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ASTRONOMY

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THINKING IN SCIENCE

ASTRONOMY AND THE DEVELOPMENT OF THEORY

INTRODUCTION

This book deals with astronomy, not to teach you a lot of facts about stars and planets but to enable you to make your own progress through the development of theory in astronomy. You will see how simple ideas developed, in the course of centuries into a great theory. And then you will find it easier to understand new theories in atomic physics and the use of theories in many sciences today.

Suppose a visitor from the last century asked you about electrons and you started to describe the picture we now have: particle behaviour and wave behaviour...motion in metals...important activities in chemistry... general nature as a universal ingredient of all matter. Your visitor would stop you and ask, "Have you seen electrons? Are they really like this?" And when you replied "No" he would object, "Then you are just describing pictures. Why do you choose those particular pictures? What are the real facts that you know about electrons; facts that make you choose those pictures?". Your visitor would be asking you about the use of theories and the relationship between experiment and theory. Those are very important matters in modern science; but if we started with theories of electrons or atoms you would find our account very complicated, rather like beginning History at the middle of a major war. That is why we offer this study of theory in Astronomy -- for you to see how theory is built and used.

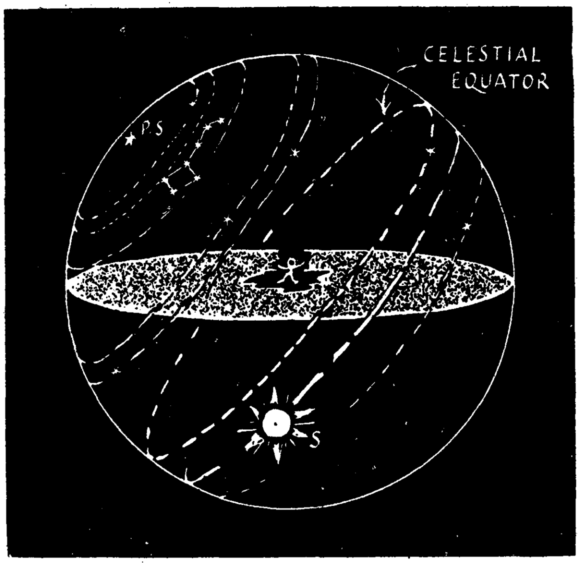
As in the story of electrons, the question "What are the facts?" asks for a clear beginning. We are going to look at theories of the motion of Sun, Moon, stars and planets; and we must start with the facts. Some readers already know a lot about these. Others may need some general descriptions: so here they are.

CHAPTER I. WHAT WE SEE HAPPENING IN THE SKY

The Pattern of Stars.

If you watch the sky for some time at night and then again on later nights you will see that the stars make unchanging patterns. A photograph of the stars in the sky will look just the same next year and for many years to come. That unchanging pattern of stars rolls across the sky during the night, turning round the pole-star as a "pivot", once in 24 hours. You can see that motion if you watch for some hours; or you can take a photo with a camera held still with the lens open for part of the night.

We can illustrate that motion by marking the stars on a "celestial sphere" * which we spin on an axis going through the pole-star. The Earth, with you the observer on it is at the centre of the sphere. Imagine the ground that you stand on continued out as a great flat sheet to meet the celestial sphere -- that is the horizon plane. Some stars can be seen to make complete circles, round the pole-star; and others, farther from the pole-star, dip down under the horizon plane for part of their journey.



THE STAR PATTERN REVOLVES

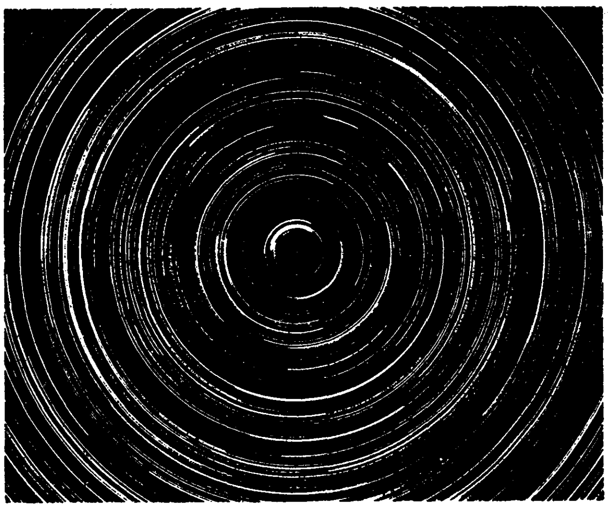


PHOTO OF THE SKY NEAR POLE STAR
Taken with an eight-hour exposure. The Pole Star itself made the very heavy trail near the center. Photo from Lick Observatory.

Moon.

On some nights you see the Moon among the stars. On other nights the stars are there but there is no Moon. The Moon must travel across the star pattern. Have you watched it do that? If not, look at the Moon one night and make a note of its position among neighboring stars. Look again an hour later, then later still, then a day or two later. In the course of the night, the Moon sweeps across the sky from east to west with all the stars, but not quite as fast as the stars. If you watch carefully, you will see that the Moon lags behind the stars (like a lazy child on a walk), more and more as the month goes on. That lagging carries it backward through the star pattern, from west to east, 90° in a week, all the way round in a month. You can see the full Moon one night (when the Sun is down below the Earth in just the opposite direction), then a fortnight later no Moon at all; and another full Moon a fortnight later still. Even in a single hour, the Moon moves by its own diameter relative to the stars.

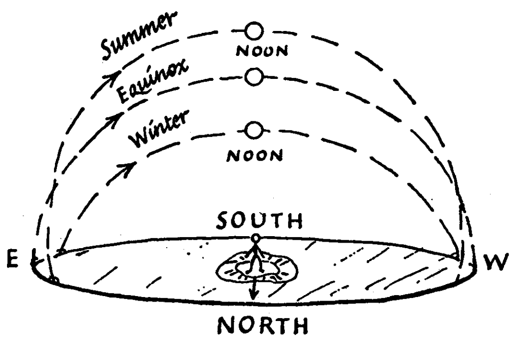
If you watch the Moon carefully and mark its lagging path from night to night throughout the month, you will find that its path is a slanting one.

* Make one for yourself with a rubber ball with a knitting needle, or an orange with a skewer through it, or just an umbrella with a few stars marked on it, spun by twisting the handle. Or, take a photograph like the picture shown here.

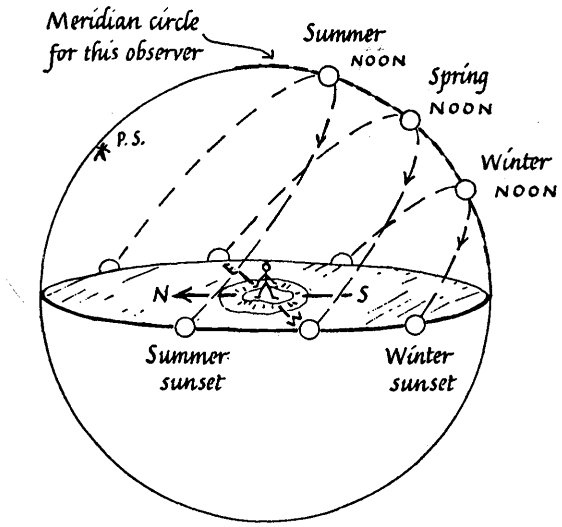
It does not just drift backwards along the same direction as its rapid nightly east-to-west forward motion during the night. It drifts backward along a slanting line close to a line that we call the "ecliptic".

Sun.

At noon, the Sun is always due south. It makes one complete revolution from noon to noon. *



THE PATH OF THE SUN IN THE SKY CHANGES WITH THE SEASONS



Instead of thinking about the Sun at noon, look at the stars at midnight, and again the next midnight and so on. You will see that, although the pattern of stars keeps exactly the same shape it changes position from night to night. In 24 hours from midnight to midnight the star pattern does not move 360°: it swings round the pole-star axis 1° more than that, 7° more in a week, 30° more in a month. So while the Sun seems to go round the Earth 365 times in a year, the stars make just 366 revolutions more than the Sun in a year.

We can look at that a different way: the Sun does not keep a fixed place in the star pattern but lags backward (like the Moon) 1° per day. Of course you do not <u>see</u> the Sun lagging like that, because the stars are too faint to be visible in daylight. But you can imagine the Sun down under the Earth at midnight and guess from the way the star pattern moves forward 1° from midnight to midnight that the Sun must be drifting backward relative to the star pattern 1° in each 24 hours.

The lagging motion of the Moon carries it right round in a month, but the lagging motion of the Sun is slower: 1° in a day, and all the way round in a year.

* Except for some minor deviations which are connected with changing speeds of the Earth's motion round the Sun during the year.

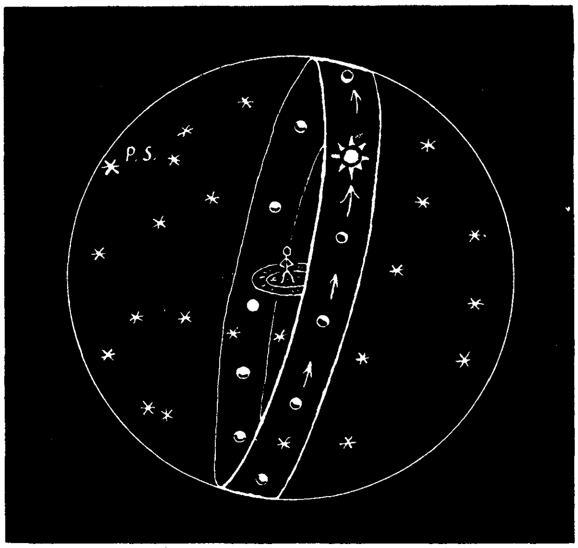
"Freezing" the Daily Motion.

Some of the earliest astronomers, watching the motion of the Sun, Moon, and stars made a very intelligent move in their thinking: they analyzed the motion into two parts: the fast daily motion and the slow backward lagging. The daily motion, shared by all, was given a speed of 361^{o} per day, or 366 revolutions per year. Then the lagging motion was 0 for the stars, 1 revolution per month for the Moon, one revolution per year for the Sun. Since the fast motion is the same for all, we can forget about it when we are trying to describe the special motions of Sun and Moon. Thus, those early astronomers started subtracting the daily motion from what they saw. In other words, they imagined the daily motion stopped or "frozen" and then catalogued the strange motions of Sun and Moon through a stationary star pattern.

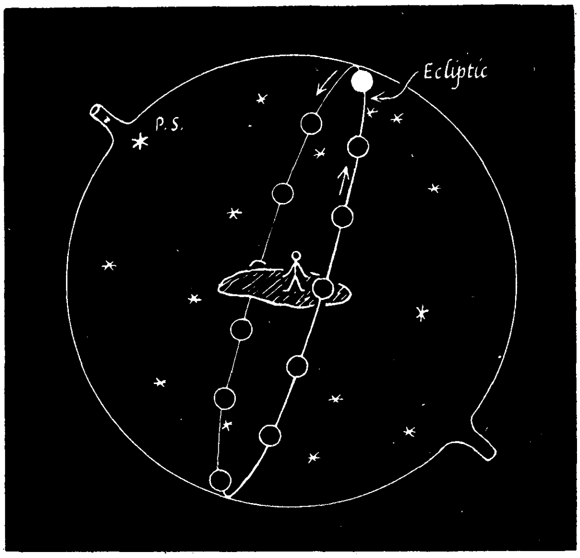
That idea of freezing the daily motion was a great step forward in science, a difficult intellectual jump, made by astronomers in early civilizations.

Sun's Yearly Motion Ecliptic.

With the daily motion frozen we see the Sun moving slowly backwards from west to east on a slanting circle through the star pattern, completing the circle in a year. That circle is inclined at $23\frac{1}{2}^{o}$ to the celestial equator; and we call it the "ecliptic". To early astonomers the ecliptic was the wandering path of the Sun through the star pattern. In terms of present-day knowledge, it represents the Earth's orbit round the Sun.



ZODIAC BELT WITH POSITIONS OF MOON, IN VARIOUS PHASES, IN THE COURSE OF A MONTH
The daily motion of the celestial sphere is "frozen" here.



THE ECLIPTIC, the Sun's track through the star pattern in the course of a year.
Here the daily motion is imagined "frozen."

The Sun does not travel at exactly constant speed round the ecliptic. It moves a little faster in our winter than in our summer; so the four seasons are not exactly equal in length.

On your celestial sphere, you should mark the horizon circle, the celestial equator, and the ecliptic. Then put a small sticky label on the ecliptic, to represent the Sun. Spin the sphere about its north-south axis through the pole-star, and watch the Sun's daily motion. Then move the Sun to another position on the ecliptic and again look at its daily motion. You will see how the Sun sweeps over the sky in a long high arc in summer and a shorter low one in winter.

Equinox. Half-way between summer and winter there are the times that we call equinox, equal day and night. There, the Sun makes just half a circle above the horizon plane. At equinox the Sun, which is always on the ecliptic, is also on the celestial equator -- it is where those two circles cross.

Planets.

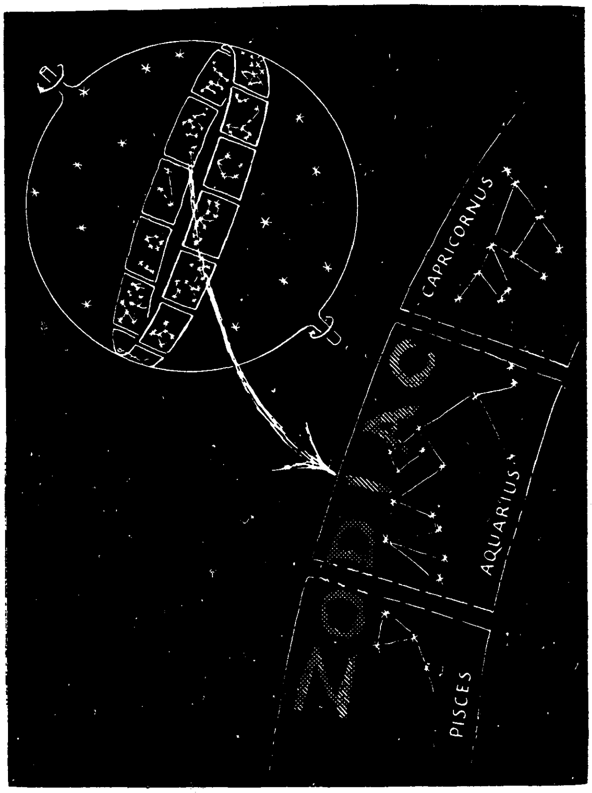
Among all the stars, early astronomers noticed a few which did not keep their places in the unchanging pattern. We call them planets, using the Greek name which means "wanderers". Like the Sun and Moon, the planets sweep round with the star pattern in a daily motion. Freezing out that daily motion, we find that each planet slips slowly backward from west to east through the star pattern in the course of years. The slanting paths of planets through the star pattern are quite close to the Sun's ecliptic path. Since the planets seemed very important to superstitious people, astrologers broadened the ecliptic circle into a wide belt, with the ecliptic along the middle. This band is called the Zodiac. The paths of Sun, Moon and all the planets lie in the Zodiac belt.

Zodiac.

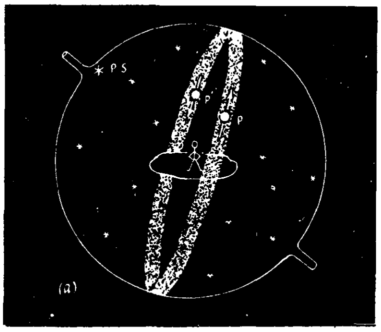
The Zodiac belt is divided into twelve sections, each named after a "constellation", a group of stars given an imaginative name by early astronomers. The Sun as it travels round the ecliptic moves from one zodiac section to the next each month.

The Moon's path in the Zodiac makes an angle of about 5° with the Sun's path, the ecliptic.

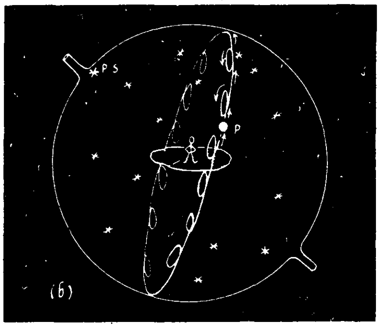
The Motion of the Planets. Unlike the almost steady motion of the Sun and Moon, each planet's motion through the star pattern is more irregular. The planet slides backward from west to east for some time, comes to a stop, then moves forward for a short time, comes to rest, moves backward and so on The backward motion from west to east predominates, carrying the planet Jupiter, for example, all the way round the zodiac in a dozen years. The short forward motions, in which the planet makes a loop, (seen almost sideways on), occur about once in every Earth-year.



THE ZODIAC, a belt of the celestial sphere, tilted 23½° from the equator. The Sun's yearly path (the ecliptic) runs along the middle line of this belt. The paths of Moon and planets lie within this belt. The Zodiac was divided into twelve sections named after prominent star-groups or constellations. (Zodiac patterns after H. A. Rey, *The Stars.*)



THE PATH OF A PLANET
All the planets wander through the star pattern in a belt near the ecliptic—the zodiac belt.
(a) General region of a planet's path—the zodiac belt.



(b) In detail, a planet's path has loops—an epicycloid seen almost edge-on.

These wandering stars, the planets, are the chief object of our present study. It was their motion that presented the greatest problem to astronomers who wanted to "explain" the movement of things in the heavens.

Two of the planets shine so brightly, (by reflected sunlight) that you can easily pick them out, and you may be able to watch them changing their place among the stars from month to month. These are:

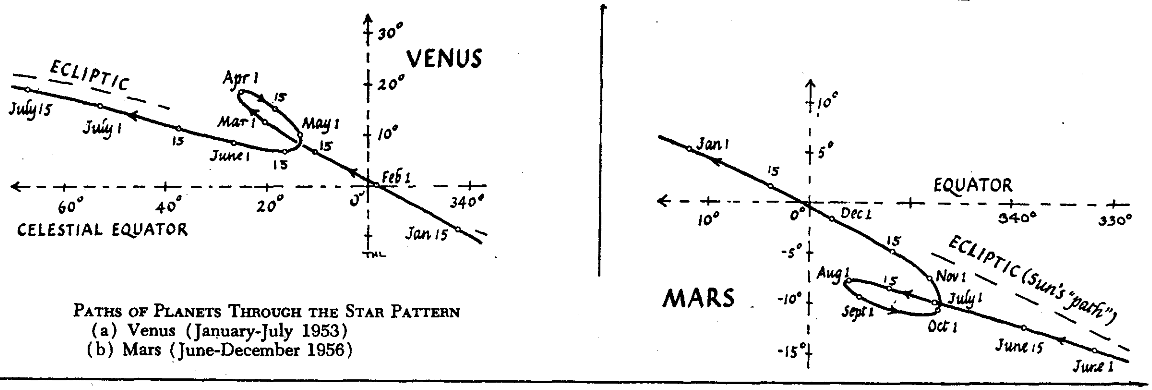
<u>Venus</u>, which never moves far away from the Sun. Venus swings out 46° to one side of the Sun and then back, disappearing when we are dazzled by the Sun nearby, and out to 46° on the other side. You can see Venus at some seasons as the "evening star" and at other seasons as the "morning star".

