Velocity and Acceleration

1965

Acknowledgment

This document is only one of many instructional materials being developed by Harvard Project Physics. Like all existing Project materials--additional text units, laboratory experiments, teachers guide, and the rest--it is now an experimental, intermediate stage. This text is based on earlier versions used in cooperating schools, and its development has profited from the help of many friends and colleagues, both within Project Physics and outisde that group. Successive revisions in this text are planned in the light of further experience and use. In the final experimental edition scheduled for 1967 , a detailed acknowledgment will appear of those contributions that were found to be of greatest use and permanence in the development of the new course.

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The material you are about to study is a programed text--often called a "program" for short. It is designed to help you learn some useful techniques and important concepts which are essential for you to know in the course you are presently taking.

Although you will be asked questions at every step in the program, there is no penalty for errors. In fact, you will often be asked to guess at answers you probably don't know. Occasionally hints will follow a question. When you can answer the original question, you may skip the rest of the hints; if you have trouble with the problem, keep working out the hints. There are no time limits for a program either. You should work at your own pace. And if you have any questions about the material, see your teacher for assistance. Remember, A PROGRAM IS NOT A TEST.

When you are asked a question in the program, try to answer it or puzzle it out on your own. Every question is answered in the program immediately after the answer space. (Sometimes below the question, or below and to the right, or on the page following the question.) If your answer does not coincide with the answer given, try to see why the program's answer is more reasonable. If you do not see why, consult with your teacher.

Since you do not want to see the answer before you have tried the question yourself, take a sheet of paper and place it below the material you are studying. When you have finished your answer, slide the sheet down to find the correct answer. You will find this "answer shield" a handy place to make calculations. In general, you should have additional <u>scrap paper</u> for doing arithmetic. For most of these programs, a <u>ruler</u> is also needed.

SAMPLE FRAMES

$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = \_\_ = \_\_$$

(Fill in the blanks.)

ANSWER: 25, 5^2

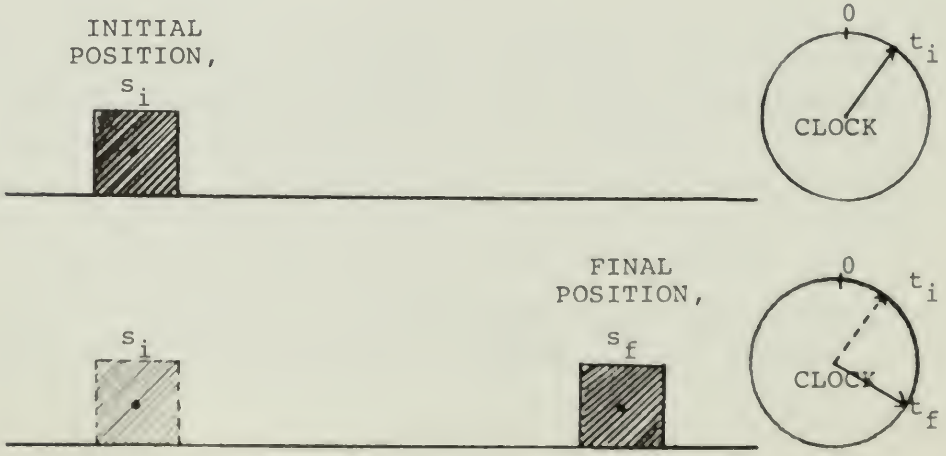
The sum of the first 8 odd numbers is ______, or ______.

ANSWER: 64, 8^2

You have seen a film about mass, acceleration, velocity, force, and related concepts. You have made some investigation of motion in the laboratory. You have read about early approaches to these phenomena and about present ways of analyzing them. Now we are going to ask you to translate these phenomena into symbols and graphs. Physicists have found this translation helpful in dealing with more complex problems involving these phenomena.

Consider a block sliding on a horizontal board, assumedly with very little friction. The diagram gives a side view of the situation; you are to fill in the missing symbolic and verbal information. When you see a blank, fill it in, then look further down the page, or on the next page, for the correct answer so that you can check your own progress. Cover up material below the dotted line until you have written your own answer.

These two diagrams simulate an experiment such as you saw in the film and/or did yourself in the laboratory:



The diagram is repeated below. Fill in the missing symbols and then check your version with the one on the previous page.

Give an algebraic expression for the elapsed time:_________.

Give an algebraic expression for the distance traveled:

______________.

(Cover up the answers below until you have answered yourself.)

Elapsed time: $t_f - t_i$.

Distance traveled: $s_f - s_i$.

(Remember: t_i means "initial time," s_i means "initial position," t_f is "final time," etc.)

Which of the problems below can be answered on the basis of information given so far? (Give the answer where possible.

(1) Give an expression for the speed at a point halfway between s_i and s_f:____________________.

(2) Give an expression for the average speed: ____________.

(3) Is a force acting on the block? ______________________.

(4) Is the block moving at a constant speed? _____________.

(1) You can't answer this one. The speed <u>at a point</u> is a complex concept which we have not covered as yet; you may understand it, but we have no way of deriving an expression for it from the information given.

(2) The average speed is the <u>distance covered</u> divided by the <u>time</u> required to cover it:

$$\frac{s_f - s_i}{t_f - t_i}.$$

(3) You can't tell from the information given.

(4) You can't tell from the information given.

For the last two answers: to determine whether a force is applied, one has to know whether or not the block has accelerated. There is no information about that: the block may have speeded up and/or slowed down between initial and final positions, or its speed may have been constant.

What further information would you need to have in order to determine whether the block accelerated?

You would have to know whether the average speed was changing; in other words, you would have to know at least one more value for both s and t.

Consider for a moment the symbolic value $(s_f - s_i)$. Rather than use two symbols with a minus sign, it is easier to have a symbol which means "the change in." The symbol used to represent change is the Greek letter "delta," which looks like this: Δ. Thus the change in position $(s_f - s_i)$ is given as follows: Δs.

How would you symbolize the elapsed time? ________________.

(Don't look until you write.)

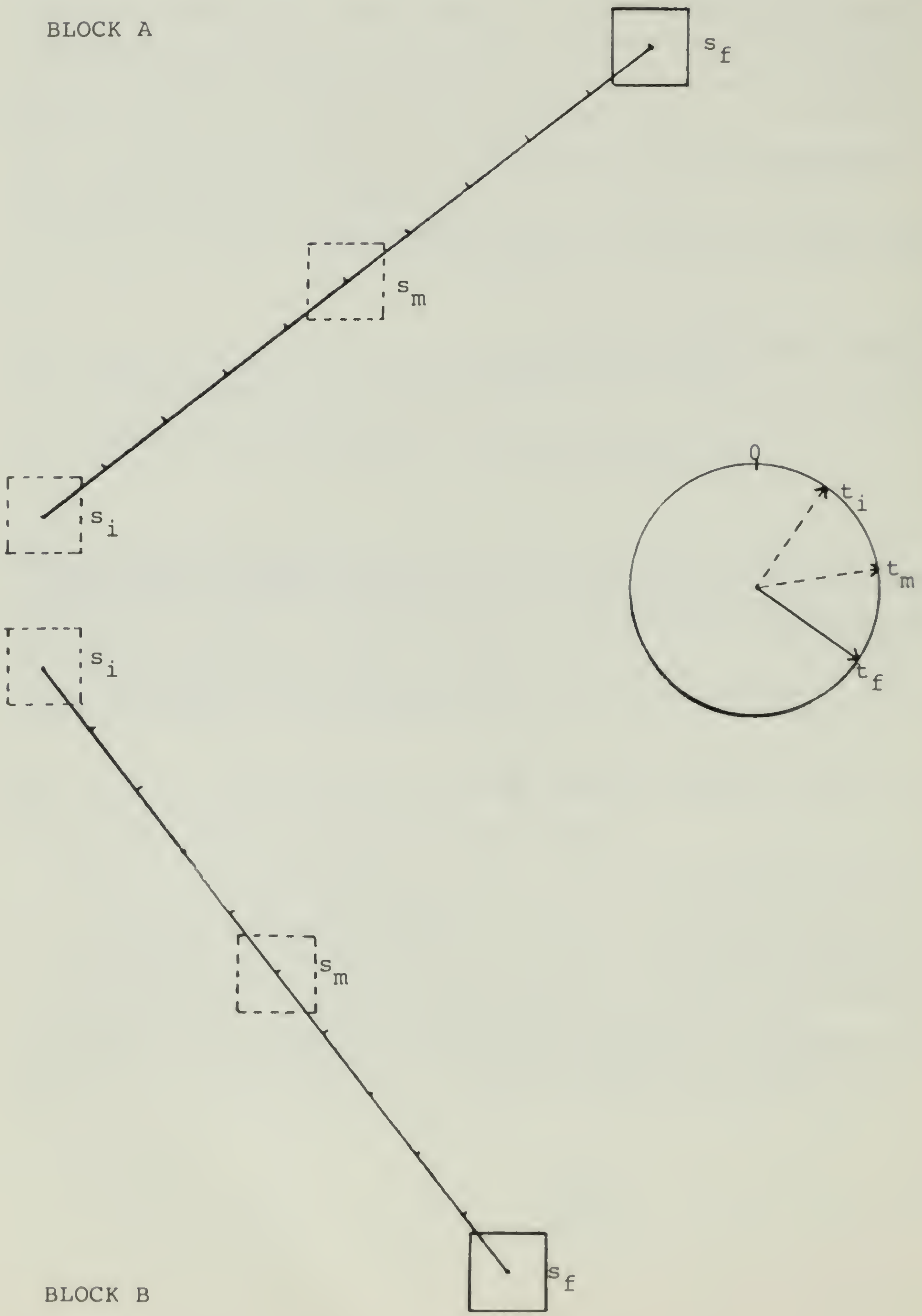
Elapsed time = Δt

Now write an expression for the average speed of the block using delta notation: _______________.

Average speed = $\frac{\Delta s}{\Delta t}$

On the following page is a diagram representing a view of two blocks seen from directly above. They are going in different directions, and the two paths of the blocks are marked off in units of measurement chosen arbitrarily for this problem. Examine the diagram, and then give answers to the questions which follow.

LOOKING VERTICALLY DOWN
ON TWO MOVING BLOCKS



How do the speeds of the two blocks compare?

Technically speaking, you can't answer until we specify whether we mean instantaneous speed at some point or the average speed from initial to final states. Assuming we mean average speeds, they are about equal (they are meant to be equal, but our drawing is only so accurate).

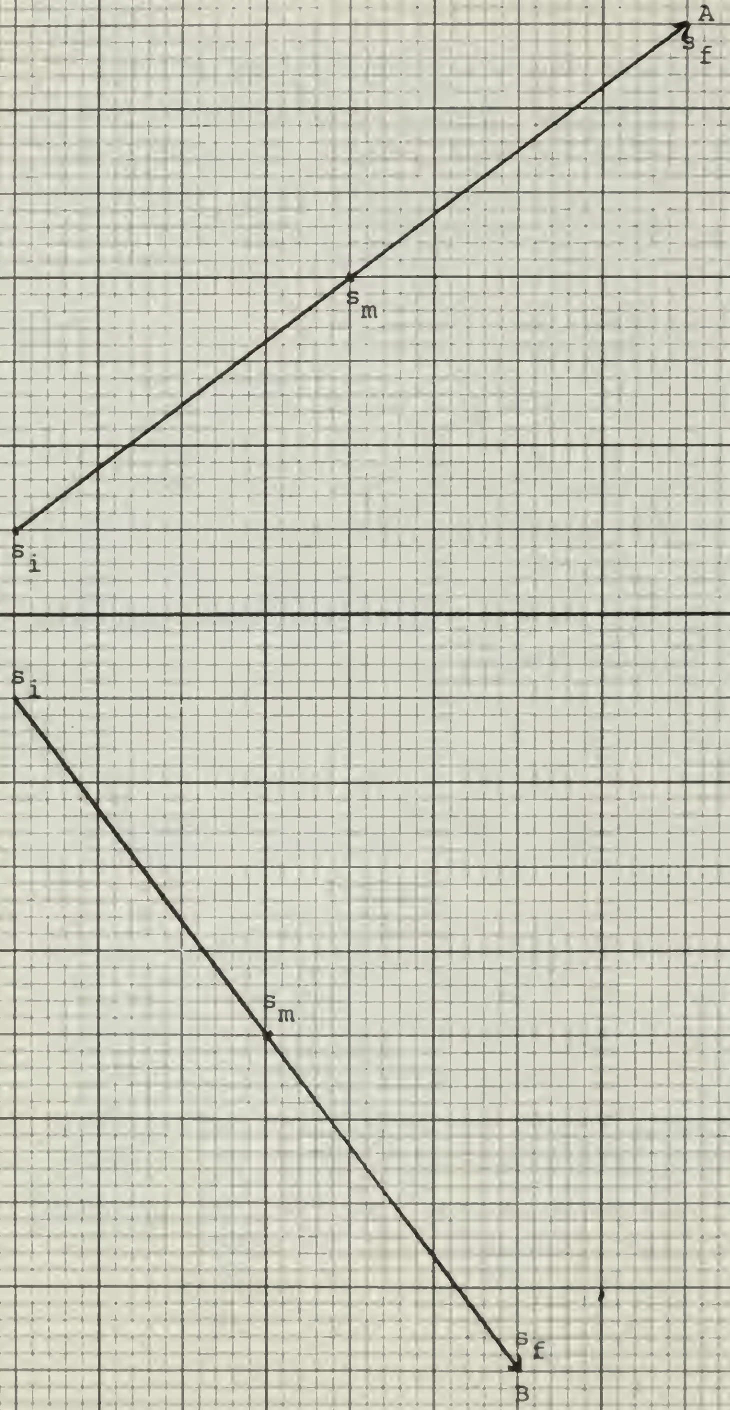
How do the velocities compare?

Assuming that we mean average velocities, you might be tempted to say that they are equal also, which is not true. Velocity is a vector quantity, involving both speed and direction; the speeds of the blocks are equal, but the velocities are not equal because __________________________.

The velocities are not equal because <u>the blocks are moving in different directions</u>.

We should like to be a little more precise, however, about the difference between the motions of A and B. It would be helpful to have some common frame of reference to measure their displacements. You have already learned how to use graphs to plot the coordinates of vectors; we can use the same method for the displacements of A and B. We consider each of them to be the vector sum of vertical and horizontal vectors drawn on suitably chosen coordinate axes. On the next page the displacements of A and B are plotted.

A GRAPH OF THE DISPLACEMENT VECTORS OF A AND B



You can draw vertical lines to the horizontal axis from the points s_i, s_m, and s_f on the vectors A and B. Now consider the initial, middle, and final positions as they are projected onto the horizontal axis. (The horizontal projection of a vector is called its "horizontal component," and the process of finding the components of a vector is called the "resolution of vectors.") Now, how does block B's displacement compare with A's when the displacements are projected on the horizontal axis; or, more precisely, how does the horizontal component of displacement B compare with the horizontal component of displacement A.

ANSWER: (In your own words.) Block B's displacement is <u>less than</u> A's when the displacements are projected on the horizontal axis; or, more precisely, the horizontal component of displacement B is <u>less than</u> the horizontal component of displacement A.

Now, what can we say about the components of the <u>velocities</u> A and B? (Remember: Δt is the same for each of the positions s_i, s_m, and s_f.)

HINT: Since $\bar{v} = \Delta s/\Delta t$, and Δt remains constant for each interval, $\bar{v}$ is directly proportional to Δs (symbolically: $\bar{v} \propto \Delta s$). Therefore, we can be confident that the horizontal ______________ of velocity A is (greater/less) than the ______________ ______________ of velocity B.

ANSWER: The horizontal <u>component</u> of velocity A is <u>greater</u> than the <u>horizontal</u> <u>component</u> of velocity B.

Remember that the speeds are equal, though the velocities are not. Now consider the vertical components of displacements A and B:

(1) How do these components compare?

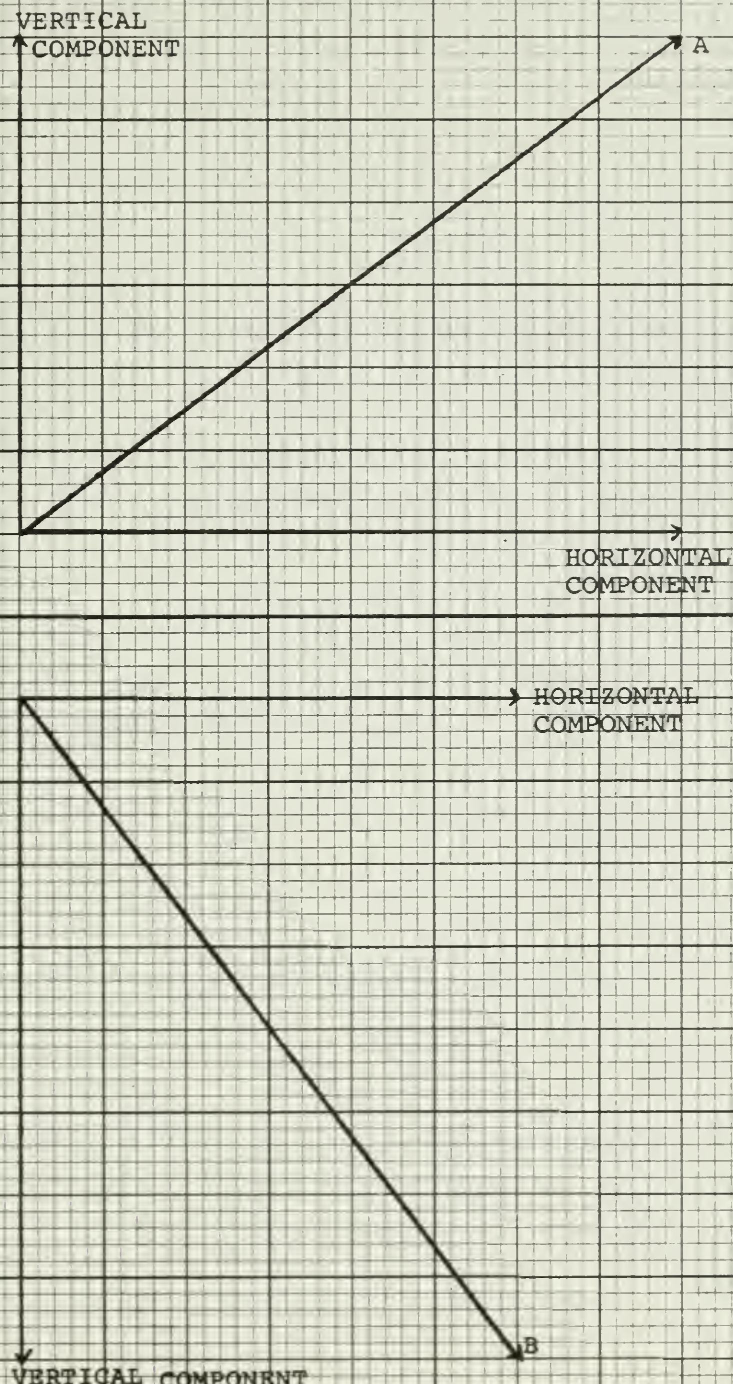
(2) How about the vertical components of velocity vectors based on the same information about A and B?

ANSWERS: (1) Displacement A's vertical component is less than displacement B's.

(2) We should confidently expect the vertical component of velocity A to be less than the vertical component of velocity B.

We can check our ideas about the velocity components by plotting the velocity "sum" vectors on a graph and resolving their components, just as we did with the displacement vectors. The velocity vectors will have the same directions as the displacement vectors; their magnitudes will be the magnitudes of the displacement vectors divided by the time elapsed during each interval ($\Delta s/\Delta t$). On the next page is a graph showing the velocity vectors. Observe that we could have chosen <u>any</u> convenient unit to measure the magnitude of $\overline{v}$, but since we chose the same scale for $\overline{v}$ as for Δs, and since $\Delta t = 1$, the velocity vectors <u>look</u> precisely the same as the displacement vectors, although they <u>mean</u> quite different things.

A GRAPH OF THE VELOCITY VECTORS A AND B



Now consider the symbols used above. The time elapsed between initial and middle is the same as that between middle and final; give three symbolic expressions for the time intervals between exposures:

$t_m - t_i$ $t_f - t_m$ Δt

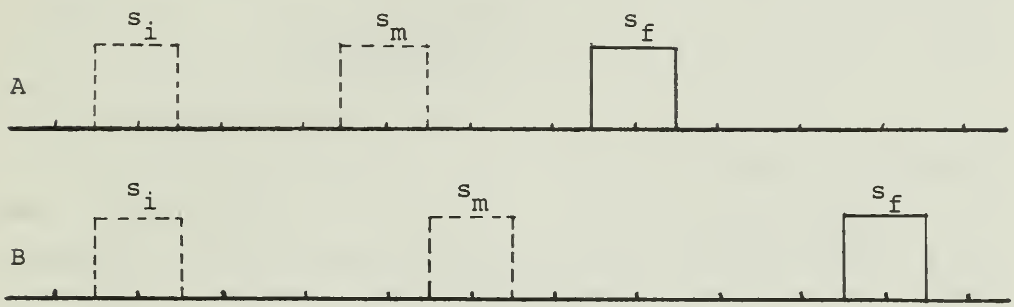
Now give the <u>simplest</u> way of expressing the change in <u>distance</u> between exposures for block A:

Δs $(\Delta s = s_m - s_i = s_f - s_m)$

Finally, $\bar{v}$ = __________.

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

Consider the two events pictured below. Two blocks are photographed simultaneously at intervals of one second. One block moves at a constant speed; the other accelerates. Which is which? (Answer only if you can on the basis of information provided here.)



A represents a block moving at constant speed; B represents a block which accelerates.

What can you deduce further about situations A or B?

HINT: What about the force acting in A? in B?

ANSWER: Since the speed in A is constant, there is no force acting. (Assuming, of course, that there is no friction; otherwise the force acting on the block A is just equal to the opposing force of friction.) In B, since the block accelerates, there is an unbalanced force acting.

Now consider for a moment the distance covered in B as compared with that in A. In B the block covers an additional ___________ (how many?) unit(s) of distance each second.

ANSWER: In each second the block in B covers <u>one</u> unit of distance more than it did the previous second. This speeding up, as you know, is called <u>acceleration</u>.

Suppose the block is thought of as gaining one additional unit of distance each second; then it has gained speed, has it not? How much average <u>speed</u> has it gained; in other words, how much speed does it gain each second?

HINT: It covers one additional unit of distance in one second, every second. Describe in your own words how much speed it gains each second.

ANSWER: It covers one additional unit of distance each second, so <u>it gains an average speed of one unit per second during each second</u>, or one unit per second per second. For instance, if the units were feet, then it would gain one foot a second, and it would gain this much every second, or one foot per second per second. This may sound a little complicated, but it should become clearer as you do some numerical problems related to it.

Suppose that the distance an object travels is 4 cm during the first second, 6 cm during the second second, and 8 cm during the third second. What is the average speed during the first second? during the second second? during the third second?

First second: 4 cm per second;
Second second: 6 cm per second;
Third second: 8 cm per second.

Now what is the change in average speed from second to second?

The change in average speed is 2 cm per second each second; that is, 2 cm per second is added to the velocity each second. The acceleration, therefore, is ____________________.

ANSWER: 2 cm/sec/sec, or 2 cm/sec^2

NOTE: $$\frac{2\,cm/sec}{sec} = \frac{2\,cm}{sec}\cdot\frac{1}{sec} = 2\frac{cm}{sec^2}$$

Now compute the acceleration of another object from the data given below.

