have done.

35. Newton's law of universal gravitation was distinctive in the history of physics because it was the first

A. explanation of the cause of gravity.
B. demonstration that the same equations can be used to describe motion on the earth and in the heavens.
C. accurate mathematical description of the planetary orbits.
D. demonstration that Kepler's laws and Copernicus' hypotheses were alternate explanations of the same observations.
E. use of algebra to describe physical phenomena.

36. If F_1 is the magnitude of the force exerted on the sun by the earth, and F_2 is the magnitude of the force exerted on the earth by the sun, then



A. F_1 is much greater than F_2.
B. F_1 is slightly greater than F_2.
C. F_1 is equal to F_2.
D. F_1 is slightly less than F_2.
E. F_1 is much less than F_2.

37. Assume that the following measurements were made on three planets revolving about a star. (Planets are listed in order of discovery.)

Planet	*Orbital Period*	*Mass*
Alpha	14 earth years	10 earth masses
Beta	188 earth years	17 earth masses
Gamma	50 earth years	½ earth mass

On the basis of Kepler's laws of planetary motion, these planets could be arranged in order of their increasing distance from the star. If the planets were listed in sequence, starting with the planet nearest the star, they would be arranged

A. Alpha, Beta, Gamma.
B. Beta, Gamma, Alpha.
C. Gamma, Alpha, Beta.
D. Beta, Alpha, Gamma.
E. Alpha, Gamma, Beta.

38. One of Newton's contributions to astronomy was his description of

A. the algebraic equations of ellipses and other conic sections.
B. the forces that produce the elliptical orbits of the planets.
C. the distances to the planets.
D. the geometry of the ellliptical orbits of the planets.
E. the theory of light.

39. Assume that the earth suddenly became one-half its original diameter, but that its mass was unchanged. The strength of the earth's gravitational pull on the moon would be

A. four times as great.
B. twice as great.
C. the same.
D. one-half as great.
E. one-fourth as great.

40. ALL EXCEPT ONE of the following state aspects of Kepler's work. Which one is the *exception*?

A. He emphasized the mathematical regularities of planetary motion.
B. He recognized the value of precise observations
C. He gave a mathematical description of the cause of planetary motion.
D. He helped to perfect the theory of the solar system in which the planets move around the sun.
E. He recognized that a planet's path was not a circle.

Unit 2 / Motion in the Heavens — TEST D

Directions

This test consists of eight questions in two groups. Answer only FOUR of the five questions in Group One, and only TWO of the three questions in Group Two. Spend about 5 min on each of the questions from Group One, and 10 min on each of the questions from Group Two.

The numerical values of some physical constants and equations that may be useful in this test are given at the end of the tests for this unit.

Group One

Answer only FOUR of the five questions in this group.

1. Indicate briefly the major contributions to astronomy of

(a) Kepler (b) Newton

2. Briefly describe what is meant by "Newton's great synthesis."

3. State and discuss briefly one hypothesis used by both Ptolemy and Copernicus, but completely abandoned by Kepler in his theory of planetary motion.

4. According to Kepler's "harmonic law," the period of revolution of a planet is related to the mean radius of its orbit by the equation $T^2 = ka^3$. The mean radius of the orbit of Saturn around the sun is 9 AU. What is its period of revolution in years? (Show your calculations.)

5. Draw a graph of the earth's gravitational attraction on a 10-kg mass versus its distance from the center of the earth. Let R equal the earth's radius. Make entries at distances R, $2R$, $3R$, and $4R$.

Group Two

Answer only TWO of the three questions in this group. Spend about 10 min on each question.

6. (a) What is retrograde motion?
(b) Explain retrograde motion (with the aid of a diagram) using either a heliocentric or a geocentric model of the solar system. State which model you are using.

7. In order to develop an equation for the gravitational force F_{grav} between the sun and a planet, Newton had to determine the way in which the force of gravitation depends upon the mass of the sun M_s and the mass of the planet M_p. Among the possible hypotheses are the following proportionalities:

A. $F_{grav} \propto (M_s + M_p)$ B. $F_{grav} \propto (M_s M_p)$ C. $F_{grav} \propto (M_s/M_p)$

(a) Which of the above is the relationship Newton chose?
(b) Pick one of the other two options and give a reasonable argument for rejecting it.

8. Two criteria for an adequate theory are:

(a) that it be based upon simple assumptions.
(b) that it be in agreement with experimental observations.

Did the Ptolemaic (earth-centered) theory of planetary motion of AD 150 satisfy these criteria? Explain your answer.

Unit 2

Physical Constant:

Acceleration of gravity on the surface of the earth (approximate value to be used in these tests)
$a_g = 10 \text{ m/sec}^2$

Radius of the earth $\quad R_E = 6{,}000{,}000 \text{ m}$

Mass of the earth $\quad m_E = 5.96 \times 10^{24} \text{ kg}$

Equations:

$$F_{grav} = G\frac{m_1 m_2}{R^2} \qquad v = \frac{2\pi R}{T}$$

$$T^2 = ka^3 \qquad a = \frac{v^2}{R}$$

$$T^2 = \left[\frac{4\pi^2}{Gm}\right] R^3 \qquad \frac{m_1}{m_2} = \left[\frac{R_1}{R_2}\right]^3 \left[\frac{T_2}{T_1}\right]^2$$

Unit 3 / The Triumph of Mechanics — TEST A

Directions

This test consists of 15 multiple-choice questions and six problem-and-essay questions divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants, definitions of certain units, and equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. When the speed of a car is doubled, the car's

A. kinetic energy is doubled.
B. potential energy is doubled.
C. momentum is doubled.
D. acceleration is doubled.
E. inertia is doubled.

2. A freight car of mass 2.0×10^4 kg standing at rest is rammed by a loaded tank car with a mass of 3.0×10^4 kg. After the collision, the two cars are locked together and move off at a speed of 0.60 m/sec. What was the speed of the tank car before the collision?

A. 0.20 m/sec B. 0.75 m/sec C. 1.0 m/sec D. 3.6 m/sec E. 4.0 m/sec

3. When two waves pass the same point at the same time, their amplitudes at this point always

A. cancel.
B. reflect off each other.
C. reinforce each other.
D. hinder each other's progress.
E. superpose.

4. In a certain medium, the speed of a group of waves has a fixed value. If the frequency of the waves is doubled, their wavelength will be

A. four times its original value.
B. two times its original value.
C. unchanged.
D. one-half its original value.
E. one-fourth its original value.

5. Which of the following three quantities have the same magnitude just before and just after a perfectly elastic collision?

1. momentum
2. kinetic energy
3. total energy

A. 1 only B. 2 only C. 3 only D. 2 and 3 only E. 1, 2, and 3

Questions 6, 7, and 8 are the names of scientists who made early significant contributions to the study of thermodynamics. Select the one statement that best describes a contribution of the particular scientist.

A. The pressure of a gas is proportional to the square of the speed of its molecules.
B. Heat is a form of energy.
C. The speeds of molecules in a gas follow a statistical law.
D. In an elastic collision, momentum is conserved.
E. The process of equalization of temperatures by the flow of heat from hot to cold bodies is always taking place in nature.

6. Joule

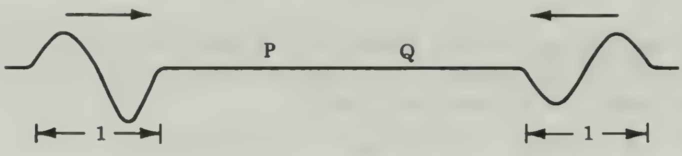
7. Carnot

8. Maxwell

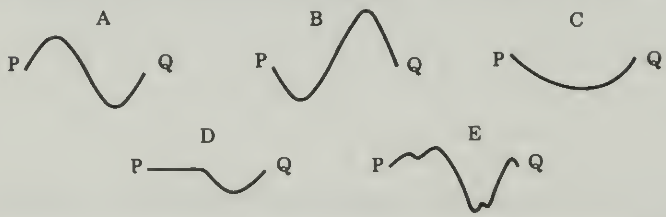
9. A girl lifts a bowling ball from the floor and places it on a rack. If you know the weight of the ball, what else must you know in order to calculate the work she does on the ball?

A. mass of the ball
B. value of a_g
C. height of the rack
D. the time required
E. nothing else

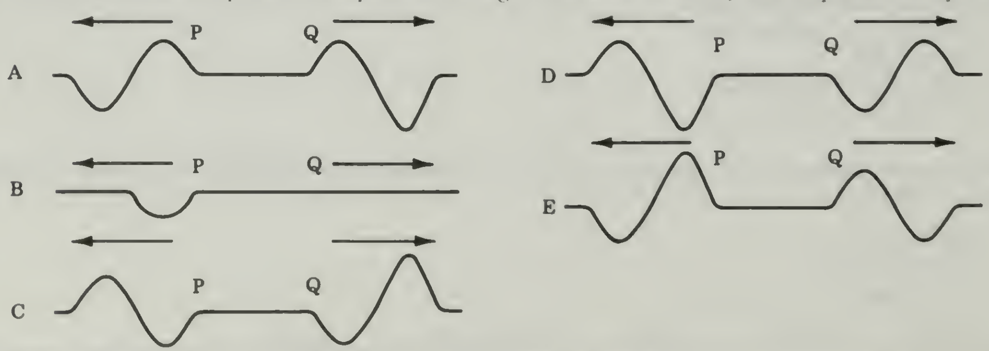
Questions 10 and 11 refer to the following statement and diagram. Two wave pulses, each of length 1, are traveling toward each other along a rope as illustrated in the diagram below.



10. When both waves are entirely in the region between P and Q, the shape of the rope will be



11. Just after both wave pulses have passed the region between P and Q, the shape of the rope will be



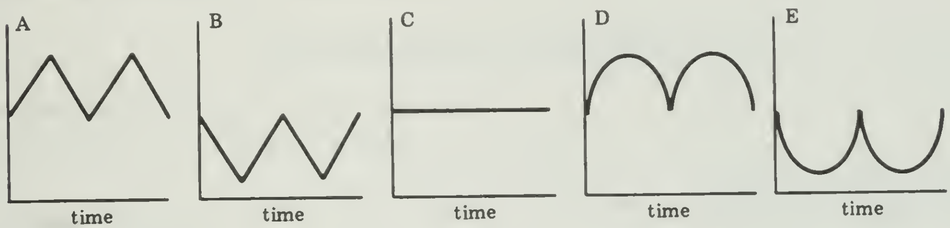
12. The prediction of a "heat death" is based on the principle that states that

A. the law of conservation of energy applies only to closed systems.
B. at some time in the future, the energy of the universe will become zero.
C. all bodies in the universe will eventually reach the same temperature by exchanging heat with each other.
D. it is impossible to think of a system in which energy is completely conserved.

13. The law of normal distribution applies in ALL EXCEPT ONE of the following cases. Which one is the *exception*?

A. the heights of a large number of 25-year-old maple trees in a certain forest
B. the speed of a falling object measured at many different times during the object's fall
C. the scores on a test taken by a large number of students

The following graphs refer to questions 14 and 15.



At time $t = 0$ a pendulum is set into motion by releasing the pendulum bob at a certain height.

14. Which of the graphs best represents the variation of the bob's kinetic energy with time?

15. Which of the graphs best represents the variation of potential energy with time?

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. A vibrating rod generates waves in a pool of water. Describe any change in the water waves that would occur if the rod's frequency of vibration was increased.

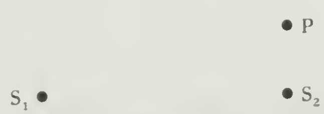
2. What is the magnitude of the momentum of

(a) a baseball (mass = 0.14 kg) as it moves at 30 m/sec?
(b) a 0.22 caliber bullet (mass = 2.0×10^{-3} kg) as it leaves the barrel of a pistol at a speed of 300 m/sec?

3. Describe the model of a gas developed in the kinetic theory of gases.

4. What is the power of a motor that can lift a 100-N weight a height of 10 m in 50 sec?

5. S_1 and S_2 are two in-phase periodic sources of waves with wavelength λ. What conditions determine whether point P lies on a nodal (destructive interference) or antinodal (constructive interference) line?



Group Two

Answer ONE of the following two questions. Allow about 10 min.

6. Ten joules of elastic potential energy is stored by compressing a spring. A 2-kg object is placed on top of this spring and the spring released so that the object is projected straight up. If we neglect the energy dissipated in the spring, how high will the object rise? (a_g = 10 m/sec^2)

7. Show that Newton's second law $\vec{F} = m\vec{a}$ can be written $\vec{F} = \Delta \vec{p} / \Delta t$.

Unit 3 / The Triumph of Mechanics — TEST B

Directions

This test consists of 15 multiple-choice questions and six problem-and-essay questions divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants, definitions of certain units, and equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. An object at rest may have a nonzero amount of

A. momentum. B. energy. C. speed. D. velocity.

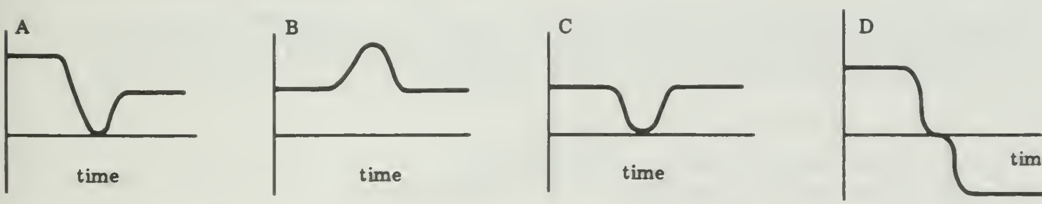
2. ALL EXCEPT ONE of the following can be adequately described by Newtonian mechanics. Which one is the *exception*?

A. the motion of a flare dropped from an airplane
B. the relationships between observable properties of gases
C. the sizes and speeds of molecules in a gas
D. the motions of atoms inside molecules

3. The first law of thermodynamics is a statement of

A. the law of conservation of energy.
B. the law of conservation of momentum.
C. the law of conservation of mass.
D. Newton's law of action and reaction.
E. Galileo's law of motion.

Questions 4 to 7 refer to the following graphs.



A ball is thrown against a wall from which it rebounds. Which of the above graphs could best represent each of the following? (Note: An elastic collision is one in which the kinetic energy is the same before and after the collision.)

4. the kinetic energy of the ball (assuming an elastic collision

5. the kinetic energy of the ball (if the collision is partly elastic

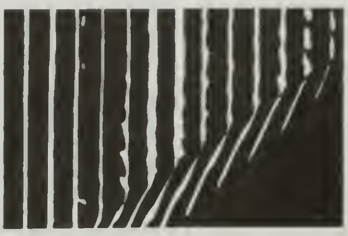
6. the magnitude of the ball's velocity during an elastic collision

7. the magnitude of the ball's velocity during a partly elastic collision

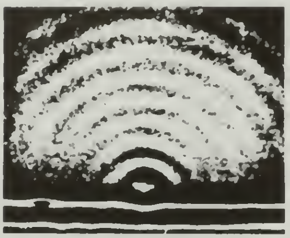
Questions 8 to 13 refer to pictures of water ripples. Use the following key to answer questions 8 to 13.

A. diffraction B. refraction C. reflection D. interference

8. Which of the above is illustrated by the picture below?



9. Which of the above is illustrated by the picture below?



10. Which of the above is illustrated by the picture below?

11. The law of normal distribution applies in ALL EXCEPT ONE of the following cases. Which one is the *exception*?

A. the heights of a large number of 25-year-old maple trees in a certain forest
B. the speed of a falling object measured at many different times during the object's fall
C. the scores on a test taken by a large number of students

12. The second law of thermodynamics suggests that

A. energy tends to transform itself into less useful forms.
B. the usual order of natural processes is from disorder to order.
C. all natural processes are reversible.
D. it is possible to determine the motion of an individual molecule in a gas.

13. Even though one may listen to a band from a considerable distance, the sound of the piccolo and that of the tuba do not get "out of step" with each other. This is evidence that, in this situation, sound waves

A. travel at the same speed for all frequencies.
B. are not polarized.
C. are longitudinal.
D. tend to be sinusoidal.
E. travel at a slower speed than light.

14. All bodies contain electrical charge (which comes in two varieties, positive and negative). A law of conservation of charge applies. Which of the following might be a consequence or statement of that law?

A. Charge is rare and must be used carefully.
B. If an object with a net positive charge explodes, each of the pieces must have a net positive charge.
C. The total net charge in an isolated system does not change with time.

15. The prediction of a "heat death" is based on the principle that states that

A. the law of conservation of energy applies only to closed systems.
B. at some time in the future, the energy of the universe will become zero.
C. all bodies in the universe will eventually reach the same temperature by exchanging heat with each other.
D. it is impossible to think of a system in which energy is completely conserved.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. A ball is dropped from the top of a tower (neglect air resistance). Draw two graphs showing (a) the change in the kinetic energy and (b) the change in gravitational potential energy of the ball with height.

2. The second law of thermodynamics summarizes our knowledge concerning the direction of natural processes and energy dissipation. Describe one physical phenomenon that is explained by the second law.

3. The orbit of the Alouette satellite is nearly circular and hence its speed is nearly constant. Is its momentum also constant? Explain.

4. Lavoisier observed chemical reactions in closed containers and carefully weighed the containers and their contents before and after the reactions.

(a) What were the results of such experiments?
(b) What was the significance of these results?

5. The kinetic theory of gases uses a model of a gas that assumes that gases consist of large numbers of very small particles (molecules) in rapid disordered motion. What is one consequence of assuming that the motion of the particles is disordered?

Group Two

Answer ONE of the following two questions. Allow about 10 min.

6. Some seventeenth-century philosophers insisted that the idea of a universe running down was incompatible with the idea of the perfection of God. They held that if motion was correctly defined it could be shown that the amount of motion in the universe is constant.

Was the law of conservation of momentum compatible with the beliefs of these seventeenth-century philosophers? Explain.

7. A hunter returned from safari in Africa and told the following tale: "Suddenly a lion jumped at me. I quickly fired my rifle. The bullet struck the lion while he was in the middle of his jump, and he fell straight down to the ground." Do you believe this? Explain, with reference to the appropriate law or laws of conservation.

Unit 3 / The Triumph of Mechanics — TEST C

Directions

Answer all 40 multiple-choice questions by marking the letter corresponding to the one best answer.

The numerical values of some physical constants, definitions of certain units, and equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. Which of the following is a vector quantity?

A. momentum B. kinetic energy C. work D. heat E. temperature

2. A 10-kg weight is dropped from a height of 3 m. Just before striking the ground, the weight's kinetic energy will be about

A. 3 J. B. 30 J. C. 300 J. D. 3000 J.

3. A number of the examples of the energy concept make use of "frictionless" systems. Why is this done?

A. Most systems are frictionless.
B. Total energy is not conserved when friction is present.
C. Friction is not as meaningful a concept as energy.
D. Friction is not present in outer space.
E. It is often possible to get useful answers by ignoring friction.

4. Which one of the following is most nearly an "elastic" collision?

A. two railway cars coupling
B. an automobile collision
C. two billiard balls colliding
D. an apple dropped on the ground
E. a hammer hitting a nail into wood

5. When the displacement pattern of a transverse wave lies in a single plane, the wave is said to be

A. reflected. B. polarized. C. diffracted. D. refracted.

6. Two steel balls collide elastically.

A. Momentum is the same before and after the collision, but kinetic energy is not
B. Momentum and kinetic energy are the same before and after the collision.
C. The temperature of both balls will increase.
D. The balls will be permanently deformed.
E. The balls will stick together.

7. ALL EXCEPT ONE of the following can be adequately described by Newtonian mechanics. Which one is the *exception?*

A. the motion of a flare dropped from an airplane
B. the relationships between observable properties of gases
C. the sizes and speeds of molecules in a gas
D. the motions of atoms inside molecules

8. The law of normal distribution applies in ALL EXCEPT ONE of the following cases. Which one is the *exception?*

A. the heights of a large number of 25-year-old maple trees in a certain forest
B. the speed of a falling object measured at many different times during the object's fall
C. the scores on a test written by a large number of students

9. The second law of thermodynamics suggests that

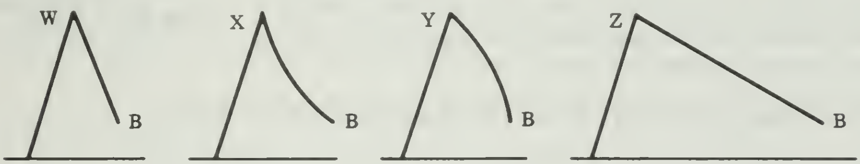
A. energy tends to transform itself into less useful forms.
B. the usual order of natural processes is from disorder to order.
C. all natural processes are reversible
D. it is possible to determine the motion of individual molecules in a gas.

10. During a baseball game, the first batter hits a waist-high pitch over the center fielder's head.

If air resistance is negligible, only one of the following statements about the ball during its flight is FALSE. Which one?

A. The higher the ball goes, the greater its gravitational potential energy.
B. The horizontal component of the ball's velocity is constant.
C. The total energy of the ball is constant.
D. The momentum of the ball is constant.

11. A girl wants to slide down a playground slide so that she will have the greatest possible speed when she reaches the bottom (point B). Which of the following frictionless slides should she choose? (Points W, X, Y, and Z are all 2 m above the ground, and point B is 0.5 m above the ground.)



A. slide W
B. slide X
C. slide Y
D. slide Z
E. The speed at B will be the same for each.

12. Leibniz's *vis viva* most closely resembles

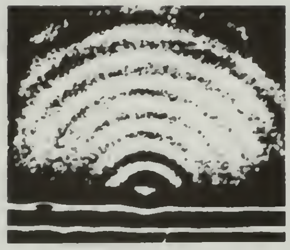
A. potential energy. B. kinetic energy. C. heat. D. velocity. E. momentum.

Questions 13 to 15 refer to pictures of water ripples. Use the following list to answer questions 13 to 15.

A. diffraction B. refraction C. reflection D. interference

13. Which of the choices is illustrated by the picture below?

14. Which of the choices is illustrated by the picture below?



15. Which of the choices is illustrated by the picture below?

Questions 16 and 17 refer to the following statement:

A 0.1-kg snowball strikes a 0.9-kg stationary skateboard and sticks to it. At the instant of impact, the snowball has a velocity of 18 m/sec in the horizontal direction. (Assume that the skateboard is on a perfectly horizontal stretch of ground and that it moves without friction.)