<u>Jupiter</u> which moves very slowly through the star pattern, close to the ecliptic, 12 times slower than the Sun. In crawling backward, all the way round in 12 years, Jupiter makes a forward loop once a year.

The other planets are more difficult to identify but you can find them with the help of star maps published in newspapers. Here is a list of all the planets known to the early astronomers:

PLANET	TIME FOR COMPLETE ORBIT ROUND THE SUN *	
Mercury	87 days	
Venus	225 days	Sun; orbit round the Earth, 365.3 days
---	---	
Mars	687 days	Moon; orbit round the Earth, 27.3 days **
Jupiter	12 years	
Saturn	30 years	

To the early astronomers, the Earth was not a planet. They were sure that the Earth remains fixed at the centre of the Universe. But they included the Sun and Moon as wanderers, making 7 planets in all. In our table above, we left a space for the Earth, but it should not be placed there yet.

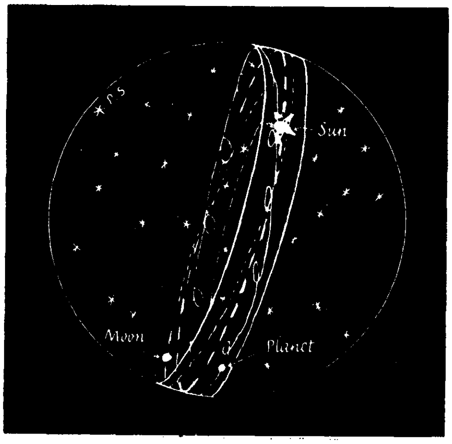


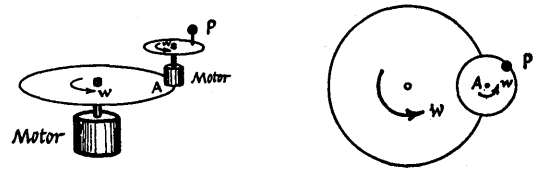
PATHS OF PLANETS THROUGH THE STAR PATTERN
(a) Venus (January-July 1953)
(b) Mars (June-December 1956)

Path of a Planet: Epicycloid. The path that we see a planet taking through the star pattern seems to be an "epicycloid": that is, a compound of motions round a small circle and a big circle. Sketch one for yourself like this: on a large piece of paper, draw a big circle with a pencil freehand, say of diameter 10 inches. Sweep your hand slowly round that circle, but as you do

* The period of a planet's motion, "its time to get round its orbit", depends somewhat on our viewpoint. The value given here is the "true" period, or the planet's "year", as an observer on the Sun would see it. However, during that "true period" of the planet's motion round its orbit, the Earth moves to a different position; so an observer on the Earth would not see the planet back at the same place among the stars. The planet's apparent period, reported by an observer on the Earth may be different. If we were giving a proper account of the planetary picture seen by earlier astronomers, we should describe the apparent periods. But then we should have to disentangle the true periods from them when we came to the Copernican picture of the Solar System. For simplicity we have just given the true periods here.

** For the Moon, 27.3 days is the "true" month, relative to the stars. But as the Earth moves on while the Moon goes down, the apparent month-- from full Moon to full Moon -- is longer, $29\frac{1}{2}$ days.



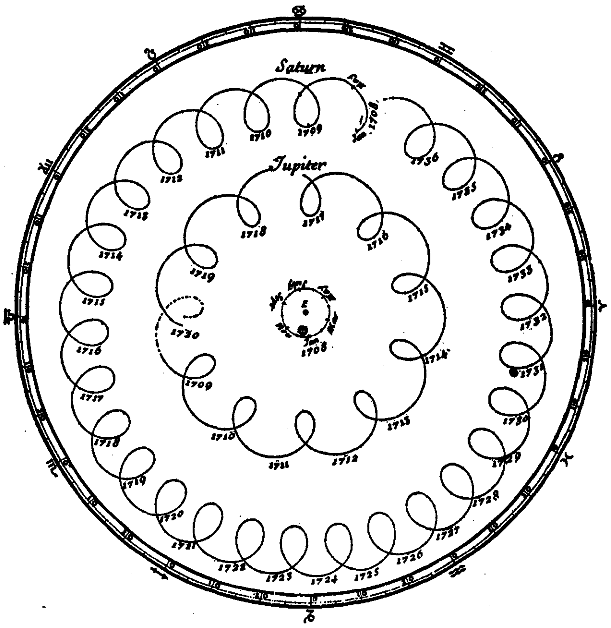


Machine for Drawing Epicycloids

← Zodiac Belt with Paths of Sun (in one year), moon (in one month), and a specimen planet (in planet's "year"). The daily motion of the celestial sphere is "frozen" here.

so move the pencil in your hand, faster, round a small circle of diameter say 3 inches. You are drawing an epicycloid. In watching a planet, we see that pattern almost edge on, and of course only part of it in the course of a few months. To take that view, tear part of the epicycloid from your sheet of paper and hold it almost at eye level so that you see it from a slanting point of view.

Or you can imitate the motion with your hand, holding your left hand in front of you to represent the Earth and sweeping your right hand round with outstretched arm in a large circle. If you twist your right wrist so that your hand makes a small circle you can imitate an epicycloid.



Paths of Planets in the Sky.
This sketch shows the apparent paths of Jupiter and Saturn, plotted for many years, as they would appear to an observer attached to the Earth but viewing them from *far* out from the Earth, so that the epicycles are seen face-on, without the foreshortening really observed. The apparent orbit of the Sun is also shown. The Earth is at the center. When the astronomer Cassini constructed this diagram in 1709 he used Copernican measurements of orbit sizes.

Please pretend you are back with our earliest ancestors, who were the first astronomers. Why did they worry about the Sun, Moon and stars? What did they find out? How did they find out? Of course, you already know a great deal about such things. You learned that the Earth goes round the Sun in a yearly orbit and the Earth spins. But please forget that for the moment and pretend that you only know what you see. Pretending you are back with those ancestors long ago needs some clever imagination, but it is necessary if you are to understand the development of astronomy.

We shall start with the earliest astronomers, many thousands of years ago; and, until we come to the great change made by Copernicus 400 years ago, we shall take the heavens as we see them and think of man on a stationary flat Earth watching the motion of the stars and Sun, etc.

The early astronomers saw the Sun moving across the sky, clearly travelling round the Earth. At night they saw the whole pattern of stars swinging across the sky making circles round the north-south axis which goes through the pole-star. If you camp out and watch the stars through the night you yourself will believe that you are lying under a great whirling dome of stars -- "Nobody but a lunatic would think anything else".

Earliest man, living by hunting and chance cropping, may have looked at the stars and wondered. He may have used them as guides at night. He may have welcomed the Moon's light for hunting. We do not know. He may have used Sun and stars unconsciously as rough clocks; but it is very unlikely that he used Moon or stars for reckoning days or weeks, or even the Sun for reckoning hours, because he lived simply and such things were not needed.

The Revolution From Food-gathering Man to Food-producing Man. Some 12,000 years ago a new level of human life developed, perhaps even new races of man, in which better stone and bone tools were used and agriculture began to supplement chance cropping, and herding began to replace chance hunting of wild animals, and pottery and cooking came into use. That was when village life developed, in the new "food-producing culture"; and simple trade was carried on.

For agriculture a calendar was needed, to tell people in the village to the proper season for sowing. And, as sheep were the first domesticated animals, herdsmen wanted a calendar for breeding. To us a calendar seems an obvious, easy thing to construct. But to our early ancestors in villages it was a new idea with no tradition to help in making one. Village priests who had acquired some rules for calendar-making were very important people, paid by food and endowed with authority. Those calendar priests were the first astronomers, because they had to use Sun, Moon and stars for calendar making.

The Revolution From Villages to City Civilizations. A second great revolution of mankind, with people gathering into cities and developing great civilizations, came about 6,000 years ago. Then astronomy was even more important, because the life of cities needed some kind of clock for time-keeping and a compass for navigating long journeys for trade. Compass, clock, and calendar were essential in the early civilizations just as they are now: astronomy provided all three.

Village life profited from a calendar to organize the work in advance. The calendar made the priests who administered it powerful and important; and, whether they liked it or not, made it easy for their knowledge to seem grand and mysterious. Things that happened in the sky took on obvious importance. Even to simplest man, the Sun must have seemed very important for warmth and growing life; so it is not surprising that a worship of the Sun developed. The Moon and stars had magical values for hunting and journeys by night. No wonder primitive people speculated about these lamps in the sky; and no wonder they worried about the few lamps which wandered, the planets.

Worship and Superstition. Sun, Moon and planets were treated with awe and came to be worshipped; also some stars which seemed to rise or set at important seasons. And as the great city civilizations developed this worship and general feeling of importance somehow produced a strong superstitious idea that Sun, Moon and planets control the individual fates of men. The positions of those bodies in the Zodiac at the time a child was born were taken to predict the child's character and important events in the child's life. Calculating the positions of Sun, Moon, and planets at the time of the child's birth and applying magical rules to make predictions from them is called "casting a horoscope". Horoscopes became important and fashionable and the astrologers who cast horoscopes were well paid and much trusted.

The early superstitious belief in horoscopes has continued right down to the present day. Scientists find it impossible to believe that the arrangement of planets far away in the sky can control a human being's character or fate; yet superstition is easily made attractive by fear and hope. We think perhaps learning more about astronomy and the laws of physics may help to lessen superstition, by giving people a more reasonable view of the heavens. As the Latin poet Lucretius wrote 2,000 years ago "reason (meaning science) frees man from the terror of the gods".

In one way astrology benefited astronomy a great deal: astrologers needed reliable rules for the motion of Sun, Moon and planets, and that need encouraged both observation and the development of astronomical theory.

Also, in some cases, superstitious royalty provided rich endownments for astronomers, hoping for help in astrology, but thereby helping to maintain good observing and thinking.

Astronomy in Early Civilizations. By the time the early civilizations -- such as those in Babylon and Egypt -- had grown up, systematic rules had been extracted from astronomical observations. The calendar priests knew the motions of Sun and Moon well and could predict the seasons and even eclipses at the Moon.

In the later ages of those civilizations men had a good knowledge of the lengths of the seasons, the length of the year, etc., good weights and measures, knowledge of algebra and geometry (including Pythagoras' theorem long before his name was put to it).

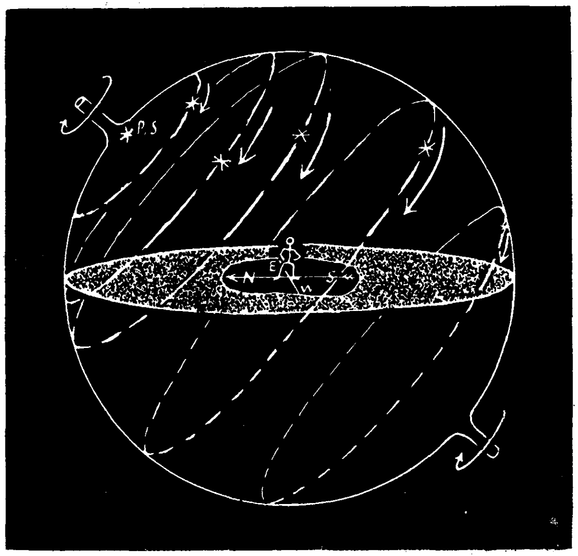
Their astronomers had schemes for predicting the slightly irregular motions of the Sun and Moon along their paths through the stars. Those schemes, in the hands of the Babylonians, amounted to zig-zag graphs that were used for calendar making. The astronomers who used them seemed to have no idea of giving any reason for the patterns or imagining any mechanism responsible for them. They were just working graphs, such as an engineer might sketch for the detailed running of a piece of machinery.

The First Theories: Gods and Spirits. As a story to tell why stars, Sun, Moon and planets move as they do, early man simply imagined gods or spirits -- the same kind of explanation that they accorded to thunderstorms, springs of hot water, plagues of flies,.... The Sun god drove his chariot across the sky each day; but the planets must have been run by very wayward gods. Such a picture or explanation was a theory in infancy, and we should not neglect or despise it. We may still find such attitudes towards events in Nature in some cultures in the world today; and we should remember how strongly those attitudes can influence the learning of science.

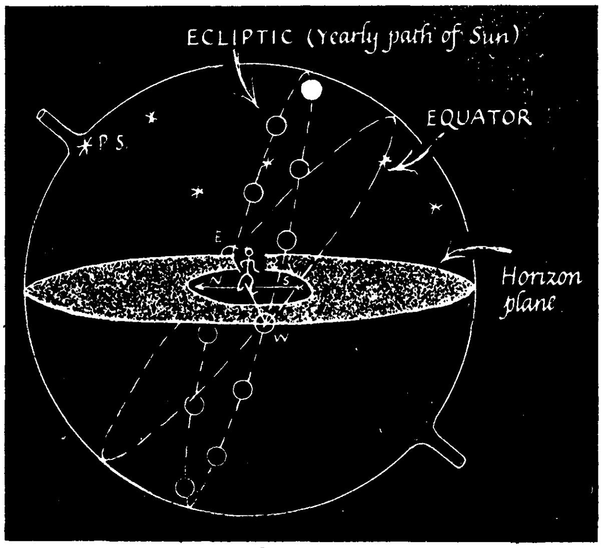
CHAPTER III. GREEK ASTRONOMY: MACHINERY TO EXPLAIN AND PREDICT

Unlike the astronomers of other early civilizations, the Greek philosophers did not just endow Sun, Moon and planets with special gods to carry them across the sky. They imagined simple machinery to move the bright objects in the heavens. This was a comforting move to lessen fears of mystery. Suppose a very young child was worried about the way the hands of clocks go round and thought there must be elves or mice inside to make them move. You, as an older brother, might try telling him there are only cog wheels and chains inside like those in a bicycle or a farm tractor. That was what the Greeks did for people's questions about Astronomy; but of course they carried their design of machinery much farther -- although it was imaginary machinery -- so that they could use it to predict future positions of planets, etc. They described things in a way that made Nature seem reasonable.

Thales (about 600 B.C.) was one of the earliest philosopher-astronomers. He thought of the Earth as flat, a great island surrounded by ocean; but he described the stars and their motion by imagining a great bowl, so that we



THE UNIVERSE ACCORDING TO THALES



EARLY GREEK VIEW
The Sun's yearly path through the star patterns was mapped. This is the tilted band called the ecliptic. The Sun is shown in one position (near mid-summer) and other positions are sketched. Here the celestial sphere is not spinning, but "frozen" with one star pattern overhead.

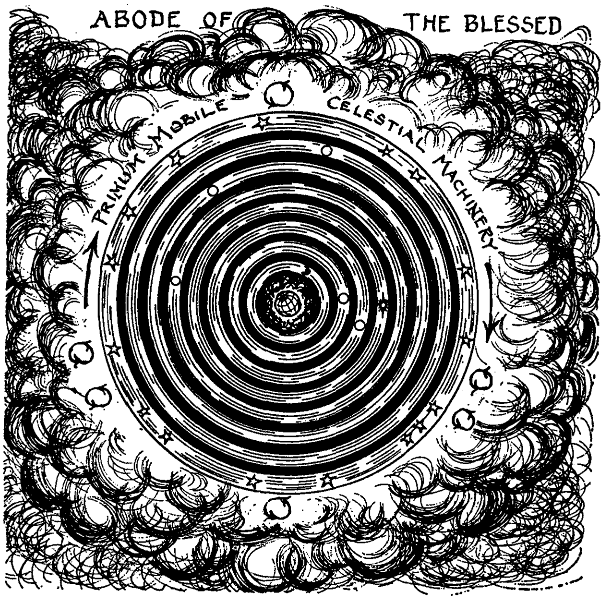
live inside the celestial sphere, which spins round, one revolution in 24 hours. That carried all the stars, like lamps stuck in it. He did not worry about the slow extra motions of Sun, Moon and planets. This was the first, simplest, machine of all: one great spinning bowl to account for the nightly motion.

He knew that the Moon shines by reflected sunlight -- and that shows that he applied reasoning to common observations. He is rumoured to have discovered electricity by rubbing amber and finding it would attract small light objects.

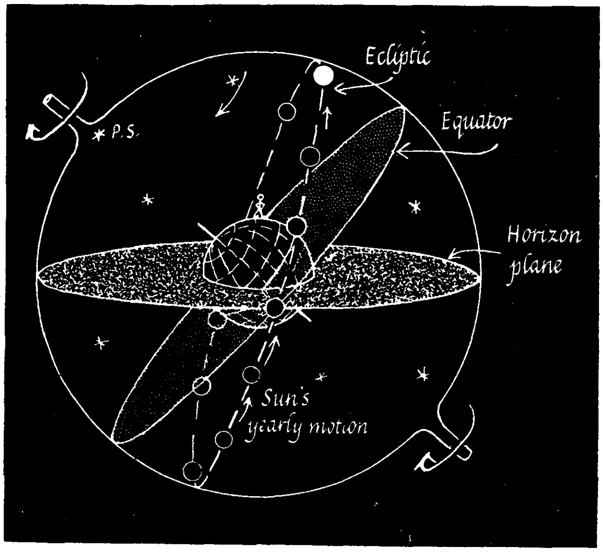
He contributed an important general idea: that "water is the basic material of which everything else is made in the world." Although we know that is not true, we should not laugh at him, because he was trying to do something that is very good in developing science; he was trying to offer a general principle, something far-reaching to guide our thinking about the whole world.

Thales was a man of science who assumed that the whole universe could be explained by ordinary knowledge and reasoning.