DISTANCE COVERED DURING

FIRST SECOND	SECOND SECOND	THIRD SECOND
7 ft	19 ft	31 ft

HINT: What is the average speed during each second? What is the increase in speed during each second? What, then, is the acceleration?

ANSWER: Since the unit of time is conveniently one second, the distance covered is numerically equal to the average speed. The increase in speed during each second, therefore, is 12 ft/sec. ([19 ft/sec - 7 ft/sec] = [31 ft/ sec - 19 ft/sec] = 12 ft/sec.) The acceleration, therefore, is 12 ft/sec/sec, or 12 ft/ sec^2.

Compute the acceleration of another object.

TOTAL DISTANCE AT THE END OF		
THREE SECONDS	SIX SECONDS	NINE SECONDS
3 m	21 m	54 m

HINT: Your first step might be to set up a revised table like the following:

DISTANCE COVERED DURING		
FIRST 3 SECONDS	SECOND 3 SECONDS	THIRD 3 SECONDS
3 m	18 m	33 m
	(21 m [total distance] - 3 m [distance covered previously] = 18 m)	(54 m - 21 m = 33 m)

Now, what was the average speed during the first three seconds? the second three? the third three? What, then, was the acceleration?

ANSWER:

Average speed during the first three seconds: 1 m/sec.
Average speed during the second three seconds: 6 m/sec.
Average speed during the third three seconds: 11 m/sec.

The increase in average speed is 5 m/sec each 3 seconds, so the acceleration is $\frac{5}{3}$ m/sec^2.

These observations and results can be presented in tabular form. (Note that s_1 is the <u>first</u> observed position, s_2 the second, and so on.)

	Δs	Δt	$\Delta s/\Delta t$ or $\bar{v}$	$\Delta(\Delta s/\Delta t)$ or $\Delta\bar{v}$	$\frac{\Delta(\Delta s/\Delta t)}{\Delta t}$ or a
s_1 0m					
	3m	3sec	$1\frac{m}{sec}$		
s_2 3m				$5\frac{m}{sec}$	$\frac{5}{3}\ \frac{m}{sec^2}$
	18m	3sec	$6\frac{m}{sec}$		
s_3 21m				$5\frac{m}{sec}$	$\frac{5}{3}\ \frac{m}{sec^2}$
	33m	3sec	$11\frac{m}{sec}$		
s_4 54m					

Some of the symbols above may be new to you, although the concepts they represent should be more or less familiar. "$\bar{v}$" stands for average velocity; it is pronounced "v-bar" and is the algebraic equivalent of $\Delta s/\Delta t$. (Be sure that you don't confuse $\bar{v}$ with $\vec{v}$.) "a" stands for acceleration, the change in average velocity during a given time interval ($\Delta\bar{v}/\Delta t$).

Below is a table giving the positions of an object (s_1, s_2, etc.) and the times between positions (Δt). The differences between positions (Δs), average speeds ($\bar{v}$), <u>changes</u> in speed ($\Delta\bar{v}$), and acceleration (a) are to be filled in.

	Δs	Δt	$\Delta s/\Delta t$ or $\bar{v}$	$\Delta(\Delta s/\Delta t)$ or $\Delta\bar{v}$	$\frac{\Delta(\Delta s/\Delta t)}{\Delta t}$ or a
s_1 -3cm					
	(a)	1 sec	(d)		
s_2 4cm				(g)	(i)
	(b)	1 sec	(e)		
s_3 11cm				(h)	(j)
	(c)	1 sec	(f)		
s_4 18cm					

(a) 7cm

(b) 7cm

(c) 7cm

(d) $7\frac{cm}{sec}$

(e) $7\frac{cm}{sec}$

(f) $7\frac{cm}{sec}$

(g) 0

(h) 0

(i) 0

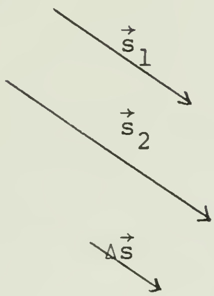
(j) 0

Now consider the data above as they describe the motion of an object; what can you say about this motion?

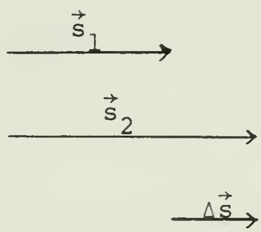
HINT: Is the speed constant? Is there any acceleration? Is there any force acting on the object?

ANSWER: The speed is constant; there is no acceleration; therefore, there is no net force acting on the object.

We have been observing an object which moved in a straight line:

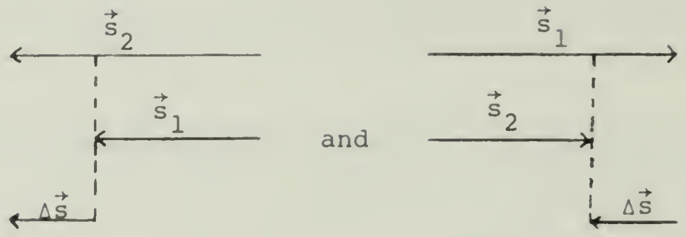


There is no objection to tilting our frame of reference so that the vectors are horizontal:

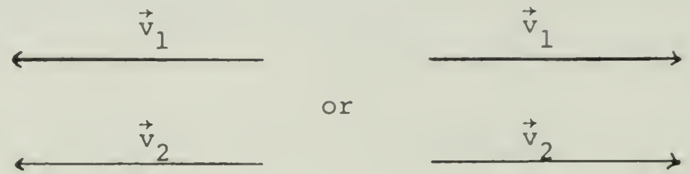


The vector Δs now points to the right. Draw different vectors $\vec{s}_1$ and $\vec{s}_2$ so that their difference $(\vec{s}_2 - \vec{s}_1)$ points to the left.

Two cases are possible:

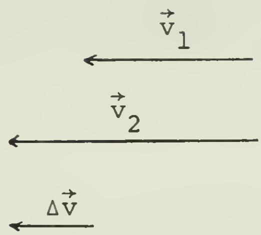


A similar situation occurs with the velocity vector. In straight-line motion, the average velocity can remain constant from interval to interval:

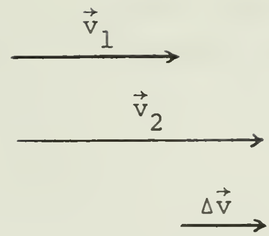


or it can change. But since the direction is fixed, only the magnitude (the speed) of the velocity vector can change. Draw the velocity vectors for the two cases where the speed <u>increases</u>:

ANSWERS:



and



Since there are only two possible directions in which these vectors can point, we can simplify computations considerably if we assign plus signs to vectors pointing in one direction, and minus signs to vectors pointing in the opposite direction. By convention, vectors pointing to the right (or up, if you use the vertical axis) are plus, and vectors pointing to the left (or down) are minus. Our computations are simplified because we can now use ordinary arithmetic on the magnitudes of our vectors. In our horizontal reference system, +2 cm is a displacement of 2 cm to the right, and -2 cm is a displacement of ______________________________.

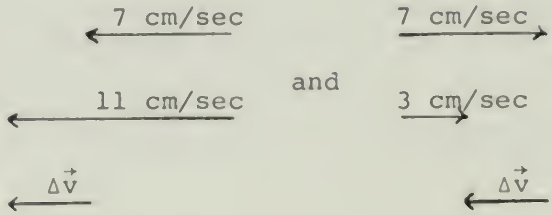
ANSWER: -2 cm is a displacement of 2 cm to the right.

Furthermore, +7 cm/sec is a velocity of ____________________, and -4 cm/sec^2 is an acceleration of ______________________.

ANSWERS: +7 cm/sec is a velocity of 7 cm/sec to the right, and -4 cm/sec^2 is an acceleration of 4 cm/sec^2 to the left.

A speed of 7 cm/sec may be a velocity of either +7 cm/sec or -7 cm/sec. Consider a body with an initial speed of 7 cm/sec. Draw a vector diagram for a change in velocity of -4 cm/sec. (Label the vector which is v.)

Again, two cases are possible:



In what follows, we shall always be dealing with vector displacement, vector velocity, and vector acceleration. To simplify the writing we shall omit the arrows over vector quantities. We shall also omit the plus sign; a number with no sign if front of it, if it is a vector quantity, is a vector pointing to the (right/left).

ANSWER: A vector quantity with no sign in front of it points to the right.

Now consider an object which moves according to the data recorded in the next table. (Fill in the blanks.)

		Δs	Δt	$\Delta s/\Delta t$ or $\bar{v}$	$\Delta(\Delta s/\Delta t)$ or $\Delta\bar{v}$	$\frac{\Delta(\Delta s/\Delta t)}{\Delta t}$ or a
s_1	108ft					
		(a)	1 min	(d)		
s_2	90ft				(g)	(i)
		(b)	1 min	(e)		
s_3	70ft				(h)	(j)
		(c)	1 min	(f)		
s_4	48ft					

ANSWER (Be sure to observe signs and units):

(a) -18ft (d) $-18\frac{\text{ft}}{\text{min}}$

(b) -20ft (e) $-20\frac{\text{ft}}{\text{min}}$

(c) -22ft (f) $-22\frac{\text{ft}}{\text{min}}$

(g) $-2\frac{\text{ft}}{\text{min}}$ (i) $-2\frac{\text{ft}}{\text{min}^2}$

(h) $-2\frac{\text{ft}}{\text{min}}$ (j) $-2\frac{\text{ft}}{\text{min}^2}$

What can you say about the motion of this object?

HINT: In addition to the information asked for previously, what can you say about the nature of the acceleration?

ANSWER: The speed changes; from what can be observed in the table, the change is uniform, that is, the acceleration is constant. Furthermore, the acceleration is such that the speed increases. The minus signs in the Δs column indicate that the object moves in a <u>negative</u> direction; Δs is a <u>vector</u>, and so has both magnitude and direction. Since velocity is also a vector quantity, it too has a direction (which depends on Δs), and so has a negative sign in this case. The change in velocity ($\Delta\bar{v}$), however, depends, in this case, not only on the direction of $\bar{v}$, but also on whether the difference between the average velocities is an increase or a decrease. Acceleration depends directly on $\Delta\bar{v}$, and so has the same sign.

Now consider the motion of an object as described in the table below:

	Δs	Δt	$\Delta s/\Delta t$ or $\bar{v}$	$\Delta(\Delta s/\Delta t)$ or $\Delta\bar{v}$	$\frac{\Delta(\Delta s/\Delta t)}{t}$ or a
s_1 0ft					
	(a)	5 sec	(d)		
s_2 35ft				(g)	(i)
	(b)	5 sec	(e)		
s_3 60ft				(h)	(i)
	(c)	5 sec	(f)		
s_4 75ft					

ANSWER:

(a) 35ft

(b) 25ft

(c) 15ft

(d) $7\frac{ft}{sec}$

(e) $5\frac{ft}{sec}$

(f) $3\frac{ft}{sec}$

(g) $-2\frac{ft}{sec}$

(h) $-2\frac{ft}{sec}$

(i,j) $-\frac{2}{5}\frac{ft}{sec^2}$ or $-.4\frac{ft}{sec^2}$

Describe the motion of this object:

HINT: Is it speeding up or slowing down?

ANSWER: The speed is changing, and the change seems to be uniform--that is, the acceleration is constant. In this case, however, the object is slowing down, so you might want to call this motion "deceleration." If an accelerating force acts in the same direction as the object's motion (regardless of whether that motion has a positive or a negative direction), then we say the object is accelerating; if the accelerating force acts in the opposite direction to that of the object's motion, then we usually say the motion is decelerating. To the physicist, however, both speeding up and slowing down are cases of acceleration, a vector quantity (with direction as well as magnitude).

Now consider this final problem:

	Δs	Δt	$\Delta s/\Delta t$ or $\bar{v}$	$\Delta(\Delta s/\Delta t)$ or $\Delta\bar{v}$	$\frac{\Delta(\Delta s/\Delta t)}{\Delta t}$ or a
s_1 150ft					
	(a)	5 min	(d)		
s_2 120ft				(g)	(i)
	(b)	5 min	(e)		
s_3 95ft				(h)	(j)
	(c)	5 min	(f)		
s_4 75ft					

(a) -30ft	(d) $-6\frac{ft}{min}$	(g) $1\frac{ft}{min}$	(i,j) $\frac{1}{5}\ \frac{ft}{min^2}$
(b) -25ft	(e) $-5\frac{ft}{min}$		or
(c) -20ft	(f) $-4\frac{ft}{min}$	(h) $1\frac{ft}{min}$	$.2\frac{ft}{min^2}$

In this problem, the direction of $\bar{v}$ is (positive/negative) [choose one]. This means that the object is moving horizontally from (right to left/left to right).

The direction of the acceleration vector is (positive/ negative); in other words, the accelerating force is acting in a direction that is (the same as/ opposite to) that of the body's velocity. Therefore, the body is speeding up/ slowing down).

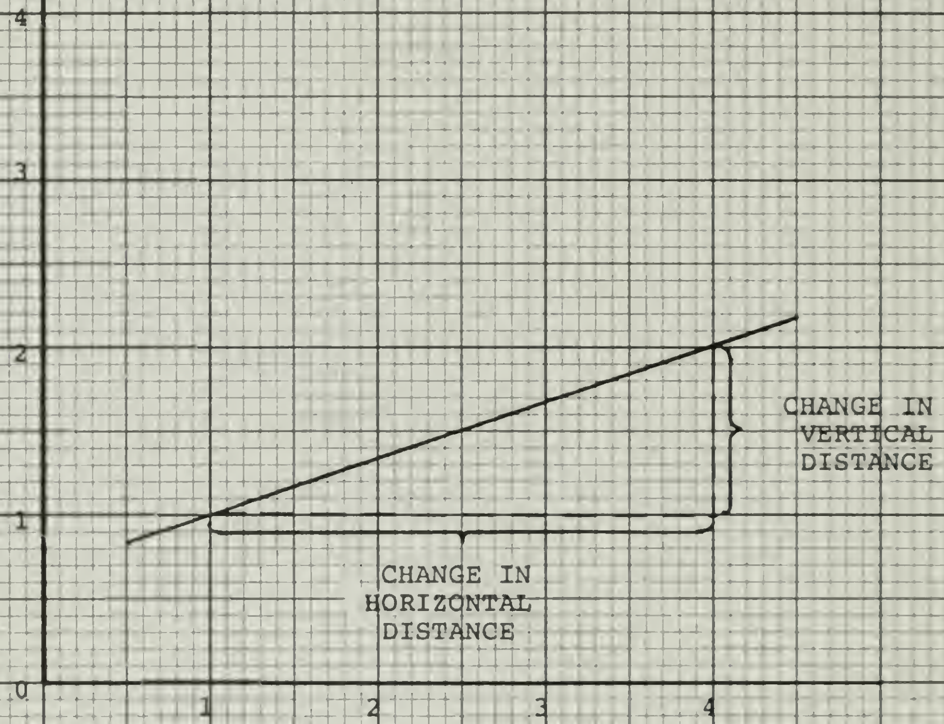
The direction of $\overline{v}$ is <u>negative</u>. In other words, the object is moving horizontally from <u>right to left</u>.

The direction of the acceleration vector is <u>positive</u>; this means that the accelerating force is acting in a direction that is <u>opposite to</u> that of the body's velocity. Therefore, the body is <u>slowing down</u>.

PART II

REVIEW

What is the slope of the line?

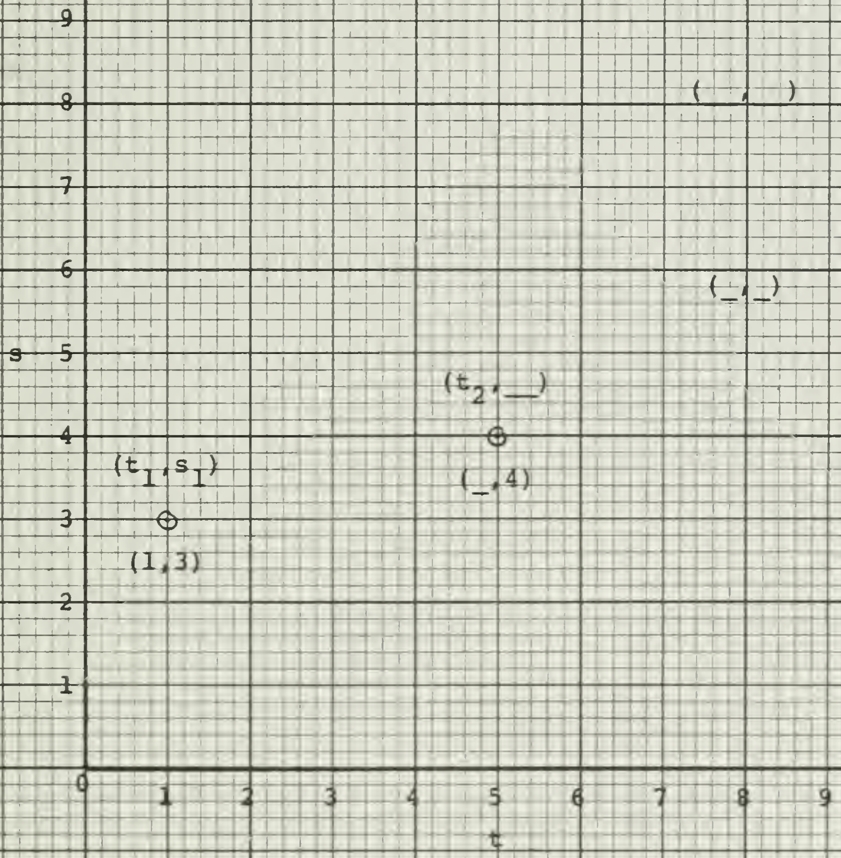


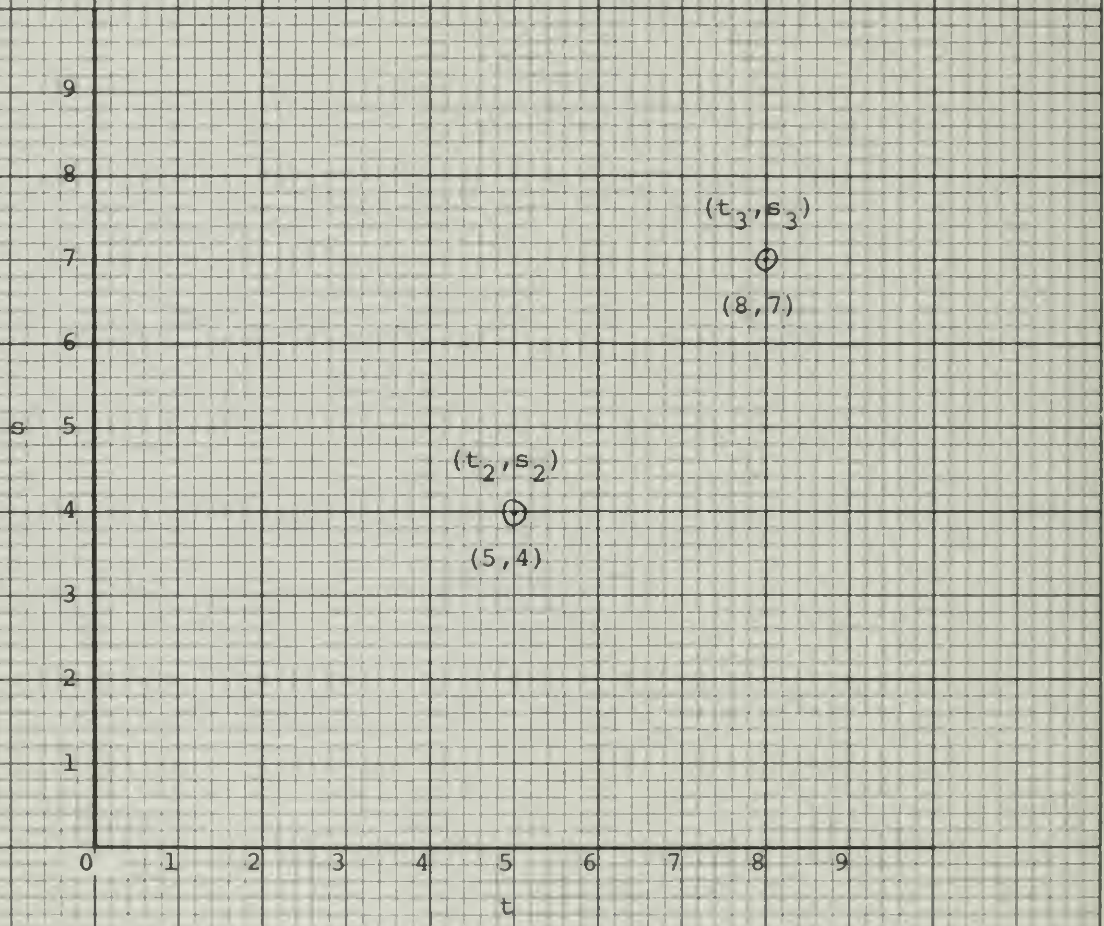
Its slope is $\frac{1}{3}$, that is, the change in vertical distance divided by the change in horizontal distance.

If this confused you, or if you have not seen it before, ask your teacher to explain it before you go on.

Fill in the missing symbols, numbers, and point, using the data in the table.

s	t
$s_1 = 3$	$t_1 = 1$
$s_2 = 4$	$t_2 = 5$
$s_3 = 7$	$t_3 = 8$



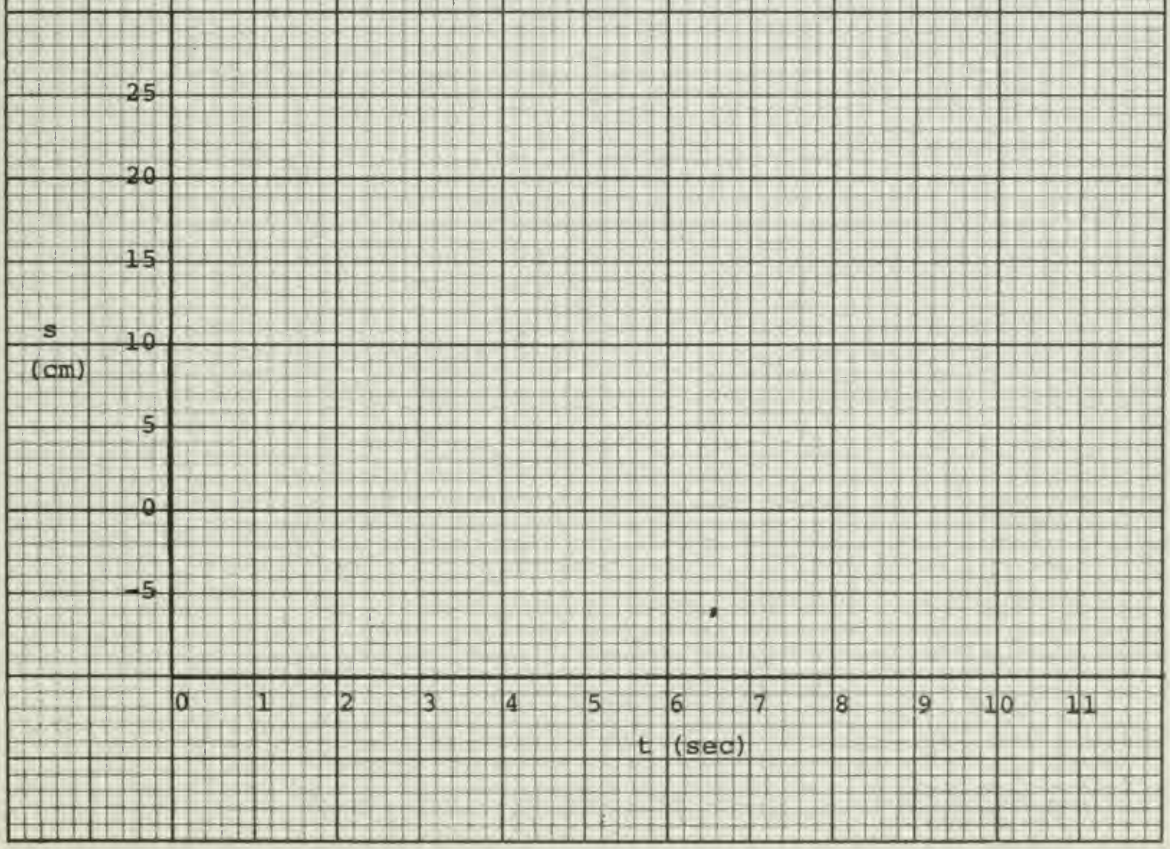


If you have difficulty here, see your teacher before going on.

