

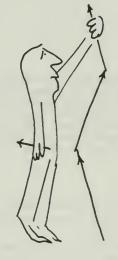
The Project Physics Course

Programmed Instruction

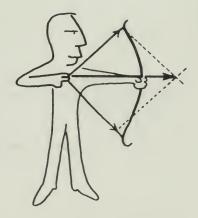
Vectors 1 The Concept of Vectors



Vectors 2 Adding Vectors



Vectors 3 Components of Vectors



INTRODUCTION

You are about to use a programmed text. You should try to use this booklet where there are no distractions—a quiet classroom or a study area at home, for instance. Do not hesitate to seek help if you do not understand some problem. Programmed texts require your active participation and are designed to challenge you to same degree. Their sole purpose is to teach, not to quiz you.

This book is designed so that you can work through one program at a time. The first program, Vectors 1, runs page by page across the top of each page. Vectors 2 parallels it, running through the middle part of each page, and Vectors 3 similarly across the bottom.

This publication is one of the many instructional materials developed for the Project Physics Course. These materials include Texts, Handbooks, Teacher Resource Books, Readers, Programmed Instruction Booklets, Film Laaps, Transporencies, 16mm films and laboratory equipment. Development of the course has profited from the help of many colleagues listed in the text units.

Directors of Project Physics

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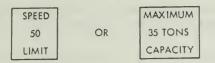


A Companent of the Project Physics Course

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Vectors 1 The Concept of Vectors

You are familiar with signs such as ONE WAY SUBWAY that indicate a direction. You have also seen signs which give a magnitude such as



This program is about quantities that have both a direction and a numerical value. These are called vectors and they are very important in physics.

You are already familiar with some examples of vectors. This part of the program will start with these examples.

Vectors 2 Adding Vectors

Adding vectors is an important technique for you to understand and be able to use. After going through this set of programmed materials you will be able to add two or more vectors together and abtain the resultant vector. The next three sample questions represent the kinds of questions you should be able to answer after you have finished Vectors 2. If you can already answer these frames, you need not take Vectors 2. In that case you can go on to Vector- 3.

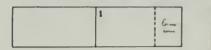
Vectors 3 Components of Vectors

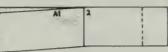
When we use a vector to represent a physical situation, we may wish to find the component of that vector in a given direction. This is Part III of the series of programmed instruction booklets on vectors. In this part, you will learn how to separate vectors into components and how to obtain a vector from its components.

The two sample questions that follow illustrate the objectives of this part of the program, Vectors 3. If you find that you can answer these two questions correctly, you need not work through the program.

INSTRUCTIONS

- 1. Frames: Each frame contains a question. Answer the question by writing in the blank space next to the frame. Frames are numbered 1, 2, 3, . . .
- 2. Answer Blocks: To find an answer to a frame, turn the page. Answer blocks are numbered A1, A2, A3, ... This booklet is designed so that you can compare your answer with the given answer by folding back the page, like this:



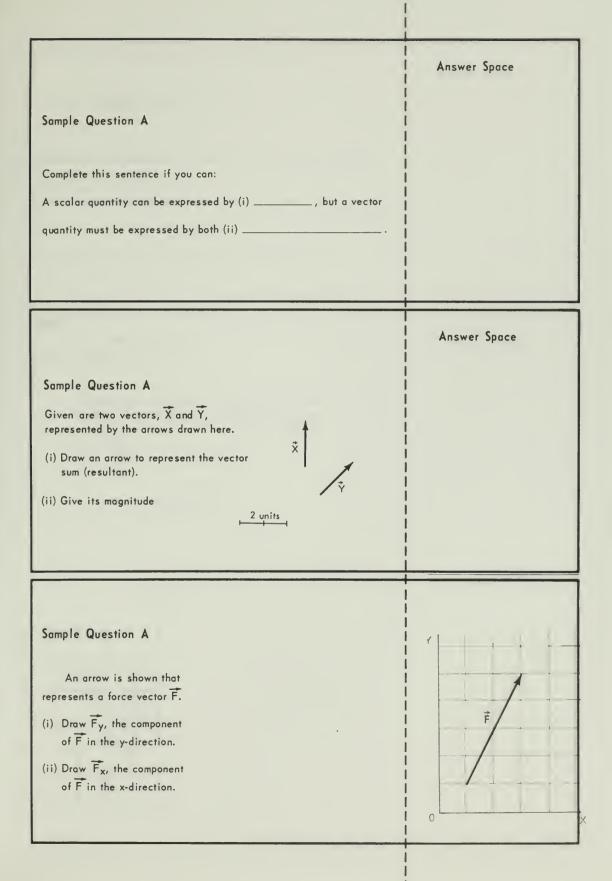


A1	2	1	٦
lan			

- 3. Always write your answer before you look at the given answer.
- 4. If you get the right answers to the sample questions, you do not have to complete the program.

INSTRUCTIONS: Same as for Program 1, above.

INSTRUCTIONS: Same as for Program 1, o' ve.

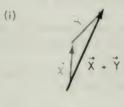




(i) a number (with or without units)

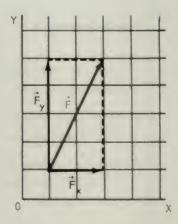
(ii) a number (with or without units) and a direction.







Answer A



Answer Space

Sample Question B

It is important to be able to distinguish between vector and scalar quantities in equations.

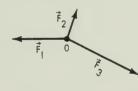
(i) List all of the vector quantities in the equation

 $\overrightarrow{T} = \overrightarrow{mo} + \overrightarrow{6P}$.

(ii) List all of the scalar quantities in the same equation.

Sample Question B

Three forces acting on an object, O, can be represented by arrows as drawn below. What is the resultant force on the object, that is, what is the vector sum of the three forces?