16. After collision, the skateboard and snowball move horizontally with a velocity of

A. 1.8 m/sec. B. 2 m/sec. C. 16.2 m/sec. D. 18 m/sec. E. 180 m/sec.

17. Kinetic energy is not conserved in the above collision because

A. the system is not closed.
B. the collision is not perfectly elastic.
C. momentum and energy cannot both be conserved in a collision.
D. the law of conservation of energy does not hold for a frictionless system.
E. heat cannot flow from a cold object to a hot object.

18. An object is hung on a vertical spring and allowed to oscillate up and down. At any instant the system's total energy is

A. $KE + PE_{elastic} + PE_{gravitational}$.
B. $KE + PE_{elastic}$.
C. $KE + PE_{gravitational}$.
D. $PE_{elastic} + PE_{gravitational}$.
E. KE.

19. Which one of the following is transferred from one place to another by a propagating wave?

A. mass B. energy C. time D. velocity

20. A body's momentum is defined as the body's mass times its velocity. The mks unit of momentum is

A. kilogram · meter.
B. kilogram · meter/sec.

C. kilogram · meter^2/sec^2.
D. kilogram^2 · meter/sec.
E. none of the above.

Questions 21, 22, and 23 are the names of scientists who made early significant contributions to the study of thermodynamics. Select the one statement that best describes a contribution of the particular scientist.

A. The pressure of a gas is proportional to the square of the speed of its molecules.
B. Heat is a form of energy.
C. The speeds of molecules in a gas follow a statistical law.
D. In an elastic collision, momentum is conserved.
E. The process of equalization of temperatures by the flow of heat from hot to cold objects is always taking place in nature.

21. Joule

22. Carnot

23. Maxwell

24. A girl lifts a bowling ball from the floor and places it on a rack. If you know the weight of the ball, what else must you know in order to calculate the work she does on the ball?

A. mass of the ball
B. value of a_g
C. height of the rack
D. the time required
E. nothing else

25. ALL EXCEPT ONE of the following are in agreement with Goethe's nature philosophy. Which one is the *exception*?

A. The methods of mechanistic science (for example, mathematical analysis and experimentation) give the wrong idea of nature.
B. Nature as it really is can be understood by direct observation.
C. One should search for the inner meaning of nature.
D. Laws that are practical and quantitative can best describe nature.

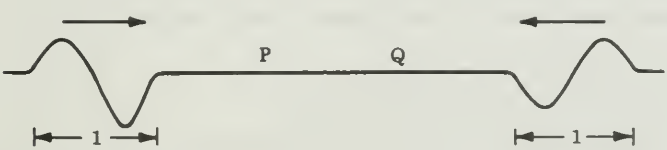
26. The kinetic energy of an object is increased the most by doubling its

A. mass. B. temperature. C. volume. D. density. E. speed.

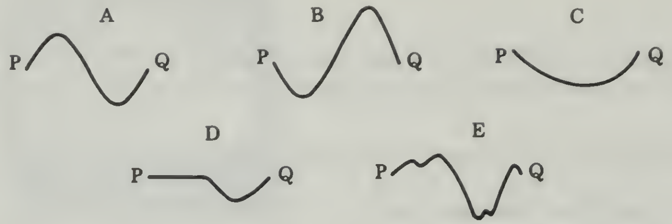
27. The first law of thermodynamics is a statement of

A. the law of conservation of energy.
B. the law of conservation of momentum.
C. the law of conservation of mass.
D. Newton's law of action and reaction.
E. Galileo's law of motion.

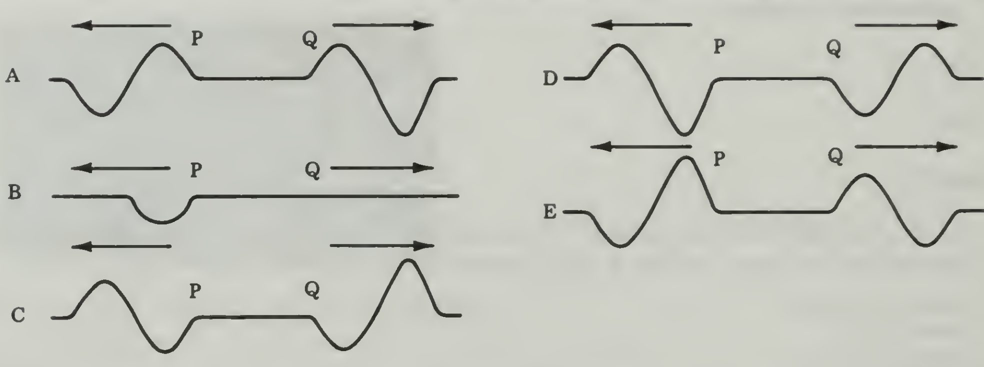
Questions 28 and 29 refer to the following statement and diagram. Two wave pulses, each of length 1, are traveling toward each other along a rope as illustrated in the diagram below.



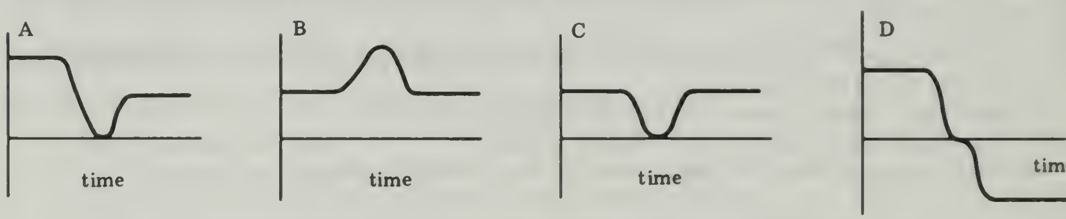
28. At the instant that both waves are entirely in the region between P and Q, the shape of the rope will be



29. Just after both wave pulses have passed the region between P and Q, the shape of the rope will be



Items 30 to 33 refer to the following graphs. A ball is thrown against a wall from which it rebounds. Which of the graphs below could best represent each of the following? (Note: An elastic collision is one in which the kinetic energy is the same before and after the collision.)



30. the kinetic energy of the ball, assuming an elastic collision

31. the kinetic energy of the ball, if the collision is partly elastic

32. the magnitude of the ball's velocity in an elastic collision

33. the magnitude of the ball's velocity in a partly elastic collision

34. The principle of superposition states that

A. the amplitudes of waves that coincide at a point may be added.
B. the wavelength of a reflected wave equals the wavelength of the incident wave.
C. every point on a wave front may be considered to behave as a point source of waves.
D. the diffraction pattern depends on the ratio of the wavelength to the slit width.

35. The prediction of a "heat death" is based on the principle that states that

A. the law of conservation of energy applies only to closed systems.
B. at some time in the future, the energy of the universe will become zero

C. all bodies in the universe will eventually reach the same temperature by exchanging heat with each other.
D. it is impossible to think of a system in which energy is completely conserved.
E. heat must flow from a cold object to a hot object.

36. Even though one may listen to a band from a considerable distance, the sound of the piccolo and that of the tuba do not get "out of step" with each other. This is evidence that in this situation sound waves

A. travel at the same speed for all frequencies.
B. are not polarized.
C. are longitudinal.
D. tend to be sinusoidal.
E. travel at a slower speed than light.

37. Two spheres of the same diameter, one of mass 5 kg and the other of mass 10 kg, are dropped at the same time from the top of a tower. When they are 1 m above the ground, the two spheres have the same

A. momentum.
B. kinetic energy.
C. potential energy.
D. total mechanical energy.
E. acceleration.

38. When a gas is held at a constant temperature, its molecules

A. have a certain constant average energy.
B. all have the same energy.
C. all have different energies that remain constant.

39. The unit "horsepower" is a measure of

A. force.
B. work.
C. work per bushel of coal.
D. work per unit of time.
E. work per steam engine.

40. In 1620, Francis Bacon wrote: "There is nothing more true in nature than the twin propositions that 'nothing is produced from nothing' and 'nothing is reduced to nothing.' ... the sum total of matter remains unchanged, without increase or diminution." This statement implies which of the following basic scientific principles?

A. conservation of momentum
B. conservation of *vis viva*
C. conservation of mass
D. conservation of mechanical energy
E. conservation of charge

Unit 3 / The Triumph of Mechanics — TEST D

Directions

This test consists of eight questions in two groups. Answer only FOUR of the five questions in Group One, and only TWO of the three questions in Group Two. Spend about 5 min on each of the questions from Group One, and 10 min on each of the questions from Group Two.

The numerical values of some physical constants, definitions of certain units, and equations that may be useful in this test are given at the end of the tests for this unit.

Group One

Answer FOUR of the five questions in this group. Allow about 5 min each.

1. The speed of sound in air at normal temperatures is about 340 m/sec. If the musical note A has a frequency of 440 Hz (cycles per second), what will be the length of the wave in air representing this note?

2. Why is mass more appropriate than volume as a measure of an object's "amount of matter"?

3. Two pulses are sent down a thin light rope that is joined to a heavy, thick rope as shown in the figure below. Describe the pulses after they have passed through the junction into the thick rope.

4. A soft rubber ball thrown against a brick wall strikes the wall and bounces back. Is the ball's momentum the same before and after the collision? Is its kinetic energy the same before and after the collision? Explain what happens to the ball's original momentum and kinetic energy.

5. How does the kinetic theory of gases explain the fact that a gas expanding while pushing a piston cools, whereas a gas expanding into a vacuum does not change temperature?

Group Two

Answer TWO of the following three questions. Allow about 10 min each.

6. Two people sit facing each other at opposite ends of a canoe on a quiet pond. One tosses a heavy lunch box to the other.

Make use of the law of conservation of momentum to explain the motion of the boat

(a) while the lunch box is in the air, and
(b) after the lunch box is caught by the other person.

7. The efforts to improve the efficiency of steam engines have produced new ideas of importance to the study of physics. Discuss one of these by-products

8. In what way did Goethe's nature philosophy influence the discovery of the law of conservation of energy? Explain.

Physical Constant:

Acceleration of gravity $a_g = 10 \text{ m/sec}^2$

$$1 \text{ newton} = \frac{1 \text{ kilogram} \cdot \text{meter}}{\text{second}^2}$$

$$1 \text{ joule} = 1 \text{ newton} \cdot \text{meter or } \frac{1 \text{ kilogram} \cdot \text{meter}^2}{\text{second}^2}$$

$$1 \text{ watt} = \frac{1 \text{ joule}}{\text{second}}$$

Equations:

$v = at$

$d = \frac{1}{2}at^2$

$\vec{F} = m\vec{a}$

$\vec{p} = m\vec{v}$

$KE = \frac{1}{2}mv^2$

$\Delta(PE)_{grav} = ma_g d$

$W = Fd$

$\text{power} = \frac{W}{t}$

$T = \frac{1}{f}$

$v = f\lambda$

Unit 4 / Light and Electromagnetism — TEST A

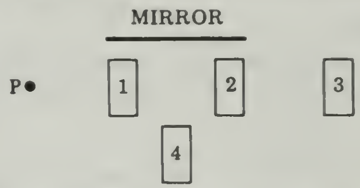
Directions

This test consists of 15 multiple-choice questions and seven problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

Equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. A person whose eyes are located at point P is looking into the mirror. Which of the numbered cards can he see reflected in the mirror?

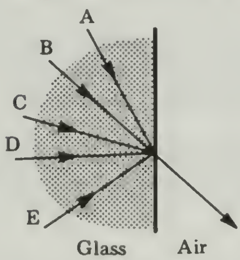


A. 1 and 4
B. 2 and 3
C. 1, 2, and 3
D. 1, 2, 3 and 4

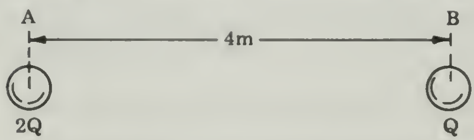
2. A point charge $+Q_1$ exerts an electrostatic force F on point charge $+Q_2$ 3 cm away. If the charges are placed 6 cm apart, the magnitude of the electrostatic force $+Q_1$ exerts on $+Q_2$ will be

A. $4F$. B. $2F$. C. F. D. $F/2$. E. $F/4$.

3. A narrow beam of light emerges from a block of ordinary glass in the direction shown in the diagram. Which arrow in the diagram best represents the path of the beam within the glass?



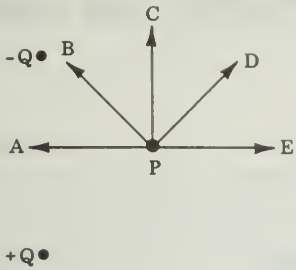
4. Two spheres, A and B, are 4 m apart. A charge of $2Q$ coulombs is distributed over sphere A and a charge of Q coulombs is distributed over sphere B. (See sketch.)



How does the magnitude of the force exerted by A on B compare with the magnitude of the force exerted by B on A?

A. The force on A is four times the force on B.
B. The force on A is two times the force on B.
C. The force on A is the same as the force on B.
D. The force on A is one-half the force on B.
E. The force on A is one-quarter the force on B.

5. Which of the arrows indicates the direction of the electric field at point P due to the stationary charges $+Q$ and $-Q$?



6. The first definite evidence that light moves at a finite speed was found by

A. Galileo. B. Römer. C. Huygens. D. Young.

7. In a vacuum, electromagnetic radiations, such as radio waves, light, X rays, and gamma rays, have the same

A. wavelength. B. frequency. C. period. D. speed. E. amplitude.

8. Newton's synthesis of terrestrial and celestial mechanics incorporated the work of Kepler and Galileo. In a similar way, the work of Oersted and Faraday was incorporated in the synthesis made by

A. Ampere. B. Hertz. C. Maxwell. D. Gilbert.

9. The direction of the electric field in a plane electromagnetic wave is

A. perpendicular to the magnetic field and in the direction of the wave's propagation.
B. perpendicular to the magnetic field and perpendicular to the direction of the wave's propagation.
C. parallel to the magnetic field.

10. Power is

A. work.
B. electrical current.
C. the rate of flow of electric charge.
D. the rate of doing work.

11. A transformer changes

A. electrical energy into mechanical energy.
B. mechanical energy into electrical energy.
C. high voltage dc to low voltage dc.
D. low voltage ac to high voltage ac.

12. An example of electromagnetic induction is the

A. magnetic field about a conductor carrying a current.
B. force between a magnet and a wire carrying a current.
C. production of a current in a wire owing to a changing magnetic field.
D. force between two wires carrying electric currents.
E. depositing of an element at the cathode of an electrolytic cell.

Questions 13, 14, and 15 list the names of scientists who made significant contributions to the study of electromagnetic phenomena. Select the statement that best describes a contribution of each of the scientists.

A. Two current-carrying wires exert forces on each other.
B. An electric field changing with time generates a magnetic field.
C. An electric current exerts a force on a magnet.
D. A magnetic field changing with time can cause a current to flow in a wire.

13. Maxwell

14. Ampère

15. Faraday

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

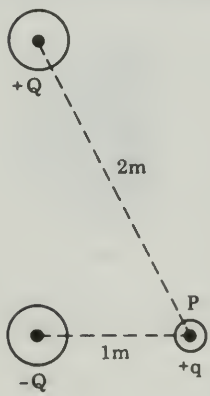
1. Explain how the results of Hertz's experiment supported Maxwell's theory of electromagnetism.

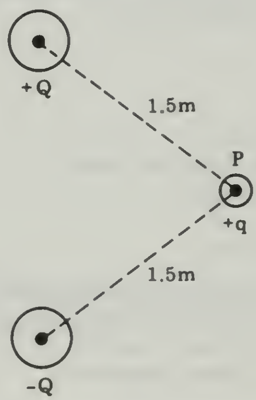
2. What is one of the predictions Maxwell made on the basis of his electromagnetic wave theory?

3. The starter motor in a car for a short time draws a fairly steady current of 100 A from a 12-V battery to turn over the engine. How many watts of power does this represent?

4. Describe two examples of experimental evidence that support the contention that a magnetic field exists in the space around a current-carrying wire.

5. Consider two bodies with charges $+Q$ and $-Q$ and a third body at P with charge $+q$, as shown in the diagrams below. In each of the diagrams, draw an arrow at P pointing in the direction of the resultant force that $+q$ experiences due to $+Q$ and $-Q$.





Group Two

Answer ONE of the following two questions. Allow about 10 min.

6. With the aid of a diagram showing the essential parts of a transformer, explain how a transformer works.

7. Young's double-slit experiment demonstrated the interference of light waves. With the aid of a diagram describe how the experiment is done and explain why interference is observed

Unit 4 / Light and Electromagnetism — TEST B

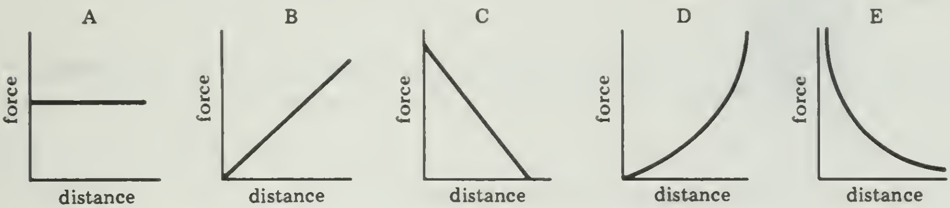
Directions

This test consists of 15 multiple-choice questions and seven problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

Equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. Which of the following graphs best represents the way that the force one small charged body exerts on another changes when the distance between their centers is changed?



2. ALL EXCEPT ONE of the following satisfy the definition of a field as given in the *Text*. Find the *exception.*

A. water temperature in Lake Michigan
B. density of smoke in the air above New York
C. noise level in a stadium during a baseball game
D. depth of snow on the ground during a blizzard
E. the total number of babies born in the United States during 1980

3. Which of the following is not an electromagnetic wave phenomenon?

A. radar
B. ultraviolet light
C. sound
D. X radiation

4. A coulomb is a unit of

A. resistance.
B. power.
C. current.
D. potential difference.
E. charge.

Questions 5, 6, and 7 list the names of scientists who made significant contributions in the study of light. Select the statement from the list below that best describes the contribution of the particular scientist.

A. showed that light exhibits the phenomenon of interference

B. found that color is not an inherent property of an object, but depends on how the object reflects and absorbs the various colored rays that strike it
C. invented a plastic sheet that would polarize light
D. developed a mathematical wave theory of light

5. Young

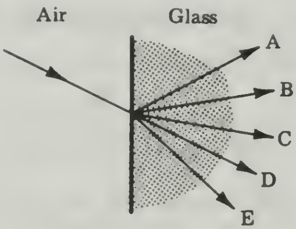
6. Fresnel

7. Newton

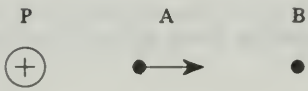
8. In a vacuum, electromagnetic radiations, such as radio waves, light, X rays, and gamma rays, have the same

A. wavelength. B. frequency. C. period. D. speed. E. amplitude.

9. A narrow beam of light strikes a block of ordinary glass at the angle shown in the diagram. Which arrow in the diagram best represents the direction of the beam within the glass?

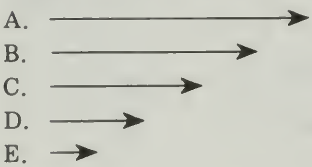


Question 10 refers to the following diagram and statement.



A positively charged pith ball is located at point P. The electric field (magnitude and direction) at point A due to the charge at P is represented by the arrow shown.

10. Which vector best represents the electric field at point B due to the charge at P?



11. Gravitational and electrostatic (coulomb) forces are similar in many ways but differ in others. Which one of the following statements is *not* true for both gravitational and electrostatic forces?

A. The force varies at $1/R^2$.
B. The force depends upon the quantity (mass or charge) on which the force acts.
C. The force can be attractive and repulsive.
D. The force law can be tested in the laboratory.

12. The angstrom (Å) is a unit of

A. mass B. time C. speed D. length

13. The direction of the electric field in a plane electromagnetic wave is

A. perpendicular to the magnetic field and in the direction of the wave's propagation.
B. perpendicular to the magnetic field and perpendicular to the direction of the wave's propagation.
C. parallel to the magnetic field.

14. The wavelength of visible light is most nearly the same as

A. the length of a football field.
B. your height.
C. the diameter of an apple.
D. the diameter of a pencil.
E. the thickness of a soap bubble.

15. A conclusion that could be drawn from the experiment of Michelson and Morley was that

A. the speed of light is greater in a vacuum than in glass.
B. light is an electromagnetic radiation.
C. the earth moves through the ether.
D. light consists of particles, not waves.
E. there is no ether.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. The introduction of inexpensively transmitted electrical power late in the nineteenth century had profound effects on the structure of American society. State and discuss briefly two of these effects.

2. Discuss the objection to Newton's theory of color raised by such nature philosophers as Schelling and Goethe.

3. Describe the term "field" as it is used in physics.

4. Describe one property of X rays that makes them suitable for medical diagnosis of bone fractures.

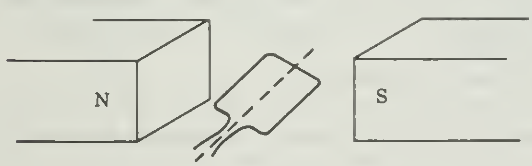
5. State one reason for the conclusion that electromagnetic waves carry energy.

Group Two

Answer ONE of the following two questions. Allow about 10 min.

6. Why is the sky blue?

7. Consider a wire loop between magnetic poles, as shown below. Discuss the physical principles involved when this apparatus is used as a generator of electric current.



Unit 4 / Light and Electromagnetism — TEST C

Directions

Answer ALL 40 multiple-choice questions by marking the letter corresponding to the one best answer. Equations that may be useful in this test are given at the end of the tests for this unit.