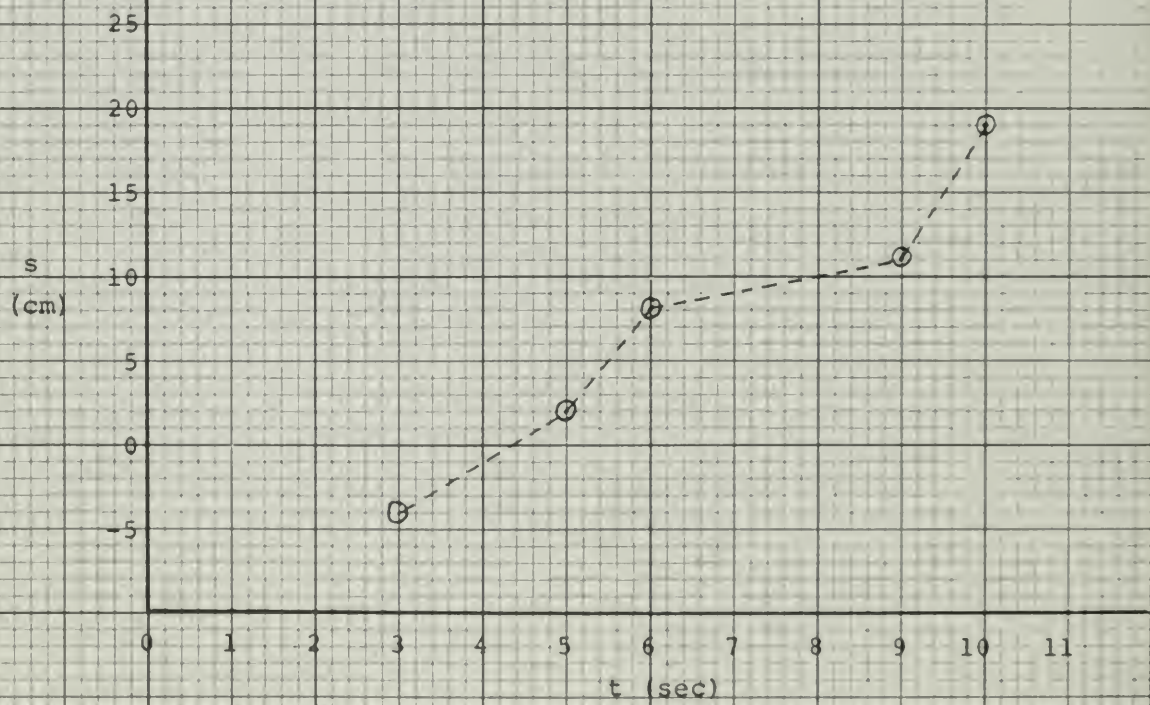
Here are s and t data for a moving object:

	cm		sec	Δs	Δt	Δs/Δt	Δ(Δs/Δt)
$s_1 =$	−4	$t_1 =$	3				
				___	___	___	
s_2	2	$t_2 =$	5				___
				___	___	___	
s_3	8	$t_3 =$	6				___
				___	___	___	
s_4	11	$t_4 =$	9				___
				___	___	___	
s_5	19	$t_5 =$	10				

Fill in the blanks, then plot the graph of s vs. t.



cm	sec	Δs	Δt	$\Delta s/\Delta t$	$\Delta(\Delta s/\Delta t)$
$s_1 = -4$	$t_1 = 3$				
		6 cm	2 sec	$3\frac{\text{cm}}{\text{sec}}$	
$s_2 = 2$	$t_2 = 5$				$3\frac{\text{cm}}{\text{sec}}$
		6 cm	1 sec	$6\frac{\text{cm}}{\text{sec}}$	
$s_3 = 8$	$t_3 = 6$				$-5\frac{\text{cm}}{\text{sec}}$ (Watch the sign!)
		3 cm	3 sec	$1\frac{\text{cm}}{\text{sec}}$	
$s_4 = 11$	$t_4 = 9$				$7\frac{\text{cm}}{\text{sec}}$
		8 cm	1 sec	$8\frac{\text{cm}}{\text{sec}}$	
$s_5 = 19$	$t_5 = 10$				

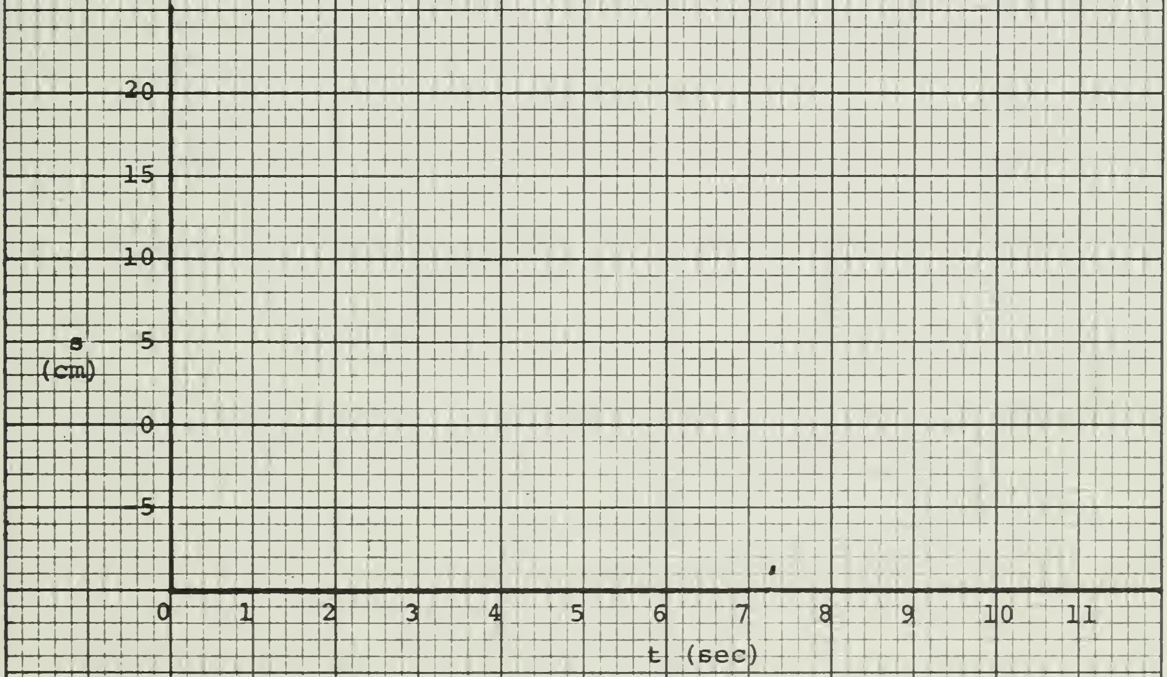


Now observe the slopes of the line segments joining points. Do they relate to any of the data in the table?

ANSWER: The slopes for each interval are the same as $\Delta s/\Delta t$, and so represent $\bar{v}$ for that interval.

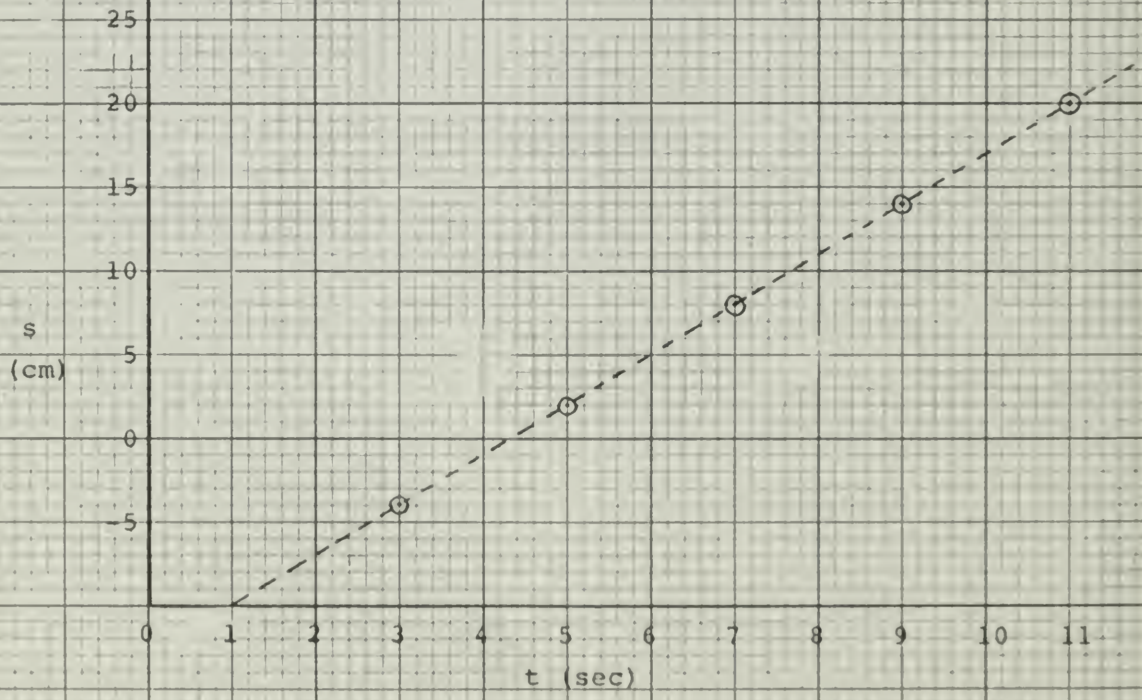
Now begin with some of the same data, but change the rest so that the graph of s vs. t will be one straight line; plot the curve to check yourself.

cm	sec	Δs	Δt	$\Delta s/\Delta t$	$\Delta \bar{v}$	$\Delta \bar{v}/\Delta t$
$s_1 =$ -4	$t_1 =$ 3					
		___	___	___		
$s_2 =$ 2	$t_2 =$ 5				___	___
		___	___	___		
$s_3 =$ ___	$t_3 =$ 7				___	___
		___	___	___		
$s_4 =$ ___	$t_4 =$ ___				___	___
		___	___	___		
$s_5 =$ ___	$t_5 =$ ___					

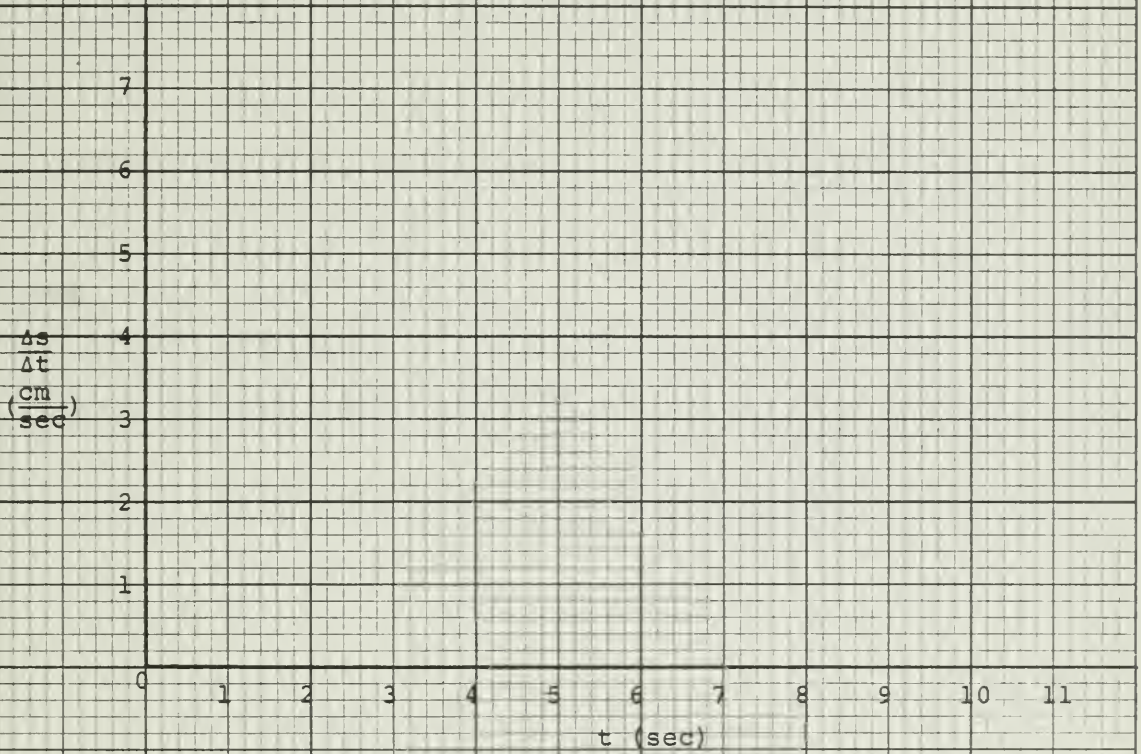


(HINT: The slopes ($\Delta s/\Delta t$) of each interval should be equal.)

	cm	sec	Δs	Δt	$\Delta s/\Delta t$	$\Delta \overline{v}$	$\Delta \overline{v}/\Delta t$
	$s_1 = -4$	$t_1 = 3$					
			6cm	2sec	$3\frac{cm}{sec}$		
	$s_2 = 2$	$t_2 = 5$				0	0
			6cm	2sec	$3\frac{cm}{sec}$		
	$s_3 = 8$	$t_3 = 7$				0	0
			6cm	2sec	$3\frac{cm}{sec}$		
	$s_4 = 14$	$t_4 = 9$				0	0
			6cm	2sec	$3\frac{cm}{sec}$		
	$s_5 = 20$	$t_5 = 11$					

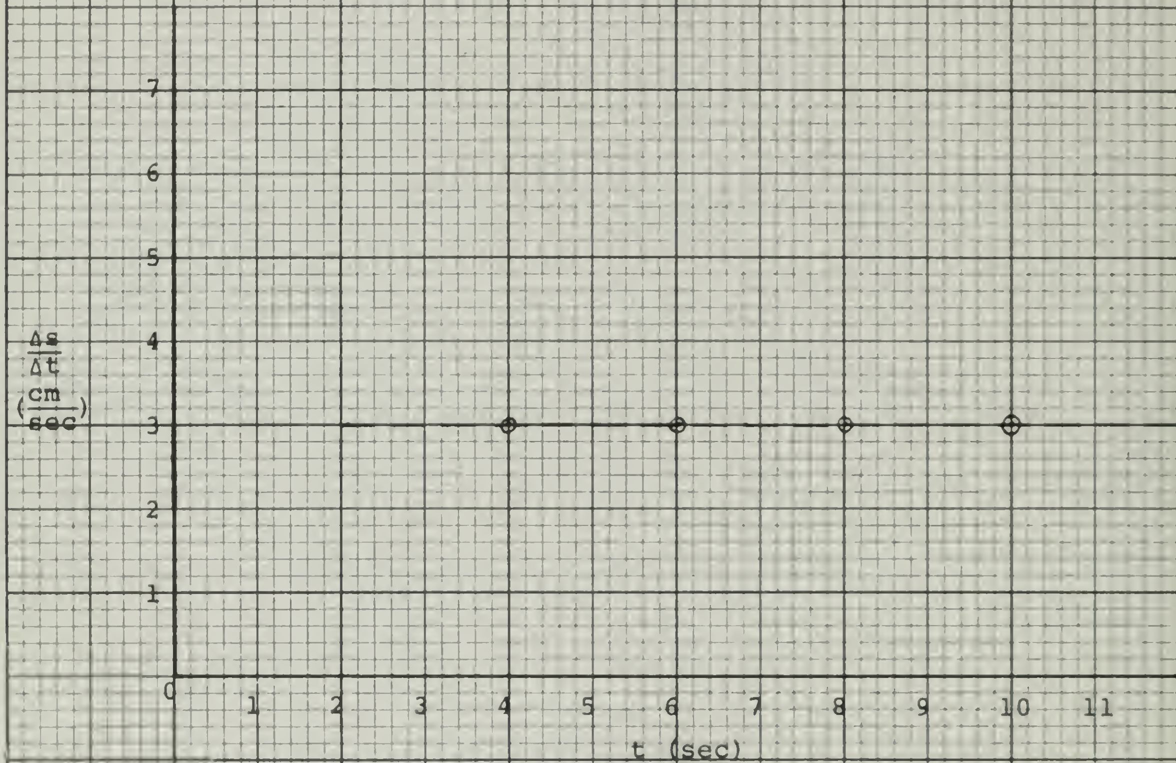


Now plot Δs/Δt against t for the same data:



HINT: One way to solve the difficulty you might have in choosing a value for t is to plot Δs/Δt against the <u>midpoint</u> of each time interval.

$$\text{Slope} = \frac{\Delta(\Delta s/\Delta t)}{\Delta t} = 0.$$



What does the slope represent? (In respect to values in the table)

What did the slope of the first graph (s vs. t) represent?

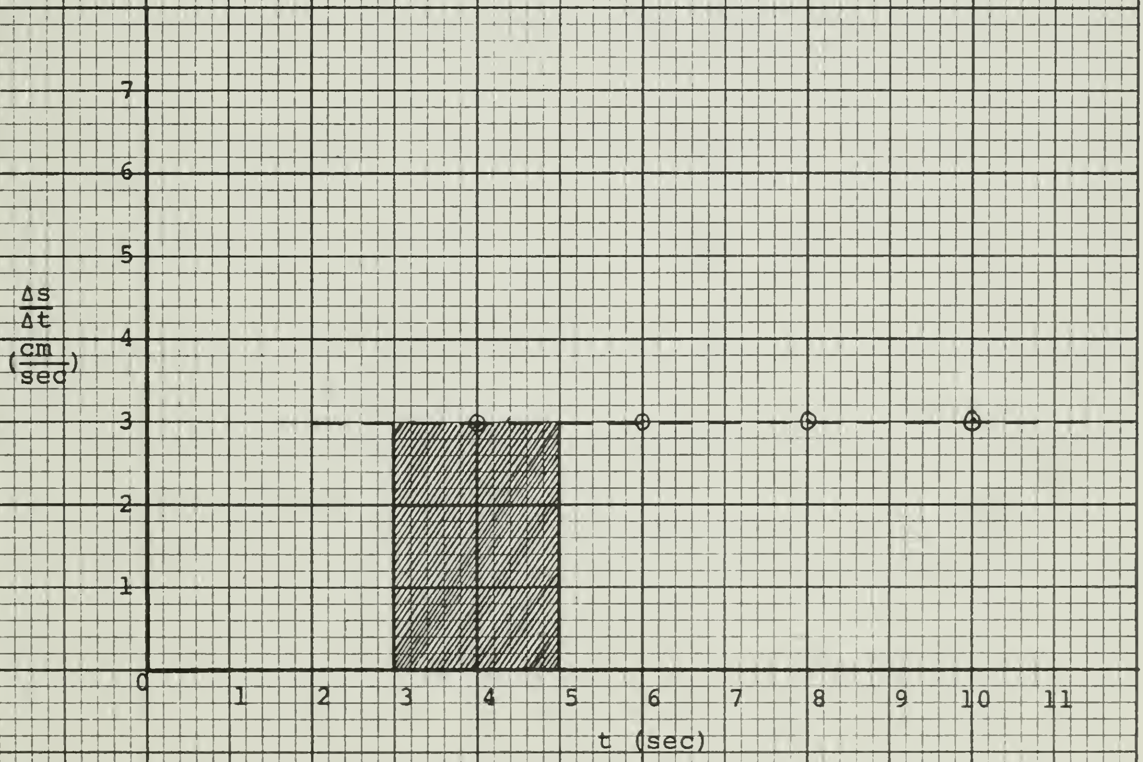
Shade in the area under the curve between t=3 and t=5; what does this area represent, in respect to values in the table?

The slope of the $\Delta s/\Delta t$ vs. t graph, which is 0, represents acceleration. Although it may seem so at first, the slope does not represent $\Delta\bar{v}$; the slope of $\Delta s/\Delta t$ vs. t is equal to $\frac{\Delta(\Delta s/\Delta t)}{\Delta t}$.

(If you have difficulty seeing this, you might consult with your teacher, although the distinction may become clearer after you have worked on the next set of data.)

The slope of the s vs. t graph represented average speed, $\Delta s/\Delta t$, which is constantly 3.

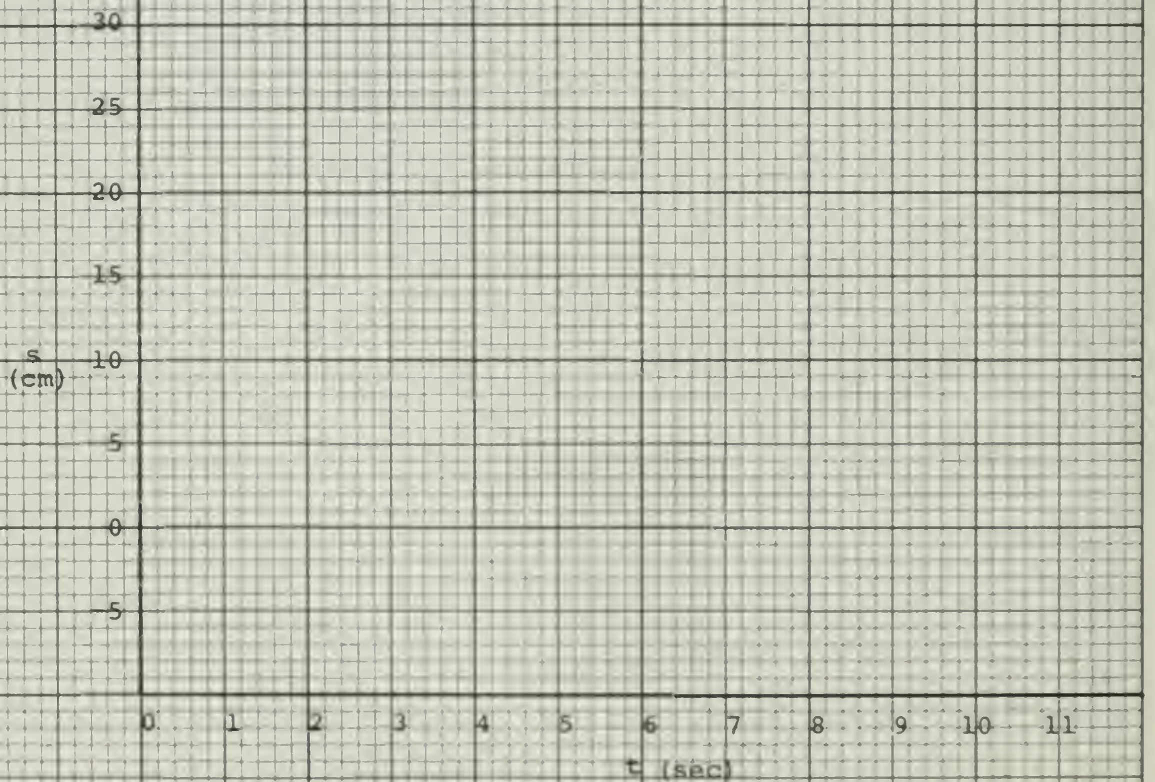
The area under the $\Delta s/\Delta t$ vs. t curve from t=3 to t=5 is 6 centimeters (3cm/sec·2sec = 6cm); this corresponds to Δs (the change in distance) for this time interval.