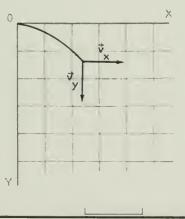


Answer Space

Sample Question B

Given \overline{v}_x and \overline{v}_y :

- (i) Construct and draw v.
- (ii) Give the direction and magnitude of v.



Answer to B

(i) \overrightarrow{T} , \overrightarrow{a} , and \overrightarrow{P}

(ii) m and 6

Answer to B

Resultant

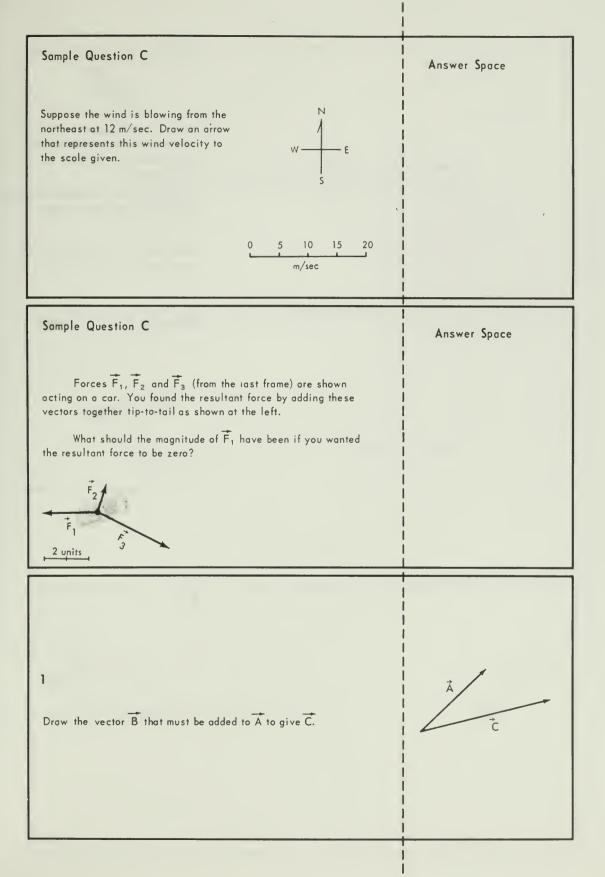
Resultant Force, F_R shown.

Answer B

(i)

- (ii) 45° below horizontal, 50 m/sec.

If your answers to the sample questions were correct, the remainder of the program is optional.



Answer to C



Your onswer is correct only if the arrow you draw points in the same direction as this one and is the same length.

If you answered all 3 sample questions correctly, you are ready for the Vectors 2 program.

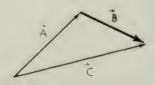
If not, begin with question 1 on the next page.

Answer to C

New F,

 $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ if the magnitude of \vec{F}_1 is 3.5 units.

Al



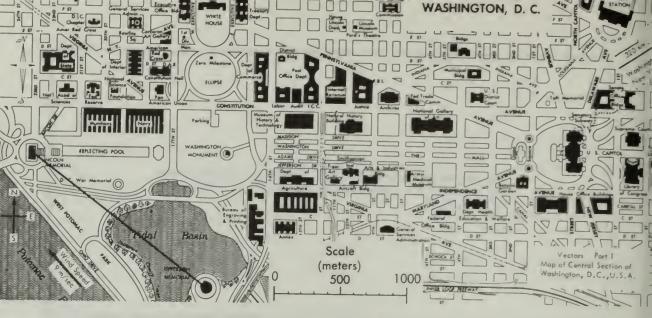
Now turn the page to begin Vectors 1. Remember to proceed through the book from left to right, confining your attention to the top frame on each page.

Now turn the page to begin Vectors 2. Remember, left to right, middle frames only.

F

Draw two perpendicular vectors that add to give F.

2



The Parallelogram Law

A vector is an entity having both magnitude and direction; vectors also have the property of addition by the parallelogram law as shown here, where A and B represent two vector quantities.



The vector sum of $\overrightarrow{A} + \overrightarrow{B}$ is \overrightarrow{C} and can be drawn in two ways. Both ways of drawing the parallelogram law shown above are equivalent, but the "tip-to-tail" method on the right will be shown to the more powerful since it can be extended easily to more than two vectors.

There are many physical quantities which have both direction and magnitude and add together according to the parallelogram law. In Part I of the vectors program the displacement vector was introduced, and Part II will begin with the addition of displacements.

A2

passible solutions:

NOTE: There are an infinite number of solutions.

1					
Questions 1 through 16 require the map of washington, D.C., shown to the left. Find the location of the Lincoln Memorial and the Jefferson Memo- rial on the map of Washington, D.C. A straight line is shown be- tween the memorials. According to the scale of the map, the dis- tance between the Lincoln and Jefferson Memorials is					
1					
Read the panel on the opposite page.					
You learned in Part 1 of the program that a vector quantity has both magnitude and direction.					
What other property will a vector quantity have?					
3					
Martha walked from the post office to the bus stop.					
Her displacement is represented by the arrow marked \overrightarrow{D} on the map.					
(i) How many blocks east did she walk?					
(ii) How many blocks south did she walk?					
Sth Ave. Sth Ave. Sth Ave. Sth Ave. Ave. Ave. Ave. Ave. Ave. Ave. Ave.					