Pythagoras*(about 530 B.C.) gathered a group of scholars to discuss the philosophy, religion, science, politics...and his pupils continued that "school of philosophy" for some 200 years. They devised better models for the heavens: imaginary machinery that would give Sun, Moon, etc. motions to



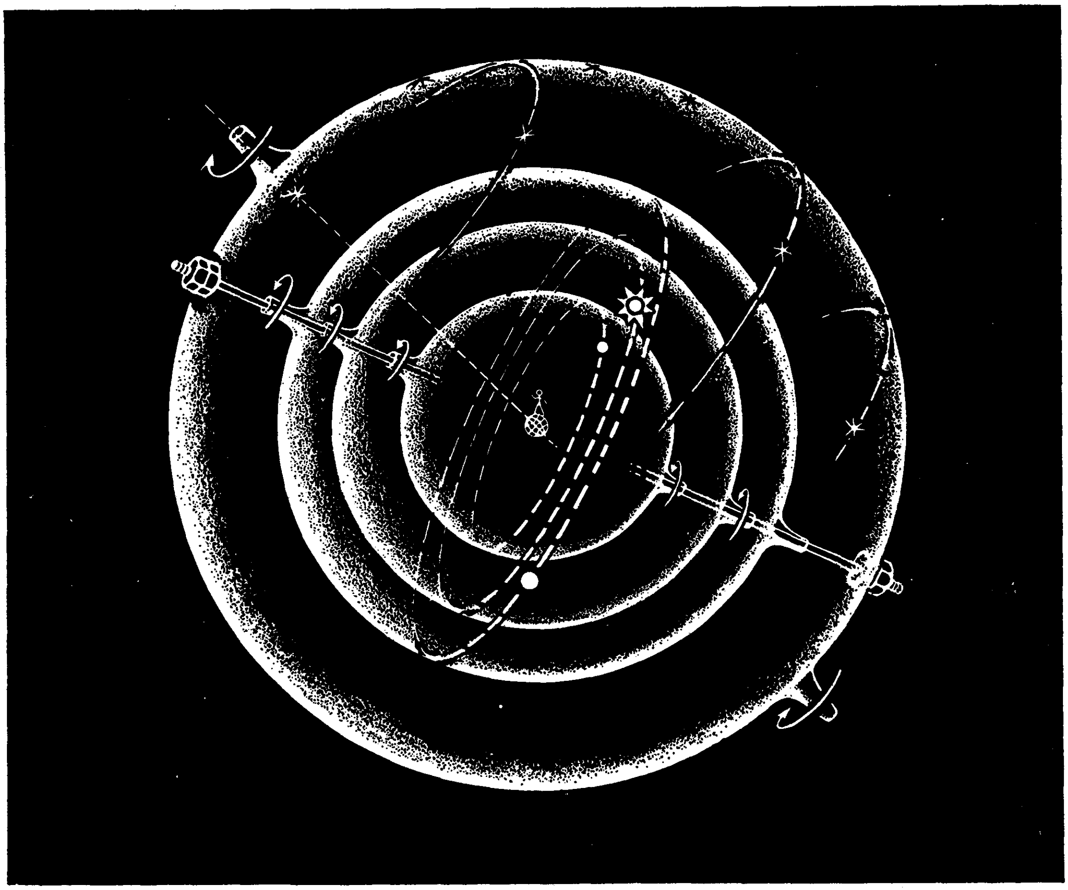
EARLY GREEK SYSTEM OF CRYSTAL SPHERES
A "section" of the whole system in the ecliptic plane.



PYTHAGOREAN VIEW
The Pythagorean school adopted spherical Earth; and separated the general daily motion of stars, Sun, Moon, and planets, from the slow, backward motion of Sun, etc., through the star pattern.

imitate the observed facts closely. The stars were placed like bright lamps on a great sphere, that rotated once in 24 hours about the pole-star axis, as in the simplest model. But the Sun was embedded in another sphere, inside the sphere of stars. That sphere rotated slowly the opposite way, taking one year to carry the Sun round the ecliptic. Another sphere carried the Moon round its Zodiac path in a month; and other spheres carried the planets in their general backward motion through the star pattern. All those spheres for Sun, Moon and planets revolved about an axis perpendicular to the ecliptic, making $23\frac{1}{2}^{\circ}$ with pole-star axis. And since their axle had its ends embedded in the outmost star-sphere, that sphere carried them all round with the daily motion. All these spheres were called "crystal spheres" -- meaning that they were made of transparent, invisible material like perspex. In the hands of able scientists they were imaginary -- and therefore completely transparent! -- but many people who learned about them thought of them as real transparent globes, grinding smoothly without friction around appropriate axles at the proper speeds to imitate what was seen in the sky. The power-house for that celestial machinery was supposed to be outside the outermost, sphere, (which carried the stars) in a heaven where gods and the souls of dead people resided.

For example, the sphere for Jupiter, with Jupiter embedded in its equator, revolved slowly backwards, making one revolution in a dozen years.



EARLY GREEK SYSTEM OF CRYSTAL SPHERES. (Pythagoras)

Part of the system, showing the rotating spheres of the Sun and two planets, carried around by the outer sphere of stars which spins daily.

Although this machinery did not give planets the "loops" in their motion it provided a reassuring picture that told people there is no frightening mystery: it is all simple machinery with clear steady motions.

Pythagoras probably knew that the Earth is round. The sight of ships sailing in over the horizon, and travellers' tales of the pole-star being higher in the sky when one was further north, suggested a round Earth.

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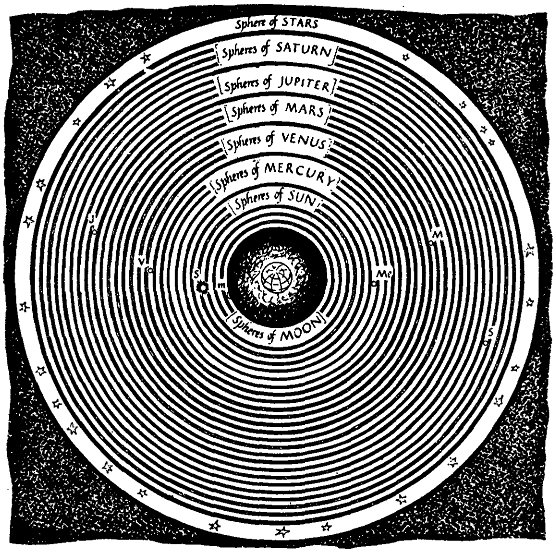
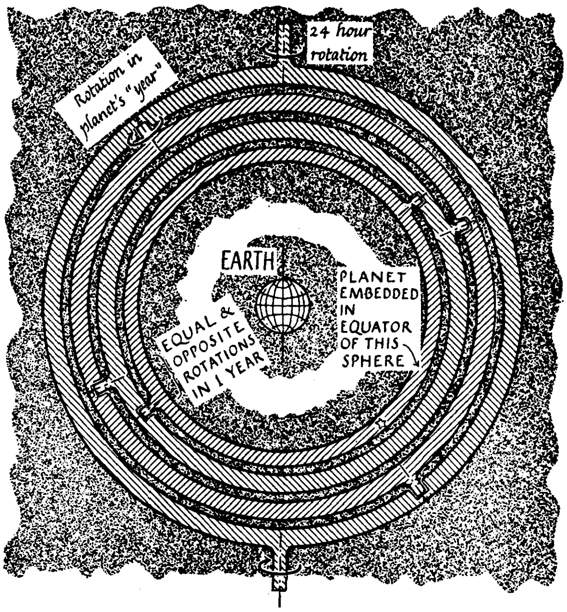
* This is the Pythagoras of the right-angled triangle theorem. The theorem was discovered much earlier, but we name it after him. He put forward a general view of nature, like Thales' suggestion of water, that <u>numbers</u> are the key to the knowledge of nature. He experimented with musical notes from a taut string (as in a harp or violin) and found that if the length of string is halved, the note goes up by an octave -- in physics, we now know the frequency of vibration is doubled. If the string is shortened to one-third the note goes up by an octave and a musical fifth above the original. So he said that 1/2, 1/3,... are the numbers that hold the secret of the pleasant harmonic scale that we still use in music. In astronomy, he again looked for simple number relationships. Among the times the planets take to travel right round the Zodiac. In his emphasis on number and relationships he encouraged an attitude which is still the backbone of physics; though we are less mystical about it today.

Eudoxus (about 370 B.C.) gathered together Greek and Egyptian knowledge of astronomy and devised a scheme that fitted the facts much more closely. He took the previous scheme of many spheres slipping round within each other and inserted more spheres, with different axes and motions. He was very clever at geometry in solid space, and he saw how to imitate the planets' looped paths by giving each planet four spheres. You need not bother about the details of that scheme; but, in case you are interested, the sketch shows the quartet of spheres for one planet.



EUDOXUS' SCHEME OF MANY CONCENTRIC SPHERES
Each body, Sun, Moon or planet, had several spheres spinning steadily around different axes. The combination of these motions succeeded in imitating the actual motions of Sun, Moon and even planets across the star pattern.



PART OF EUDOXUS' SCHEME:
FOUR SPHERES TO IMITATE THE MOTION OF A PLANET
The sketch shows machinery for one planet. The outermost sphere spins once in twenty-four hours. The next inner sphere rotates once in the planet's "year." The two innermost spheres spin with equal and opposite motions, once in our year, to produce the planet's epicycloid loops.

Note that the axle of each sphere is embedded in the next sphere outside it, tilted in such a way as to produce a good imitation of the real motion of the planet, which is fixed in the equator of the innermost sphere of the four. Thus the outermost sphere carries the three inner ones with its motion; the next inner sphere carries the innermost pair with its motion; and the innermost pair revolving with opposite motions, in an Earth-year, about axles that make only a small angle, manage to imitate the loops in the planet's observed path.

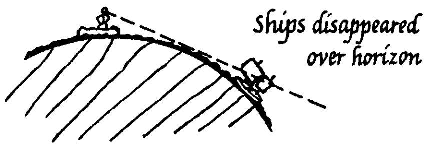
This model needed one sphere for the stars, three for the Sun, three for the Moon, four for each planet, 27 concentric spheres in all, like the layers of an onion, with a stationary Earth at the centre. This was a fantastic, successful model, with no shapes except simple spheres and no motions except rotations at constant rates. To fit the facts more closely, later thinkers added still more spheres.

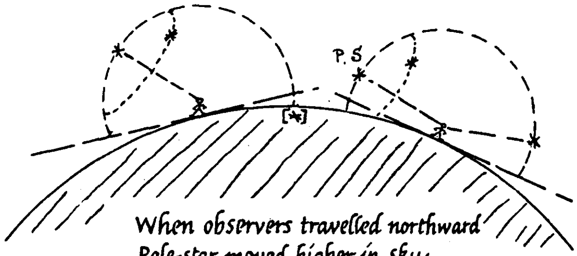
If it seems artificial and silly to imagine the heavens full of invisible smooth spinning spheres, with us at the centre of the onion, remember that this model did fit the facts: it gave a satisfying assurance that the events in the sky are reasonable and need not be viewed with superstitious fear. *

Aristotle (about 340 B.C.) favoured the scheme of spheres for a dogmatic reason: "the sphere is the perfect solid shape;" -- and this prejudiced astronomical thinking about orbits for centuries. He insisted that the heavens are perfect and unchangeable, and have circular motions as "natural".

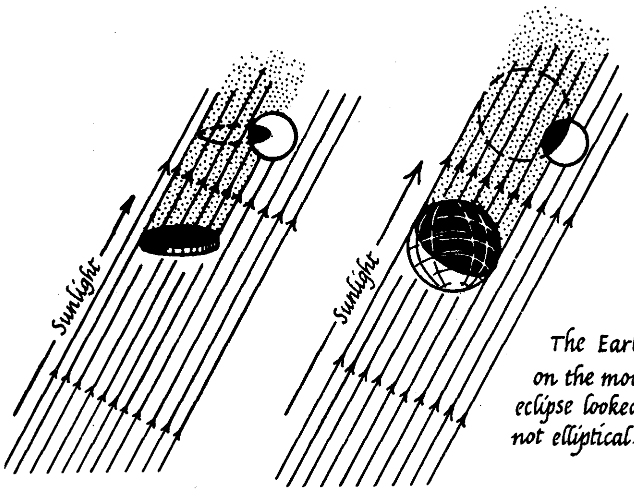
EVIDENCES FOR ROUND EARTH

ANCIENT





When observers travelled northward
Pole-star moved higher in sky;
other stars' paths tilted also, and
some southern stars became invisible.



The Earth's shadow
on the moon in an
eclipse looked circular,
not elliptical.

MODERN

Photographs from rockets

Flights around world

Geodetic surveys

. . . . . .

* And in fact Eudoxus was doing, on a grand scale, something that we do in modern engineering and physics: we add up circular motions -- or their components, simple harmonic motions, -- to imitate any complicated repeating motion. We do that to predict ocean tides at a seaport; we do that to imitate the behaviour of electrons in an atom. It is called "Fourier synthesis". Eudoxus used that 2,000 years before scientists and mathematicians re-invented it.

He also made a strong case for the Earth being round. He gave theoretical reasons:

(i) Symmetry: a sphere is symmetrical, perfect.

(ii) Pressure: the Earth's component pieces, falling naturally towards the centre, would press into a round form.

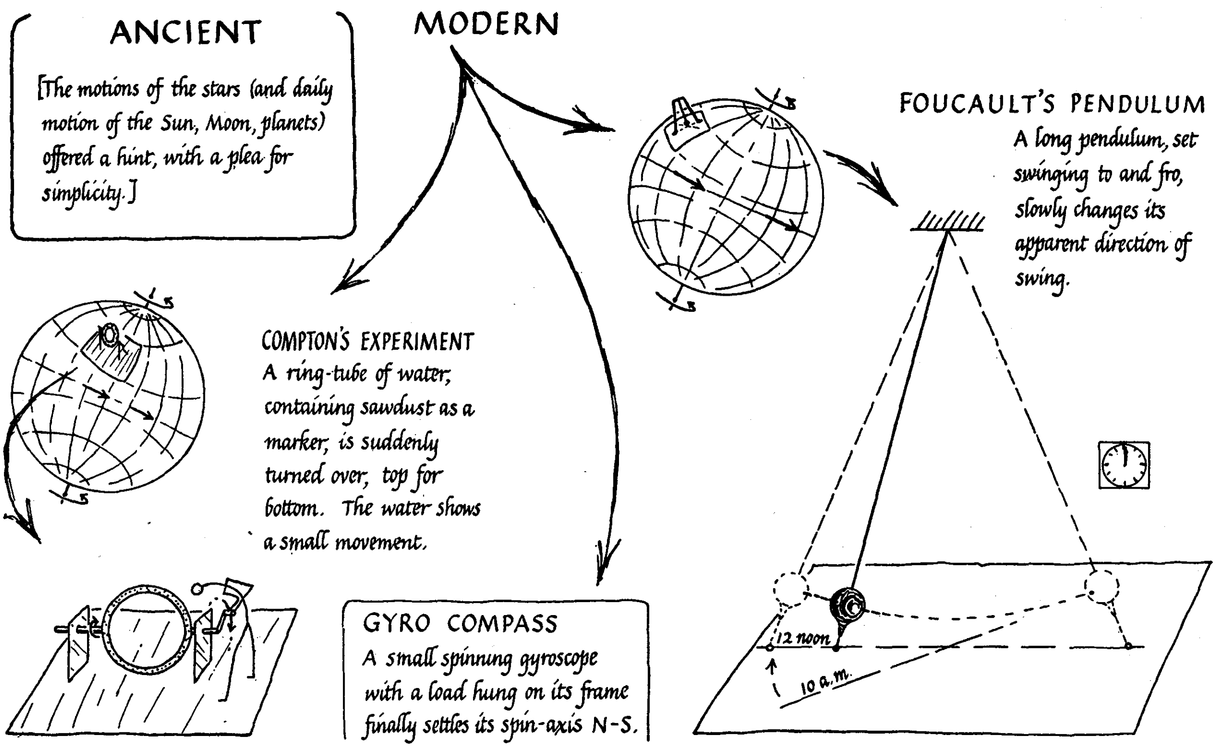
and experimental reasons:

(iii) Shadow: in an eclipse of the Moon, the Earth's shadow is always circular: a flat disc could cast an oval shadow.

(iv) Star heights: even in short travels Northward or Southward, one sees a change in the height of the pole-star.

Aristotle did much to set science on its feet. He catalogued scientific information and listed good questions. And he emphasized the basic problems of science, distinguishing between "true physical causes" of things and "imaginary machines to fit the facts".

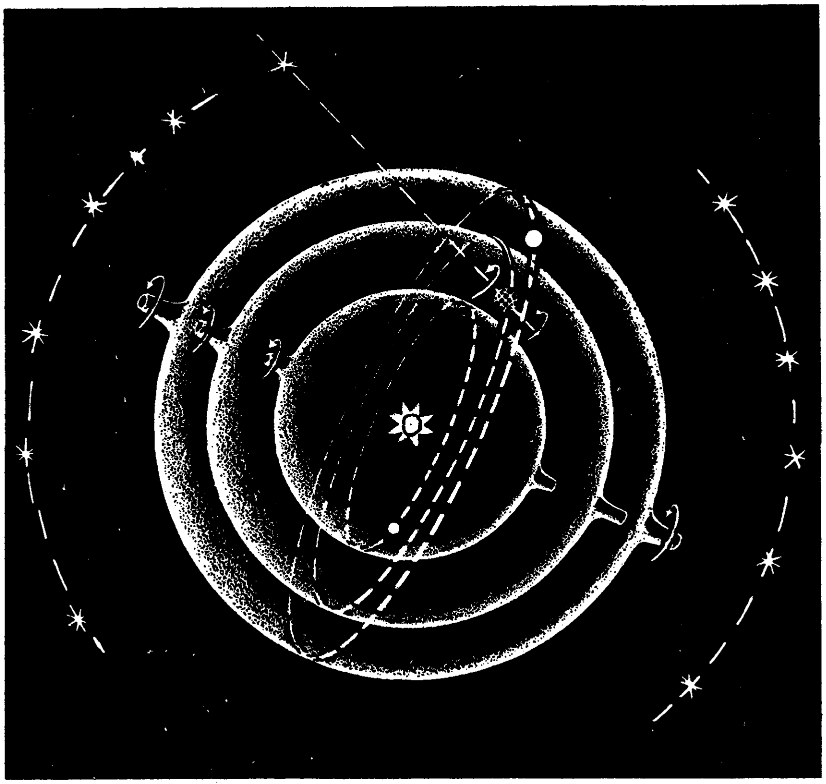
EVIDENCES FOR SPINNING EARTH



<u>A Different Greek View.</u> A few astronomers at Alexandria suggested quite a different scheme for the heavens. They pointed out that it was unnecessary to have the outer sphere of stars whirling round once in 24 hours, and carrying the other spheres for Sun, Moon and planets with it. All that daily motion could be removed if the Earth is spinning instead.

Although it was suggested, that was an unpopular idea because people did not understand the mechanics of living on a rotating planet with a central gravity pull. They thought that all moveable things would fly off a spinning Earth; or at least a falling object would not travel vertically down to the ground.

The other suggestion made by the same astronomers was that the Sun is the fixed centre instead of the Earth and the Earth is travelling in a circular orbit round the Sun. They pointed out that the Earth's motion would compound with the motion of a planet to manufacture the pattern of loops that we see. But that suggestion was equally unpopular: a moving Earth, hurtling through space on a great orbit round the Sun would leave moveable objects behind -- men, birds, clouds, ... would trail away in the wake of the Earth.



ARISTARCHUS' SCHEME
Only two specimen planets are shown

These early suggestions of what we now believe, a Solar System, were made by Aristarchus and others; and many centuries later they were revived and put into convincing form by Copernicus.

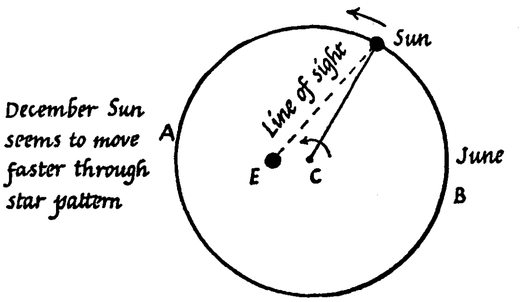
Simpler Earth-in-Centre Schemes:

"Wheels Within Wheels" and an Off-Centre Viewpoint.

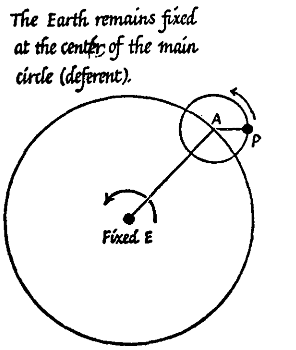
Although the concentric spheres gave a good imitation, that model was difficult to use for accurate predictions -- which were wanted for calendar keeping, navigation, and superstitious astrology. Simpler machinery was devised by astronomers in the great University at Alexandria.