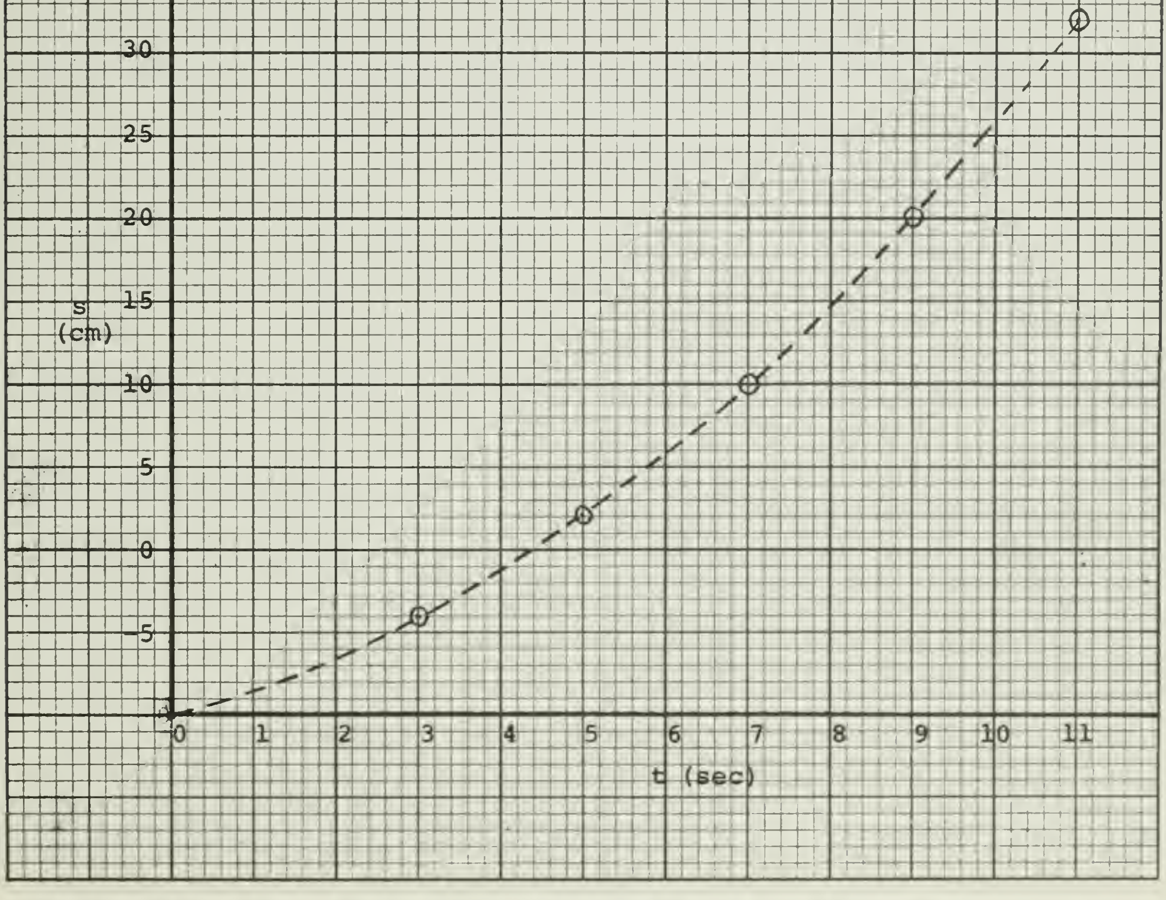
Now begin with some of the same data, but make the table describe an object which has a constant acceleration:

cm	sec	Δs	Δt	$\Delta s/\Delta t$	$\Delta \bar{v}$	a
$s_1 = -4$	$t_1 = 3$					
		6 cm	2 sec	$3\frac{cm}{sec}$		
$s_2 = 2$	$t_2 = 5$				$1\frac{cm}{sec}$	$\frac{1}{2}\frac{cm}{sec^2}$
		___	2 sec	$4\frac{cm}{sec}$		
$s_3 =$ ___	$t_3 = 7$				___	$\frac{1}{2}\frac{cm}{sec^2}$
		___	2 sec	___		
$s_4 =$ ___	$t_4 =$ ___				___	$\frac{1}{2}\frac{cm}{sec^2}$
		___	2 sec			
$s_5 =$ ___	$t_5 =$ ___					

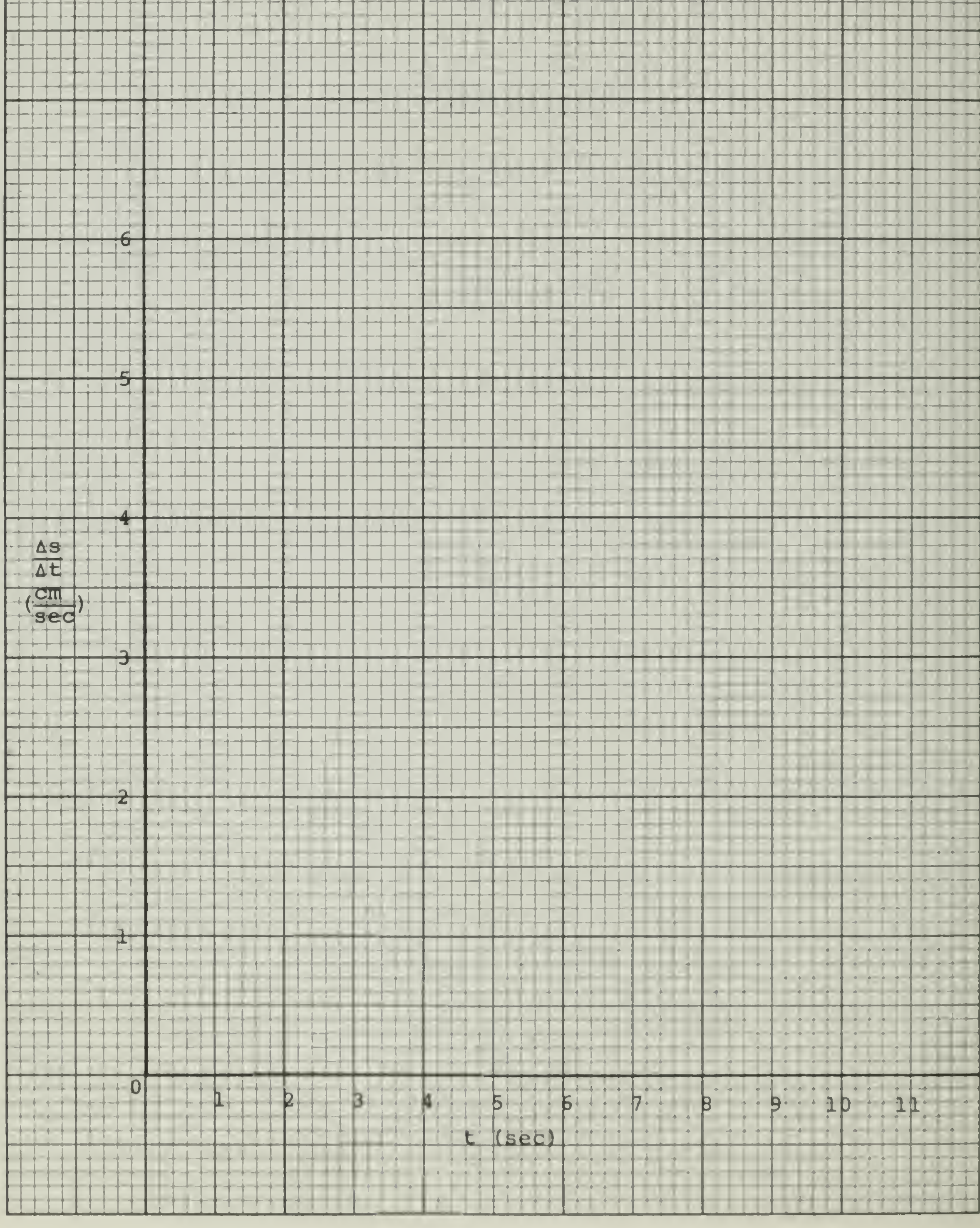
Plot the curve starting from t = 0:

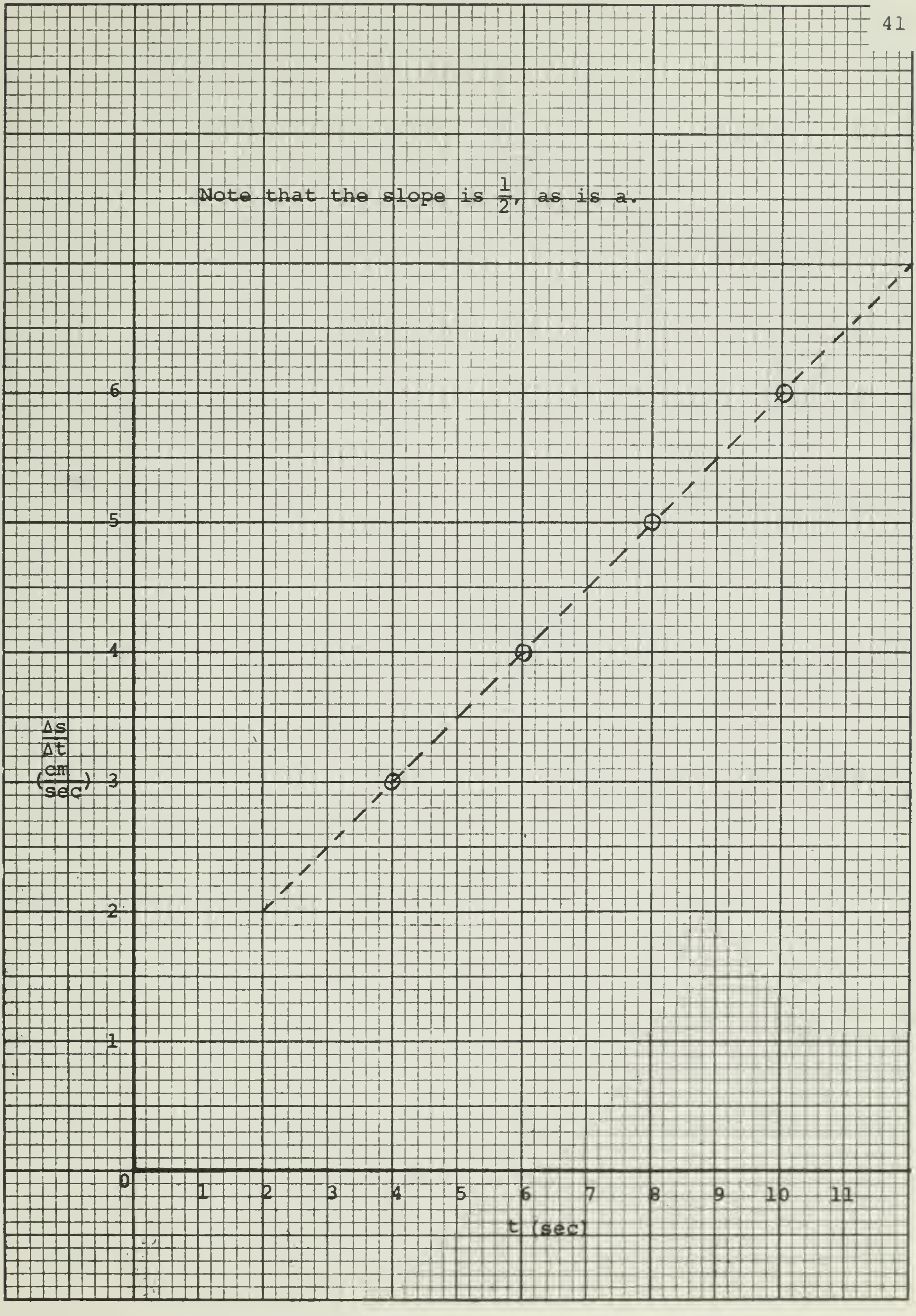


cm	sec	Δs	Δt	$\Delta s/\Delta t$	$\Delta\bar{v}$	a
$s_1 = -4$	$t_1 = 3$					
		6 cm	2 sec	$3\frac{cm}{sec}$		
$s_2 = 2$	$t_2 = 5$				$1\frac{cm}{sec}$	$\frac{1}{2}\ \frac{cm}{sec^2}$
		8 cm	2 sec	$4\frac{cm}{sec}$		
$s_3 = 10$	$t_3 = 7$				$1\frac{cm}{sec}$	$\frac{1}{2}\ \frac{cm}{sec^2}$
		10 cm	2 sec	$5\frac{cm}{sec}$		
$s_4 = 20$	$t_4 = 9$				$1\frac{cm}{sec}$	$\frac{1}{2}\ \frac{cm}{sec^2}$
		12 cm	2 sec	$6\frac{cm}{sec}$		
$s_5 = 32$	$t_5 = 11$					

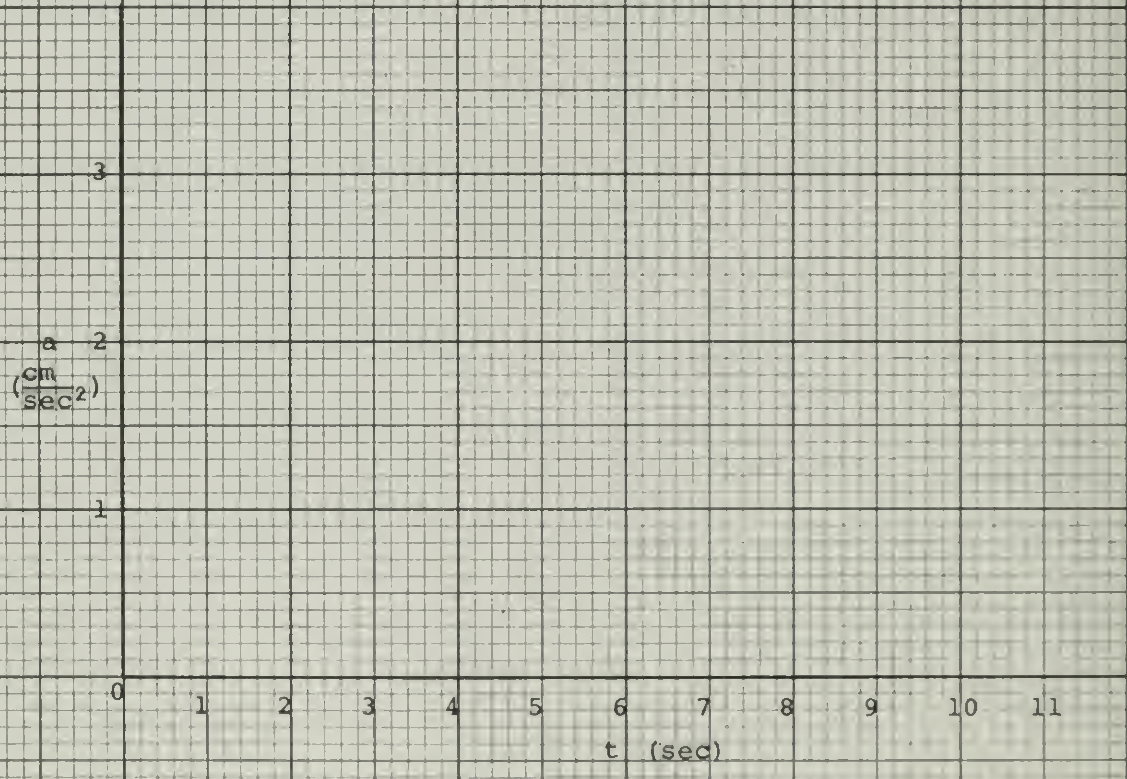


Now plot the graph of Δs/Δt vs. t for the same data:



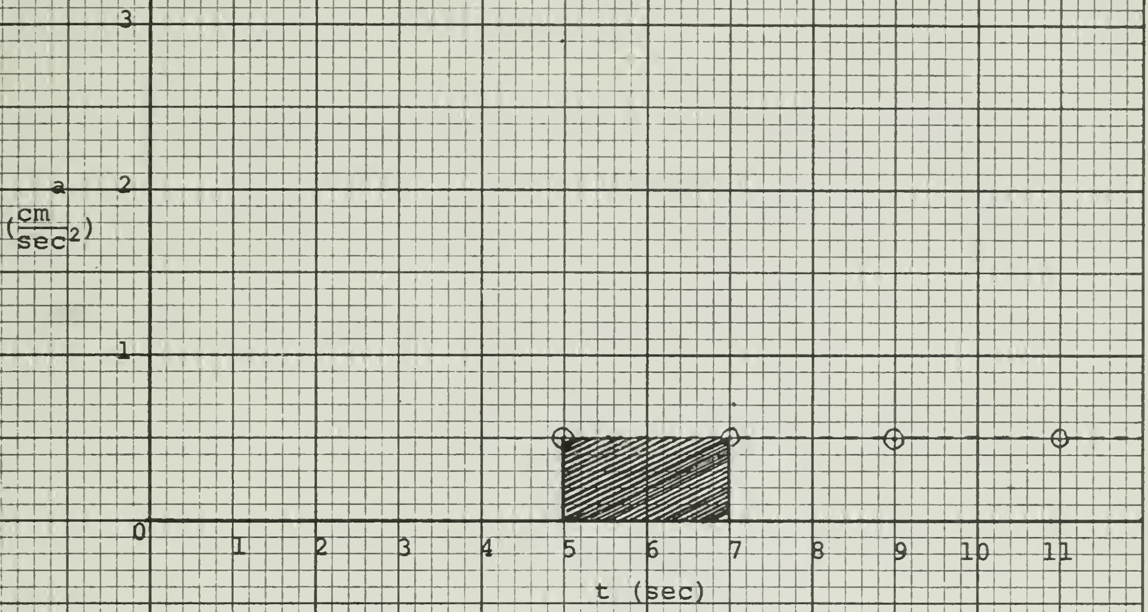

Note that the slope is $\frac{1}{2}$, as is a.
$\frac{\Delta s}{\Delta t}$ $(\frac{cm}{sec})$
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
t (sec)

What curve do you obtain when you plot a against t?



Shade in the area under the curve from t=5 to t=7.
What quantity in the data does this area represent?
(Remember: This is a plot of a $(\Delta\vec{v}/\Delta t)$ against t.)

You should have a horizontal straight line:



The area under the curve from t=5 to t=7 is 1; it represents $\Delta(\Delta s/\Delta t)$, or $\Delta\bar{v}$, the change in average velocity for the entire interval from t=5 to t=7.

The change in average velocity, then, is the product of the acceleration, a, and which of the following?

(1) Δs
(2) $\Delta s/\Delta t$
(3) Δt
(4) $\Delta(\Delta s/\Delta t)$

ANSWER: (3) Δt

Symbolically, if v_0 is the velocity at the beginning of the time interval Δt, and a is the constant acceleration during Δt, the final velocity, v_f, at the end of the interval is:

$$v_f = v_0 + \_\_\_\_\_\_\_\_\_\_.$$

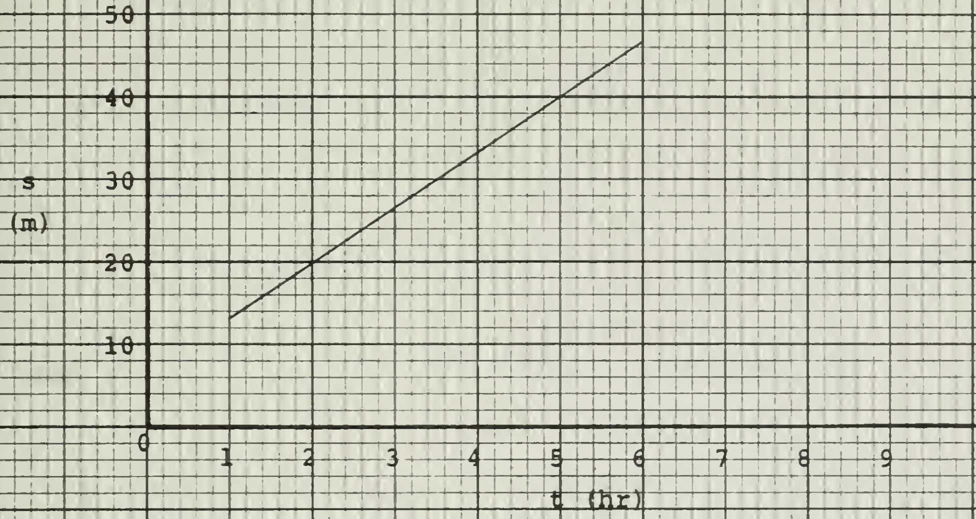
$$v_f = v_0 + a\Delta t.$$

The area under a <u>$\bar{v}$ against t</u> curve represents ______________.

The area under an <u>a against t</u> curve represents ____________.

The area under a $\bar{v}$ against t curve represents <u>distance covered</u>.

The area under an a against t curve represents <u>change in average speed</u>.



What can you tell about the motion of the object graphed above?

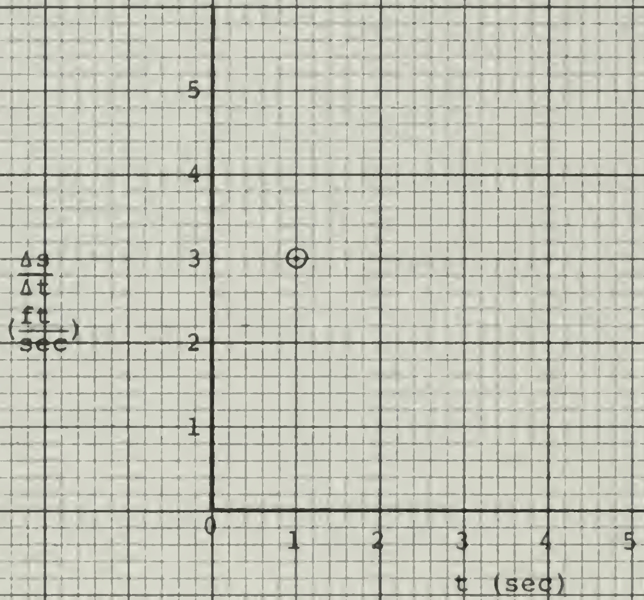
ANSWER: The speed is constant; s/ t is roughly $\frac{20}{3}\frac{\text{m}}{\text{hr}}$, or $6\frac{2}{3}\frac{\text{m}}{\text{hr}}$. The acceleration is zero.