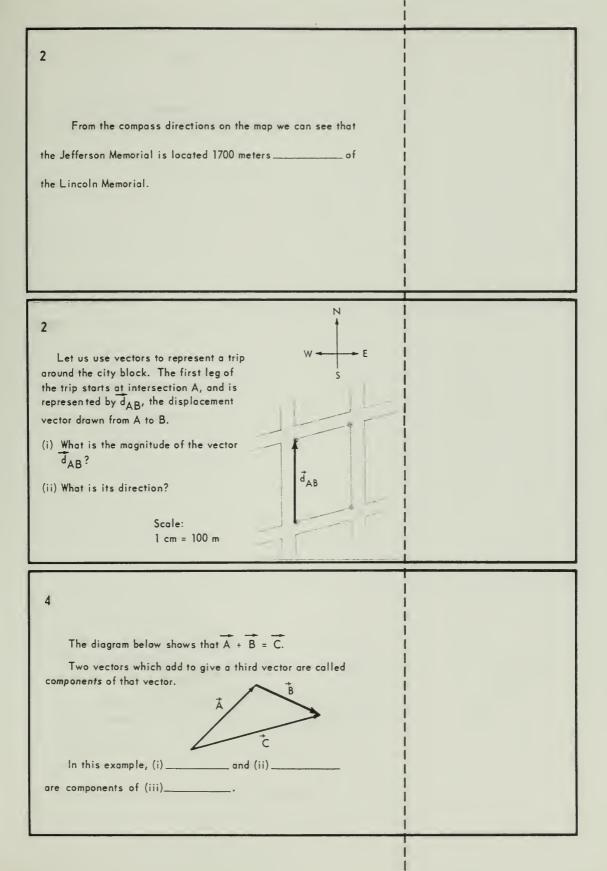
about 1700 meters, measuring center to center

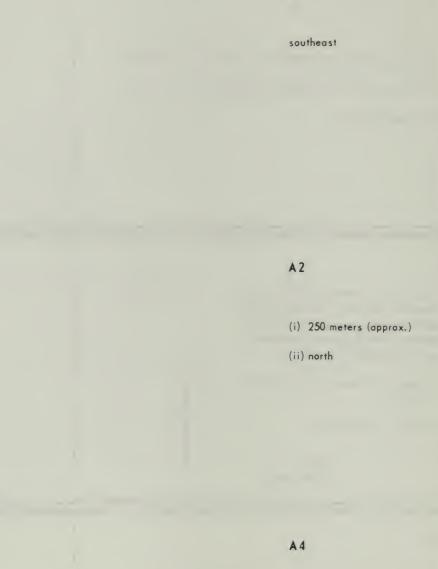
Al

Vector quantities add according to the parallelogram law.

A3

(i) 6 blocks east(ii) 2 blocks south





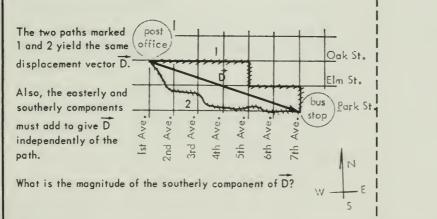
(i) \overrightarrow{A} (or \overrightarrow{B}) (ii) \overrightarrow{B} (or \overrightarrow{A}) (iii) \overrightarrow{C} Locate the White House, and find the distance and direction of the White House from the Jefferson Memorial.

3

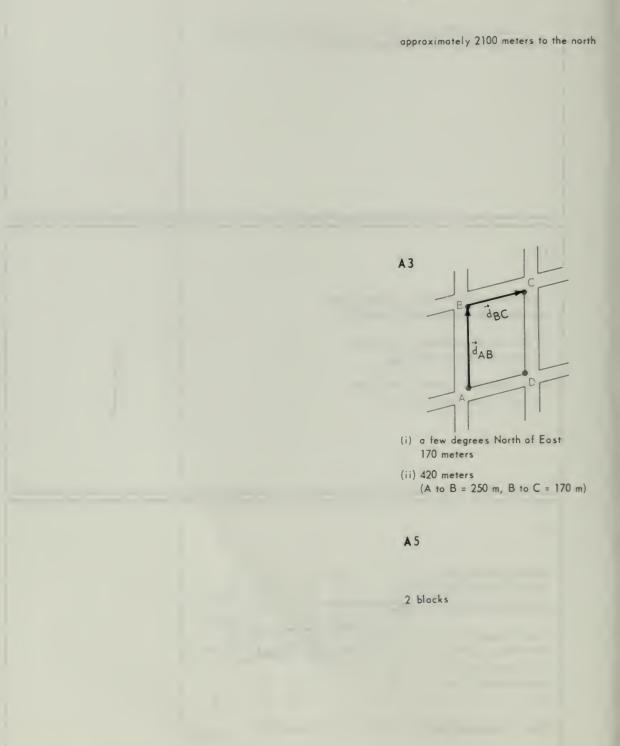
On the panel draw the second leg of the trip around the block, namely from B to C.

- (i) Give the direction and magnitude of the displacement vector d_{BC}.
- (ii) Give the total distance traveled on the first two legs of this trip.





3



One of the important concepts of physics is that of displacement: it is the straight line distance and direction between the initial and finol locations of an object. Use the map of Washington, D.C., to answer the following questions:

(i) What building will you reach if you start at the Woshington Monument and trovel 2600 meters due east?

d BC

(ii) What was your displacement?

4

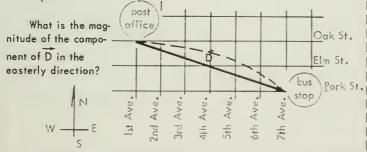
4

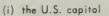
Draw the vector \overline{d}_{AC} between points A and C. (This goes diagonally across the block.)

- (i) Give the magnitude and direction of $\overline{d_{AC}}$.
- (ii) What is the difference (in meters) between the distance traveled from points A to B to C, and the magnitude of the vector dAC?

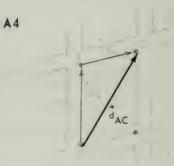
6

The dashed line represents the actual poth Martha took from the post office to the bus stop. Her displacement \overrightarrow{D} does not depend on her path and the components of \overrightarrow{D} likewise do not depend on her path.