Alexander the Great, building a huge empire, founded the city of Alexandria at the mouth of the Nile. Greek scholars collected there and set up a University which grew to be a great centre of learning. Astronomers there made actual measurements of the size of the Earth, and the distances of Sun and Moon from the Earth, turning astronomy into a more real science. (See a few pages later for an account of these measurements). They also devised simpler machinery to "fit the facts", again using spheres, though we shall use circles in our description. They had an outer sphere for the stars as usual and a stationary Earth at the centre. Their new machinery was devised to deal with Sun, Moon and planets in their motion through the star patterns, with the daily motion frozen out.

The Sun's motion round the ecliptic in one year is not quite at a constant rate. The Sun seems to move through the stars a little faster in our winter than in our summer, so the intervals between winter and equinox and equinox and summer are not quite equal. To imitate this uneven motion, the Sun was imagined to move round a circle at constant rate, but the Earth was placed a little off-centre. Then the Sun, viewed along a line of sight from the Earth, would seem to move a little faster at A in the diagram than at B.

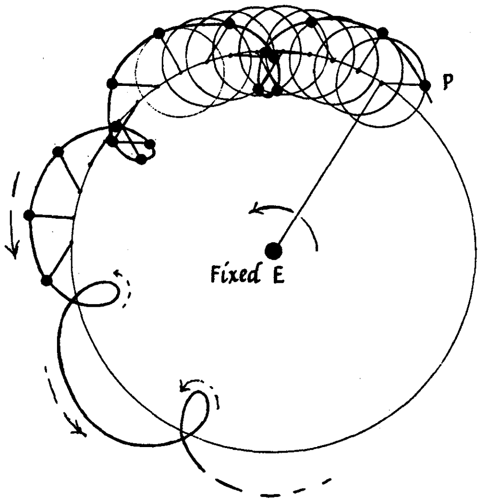


THE ECCENTRIC SCHEME FOR THE SUN
The Sun is carried around a circular path by a radius that rotates at constant speed, as in the simplest system of spheres. The observer, on the Earth, is off-center, so that he sees the Sun move unevenly—as it does—faster in December, slower in June.



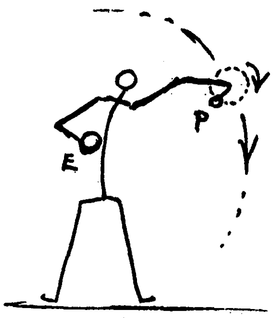
EPICYCLE SCHEME FOR A PLANET

The new machinery for a planet is shown in the sketch. The fixed Earth is placed at the centre of a large main circle (or a little way off-centre in some later improvements). A radius of that circle acts as an arm to carry, at its end, a small circle ("epicycle"). A radius of that small circle carries the planet round its circumference at a steady rate, while the arm of the large circle revolves at a smaller steady speed.

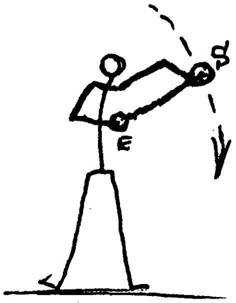


Making the Path of a Planet by the Epicycle Scheme. This sketch shows how the two circular motions combine to produce the epicycloid pattern that is observed for a planet.

Try imitating that. Hold a tennis ball or orange for the Earth at rest in front of your chest, with your left hand. Stretch your right hand out, holding a small ball (ping pong) to represent a planet. Make that planet move fairly quickly in a small circle by twisting your hand round and round your wrist. At the same time sweep your outstretched arm slowly to carry your hand round a large circle. The combined motion will show the planet moving in a circular path with loops. Then, to see the motion as we see it for a planet, an observer should watch that motion from one side, almost in the plane of the big circle.



Simple model for epicycle scheme. "Planet" held in hand moves fast round small circle, while hand sweeps slowly round large arc whose centre is fixed "Earth".
Hand simple revolves round wrist.



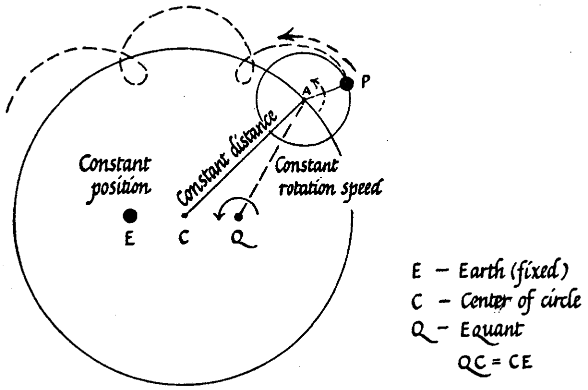
Simple model for eccentric scheme for "Sun". Hand carries "Sun" round large arc. "Earth" held fixed, a small distance off centre of arc. Elastic thread joining E and S shows speed changing with seasons.

Here was a simpler model for planets, just a large revolving arm carrying a small revolving arm. Yet as time went on it did not account for the details of planets' motions accurately enough to fit observations, which were growing in both number and accuracy.

Hipparchus (about 140 B.C.) collected careful observations and devised improved machinery to describe the heavenly motions. He is said to have catalogued the positions of a thousand stars, measuring angles to the nearest $1/6^{\circ}$. Remember he had no telescope -- those were invented seventeen centuries later -- but had to make measurements by sighting through peep holes on long jointed sticks, like a giant pair of dividers.

Precession of the Equinoxes. By comparing his measurements with older records dating back some centuries, Hipparchus discovered a strange slow motion of the whole star system "The Precession of the Equinoxes". At the spring equinox, midway between winter and summer, the Sun is at a definite place in the zodiac, in a well known constellation there. But Hipparchus found that the Sun does not return to exactly the same place at the next spring equinox but to a place about $1/70^{\circ}$ earlier in the zodiac. He saw that the zodiac girdle must be slipping round the celestial sphere very slowly, carrying all the stars with it, leaving the celestial equator fixed with a fixed Earth. This motion of the whole sphere of stars round the ecliptic axis (not the usual north-south axis) takes 26,000 years for a complete revolution -- yet it matters in astronomical measurements and has always been allowed-for since Hipparchus discovered it. This strange extra motion is not something you need to remember, yet later it will have a marvellous explanation.

Ptolemy's Successful Machinery (about 120 A.D.). The mathematician and scientist Ptolemy reviewed the records and studied the machinery and produced a machine that did match the facts with wonderful accuracy. It was the same as the previous epicycle scheme except that the Earth was set at a small distance from the centre of the main circle and a new point was marked an equal distance on the other side. That point, the equant point, Q , was imagined to carry an arm that ran out to A , the end of the arm of the main



THE PTOLEMAIC SCHEME
This system imitated the motions of Sun, Moon, and planets very accurately.

circle. Instead of having the main circle's radius CA revolve at constant rate, Ptolemy had the equant arm QA rotate at constant rate. You can see from the sketch that QA will change in length as A goes round the circle; so one has to imagine that arm sliding through a knob at A.

Ptolemy devised this more complicated scheme because he found it would fit the facts very well. He described it in detail in a great book "The Algamest"; and that book and Ptolemy's machinery remained the authority for describing and predicting the motions of Sun, Moon and planets for many centuries.

Comparing this machinery with the first crude model of a single sphere, we do not find it so easy to picture, yet it has the essential characteristics of good science; it runs by keeping certain things constant. * The main circle has constant radius, the arm from the equant point Q revolves with constant speed, the Earth is in a constant position, at a constant distance off-centre; the radius of the sub-circle stays constant and revolves with constant speed.

No "ultimate cause" was given for this machinery or its motions. Planetary motions were presumably started by gods and perhaps maintained by gods; and there was no link between them and the motions observed on Earth.

The planets were just bright stars moving in the starry pattern. Their real distances were unknown and no one knew whether they were much nearer to us than the stars, or even which ones were nearer than others. Ptolemy's system could place Jupiter nearer than Mars, or Mars nearer than Jupiter, equally easily. However, since Jupiter moves backward through the star pattern so much slower than Mars, Greek astronomers guessed that Jupiter is

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* Scientists make descriptions of the behaviour of things in nature, and then make use of those descriptions in designing engines and developing further knowledge. Many of those descriptions take the form of laws or rules which sum up the results of many experiments or observations.

In finding general laws or rules for the ways in which things behave, scientists are continuing what a child does when he first explores the world around him. He gathers general rules, even if he does not put them in words, "Hot stoves burn." "Knives cut." "Ice cream is cold and tastes nice." Those beginnings of science are not bits of useful knowledge: they are much needed insurance against insecurity. If ice cream were cold on some days, hot on others, sour on still others -- all in a random way -- children might avoid ice cream. And, if many things in the world around changed in a wild way from day to day or place to place, children would feel insecure in such an unreliable world -- that way madness lies. We are kept sane by finding rules which say things like "Ice cream is usually very cold and nearly always tastes nice".

In scientific laws we press the certainty farther: "All springs always stretch proportionally to load -- up to a limit." And those laws can be stated with the word constant as the key word in the account they give of nature. You will find that you can reword almost any law in physics, chemistry, and most other sciences with the word constant in it. e.g. stretch/ For example, stretch/load = constant, for wires; P.V = constant, for gases at constant temp.

Look for the word "constant" in the laws and models for the planetary system: it is the sign of scientific simplicity and certainty, the offer of a guarantee against insecurity.

much farther away, just as we guess relative distances of cows, trees, etc., seen from a moving train. In fact the order of planetary distances they guessed at agreed with what we know today; but they had neither an experimental reason for it nor any idea of the proportions within that order.

Thus, Ptolemy had a magnificent scheme of circles, arms, sub-circles and arms, which could reproduce the heavenly motions so accurately that it could be trusted to predict the positions of Sun, Moon and planets for centuries to come. In fact it continued in use, with occasional corrections, for more than a dozen centuries. And for practical purposes, navigators and astronomers would use a scheme of that form today. It was neither stupid nor clumsy, it was very clever and accurate, a successful machine.

Yet as machinery the complete Ptolemaic system was quite complicated. Remember that the line ECQ can take different directions from the fixed Earth E for different planets, and the "eccentric distance" EC can have a different value for different planets. So, if you sketch Ptolemy's arrangement for the Sun and several planets all on one picture you will find it complicated. You should not despise it but if you develop a headache over its complexity you are ready to be sympathetic towards the great change suggested by Copernicus.

<u>Greek Measurements.</u>

Nowadays we can find out how far the Moon is from the Earth by radar (timing a pulse of radio waves to the Moon and back). Or, by placing a reflector on the Moon and timing a flash of laser light there and back we can find the Moon's distance with fantastic precision, to the nearest foot.

How did the Greeks make even rough measurements twenty-two centuries ago, with no lasers, no radar, no wireless time, no telescopes, and only a small part of the world explored? Before Hipparchus and Ptolemy perfected their machinery, Greek astronomers at Alexandria estimated the size of the Earth (known to be a sphere); estimated the distance, and therefore the size, of the Moon by an ingenious method based on eclipse shadows; and attempted a rough estimate of the distance of the Sun.

<u>Radius of the Earth.</u> <u>Eratosthenes</u> (about 240 B.C.) made an early estimate of the size of the Earth. He used parallel rays of the sunlight as a standard direction and measured shadows at two stations a known distance apart. He needed to measure the shadows at the same instant of time at both stations. So he chose Alexandria and Syene *, 500 miles farther south, because he already had a special piece of information about Syene in the Library at Alexandria: a record by a traveller told him that at Syene at noon on a midsummer day, 22 June, sunbeams falling on a deep well go down to the water and are reflected straight up again. Therefore, the noonday Sun must be vertically overhead at Syene on that day. At noon on the same day

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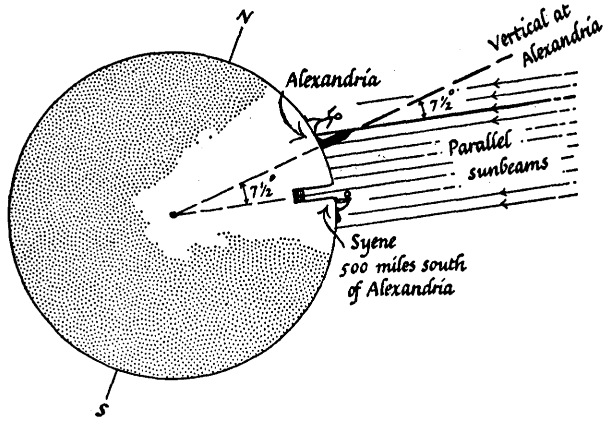
* Modern name: Aswân, where the great dam has been built on the Nile.



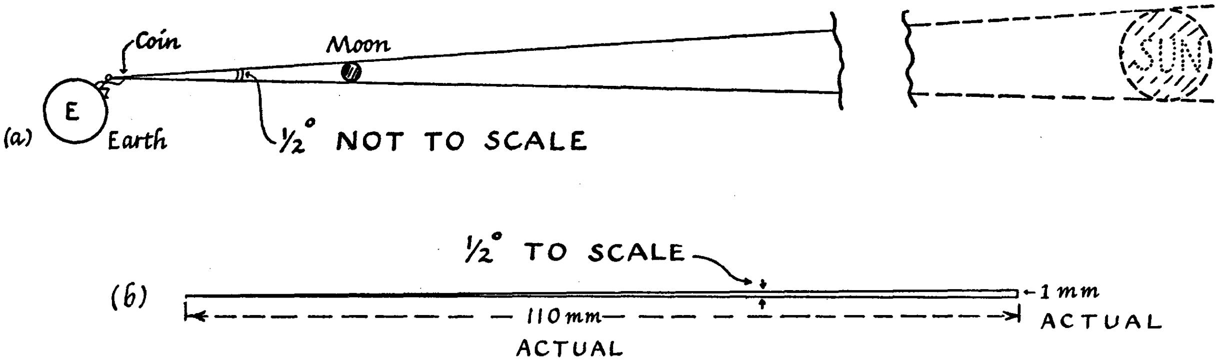
of the year, Eratosthenes measured the shadow of a tall pillar at Alexandria and found that the Sun's rays made $7\frac{1}{2}^{\circ}$ with the vertical. He assumed that all sunbeams reaching the Earth are parallel, so the Earth's radii, which have different directions at those two stations, in fact must make an angle of $7\frac{1}{2}^{\circ}$, as you can see from the diagram. Then, if 500 miles of Earth's circumference subtends $7\frac{1}{2}^{\circ}$ what length subtends 360°? From the circumference, he calculated the radius of the Earth. What radius do you get from this simple sum?

Measuring the 500 miles was difficult -- probably a military measurement done by professional pacers. Eratosthenes' error in the Earth's radius may have been as small as 5% -- a remarkable success for an early attempt.

<u>Your Own Measurement.</u> You can make your own estimate of the size of the Earth if your school co-operates with another school several hundred miles away, more or less on a north-south line. Each school should set up a pole of known height, say 10 feet, vertical, as shown by a plumb line. Then pupils at each school observe the direction of sunlight at a particular instant, (preferably noon), on the same day; by measuring the shadow of the pole. Then the two teams compare notes. That can be done by postcard correspondence, but it is much more thrilling to do it at the time, by arranging a telephone call to link the two stations.

The fraction (shadow length)/(pole height) gives the tan of the angle the Sun's rays make with the local vertical. Each team calculates that fraction, and finds the actual angle from trig. tables. The difference between those angles is the angle at the centre of the Earth. The teams also need to know the distance between them. One way of finding that is from a railway timetable. Or a long journey by car can be measured. It is not good to take the distance from the latitude-lines on a map, because in placing those lines the map-makers assumed Eratosthenes' measurement!

Size and Distance of the Moon. Look at the full Moon and hold a small coin at arm's length, moving it nearer and farther until it just blots out the Moon. You will find the coin is then about 110 coin-diameters from your eye. By the same proportion, the Moon must be 110 Moon-diameters away.

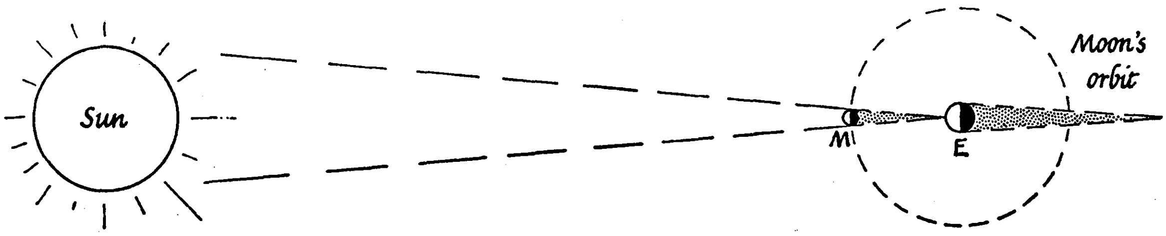


RELATION BETWEEN SIZE AND DISTANCE can be found by holding measured coin at measured distance. This does not tell us absolute size or distance. Sketch (a) is *not* to scale. Sketch (b) shows the "angular size" of Sun and Moon *drawn to scale*. Measurements show the Sun and Moon each subtend about ½° at Earth. Measurements or trigonometry tables give a proportion of about 1:110 for base:height.

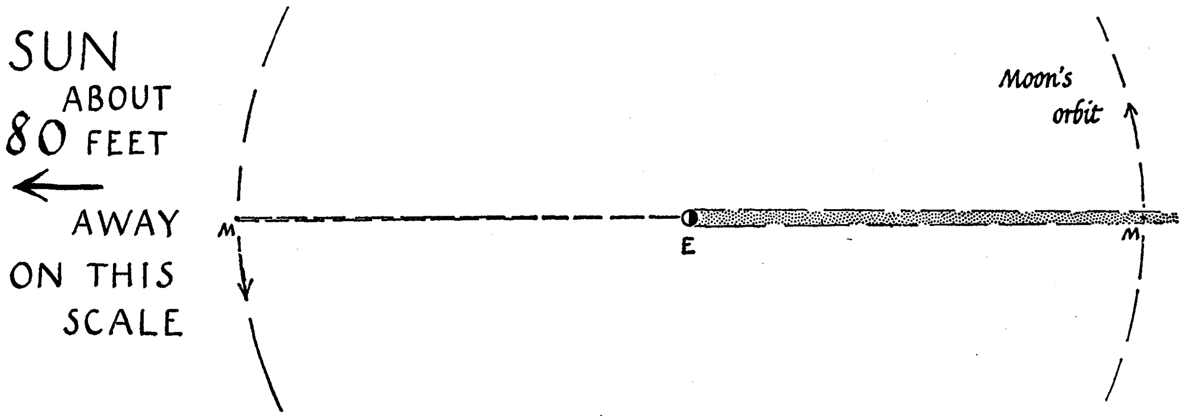
By an odd chance, the Sun looks just about the same size as the Moon. A coin held up to blot out the Sun is about 110 diameters away; so the Sun is about 110 Sun-diameters away -- although it is much bigger and farther than the Moon.