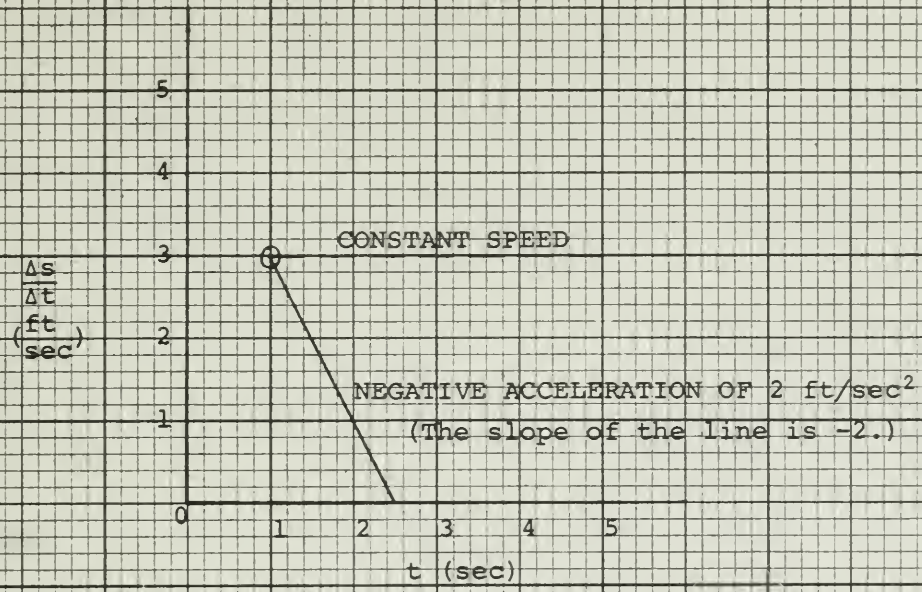
One point is given on the graph below.

Plot two lines, one dotted, the other continuous.

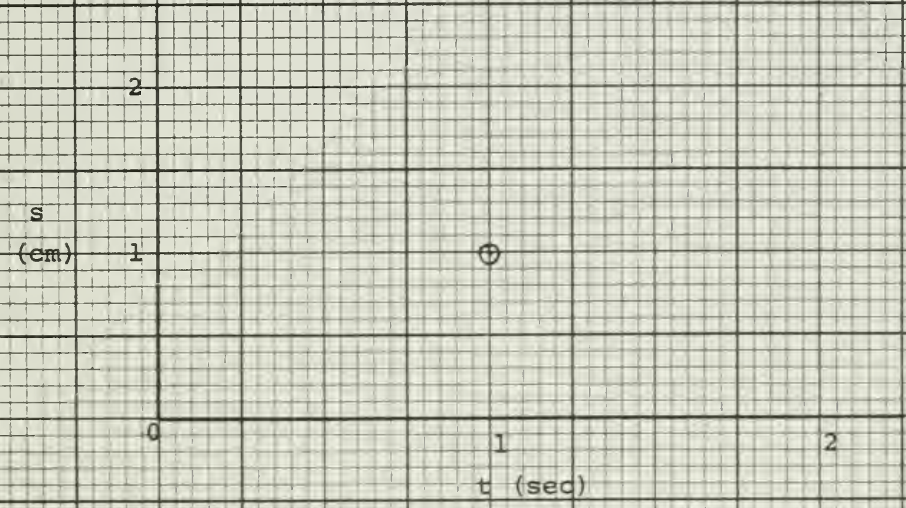
(1) Let the dotted line show constant speed.

(2) Let the continuous line show a decreasing acceleration of 2 ft/sec^2.





Imagine an object which is at rest at $t_0 = 0$ and $s_0 = 0$. Suppose you know that its position at $t_1 = 1\text{sec}$ is $s_1 = 1\text{cm}$.



Can you, on the basis of <u>only</u> the above information, give any of the following?

(a) average speed

(b) acceleration

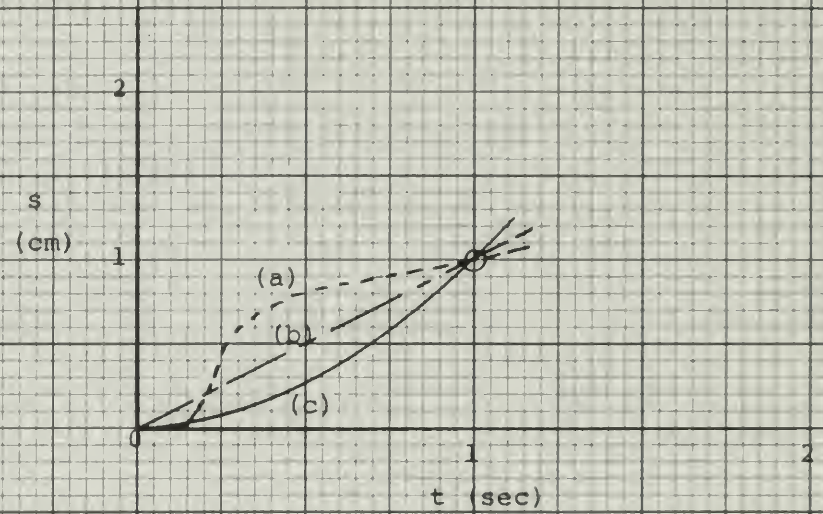
(c) speed at $t_1 = 1\text{sec}$

(d) the graph from $t_0 = 0$ to $t_1 = 1$

(a) Average speed = $\Delta s/\Delta t = \frac{1cm}{1sec} = 1\frac{cm}{sec}$

(b) One cannot determine the acceleration.

(c) Nor its speed at t = 1sec.



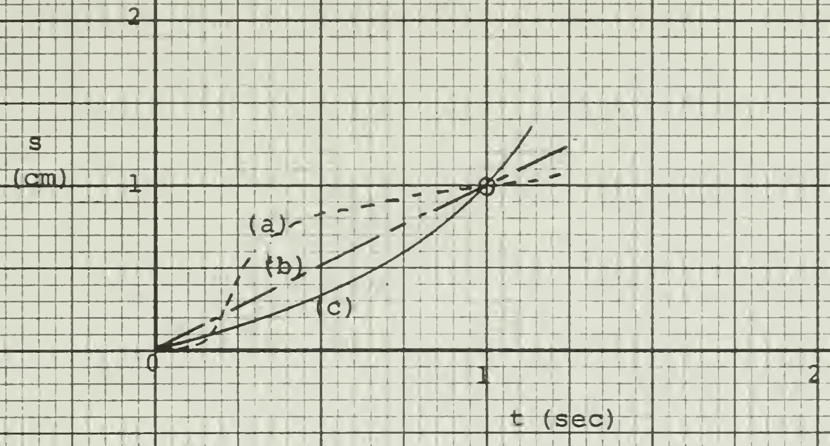
(d) The graph could be any of a number of lines; two of the ones shown are possible, one not.
Discuss the physical meaning of these graphs. (Which one is impossible?)

(1)

(2)

(3)

HINT: Remember, slope = velocity.

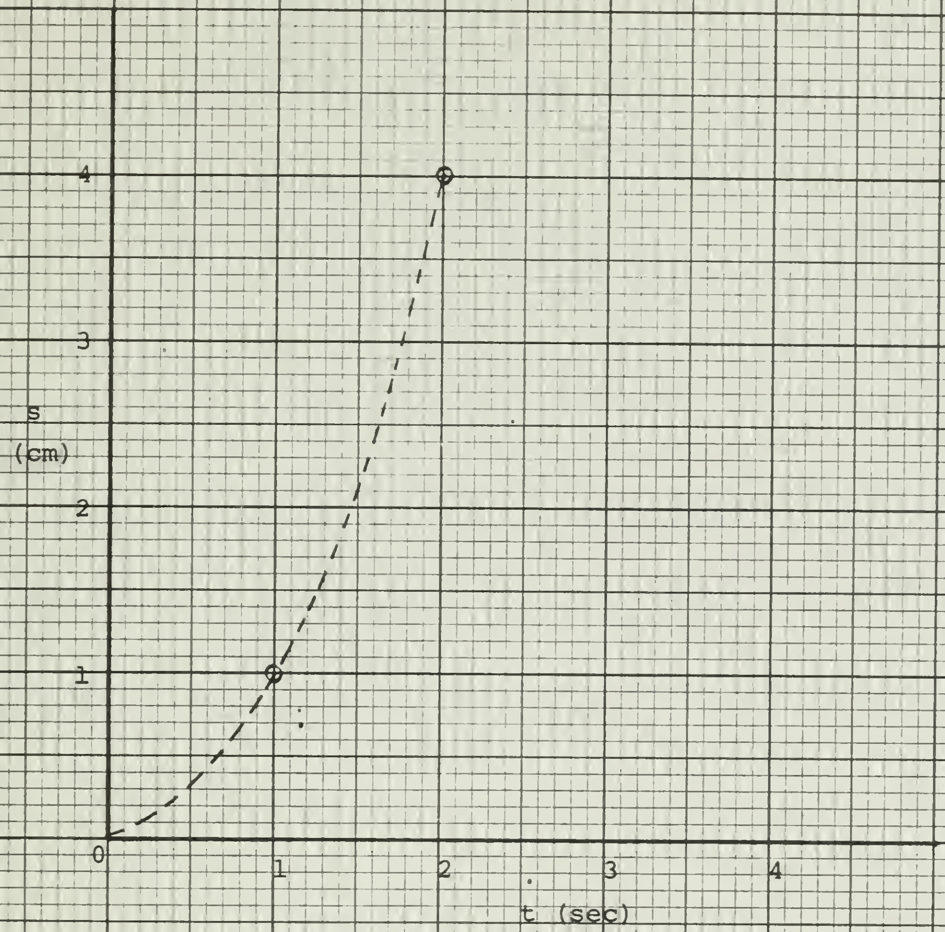


(a) The object gains speed very rapidly, then slows down (it goes half of the way in about .25 sec).

(b) At face value, this implies that the object maintains a constant velocity, which is impossible if its velocity at $t_0=0$ is 0.

(c) This curve shows the object speeding up gradually; it is close to a constant acceleration.

PART III



If we were to have a situation in the laboratory such that a constant force applied to an object initially at rest moved it to s=1 at t=1, then at t=2 the block would be in the position shown. Can you calculate its acceleration?

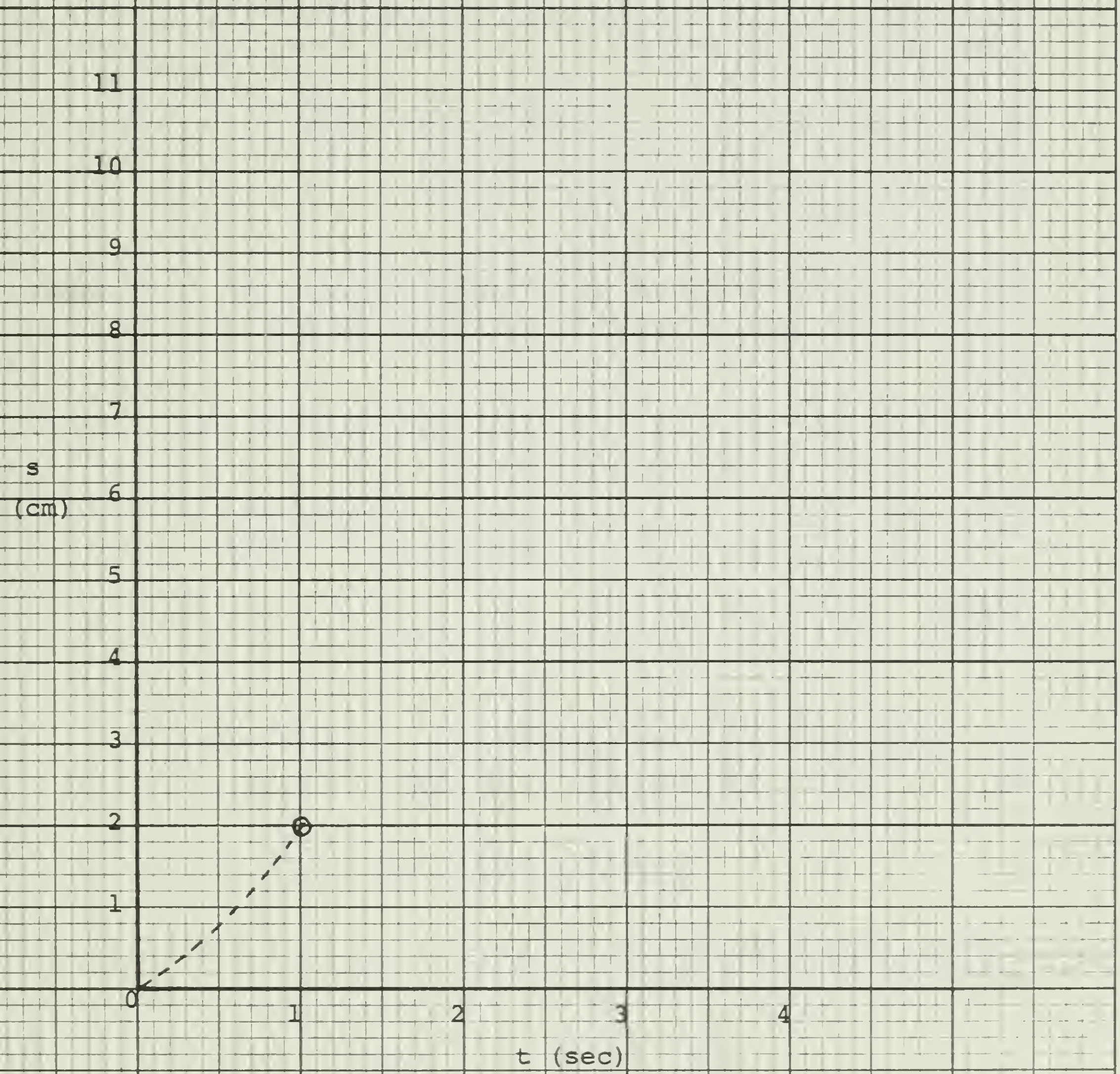
HINT: What is $\Delta s/\Delta t$ for the first second?
What is it for the next second?

$$\left.\begin{array}{l}\text{First second: } \dfrac{\Delta s}{\Delta t} = 1\dfrac{\text{cm}}{\text{sec}} \\[2ex] \text{Second second: } \dfrac{\Delta s}{\Delta t} = 3\dfrac{\text{cm}}{\text{sec}}\end{array}\right\} \Delta\left(\frac{\Delta s}{\Delta t}\right) = 2\frac{\text{cm}}{\text{sec}}, \text{ so } \frac{\Delta(\Delta s/\Delta t)}{\Delta t} =$$

$$a = 2\frac{\text{cm}}{\text{sec}^2}$$

It seems easy enough, then, to calculate the acceleration when we have s and t data for two or more intervals. But what happens if we have information about only one interval?

Suppose you observed an object which started at rest and was accelerated by a constant force from $t_0 = 0$ to $t_1 = 1$; s_0 also = 0, and $s_1 = 2$cm.



Could you, on the basis of only this information, determine the magnitude of the acceleration? Could you tell where the object would be at $t_2 = 2$sec? ($s_2 = ?$)

You might be able to puzzle out (or guess) the acceleration, or you might very well not; in either case, you would probably find it a long and complicated process, involving a great deal of untidy algebraic juggling. We'd like to help you over some of the rough spots, if you'll bear with us for a little. What we are after is an equation, or set of equations, for determining (1) constant acceleration over a given interval, and (2) an object's position at the end of successive intervals. Somewhere along the way we should also be happy to pick up some information about the velocity at various odd moments.

We have already developed one equation (containing a as one of its terms) for the final velocity at the end of an interval.

$$v_f = \underline{\hspace{6cm}}.$$

ANSWER: (1) $v_f = v_0 + a\Delta t$

Now, given v_0 and v_f, what would you do if you wanted to discover the average velocity over the interval?

ANSWER: Divide the sum of the initial (v_0) and final (v_f) velocities by 2. (Elementary, my dear Watson.)

$$(2) \quad \bar{v} = \frac{v_0 + v_f}{2}.$$

Now substitute the right side of equation (1) for v_f in equation (2). (You may not be able to see, yet, why we are doing all this, but keep in mind that we are trying to develop a formula which will give us the value of a when we know only one value for s and t; in other words, we should like to develop an equation containing only the terms s, t, and a.) Now, what do you get when you substitute the value of v_f from (1) in (2)?

ANSWER: (3) $\bar{v} = \dfrac{v_0 + (v_0 + a\Delta t)}{2} = \dfrac{2v_0 + a\Delta t}{2} = v_0 + \frac{1}{2}a\Delta t.$

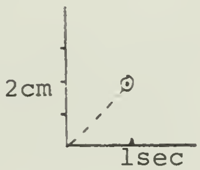
We're getting warm! Let's try to get our friend Δs into the ring:

$$\frac{\Delta s}{\Delta t} = \bar{v} = v_0 + \frac{1}{2}a\Delta t,$$

So: $\Delta s =$ ____________________.

ANSWER: (4) $\Delta s = \bar{v}\Delta t = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$

Now can you figure out the value of a?



HINT: You can clean up the general formula a bit by getting rid of one of the terms:

$$(4)\quad \Delta s = v_0\Delta t + \frac{1}{2}a(\Delta t)^2;$$

If $t_0 = 0$, then $\Delta t = t$, so $\Delta s = v_0 t + \frac{1}{2}at^2$.

But even more important, what about v_0?

ANSWER: When the object starts from rest, $v_0 = 0$, so $\Delta s = \frac{1}{2}at^2$. (The term with v_0 drops out, giving us an equation with only the terms s, t, and a.)

Now you should be able to find a with little difficulty:

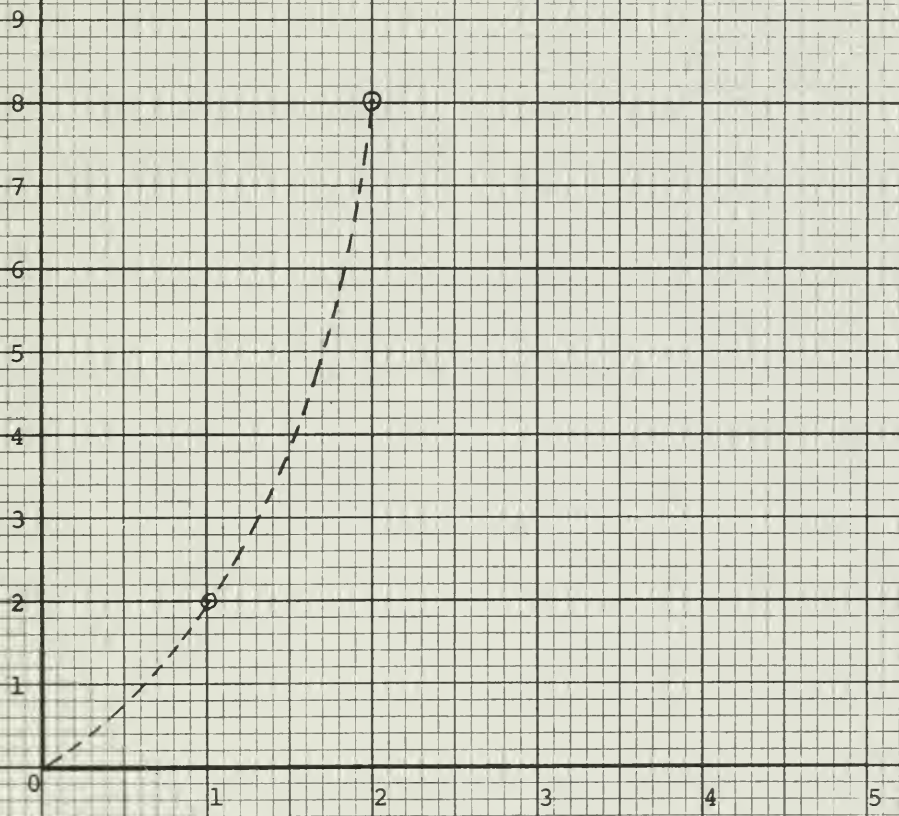
To find a:

$$\frac{1}{2}at^2 = \Delta s$$
$$at^2 = 2\Delta s$$
$$a = \frac{2\Delta s}{t^2} = \frac{2\cdot 2\,\text{cm}}{1\,\text{sec}^2} = 4\frac{\text{cm}}{\text{sec}^2}$$

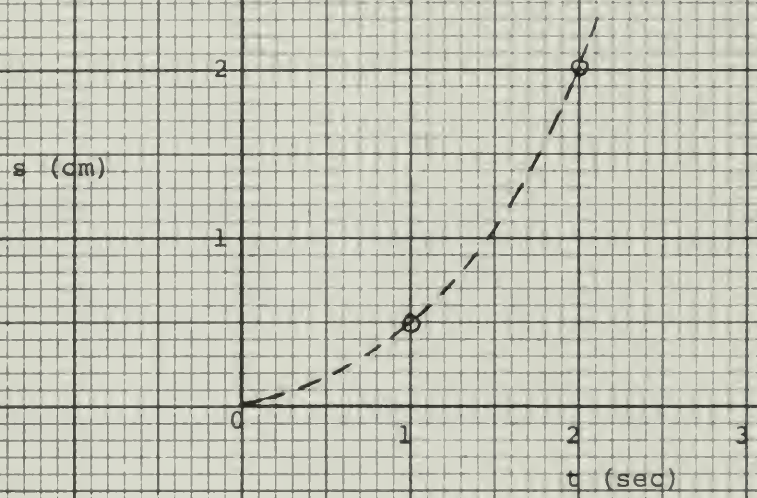
Now what about the position of the object at $t_2 = 2$, if the acceleration remains constant?

ANSWER: $\Delta s = \frac{1}{2}at^2$ (A very useful companion, this fellow.)

$$\Delta s = \frac{1}{2}\cdot 4\frac{cm}{sec^2}\cdot 4sec^2 = 8cm.$$



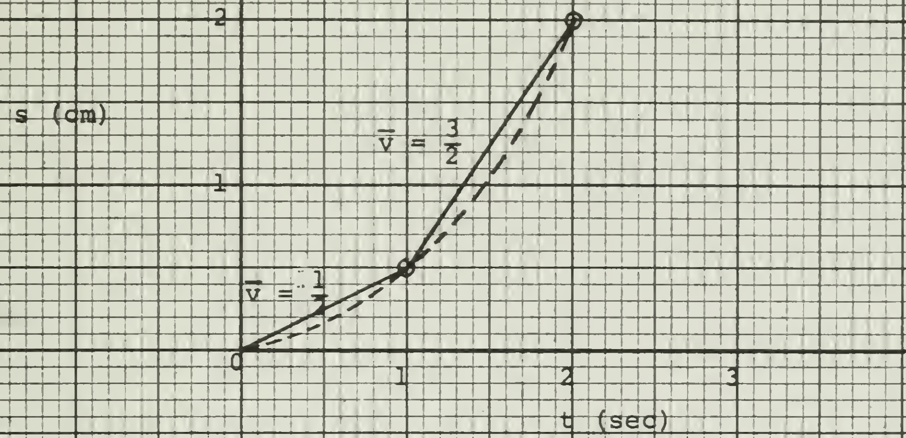
Try to remember the general formula: (4) $\Delta s = v_0t + \frac{1}{2}at^2$, and his helpful offspring: $\Delta s = \frac{1}{2}at^2$; but even more importantly, try to recall how you derived these formulae.



Assume that the object described above starts at rest and is accelerated by a constant force

What is the acceleration?

Do the data confirm the equation $s = v_0 t + \frac{1}{2}at^2$?