(ii) 2600 m east from the Washington Monument



(i) 330 m
 a few degrees North of Northeast

(ii) 90 m difference

A6

6 blocks

5

- (i) What would be your displacement if you traveled from the Capitol to the White House?
- (ii) What is the displacement if something is moved from the White House to the Washington Monument?

5

The displacement vector from A to C, d_{AC} , is the resultant of adding d_{AB} and d_{BC} .

The displacement vector \vec{d}_{AD} is the resultant of adding \vec{d}_{AC} and (i) _______. (ii) What is the resultant of \vec{d}_{BC} and \vec{d}_{CD} ?

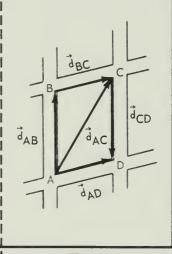
(iii) Draw the resultant of d_{BC} and d_{CD} on the diagram at the right.

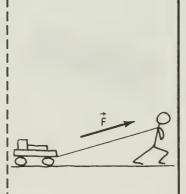
7

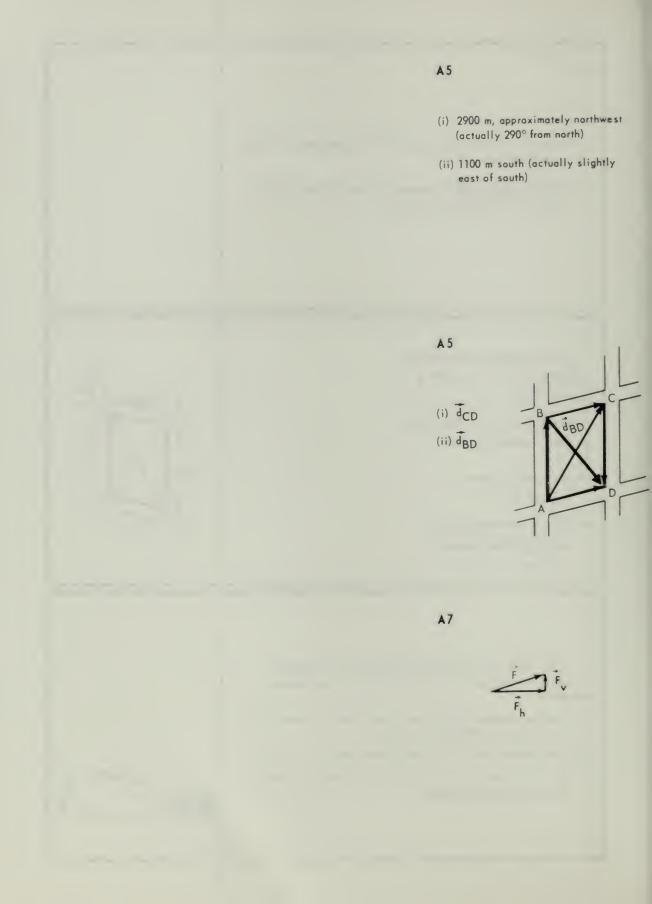
The vector $\overrightarrow{\mathsf{F}}$ represents the force exerted by the rope on the wagon. We can separate the force into vertical and horizontal components.

- (i) Draw the component of \overline{F} in the vertical direction. Label it $\overline{F_v}$. This component tends to lift the wagon.
- (ii) Draw the component of F in the horizontal direction. Label it

 \overline{F}_{h} . This component of the force is responsible for the motion of the wagon along the ground

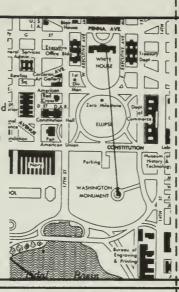






A displacement can be represented by an arrow in a map. The length of the arrow represents a scale drawing of the actual displacement.

What displacement is shown?



6

6

The final leg of the trip around the block, from intersection D to A, is given by the displacement vector d_{DA}.

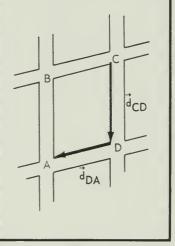
Draw the vector sum of d_{CD} and \overline{d}_{DA} .

8

The arrow labeled F_{grav} represents the force of gravity on this railroad hopper car. The component of F_{grav} per-

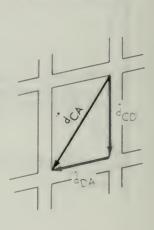
pendicular to the track is balanced by the opposite force of the track on the wheels.

- (i) Draw the component of $\overrightarrow{F_{grav}}$ that is perpendicular to the track. Label it $\overrightarrow{F_{\perp}}$.
- (ii) Draw the component of $\overline{F_{grav}}$ that is parallel to the track. Label it $\overline{F_n}$.





White House to Washington Monument (1100 m south)





grav

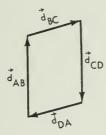
A 6

- (i) Draw an arrow an the map to represent the displacement of a person who has walked from the Washington Monument to the Jefferson Memorial. (Hint: If you are not sure how to do this, recall the definition of displacement in Frame 4.)
- (ii) Draw a broken line on the map to show the shortest path for walking on dry ground from the Washington Monument to the Jefferson Memorial.
- (iii) Is the path length the same as the displacement?
- (iv) Does the choice of path change the displacement?

7

7

The four legs of the trip around the block can be represented by the four separate vectors shown here.



What is the sum of these four vectors?

9

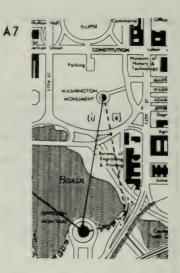
Here is an expanded diagram from Frame 8. The magnitude of \vec{F}_{grav} is 120,000 N.