To find the Moon's actual distance we need another measurement as well. An early Greek method was to measure the time the Moon spends in a total eclipse and work out from that the size of the Earth's shadow in Moon-diameters.

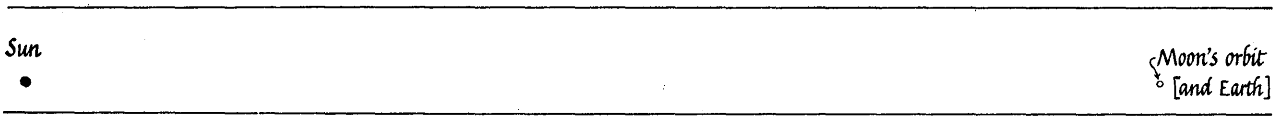
In most books that describe eclipses the diagrams are drawn with quite incorrect proportions, like diagram (a) here. The only good thing about that sketch is that it does show how the Moon's shadow just tapers to a point by the time it reaches the Earth. (We know that because the Moon and Sun look the same size to us; and, in eclipses of the Sun, the Moon only just manages to blot out the Sun and make a total eclipse occasionally). The



a. SUN, MOON, EARTH. Sketch *not* to scale. The Sun is shown much too near, and the Moon is too big and too near.

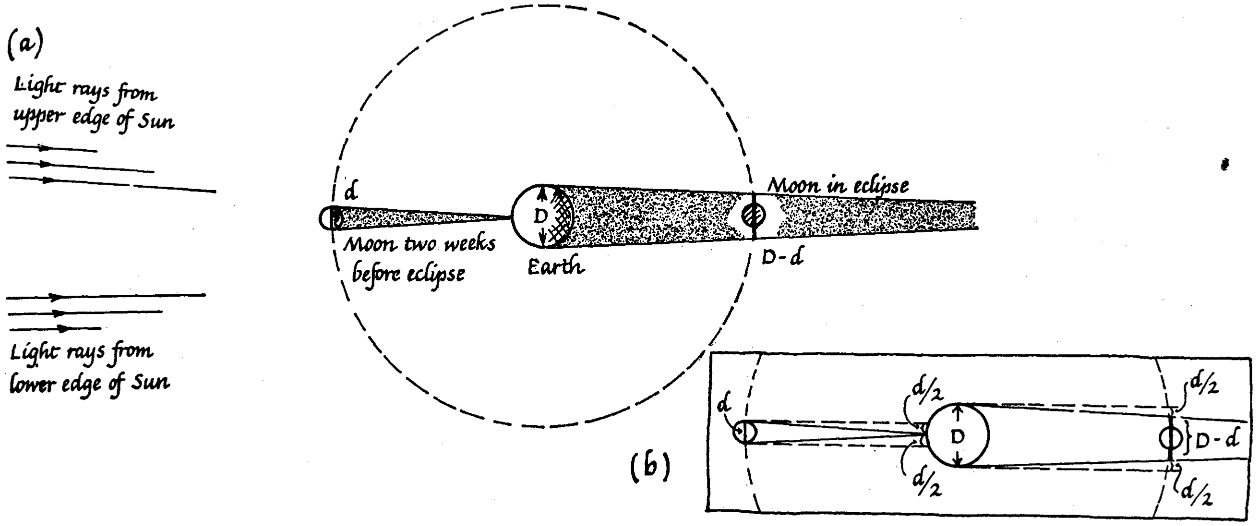


b. Sketch to scale showing the shadow cones of Moon and Earth.



c. Sketch to scale. Here the scale has been reduced so that Sun, Moon and Earth are in the picture. The small circle is the Moon's orbit. The Earth, at the center of that circle, is too small to show. On this scale it is a dot 1/1000 inch in diameter. The Moon is much too small to show.

true proportions are shown in diagram (b). There you must imagine that you can see the Moon's shadow tapering to nothing at the Earth and the Earth's shadow also growing smaller, with the same angle of taper.



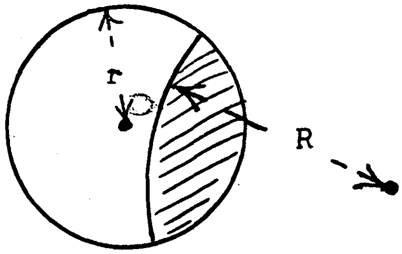
Early Greek Measurement of Size of the Moon (and Therefore Its Distance). Observations of eclipses showed that the width of the Earth's shadow at the Moon is 2.5 Moon-diameters. However, the Earth's shadow narrows as its distance from Earth increases because the Sun is not a point-source. Since the Moon's shadow almost dies out in the Moon-Earth distance, the Earth's shadow must narrow by the same amount—one Moon-diameter—in the same distance. Then Earth-diameter must be 3.5 Moon-diameters.

By watching a total eclipse of the Moon, the Greeks found that the Earth's shadow is $2\frac{1}{2}$ Moon-diameters wide, out at the Moon. That shadow has had the whole radius of the Moon's orbit in which to taper, so it is wider just behind the Earth. And, since we know that the tapering makes a shadow lose one Moon-diameter in that distance, we expect the Earth's shadow to be $3\frac{1}{2}$ Moon-diameters just behind the Earth. Therefore the Earth's diameter is $3\frac{1}{2}$ Moon-diameters. The Greeks, already knowing that the Earth's diameter is about 8,000 miles, could then calculate the Moon's diameter. Multiplying by 110 gave the Moon's distance, about 240,000 miles.

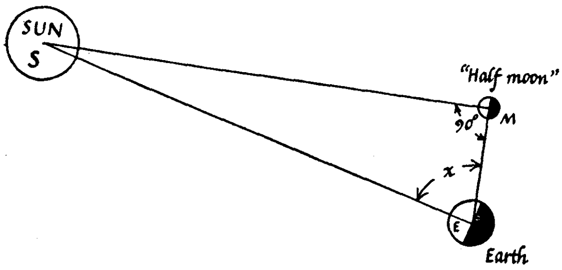
<u>Your Own Estimate of the Moon's Size and Distance.</u> You could try this yourself with a photograph of a partial eclipse of the Moon. By making measurements on the photograph you could estimate the proportion between the radius of the Moon and the radius of the Earth's shadow which you see taking a bite out of the Moon. That would tell you the proportion of Moon-diameter to Earth-diameter, but for the effect of tapering shadow which makes the Earth's shadow lose one Moon-diameter.

Therefore

$$\frac{r}{R} = \frac{\text{Moon-diameter}}{\text{Earth-diameter -- one Moon-diameter}}$$



<u>Estimating the Sun's Size and Distance.</u> Greek astronomers made a clever attempt but the result was hopelessly inaccurate. They waited until they saw the Moon <u>exactly</u> at half-moon and tried to measure, at that instant, the angle between the Moon's direction and the Sun's direction.
As you can see from the diagram, when the Moon is exactly half-moon to an observer on the Earth, the angle at M must be 90°. If angle X is measured, all three angles of the triangle are known. Then one can draw a scale diagram, or one can use trigonometry, to find the proportion of ES to EM. The Greek estimate of angle X was 87°, but the correct angle is about 89.85° -- much nearer to 90°. So the Greeks were wrong by a factor 20; and this error in the Sun's distance -- and therefore in the scale of the whole Solar System -- held for many centuries.



SUN'S DISTANCE
Early Greek estimate of the Sun's distance from the Earth, in terms of the Moon's known distance. Greek astronomers tried to measure the angle x (or SEM), which is itself nearly 90°.

Astronomy After the Greek Age.

For many centuries, astronomy was taught with the authority of books, in the form that Ptolemy had set it forth. That was part of the general teaching of mathematics that was given to church scholars; and it was kept alive by the need to train navigators and by strong superstitious interest in astrology. But people were occupied with other matters; and astronomy was largely taught without question.

From time to time Ptolemaic machinery was given new radii and revised periods of rotation, to bring it into still better gear with observations.

Then, after a dozen centuries, the devout monk Nicholaus Copernicus made a far-reaching suggestion which changed the whole picture of the heavens.

CHAPTER IV. NICHOLAUS COPERNICUS (1473-1543)

Nicholas Copernicus was born in what is now Poland. He was brought up by his uncle who was bishop of the cathedral nearby, practically the ruling prince of the district. After school and university, Nicholas was sent on a long trip abroad to continue his education. He went to Italy to study church law because his uncle planned to secure him an administrative post in the Church.

He stayed on in Italy, reluctant to leave because he enjoyed his life there and had developed a tremendous interest in astronomy. He visited Rome and gave lectures on mathematics, which included astronomy. He read descriptions of Greek schemes for the Sun and Moon and planets -- not just Ptolemy's system, which was generally taught, but earlier schemes as well. He found that a *few* Greeks had suggested a very unpopular model with the Sun as a fixed centre instead of the all-important Earth, and the Earth travelling round the Sun in a circular orbit, and at the same time spinning round its own tilted axis once in twenty-four hours. That had seemed unbelievable to most Greek astronomers because they thought a spinning and moving Earth would fling things off and leave the clouds behind as it moved. And people felt it undignified and uncomfortable not to have all-important man and the Earth at the very centre.

But Copernicus saw that such a scheme would provide a simpler picture of the heavens. He thought about it more and more, trying to devise tests and to show how such a model could be described in detail. If the Earth is spinning, that would account for the motion of the whole star pattern round the pole-star; and it would explain why the Sun, Moon and planets also have that motion, apart from their individual backward motions. If the Earth and all the planets travel round the sun they can all move steadily round circular orbits.

Copernicus was called back to Poland and settled down as a quiet devoted monk, looking after church affairs. But he continued to think about his idea for a simpler model of the World. He felt that God's scheme in creating the World would be a simple one, not something as complicated and unsymmetrical as Ptolemy's machinery.

So Copernicus felt that his work in describing a simpler astronomical scheme was all to the Glory of God; it was a search for the real truth.

Copernicus' Solar System

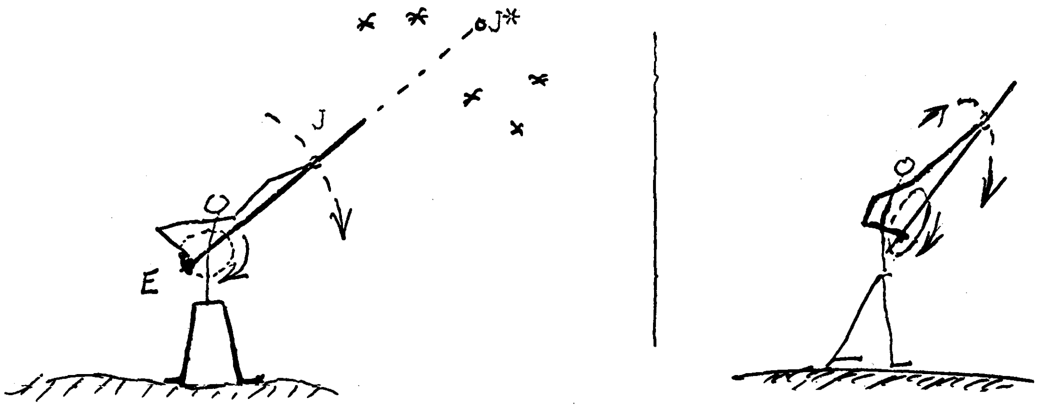
As time went on, this quiet, sincere monk, who believed so strongly in truth, described the heavens as he felt God had built them: the Sun is the fixed centre and each planet moves round the Sun in a circular orbit -- Jupiter in a dozen years, Mars in a couple of years, Venus in less than a year, and so on. The Earth itself is a planet, moving in a circular orbit round the Sun in one year. The Earth also spins about its north-south axis making one revolution in 24 hours.

The Earth's spinning accounted for the nightly motion of all the stars and the Sun, Moon, and planets together. It was no longer necessary to think of the stars as stuck on a great revolving shell that whirled them across the sky every night; they could just hang in space while we watch them from our spinning Earth.

Looped Paths of Planets

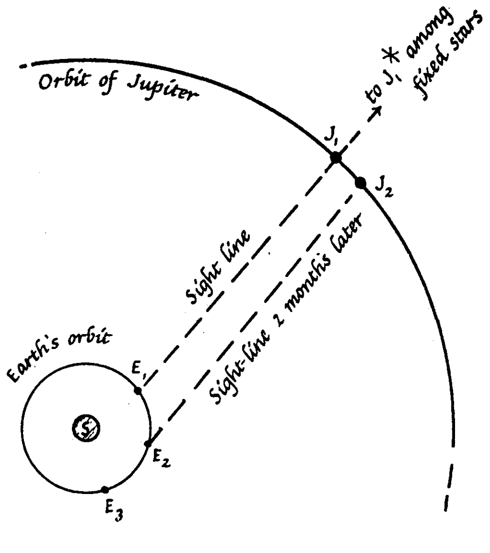
If the Earth goes round the Sun in a circular orbit, and the other planets move in similar circles, the Earth's motion will account for the "loops" that we see a planet make as it travels along the zodiac. The planet itself is moving steadily along its circle but the motion of the Earth (from which we observe the planet) tilts our sight-line (which runs from Earth to planet and on to remote stars) to-and-fro. That tilting motion added to the planet's steady motion gives the planet's path a "loop" for each of our years.

Try this experiment yourself with a long pole. Pretend the Sun is just

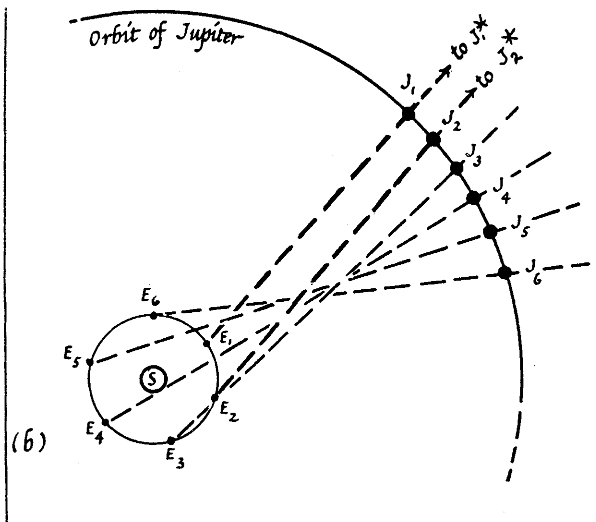


Copernicus' Explanation of Planetary Epicycloids

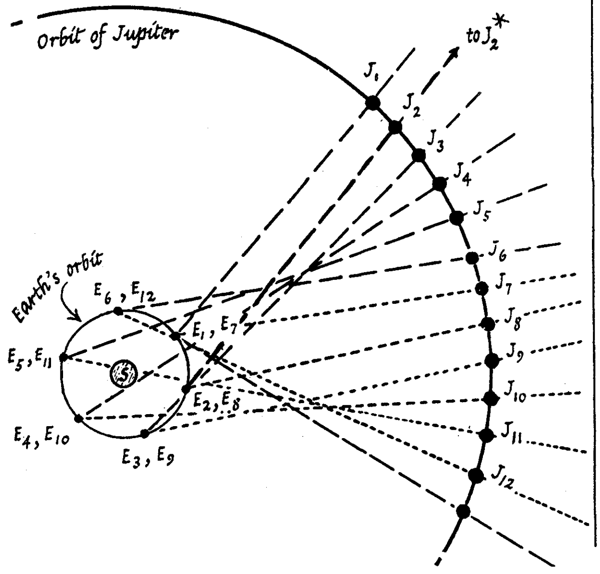
The lines E_1J_1, E_2J_2, etc., are sight-lines from positions of the Earth every two months through Jupiter's position towards the stars.



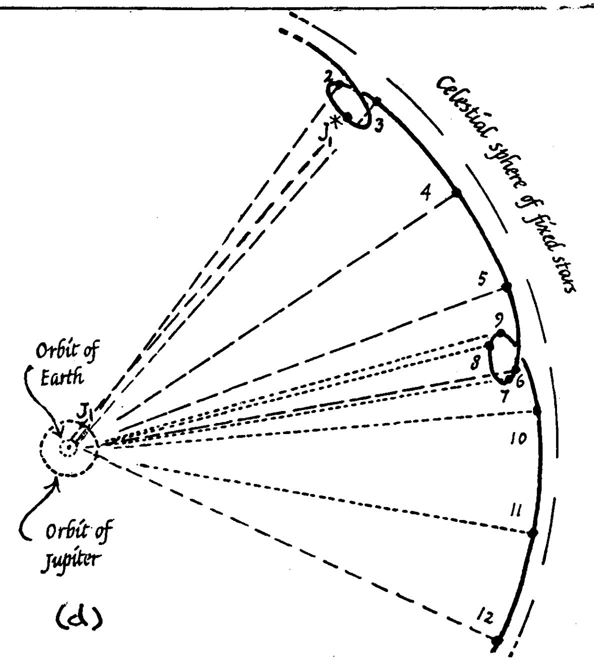
(a) Two stages sketched



(b) More stages sketched.



(c) Many stages sketched. The sight-line EJ wags up and down in a complicated way.



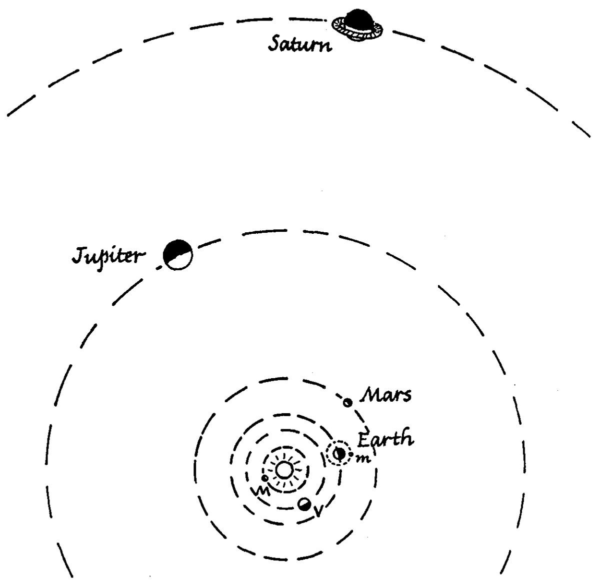
Copernicus' Explanation

The apparent positions of Jupiter in the background of fixed stars. This shows Fig. (c) redrawn on a much more condensed scale with the sight-lines from Earth to Jupiter continued on out to the stars. (E.g. the line to J_1^* here is continuation of EJ_1.) The specimen sight-lines are drawn parallel to the corresponding ones in Fig. (c).

in front of your chest. Move your right hand round the "Sun" in a vertical circle, fairly quickly, to represent the motion of the Earth in an orbit round the Sun. Stretch your left arm out and move your left hand round a larger circle, more slowly, to represent the motion of a planet, "Jupiter". When you have practised making these two motions, make them while you hold one end of the pole in your right hand and let the pole slide through the fingers of your left hand. Then the pole represents a line-of-sight from an astronomer on the Earth to Jupiter and on out towards the distant stars. Watch the other end of the pole, as you make the motions for "Earth" and "Jupiter", and imagine how that astronomer would see Jupiter moving through the pattern of stars. Imagine that the long pole continues out to the walls of your room and that the stars are hung all over those walls.