1. A unit of electric potential difference is the

A. ampere. B. ohm. C. volt. D. joule. E. coulomb.

2. Arrange the following units of length in order of increasing magnitude.

1. centimeter
2. nanometer
3. meter

The correct arrangement is

A. 1, 2, 3.
B. 2, 3, 1.
C. 3, 1, 2.
D. 2, 1, 3.
E. 3, 2, 1.

3. The equations that led to the prediction that light is an electromagnetic phenomenon were derived by

A. Coulomb. B. Oersted. C. Faraday. D. Maxwell. E. Ampère.

4. ALL EXCEPT ONE of the following satisfy the definition of a field as given in the *Text*. Find the *exception*.

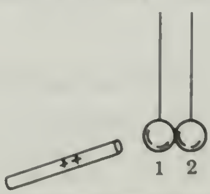
A. water temperature in Lake Michigan
B. density of smoke in the air above New York
C. noise level in a stadium during a baseball game
D. depth of snow on the ground during a blizzard
E. the total number of babies born in the United States during 1980

5. Which one of the following statements is correct?

A. Electricity and magnetism are unrelated phenomena.
B. Magnets can produce electric currents, but electric currents cannot produce magnetic fields.
C. Magnets can produce electric currents and electric currents can produce magnetic fields.
D. Electric currents cannot produce magnetic fields.
E. Electricity and magnetism are identical properties of lodestones.

6. Two uncharged conducting spheres are suspended by nylon threads and touch each other. With a positively charged rod held *near* Sphere 1, the two spheres separate. The charges on the two spheres will be

	Sphere 1	*Sphere 2*
A.	none	positive
B.	negative	positive
C.	none	none
D.	negative	none



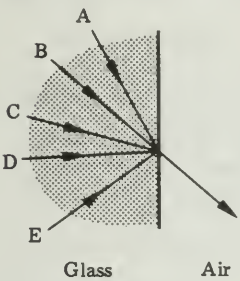
7. The electric field vector at a point in an electrostatic field indicates

1. the magnitude of the electrostatic force exerted per unit charge at that point
2. the direction of the electrostatic force exerted per unit charge at that point.
3. the electric charge at that point.

Which of the above correctly describe(s) a property of the electric field vector?

A. 3 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. 1, 2, and 3

8. A narrow beam of light emerges from a block of ordinary glass in the direction shown in the diagram. Which arrow in the diagram best represents the path of the beam within the glass?



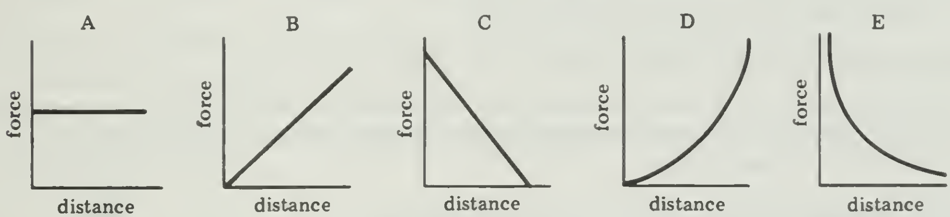
9. Newton's synthesis of terrestrial and celestial mechanics incorporated the work of Kepler and Galileo. In a similar way, the work of Oersted and Faraday was incorporated in the synthesis made by

A. Ampère. B. Hertz. C. Maxwell. D. Gilbert.

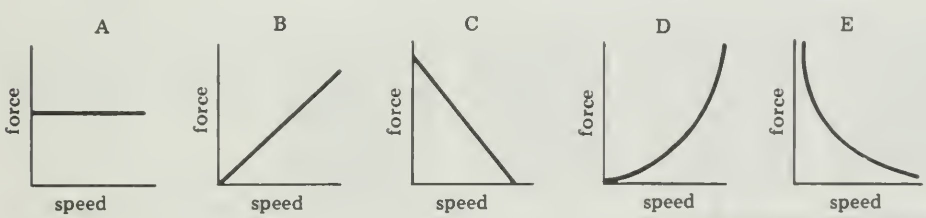
10. A glass prism separates white light into the colors of the spectrum because

A. light is reflected inside the prism.
B. different frequencies of light move with different speeds in the prism.
C. different frequencies of light superpose in the prism.
D. electromagnetic energy is dissipated inside the prism.

11. Which of the following graphs best represents the way that the force one small charged body exerts on another changes when the distance between their centers is changed?



12. Which of the following graphs best represents the force on a charged particle moving across a uniform magnetic field when the particle's speed increases?



13. A conclusion drawn from the experiment of Michelson and Morley was that

A. the speed of light is greater in a vacuum than in glass.
B. light is an electromagnetic radiation.
C. the earth moves through the ether with the speed of light.
D. light consists of particles, not waves.
E. there is no ether.

14. Which of the following is *not* an electromagnetic wave phenomenon?

A. radar B. ultraviolet light C. sound D. X radiation

15. Which of the following produce(s) a magnetic field?

1. an electric current in a wire
2. a moving charged particle
3. a changing electric field

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. 1, 2, and 3

16. In a vacuum, electromagnetic radiations such as radio waves, light, X rays, and gamma rays, have the same

A. wavelength. B. frequency. C. period. D. speed. E. amplitude.

17. P watts of power are dissipated in the form of heat when a current I flows through a heater coil whose resistance is R. If the current through the coil is doubled, how much power will be dissipated in the form of heat?

A. 1/4 P watts B. 1/2 P watts C. P watts D. $2P$ watts E. $4P$ watts

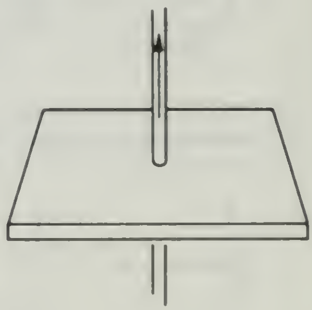
18. Gravitational and electrostatic (Coulomb) forces are similar in many ways but differ in others. Which one of the following statements is *not* true for both gravitational and electrostatic forces?

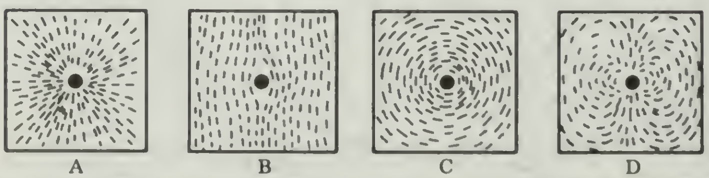
A. The force varies as $1/R^2$.
B. The force depends upon the quantity (mass or charge) on which the force acts.
C. The force can be attractive and repulsive.
D. The force law can be tested in the laboratory.

19. The direction of the electric field in a plane electromagnetic wave is

A. perpendicular to the magnetic field and perpendicular to the direction of the wave's propagation
B. perpendicular to the magnetic field and in the direction of the wave's propagation.
C. parallel to the magnetic field.

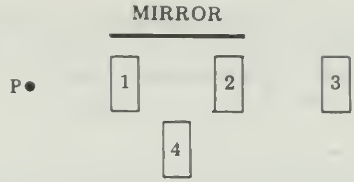
20. A wire carrying a large and constant electric current passes through the center of and perpendicular to a piece of cardboard, as shown at right. If iron filings are sprinkled on the cardboard, how will they arrange themselves?





21. A person whose eyes are located at point P is looking into the mirror. Which of the numbered cards can he see reflected in the mirror?

A. 1 and 4
B. 2 and 3
C. 1, 2, and 3
D. 1, 2, 3, and 4



22. The wave and particle models of light predict contradictory values of the velocity of light when used to explain

A. reflection of light. B. refraction of light. C. polarization. D. superposition.

23. Consider the following:

1. a wire loop surrounding a wire with a steady current
2. a magnet dropping through a wire loop
3. a stationary charged sphere at the center of a wire loop

In which of the above is a current produced in the wire loop?

A. 1 only
B. 2 only
C. 3 only
D. 1 and 3 only
E. 2 and 3 only

24. Which of the following is the chief physical principle on which the operation of an electric generator depends?

A. A current is induced in a wire moving through a magnetic field.
B. The electric field strength varies as the inverse square of the distance from a charge.
C. Two current-carrying wires exert forces on one another.
D. An alternating current produces electromagnetic radiation.

25. The angstrom (Å) is a unit of

A. mass. B. time. C. speed. D. length.

26. Which of the following could you measure to find your true motion through space?

A. apparent speed of light
B. speed of the motion relative to the ether
C. speed of the motion relative to some stationary object
D. none of the above

27. A vertical wire hidden in a wall is carrying a direct current. What piece of equipment might help you find the location of the wire?

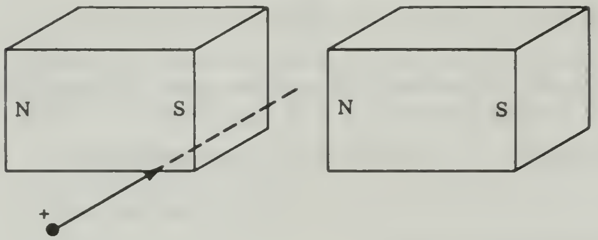
A. transformer B. dc generator C. compass D. radio receiver

28. Two identically charged small spheres are at a distance r meters apart. If the distance is doubled to $2r$ meters, the force exerted on each sphere will

A. change to four times the original value.
B. change to two times the original value.
C. not change.
D. change to one-half the original value.
E. change to one-fourth the original value.

29. In the apparatus shown at right, a beam of positively charged particles is directed horizontally into the field between two magnets. What is the effect of the magnetic field?

A. The particles continue in the same direction with the same speed.
B. The particles are accelerated toward the S magnetic pole.
C. The particles are accelerated toward the N magnetic pole.
D. The particles are accelerated upward.



Questions 30, 31, and 32 list the names of scientists who made significant contributions in the study of light. Select the statement from the list below that best describes the contribution of the particular scientist.

A. Light exhibits the phenomenon of interference.
B. Color is not an inherent property of an object, but depends on how the object reflects and absorbs the various colored rays that strike it.
C. invented a plastic sheet that would polarize light
D. developed a mathematical wave theory of light

30. Young

31. Fresnel

32. Newton

33. Three identical metal balls A, B, and C are mounted on insulating rods. Ball A has a charge $+q$, whereas balls B and C are uncharged. Ball A is brought into contact momentarily with ball B, and then with ball C. At the end of this experiment, the charge on ball A will be:

A. $+q$.
B. $+q/2$.
C. $+q/3$.
D. $+q/4$.
E. No charge remains on A.

Questions 34, 35, and 36 list the names of scientists who made significant contributions to the study of electromagnetic phenomena. Select the statement that best describes a contribution of the particular scientist.

A. Two current-carrying wires exert forces on each other.
B. An electric field changing with time generates a magnetic field.
C. An electric current exerts a force on a magnet.
D. A magnetic field changing with time can cause a current to flow in a wire.

34. Maxwell

35. Ampère

36. Faraday

37. An electric motor that draws a 2-A current when operating at 100 V can do work at the rate of

A. 55 W. B. 110 W. C. 220 W. D. 440 W.

38. Which one of the following scientists first demonstrated experimentally that the earth behaves like a large magnet?

A. Gilbert B. Oersted C. Faraday D. Maxwell E. Ampère

39. A charged particle moves through a uniform magnetic field. The effect of the field can change the particle's

A. velocity. B. speed. C. energy.

40. A transformer can be used to change

A. electrical energy into mechanical energy.
B. mechanical energy into electrical energy.
C. high-voltage dc to low-voltage dc.
D. low-voltage ac to high-voltage ac.

Unit 4 / Light and Electromagnetism — TEST D

Directions

This test consists of eight questions in two groups. Answer only FOUR of the five questions in Group One, and only TWO of the three questions in Group Two. Spend about 5 min on each of the questions from Group One, and 10 min on each of the questions from Group Two.

Equations that may be useful in this test are given at the end of the tests for this unit.

Group One

Answer only FOUR of the five questions in this group. Allow about 5 min each.

1. Describe any changes that occur in the velocity, wavelength, and frequency of light as it passes from air into a block of glass.

2. Explain why there are belts of rapidly moving charged particles (Van Allen belts) around the earth.

3. Was the postulation of an ether a necessary part of Maxwell's electromagnetic theory? Explain briefly.

4. Describe two similarities between X rays and radio waves.

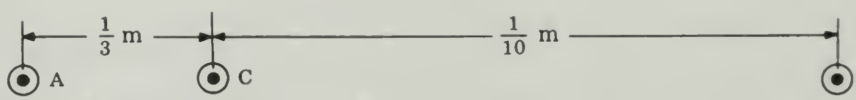
5. Describe how to determine whether a given material is an electrical insulator or conductor.

Group Two

Answer only TWO of the three questions in this group. Allow about 10 min each.

6. Three charged objects A, B, and C are situated as indicated in the diagram below. Use Coulomb's law to calculate the net electrostatic force on C.

   ($k = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$)



$q_A = +1 \times 10^{-6}$ C $\quad q_C = -1 \times 10^{-6}$ C $\quad q_B = +2 \times 10^{-6}$ C

7. Why are brilliantly colored sunsets sometimes seen in highly industrial areas where there is the problem of air pollution?

8. What are the two main assumptions of Einstein's special theory of relativity?

Equations:

$$F_{AB} = \frac{kq_A q_B}{R^2}$$

$$v = f\lambda$$

$$P = IV = I^2R$$

$$F = qvB$$

Unit 5 / Models of the Atom — TEST A

Directions

This test consists of 15 multiple-choice questions and seven problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants, a definition, and equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. Which one of the following equations relates an increase in an object's mass with an increase in the object's speed?

A. $m = F/a$ B. $\frac{q}{m} = \frac{v}{BR}$ C. $\frac{1}{2}mv^2 = hf - W$ D. $m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$ E. $mv = \frac{hf}{c}$

2. Which of the following could not be explained in terms of classical physics?

1. the photoelectric effect
2. variation of mass with speed
3. the Compton effect

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 1, 2, and 3

3. An electron from a hydrogen atom

A. is identical to an electron from an oxygen atom.
B. has greater rest mass than an electron from an oxygen atom.
C. is larger than an electron from an oxygen atom.
D. has greater charge than an electron from an oxygen atom.

4. A reasonable prediction, based on the evolution of previous scientific theories, is that in the future the quantum theory will

A. be replaced by a theory based on a mechanical model.
B. be replaced by a more general theory.
C. be shown to be wrong.
D. explain everything about nature.

5. A clean surface of potassium metal will emit electrons when exposed to blue light. If the intensity of the blue light is increased, which of the following will increase also?

1. the number of electrons ejected per second
2. the maximum kinetic energy of the ejected electrons
3. the charge of each ejected electron

A. 1 only B. 2 only C. 3 only D. 1 and 2 only E. 1, 2, and 3

6. In an electrolysis experiment, a certain amount of hydrogen is collected. If the experiment were repeated with one-third as much electric current, and one-fifth as much time, how much hydrogen would be collected?

A. 1/15 as much B. one-eighth as much C. one-fifth as much D. one-third as much E. one-half as much

7. ALL EXCEPT ONE of the following terms can be applied to both an X ray and an atom of hydrogen. Find the *exception.*

A. wavelength B. momentum C. velocity D. rest mass E. energy

8. Bohr's atomic model

A. allows only certain values of angular momenta for the orbital electron of hydrogeen.
B. explains the spectra of elements whose atoms have more than one electron in the outermost shell.
C. assumes that electrons have wave properties.

9. In the modern periodic table, the elements are arranged in order of increasing

A. atomic mass. B. atomic number.

10. Which of the following entered significantly into the determination of q/m for electrons in J. J. Thomson's experiment?

1. A force acts upon a moving electron in a gravitational field.
2. A force acts upon a moving electron in an electric field.
3. A force acts upon a moving electron in a magnetic field.

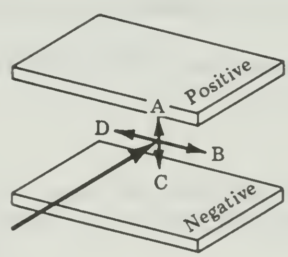
A. 1 and 2 only
B. 1 and 3 only
C. 2 and 3 only
D. 1, 2, and 3

11. In a scattering experiment, some α particles directed toward a gold foil come straight back. At the point of closest approach of an α particle to the nucleus of the gold atom, the α particle must have had zero

A. kinetic energy. B. potential energy. C. electrical energy. D. acceleration. E. charge.

12. A beam of electrons is directed between two charged plates as indicated in the diagram. Once the beam is between the plates it will

A. curve in direction A.
B. curve in direction B.
C. curve in direction C.
D. curve in direction D.
E. continue in a straight line.



13. When the speed of an electron increases, the measured value of the charge-to-mass ratio is

A. increased because the mass decreases.
B. increased because the charge increases.
C. decreased because the mass increases.
D. decreased because the charge decreases.
E. unchanged.

14. No physicist has been able to think of an experiment that could reveal the exact position of an electron in a given atom. Therefore, modern physicists

   A. assume that the electrons take positions predicted by Bohr's theory.
   B. have developed a theory that states that the position of an electron in an atom cannot be found precisely.
   C. look forward to the time when such experiments will be done.

15. Which statement about electrons is *false*? Moving electrons

   A. have masses that are independent of speed.
   B. may be diffracted.
   C. can be deflected by a magnetic field.
   D. can be deflected by an electric field.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. It was found by experiment that the ratio of charge to mass of a certain particle was 1/1,836 times the ratio of charge to mass of an electron. State at least two different hypotheses that might account for this observation.

2. The compound zinc oxide (ZnO) contains equal numbers of atoms of zinc and oxygen. The atomic mass of zinc is 65.37 and the atomic mass of oxygen is 15.99. Calculate the percentage by mass of zinc in zinc oxide.

3. What factors influence the amount of deflection of a beam of electrons by a magnetic field?

4. Explain the meaning of the equation $mvr = \dfrac{nh}{2\pi}$ in Bohr's model of the atom.

5. Calculate the de Broglie wavelength of a neutron (mass $= 1.67 \times 10^{-27}$ kg) traveling at 10^{6} m/sec.

Group Two

Answer ONE of the following two questions. Allow about 10 min.

6. The generalized Balmer formula that describes the hydrogen spectrum is

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

   In the Bohr model, the energy of the radiation emitted or absorbed when a hydrogen atom goes from an initial energy state to a final energy state is

$$hf = \frac{E_1}{n_f^2} - \frac{E_1}{n_i^2}$$

   If $E_1 = R_H \cdot hc$, show that the Balmer formula may be derived from the Bohr formula.

7. (a) Write a brief statement of Heisenberg's uncertainty principle.
   (b) If the uncertainty in the position of an electron is 10^{-10} m, what is the uncertainty in its momentum?

Unit 5/ Models of the Atom — TEST B

Directions

This test consists of 15 multiple-choice questions and eight problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

The numerical values of some physical constants, a definition, and equations that may be useful in this test are given at the end of the tests for this unit.

MULTIPLE-CHOICE QUESTIONS

1. An electron has a rest mass of 9.1×10^{-31} kg. Each electron in a certain beam has a mass of 9.6×10^{-31} kg. Therefore, we can conclude there has been an increase in the electron's

1. kinetic energy.
2. speed.
3. rest mass.

A. 1 only
B. 2 only
C. 3 only
D. 1 and 2 only
E. 1, 2, and 3

2. An oxygen molecule is made up of atoms, nuclei, and electrons. If one lists these for oxygen in order of decreasing mass with the most massive listed first, which one of the following lists is correct?

A. electron, nucleus, atom
B. nucleus, atom, electron
C. nucleus, electron, atom
D. atom, nucleus, electron
E. atom, electron, nucleus

3. Most gases can be analyzed by means of a spectroscope because each element

A. can be recognized when magnified up to 100,000 times its normal size.
B. occupies a unique position in the periodic table.
C. when heated to a high temperature emits light with a characteristic set of wavelengths.
D. has a different atomic mass.

4. An electron from a hydrogen atom

A. is identical to an electron from an oxygen atom.
B. is more massive than an electron from an oxygen atom.
C. is larger than an electron from an oxygen atom.
D. has greater charge than an electron from an oxygen atom.

5. Which of the following statements is (are) correct?

1. X rays travel at the speed of light.
2. X rays may be produced when high-energy electrons are stopped by a target.
3. X rays are high-energy electrons.

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. 1, 2, and 3

6. An understanding of the photoelectric effect was most important to the development of

A. the quantum theory of light.
B. Thomson's atomic model.
C. Faraday's second law of electrolysis.
D. the periodic table of elements.

7. Evidence that atoms might have structure was found in

1. electrolysis experiments.
2. the periodic properties of elements.
3. cathode-ray experiments.

A. 1 only
B. 2 only
C. 3 only
D. 1 and 3 only
E. 1, 2, and 3

8. Rutherford's model of the atom accounted for the

A. stability of the nucleus.
B. stability of the electron orbits.
C. line spectra of elements.
D. scattering of α particles by metal foils.
E. scattering of X rays by metal foils.

9. Which one of the following electromagnetic radiations has photons of the greatest energy?

A. radio
B. infrared
C. visible light
D. ultraviolet
E. X rays

10. In a letter to Max Born in 1926, Einstein wrote:

> The quantum mechanics is very imposing. But an inner voice tells me that it is still not the final truth. The theory yields much, but it hardly brings us nearer to the secret of the Old One. In any case, I am convinced that He does not throw dice.

In this statement, what characteristic of quantum mechanics was Einstein objecting to?

A. The predictions of quantum mechanics can only be expressed as probabilities
B. Quantum mechanics considers both wave and particle properties of matter.
C. The development of quantum mechanics involved very complicated mathematics

11. Physicists refer to the dual nature of matter: Matter has both particle properties and wave properties However, the wave property of large, massive objects is NOT observed because

A. this dual nature applies only to matter on the atomic scale.
B. their accelerations are too small.
C. their wavelengths are too small to detect.
D. their speeds are too small.
E. they do not emit photons.

12. The following men made important contributions to our understanding of atomic structure.

   1. Bohr
   2. Dalton
   3. Schrödinger

   If one were to list the names in order of their contribution, with the earliest listed first, they would be arranged

   A. 1, 2, 3.
   B. 2, 1, 3.
   C. 2, 3, 1.
   D. 3, 1, 2.
   E. 3, 2, 1.

13. The model of the atom used in quantum mechanics is

   A. the planetary model described by Bohr.
   B. similar to Bohr's model, but with elliptical orbits for the electrons.
   C. a mathematical wave equation.
   D. the "raisin pudding" model of electrons imbedded in positive electricity.

14. The Millikan oil-drop experiment was the first conclusive experimental demonstration that

   A. electric charge is found as multiples of a certain unit of charge.
   B. all electrons have a negative charge.
   C. electrons are particles.
   D. electrons have wave properties.
   E. all atoms contain electrons.

15. The combining capacity of an element is called its

   A. atomic number.
   B. valence.
   C. atomic mass.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. What is meant by the expression "wave–particle dualism"?