- (i) Find the magnitude of $\overline{F_{\perp}}$.
- (ii) Find the magnitude of F₁₁.

scale: 50,000N 0

F⊥ F_{grav}

(iii) no (it changes the path length, but not the displacement, which is defined as the straightline distance.)



A7

zero

A9

(i) 120,000 N(ii) 30,000 N

On the map of Washington	, D.C.,	there is	s an	arrow	which
--------------------------	---------	----------	------	-------	-------

direction?

indicated that the displacement of New York City from

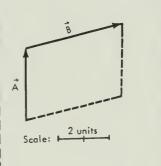
distance?

Washington is _

8

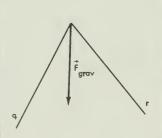
If the vector C is the sum of vectors A and B, we can write: $\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{C}$

- (i) Given A and B as shown, draw the vector sum C.
- (ii) Find the direction and magnitude of C by measuring the scale drawing.



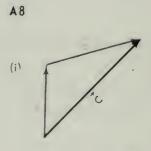
10

In general, components of a vector are constructed as the sides of a parallelogram which has the vector as the diagonal. The angle between the sides of the parallelogram may be any value; however, the physical analysis is often easiest if this is chosen to be 90°. The preceding examples of the wagon and the hopper car illustrate the usefulness of components that are at right angles. As an example of non-perpendicular components, take the vector Farav from before and resolve it into components in the g and r directions. Label the components $\overline{F_q}$ and $\overline{F_r}$. Be sure to draw these components as vectors.



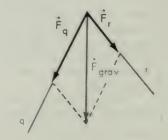
8

320 km northeast



(ii) direction: 43° from A. magnitude: 5.2 units.





Note that the distance scale at the bottom of the map is for measurements inside Washington, and the displacement to more remote places such as New York City is represented with another scale. It is not essential that the arrow representing a displacement vector be drawn to the same scale as the map.

Pittsburgh, Pennsylvania, is approximately 320 kilometers to the northwest of Washington. Draw an arrow by which you can represent this displacement.

(Use the same scale as the arrow showing the displacement of New York City.)

9

Two arrows representing the vectors \vec{S} and \vec{T} are drawn separately. \vec{S} and \vec{T} cannot be added without shifting them so that they touch. The most useful way to make this shift is so that the pointed "tip" of one touches the blunt "tail" of the other.

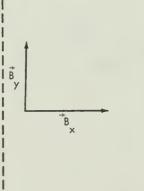
 (i) Redraw S and T with the tip of S touching the tail of T.

 (ii) Draw the vector sum of S and T on the tipto-tail drawing.

11

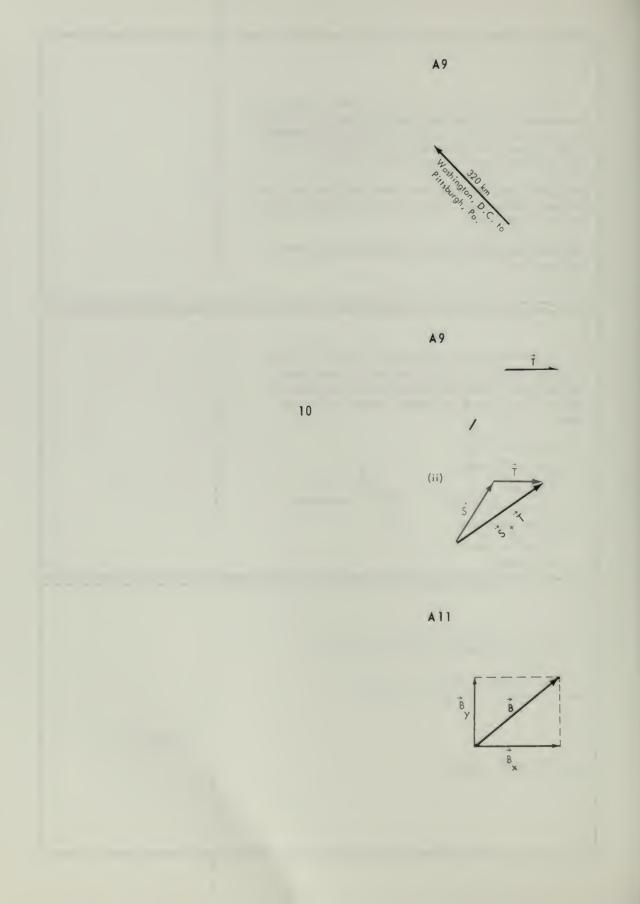
The previous frames have shown that a vector may be resolved into components along any chosen axis.

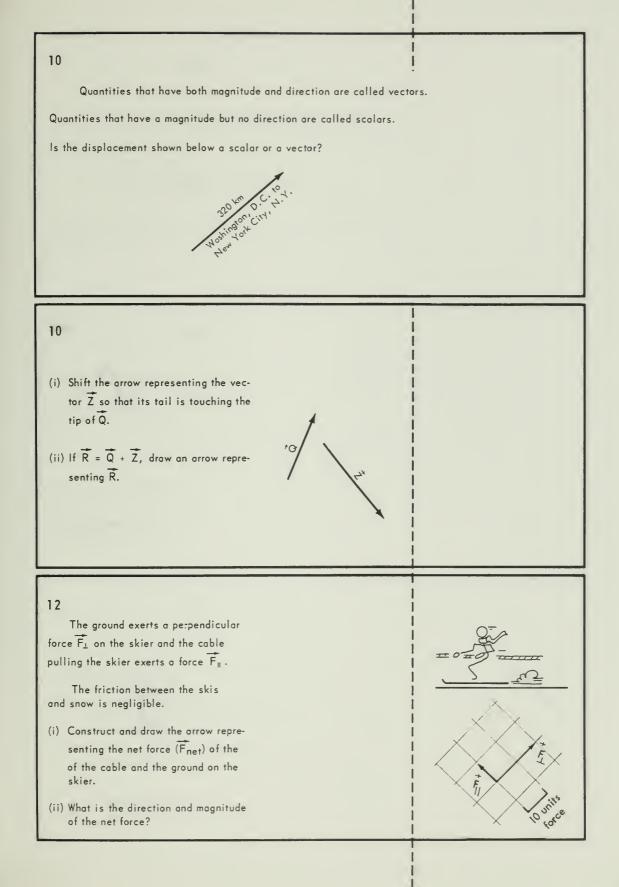
Naw, given the components, it can be seen that a vector is the (vector) sum of its components.