Copernicus' Scheme

Here is a sketch of the scheme that Copernicus suggested.



COPERNICUS' PLANETARY SYSTEM

So far, Copernicus was merely following the suggestions of some almost-forgotten Greek astronomers. But now see what he added by thinking about his model and using some observations.

Other Successes

Mars' brightness explained. The planet Mars, the next one out beyond the Earth, looks much brighter at some times than others. Greek models could not give a reason for this great change of brightness; but Copernicus pointed out that on his scheme Mars would be much nearer the Earth when they are both out to the same side of the Sun, than when they are on opposite sides of the Sun -- and those are the stages at which Mars looks brightest.

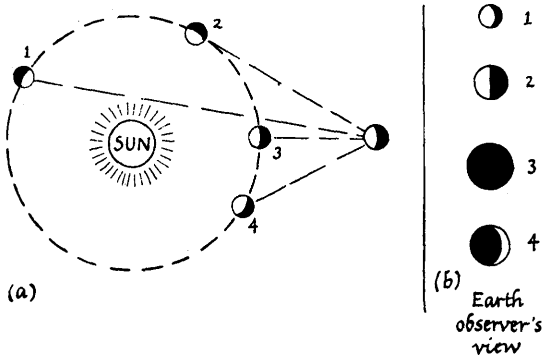
Venus' phases predicted. Copernicus knew that, according to his scheme, the planet Venus would show "phases", like our Moon, changing from a crescent to "half-moon" to "full-moon" and so on in the course of travelling round the Sun. With Ptolemy's scheme, Venus would never show that full series of phases. Unfortunately, Venus is too small and far away for us to see such patterns of sunlight and shadow with our naked eye; and Copernicus could not foresee the invention of telescopes which now show us just those "phases of Venus". Outer planets, with orbits beyond the Earth would not show phases, because we see them always in full sunlight, from our position in an inner orbit.

Copernicus also pointed out that a planet must seem to make a "backward loop" whenever the Earth is on the same side of the Sun as the planet. Think carefully about your experiment with the long pole, you will see that this should be so -- and records of observations show it is so for every planet (except, of course, the Earth itself!)

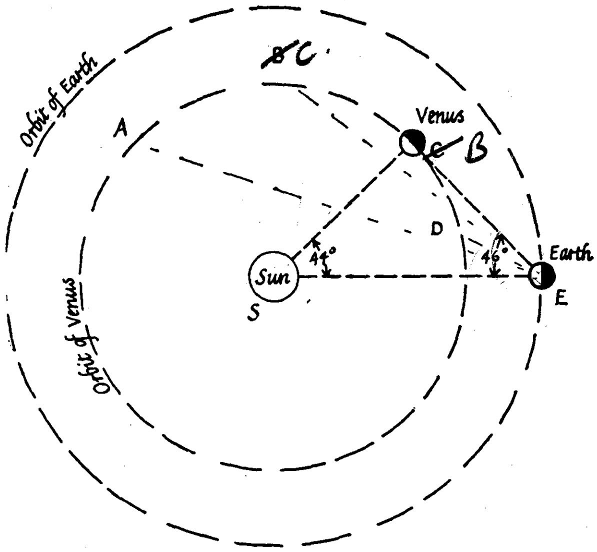
Orbit Sizes

Now see how Copernicus' model gives us new information. That is one of the characteristics of a good theory or model of Nature. We pour ideas and assumptions and facts into the building of a theory and then the theory gives us back information -- measurements or relationships -- which are true if the assumptions, etc., are true. The theory, so to speak, continues its own story; it adds a new chapter, but necessarily a chapter about the people in its own story. Copernicus was able to say: If my model is correct, I can tell you the sizes of the orbits of planets; or, rather, the relative sizes. He could draw a scale model of his Solar System, giving the planetary orbits correct proportions. None of the Greek models could do that. The quartet of Eudoxus' spheres for one planet could go inside the quartet for another, or outside. The Greeks could not even tell the order in which the planets were arranged, going outward from the Earth. Ptolemy's main circle and sub-circle for a planet could be enlarged to place any planet beyond any other planet. The Greeks did guess at the relative order; they thought that the longer a planet takes to go round the zodiac the farther away it must be -- an intelligent guess; but it was pure good luck, for all they knew, that it did place the planets in the right order. So when Copernicus calculated relative sizes of orbits he made a model that seemed more real and convincing: he placed the planets in a definite order because he knew their relative distances from the Sun.

To see how Copernicus found the relative sizes of orbits, look at his method for the planet Venus. We see Venus as a bright morning star or evening star, never very far away from the Sun. If you watch Venus carefully and make rough measurements you will find that Venus is never farther away from the Sun than 46°. From that Copernicus calculated relative orbits. In the diagram, when Venus is farthest from the Sun at B, the sight-line from Earth to Venus must just be a tangent to the orbit of Venus. Then that line, EB, is perpendicular to the radius SB . From the right angle triangle,



PHASES OF VENUS, AS SEEN FROM THE EARTH



ESTIMATING RELATIVE RADII OF ORBITS
Venus is shown farthest from the sun.

SB/SE must be the sine of 46° or 0.72. * Then the radius of Venus' orbit must be 72/100 of the radius of Earth's orbit.

The same method was used for the planet Mercury. For the planets outside the Earth's orbit, Mars, Jupiter, and Saturn, the geometrical argument was a little more complicated but essentially the same. Then Copernicus could draw a scale model of his Solar System. Here is his list of planets with the proportions of orbit radii, taking the Earth's orbit radius as 100.

Mercury 39 Venus 72 Earth 100 Mars 152 Jupiter 520 Saturn 960

Although Copernicus could draw a scale model of the Solar System, he did not know its real size at all accurately. For that he needed a measurement of one distance and all he had was a very inaccurate Greek estimate of the distance from Earth to Sun.

If you wished to make a scale model of the Solar System you would need to know the sizes of the planets themselves and those were not measured until much later, when telescopes were available and the magnified images of planets could be measured. If you like to anticipate those measurements you might arrange the following for a rough model: place a football to represent the Sun; then a grain of sand the size of a small pinhead 40 feet away, to represent Mercury; and an apple-pip 72 feet away to represent Venus, etc., thus:

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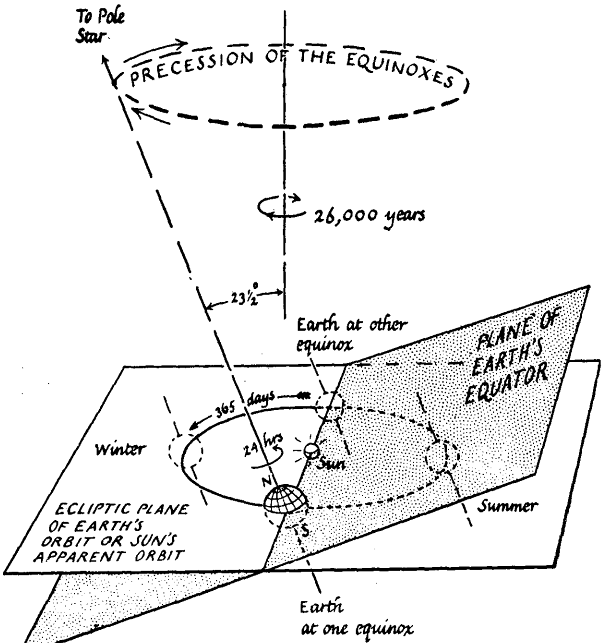
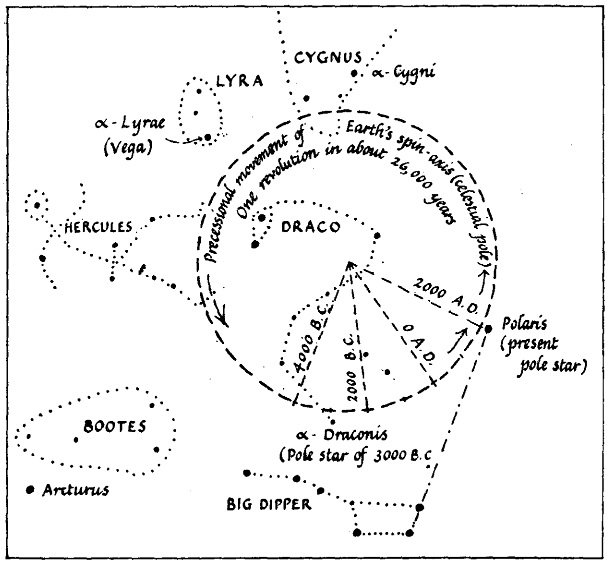
* If trig. tables are not available you could still draw a right-angled triangle with one angle 46° and measure its sides to find the proportion 72/100. Copernicus had trig. tables, as did the Greeks long before him; and for some of his work he had to extend his trigonometry to formulae for triangles on a sphere.

Sun	football	
Mercury	small pinhead	40 ft. from "Sun"
Venus	apple-pip	72 ft.
Earth	apple-pip	100 ft.
Mars	pinhead	150 ft.
Jupiter	ping-pong ball	520 ft.
Saturn	ping-pong ball	960 ft.
Our Moon	pinhead	3 ½ inches from "Earth"

Precession. Copernicus made his model yield one more piece of help; it provided a simpler description of the precession of the equinoxes. The Greek astronomer, Hipparchus, had discovered that slow creeping motion sixteen centuries earlier, and astronomers had continued to make allowances for it; but it was a difficult motion to picture. Copernicus pointed out that it means just this: The Earth's north-south axis, about which the Earth spins daily, slowly swings round a 23½° cone, taking 26000 years. The axis of that cone is the axis of the ecliptic, an axis perpendicular to the ecliptic. Till now, we have called the ecliptic the Sun's yearly path; but now, as Copernicus would put it, the ecliptic is the plane of the Earth's yearly path round the Sun. That conical motion of the spinning Earth, Copernicus' picture of precession, seemed simpler, but he could give no reason for it, any more than did the Greeks.



Sketch Showing Motion Called the Precession of the Equinoxes



The Precession of the Equinoxes
Sketch of a large patch of Northern sky (about 90° by 90°), showing the slow movement of the celestial North Pole among the stars. The point where the Earth's spin-axis cuts the pattern of the stars moves slowly around a roughly circular path making one revolution in about 26,000 years. (After Sir Robert Ball.)

Copernicus' Great Book. Copernicus stayed at home, working for the Church and thinking about his system of the world. Scholars interested in astronomy came to visit him and discuss his ideas. He was persuaded to publish a small booklet describing his scheme; but he was modest and hesitant about publishing a scheme so different from the one the people held. He wanted to wait until he was sure. But he finally put all his work into a great book. We still have the actual manuscript in his handwriting. In that he described his system, gave his reasons in support of it, worked out tables to show how it could be used for predictions, and added a chapter on spherical trigonometry. The manuscript was sent across to Nuremberg in Germany to be printed. Remember that Ptolemy's great book had had to be copied by hand from generation to generation but by now (1541) printing with moveable type was available. The book was set up in type, corrections were added, and it was ready to be printed. Copernicus was a very old man, in poor health. A cowardly editor added a preface saying that the scheme was a hypothesis, an imaginary idea, and not necessarily true. Nothing could be farther from Copernicus' own view. He was devoutly sure that his scheme was true, God's work of great simplicity. In his own dedication of the book to the Pope he showed no doubt, though he worried about people objecting.

When the book was finally printed, Copernicus was dying. It is said that the first copy was rushed across Europe to him and that he touched it on the day he died.

The book was called De Revolutionibus Orbium Coelestium, meaning "Concerning the Revolutions of the Heavenly Spheres"; and we regard it still as one of the great books of science.

The book was a bombshell: an unexploded one in its original form, which was difficult to read. Copernicus wrote in Latin -- he probably thought in Latin when he was considering astronomy -- because Latin was the international language of scholars. So his book was only read by people with special interests. Not until his scheme was published in popular language, almost half a century later, did the bombshell explode. We shall meet that when we come to Galileo. Then people found a great change of ideas forced upon them, to the dismay of many. They were appalled to find the Earth moved out from being the glorious Center of the Universe and placed as a planet among the rest -- the earth made common and ordinary.

To fit the facts closely, Copernicus had to place the Sun a little off center, and to add some small circles to modify the main-circle motion a little. But those were minor additions to his simple arrangement of circles -- in contrast with Ptolemy's sub-circles which were major necessities.

Copernicus' scheme, the Solar System, is the scheme we believe today. We now know that the planets' orbits are ellipses and not circles -- and that is why Copernicus had to make minor shifts. But, for the actual orbits of the planets, each ellipse is so near to a circle that it is small wonder that Copernicus' simple scheme of circles almost fitted.

Scientists use models or theories like maps* to help them find their way or to describe their explorations, or to direct another scientist on his way. We like to choose the simplest model that suits our purpose -- just as a tourist chooses his map -- and we do not worry too much whether it is true, so long as it fits the facts. Ptolemy's scheme for the heavens fitted the facts; and in making tables or calculations for navigation of ships or aeroplanes we would still use a Ptolemy pattern of "things as seen from the Earth." On the other hand, a space traveller voyaging among the planets would find it easier to use a Copernican model, a Solar ("helio-centric") System.

And when we come to Newton's general theory, we find that the Copernican model fits in well with the rest of our scientific knowledge, whereas a Ptolemaic ("geo-centric") system, with the Earth at the centre, would still need special machinery to keep it going — machinery quite different from the machinery of science on the Earth. So we are glad that Copernicus, that quiet thinker who loved the truth, wrote his great book to give us his model of the Solar System.

CHAPTER V. TYCHO BRAHE (1546-1601)

The next development of astronomical theory owed much to an astronomer with tremendous skill and success at observing. So far, observations had been gathered and recorded in a rather haphazard way. Astronomers made observations when it was convenient and though they took reasonable care their records were not always very accurate. Tycho Brahe spent his life making good instruments and using them with fantastic skill and tremendous enthusiasm, to measure the position of planets on a systematic timetable of work. As a result his student and successor, Kepler, could discover precise reliable laws of planetary motion that then served as foundations for Newton's theory.

* If you visit London, or live there, you will find the map of London UndergrounD very helpful. On that map in many colors, the lines run due north and south or east and west, with a few lines slanted at 45°, and even the River Thames follows a simple course. If you look at a map of bus routes instead, you see all the crooked complexity of the streets. That map can be useful, too. But if someone asks you "Which of the two is the right map?" You would call it a silly question and say that the answer depends on what you want the map for. Theories in science are maps of our thinking and knowledge. Which theory -- or which model, of the heavens, or a molecule, of an atom -- is the right one depends on what we want to use it for. We try to choose the simplest model.

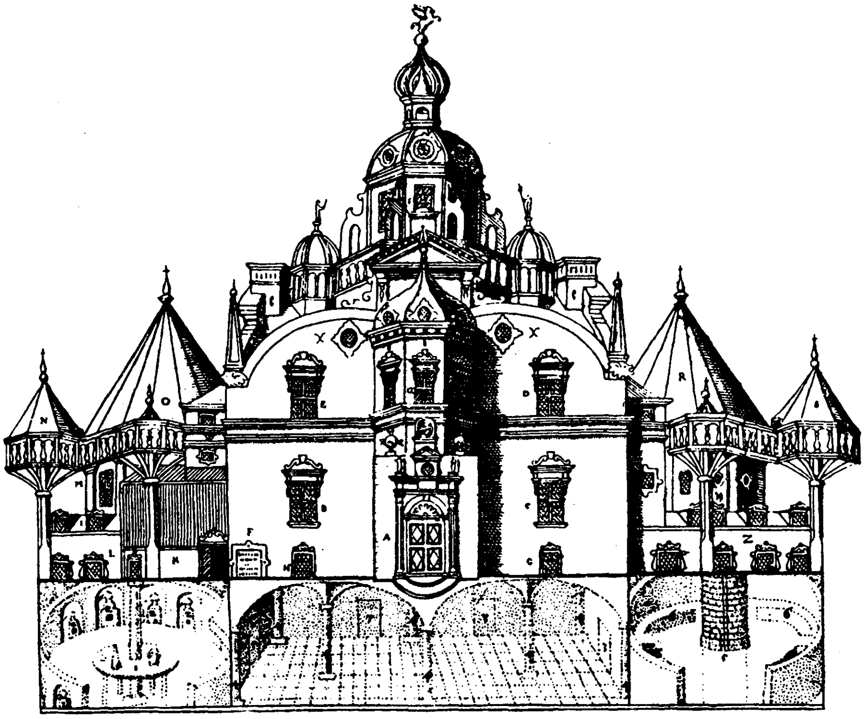
Tycho's family were noble people in the Court of Denmark in Copenhagen. Hunting and fighting were main interests of court life but Tycho was brought up by an uncle with intellectual interests, who valued good education. Tycho began Latin at 7, because his uncle considered that an early start towards a possible career as a lawyer. At 13 he went to the University to learn philosophy and law; but there he suddenly became fascinated with astronomy.

Astronomers had predicted an eclipse of the Sun; and crowds turned out to watch for it with great excitement. When it happened at the predicted time, young Tycho was thrilled, not just with the wonder of the eclipse itself but with the idea that astronomers could predict it. It seemed to him amazing that men could know beforehand the exact time of an event high up in the sky, far out of man's reach or influence. He longed to understand the science that could do such wonders and he decided astronomy was to be his work -- and he made it a magnificent life's work. He continued to study law; but his heart was set on astronomy, and he spent pocket money on tables and a translation of Ptolemy's book.

After the University, his uncle sent him on a tour of foreign travel with a tutor, to continue his study of law. But he sat up secretly at night observing stars and planets; and he bought books and instruments with all the money he could get.

Then another astronomical event changed his enthusiasm. He foresaw that the planets Jupiter and Saturn would soon <u>seem</u> to pass very close to each other in the star pattern. That "clashing" of two planets was called a "conjunction". It could be predicted with the help of tables such as Ptolemy's or Copernicus'. Superstitious people considered a conjunction fateful, threatening strong good luck or bad luck. (Tycho himself, as superstitious as any, later believed that this conjunction foretold the great plague that swept across Europe). Tycho watched impatiently for the conjunction to occur, as the two planets seemed to move closer to each other in the star pattern night after night. To his disappointment, he found Ptolemy's tables predicted the date wrong by a month; and Copernicus' table was wrong by several days. Like many a young person he was angry at those inaccuracies and he felt a fiery determination to set things right. He decided to spend his life measuring the motions of the planets with such care and such skill that he could make really accurate predictions and give the rival theories such precise tests that he could decide between them. And he succeeded with a vengeance.