2. Describe two shortcomings of Bohr's model of the atom.

3. How is the energy of a photon related to properties of the electromagnetic wave with which it is associated?

4. State one important consequence of the periodic table of elements formulated by Mendeleev.

5. Although the alchemists failed in their efforts to transmute ordinary metals into gold, their work has had a profound influence on the development of certain areas of science as we know them today. Describe briefly the role played by alchemy in the process by which modern chemistry evolved.

Group Two

Answer ONE of the following three questions. Allow about 10 min.

6. Select ONE of the following experiments.

J. J. Thomson's q/m experiment
Millikan's oil-drop experiment
Photoelectric effect experiments
Faraday's electrolysis experiment
Rutherford's α-particle scattering experiment

(a) Sketch a diagram of the apparatus used in the experiment.
(b) Explain the significance of the experiment in the development of present ideas about the atom.

7. The atomic mass of element A is six times that of element B. In a compound containing only A and B, it is found that there is three times as much A as there is B (by weight). That is, an 8-g sample of the compound contains 6 g of A and 2 g of B. What is a possible formula for this compound?

8. Franck and Hertz found that electrons lose only certain amounts of kinetic energy in collisions with atoms of a gas. The experiment involved measuring the kinetic energy of electrons before and after they passed through a sample of gas.

Sketch a diagram of an apparatus that could be used to make these measurements.

Unit 5 / Models of the Atom — TEST C

Directions

Answer ALL 40 multiple-choice questions by marking the letter corresponding to the one best answer.
The numerical values of some physical constants, a definition, and equations that may be useful in this test are given at the end of the tests for this unit.

1. In a scattering experiment, some α particles directed toward a gold foil come straight back. At the point of closest approach of an α particle to the nucleus of the gold atom, the α particle must have had zero

A. kinetic energy. B. potential energy. C. electrical energy. D. acceleration. E. charge.

2. ALL EXCEPT ONE of the following are predictions of the special theory of relativity. Which one is the *exception*?

A. Photons have momentum.
B. The mass of a body increases with its speed.
C. Electrons in an atom have certain discrete energies.
D. Kinetic energy can be converted into matter.

For questions 3 to 6, use the following list to select the phenomenon that correctly completes the sentence.

A. scattering of α particles by gold foil.
B. bright-line spectra of hydrogen atoms.
C. emission of electrons from metal surfaces struck by electromagnetic radiation of different frequencies.
D. diffraction of electrons by crystals.
E. scattering of X rays by electrons.

3. The concept of a nuclear atom was established from experiments on the

4. The momentum of a photon was demonstrated in experiments on the

5. The wave character of matter was confirmed by

6. Bohr's theory was successful in explaining

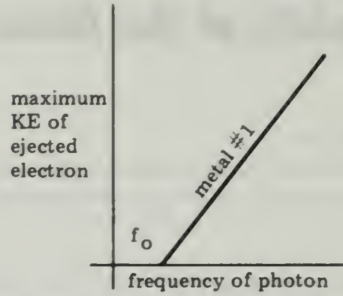
7. In an electrolysis experiment, 10.0 mL of hydrogen gas is collected. If the experiment were repeated using the same amount of water, one-third as much electric current, and one-fifth as much time, how much hydrogen would be collected?

A. 0.67 mL
B. 1.00 mL
C. 1.67 mL
D. 2.00 mL
E. 3.33 mL

8. The success of the Bohr theory rested primarily on the fact that it

A. had a firm theoretical basis in quantum physics.
B. was a consequence of Einstein's relativity theory.
C. explained the observed spectrum of hydrogen.
D. explained the properties of the nucleus.

Questions 9 and 10 refer to the graph that displays the results of a photoelectric effect experiment.



9. The symbol f_o represents

A. Planck's constant.
B. the energy required for ejection from metal #1.
C. the threshold frequency for metal #1.
D. the energy of an ejected electron.

10. The slope of the line labeled metal #1 equals

A. Planck's constant.
B. the energy required for ejection from metal #1.
C. the threshold frequency for metal #1.
D. the energy of an ejected electron.

11. The unexpected finding about the scattering of α particles by gold foil was that

A. most particles went through the foil.
B. some particles were deflected through large angles.
C. scintillations were observed in the detector.
D. scattering varied with foil thickness.
E. α particles were more massive than cathode rays.

12. The "electron volt" is a unit of

A. electric current.
B. energy.
C. potential difference.
D. rate of flow of electricity.

Questions 13 and 14 refer to the following diagram that gives the energies of some stationary states of hydrogen.

$n = \infty$	0.0 eV
$n = 4$	−0.8
$n = 3$	−1.5
$n = 2$	−3.4
$n = 1$	−13.6

13. How much energy is emitted when an atom makes a transition between the stationary states designated by $n = 3$ and $n = 2$?

A. 1.5 eV
B. 1.9 eV
C. 3.4 eV
D. 4.9 eV
E. 10.2 eV

14. If hydrogen atoms, as described by the Bohr model, are excited to the stationary state designated by $n = 3$, how many different frequencies of radiation may be emitted by the atoms?

A. 1
B. 2
C. 3
D. 4
E. 5

15. Which of the following three statements is (are) true of cathode rays?

1. They are emitted by a variety of cathode materials.
2. Their paths may be bent by magnetic fields.
3. Their paths may be bent by electric fields.

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. 1, 2, and 3

16. According to classical electromagnetic theory, which of the following should occur in an atomic model that has electrons revolving in orbits around the nucleus?

1. Electrons should lose energy and fall into the nucleus.
2. Electrons should emit radiation continually.

A. 1 only
B. 2 only
C. 1 and 2

17. A beam of α particles with kinetic energy 3 MeV is directed at a <u>gold</u> foil 1,000 atoms thick. A second beam of α particles with kinetic energy 3 MeV is directed at a <u>silver</u> foil 1,000 atoms thick.

A. The number of α particles scattered back by both foils will be identical.
B. The number of α particles scattered back by the gold foil will be different from the number scattered back by the silver foil.
C. Each foil will scatter all the particles directed at it.
D. There will be no scattering by either foil.

18. A clean surface of potassium metal will emit electrons when exposed to blue light. If the intensity of the blue light is increased, which of the following will also increase?

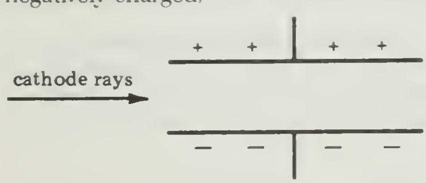
1. The number of electrons ejected per second.
2. The maximum kinetic energy of the ejected electrons.

A. 1 only
B. 2 only
C. 1 and 2

19. X rays are

A. low-energy cathode rays.
B. high-energy photons.
C. ionized gas molecules.
D. waves accompanying photoelectrons.
E. particles traveling at speeds just below the speed of light.

20. A beam of cathode rays traveling between two parallel plates, one positively charged and the other negatively charged,



A. is deflected toward the positive plate.
B. is deflected toward the negative plate.
C. is not deflected.

21. CuO and Cu_2O are two compounds of copper and oxygen. If 4 g of copper combine with 1 g of oxygen to form CuO, what weight of copper will combine with 1 g of oxygen to form Cu_2O?

A. 1/4 g
B. 1/2 g
C. 2 g
D. 4 g
E. 8 g

22. Mercury vapor, when conducting a current, appears bluish-green. What is observed when the light from glowing mercury vapor is analyzed in a spectroscope?

A. a series of discrete lines
B. a series of irregular bluish-green flashes
C. a bluish-green glow
D. the entire visible light spectrum with some dark lines

23. Millikan's charged oil-drop experiment was the first conclusive experimental demonstration that

A. electric charge is found as multiples of a certain unit charge.
B. all electrons have a negative charge.
C. electrons are particles.
D. electrons have wave properties.
E. all atoms contain electrons.

Questions 24 to 26 refer to the following table that gives some data from electrolysis experiments.

Element	*Atomic Mass*	*Valence*	*Quantity of element produced by 1 F of charge*
Hydrogen (H)	1.0	1	1.0 g
Zinc (Zn)	65.0	2	
Phosphorus (P)		3	10.3 g

24. The quantity of zinc deposited by 1 F of electric charge is

A. 21.7 g. B. 32.5 g. C. 65 g. D. 130 g.

25. The atomic mass of phosphorus is

A. 3.4. B. 10.3. C. 20.6. D. 30.9.

26. The most obvious formula for a compound of hydrogen and phosphorus is

A. HP B. HP_2 C. HP_3 D. H_3P E. H_2P_3

27. Physicists are willing to accept the wave–particle dualism because

A. the waves associated with particles are too small to be measured.
B. two theories are always better than one.
C. both wave and particle descriptions are needed to understand experimental results.
D. the dualism is confirmed by the theory of relativity.

28. Einstein explained the photoelectric effect by assuming that

A. the charge of an electron increases with speed.
B. atoms do not radiate energy from stationary states
C. the mass of an electron increases with speed.
D. light consists of quanta of energy.
E. the energy of light increases with speed.

29. ALL EXCEPT ONE of the following are properties of X rays. Which one is the *exception*?

A. They penetrate light materials.
B. They ionize gases.
C. They are deflected by magnetic fields.
D. They discharge electrified bodies.
E. They are diffracted by crystals.

30. A reasonable prediction, based on the evolution of previous scientific theories, is that in the future the quantum theory will

A. be replaced by a theory based on a mechanical model.
B. be replaced by a more general theory.
C. be shown to be wrong.
D. explain everything about nature.

31. ALL EXCEPT ONE of the following are true of an electron of rest mass m_o moving with high speed. Which one is the *exception*?

A. Its mass is greater than m_o.
B. Its momentum is greater than $m_o v$.
C. It behaves also like a wave train of wavelength h/p.
D. Its kinetic energy is greater than $\frac{1}{2} m_o v^2$.
E. Its charge is greater than q_e.

32. The model of the atom used in quantum mechanics is

A. the planetary model described by Bohr.
B. similar to Bohr's model, but with elliptical orbits for the electrons.
C. mathematical.
D. the "raisin pudding" model of electrons dispersed in positive electricity.
E. a small solid sphere.

33. Bohr dealt with the dilemmas of the planetary model of atoms by

A. adjusting the data to fit his theory.
B. postulating that parts of classical theory did not apply.
C. postulating that atoms are unstable.
D. postulating that electrons have no energy.
E. disproving the Balmer formula.

34. The wave–particle dualism of matter can be confirmed experimentally for

A. electrons.
B. baseballs.
C. planets.
D. stars.
E. water waves.

35. The conclusion that the atom has a tiny, charged nucleus was first reached from

A. the evidence that X rays can ionize molecules.
B. the evidence that X rays can pass through matter.
C. the calculation of the distance between the nucleus and the electron in hydrogen atoms.
D. the calculation that 11 series of the hydrogen spectrum are described by the equation

$$1/\lambda = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

E. the evidence that some α particles are deflected through large angles by thin slices of matter.

36. ALL EXCEPT ONE of the following are conclusions that can be drawn from a quantitative study of the electrolysis of water. Which one is the *exception*?

A. Water is not an element.
B. Matter has electricity associated with it.
C. Hydrogen and oxygen are elements.
D. In water, hydrogen and oxygen carry opposite charges.

Questions 37 to 39 are statements that relate most directly to one of the following theories. Select the appropriate theory.

A. Bohr's theory
B. Heisenberg's uncertainty principle
C. Newton's universal theory of gravitation
D. Einstein's relativity theory

37. The mass of a moving object increases as its speed increases.

38. There is a limit to the accuracy of the simultaneous measurement of the velocity and position of a moving electron.

39. The angular momentum of an electron in a hydrogen atom can have only the values $h/2\pi$, $2h/2\pi$, $3h/2\pi$, . . .

40. The Franck–Hertz experiment on the energy of electrons after passing through a gas provided evidence for the concept of

A. discrete atomic energy levels.
B. momentum of photons.
C. a plum pudding atom.
D. Compton scattering.
E. electron wavelengths.

Unit 5/ Models of the Atom — TEST D

Directions

This test consists of eight questions in two groups. Answer only FOUR of the five questions in Group One, and only TWO of the three questions in Group Two. Spend about 5 min on each of the questions from Group One, and 10 min on each of the questions from Group Two.

The numerical values of some physical constants, a definition, and equations that may be useful in this test are given at the end of the tests for this unit.

Group One

Answer only FOUR of the five questions in this group. Allow about 5 min each.

1. Faraday's law of electrolysis relates a particular amount of electric charge (the faraday) to the atomic mass and valence of an element. What is this relation? Explain how this relation implies that electricity may be atomic (quantized) in nature.

2. What is one implication of the Millikan oil-drop experiment?

3. What aspects of the atom not included in Rutherford's idea of a nuclear atom were later explained by Bohr's model?

4. If the energy required for an electron to escape a certain metallic surface is 2×10^{-18} J, what is the lowest frequency of light that will release electrons from this surface?

5. What happens to the relativistic mass of an electron as its speed approaches the speed of light?

Group Two

Answer only TWO of the three questions in this group. Allow about 10 min each.

6. When 96,500 C of charge (1 F) pass through water, 1.00 g of hydrogen and 8.00 g of oxygen are released. How much hydrogen and how much oxygen will be produced when a current of 3.00 A is passed through water for 60 min (3,600 sec)?

7. Each of the following equations symbolically represents a key advance in physics. Select two of these equations and describe their roles in the development of modern physics.

I. $KE_{max} = hf - W$

II. $\dfrac{1}{\lambda} = R_H \left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2} \right)$

III. $m = \dfrac{m_o}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

IV. $\lambda = \dfrac{h}{mv}$

V. $(\Delta x)(\Delta p) \geq \dfrac{h}{2\pi}$

8. Compare the atomic theory of the Greeks with the atomic theories developed by scientists late in the nineteenth century.

Physical Constants:

velocity of light in vacuum (c) $= 3.00 \times 10^{8}$ m/sec
Planck's constant (h) $= 6.61 \times 10^{-34}$ J-sec

Definition:

1 F = 96,500 C

Equations:

$$KE_{max} = hf - W$$

$$\lambda = \frac{h}{mv}$$

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$(\Delta x)(\Delta p) > \frac{h}{2\pi}$$

$$I \text{ (amperes)} = \frac{q \text{ (coulombs)}}{t \text{ (seconds)}}$$

Unit 6/ The Nucleus — TEST A

Directions
This test consists of 15 multiple-choice questions and eight problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

MULTIPLE-CHOICE QUESTIONS

Questions 1, 2, and 3 refer to the following statement: An isotope of neon is represented by the symbol ${}_{10}Ne^{21}$.

1. How many electrons are there in a neutral atom of this isotope?

A. 0
B. 10
C. 11
D. 21
E. 31

2. How many neutrons are in an atom of this isotope?

A. 0
B. 10
C. 11
D. 21
E. 31

3. How many protons are in an atom of this isotope?

A. 0
B. 10
C. 11
D. 21
E. 31

4. The charge-to-mass ratio of an α particle is the same as the charge-to-mass ratio of a

A. β particle.
B. neutron.
C. proton.
D. ${}_1H^2$ nucleus.
E. ${}_3Li^7$ nucleus.

5. While looking for the emission of X rays from fluorescent materials, Henri Becquerel discovered a new type of radiation. Which one of the following facts led Becquerel to suspect that the newly discovered rays were different from X rays?

A. The rays could penetrate thick black paper.
B. The rays were capable of producing ionizations in the air.
C. The rays were invisible to the naked eye.
D. The rays could not be started and stopped by the investigator.
E. The rays affected photographic plates.

6. Gamma rays are

A. high-frequency electromagnetic radiation.
B. identical to electrons.
C. like electrons, but with a positive charge.
D. nuclei of the element helium.
E. neutral particles with mass number 1.

7. The major function of a cyclotron is

A. to separate isotopes from one another.
B. to detect neutrons.
C. to produce neutrons.
D. to accelerate charged particles.
E. to maintain a chain reaction.

8. Which one of the following processes is an example of nuclear fusion?

A. the formation of water from hydrogen and oxygen
B. the formation of helium from hydrogen
C. the formation of barium and krypton from uranium
D. the formation of lead from radium by radioactive decay
E. the formation of potassium from potash

9. Over a period of time a certain radioactive atom emits the following particles in succession: α, α β, α, β.

The atomic mass of the end product of this radioactive decay is less than the atomic mass of the original atom by approximately

A. 1 amu.
B. 3 amu.
C. 6 amu.
D. 10 amu.
E. 12 amu.

10. Imagine that a new isotope of lithium with atomic number 3 and mass number 5 has been discovered among the radiations emitted by radioactive plutonium. Which one of the following nuclear equations describes its emission from a ${}_{94}Pu^{239}$ nucleus?

A. ${}_{94}Pu^{239} \rightarrow {}_{3}Li^{5} + {}_{91}Pa^{234}$
B. ${}_{94}Pu^{239} \rightarrow {}_{3}Li^{5} + {}_{97}Bk^{244}$
C. ${}_{94}Pu^{239} \rightarrow {}_{3}Li^{5} + {}_{91}Pa^{244}$
D. ${}_{94}Pu^{239} \rightarrow {}_{5}Li^{3} + {}_{89}Ac^{236}$
E. ${}_{94}Pu^{239} \rightarrow {}_{5}Li^{3} + {}_{91}Pa^{234}$

11. A standard way of representing a given nuclide of element X is ${}_{Z}X^{A}$. Which of the following symbols can identify the nuclide completely?

A. A only
B. X only
C. X and Z only
D. A and Z only
E. A, X, and Z

12. The chemical properties of an atom are determined by its

A. mass number. B. number of isotopes. C. atomic number. D. nuclear binding energy.

13. A proton of mass m_p and a neutron of mass m_n combine in a fusion process to form a stable deuterium nucleus. The mass of this nucleus is

A. greater than m_p plus m_n.
B. equal to m_p plus m_n.
C. less than m_p plus m_n.
D. sometimes less than and sometimes equal to m_p plus m_n.
E. sometimes greater than and sometimes equal to m_n.

14. According to the proton–neutron theory of the atomic nucleus, β-particle emission results from

A. a proton changing into an α particle.
B. a neutron changing into a proton.
C. a proton expelling an electron from the electron shells of the atom.
D. a γ ray producing an electron and positron.
E. the loss of one of the electrons in the nucleus.

15. The purpose of a moderator in an atomic reactor is to

A. provide neutrons for the fission process.
B. react with the uranium to release energy.
C. slow down fast neutrons to increase the probability of fission.
D. absorb the dangerous γ radiation.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. The atomic masses of H^1, Li^7, and He^4 are 1.0080 amu, 7.0160 amu, and 4.0026 amu, respectively (1 amu = 931 MeV). Calculate (in MeV) the amount of energy liberated in the following nuclear reaction.

$$_1H^1 + {}_3Li^7 \rightarrow {}_2He^4 + {}_2He^4$$

2. The following diagram contains the first four members of the uranium–radium series. Supply the missing data.

$$_{92}M^{238} \xrightarrow{\alpha} {}_?Th^? \xrightarrow{\beta} {}_?Pa^? \xrightarrow{?} {}_{92}M^{234}$$

3. What is meant by the statement "the law of disintegration of a radioactive substance is a satistical law?"

4. A physicist prepares 8 mg of pure Po^{218}. If the half-life of Po^{218} is 3.05 min, after what time is there only 1 mg of Po^{218} left?

5. Comment briefly on one of the social consequences of our ability to control and use atomic energy.

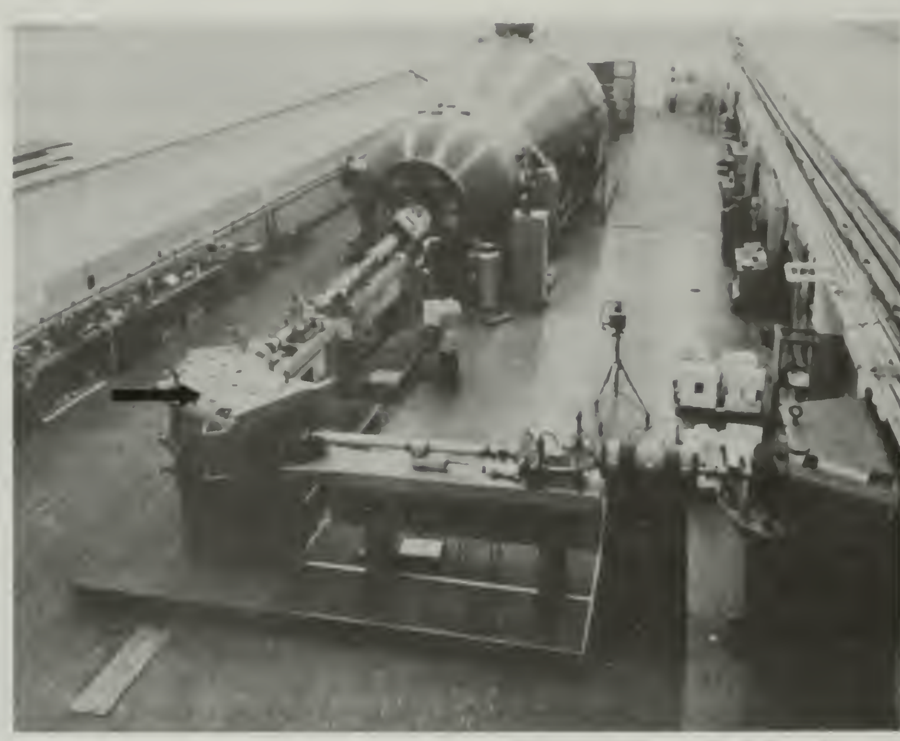
Group Two

Answer ONE of the following three questions. Allow about 10 min.

6. (a) Explain the function of the moderator in a nuclear reactor.
(b) Why is heavy water an effective moderator?

7. (a) Describe briefly the liquid-drop model of the nucleus.
(b) List one nuclear phenomenon that is "explained" by this model.

8. The picture shows the "Emperor" tandem Van de Graaff particle accelerator at Yale University and some of its associated apparatus.



(Courtesy Yale University)

What is the piece of equipment at the left marked by the black arrow? What do you think its functions are?