$\Delta\bar{v} = 1\frac{cm}{sec}$

so $a = 1\frac{cm}{sec^2}$

$v_0 = 0$

so $s = v_0t + \frac{1}{2}at^2 = \frac{1}{2} \cdot 1\frac{cm}{sec^2} \cdot t^2$

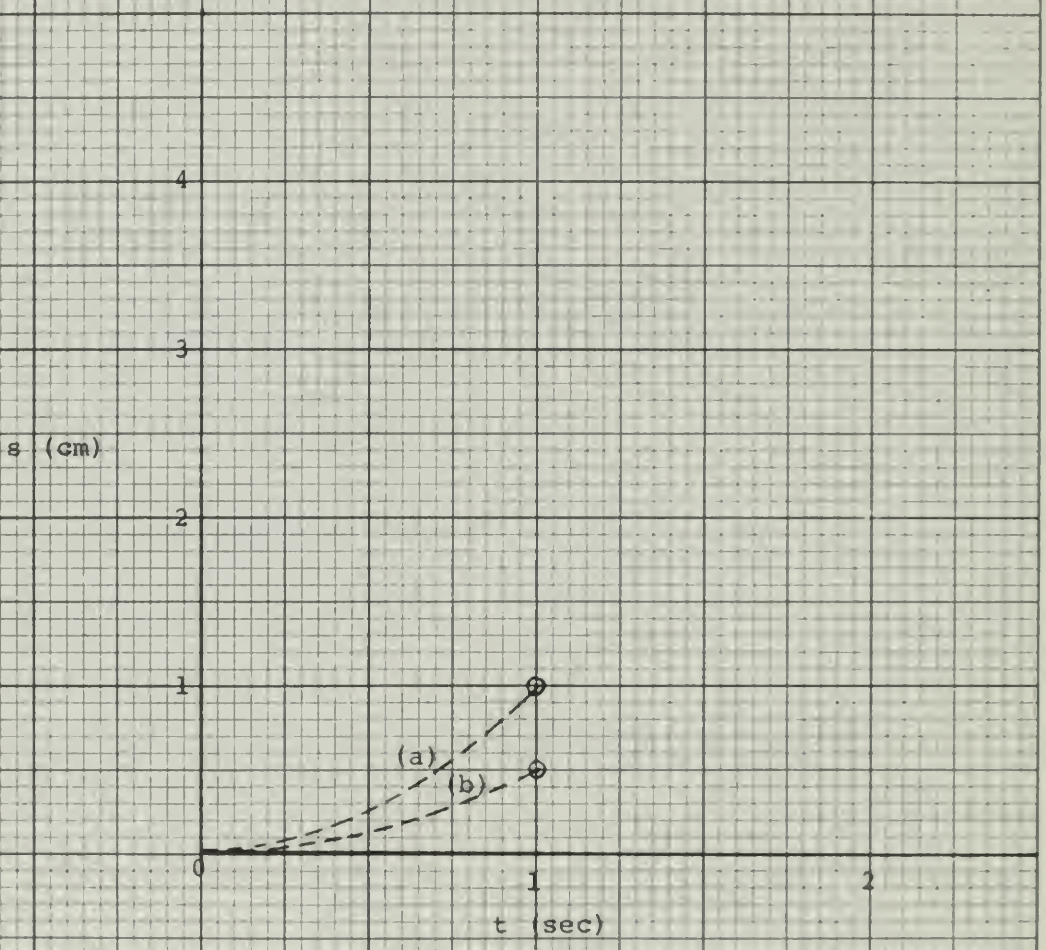
For $t = 1$:

$s = \frac{1}{2} \cdot 1\frac{cm}{sec^2} \cdot 1sec^2 = \frac{1}{2}cm$ ✓.

For $t = 2$:

$s = \frac{1}{2} \cdot 1\frac{cm}{sec^2} \cdot 4sec^2 = 2cm$ ✓.

The two situations analyzed previously are partially graphed below. Remember that the motion is uniformly accelerated and that the objects start from rest.



Without looking back, compute what would happen in each case if the acceleration remained constant to t = 2.

HINT: Use the distance equation previously developed.

HINT: The equation is $\Delta s = v_0 t + \frac{1}{2}at^2$.

But $v_0 = 0$, so $\Delta s = \frac{1}{2}at^2$,

from which you can compute a.

For (a): When t = 1 sec; s = 1 cm; then, a = ?

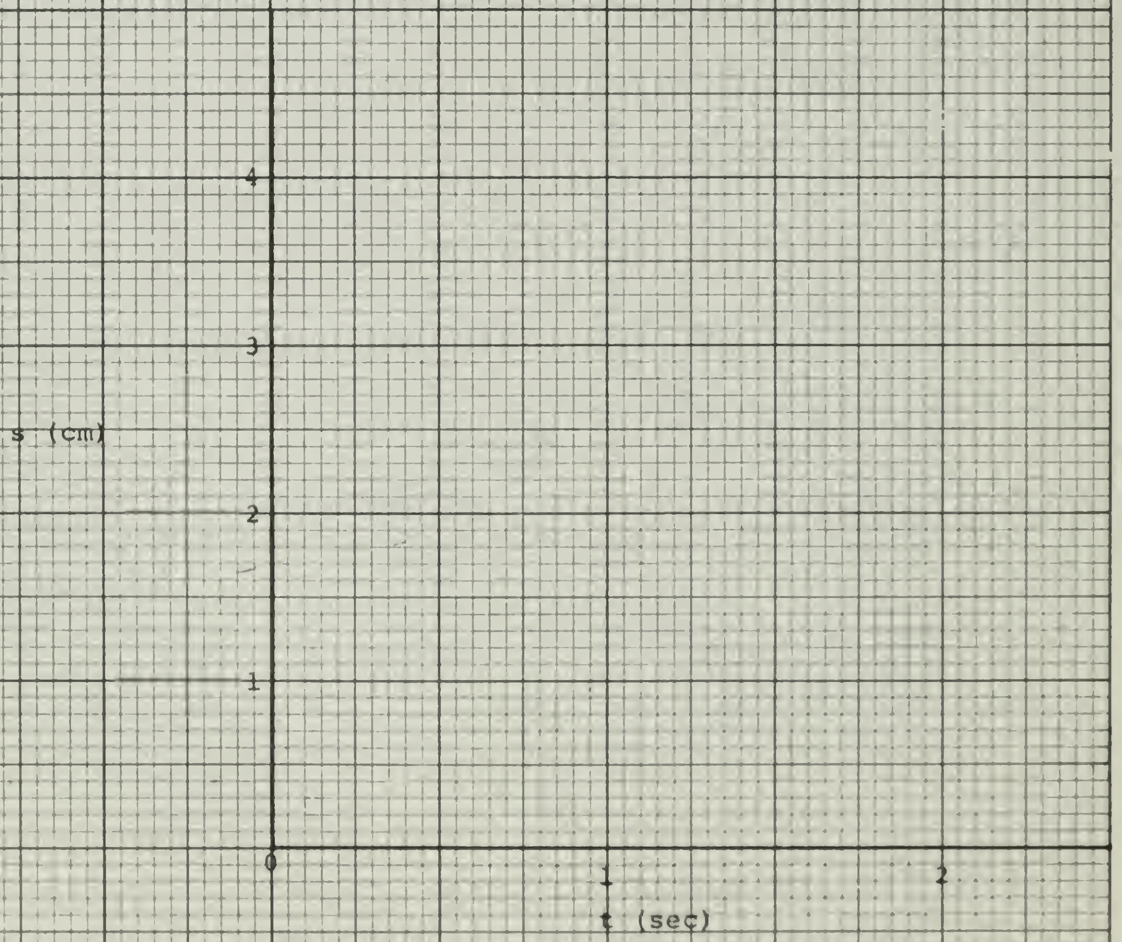
$$\Delta s = \frac{1}{2}at^2;\ 2\Delta s = at^2;\ a = \frac{2\Delta s}{t^2}$$

$$a = \frac{2 \cdot 1\text{cm}}{1\text{sec}^2} = 2\frac{\text{cm}}{\text{sec}^2}$$

Then, to find the position of the object at t = 2sec, if the acceleration ($a = 2\frac{\text{cm}}{\text{sec}^2}$) remains constant:

$$\Delta s = \frac{1}{2}at^2 = \frac{1}{2} \cdot 2\frac{\text{cm}}{\text{sec}^2} \cdot 4\text{sec}^2 = 4\text{cm}.$$

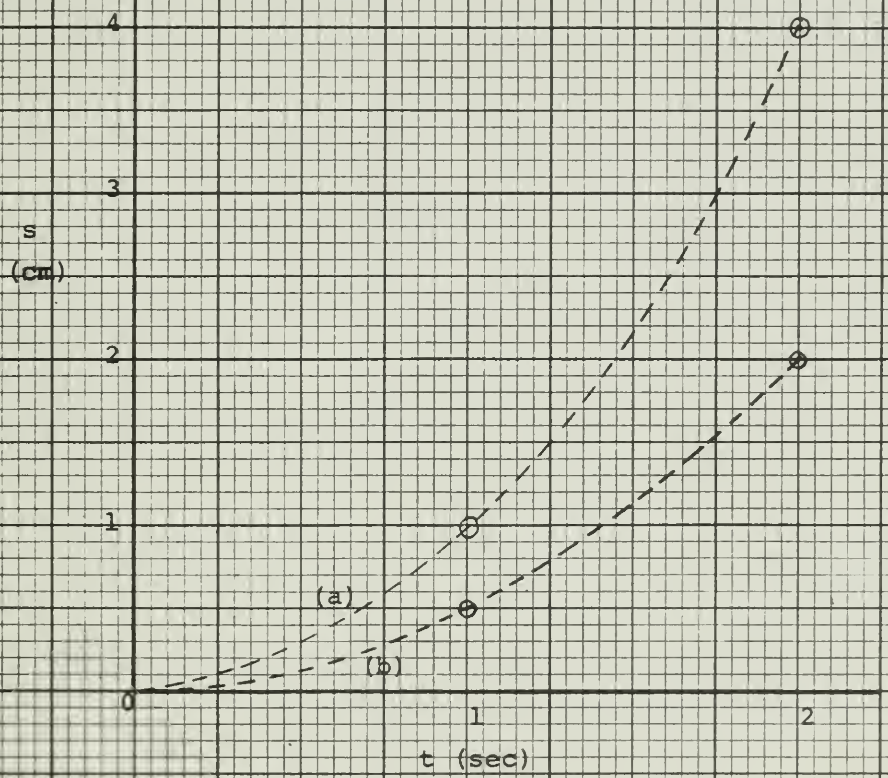
Now do the same for (b), then sketch the curves for (a) and (b):



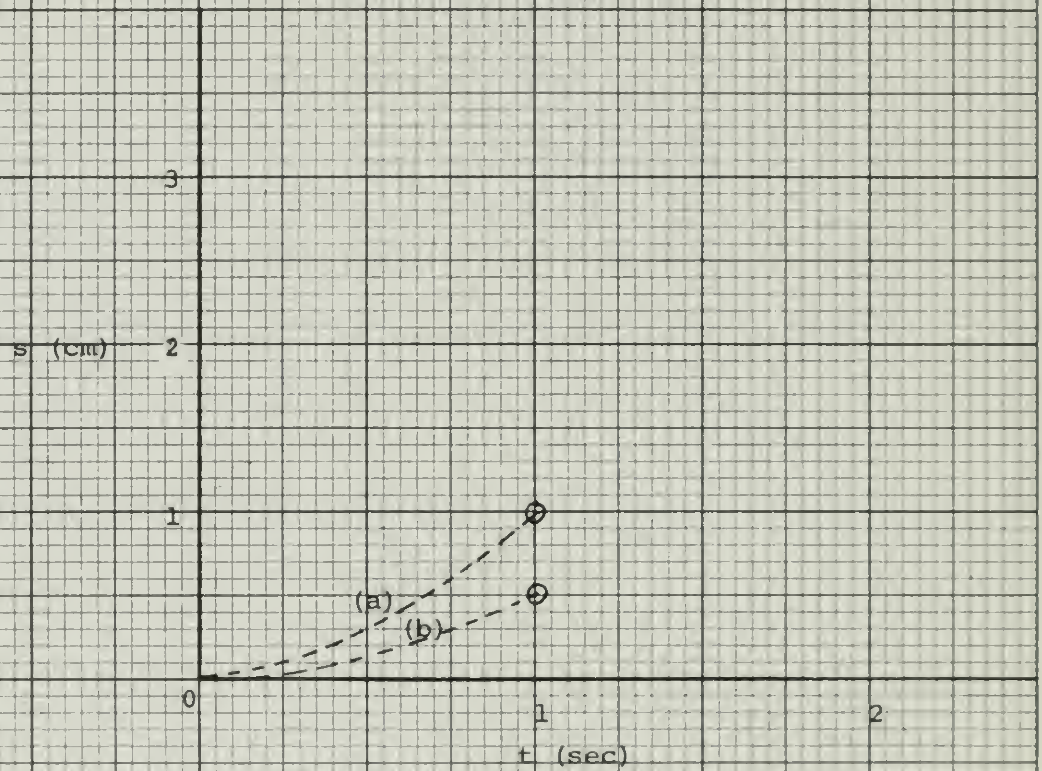
For (b): When $t = 1$ sec, $s = \frac{1}{2}$cm; $a = ?$ (constant)

$$a = \frac{2s}{t^2} = 1\frac{\text{cm}}{\text{sec}^2}$$

So at $t = 2$sec, $s = \frac{1}{2}at^2 = \frac{1}{2}\cdot 1\frac{\text{cm}}{\text{sec}^2}\cdot 4\text{sec}^2 = \underline{2\text{cm}}$



Now suppose that the acceleration stops at t = 1.
What are the positions of (a) and (b) at t = 2?



HINT: If you know the value of a, does this information tell you anything about the <u>speed</u> at t=2?

In other words, if a = $2cm/sec^2$, then in the first second the object accelerates to a speed of ________.

Remember, $v_f = v_0 + a\Delta t$

So for $t = 0 \rightarrow 1$, $v_0 = 0$, $a = 2\frac{cm}{sec^2}$

And $v_f = 2\frac{cm}{sec^2} \cdot 1sec = \underline{2\frac{cm}{sec}}$

(We assume for the moment that the final velocity of the interval $t = 0 \rightarrow 1$ is the instantaneous velocity at $t = 1$.)

Now, what about the average velocity over the following interval? (Remember, $a = 0$.)

$\overline{v}_2 = v_{f_1} + \Delta\overline{v}_2 = v_{f_1} + a\Delta t_2$

But $a = 0$ (There is no change in speed.)

So: $\overline{v}_2 = v_{f_1} = \underline{2\frac{cm}{sec}}$

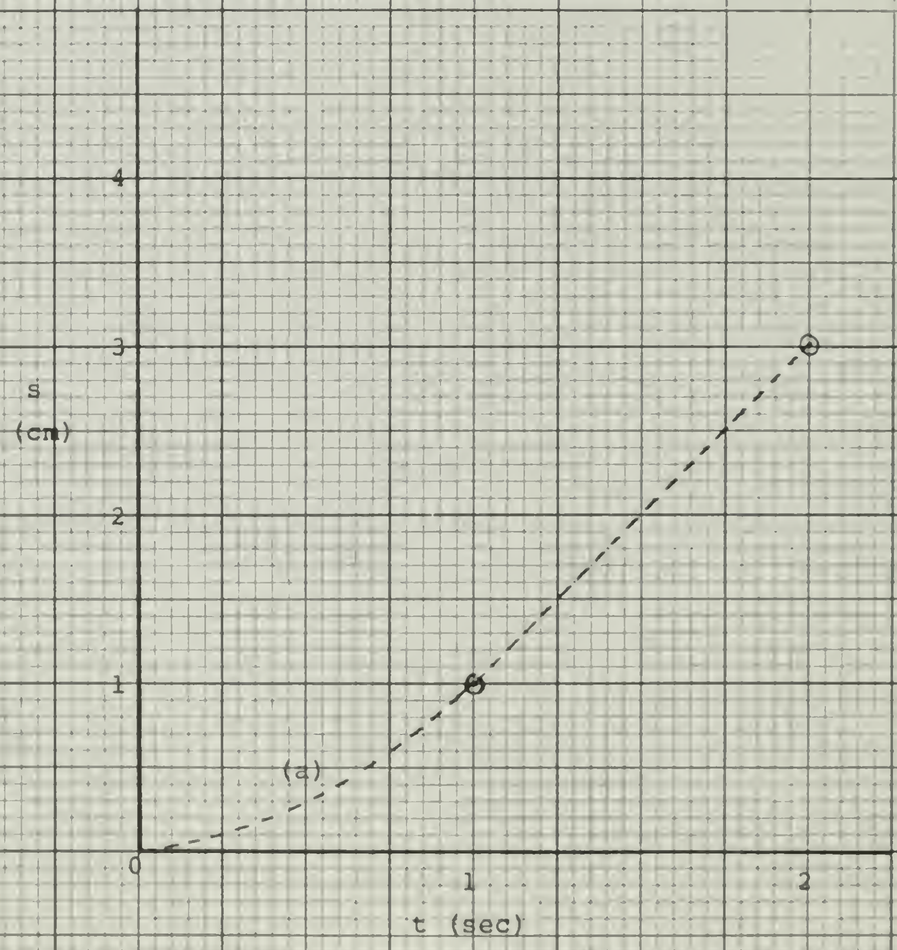
In other words, if there is no acceleration, the object continues through the second interval at the velocity which it had attained by the end of the first interval.

Now, if you have not already done so, calculate the position of (a) at $t = 2$ and sketch the graph on the preceeding page. (What will the section of the curve between $t = 1$ and $t = 2$ look like?)

$$\bar{v}_2 = \frac{\Delta s_2}{\Delta t_2}$$

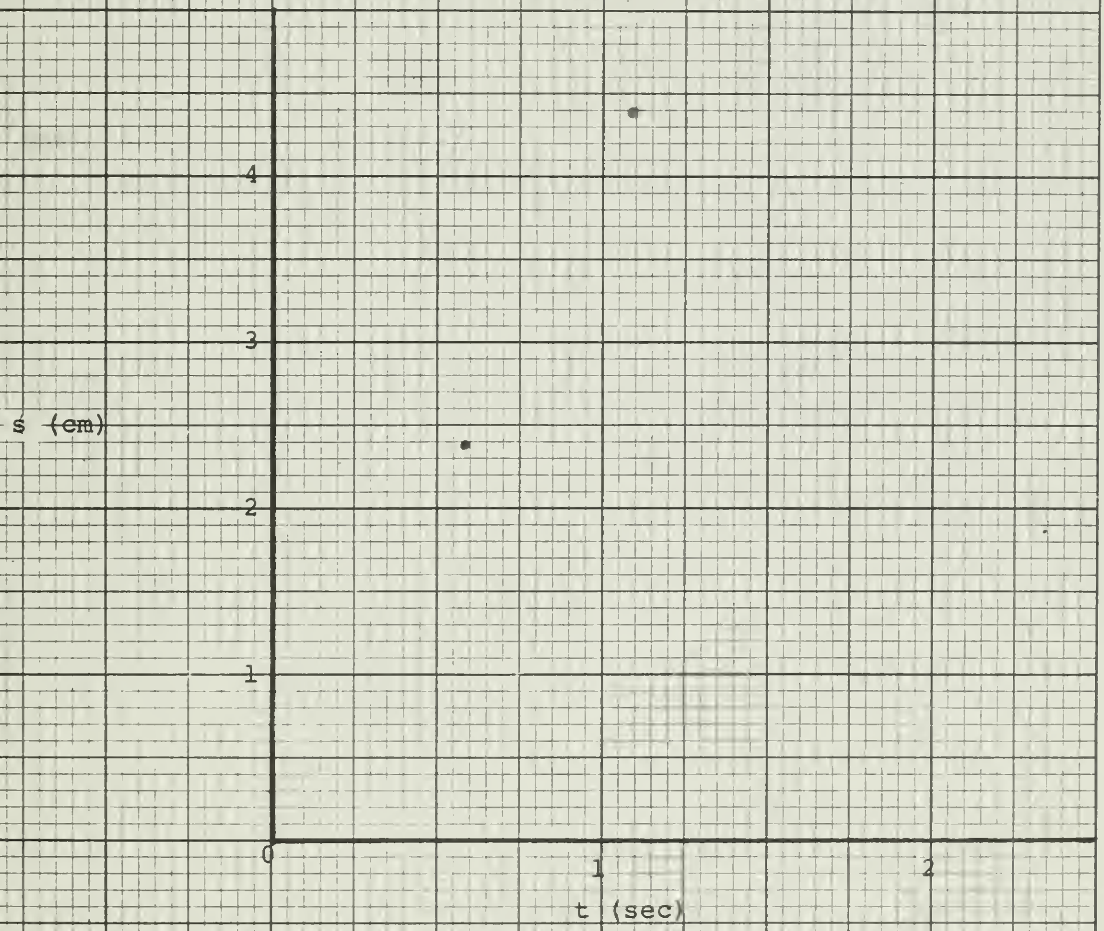
$$\text{so } \Delta s_2 = \bar{v}_2 \Delta t_2 = 2\frac{\text{cm}}{\text{sec}} \cdot 1\text{sec} = \underline{2\text{cm}}$$

$$\text{and } s_{tot} = \Delta s_1 + \Delta s_2 = 1\text{cm} + 2\text{cm} = \underline{3\text{cm}}$$



Now do the same for (b), if you have not already done so.