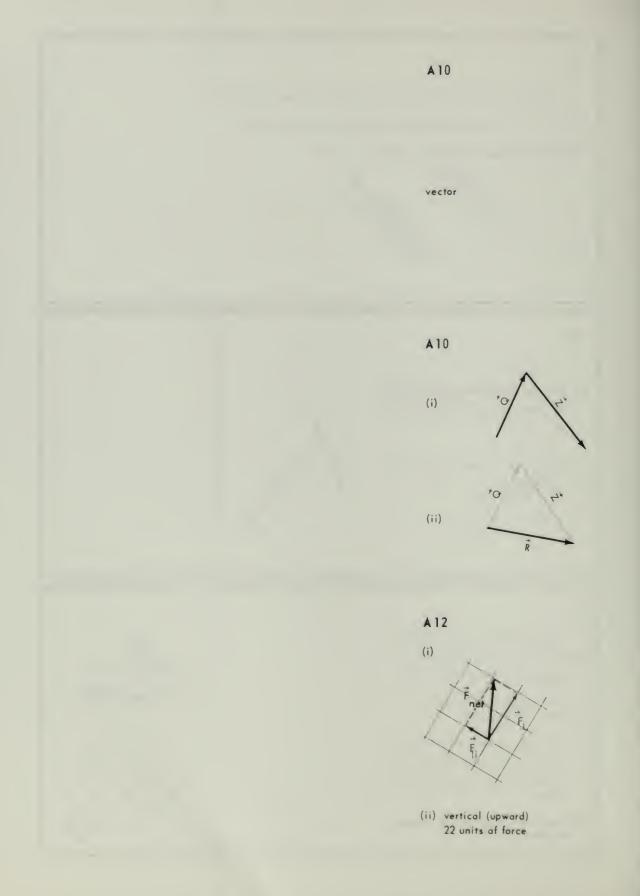


Given \overrightarrow{B}_x and \overrightarrow{B}_y , find \overrightarrow{B} .

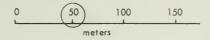
9







Quantities that have only a magnitude are called scalars. Those quantities that have both magnitude and direction are called vectors.



Is the position of the 50 meter mark on the scale a vector or a scalar?

11

 $\vec{H} = \vec{F} + \vec{G}$. Find \vec{H} by adding \vec{F} and G with the tip-to-tail method in both of these ways:

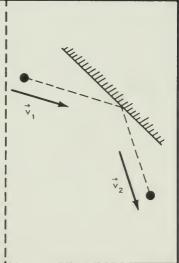
- (i) shifting \overrightarrow{F} to the tip of \overrightarrow{G} .
- (ii) shifting G to the tip of F.
- (iii) Do both procedures give the same result?

13

The diagram shows a particle striking a barrier and rebounding elastically.

- (i) Resolve each of the velocity vectors into components which are perpendicular to the wall and parallel to the wall.
- (ii) Which component of velocity did not change during the interaction?





11

scalar

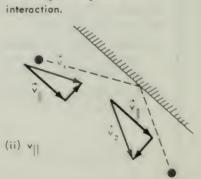
ŧ

A13

(iii) Yes

(ii)

The component of velocity parallel to the wall does not change during the interaction.



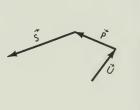
A scalar quantity can be expressed by a single number

(with or without units), but a vector must have both

12

The clear advantage of using the tip-to-tail method of graphically | adding vectors can be seen when three or more vectors are to be added. We have already seen this in the example of the city block. The addition is performed by making a "chain" of

vectors. Then the sum (or resultant) is found by drawing the arrow from the tail of the first to the head of the last arrow in the chain.

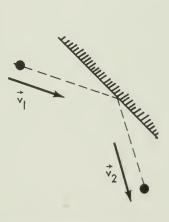


Draw the resultant for U + P + S

14

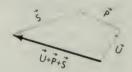
Here is the same event again.

Describe the change of the component of velocity perpendicular to the wall.