Unit 6 / The Nucleus

TEST B

Directions

This test consists of 15 multiple-choice questions and eight problem-and-essay questions, divided into two groups. Answer ALL multiple-choice questions by marking the letter corresponding to the one best answer. Answer THREE of the problem-and-essay questions from Group One and ONE from Group Two. Spend about 15 min on the multiple-choice questions, 5 min on each of the problem-and-essay questions from Group One, and 10 min on the problem-and-essay question from Group Two.

MULTIPLE-CHOICE QUESTIONS

1. Biologists are learning more about the metabolism of plants and animals through the use of

A. high-energy particle accelerators.
B. cloud-chamber photography.
C. isotopic tracers.
D. mass spectroscopy.
E. cosmic rays.

2. Alpha particles

A. are electromagnetic radiation of high frequency.
B. are negatively charged particles.
C. have the highest penetrating power of the three types of radiation emitted by radioactive elements.
D. produce the greatest amount of ionization per centimeter of the three kinds of emissions from radioactive nuclei.
E. have the same properties as electrons.

3. Which of the following is (are) true of β particles?

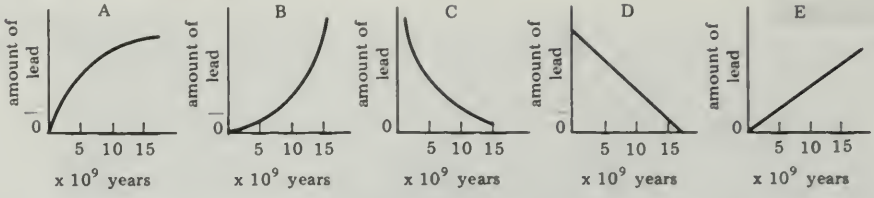
1. They originate from atomic nuclei.
2. They have the same properties as electrons.
3. If they travel through air, they produce ionization.

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. 1, 2, and 3

4. The electron volt (abbreviated eV) is a unit of

A. energy.
B. speed.
C. voltage.
D. radioactivity.
E. force.

5. Which graph best represents the change with time of the amount of stable lead present in a sample that was originally pure uranium-238?



6. ALL EXCEPT ONE of the following particles can be accelerated by an electric or magnetic field. Which one is the *exception*?
   A. electron
   B. proton
   C. neutron
   D. α particle
   E. deuteron (${}_1H^2$ nucleus)

Questions 7 and 8 list key discoveries in the field of nuclear physics. From the list below, select the person most reponsible for the discovery.

A. Becquerel
B. Curie
C. Soddy
D. Chadwick
E. Fermi

7. discovery of radium

8. discovery of the neutron

9. Isotopes of an element have
   A. different masses and different atomic numbers.
   B. the same chemical properties but different masses.
   C. the same chemical properties but different atomic numbers.
   D. the same mass but different atomic numbers.
   E. the same atomic number but different chemical properties.

10. The law of decay of radioactive samples is a statistical law. This implies that
    1. it is only applicable to samples containing a large number of atoms.
    2. it can predict little about the time of decay of an individual atom.
    3. it makes no assumptions as to why atoms disintegrate.

    A. 1 only
    B. 2 only
    C. 1 and 2 only
    D. 1 and 3 only
    E. 1, 2, and 3

Questions 11 and 12 are statements that identify one of the equations in the list below. Select the equation identified by each statement.

A. ${}_0n^1 + {}_{94}Pu^{239} \rightarrow {}_{56}Ba^{141} + {}_{38}Sr^{96} + 3{}_0n^1$
B. $4{}_1H^1 \rightarrow {}_2He^4 + 2{}_{+1}e^0 + 2\nu + \gamma$
C. ${}_0n^1 + {}_{92}U^{238} \rightarrow {}_{92}U^{239}$; ${}_{92}U^{239} \rightarrow {}_{93}Np^{239} + {}_{-1}e^0 + \nu$
D. ${}_0n^1 + {}_{13}Al^{27} \rightarrow {}_{13}Al^{28}$
E. ${}_1H^2 + {}_{80}Hg^{199} \rightarrow {}_{79}Au^{197} + {}_2He^4$

11. the formation of a transuranium element

12. release of energy in stars

13. In 1939, Hahn and Strassman identified barium as one of the disintegration products produced when uranium was bombarded with neutrons. The importance of this discovery was that it suggested that

A. the nucleus of the uranium atom could be split apart.
B. the uranium atom is really several barium atoms bound together.
C. uranium could be made radioactive.
D. uranium and barium were isotopes of the same element.
E. neutrons were converted into barium atoms.

14. Rutherford identified three types of radiation emitted from radium: alpha, beta, and gamma. If these radiations are listed in order of increasing penetrating power, with the least penetrating listed first, the order is

A. α, β, γ.
B. β, γ, α.
C. γ, α, β.
D. β, α, γ.
E. α, γ, β.

15. The purpose of a moderator in an atomic reactor is to

A. provide neutrons for the fission process.
B. react with the uranium to release energy.
C. slow down fast neutrons to increase the probability of fission.
D. release energy by combustion to keep the reactor "critical."
E. absorb the dangerous γ radiation.

PROBLEM-AND-ESSAY QUESTIONS

Group One

Answer THREE of the following five questions. Allow about 5 min each.

1. "Since the half-life of radium is 1,620 years, this element will have vanished from the earth by the year 6000." Do you agree with this statement? Discuss your answer.

2. By bombarding mercury-198 with neutrons, a modern physicist can produce gold. Does this satisfy the ancient alchemists' dream of producing gold from other metals? Discuss your answer.

3. The average binding energy per particle in the nuclide Ba^{141} is greater than the average binding energy per particle in the nuclide Ra^{222}. State one implication of these data.

4. Explain the statement "Pb^{214} and Pb^{206} are isotopes of lead."

5. What is the man in the picture doing with the instrument that he is holding?

Courtesy AccuRay

Group Two

Answer ONE of the following three questions. Allow about 10 min.

6. Discuss the differences and similarities between fusion and fission.

7. Describe briefly the theory of radioactive transformation as proposed by Rutherford and Soddy.

8. In 1963, most nations agreed to end nuclear bomb tests in the atmosphere. This agreement was reached because of concern over the undesirable effects of radioactive fallout from such tests. Comment briefly on the reasons for this concern over the effects of radioactive fallout.

Unit 6 / The Nucleus — TEST C

Directions

Answer ALL 40 multiple-choice questions by marking the letter corresponding to the one best answer.

Questions 1, 2, and 3 refer to the following statement: A certain isotope of carbon is represented by the symbol $_6C^{13}$.

1. How many electrons are there in a neutral atom of this isotope?

A. 0 B. 6 C. 7 D. 13 E. 19

2. How many protons are there in a neutral atom of this isotope?

A. 0 B. 6 C. 7 D. 13 E. 19

3. How many neutrons are there in a neutral atom of this isotope?

A. 0 B. 6 C. 7 D. 13 E. 19

4. The charge on a proton is expressed in

A. amperes. B. coulombs. C. electron volts. D. ohms. E. joules.

5. Radioactive bismuth has a half-life of 5 days. A 1-g sample of bismuth is prepared. After 5 days, the amount of bismuth in the sample is very close to

A. 1/16 g. B. 1/8 g. C. 1/4 g. D. 1/2 g. E. 1 g.

6. ALL EXCEPT ONE of the following statements are true. Which one is the *exception*?

A. Radioactivity is a natural characteristic of some elements.
B. Radioactive isotopes can be produced in the laboratory.
C. Radioactive isotopes decay by the emission of particles from the nucleus.
D. All isotopes are radioactive.
E. There is a wide variety of decay rates for radioactive elements.

7. Three of the names listed below refer to the same thing. Which one does NOT?

A. electrons
B. β particles
C. cathode rays
D. α particles

8. The radiation from a sample of Kr^{85} decreases to one-third of the original intensity I_o in a period of 18 years. What would be the intensity after 18 *more* years?

A. I_o
B. $1/2\ I_o$
C. $1/3\ I_o$
D. $1/6\ I_o$
E. $1/9\ I_o$

9. Suppose that a new type of radiation, the T particle, is discovered among the radiations emitted by certain elements. Suppose measurements yield the following information:

1. The charge-to-mass ratio of the T particle is one-third that of the charge-to-mass ratio of a proton.
2. T particles are deflected in a magnetic field in the same direction as α particles

Which one of the following symbols could describe the particle?

A. ${}_{-1/3}T^1$ B. ${}_{-1}T^3$ C. ${}_{1}T^3$ D. ${}_{-3}T^1$ E. ${}_{3}T^1$

10. Rutherford identified three types of radiation emitted from radium: alpha, beta, and gamma. If these radiations are listed in order of increasing penetrating power, with the least penetrating radiation listed first, the order is

A. α, β, γ.
B. β, γ, α.
C. γ, α, β.
D. β, α, γ.
E. α, γ, β.

Question 11 refers to the following information:

An isotope of polonium is gradually transmuted to lead according to the following decay scheme. (The half-life for each member is listed below its symbol.)

$Po^{218} \rightarrow$	$Pb^{214} \rightarrow$	$Bi^{214} \rightarrow$	$Po^{214} \rightarrow$	$Pb^{210} \rightarrow$	$Bi^{210} \rightarrow$	$Po^{210} \rightarrow$	Pb^{206}
3.05 min	26.8 min	19.7 min	1.64×10^{-4} sec	21.4 yr	5.0 days	138.4 days	stable

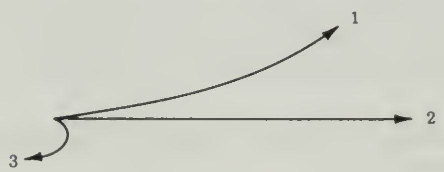
11. If we have a pure sample of Po^{218} initially, which member of the series will be present in the greatest amount after 1 day?

A. Pb^{206} B. Po^{210} C. Pb^{210} D. Bi^{214} E. Pb^{214}

12. ALL EXCEPT ONE of the following particles leave a track of condensed vapor when they pass through a cloud chamber. Which one is the *exception?*

A. electron
B. neutron
C. positron
D. α particle
E. proton

Question 13 refers to the following diagram:



13. Assume you know nothing about the rays from a radioactive isotope except that, in a uniform magnetic field, they behave as shown in the diagram above. (The magnetic field is directed into the paper.) Which one of the following statements *must* be true about the rays that move along paths 1, 2, and 3?

A. 3 has a charge of larger magnitude than 2.
B. 1 consists of electromagnetic radiation.
C. The speed of 3 is different from that of 2.
D. 3 consists of particles of smaller mass than 2.
E. 1 and 3 both consist of charged particles.

14. Which one of the following is true of the most stable nuclides? They have

A. even numbers of protons and neutrons.
B. odd numbers of protons and neutrons.
C. even numbers of protons, odd numbers of neutrons.
D. odd numbers of protons, even numbers of neutrons.
E. equal numbers of protons and neutrons, regardless of whether odd or even.

15. Nuclide A decays by emitting a β particle and forms nuclide B. Compared to nuclide A, nuclide B has

A. a charge of one unit more, and practically the same mass.
B. a charge of one unit more, and a mass of one unit less.
C. a charge of one unit less, and a mass of one unit more.
D. a charge of one unit less, and a mass of one unit less.
E. a charge of one unit less, and a mass of two units less.

16. Which of the following pairs of nuclei are isotopes of the same element?

A. two nuclei with the same numbers of neutrons, but different numbers of protons
B. a nucleus of carbon and a nucleus of nitrogen, both nuclei with the same mass
C. two nuclei that carry different electric charges, but have the same mass
D. two nuclei in which the number of protons equals the number of neutrons
E. two nuclei that have the same numbers of protons, but with different masses

17. ALL EXCEPT ONE of the following particles can be accelerated by an electric or magnetic field. Which one is the *exception*?

A. electron B. proton C. neutron D. α particle E. deuteron ($_1H^2$ nucleus)

18. An atom of mass number 11 and atomic number 5 captures an α particle and then emits a proton.

The mass number of the resulting atom will be

A. 10.
B. 11.
C. 12.
D. 13.
E. 14.

19. The discovery of isotopes was difficult because

A. isotopes of one element have the same chemical properties.
B. only obscure elements have isotopes.
C. isotopes decay rapidly.
D. isotopes are found only in the three radioactive series.

Questions 20 and 21 require the completion of nuclear equations. From the following key, select the particle that must be inserted in the space between the parentheses to balance the equation.

A. $_{-1}e^0$
B. $_{-1}e^0$
C. $_1H^1$
D. $_0n^1$
E. $_2He^4$

20. $_{13}Al^{27} + {}_1H^2 \rightarrow (\qquad) + {}_{12}Mg^{25}$

21. $_5Be^{11} + {}_2He^4 \rightarrow (\qquad) + {}_7N^{14}$

22. The chemical properties of an atom are determined by its

A. mass number.
B. number of isotopes.
C. atomic number.
D. nuclear binding energy.

23. Electrons moving at right angles to a uniform magnetic field travel in a circular path. The radius of the circle is 1.2 m. If the electrons had twice the speed while moving through the same magnetic field, the radius of their circular path would be

A. 0.3 m.
B. 0.6 m.
C. 1.2 m.
D. 2.4 m.
E. 4.8 m.

$$\left[\begin{array}{l} F_{\text{magnetic}} = qvB \\ F_{\text{centripetal}} = \dfrac{mv^2}{R} \end{array}\right]$$

24. ALL EXCEPT ONE of the following developments of modern physics date from the period 1890–1915. Which one is the *exception*?

A. discovery of radioactivity
B. nuclear model of the atom
C. discovery of isotopes
D. discovery of nuclear fusion
E. relativity theory

Questions 25 and 26 refer to important advances in the field of nuclear physics. From the list below, select the physicist whose contribution was most important to the event described.

A. Becquerel
B. Fermi
C. Chadwick
D. Compton
E. Rutherford

25. first self-sustaining nuclear reaction

26. first artificial transmutation

27. The concept of binding energy explains the

A. mass lost by protons and neutrons when they combine to form an atomic nucleus.
B. energy of α particles emitted by a radioactive nuclide.
C. relativistic mass gained by accelerated particles.
D. minimum energy of neutrons that collide with uranium or plutonium to produce fission
E. mass gained by a proton to produce a neutron.

Questions 28, 29, and 30 are statements that identify one of the equations in the key below. Select the equation identified by each statement.

A. ${}_0n^1 + {}_{94}Pu^{239} \rightarrow {}_{56}Ba^{141} + {}_{38}Sr^{96} + 3\,{}_0n^1$
B. $4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\nu + \gamma$
C. ${}_0n^1 + {}_{92}U^{238} \rightarrow {}_{92}U^{239}$; ${}_{92}U^{239} \rightarrow {}_{93}Np^{239} + {}_{-1}e^0 + \overline{\nu}$
D. ${}_0n^1 + {}_{13}Al^{27} \rightarrow {}_{13}Al^{28}$

28. the formation of a transuranium element

29. production of energy in stars

30. nuclear fission

31. If these radiations are listed in order of increasing deflection by a given magnetic field, starting with the radiation least deflected, the order is

A. α, β, γ.
B. β, γ, α.
C. γ, α, β.
D. β, α, γ.
E. α, γ, β.

32. Biologists are learning more about the metabolism of plants and animals through the use of

A. high-energy particle accelerators.
B. cloud-chamber photography.
C. isotopic tracers.
D. mass spectroscopy.
E. cosmic rays.

33. The "decay constant" is defined as the fraction of remaining atoms that decays in a unit time interval. The decay constant of a bismuth isotope, with a half-life of 5 days, is 0.14/day. After 10 days the decay constant will have a value

A. two times larger than the present value.
B. the same as the present value.
C. one-fourth the present value.
D. one-eighth the present value.

34. All isotopes of hydrogen

A. have the same mass.
B. are artificial.
C. are radioactive.
D. have identical physical properties.
E. have the same electric charge on the nucleus.

35. The following are related events:

1. Becquerel's discovery of radioactivity
2. the discovery of X rays
3. Rutherford's discovery of the nucleus

Order these three events in time, with the earliest event listed first.

A. 1, 2, 3
B. 1, 3, 2
C. 2, 1, 3
D. 2, 3, 1
E. 3, 1, 2

36. A plasma is

A. the shield of concrete surrounding nuclear reactors.
B. the carbon rods inserted inside nuclear reactors.
C. an ionized gas containing both positive and negative ions.
D. the fluid required to cool nuclear reactors.
E. a region in a reactor occupied only by neutrons.

Questions 37–39 describe some technical features of nuclear reactors that are also identified by the technical terms in the list. Match the descriptive statements with the technical terms in the list.

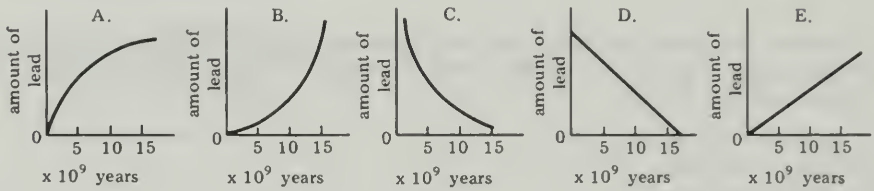
A. half-life
B. critical size
C. control rods
D. moderator
E. plasma

37. There is an approximate balance between production of neutrons and loss of neutrons due to capture or escape.

38. Neutrons are slowed down by substances such as water, heavy water, or carbon.

39. Substances such as cadmium or boron, which readily absorb neutrons, are present.

40. Which graph best represents the change with time in the amount of stable lead present in a sample that was originally pure uranium-238?



Unit 6 / The Nucleus

TEST D

Directions

This test consists of eight questions in two groups. Answer only FOUR of the five questions in Group One, and only TWO of the three questions in Group Two. Spend about 5 min on each of the questions from Group One, and 10 min on each of the questions from Group Two.

Group One

Answer FOUR of the following five questions. Allow about 5 min each.

1. A nucleus of ${}_8O^{16}$ will absorb a neutron to form a new stable nuclide.

(a) Write the equation for this nuclear reaction.
(b) How is the new nuclide related to ${}_8O^{16}$?

2. Calculate the binding energy of ${}_2He^4$ (in MeV)
given the following data: mass of ${}_2He^4$ atom = 4.002403 amu
mass of e^- = 0.000549 amu
mass of p = 1.007276 amu
mass of n = 1.008665 amu
1 amu = 1 MeV

3. How did the observation of radioactivity conflict with the traditional atomic-molecular theory of matter?

4. What evidence indicated that radioactive emissions, as first observed by Becquerel, were not the same as X rays?

5. Describe briefly two biological or medical applications of nuclear physics.

Group Two

Answer TWO of the following three questions. Allow about 10 min each.

6. Why is the decay rate of a radioactive sample expressed in terms of "half-life" rather than "total lifetime"?

7. In terms of the Rutherford–Bohr model of the atom, discuss

(a) how an atom changes when it emits an α particle;
(b) how an atom changes when it emits a β particle.

8. (a) Describe briefly the shell model of the nucleus.
(b) List one nuclear phenomenon that is "explained" by this model.

Suggested Answers to Unit Tests

Unit 1 / Test A

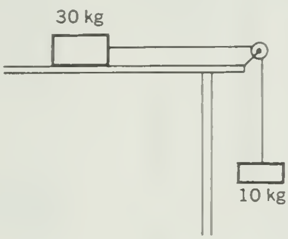
MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	A	3.4	0.472
2	C	3.2	0.674
3	B	4.10	0.416
4	E	4.11	0.483
5	D	1.6	0.944
6	D	1.6	0.876
7	E	1.0	0.404
8	A	1.5	0.640
9	E	1.5	0.933
10	B	3.5	0.607
11	C	1.0	not available
12	A	1.0	0.640
13	C	4.11	0.438
14	D	2.8	0.551
15	B	4.11	0.730

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 3.10



$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{(10\text{ kg})\left(10\ \frac{\text{m}}{\text{sec}^2}\right)}{10\text{ kg} + 30\text{ kg}}$$

$$= \frac{100}{40}\ \frac{\text{m}}{\text{sec}^2}$$

$$= 2.5\ \frac{\text{m}}{\text{sec}^2}$$

The 10-kg mass accelerates downwards at 2.5 m/sec^2. The 30-kg mass accelerates to the right at 2.5 m/sec^2.

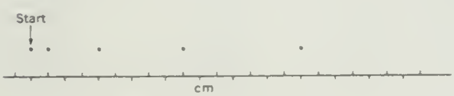
2. Section of Unit: 1.7

(a) The average speed of the car is the total distance traveled from the stoplight divided by the total time traveled. The instantaneous speed is an "average" speed calculated over an infinitesimally small time interval.

v = average speed
v_i = instantaneous speed
Δd = distance traveled during Δt = the time interval

(b) $v = \dfrac{\Delta d}{\Delta t}$

$$v_i = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$



3. Section of Unit: 2.3

$$d = \frac{at^2}{2}$$

$$a = \frac{2d}{t^2}$$

$$= \frac{2(8\text{ cm})}{(0.4\text{ sec})^2}$$

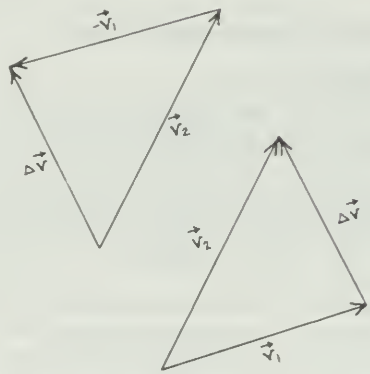
$$= \frac{16\text{ cm}}{0.16\text{ sec}^2}$$

$$= 100\ \frac{\text{cm}}{\text{sec}^2}$$

4. Sections of Unit: 3.2, 4.6

Find Δv, where $\Delta v = v_2 - v_1$.

Method I
(by adding $-v_1$ to v_2)



Method II
(from the head of v_1 to the head of v_2)

Group Two

5. Section of Unit: 4.3

A satisfacotry answer may involve:
(a) a discussion in depth of one or two relevant points, or
(b) a comprehensive overview of many relevant points.

A suitable answer may involve any of the following points. This list does not include all possibilities.

(a) Precise description: allowing comparison, measurement, graphical representation, transmission of knowledge.
(b) Functional relationships: allowing prediction, specific tests, deduction, interference, combination of independent equations, mathematical operations.

6. Section of Unit: 1.6

During the first 10 min the man walks 0.5 mile at a constant speed of $v = d/t$ = 0.5 mile/10 min = 0.05 mile/min.

During the next 10 min he does not move.