Plot the graph:



$$\bar{v}_2 = (v_{f_1}) + (\Delta\bar{v}_2) = (v_{f_1}) + (a\Delta t_2)$$

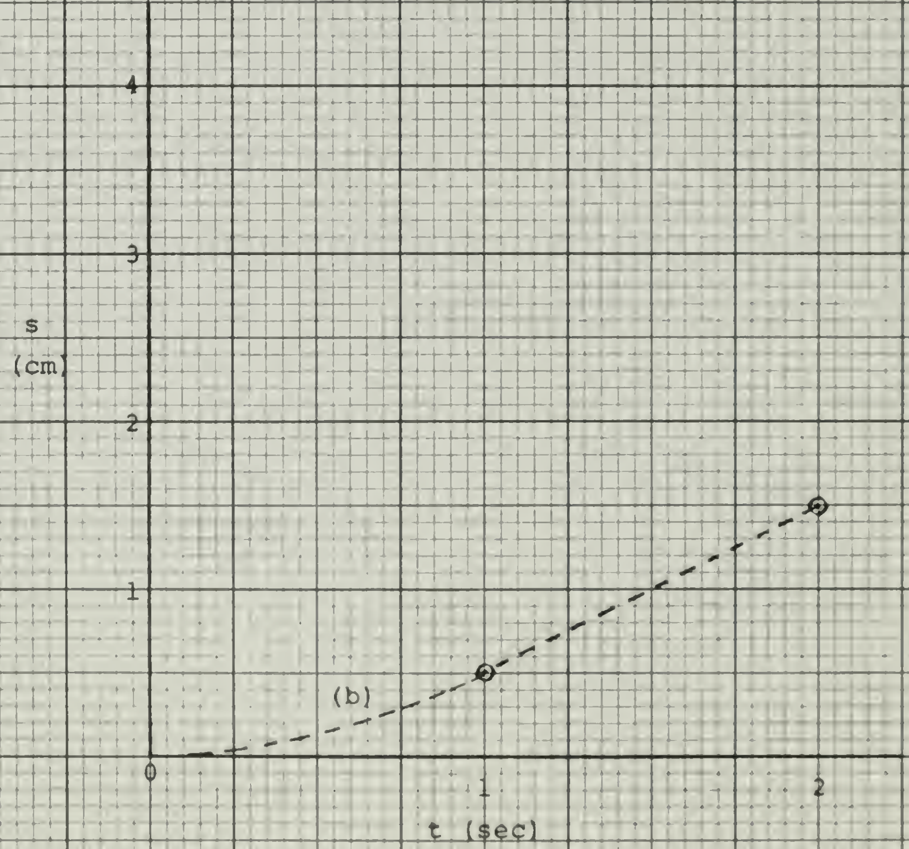
But $a = 0$,

so $\bar{v}_2 = (v_{f_1}) = v_0 + a\Delta t_1$

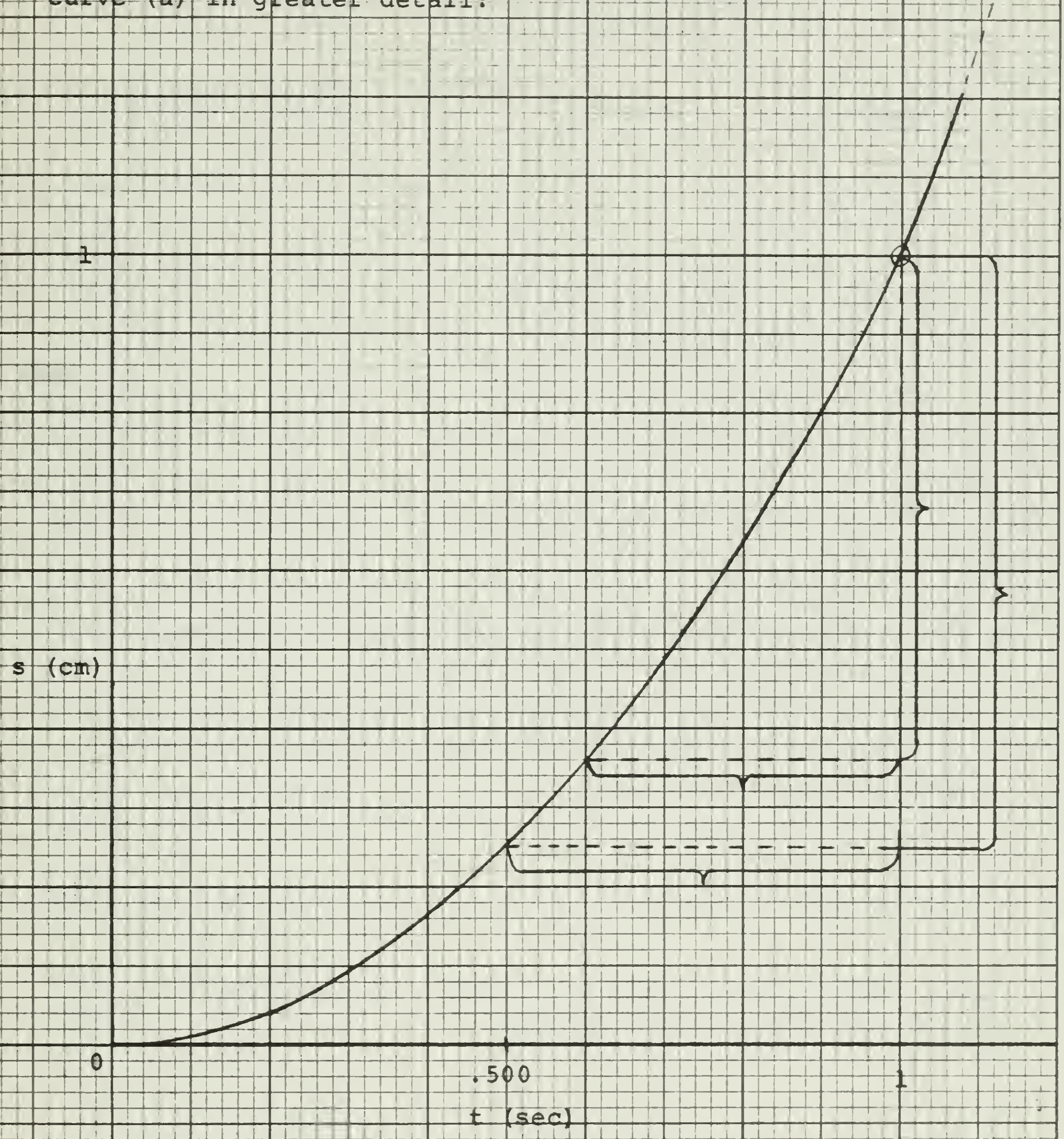
$$\bar{v}_2 = 0 + 1\frac{cm}{sec^2}\cdot 1sec = \underline{\underline{1\frac{cm}{sec}}}$$

So: $\Delta s_2 = \bar{v}_2\Delta t_2 = 1\frac{cm}{sec}\cdot 1sec = \underline{1cm}$

Finally: $s_{tot} = \Delta s_1 + \Delta s_2 = \frac{1}{2}cm + 1cm = \underline{\underline{\frac{3}{2}cm}}$



Now let's take some time to check our assumption that the final velocity at the end of the interval t = 0→1 is the instantaneous velocity at t = 1. Suppose we examine the curve (a) in greater detail:



By "instantaneous velocity at a moment" we mean the average velocity during an extremely small time interval containing that moment. Measure the graph to find the necessary data and fill in the table below.

DISPLACEMENT INTERVAL	Δs_{cm}	TIME INTERVAL	Δt_{sec}	$\Delta s/\Delta t = \bar{v}$ (cm/sec)
0 → 1	1	0 → 1	1	1
____ → 1	____	0.5 → 1	0.5	____
0.36 → 1	0.64	____ → 1	____	____
____ → 1	____	____ → 1	____	____
0.81 → 1	0.19	0.9 → 1	0.1	1.9

Since there is always some variability in measurement, we shall not supply the "correct" answers. The column for $\bar{v}$, however, should show gradually increasing values. The limit value, as the time intervals become smaller and smaller, appears to be ________________________.

The limit value, as the time intervals become smaller and smaller, appears to be 2cm/sec.

What is $\bar{v}$ for the interval from 1 sec to 1.05 sec?

$\Delta t = 0.05$sec, and Δs 0.13cm. Hence:

$$\bar{v} \simeq \frac{0.13\text{cm}}{.05\text{sec}} = 2.6\frac{\text{cm}}{\text{sec}}.$$

PART IV

P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9 P_{10} P_{11} P_{12} P_{13}

This is a stroboscopic photograph of a falling sphere. [Source: <u>PSSC Physics</u>, D.C. Heath & Co., 1960.]

The time interval between exposures is 1/30 sec. The scale on the left is calibrated in centimeters.

There are a number of questions we might want to ask about the picture. To begin with, you might ask yourself if you can tell, simply by looking at the picture, whether the velocity is constant or not.

REMOVE THIS PAGE AND REFER TO IT FOR THE NEXT FEW PAGES.

The velocity is increasing, but what about the acceleration? How would you go about determining whether it is constant or changing?

You probably decided that you would have to make some measurements and some calculations to determine whether the acceleration was constant or not. Here are some data to help you get started:

INTERVAL	Δs	$\bar{v}$	$\Delta \bar{v}$	a
$P_{1 \to 2}$	6.3cm	$189 \frac{cm}{sec}$		

How much more information will you need to determine whether a is constant? Which intervals would you choose to measure?

(When taking your measurements, you will find it easiest to measure between the bottoms of each ball. And you might find it helpful to use a caliper to transfer your readings to the scale.)

Your calculations might look something like this:

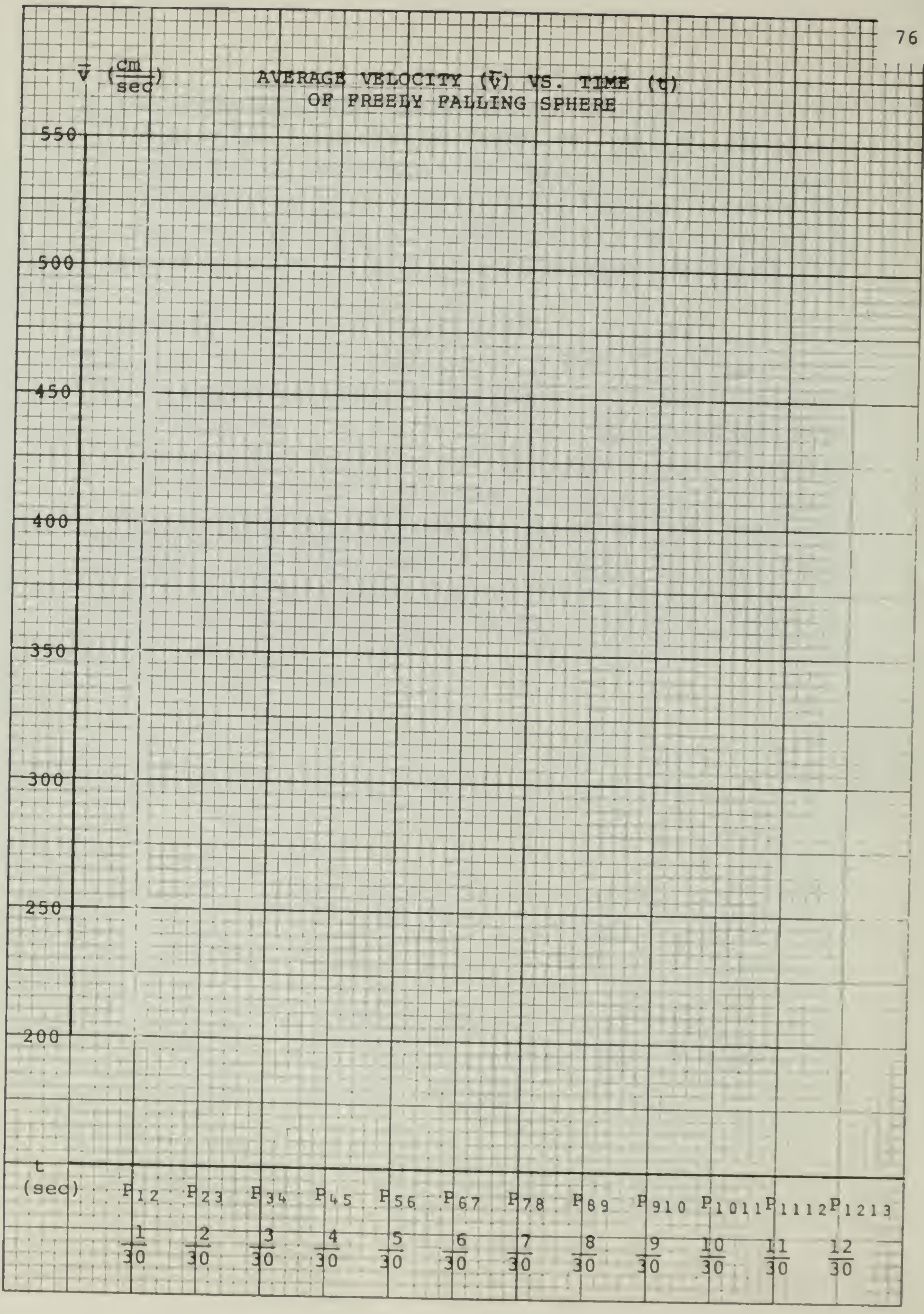
INTERVAL	Δs	$\bar{v}$	$\Delta\bar{v}$	a
$P_{1\rightarrow 2}$	6.3cm	$189\frac{cm}{sec}$		
			$162\frac{cm}{sec}$	$9.72\frac{m}{sec^2}$
$P_{6\rightarrow 7}$	11.7cm	$351\frac{cm}{sec}$		
$P_{7\rightarrow 8}$	12.9cm	$387\frac{cm}{sec}$		
			$165\frac{cm}{sec}$	$9.90\frac{m}{sec^2}$
$P_{12\rightarrow 13}$	18.4cm	$552\frac{cm}{sec}$		

The acceleration would seem to be nearly constant. (If your values for a were within 1 or 2 tenths of a meter/sec^2 of ours, your measurements from the scale were quite accurate.) As a check you might want to calculate a for $\Delta\bar{v}$ between $P_{1\rightarrow 2}$ and $P_{12\rightarrow 13}$:

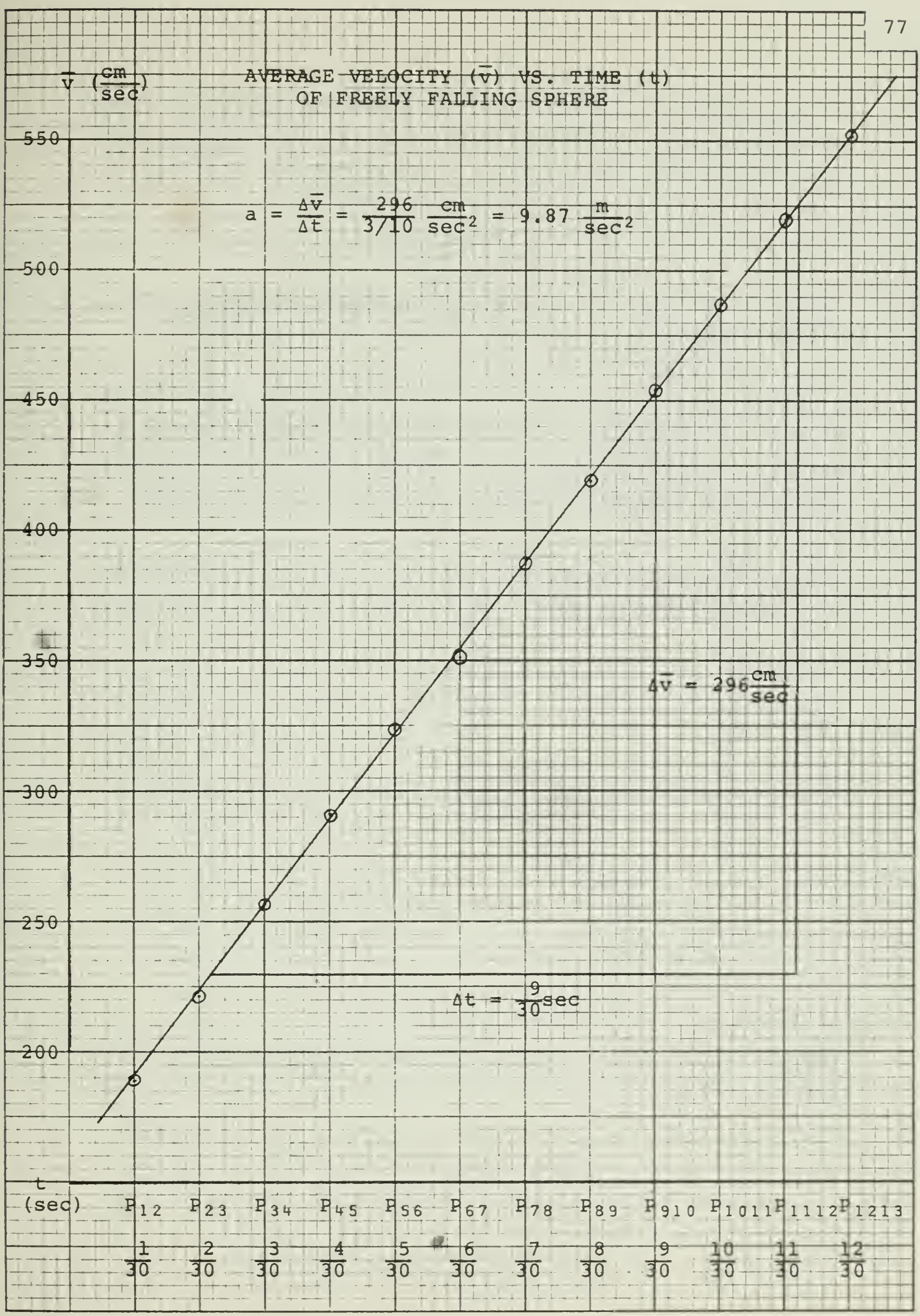
INTERVAL	Δs	$\bar{v}$	$\Delta\bar{v}$	a
$P_{1\rightarrow 2}$	6.3cm	$189\frac{cm}{sec}$		
			$363\frac{cm}{sec}$	$9.9\frac{m}{sec^2}$
$P_{12\rightarrow 13}$	18.4cm	$552\frac{cm}{sec}$		

The last value for a seems in fairly close agreement with the others, but the fact that the value of a varies does suggest that there is some error involved in the problem. You probably observed how difficult it is to measure Δs on the small scale in the picture, and you may have considerable doubt about whether a is really constant or not.

In fact, if you chose to calculate $\Delta\bar{v}$ over shorter intervals (between $P_{1\to2}$ and $P_{2\to3}$, $P_{2\to3}$ and $P_{3\to4}$, etc.) you might have found that your values for a varied so much that the acceleration did not seem constant at all. We chose the largest intervals possible in an effort to increase the accuracy of our algebraic calculation. But there is an even more accurate way of finding a, which depends on the properties of a graph. What happens if you plot $\bar{v}$ against t? (A graph with suitably chosen units is provided on the next page; plot the values of $\bar{v}$ and try to draw the curve.)



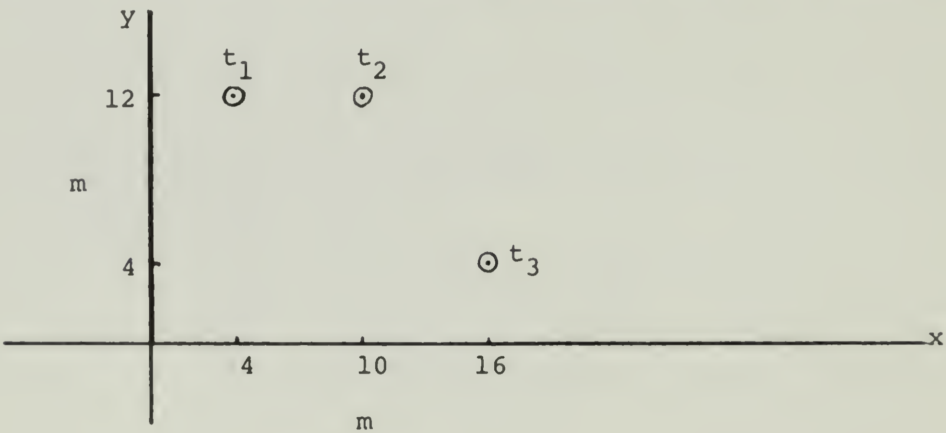
AVERAGE VELOCITY ($\bar{V}$) VS. TIME (t)
OF FREELY FALLING SPHERE
$\bar{V}$ ($\frac{cm}{sec}$)
550
500
450
400
350
300
250
200
t (sec)
P_{12} P_{23} P_{34} P_{45} P_{56} P_{67} P_{78} P_{89} P_{910} P_{1011} P_{1112} P_{1213}
$\frac{1}{30}$ $\frac{2}{30}$ $\frac{3}{30}$ $\frac{4}{30}$ $\frac{5}{30}$ $\frac{6}{30}$ $\frac{7}{30}$ $\frac{8}{30}$ $\frac{9}{30}$ $\frac{10}{30}$ $\frac{11}{30}$ $\frac{12}{30}$


AVERAGE VELOCITY ($\bar{v}$) VS. TIME (t)
OF FREELY FALLING SPHERE
$\bar{v}$ ($\frac{cm}{sec}$)
$a = \frac{\Delta\bar{v}}{\Delta t} = \frac{296}{3/10}\frac{cm}{sec^2} = 9.87\frac{m}{sec^2}$
$\Delta\bar{v} = 296\frac{cm}{sec}$
$\Delta t = \frac{9}{30}$sec
550
500
450
400
350
300
250
200
t
(sec)
P_{12} P_{23} P_{34} P_{45} P_{56} P_{67} P_{78} P_{89} P_{910} P_{1011} P_{1112} P_{1213}
$\frac{1}{30}$ $\frac{2}{30}$ $\frac{3}{30}$ $\frac{4}{30}$ $\frac{5}{30}$ $\frac{6}{30}$ $\frac{7}{30}$ $\frac{8}{30}$ $\frac{9}{30}$ $\frac{10}{30}$ $\frac{11}{30}$ $\frac{12}{30}$

In straight-line motion, changes in motion have been either a speeding up or a slowing down, that is, the velocity vector has changed its ___________, but not its ___________.

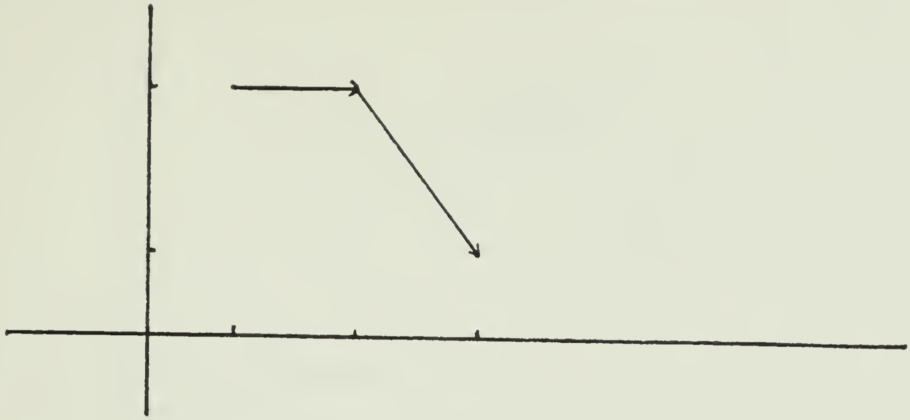
ANSWER: The velocity vector has changed its magnitude, but not its direction.

Below is a schematic representation of the position of an object. A reference frame is provided for distance and direction measurements. Notice that this is not a graph.

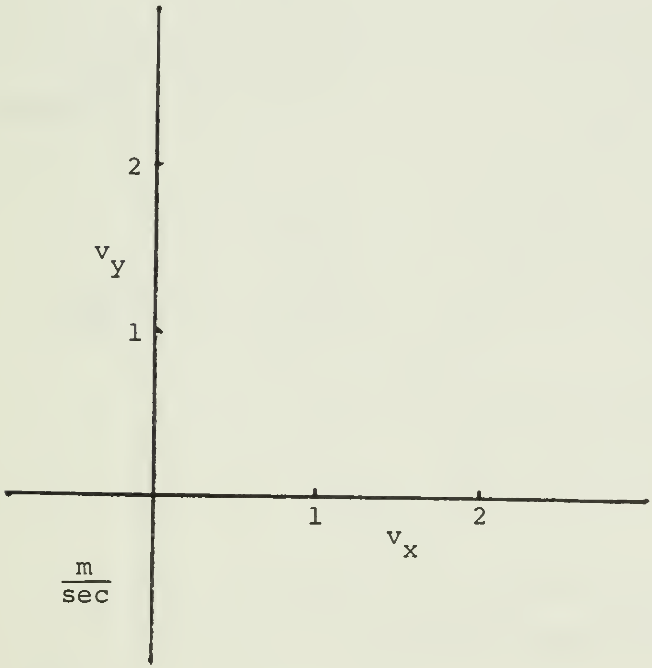


t_1 = 12sec
t_2 = 18sec
t_3 = 23sec

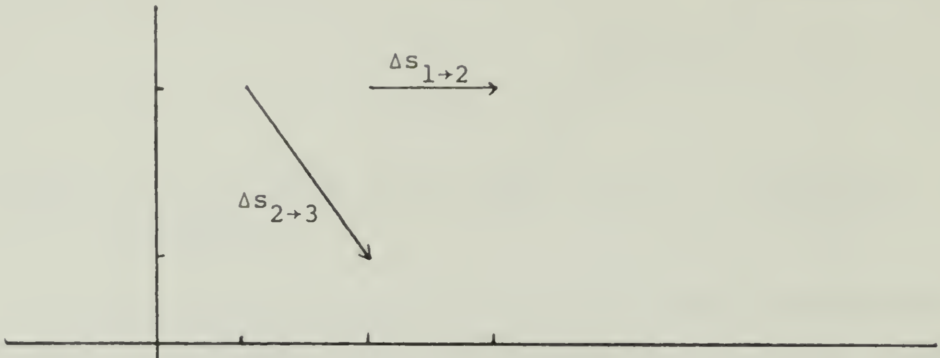
We can easily draw a vector diagram of displacements on this sketch. Do so.



Now construct a vector diagram for the average velocities, using the frame of reference and scale provided below.