When he returned home his family gave a chilly welcome to this young man with a strange interest in stars; so he went back to Germany alone and gathered other young astronomers to help him make huge instruments for accurate observing and use them for measurements. He needed to measure each planet's celestial latitude and longitude very accurately; those precise measurements of angles would need huge instruments so that each degree

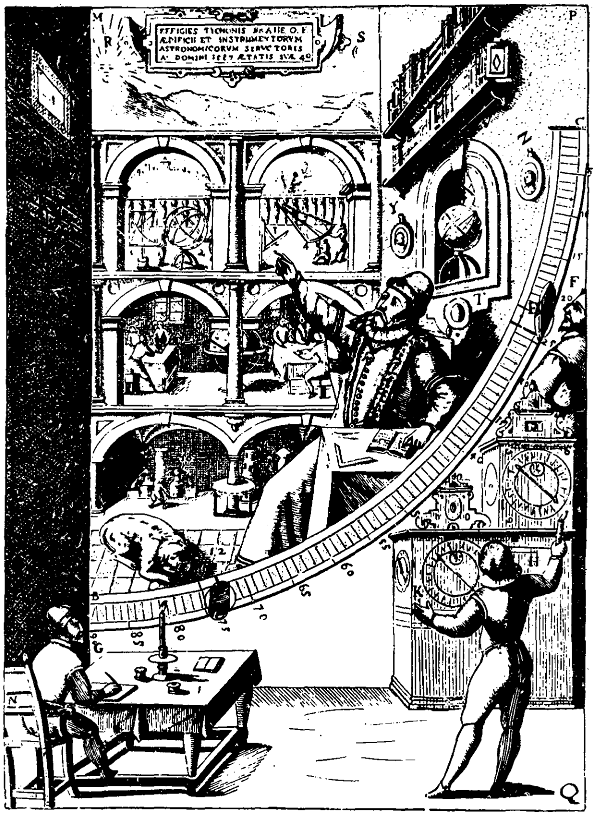


URANIBORG. Design of the main building, built about 1580.

on their scale of angles could be subdivided into small fractions. He saw that if his instruments were to yield reliable measurements he must make them very robust as well as large. And then he must mark them with fine marks for the graduations of angles. But he knew that those marks could not be placed with perfect accuracy. So he made his instruments with very fine clear marks and then tested them, "calibrated" their scales, making a careful table of corrections to be applied at each angle reading.

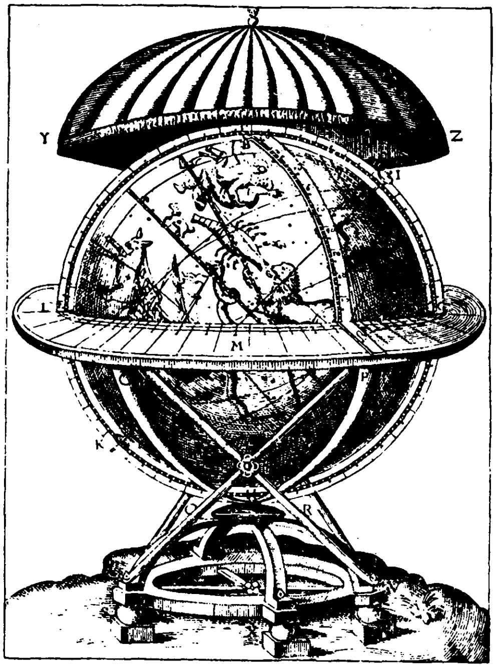
As a boy of seventeen he realized what the professional astronomers had missed, that a long series of precise observations was needed to establish astronomical theory. Haphazard records of observations made at random could not decide between one system for the heavens and another. Here was the start of his life's work.

Tycho again returned home; was better received, as a scientist of growing fame; but he soon threatened to travel abroad again and continue astronomy. The King of Denmark, hearing of Tycho's fame offered to provide him with a magnificent observatory if he would stay. So, using royal endowments and his own fortune, Tycho built a palatial observatory on an island off the shore of Copenhagen. He spent 20 years there, building a tremendous record of accurate observations, all made with naked eyes (telescopes had not yet been invented) using instruments which he designed and constructed in the workshops of his palace. It was a fantastic life of



Tycho's Mural Quadrant
The huge brass arc was firmly fixed in a western wall, with its center at an open window in a southern wall. The empty wall above the arc was decorated by a huge painting showing: Tycho observing; students calculating; Tycho's globe, books, dog; and some of Uraniborg's main instruments. An observer sighted the star (or Sun) by pinholes at F and a marker in the window. The brass arc (radius over 6 ft.) could be read to 1/360 degree. This sketch, from Tycho's own book, shows an observer at F, a recorder, and a timekeeper with several clocks. Good clocks had not been invented, but these were the best Tycho could make.

GLOBUS MAGNUS ORICHALCICUS

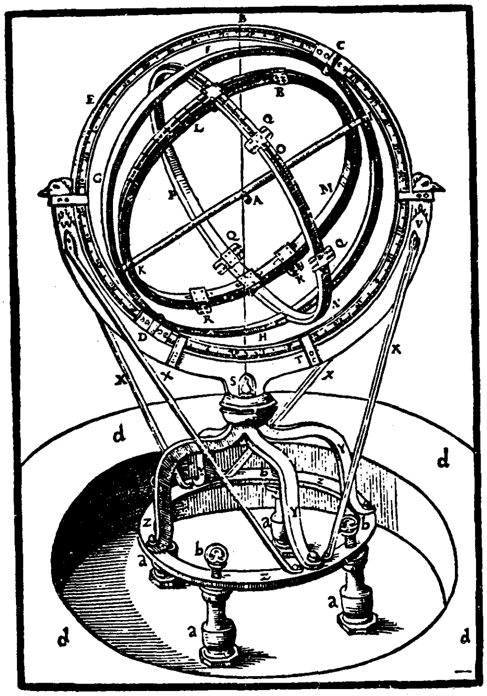


Tycho's Great Globe
Tycho had this globe made very carefully, at great expense, so that he could mark his measurements of star positions on its polished brass surface. He ordered it before he started Uraniborg, had it brought and installed there, and took it with him when he moved to Prague.

constant work and observing with fanatical insistence on accuracy. Students came from far across Europe to work with him; and he drove them hard to make observations and reduce them to records, cross-checking between one observer and another, and between one instrument and another. It was a fantastic life of enthusiastic work with fanatical insistence on accuracy.

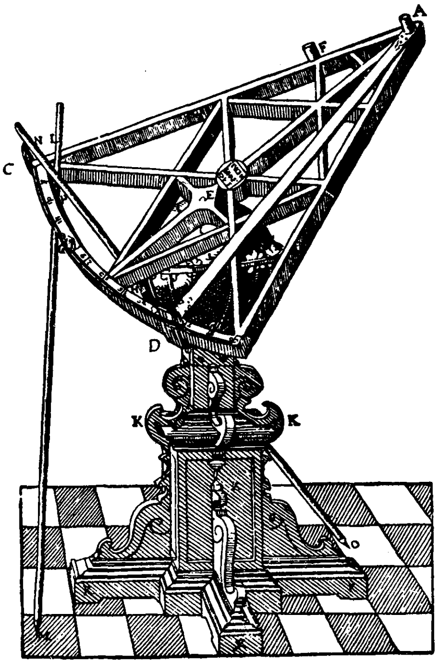
After 20 years, the King died and complaints about Tycho's management of his estates grew stronger until Tycho was forced to close his observatory and leave his island. To find a new royal patron, he published a book describing his instruments -- from which some pictures are shown here. The Emperor Rudolph , who was then living in Prague invited Tycho to be Imper-

ARMILLÆ ZODIACALES



AN "ASTROLABE" BUILT BY TYCHO, following the design used by Hipparchus. This instrument measures the latitude and longitude of a star or planet directly. (Diameter of circles: about 4 ft.) Tycho built several improved forms, with one axis parallel to the Earth's polar axis.

SEXTANS ASTRONOMICUS TRIGONICUS PRO DISTANTIIS RIMANDIS



ONE OF TYCHO'S SEXTANTS
This instrument, with brass scales and wooden frame, was used to measure the angle between the directions of two stars, by two observers sighting simultaneously along arms AD, AC. It was carried on a globe which could twist in firm supports, so that it could be tilted in any direction. (Length of arms: about 5 ft. Angles estimated to 1/240°.)

ial Astronomer (and Astrologer!); so Tycho moved his household and many of his great instruments to a castle near Prague. There he carried on the same wild life of observing and recording, though his health was failing.

The young mathematician, John Kepler, asked if he might come and visit Tycho to experiment with some theories. Tycho welcomed him, and finally bequeathed his records to him, begging Kepler to see his great records published. Those records, which Kepler did publish, catalogued the motions of Sun and planets with an accuracy quite unknown before. Tycho himself considered his measurements trustworthy to one minute, 1/60 of a degree.

Tycho in his last illness cried out "Oh, that I may not appear to have lived in vain!". This doubt was undeserved by an astronomer who had catalogued a thousand stars so accurately that his observations are still used, a man who recorded the planets' positions for 20 years with superb accuracy a man who gave Kepler, and Newton in turn, the essential basis for their development of astronomical theory.

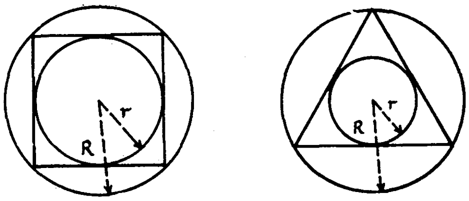
John Kepler was the young mathematician who travelled across Europe to visit Tycho and ask for his records of Mars, to test theories of planetary orbits. They worked together in the last few years of Tycho's life, forming a strange contrast: Tycho, "rich, noble, vigorous, passionate, strong in mechanical ingenuity and experimental skill, but not above the average in theoretical power and mathematical skill"; and Kepler, "poor, sickly, devoid of experimental gifts, and unfitted by nature for accurate observation, but strong almost beyond competition in speculative subtlety and innate mathematical perception". Tycho's work was well supported by royalty, at one time magnificently endowed; Kepler's material life was largely one of poverty and misfortune. They had in common a profound interest in astronomy and a consuming determination in pursuing that interest.

Kepler had grown up as a poor boy with poor health, struggling as a Protestant in a largely Roman Catholic world. He studied philosophy and religion in the university and was offered a post to teach mathematics and astronomy, which he accepted unwillingly, saying he hoped for something better. Once started, however, he threw himself into a study of the planets and, as he said, he brooded with the whole energy of his mind on the subject.

He was fascinated by puzzles concerning numbers and sizes; and he was determined to find the mathematical scheme underlying the behaviour of the planets. He was sure that God had created the world with a system of numbers according to mathematical laws and he set out to discover those laws. His mind burned with questions: "Why are there only 6 planets?" Why do their orbits have just the proportions and sizes they do?" "Are the times of the planets' 'years' related to their orbit-sizes?"

Orbit Sizes? Kepler's first question "Why just 6" is characteristic of Kepler's times. Nowadays we should just hunt for a seventh; but then there was a magic in numbers. *

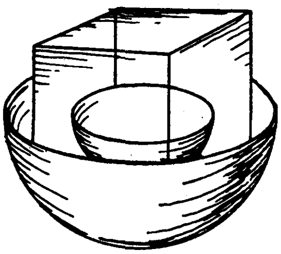
Kepler tried to find geometrical patterns that would predict the relative proportions of planetary orbits, which he obtained from Copernicus. Geometry was the fashionable form of mathematics, so he tried fitting a square between two circles, as a promising way of predicting the proportion of one pair of orbits. You can see that this pattern would make the orbit ratio R/r have a value $\sqrt{2}$ or 1.41. No pair of orbits among the planets



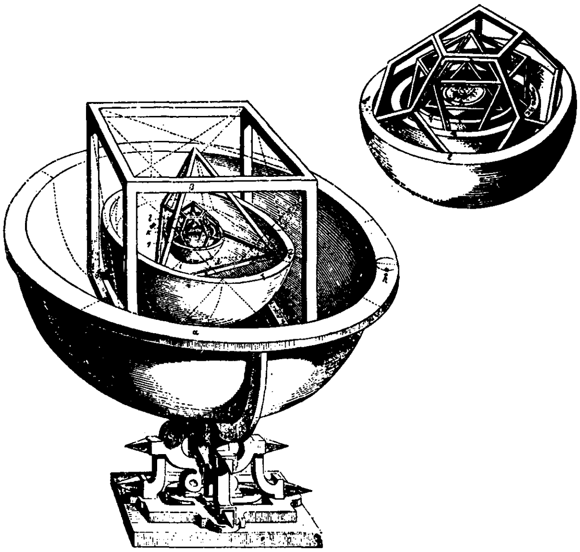
KEPLER'S FIRST GUESS
A regular plane figure (such as a square) can have a circle inscribed, to touch its sides. It can also have an outside circle, through its corners. Then that outside circle can be the inner circle for another, larger plane figure. The ratio of radii, R/r, is the same for all squares; and it has a different fixed value for all triangles. Geometrical puzzle: what is the fixed value of R/r for the inner and outer circles of a square?

* The Ptolemaic system counted seven planets (including Sun and Moon, excluding the Earth) and even had reasons to prove seven must be right, as a lucky number. On Copernicus' system only six planets were known: Mercury, Venus, Earth, Mars, Jupiter, Saturn.

will give the proportion 1.41, so Kepler had to start again with other shapes such as a triangle or hexagon between two circles. Again he failed and then says he had a brilliant inspiration: to use solid figures enclosed between spherical shells. Greek mathematicians had proved long before that only five regular solids are possible in our three-dimensional world -- a regular solid is a shape that has all edges equal, all face angles equal, all corners the same and all faces the same. Look at the pictures and try the problem if you wish to see why there must be a limit of 5 such solids. Kepler imagined a cube sitting in a sphere, its eight corners just touching the sphere, and another sphere inside the cube just touching the centres of the cube's faces. The radii of those two spheres gave him just about the right proportion for the orbits of Saturn and Jupiter. Using the other four solids to space spheres apart, he had a scheme of five solids, bounded by six spheres, which predicted the planetary orbit sizes reasonably well, and indeed gave a "reason" for there being only six planets.



This shows the basis of Kepler's final scheme. He chose the order of regular solids that gave the best agreement with the known proportions of planetary orbits.



Kepler's Scheme of Regular Solids, From his Book

The relative sizes of planetary orbits were shown by bowls separating one solid from the next. The bowls were not thin shells but were just thick enough to accommodate the *eccentric* orbits of the planets.

As a young astronomer he was simply delighted by this discovery and determined to extract other secrets from records of planetary observations. That was when he decided to visit Tycho.

We might laugh at Kepler's mystical scheme of solids; yet to him it was a wonderful discovery and good theory, in keeping with the spirit of the time. In fact we even have some such mystical schemes in modern physics. We do now discredit Kepler's scheme for two reasons: first, he could only provide for the six planets then known -- two centuries later another planet was discovered, and that spoiled the scheme. Second, when, a century later, Newtonian theory developed, it linked Kepler's other three laws with our general knowledge of mechanics on Earth; but it offered no link with the 5-solid law. Then, a law that remained like an orphan outside the family

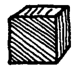
THE REGULAR SOLIDS. A geometrical intelligence test

How many different shapes of regular solid are possible? To find out, follow argument (a); then try (b).

A regular solid is a geometrical solid with identical regular plane faces; that is, a solid that has:

all its edges the same length
all its face angles the same
all its corners the same
and all its faces the same shape.
(See opposite for shapes that do not meet the requirements.)

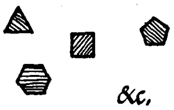
FIG. 18-2.



For example, a cube is a regular solid.

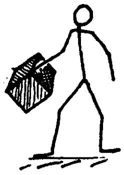
The faces of a regular solid might be:

all equilateral triangles
or all squares
or all regular pentagons
or . . . and so on . . .



(a) Here is the argument for square faces. Try to make a corner of a regular solid by having several corners of squares meeting there.

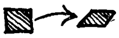
We already know that in a cube each corner has three square faces meeting there. Take three squares of cardboard and place them on the table like this, then try to pick up the place where three corners of squares meet. The squares will fold to make a cube corner.



Therefore we can make a regular solid with three square faces meeting at each of the solid's corners. (We need three more squares to make the rest of the faces and complete the cube.)

Could we make another regular solid, with only one, or two, or four square faces meeting at a corner?

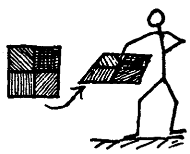
With *one square*, we cannot make a solid corner.



With *two squares*, we can only make a flat sandwich.

With *three squares*, we make a cubical corner, leading to a cube.

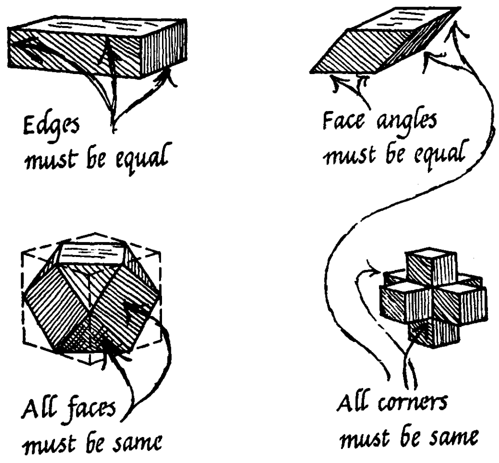
With *four squares* meeting at a corner, they make a flat sheet there, and cannot fold to make a corner for a closed solid.



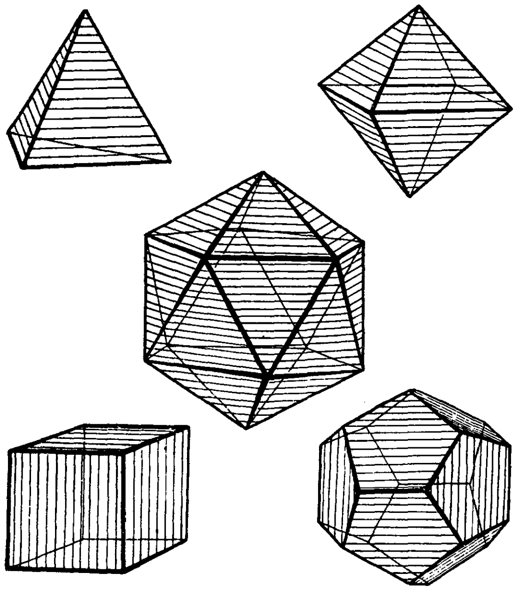
Thus, SQUARES CAN MAKE ONLY *ONE* KIND OF REGULAR SOLID, A CUBE.

(b) Now try for yourself with regular pentagons, and ask how many regular solids can be made with such faces.

THE SOLIDS BELOW ARE
NOT REGULAR SOLIDS



THE REGULAR SOLIDS



Then try hexagons, and other polygons.

Then return to triangles and carry out similar arguments with triangular faces.

THE RESULT: Only FIVE varieties are possible in our 3-dimensional world.

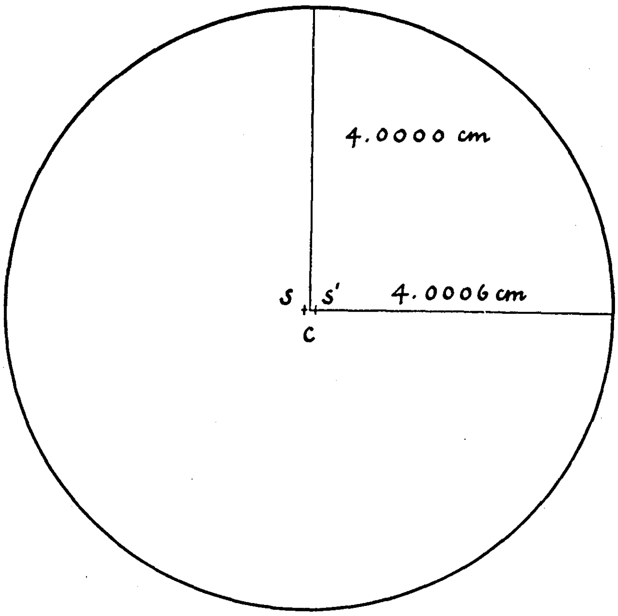
(NOTE that these arguments need pencil sketches but can be carried out in your head without cardboard models.)

of general theory seemed little more than a special explanation made for just one job. Those two objections could not operate until long after Kepler's time. The only objection then was that the measurements of orbits given by the Copernican scheme did not fit the pattern of solids accurately. Kepler himself was obliged to thicken up the spherical shells between solids; but many theories throughout the ages have had to adopt minor modifications to escape criticisms of inaccuracy.