In the following 5 min he walks 0.25 mile at a constant speed of 0.25 mile/5 min = 0.05 mile/min.

Beyond the 25th minute the man is at rest.

Unit 1 / Test B

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	B	1.5	0.889
2	B	1.5	0.750
3	D	3.5	0.792
4	A	2.0	not available
5	E	4.13	0.431
6	C	4.11	not available
7	D	1.7	0.500
8	B	4.6	0.458
9	C	2.7	0.736
10	D	3.5	0.944
11	E	3.12	0.375
12	A	general	0.944
13	B	3.4	0.250
14	B	3.11	0.819
15	E	3.4	0.653

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 2.9

The answer must depend on your assessment of the accuracy of Galileo's instruments.

The extreme values in this column of data are 0.00192 and 0.00182.

They differ by less than 5%. Assuming the crudeness of Galileo's equipment accounts for this variation, we can conclude that d/t^2 is constant.

2. Section of Unit: 4.4

(a) As seen by the pilot, the flare drops straight down.
(b) As seen by an observer on the ground, the flare is a projectile whose trajectory is a parabola.

3. Section of Unit: 3.8

Weight is a measure of the gravitational force on an object. It is defined by Newton's second law: $w = ma_g$. Its unit is the newton (1 kg·m/sec^2).

Mass is a measure of the resistance of an object to changes in motion, a measure of inertia. It is also defined by Newton's second law: $m = w/a_g$. Its unit is the kilogram.

4. Section of Unit: 3.8

$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{a_g}{m}$$

$a = a_g$ or about 10 m/sec^2

Group Two

5. Sections of Unit: 2.7–2.10

(a) Joe's point of view is stated mathematically as $v = at$. If an object is accelerating uniformly, the longer it goes, the faster it goes. Maria's point of view is stated mathematically as $v^2 = 2ad$. If an object is accelerating uniformly, the farther it goes, the faster it goes.

(b) Both points of view are correct. Maria's equation may be derived from Joe's.

square $v = at$ $\qquad d = \frac{1}{2}at^2$

$$v^2 = a^2t^2 \qquad t^2 = \frac{2d}{a}$$

substitute $\frac{2d}{a}$ for t^2

$$v^2 = a^2\,\frac{2d}{a}$$

$$v^2 = 2ad$$

6. Sections of Unit: 2.2–2.4

A satisfactory answer may involve the following points. These suggestions do not exhaust the list of possibilities.

(a) Precise description: allowing comparison, measurement, graphical representation, transmission of knowledge.
Functional relationships: allowing prediction, tests, deduction, inference, mathematical operations.
(b) Thought experiments
"Ideal" motion: frictionless surface. Real situations are often too complicated
Newton's laws, Galileo, etc.

Unit 1 / Test C

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	C	1.4	0.857
2	B	general	0.896
3	A	3.2, 4.4, 4.5	0.700
4	E	1.6	0.700
5	B	1.7	0.632
6	D	3.2	0.814
7	C	4.5	0.779
8	B	3.7	0.788
9	A	3.6	0.590
10	C	general	0.697
11	A	3.10	0.554
12	B	3.10	0.482
13	B	2.3	0.423
14	B	general	0.469
15	E	1.7	0.821
16	A	1.4	0.749
17	D	1.7	0.736
18	A	4.4	0.893
19	C	2.8	0.557
20	B	2.8	0.661
21	D	1.6	0.847
22	D	1.5	0.785
23	A	3.7	0.436
24	E	4.3	0.528
25	E	4.3	0.362
26	E	3.2	0.622
27	C	2.10	0.371
28	D	1.4	0.788
29	C	1.7	0.436
30	C	4.5	0.928
31	A	4.4	0.678
32	A	3.2	0.436
33	D	3.8	not available
34	C	4.3	0.629
35	A	2.1	0.326
36	C	3.2	0.394
37	B	2.8	0.557
38	A	3.2	0.391
39	D	4.6	0.414
40	C	4.6	0.404

Unit 1 / Test D

Group One

1. Sections of Unit: 4.4, 4.5

   Force: centripetal, due to gravitational interaction between earth and satellite, constant magnitude

   Acceleration: in direction of force, constant magnitude

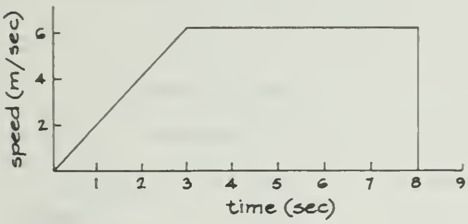
   Velocity: perpendicular to acceleration, constant magnitude

   Speed: constant

2. Sections of Unit: 1.5, 2.7

3. Section of Unit: 2.3

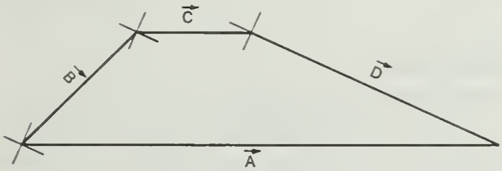
   1. Photography
      (a) slow motion
      (b) strobe light
      (c) "fast" motion



   2. Doppler effect
   3. Accurate measurement of time
      (a) "atomic" clocks
      (b) photography
   4. Elimination of friction: approaches ideal motion
      (a) dry-ice puck
      (b) air track
   5. Vector mathematics
   6. The calculus

4. Section of Unit: 3.2

   Parallelogram method



5. Section of Unit: 3.4

   Newton's first law states that a body will remain at rest or in uniform motion unless acted on by an external unbalanced force.

   A Newtonian would not accept an *a priori* statement as fact, but would expect experimental verification.

Group Two

6. Section of Unit: 4.2

   (a) $d = \frac{1}{2} a_g t^2$

   $$t^2 = \frac{2d}{a_g}$$

   $$= \frac{2\,(2{,}000\text{ m})}{10\text{ m/sec}^2}$$

   $$= 400\text{ sec}^2$$

   $$= 20\text{ sec}$$

   (b) $d = vt$

   $$= 200\text{ m/sec} \times (20\text{ sec})$$

   $$= 4{,}000\text{ m}$$

7. Section of Unit: 1.8

   (a) $surge_{av} = \dfrac{\Delta a}{\Delta t}$

   (b) $\dfrac{\text{m}}{\text{sec}^3}$ or $\dfrac{\text{distance}}{\text{time}^3}$

   (c) Instantaneous surge is the slope of an acceleration–time graph at any point.

8. Section of Unit: 2.3

   (a) Three assumptions are:

   (i) If a heavy and light body are tied together the natural speed of the new body will be the "average" of the natural speeds of the two original bodies.

   (ii) Bodies have natural speeds that differ from body to body.

   (iii) The natural speed of a heavy body is greater than that of a light body.

   (b) More appropriate assumptions are:

   (i) If a heavy and light body are tied together, the natural speed of the new body will be the "sum" of the natural speeds of the two original bodies.

   (ii) Bodies do not have natural speeds.

   (iii) All bodies accelerate toward the earth at the same rate.

   (iv) All bodies have the same natural speeds.

   (c) The following are conclusions based on the assumptions suggested in part (b).

   (i) A heavy object has a natural speed that is greater than that of a light object.

   (ii) On the basis of this premise we can make no conclusion concerning the speeds with which heavy and light objects fall.

   (iii) Heavy and light bodies should fall at the same speed.

Unit 2 / Test A

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	C	5.4	0.71
2	A	5.6	0.98
3	C	8.7	0.74
4	C	7.1	0.60
5	E	7.8	0.67
6	A	8.7	0.50
7	D	7.4	0.86
8	A	8.5	0.26
9	E	5.7	0.75
10	B	6.4	0.77
11	B	6.8	0.67
12	E	7.5	0.79
13	D	8.3	0.76
14	E	5.7	0.64
15	A	8.6	0.68

Group One

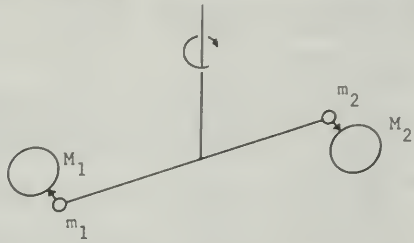
1. Sections of Unit: 5.7, 6.1–6.4

   (a) Ptolemy applied geometry in an attempt to explain the retrograde motion of planets.

   (b) Copernicus proposed a sun-centered system in which the earth's orbit replaced the epicycles in the motions of the other planets.

2. Section of Unit: 8.15

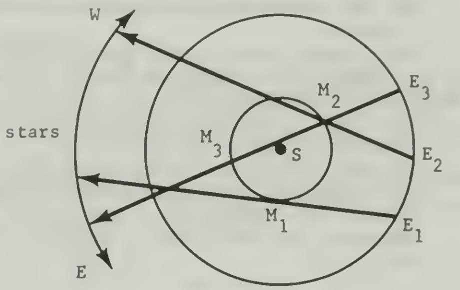
The gravitational force of attraction between m_1 and M_1 and m_2 and M_2 caused the beam holding m_1 and m_2 to twist about the vertical axis. Cavendish calibrated the force required to twist the vertical supporting wire and thus determined F_{grav}.



3. Section of Unit: 5.6

Mercury will orbit completely in the same time as the earth covers one-quarter of its orbit.

As the earth moves from E_1 to E_2 the line of sight to Mercury goes generally westward from M_1 to M_2. But as the earth moves from E_2 to E_3 the line of sight to Mercury goes generally eastward from M_2 to M_3.



4. Sections of Unit: Chapters 5 and 6

Although nature may be validly observed and described from all frames of reference, a particular phenomenon may appear simpler when viewed from a certain frame of reference. When observed from a heliocentric frame of reference rather than a geocentric frame of reference, the motions of the planets do appear simpler.

5. Section of Unit: 5.1

Each day the sun rises above the horizon on the eastern side of the sky and sets on the western side. At noon the sun is highest above the horizon. From July through November the noon height of the sun above the horizon decreases. Near December 22 the sun's height at noon is at a minimum. From January to June the sun's height at noon slowly increases. About June 21 the sun's height at noon is at a maximum.

Group Two

6. Sections of Unit: 8.6–8.9

The word "falling" is used in describing the motion of the moon relative to the earth in the same sense as it is used to describe the motion of an object, for example a ball, falling freely near the earth's surface.

The explanation of this statement should involve discussion of at least the following points.

(a) the acceleration of the moon toward the earth as compared to the acceleration of an object falling freely near the earth's surface

(b) the nature of the force accelerating both the moon and the freely falling object toward the earth

7. Sections of Unit: 8.3, 8.5

Given: $F \propto \frac{1}{R^2}$, $D = \frac{1}{2} at^2$

Derive: $\frac{(T_s)^2}{(T_p)^2} = \frac{(R_s)^3}{(R_p)^3}$

From geometry

$$\frac{R_s}{R_p} = \frac{D_s}{D_p}$$

$$\frac{R_s}{R_p} = \frac{\frac{1}{2} a_s T_s^2}{\frac{1}{2} a_p T_p^2} = \frac{a_s T_s^2}{a_p T_p^2}$$

But since $F \propto \frac{1}{R^2}$ and $F = ma$, then $a \propto \frac{1}{R^2}$

$$\text{So } \frac{R_s}{R_p} = \frac{\frac{1}{R_s^2} \cdot T_s^2}{\frac{1}{R_p^2} \cdot T_p^2} \text{ or } \frac{R_s^3}{R_p^3} = \frac{T_s^2}{T_p^2}$$

Unit 2 / Test B

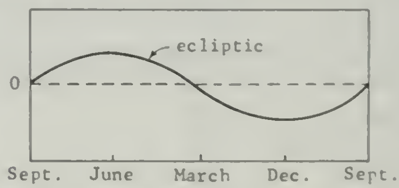
MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	A	7.4
2	B	7.1
3	D	6.8
4	D	5.7
5	C	7.1
6	A	6.6–6.8
7	E	7.8
8	C	5.7
9	C	8.7
10	A	8.5
11	E	8.6
12	B	8.19
13	C	7.5
14	C	Unit 1
15	B	7.8

Group One

1. Section of Unit 5.1 and *E14*

The sun moves eastward by about 1° per day relative to the stars. It completes one revolution through the stars in one year. There is an observable north–south oscillation completed in one year also. The path among the stars is called the ecliptic.



2. Sections of Unit: 5.6, 5.7, 6.4, 6.6

If the earth is at rest, one would not expect a stellar parallax, and indeed it was not observed until well into the nineteenth century. If the earth moves around the sun, then one should observe a parallax unless the stars are too far away. Opponents of Copernicus used the absence of a parallax as an argument against his theory and were not convinced that the stars could be far enough away to account for the absence of a parallactic shift.

3. Section of Unit: 8.7

Let m_1 be the mass of the earth, m_2 the mass of a falling body.

Then $F = m_2a = \dfrac{Gm_1m_2}{R^2}$

$$a = \frac{Gm_1}{R^2}$$

Therefore, a does not depend upon m_2, the mass of the falling body.

4. Sections of Unit: 7.10, 7.11

The answer could be yes, no, or somewhere between depending upon the point of view taken. There were some rational criteria used. For example, the absence of stellar parallax is a reasonable argument against the heliocentric theory. However, religious and philosophical beliefs played a large role, and many people refused even to consider new theories on their own merits. One possible scientific criterion of the time was agreement with observations. For the most part, the scholastics would not even consider the observations, but at that time scientific criteria were nebulous if not inadequate. There was no conclusive way to judge the relative merits of the competing theories at the time.

5. Sections of Unit: 8.5 and 8.6

(a) $F_g \propto \dfrac{\text{mass of planet}}{r^2}$

Since the mass of planet A equals the mass of planet B, and since the radius of planet A is greater than the radius of planet B, then F_g on the surface of B is greater than F_g on the surface of A.

(b) $$\frac{F_{gA}}{F_{gB}} = \frac{\dfrac{M_A}{r_A^2}}{\dfrac{M_B}{r_B^2}} = \frac{r_B^2}{r_A^2} = 1/4$$

Group Two

6. Sections of Unit: General

Galileo

(a) first of modern scientists: experimentation and thought experiments

(b) gave kinematic description of freely falling bodies

(c) used telescope to gather evidence in support of heliocentric system

(d) helped popularize the Copernican point of view through his writings

Kepler

(a) three laws gave a kinematic description of the solar system

(b) helped to make the heliocentric theory description more simple than the geocentric model

(c) first to break away from Plato's uniform circular motion

(d) three laws were used by Newton in his search for a force law of gravitation

Newton

(a) three laws of motion

(b) synthesized terrestrial and celestial theories of motion

(c) developed gravitational force law

(d) universal law of gravitation

(e) *Principia* and Rules of Reasoning in Philosophy

7. Sections of Unit: 5.7, 6.3, 6.4

(a) Yes and no. It could have been simpler if it had not used combinations of circular motions. It was in fair agreement with observations.

(b) Agreement with observations was about the same for both theories. The Copernican theory eliminated the major epicycle of the Ptolemaic theory but it was not clear that this was a simplification. Actually, the Copernican theory seemed in its components to be very complicated because it upset very fundamental beliefs without resolving these problems satisfactorily. Some of Copernicus' contemporaries regarded religious knowledge through teachings or the Bible as facts that must be accounted for in a satisfactory theory, whereas we would probably not regard Biblical quotations as evidence.

Unit 2 / Test C

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	A	5.6	0.93
2	E	5.1	0.70
3	E	6.4	0.77
4	B	7.8	0.76
5	C	5.3	0.79
6	B	8.6	0.74
7	A	6.4	0.54
8	B	6.7	0.78
9	B	8.13	0.84
10	B	7.4	0.63
11	B	6.3	0.67
12	E	8.14	0.58
13	D	6.7	0.88
14	E	5.4	0.75
15	D	7.4	0.92
16	E	8.4	0.80
17	A	8.6	0.73
18	A	7.5	0.85
19	C	7.5	0.64
20	A	6.1	0.74
21	D	8.18	0.86
22	A	8.5	0.46
23	D	5.7	0.37
24	B	5.5	0.59
25	C	8.7	0.63
26	D	8.5	0.36
27	E	6.3	0.58
28	D	5.3	0.57
29	C	5.7	0.68
30	C	7.5	0.44
31	C	7.5	0.33
32	B	8.19	0.91
33	D	8.5	0.51
34	D	8.0	0.90
35	B	8.6	0.65
36	C	8.3	0.64
37	E	7.5	0.69
38	B	8.4	0.57
39	C	8.7	0.65
40	C	7.6	0.54

Unit 2 / Test D

1. Sections of Unit: 7.2–7.5, 8.1–8.9

(a) Kepler produced a new geometrical theory based on three empirical laws to explain Tycho Brahe's astronomical observations.

(b) Newton produced a theory of universal gravitation that explained the dynamics of planetary motion by uniting Kepler's laws and terrestrial gravitation.

2. Sections of Unit: 8.2, 8.3, 8.5, 8.6

Newton's synthesis brought his work together with the work of Galileo and Kepler to build a system of dynamics that described the motion of all bodies. It synthesized the laws of motions of both terrestrial and celestial objects.

3. Section of Unit: 7.1

Ptolemy and Copernicus both used Plato's assumption of uniform circular motion. Kepler was the first to abandon this assumption. He constructed the orbits directly and determined that the orbits were ellipses.

4. Section of Unit: 7.5

$T^2 = ka^3$

$\frac{T_s^2}{T_e^2} = \frac{a_s^3}{a_e^3}$

$\frac{T_s^2}{1^2} = \frac{9^3}{1^3}$

$T_s = 9^{3/2} = 27$ yr

or

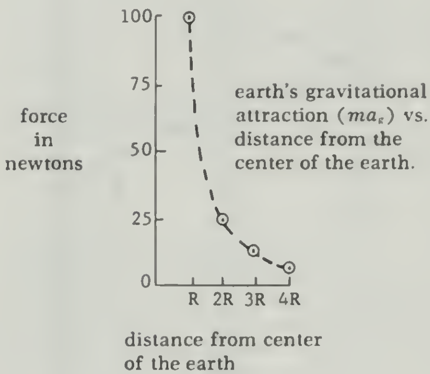
$T_s^2 = ka_s^3$

$k = 1 \frac{(\text{year})^2}{(\text{astronomical unit})^3}$

$T_s^2 = 729\ (\text{yr})^2$

$T_s = 27$ yr

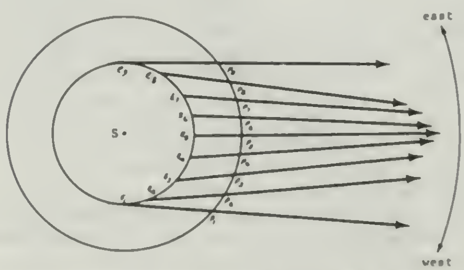
5.



6. Section of Unit: Chapter 5

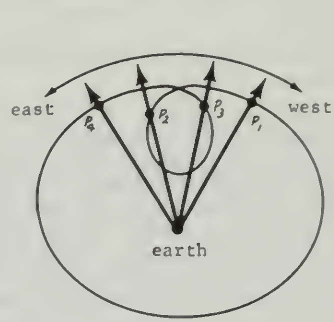
(a) The motion of a planet appears sometimes to reverse its original direction with respect to the fixed stars.

(b) *Heliocentric model:* A plane more distant from the sun than the earth will have a smaller period. As the earth laps the planet, the apparent motion of the planet changes direction, generally eastward from E_1 to E_4 and westward from E_4 to E_8, then eastward again.



Geocentric model: A planet would be traveling along an epicycle. Therefore, an orbit around the earth might look something like the diagram below.

From P_1 to P_2 and P_3 to P_4 the planet tends to move eastward across the sky. From P_2 to P_3 it appears to move westward in retrograde.

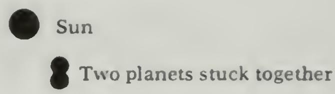


7. Section of Unit: 8.7

(a) Newton chose hypothesis B

(b) If $M_p \rightarrow 0$ as a limit, F is non-zero in case A and infinite in case C, both of which are not sensible.

Another argument:



The force on two planets stuck together should be the sum of the forces on each planet $F = M_s + M_p$ would not predict this:

$F_1 + F_2 \propto (M_s + M_p) + (M_s + M_p)$

$F_{1,2} \propto M_s + (2M_p)$

$F_1 + F_2 \neq F_{1,2}$

Similarly for $F \propto \frac{M_s}{M_p}$.

8. Section of Unit: 5.7

Yes and no.

The theory was not as simple as the one we use today, but given the philosophical conditions of the time, it was as simple as possible. Even modern theory rests on firmly held beliefs. The theory agreed pretty well with most observations. One exception was that the moon according to Ptolemy, should change apparent size by a factor of 2.

An alternative approach would be that it did not satisfy the criteria because it was a reconciliation of observation with the religious–scholastic concern of the importance of humanity and the earth. At the time, Ptolemy was too burdened with the belief that the purpose of all motion was the divinity of humanity.

Unit 3 / Test A

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	C	9.2	0.66
2	C	9.3	0.72
3	E	12.4	0.78
4	D	12.3	0.70
5	E	9.3, 10.1, 10.3	0.69
6	B	10.7	0.60
7	E	11.6	0.32
8	C	11.3	0.46
9	C	10.3	0.55
10	D	12.4	0.50
11	A	12.4	0.82
12	C	11.6	0.74
13	B	11.3	0.74
14	D	10.3	0.57
15	E	10.3	0.49

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 12.3

Since the speed of the wave cannot change, the only change is a decrease in wavelength.

2. Section of Unit: 9.2

(a) $p = mv$
$= (0.14\ \text{kg})(30\ \text{m/sec})$
$= 4.2\ \text{kg·m/sec}$

(b) $p = (2.0 \times 10^{-3}\ \text{kg})(300\ \text{m/sec})$
$= 0.6\ \text{kg·m/sec}$

3. Section of Unit: 11.2

A gas consists of a large number of very small particles (molecules) which move in rapid disordered motion.

Forces between these particles act only at very short distances.

Collisions among these particles are perfectly elastic.

4. Section of Unit: 10.6

$$\text{power} = \frac{\text{work}}{\text{time}} = \frac{Fd}{t} = \frac{(100\ \text{N})(10\ \text{m})}{50\ \text{sec}} = 20\ \text{W}$$

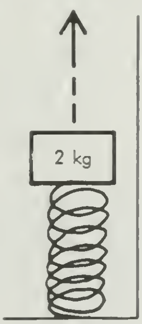
5. Section of Unit: 12.4

If the distance $PS_1 - PS_2 = (n + \frac{1}{2})\lambda$, P will lie on a nodal line.