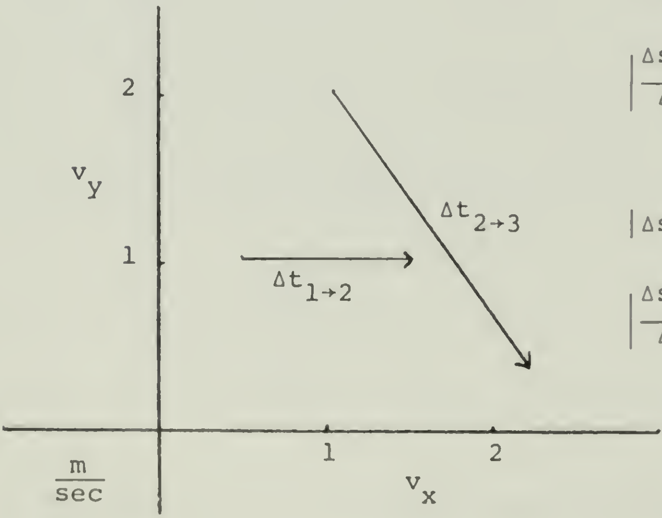


HINT: Our displacement diagram could have looked like this:



The location of a vector is irrelevant; only the direction and magnitude are important.

ANSWER:



$$|\Delta s_{12}| = 6\text{m}$$

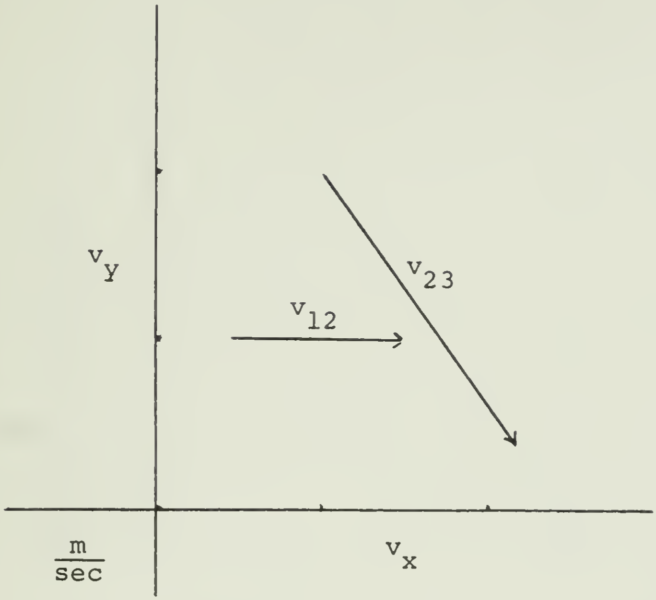
$$\left|\frac{\Delta s_{12}}{\Delta t}\right| = \frac{6}{6} = 1\frac{\text{m}}{\text{sec}}$$

$$|\Delta s_{23}| = 10\text{m}$$

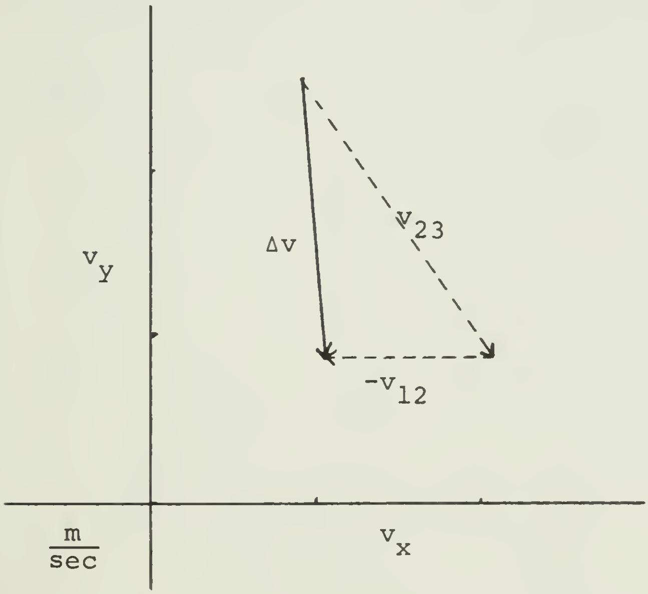
$$\left|\frac{\Delta s_{23}}{\Delta t}\right| = \frac{10}{5} = 2\frac{\text{m}}{\text{sec}}$$

The directions of these vectors are parallel to the directions of the displacement vectors.

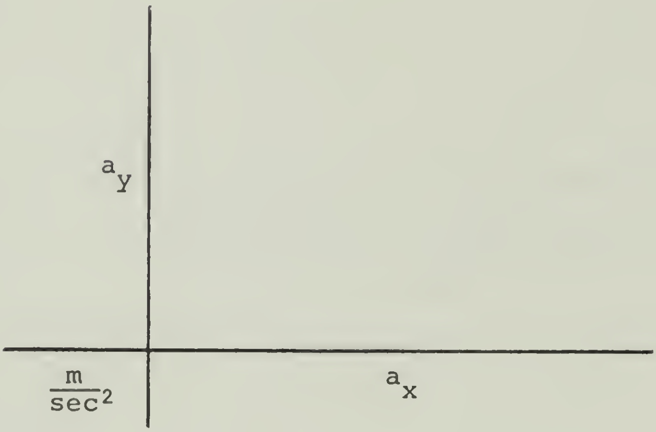
What is Δv?



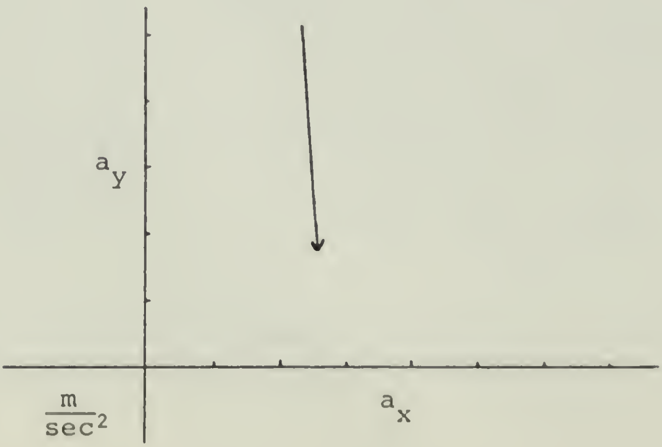
ANSWER:



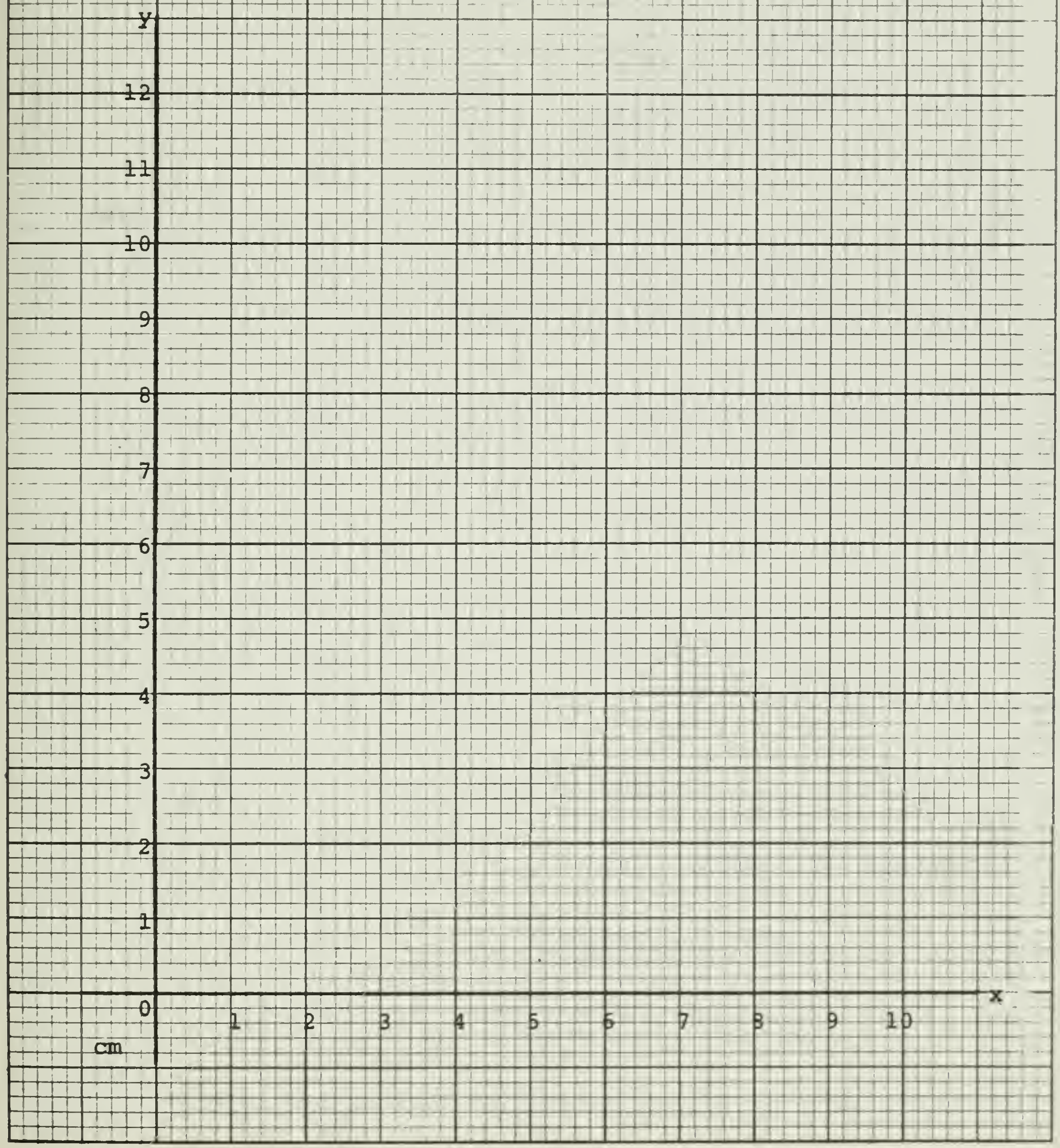
Suppose this change had taken place in a time interval of $\frac{1}{2}$ second. What was the magnitude of the acceleration? What was its direction. (Fill in the scale below.)



ANSWER: $|\Delta v| \approx 1.7\frac{m}{sec}$. Therefore, $|a| = 3.4\frac{m}{sec^2}$. The direction is the same as that of Δv from the scale above.



An object at $t_0 = 0$ has a uniform velocity in the horizontal direction, $\vec{v}_x$, of 2cm/sec, and no velocity in the vertical direction, $\vec{v}_y = 0$. At t_0 and for all times afterwards, it is constantly accelerated upward so that $\vec{a} = \vec{a}_y = 5\text{cm/sec}^2$. If $\vec{s}_0 = (0,0)$, plot the position of the object at $t_1 = 1\text{sec}$ and $t_2 = 2\text{sec}$.



HINT #1: If $\vec{v}_x$ is constant, how far does the object travel <u>horizontally</u> in 1 second? in 2 seconds?

ANSWERS: $\vec{s}_x = \vec{v}_x t$

$2\frac{cm}{sec} \cdot 1sec = \underline{2cm}$

$2\frac{cm}{sec} \cdot 2sec = \underline{4cm}$

HINT #2: What is the formula for displacement under constant acceleration when the initial velocity is 0?

ANSWER: $\vec{s} = \frac{1}{2}\vec{a}t^2$

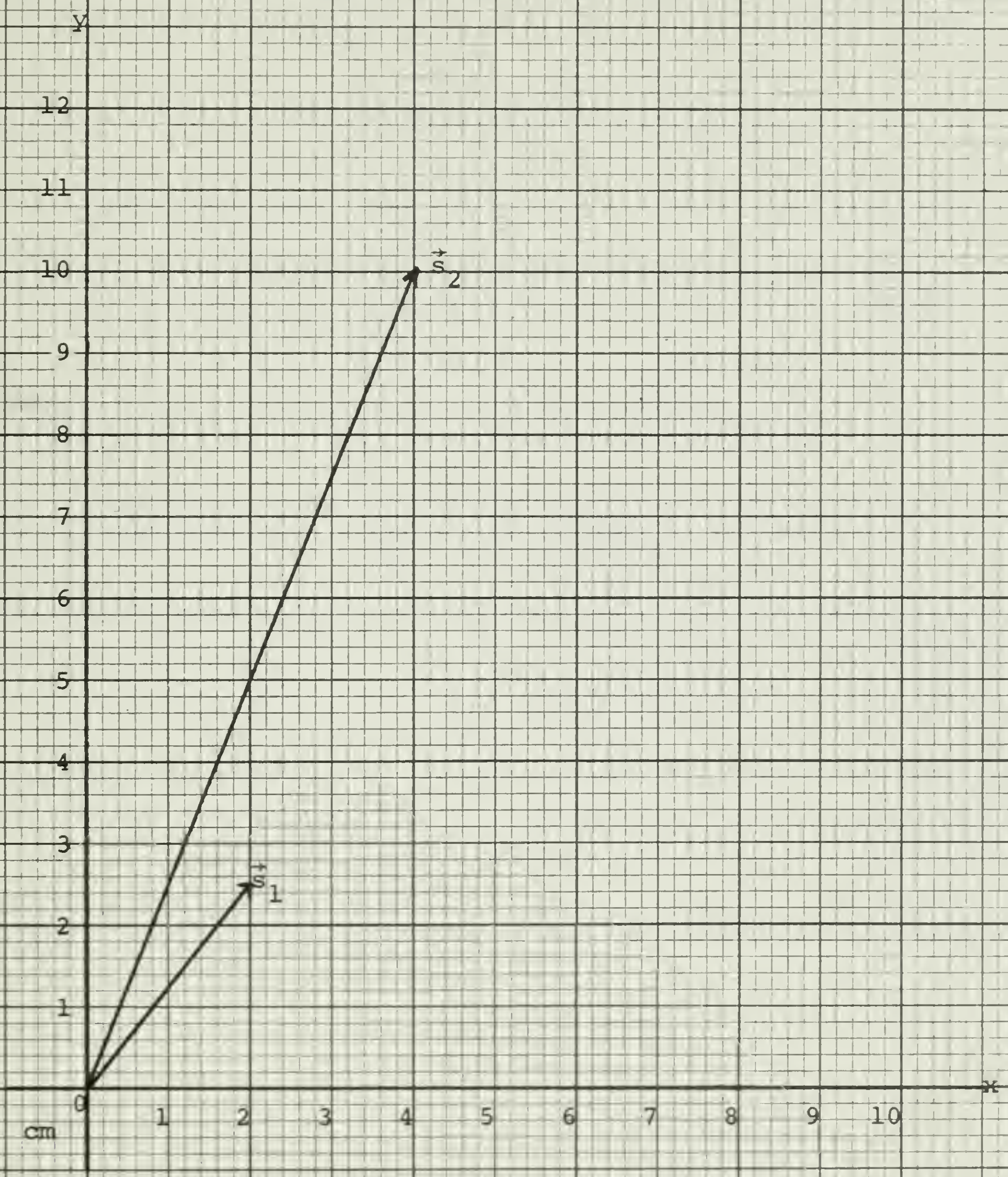
HINT #3: What is the vertical displacement after 1 second? 2 seconds?

ANSWERS: $\vec{s}_y = \frac{1}{2}\vec{a}_y t^2$

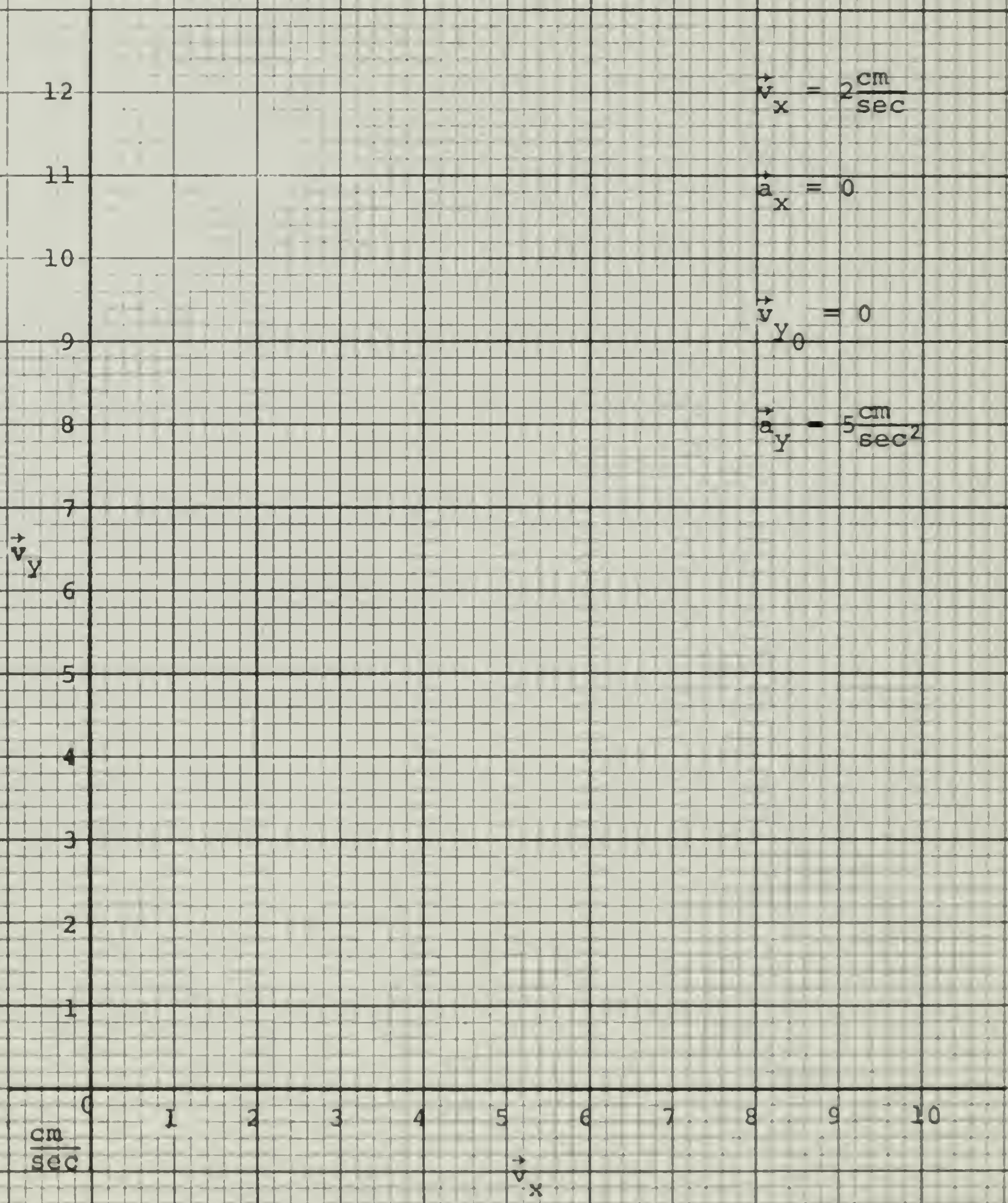
$\frac{1}{2} \cdot 5\frac{cm}{sec^2} \cdot 1sec^2 = \underline{\frac{5}{2}cm}$

$\frac{1}{2} \cdot 5\frac{cm}{sec^2} \cdot 4sec^2 = \underline{10cm}$

FINAL SOLUTION OF THE PROBLEM:



Plot the resultant instantaneous velocity at t_1 (= 1sec) and t_2 (= 2sec). (Let the origin of both vectors be the point [0,0].)



HINT #1: What is the velocity component in the x direction, $\vec{v}_x$, for both $\vec{v}_1$ and $\vec{v}_2$?

ANSWER: $\vec{v}_x$ is constant for the whole motion and is $2\frac{cm}{sec}$.

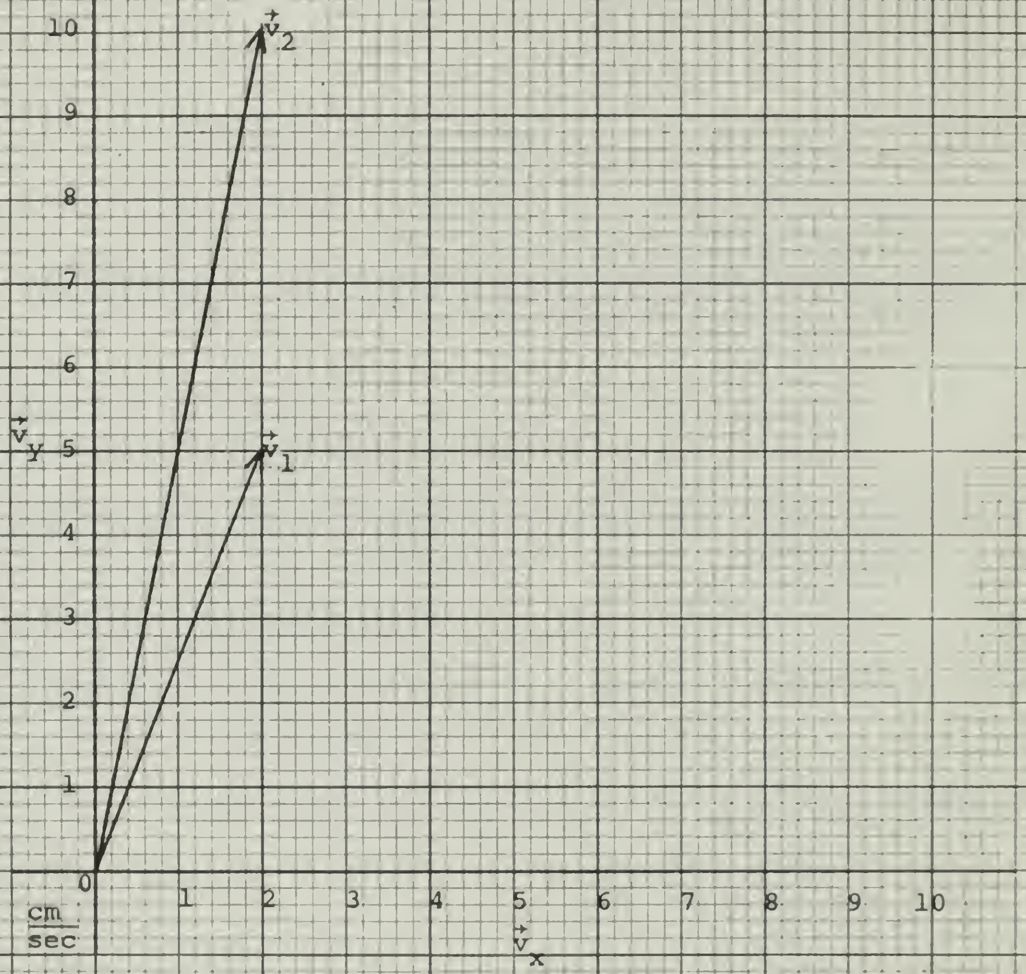
HINT #2: What is the velocity component in the y direction, $\vec{v}_y$, at the end of 1 second? 2 seconds?

ANSWERS: $\vec{v}_y = \vec{a}_y t$

$$5\frac{cm}{sec^2} \cdot 1sec = \underline{5\frac{cm}{sec}}$$

$$5\frac{cm}{sec^2} \cdot 2sec = \underline{10\frac{cm}{sec}}$$

FINAL SOLUTION OF THE PROBLEM:



Finally, you should notice that $\Delta\vec{v}$ is parallel to ________.

ANSWER: $\Delta\vec{v}$ is parallel to the acceleration, $\vec{a} = \vec{a}_y$.

Cover photograph by Mark Kaufmann, Time, Inc.