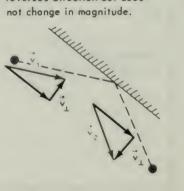
magnitude and direction

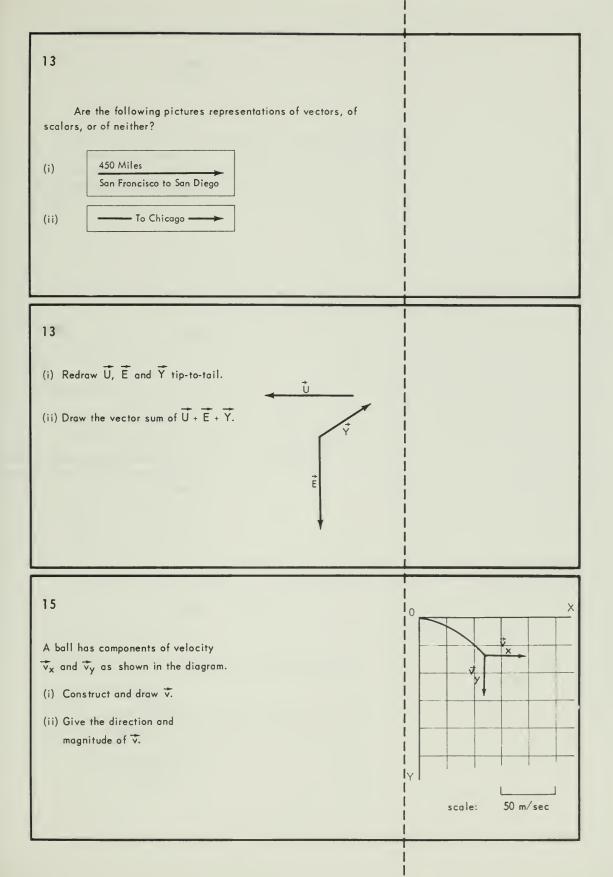




A14

The component of velocity perpendicular to the wall reverses direction but does not chonge in magnitude.





A13

(i) vector (a displacement)

(ii) neither (only direction)

A13

NOTE: As the reduced sketches below indic any sequence of V, E. and Y will give the s resultant.

1 1

Sum (ii)

A15

1



(ii) 45° below horizontal.
 50 m/sec

14 On the map of Washington, D.C., there is an arrow representing the wind velocity. The arrow indicates that the wind is blowing from the (i) at a speed of (ii)	
 14 (i) Redraw M, N and O tip-to-tail. (ii) Draw the vector sum W, where M + N + O = W. 	
You have now completed all three programs in this book. Understanding ond being able to use vectors should be helpful to you in many ways. If ever you wish to refresh your memory on Vectors, you can cover up the answer space with a sheet of blank paper and quickly run through the frames again.	

(i) southeast

(ii) 9 m/sec (about 20 miles/hr)

A14

õ Ŵ Ň

NOTE: Any sequence of \overline{M} , \overline{N} , and \overline{O} will give the same W.

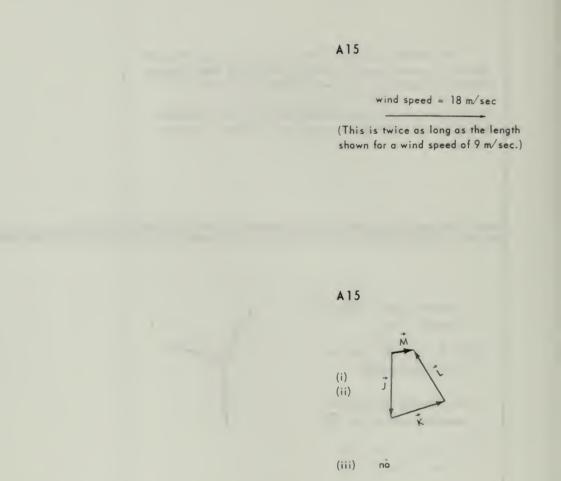
The speed and direction of the wind is a vector quantity, and therefore it can be represented by an arrow drawn to scale. Suppose the wind changed and is now coming from the west at 18 m/sec.

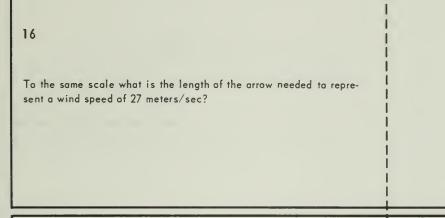
Draw the new wind direction, and indicate the new wind speed by making the arrow of the proper length (using the other wind arrow as a guide).

15

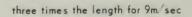
- (i) Redraw the vectors J, K and L tip-to-tail.
- (ii) $\overrightarrow{J} + \overrightarrow{K} + \overrightarrow{L} = \overrightarrow{M}$. Draw the arrow representing \overrightarrow{M} .
- (iii) Does the order in which you redraw the vectors affect \overline{M} ?







16 Given \overline{M}_1 , \overline{M}_2 , \overline{M}_3 as shown, and $\overline{M}_1 + \overline{M}_2 + \overline{M}_3 = \overline{M}_4$ Find \overline{M}_4 .









Whenever we encounter a physical quantity—such as speed, force, energy, or whatever—it is useful for us to know whether or not it involves direction. Those quantities that involve direction as well as magnitude are called

(ii) Does the pull each team exerts on the rope in the tug-of-war involve a direction?

17

17

(i) _

If $\overrightarrow{A_1} + \overrightarrow{A_2} + \overrightarrow{A_3} = \overrightarrow{0}$, and $\overrightarrow{A_1}$ and $\overrightarrow{A_2}$ are as shown, construct the vector $\overrightarrow{A_3}$ that satisfies this equation. Å₁ Å₂



When we encounter a physical quantity that is a scalar we mean it has no

(i) _

(ii) Is the diameter of the water wheel shown here a vector or a scalar?

_ •

18

Force is a vector quantity. Each of the cars shown here is exerting a force on the large wooden box.

Below each car draw an arrow to indicate the direction of the force each car exerts on the object to which it is hitched.





19 Four boys are shown pushing a car. The force each boy exerts on the car is a (i) quantity, and the number of boys pushing the car is a (ii) quantity.	
19 Suppose the small car (1) pulls with half the force the other car (2) exerts.	





(ii) scalar

A19



NOTE: These arrows can be of any length except that (1) must be just one-half the length of (2). When writing one usually draws a small arrow over the symbol used for vector quantities. For example, in the equation

 $\overline{F} = m \overline{a}$,

F represents a vector quantity, the force, and a represents an acceleration in the same direction as F. The letter m represents a

a scalar, mass.

(i) List all vector quantities in the equation

 $\overrightarrow{T} = \overrightarrow{ma} + \overrightarrow{6N}$

(ii) List all of the scalar quantities in the same equation.