If the distance $PS_1 - PS_2 = n\lambda$, P will lie on an antinodal line.

Group Two

6. Section of Unit: 10.3



Since we assume that mechanical energy is being conserved, the potential energy of the 2-kg mass at the peak of its trajectory must equal the potential energy originally stored in the spring.

$$10\ \text{J} = ma_g h$$

$$h = \frac{10\ \text{kg·m/sec}^2}{(2\ \text{kg})(10\ \text{m/sec}^2)} = 0.5\ \text{m}$$

7. Section of Unit: 9.4

$$F = ma \qquad = m\frac{\Delta v}{\Delta t}$$

If the mass of the body is constant,

$m\Delta v = \Delta(mv) = \Delta p$.

Therefore,

$$F = \frac{\Delta p}{\Delta t}$$

Unit 3 / Test B

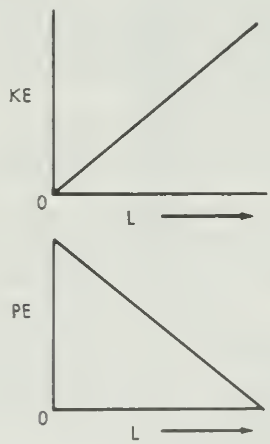
MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	B	9.2
2	D	11.1
3	A	10.10
4	C	10.3, 9.6
5	A	10.3, 9.6
6	C	9.6
7	A	9.6
8	B	12.4
9	A	11.3
10	D	11.6
11	B	12.8
12	A	general
13	A	11.6
14	C	12.7
15	C	12.5

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 10.3



2. Section of Unit: 11.6

An infinitely long list of physical phenomena are explained by the second law of thermodynamics. The student should pick one and then describe its connection with the second law.

For example: A cold spoon is dropped into hot water. Soon both the spoon and water will be at the same middle temperature.

This situation is described by the second law in that (a) the process is not reversible: the spoon and water will not return to their previous temperatures, and **(b)** the order of the process is from order (two energies) to disorder (one distribution of molecular kinetic energies). The entropy of the system has increased.

3. Section of Unit: 9.2–9.3

Although Alouette's speed is almost constant, its velocity (a vector) is not. Since Alouette's path is circular, the direction of its velocity is always changing. In fact, the direction of this change is always toward the center of the circular orbit.

Therefore, since momentum (p) = mv, Alouette's momentum cannot be constant.

4. Section of Unit: 9.1

(a) The masses measured before and after the reactions were always the same.

(b) These results lead to the conclusion that mass is conserved in a closed system.

5. Section of Unit: 11.2

The assumption of disordered or random motion of gas molecules allows only a consideration of "net" effects, the average effect of the motions of large numbers of molecules. This statistical treatment leads to a description of the properties of the gas as a whole rather than a description of the motions of the individual constituent particles.

Group Two

6. Section of Unit: 9.1

This question may be dealt with in many different ways. Students may answer yes or no and explain their answers in terms of

(a) the points of view that motivated the discovery of the law of conservation of momentum.

(b) a discussion of deterministic versus nondeterministic points of view.

(c) a discussion of whether the universe may be viewed as a closed system in the same way as the isolated collision of two billiard balls.

(d) questioning the applicability of our physical laws throughout the universe.

7. Sections of Unit: Chapters 9 and 10

Students may indicate belief or disbelief in this story. In either case, however, they should justify their decisions.

(a) with a qualitative discussion of the nature of the impact, the meaning of the word "abrupt," the phrase "straight down," and the applicability of the laws of conservation of momentum and conservation of energy.

(b) with a quantitative approach, assigning realistic numerical values to the respective masses and speeds of the bullet and the lion. Students should indicate that the collision is inelastic, and discuss the applicability of the laws of conservation of momentum and conservation of energy.

Unit 3 / Test C

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	A	9.2	0.81
2	C	10.3	0.51
3	E	9.5	0.33
4	C	9.6	0.86
5	B	12.2	0.68
6	B	9.6	0.67
7	D	11.1	0.58
8	B	11.3	0.74
9	A	11.6	0.58
10	D	9.2	0.41
11	E	10.3	0.41
12	B	10.1	0.50
13	B	12.7	0.57
14	A	12.5	0.55
15	D	12.4	0.71
16	A	9.3	0.53
17	B	9.6	0.68
18	A	10.3	0.40
19	B	12.2	0.73
20	B	9.2	0.60
21	B	10.7	0.61
22	E	11.6	0.39
23	C	11.3	0.38
24	C	10.3	0.95
25	D	10.10	—
26	E	10.1	0.69
27	A	10.10	0.73
28	D	12.4	0.60
29	A	12.4	0.78
30	C	10.3, 9.6	0.67
31	A	10.3, 9.6	0.79
32	C	9.6	0.40
33	A	9.6	0.46
34	A	12.5	—
35	C	11.6	0.60
36	A	12.8	0.74
37	E	general	0.72
38	A	11.3	0.66
39	D	10.6	0.42
40	C	9.1	0.75

Unit 3 / Test D

Group One

1. Section of Unit: 12.3

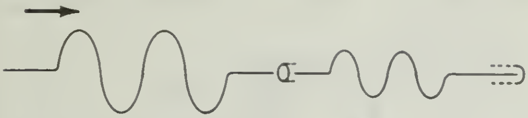
$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/sec}}{440 \text{ 1/sec}} = 0.77 \text{ m}$$

2. Section of Unit: 9.1

The volume of a substance may change without an accompanying change in the amount of substance. For example, if water is put in a container, the water level marked, and the water then frozen, we find that the volume of the ice is larger than the volume of the water we started with.

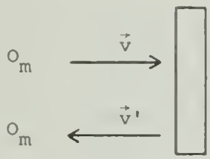
3. Section of Unit: 12.7



The pulses will be traveling at lower speed, with shorter λ and with smaller amplitude.

4. Section of Unit: 9.3

The ball's momentum is not the same before and after the collision.



M = mass of wall (and earth)

Momentum is conserved, therefore

mv	=	mv'
momentum of ball before collision		momentum of ball after collision
	+	MV
		momentum of wall (and earth) due to collision

The ball's kinetic energy is not the same before and after the collision. Since the ball is soft, the collision was not perfectly elastic. The energy was used to deform the ball during the collision; this energy, stored in the ball like energy stored in a compressed spring, was not completely returned when the ball was restored to its original shape. This lost energy reappeared as heat. In addition, a very small amount of energy ($MV^2/2$, $V \approx 0$) is transferred to the wall, in the form of mechanical kinetic energy.

5. Section of Unit: 11.2

A gas expanding while pushing a piston does work in pushing the piston, decreasing its internal energy. The gas expanding in a vacuum does no work; its internal energy remains the same.

Group Two

6. Section of Unit: 9.3

The system is isolated, therefore the total momentum of the system P_{total} must be constant throughout.

(**a**) Initially the whole system is at rest. Therefore,

$$P_{total} = 0$$

(**b**) While the box is in the air

$$P_{box} + P_{boat+men} = P_{total} = 0$$

Therefore, the boat moves in the opposite direction to the box.

(**c**) After the box is caught, $P_{box} = 0$,
and since $P_{total} = 0$,
also $P_{boat+men} = 0$, and the boat is at rest.

7. Section of Unit: 10.8

The engineering analysis of the steam engine led to several new important discoveries in physics. A student in responding to this question may select any one of the following list of discoveries, which is by no means inclusive, and explain its link to the steam engine and its importance to the study of physics.

(**a**) the nature of heat: J. P. Joule's contribution to physics
(**b**) the discovery of the law of conservation of energy
(**c**) energy conversion: Carnot's heat engine, the second law of thermodynamics
(**d**) energy conversion: efficiency, power, etc.

8. Section of Unit: 10.10

A satisfactory answer to this question should include

(**a**) a brief description of nature and of science as viewed by nature philosophy.

(**b**) an explanation of the link with the law of conservation of energy. For example:

(**i**) nature philosophy searches for the underlying reality of nature: a sweeping generalization to fully describe nature.

(**ii**) nature philosophy states that the various forces of nature (gravity, electricity, magnetism, etc.) are not really separate from one another, but are simply different manifestations of one basic force.

(**c**) a statement that by encouraging scientists to look for connections between different forms of energy, nature philosophy stimulated the experiments and theories that led to the law of conservation of energy.

Unit 4 / Test A

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	B	13.3	0.73
2	E	14.3	0.67
3	C	13.3	0.43
4	C	14.3	0.40
5	C	14.4	0.27
6	B	13.2	0.57
7	D	16.5	0.73
8	C	16.2	0.70
9	B	16.2	0.40
10	D	14.10	0.77
11	D	15.4	0.67
12	C	15.4	0.83
13	B	16.2	0.63
14	A	15.3	0.47
15	D	15.4	0.37

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 16.4

Hertz found:

(**a**) that electromagnetic waves reflect, refract, and interfere in a manner similar to light waves

(**b**) that the velocity of electromagnetic waves and light waves are the same.

He concluded that both light waves and electromagnetic waves could be accounted for by Maxwell's electromagnetic theory.

2. Sections of Unit: 16.2, 16.3

Maxwell predicted:

(**a**) that electromagnetic waves of many different frequencies could exist.

(**b**) that electromagnetic waves exert a pressure on any surface that reflects or absorbs them

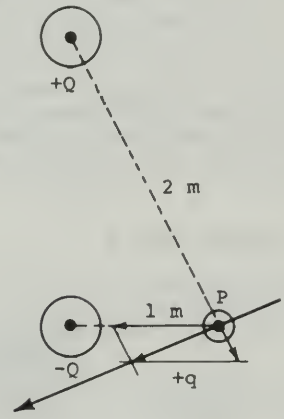
3. Section of Unit: 14.10

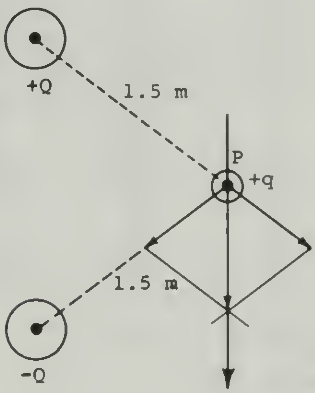
power = VI
$= (12\ V)(100\ A)$
$= 1{,}200\ W$

4. Sections of Unit: 14.11–14.13

(a) A magnetic compass needle held close to a current-carrying wire will align itself perpendicular to the wire.
(b) Two parallel current-carrying wires held close together will exert forces on each other.
(c) An insulated current-carrying coiled wire around a piece of iron will cause the iron to become magnetic.

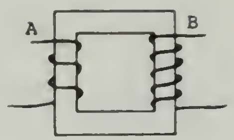
5. Sections of Unit: 14.3, 14.4





Group Two

6. Section of Unit: 15.8

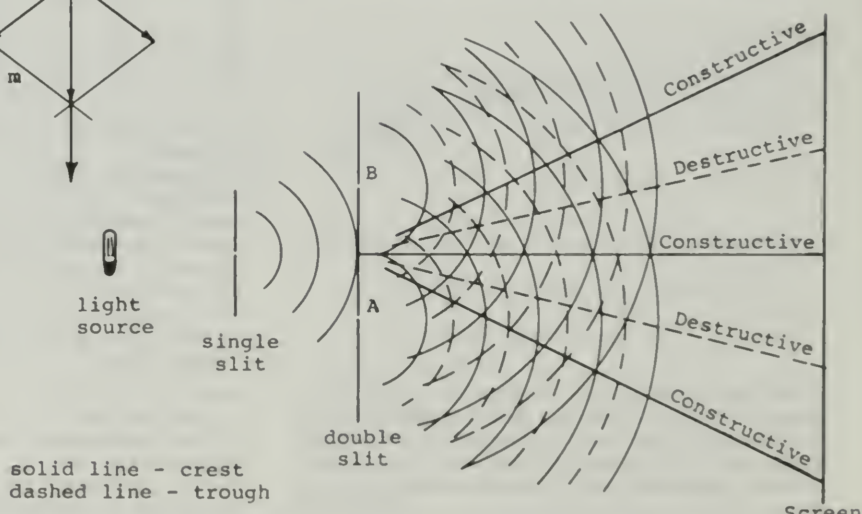


An alternating current in the primary coil (A) changes the magnetic field in the iron ring. The change in the magnetic field in the part of the ring near the secondary coil (B) induces a current in the secondary coil. If the secondary coil has more turns than the primary, the voltage across the secondary will be greater than the voltage across the primary. If the secondary has fewer turns than the primary, the voltage across the secondary will be less than the voltage across the primary.

7. Section of Unit: 13.4

An adequate answer to this question should involve some discussion of most of the following facets of Young's experiment:

(a) the nature of the light source
(b) the function of the single slit
(c) the function of the double slits
(d) the regions of constructive and destructive interference
(e) the pattern seen on the screen
(f) relative dimensions of parts of the apparatus where this information is vital to the experiment



Unit 4 / Test B

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	E	14.3	0.34
2	E	14.4	0.75
3	C	16.5	0.73
4	E	14.3	0.69
5	A	13.4	0.60
6	D	13.4	0.37
7	B	13.5	0.72
8	D	16.3	0.64
9	C	13.3	0.39
10	E	14.3	0.58
11	C	14.3,14.4	0.69
12	D	13.6	0.85
13	B	16.2,16.3	0.52
14	E	13.6	0.60
15	E	16.7	0.51

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 15.9

A satisfactory answer to this question may involve a brief discussion of two of the following effects of the introduction of electrical power. This list does not include all possible answers.

(a) the impact of electricity on industry
(b) the social effects of replacement of physical labor by machinery
(c) the trend toward a decentralized population brought about by rural electrification
(d) the unification of the country through the use of electrically operated systems of communications

2. Section of Unit: 13.5

The nature philosophers objected to Newton's theory of color because they found the scientific procedure of dissecting and analyzing natural phenomena distasteful. They felt that it was necessary to consider the unifying principles of all natural forces in order to understand nature as a whole.

3. Section of Unit: 14.4

A *field* is a set of values attached to every point in space; the values are, in general, functions of both position and time. The field idea is often invoked intuitively or for mathematical convenience to describe a region of interaction in which there are different amounts of influence produced by some source, and to avoid direct action at a distance.

4. Section of Unit: 16.5

X rays are readily absorbed by bone, whereas they pass through other types of organic matter such as flesh. In addition, X rays have the ability to expose photographic film.

5. Section of Unit: 16.5

The following list is not intended to provide all the possible reasons for believing that electromagnetic waves carry energy.

(a) Energy must be supplied to a source of electromagnetic radiation.
(b) When electromagnetic waves are absorbed, the absorbing material is heated.
(c) Electromagnetic radiations exert pressure on targets.
(d) Hertz discovered that the sparking of an induction coil caused similar sparking to occur in his receiver loop.

Group Two

6. Section of Unit: 13.6

The scattering of waves by small obstacles depends on the wavelength of these waves. The longer a wave is compared to the size of the obstacle, the less it is scattered by the obstacle. Light from the sun is scattered by air molecules and particles of dust or water vapor in the atmosphere. Since these particles are much smaller than the wavelength of visible light, the light of shorter wavelength (blue) will be more strongly scattered by the particles than light of longer wavelengths. Because the rays of longer wavelength are not scattered very much, they reach our eyes only when we look directly at the sun.

7. Section of Unit: 15.5

If the loop is rotated about its axis (dotted line), the loop will have a component of its motion perpendicular to the magnetic field. Because the charges in the wire are being moved across the magnetic field, they experience a magnetic force (qvB) "off to the side." The "free electrons" will be pushed along the wire by this force. Therefore, an electric current will be induced in the loop.

Unit 4 / Test C

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	C	14.8	0.69
2	D	13.6	0.85
3	D	16.1	0.58
4	E	14.4	0.64
5	C	14.16	0.90
6	B	14.3	not available
7	B	14.4	0.57
8	C	13.3	0.46
9	C	16.2	0.63
10	B	13.3	0.57
11	E	14.3	0.59
12	B	14.13	not available
13	E	16.7	0.38
14	C	general	0.76
15	E	14.13, 16.2	0.44
16	D	16.5	0.67
17	E	15.8	0.44
18	C	general	0.59
19	A	16.3	0.43
20	C	14.11	0.67
21	B	13.3	0.73
22	B	13.3	0.89
23	B	15.4	0.60
24	A	15.4	0.70
25	D	13.6	0.85
26	D	16.7	0.44
27	C	14.11	0.77
28	E	14.3	0.67
29	D	14.13	0.42
30	A	13.4	0.55
31	D	13.4	0.34
32	B	13.5	0.53
33	D	14.3	0.43
34	B	16.2	0.47
35	A	14.12	0.50
36	D	16.1	0.37
37	C	15.8	0.74
38	A	14.2	0.49
39	A	14.13	0.67
40	D	15.8	0.51

Unit 4 / Test D

Group One

1. Section of Unit: 13.3

As light enters the block of glass, its velocity decreases, its wavelength decreases, and its frequency remains constant.

2. Sections of Unit: 14.13

A stream of charged particles, mainly from the sun but also from outer space, continually sweeps past the earth. Many of these particles are deflected into spiral paths by the earth's magnetic field and are trapped.

3. Section of Unit: 16.2, 16.3

In developing his electromagnetic theory, Maxwell devised a mechanical model that provided the connections among the electrical and magnetic quantities observed by Faraday and others. The ether was a constituent part of this mechanical model. However, once Maxwell found the desired relations between the electric and magnetic fields, he was free to discard the mechanical model. Maxwell's electromagnetic theory is independent of any assumption concerning the ether. His model developed for matter can be applied to space that is free of matter. Under all circumstances, an electric field that is changing with time generates a magnetic field.

4. Section of Unit: 16.5

Because both X rays and radio waves are electromagnetic radiations, they can both be refracted, reflected, and diffracted, and both exhibit interference.

5. Section of Unit: 16.2

Include a sample of the material in a circuit with a battery and a current detector. If the current flow is constant, or changes very little, the substance is a conductor. If a current cannot be detected, the substance is an insulator.

Group Two

6. Section of Unit: 14.3

Force on C due to A:

$$F_{CA} = \frac{kg_C g_A}{R^2}$$

$$= \frac{(9 \times 10^9\ \text{N·m}^2/\text{C}^2)(1 \times 10^{-6}\ \text{C})(-1 \times 10^{-6}\ \text{C})}{\left(\frac{1}{10}\ \text{m}\right)^2}$$

$= -0.90$ N (a 0.9-N force toward A)

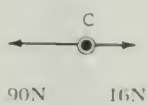
Force on C due to B:

$$F_{CB} = \frac{kg_C g_B}{R^2}$$

$$= \frac{(9 \times 10^9\ \text{N·m}^2/\text{C}^2)(+2 \times 10^{-6}\ \text{C})(-1 \times 10^{-6}\ \text{C})}{\left(\frac{1}{3}\ \text{m}\right)^2}$$

$= -0.16$ N (a 0.16-N force toward B)

Net force on C:



$F_C = (0.90 - 0.16)$ N

$= 0.74$ N (a 0.74-N force toward A)

7. Section of Unit: 13.6

The longer a wave is compared to the size of an obstacle, the less it is scattered. Thus in areas where the air is clear, sunlight is scattered by air molecules and particles of dust or water vapor. These particles are smaller than most wavelengths of light. Consequently, only light of short wavelengths (blue) is scattered, and the sky seems blue. However, when the air is polluted, larger particles are suspended in the air. These particles scatter sunlight of longer wavelengths. Consequently, reds and oranges mixed with blue are seen in the sky.

8. Section of Unit: 16.7

(a) The special theory of relativity assumes that all of physics and electromagnetism, as well as mechanics, satisfies the relativity principle. There is no way to distinguish among frames of reference that have a constant relative velocity.

(b) The speed of light in free space is the same for all observers, whatever their velocities relative to each other or to the light source.

Unit 5 / Test A

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	D	20.1
2	E	18.4,20.1,20.2
3	A	18.2
4	B	general
5	A	18.4
6	A	17.3
7	D	18.6,20.1
8	A	19.4,19.5
9	B	17.3
10	C	18.2
11	A	19.2
12	A	18.2
13	C	20.1
14	B	20.5
15	A	20.1

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 18.2

The particle has a mass that is 1,836 times the mass of an electron.

The particle has a charge that is 1/1,836 the charge of the electron.

The mass and charge of the particle are different from the mass and charge of an electron.

2. Section of Unit: 17.1

Molecular mass of ZnO = atomic mass of zinc + atomic mass of oxygen

$= 65.37 + 15.99$

$= 81.36$

$$\%\ \text{by mass of zinc} = \frac{(65.37)}{81.36} = 80.3\%$$

3. Section of Unit: 18.2

The deflection of the electron beam in a magnetic field depends upon the direction of the beam relative to the magnetic field, the speed of the electrons in the beam, and the strength of the magnetic field.

4. Sections of Unit: 19.4, 19.5

The term mvr is the angular momentum of an electron as it orbits about the positive nucleus of a hydrogen atom. Equating the electron's angular momentum with the term $nh/2\pi$ indicates that the angular momentum of the electron must be quantized, for n is an integer, whereas $h/2\pi$ is constant.

5. Section of Unit: 20.3

$$\text{de Broglie } \lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec}}{(1.67 \times 10^{-27}\ \text{kg})(10^{8}\ \text{m/sec})}$$

$$\lambda = 3.95 \times 10^{-15}\ \text{m}$$

Group Two

6. Sections of Unit: 19.4, 19.5

Balmer	Bohr
$\frac{1}{\lambda} = R_{H}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$	$hf = \frac{E_1}{n_i^2} - \frac{E_1}{n_i^2}$
	$h\frac{c}{\lambda} = E_1\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$
	$\frac{1}{\lambda} = \frac{E_1}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$
	$R_H = \frac{E_1}{hc}$
	$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

7. Section of Unit: 20.5

(a) The uncertainty principle states that it is not possible to measure simultaneosuly an electron's position and velocity (momentum) to any prescribed accuracy.

(b) $(\Delta x)(\Delta p) \geq \frac{h}{2\pi}$

$$\Delta p \geq \frac{h}{2\pi}\left(\frac{1}{\Delta x}\right)$$

$$\Delta p \geq \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec}}{2(3.14)(10^{-10}\ \text{m})}$$

$$\Delta p \geq 1.05 \times 10^{-24}\ \text{kg} \cdot \text{m/sec}$$

Unit 5 / Test B

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	D	20.1
2	D	general
3	C	19.1
4	A	18.2
5	B	18.6
6	A	20.4
7	E	general
8	D	19.2
9	E	18.6
10	A	20.6
11	C	20.3
12	B	general
13	C	20.4
14	A	18.3
15	B	17.2

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: Chapter 20

One can attribute both wavelike behavior and particle-like behavior to everything in the universe. For example the diffraction of an electron can be explained by its wave properties and its momentum can be explained by its mass and velocity properties.

2. Section of Unit: 19.10

(a) Although the Bohr model accounted for the spectra of atoms with a single electron in the outermost shell, serious discrepancies between theory and experiment appeared in the spectra of atoms with two or more electrons in the outermost shell.

(b) Bohr's theory did not account in a quantitative way for the splitting of spectral lines that occurred when the sample being studied was in an electric or magnetic field.

(c) Bohr's theory supplied no method for predicting the relative intensity of spectral lines.

3. Section of Unit: 18.4

The energy of a photon is directly proportional to the frequency of the electromagnetic wave. This can be said more succinctly by means of the equation $E = hf$, where h, the constant of proportionality, is Planck's constant.

4. Section of Unit: 17.3

(a) The periodic table provided (through the introduction of a system of "numerical characterization" of the elements) a dependable means of correlating the elements and their properties. It established the regular occurrence of physical and chemical properties, and suggested some periodic recurrence of structure in atoms.

(b) Gaps in the periodic table led Mendeleev to predict the existence of undiscovered elements, and furthermore allowed him to describe accurately many of their properties.

5. Section of Unit: Prologue

Alchemy, the futile attempt to transmute base metals into gold, was the forerunner of modern chemistry. Its importance lies in its by-products such as the development of methods of chemical analysis, the study of the properties of many substances and processes such as calcination, distillation, fermentation, and sublimation, and the invention of many pieces of chemical apparatus that are still used today.

Group Two

6. A diagram of the apparatus used in each experiment will be found in the appropriate *Text* section.

J. J. Thomson's q/m experiment: Sec. 18.2

Thomson showed that cathode-ray particles (electrons) were emitted by many different materials. Their charge was similar in magnitude to that of a hydrogen ion, but they were considerably less massive than the hydrogen ion. He concluded that these particles form a part of all kinds of matter, and in so doing suggested that the atom is not the ultimate limit to the subdivision of matter.

Millikan's oil-drop experiment: Sec. 18.3

Millikan's experiment, by showing that the electric charge picked up by an oil drop is always an integral multiple of a certain smallest value, demonstrated that charge is quantized.

Photoelectric effect experiments: Sec. 18.4

These experiments show that the maximum kinetic energy of photoelectrons increases linearly with the frequency of the incident light, provided the frequency is above the threshold frequency. This threshold frequency is different for different metals. Photoelectrons are emitted at frequencies just above the threshold no matter how low the intensity of the incident light. In addition, there is practically no time lag between the instant the incident light strikes the target and the emission of photoelectrons. At frequencies just below the threshold, no electrons are emitted no matter how intense the incident light. In summary, these experiments show that the energy of light is a function of its frequency and that light energy is quantized.

Faraday's electrolysis experiments: Sec. 17.3

Faraday's experiments showed that a given amount of electric charge is closely related to the atomic mass and combining capacity of an element and, in so doing, implied that (1) matter is electrical in nature, and (2) electricity is atomic (quantized) in nature.

Rutherford's α-particle scattering experiments: Sec. 19.2

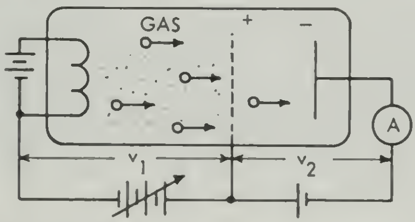
Rutherford's α-particle scattering experiments showed that the atom is mostly empty space. The experiments demonstrated that there was a positive charge within the atom that occupies a very small amount of space, and furthermore it is this charge that scatters α particles by a coulomb force of repulsion.

7. Section of Unit: 17.1

The weight of A relative to B is 6 to 1. However, in a compound of only A and B it is found that there is 3 times as much A as there is B (by weight). Consequently, the only possible formula for this compound is AB_2.

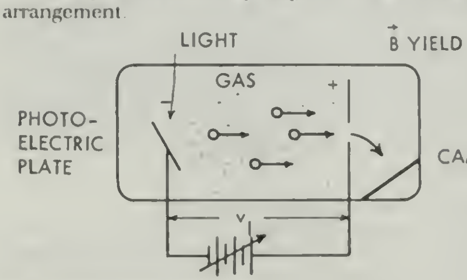
8. Section of Unit: 19.9

The following is a greatly simplified schematic diagram of the Franck–Hertz apparatus. For the purposes of this question it should be considered a more than adequate answer.



v_1 = accelerating potential
v_2 = small stopping potential

However, devices other than those shown in the diagram above can be used to make the necessary measurements. For example, the following diagram shows a satisfactory arrangement.



v = accelerating potential

The amount of bending of the electron beam is inversely related to the kinetic energy of the emerging electrons.

A workable apparatus would have to include:

(1) a source of electrons
(2) a means of accelerating the electrons
(3) a chamber where the electrons pass through a gas
(4) an electron detector (ammeter)
(5) some means of measuring the kinetic energy of the electrons after they emerge from the gas

Unit 5 / Test C

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit	Proportion of Test Sample Answering Item Correctly
1	A	19.2	0.46
2	C	20.1	0.42
3	A	19.2	0.65
4	E	20.2	0.32
5	D	20.3	0.61
6	B	19.4, 19.5	0.53
7	A	17.3	0.66
8	C	19.4, 19.5	—
9	C	18.4	0.70
10	A	18.4	0.45
11	B	19.2	0.67
12	B	general	—
13	B	19.4, 19.5	0.80
14	C	19.8	0.38
15	E	18.2	0.53
16	C	19.4	—
17	B	19.2	0.71
18	A	18.4	—
19	B	18.6	0.48
20	A	18.2	0.65
21	E	17.1	0.76
22	A	19.1	0.52
23	A	18.3	0.72
24	B	17.3	0.64
25	D	17.3	0.67
26	D	17.1	0.58
27	C	Chapter 20	0.74
28	D	18.4	0.53
29	C	18.6	0.42
30	B	general	—
31	E	20.1	0.48
32	C	20.4	0.45
33	B	19.4	0.54
34	A	Chapter 20	0.71
35	E	19.2	0.62
36	C	17.3	—
37	D	20.1	0.76
38	B	20.5	0.78
39	A	19.4	0.71
40	A	19.9	0.59

Unit 5 / Test D

Group One

1. Section of Unit: 17.3

This argument has three steps. The question provides the first and the last, the student must supply the missing middle step.

Step 1. A given amount of electric charge is related to the atomic mass and valence of an element

Step 2. Atomic mass and valence are characteristics of the atom of the element.

Step 3. Therefore, a certain amount of electric charge is associated with an atom of the element.

2. Section of Unit: 18.3

Millikan's experiment, in showing that the electric charge picked up by an oil drop is always an integral multiple of a certain minimum value, demonstrated that charge is quantized.

3. Section of Unit: 18.5

Rutherford could not explain the bright-line spectrum of hydrogen. In addition, he had nothing to say about the details of distribution of negative charge. Bohr explained the bright-line spectrum of hydrogen by suggesting that a spectral line may be attributed to the quantized release of energy by the change in the nucleus–electron energy state. Bohr also described in some detail the orbiting electron in terms of permissible quantized distances from the center of the nucleus, quantized angular momentum, and quantized energy.

4. Section of Unit: 18.5

Einstein's formula: $KE_{max} = hf - W$

When the emitted photoelectron has practically no KE, incident light with a minimum f has caused this emission. In this case,

$$hf = W$$

$$f = \frac{W}{h}$$

$$f = \frac{2 \times 10^{-18}\ \text{J}}{6.6 \times 10^{-34}\ \text{J} \cdot \text{sec}}$$

$$f = 3 \times 10^{15}\ \text{sec}^{-1}\ (\text{Hz})$$

5. Section of Unit: 20.1

As the speed of an electron increases, its mass increases without limit. This can be stated mathematically as follows:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Group Two

6. Section of Unit: 17.3

Faraday's second law of electrolysis states that 96,500 C will produce 1.00 g of hydrogen. The problem indicates that a current of 3 A flows through water for 60 min (3,600 sec). This is equivalent to the passage of 10,800 C ($q = It$).

The mass of hydrogen produced = $(1.00)\dfrac{(10{,}800)}{(96{,}500)} = 0.112$ g

The ratio of the amounts of oxygen and hydrogen liberated in the electrolysis of water is 8:1.

Therefore, the mass of oxygen produced = $(8)(0.112) = 0.896$ g

7. I. $KE_{max} = hf - W$ (Sec. 18.5)

Einstein showed that photoelectric emission could be explained by the quantization of light energy, thus paving the way to quantum mechanics.

II. $\dfrac{1}{\lambda} = R_H\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$ (Sec. 19.1)

Balmer's empirically derived formula summarized some regularity and predicted the existence of other spectral line series in the hydrogen spectrum. Agreement with the predictions of this formula led to Bohr's theoretically derived model of the atom.

III. $m = \dfrac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ (Sec. 20.1)

This equation, showing the relationship between an object's mass and its speed, is one of the more popular consequences of Einstein's special theory of relativity. The introduction of this theory prompted a total reassessment of most areas of physics involving the study of objects in motion. It has been immensely important in the field of high-energy physics, where very small particles are accelerated to speeds that approach the speed of light, and also in astronomy and atomic and nuclear physics.

IV. $\lambda = \dfrac{h}{mv}$ (Sec. 20.3)

The de Broglie equation suggested that matter has wave properties. This prediction found experimental verification when Davisson and Germer demonstrated that electrons could be diffracted.

V. $(\Delta x)(\Delta p) \geq \dfrac{h}{2\pi}$ (Sec. 20.5)

The Heisenberg uncertainty principle states that we are unable to measure simultaneously the position and velocity of an electron. This same reasoning holds for all moving objects, but is of no practical consequence for relatively massive objects. Heisenberg's principle, one of the early consequences of quantum theory, emphasized the nondeterministic probabilistic nature of physics.

8. Section of Unit: General

The idea that matter is composed of atoms was proposed by Greek philosphers between 500 and 400 B.C. This theory was devised in an "armchair" fashion with no attempt at confirming the theory in terms of physical experimentation. The nineteenth century scientists developed theories that they felt would account for the observed properties of matter and tested these theories experimentally. In addition, these atomic theories included a predictive function that would include behavior not yet observed.

Unit 6 / Test A

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	B	21.6
2	C	21.6
3	B	21.6
4	D	21.4
5	D	21.1
6	A	21.3, 21.4
7	D	23.7
8	B	24.8
9	E	22.1, 22.2
10	A	22.1, 22.2
11	D	22.6
12	D	21.3, 21.4
13	C	24.2
14	B	23.5
15	C	24.5, 24.6

Group One

1. Section of Unit: 24.4

$$\begin{aligned}
\text{Given: atomic mass of } H^1 &= 1.0080 \text{ amu}\\
\text{atomic mass of } Li^7 &= 7.0160 \text{ amu}\\
\text{atomic mass of } He^4 &= 4.0026 \text{ amu}\\
1 \text{ amu} &= 931 \text{ MeV}\\
{}_1H^1 + {}_3Li^7 &\rightarrow {}_2He^4 + {}_2He^4\\
1.0080 + 7.0160 &\rightarrow 2(4.0026)\\
8.0240 &\rightarrow 8.0052\\
\Delta m &= 0.0188 \text{ amu}\\
(0.0188 \text{ amu})(931 \text{ MeV/amu}) &= 17.5 \text{ MeV}
\end{aligned}$$

2. Sections of Unit: 22.1, 22.2

$${}_{92}M^{238} \xrightarrow{\alpha} {}_{90}Th^{234} \xrightarrow{\beta} {}_{91}Pa^{234} \xrightarrow{\beta} {}_{92}M^{234}$$

3. Section of Unit: 21.8

"The law of disintegration of a radioactive substance is a statistical law" because:

(i) it applies to a large population.
(ii) it makes no prediction regarding an individual atom.
(iii) it does not attempt to explain cause.

4. Section of Unit: 21.8

$$8 \text{ mg} \longrightarrow 4 \text{ mg} \longrightarrow 2 \text{ mg} \longrightarrow 1 \text{ mg}$$

$$\begin{aligned}
\text{This reduction will take 3 half-lives} &= 3(3.05) \text{ min}\\
&= 9.15 \text{ min}
\end{aligned}$$

5. Sections of Unit: 24.5–24.8

The list of topics suitable for discussion within the framework of this question is long. The following examples are by no means intended to exhaust all the possibilities. A sufficient answer could involve a brief discussion of the effects of using atomic energy in electric power production, transportation, or water desalination. In addition, a student may wish to consider the effects of military applications or the consequences of fallout.

Group Two

6. Sections of Unit: 24.6, 24.7

(a) The function of the moderator in a nuclear reactor: fission is caused by the "capture" of a neutron by a heavy nucleus. There is a greater probability of this occurring if the fissionable material is bombarded with slow neutrons. Fast neutrons lose energy and are slowed through collisions as they pass through a moderator.

(b) Heavy water is an effective moderator since the mass of the atoms it contains is approximately equal to the mass of a neutron. Consequently, a neutron will lose a large fraction of its energy in a collision with an H nucleus. In addition, the density of H atoms in heavy water is high. Neutrons passing through heavy water are not absorbed by the nuclei and are thus available to produce fission. Other moderators, for example, ordinary water, are less effective because they absorb neutrons.

7. Section of Unit: 24.11

(a) The nucleus is regarded as analogous to a charged drop of liquid. Particles in the nucleus, like the molecules in a liquid drop, are in continual random motion.

As in the evaporation of molecules from the surface of a liquid drop, nuclear particles may gain sufficient energy through chance collisions with other nucleons to overcome the attractive nuclear forces and escape from the nucleus.

(b) **(i)** describes nuclear reactions
(ii) accounts for fission

8. Section of Unit: General

The elbow-shaped object is an electromagnet. Its function is to bend the particle beam, and in doing this to separate out the particles of interest. In addition, the magnet might be used to focus and aim the particle beam.

Unit 6 / Test B

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	C	24.13
2	D	21.3, 21.4
3	E	21.3, 21.4
4	A	Chapter 24,Unit 5
5	A	21.8
6	C	23.7
7	B	21.2
8	D	23.4
9	B	22.1, 22.2
10	E	21.5
11	C	24.5, 24.6
12	B	24.8
13	A	24.5, 24.6
14	A	21.3
15	C	24.5, 24.6

Group One

1. Section of Unit: 21.5

No. The decay constant is the fraction of decaying atoms per unit time. This quantity is observed to be constant for a particular element. Consequently, the decay activity is a function of the number of surviving atoms of the element and must decrease as the number of survivors decreases. Thus, the "total lifetime" of a sample is indefinitely long, for the fewer atoms that are left unchanged in the sample, the fewer will disintegrate per unit time.

2. Section of Unit: 23.8

No. Granted, gold has been produced from other materials. However, these transformations vary greatly in method and in purpose, from the attempts of the alchemists.

3. Section of Unit: 24.2

The Ba^{141} nuclide is more stable than the Ba^{222} nuclide

4. Section of Unit: 22.1

Nuclides of lead-206 and -214 have the same number of protons but different numbers of neutrons. Thus, they have the same chemical properties but have different atomic masses. In addition, they have different radioactive behaviors.

5. Section of Unit: General

The probe that the man is holding includes both a radioactive source and a counter tube. With this he is able to measure the thickness of the pipe walls. However, since the details of the probe are not shown, a satisfactory answer might suggest that a tracer had been introduced earlier and the man was measuring the rate of flow, density, pressure, etc., of the fluid in the pipe.

Group Two

6. Sections of Unit: 24.5–24.8

Fission: A heavy nucleus is split into two nuclei of intermediate mass number.

The sum of the binding energies of the two product nuclei is greater than the binding energy of the heavy nucleus. Consequently, energy is released.

Fusion: Two or more nuclei with low mass numbers are joined together to form a more massive nucleus.

The heavy nucleus has more binding energy than the nuclei from which it is formed. Consequently, energy is released.

Both processes can be made to occur on a large scale, and very rapidly, resulting in nuclear explosions. The speed of the fission process can be controlled, resulting in the production of energy at a desired rate. Scientists are now trying to control the rate of the fusion process.

7. Section of Unit: 21.6

When a radioactive atom emits an α or β particle it really breaks into two parts: the α or β particle and a heavy left-over part that is physically and chemically different from the original atom.

8. Sections of Unit: 24.6, 24.7

Nuclear explosions release large amounts of radioactive materials. Winds carry these materials long distances and precipitation brings them down to earth. Some of the radioactivities are long-lived, and are absorbed by growing foodstuffs and eaten by animals and people. Under certain conditions these radioactive materials can cause harmful genetic and somatic effects. For example, one of the long-lived products of a nuclear explosion is strontium-90. This isotope of strontium is similar to calcium-40 in its chemical properties and, hence, when taken into the body, it finds its way into bone material. If present in large quantities, its radioactive decay can cause leukemia, bone tumors, and other forms of damage.

Unit 6 / Test C

MULTIPLE-CHOICE QUESTIONS

Item	Answer	Section of Unit
1	B	21.6
2	B	21.6
3	C	21.6
4	B	21.3, 21.4
5	D	21.8
6	D	Chapter 22
7	D	21.3, 21.4
8	E	21.8
9	C	21.3, 21.4
10	A	21.3, 21.4
11	C	21.8
12	B	21.8
13	E	22.5
14	A	22.7
15	A	22.1, 22.2
16	E	Chapter 22
17	C	23.7
18	E	23.8
19	A	Chapter 22
20	E	Chapter 23
21	D	Chapter 23
22	D	Chapter 23
23	D	21.3, 21.4
24	D	general
25	B	24.5
26	E	23.3
27	A	24.2
28	C	24.5
29	B	24.8
30	A	24.8
31	C	21.3, 21.4
32	C	21.8
33	B	21.8
34	E	22.1, 22.2
35	C	Chapter 21
36	C	24.8
37	B	24.6
38	D	24.6
39	C	24.6
40	A	21.8

Unit 6 / Test D

Group One

1. Section of Unit: Chapter 23

(a) ${}_8O^{16} + {}_0n^1 \rightarrow {}_8O^{17}$

(b) ${}_8O^{16}$ and ${}_8O^{17}$ are both isotopes of oxygen.

2. Section of Unit: 24.2

Given: mass of ${}_2He^4$ atom $= 4.002403$ amu

mass of e^- $= 0.000549$ amu

mass of p $= 1.007276$ amu

mass of n $= 1.008665$ amu

1 amu $= 931$ MeV

An ${}_2He^4$ atom consists of 2 electrons, 2 protons, and 2 neutrons.

$$\begin{aligned} 2\,p &= 2(1.007276) = 2.014552 \\ 2\,n &= 2(1.008665) = 2.017330 \\ 2\,e^- &= 2(0.000549) = \underline{0.001098} \\ & \qquad 4.032980 \\ 2He^4 &= -\underline{4.002603} \\ \Delta m &= 0.030377 \text{ amu} \end{aligned}$$

$(0.030377 \text{ amu})(931 \text{ MeV/amu}) = 28.28$ MeV

3. Section of Unit: 21.6

A radioactive atom undergoes change on emitting an α or β particle. The original atom is transmuted to one with new physical and chemical properties. Traditionally, atoms were considered indestructable and unchangeable.

4. Section of Unit: 21.1

In his studies of uranium, Becquerel found that:

(a) whether or not the uranium compound was being excited, it continued to emit radiations that could penetrate substances opaque to light.

(b) the amount of exposure of a photographic plate due to the radiations from the uranium compound was only a function of the amount of uranium present.

5. Section of Unit: 24.13

(a) The physical and chemical effects of various kinds of radiations on biological materials are being studied.

(b) The metabolism of plants and animals is being studied with the aid of minute amounts of radioactive nuclides called isotopic tracers.

(c) Agricultural experiments with fertilizers containing radioactive isotopes have shown at what point in the growth of a plant the fertilizer is essential.

(d) Radioactive isotopes help to determine the details of chemical reactions and of the structure of complex molecules, such as proteins, vitamins, and enzymes.

(e) Tracers help to determine rate of flow of blood through the heart and to the limbs, thus aiding in the diagnosis of abnormal conditions.

(f) Certain radioisotopes have been used in the treatment of cancer, blood diseases, brain tumors, and in the diagnosis of thyroid, liver, and kidney ailments.

Group Two

6. Section of Unit: 21.8

The fraction of the total number of atoms in a sample that decays per unit time is constant for any type of radioactive atom. Therefore, radioactivity must decrease in proportion to the number of surviving atoms. Thus, the "total lifetime" of a sample is indefinitely long because the fewer atoms that are left unchanged in a sample, the fewer will disintegrate per unit time. Consequently, "total lifetime" is not a useful measure of decay rate.

7. Sections of Unit: 22.1, 22.2

(a) An α-particle emission reduces the positive charge of the nucleus by two units. Consequently, the resulting nuclide holds two fewer electrons in its outer shells. This new nuclide acts chemically like an atom with an atomic number two units less than that of the atom before the α emission occurred.

(b) A β-particle emission increases the positive charge of the nucleus by one unit. Consequently, the resulting nuclide holds one more electron in its outer shells. This new nuclide acts chemically like an atom with an atomic number one unit greater than that of the atom before the β emission occurred.

8. Section of Unit: 24.12

(a) The nuclear shell model assumes that protons arrange themselves in shells and that neutrons can, independently, do likewise. In nuclei with even numbers of neutrons and protons (the more stable nuclei), these shells are filled. This model has been worked out in great detail on the basis of quantum mechanics and is analogous in many ways to the quantum mechanical model of the atom.

(b) The nuclear shell model has successfully correlated the properties of nuclides that emit α or β particles and photons. Futhermore, it has been useful in describing the electric and magnetic fields that surround nuclei.

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