20

(i) What is the sum of the two pulls of the cars, namely the resultant force exerted on the box by both cars pulling together? Assume the pulling forces: $\vec{F_1} = 5$ units (to the left) $\vec{F_2} = 10$ units (to the left)

(ii) Draw the resultant force (\overline{F}_R) ?

20



 (i) T, a, N
 (Did you put the arrows over the symbols?)

(ii) m, 6

A 20

(i) 15 units of force to the left

(ii)	(1)	(2)	
	Resultant		
	i	FR	

The negative of a vector quantity is represented by an arrow

in the reverse direction. For example if \overrightarrow{A} is represented by

 \overrightarrow{A} then $-\overrightarrow{A}$ is represented by $\overrightarrow{-\overrightarrow{A}}$ If \overrightarrow{B} is $\overrightarrow{/}$, drow $-\overrightarrow{B}$.

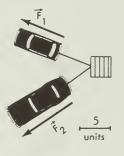
21

Two cors are shown pulling on a wooden box. The pulling force of each

car is represented by the vectors \overline{F}_1

and \overline{F}_2 (note the units).

- (i) Construct the vector sum F_R of these forces using the tip-to-tail method. (If you ore not sure how to do this, refer to Frame II.)
- (ii) What is the direction and magnitude of the sum $\overrightarrow{F_R}$?
- (iii) Write an equation to represent the relation between $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_R}$.



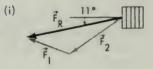
21

A21

10/

Did you draw $-\overrightarrow{B}$ to the proper length? It is a vector in the direction opposite to B but having the same magnitude.

A21



(ii) to the left and a few degrees below horizontal; magnitude about 15 units

(iii)
$$\vec{F}_1 + \vec{F}_2 = \vec{F}_R$$

22 If $-\vec{C}$ is give a full label to:

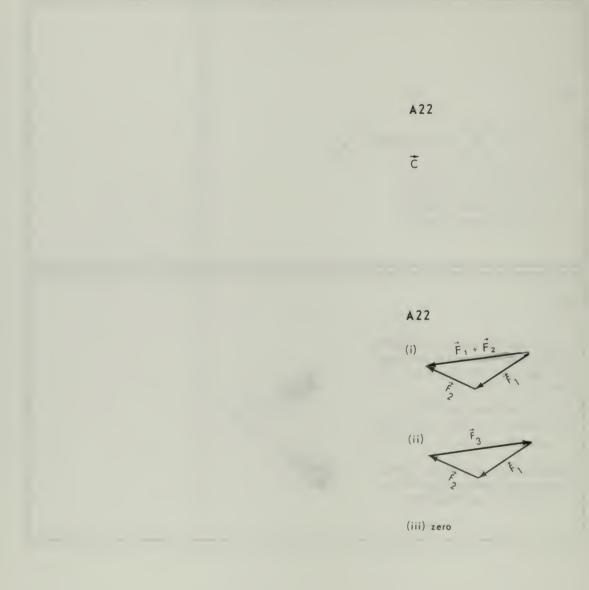
Suppose that two cars were pulling an object, and that each is exerting a force represented by the arrowns shown here.

- (i) Find the vector sum $\overline{F}_1 + \overline{F}_2$.
- (ii) Draw an arrow representing a force vector F_3 such that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$.

T.

F2

(iii) If \vec{F}_3 is the force exerted on the object by a third car, what is the resultant force on the object?



This ends Vectors 1.

You have learned to distinguish between vectors and scalars. You have drawn vector quantities to scale, and you have learned that a negative vector is in the opposite direction from the corresponding positive vector.

You are now ready to learn to add vector quantities. See the program Vectors 2. It begins at the front of this book and occupies the middle of each page.

23

Three forces acting on an object 0 can be represented by arrows as drawn below.

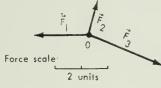
Draw an arrow to represent the resultant force $\overline{F}_{\mathsf{R}}$ on the object.



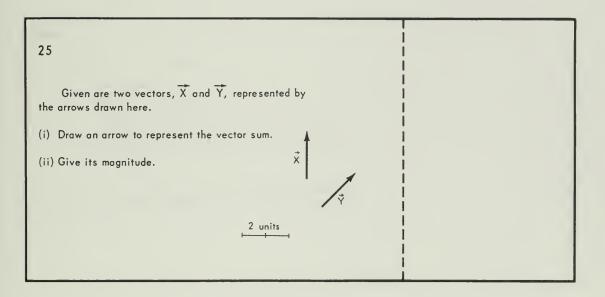
(Hint: If you are not sure how to do this, refer to Frame 15.)

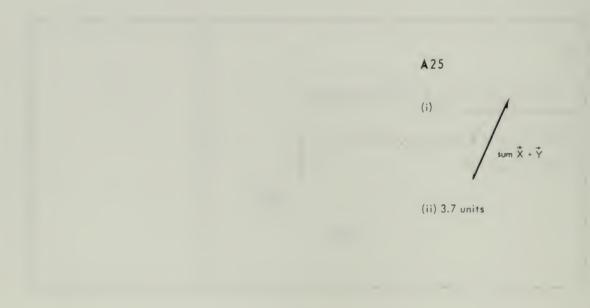


Forces \vec{F}_1 , \vec{F}_2 and \vec{F}_3 (from the last frame) are shown acting on object 0. You found the resultant force \vec{F}_R by adding these three vectors together "tip-to-tail" in Frame 23. What magnitude should \vec{F}_1 have in order to make the resultant force zero?









This ends Vectors 2.

You have learned how to add two or more vectors together and to draw the resultant vector. Also, given two vectors, you have practiced finding a third vector that would just balance the first two vectors so that the sum of the three was zero.

If you would now like to learn about components of vectors, see the program Vectors 3. It begins on the bottom part of the first page of this book.

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