Mars, the Difficult Planet

Working with Tycho and continuing after Tycho's death, Kepler tried and tried to find the shape of the orbit of Mars, "the difficult planet".

We now know that the orbit of Mars is an ellipse; but it is very close to a circle -- the ratio of maximum radius to minimum is only 1.0043:1.000. Yet the observed motion of Mars differed enough from simple motion round a circle with constant speed to show up clearly in the observations and make Mars the difficult planet.



Ellipse: The Earth's Orbit Drawn to Scale

The actual eccentricity of planetary orbits is very small. The orbits are almost circles, yet Tycho's observations enabled Kepler to show that they are not circles but ellipses. The sketch above shows the Earth's orbit drawn to scale. If a 4.0000 centimeter line is used, as here, to represent the minimum radius, which is really some 93,000,000 miles, the maximum radius needs a line 4.0006 centimeters long. The eccentricity of Mars' orbit is over thirty times as big, but even then the ratio of radii is only 1.0043 to 1.0000. Mercury is the only planet with a much greater eccentricity of orbit, with radii in proportion 1.022 to 1.000. Even this eccentricity of orbit seems small, but it is sufficient to involve Mercury in such speed changes around the orbit that Relativity mechanics predicts a very slow slewing around of the orbit—a precession of only 1/80 of a degree per *century*, discovered and measured long before the Relativity prediction!

Kepler was sure that the Copernican Solar System would turn out to be the true model. Under the influence of the Greek tradition, he tried circular orbits with the Sun a short distance off-centre and the planet carried round by a constant-speed arm from another point (like the equant) a small distance off-centre. He made dozens of trials with different amounts and directions of eccentric placing. In each trial he used some of Tycho's observations to determine the circle, then continued the motion of his theoretical planet and predicted its position at some later date, and then compared that with Tycho's observation.

Scheme after scheme failed to fit and had to be thrown away. After many trials, Kepler found an eccentric-circle scheme that fitted well. Delighted, he made one further test and found his predicted direction for Mars differed from the observed position by 8 minutes of angle.

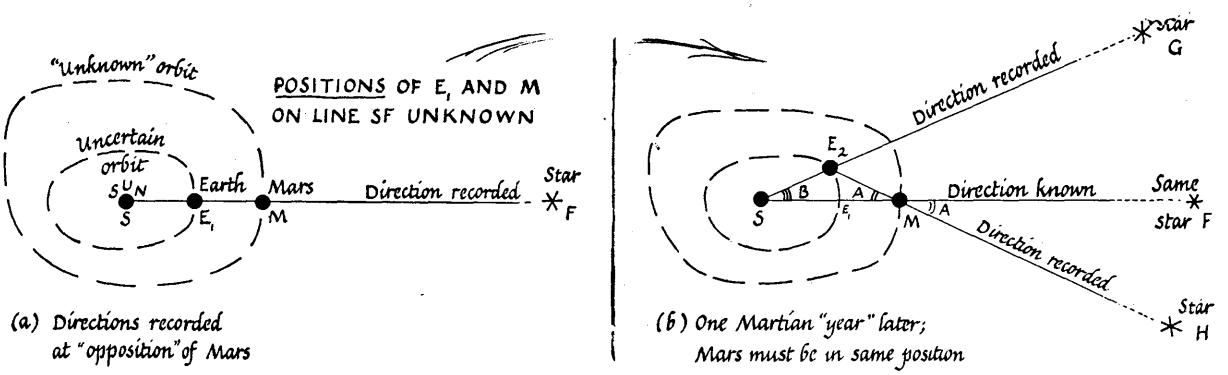
Might not the observations be wrong by this small amount? Would not "experimental error" take the blame? No. Kepler knew Tycho, and he was sure Tycho was never wrong by this amount. Tycho was dead, but Kepler trusted his record. This was a great tribute to his friend and a just one. Faithful to Tycho's memory, and knowing Tycho's methods, Kepler set his belief in Tycho against his own hopeful theory. He bravely set to work to go the whole weary way again, saying upon these 8 minutes he would yet build up a theory of the universe.

Plotting Mars' Orbit

It was clear that a circular orbit could not be made to fit the facts. Kepler realized he must obtain an accurate picture of Mars' real orbit from the observations -- not so easy, since we only observe the apparent path of Mars from a moving Earth. The true distances were unknown; only angles were measured and those gave a foreshortened compound of Mars' orbital motion and the Earth's. So Kepler attacked the Earth's orbit first -- by a method which Einstein once said was Kepler's real mark of genius.

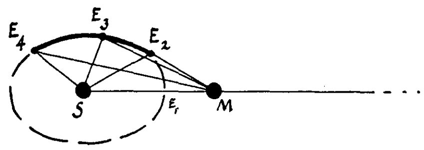
You need not spend time following Kepler's special method for the Earth's orbit; but in case you wish to see how he disentangled it, here is a short account.

To map the Earth's orbit round the Sun on a scale diagram, we need many sets of measurements, each set giving the Earth's bearings from two fixed points. Kepler took the fixed Sun for one of these, and for the other he took Mars at a series of times when it was in the same position in its orbit. He proceeded thus: he marked the position of Mars in the star pattern at one opposition (opposite the Sun, overhead at midnight). That gave him the direction of a base line Sun-(Earth)-Mars, SE_1M. Then he turned the pages of Tycho's records to a time exactly one Martian year later. (That time of Mars' motion round its orbit was known accurately, from records over centuries.)



(a) Directions recorded at "opposition" of Mars

(b) One Martian "year" later; Mars must be in same position



(c) Construction of Earth's orbit

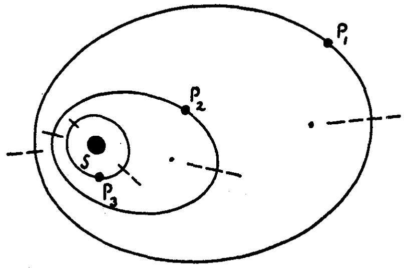
Then Kepler knew that Mars was in the same position, M, so that SM had the same direction. By now, the Earth had moved on to E_2 in its orbit. Tycho's record of the new position of Mars in the star pattern gave him the new apparent direction of Mars, E_2M; and the Sun's position gave him the direction E_2S. Then he could calculate the angles of the triangle SE_2M from the record, thus: since he knew the directions E_1M and E_2M (marked on the celestial sphere of stars) he could calculate the angle A between them. Since he knew the directions E_1S and E_2S, he could calculate the angle B between them. Then on a scale diagram he could choose two points to represent S and M and locate the Earth's position, E_2, as follows: at the ends of the fixed base line SM, draw lines making angles A and B and mark their intersection E_2. One Martian year later still, he could find the directions E_3M and E_3S from the records, and mark E_3on his diagram. Thus Kepler could start with the points S and M and locate E_2, E_3, E_4,... enough points to show the orbit's shape.

Law I. Orbits are Ellipses

Finding the Earth's true orbit to be an oval shape, he returned to Mars' orbit and found that also an oval. He wrestled with several ideas for describing that oval and at last found that it is clearly an ellipse, with the Sun in one focus.

If you have not tried drawing an ellipse yourself, it is well worth while. Use two drawing pins and a loop of string.

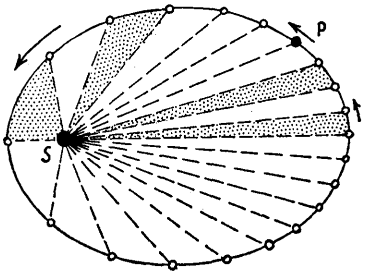
Kepler found the same rule must hold for the other planets. Thus he had the first of his three great laws.



A SOLAR SYSTEM WITH ELLIPTICAL ORBITS AROUND A COMMON SUN
(The planets' orbits in our own Solar System have much smaller eccentricities. But some comets move in elliptical orbits with great eccentricity.)

Law II. Constant Areas and Variable Speed

In the course of trying different orbits for Mars, Kepler discovered another guiding rule, now called his second law. He could not arrange a spoke from any eccentric point to sweep round with constant speed and carry the planet along the orbit in agreement with the facts, But, instead, he found that an imaginary spoke, running straight from the Sun to the planet, does sweep out area at a constant rate.

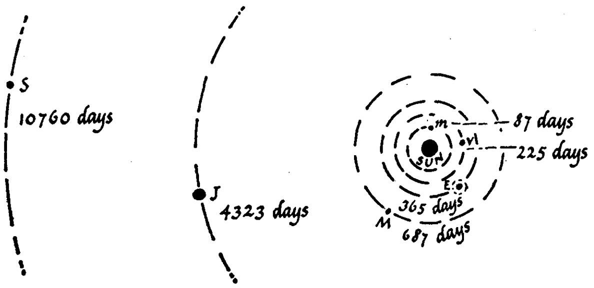


KEPLER'S DISCOVERIES FOR MARS
An ellipse with the Sun in one focus fits the orbit of Mars. The spoke from Sun to Planet sweeps out equal areas in equal times. The positions marked here show planet's positions at equal intervals of time, 1/20 of its "year" apart. The planet moves with such speeds that all the sectors marked here—a few of them shaded—have equal areas.

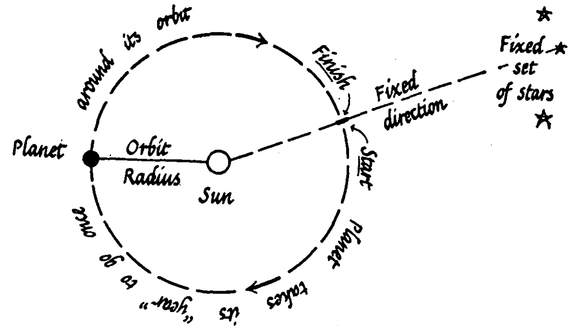
As with Ptolemy and the Greeks before him, something had to stay constant in the machinery, or it was useless as a scientific theory. Kepler replaced constant rate of revolution by constant rate of sweeping out area.

Law III. Connecting the Motions of all the Planets

Kepler continued to brood on one of his early questions: What connection is there between the sizes of the planets' orbits and the times of their "years"?



? RELATIONSHIP BETWEEN *RADIUS* AND *"YEAR"* FOR PLANETARY ORBITS ?
(Planetary orbits roughly to scale.)



PLANET'S "YEAR"
The planet's year is the time it takes to go once around its orbit. This is the time-interval from the moment when its direction hits some standard mark in the star-pattern until it returns to the same mark. (The Earth moves too. An allowance for the Earth's motion must be made when extracting the planet's true year from observations.)

On the next page there is a table of measurements(these are modern data more accurate than those Kepler had). Can you see any simple relation connecting orbit radius and the time a planet takes to go round its orbit? Suppose those went up in the same proportion from planet to planet so that orbit radius time of revolution is the same for all. Is that so? As R almost doubles from Mercury to Venus, T almost triples. As R grows almost 10 times from Earth to Saturn, T grows about 30 times. Simple proportion will not do.

Planet	*Radius of planet's orbit* *R* (*miles*)	*Time of revolution* (*planet's* "*year*") *T* (*days*)
Mercury	3.596×10^7	87.97
Venus	6.720×10^7	224.7
Earth	9.290×10^7	365.3
Mars	14.16×10^7	687.0
Jupiter	48.33×10^7	4332.
Saturn	88.61×10^7	10760.

Is there something which you could work out for Jupiter's orbit size and orbit time, again for Mars' orbit size and orbit time, again for Earth's orbit size and orbit time, and so on... and get the same answer for each planet? If so, you would have discovered a fine law connecting R and T. Kepler wrestled with that problem for a very long time, trying different combinations such as R/T (which we saw above will not succeed), R^2/T, and so on. At last he found that R^3/T^2 does succeed. Here is the full story.

PLANETARY DATA — TEST OF KEPLER'S THIRD LAW

(These are modern data, more accurate than Kepler's)

Planet	*Radius of planet's orbit* *R* (*miles*)	*Time of revolution* (*planet's* "*year*") *T* (*days*)	R^3 (*miles*)^3	T^2 (*days*)^2	$\frac{R^3}{T^2}$ $\frac{(miles)^3}{(days)^2}$
Mercury	3.596×10^7	86.96	46.49×10^{21}	7734.	6.009×10^{18}
Venus	6.716×10^7	224.7	303.3×10^{21}	50490.	6.008×10^{18}
Earth	9.290×10^7	365.3	801.7×10^{21}	133500.	6.009×10^{18}
Mars	14.16×10^7	687.1	$2836. \times 10^{21}$	472100.	6.008×10^{18}
Jupiter	48.33×10^7	4323.	$112900. \times 10^{21}$	18780000.	6.012×10^{18}
Saturn	88.61×10^7	10760.	$695800. \times 10^{21}$	115800000.	6.011×10^{18}

The test of Kepler's guess is shown in the last column.

Kepler was wild with delight and published this law as well as his others claiming that he was so happy with success that he did not mind how long his book had to wait for readers.

Kepler was a mathematical speculator. He looked for many kinds of connections among planetary data and found some that he considered successful. His three great Laws were clear, simple, and powerful. We still hold them as descriptions that fit the facts very accurately. If all the planets were controlled by the Sun alone and exerted no disturbing effect on each other, we should expect inverse-square-law gravity to hold them in elliptical orbits, fitting these Laws perfectly:

Law I The orbit of each planet is an ellipse with the Sun in one focus.

Law II The radius (arm) from Sun to planet sweeps out equal areas in equal times as the planet travels round its orbit.

Law III For all the planets, R^3/T^2 has the same value.

For these laws we call Kepler the law-giver of the heavens.

CHAPTER VII. GALILEO GALILEI (1564-1642)

The brilliant teacher Galileo did much to prepare physics for Newton's development. He insisted on making theory realistic by tying it to experiment, and on putting the laws of physics into mathematical form as far as possible. Contrary to the popular myth, he was not so much a precise experimenter as a great mathematical codifier and arguer. He seemed content to quote rough experiments in support of his arguments about nature. He argued in a clever way; he first learned his opponents' views and arguments, then expressed those arguments even more clearly and convincingly than his opponents could; then when they rejoiced at what looked like his clear understanding of their story he turned round and demolished it with his own powerful arguments. In doing that he often made bitter fun of his opponents and nearly always annoyed them very much.

In sorting out mechanics from mystical speculation, Galileo arrived at Newton's first Law by an ingenious argument. And he knew about motion with constant acceleration and had the beginnings of Newton's second Law in his thoughts. So he laid the foundation, by arguments and experiments, for Newton's relationships between force, mass and motion.

Teaching the Copernican System: The Powerful Advocate.

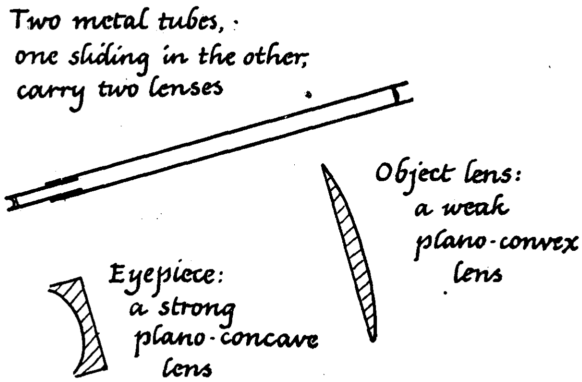
Galileo and Kepler wrote to and fro to each other. While Kepler was at work on the planets, Galileo was preaching and teaching the Copernican System with great enthusiasm. As a teacher of tremendous power he was able to put the case for Copernicus so clearly that for the first time readers far and wide understood and saw its full meaning. And when he had built his telescope he showed Jupiter's moons going round Jupiter and said **that was** a scale model of the Copernican System itself. His lecturing and writing made clear what Copernicus' book had given to only a few scholarly readers, that the Earth is just an ordinary planet like the rest, not a wonderful, special, privileged, central home for all-important man. The Earth is ordinary. And his telescope showed that the Moon is ordinary, just rocky like the Earth and not a marvellous silver ball placed specially for man's enjoyment.

All this was a violent and disturbing attack on established thinking: and the central position given by scholars to the Earth and man; on the teaching of the Ptolemaic system by the Church; and on the general public's idea of heaven as a comfortable future home outside the sphere of the stars. If the Earth spins and the stars are at rest, the starry sphere is no longer required and the stars can be placed far out in space at many different distances, perhaps without limit, certainly without any visible place for heaven. The matter-of-fact heaven beyond the stars was destroyed; a very disturbing discovery when people understood it.

When Galileo insisted the Copernican system was <u>true</u>, he was not only upsetting people's ideas, he was attacking the general authority of the Church. That brought Galileo, himself a devout member of the Roman Catholic Church, into grave trouble with the Church authorities. He was ordered to stop his teaching of Copernican theory as true and he spent the later part of his life writing a great book on elasticity, mechanics of motion, other non-controversial parts of physics -- after arguing by "thought-experiments".

<u>Telescope.</u>

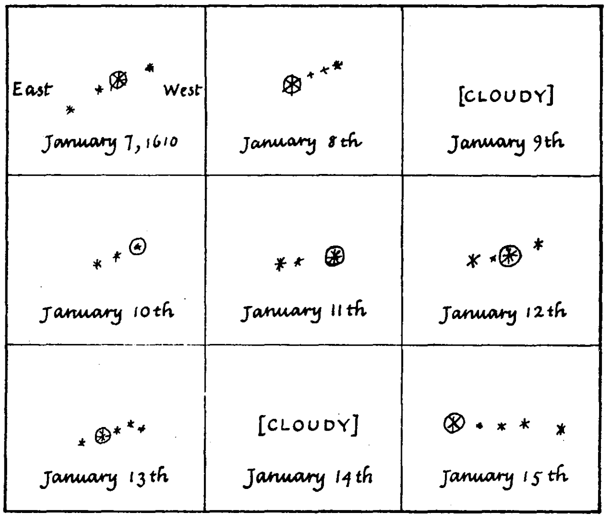
In the middle of his teaching life, Galileo heard rumours of an optical instrument that would make distant things look closer. He designed a telescope himself and made a small, weak one. Then he made larger and stronger telescopes, always grinding the lenses himself.



GALILEO'S TELESCOPE

When he turned his telescope to the heavens, he saw many surprising things, some of them very disturbing to the traditional view that was being taught by Church authorities and other astronomers. He saw the Milky Way resolved into a cloud of tiny stars. He saw that the Moon is rocky, with mountains and craters — that was a shock to astronomers, who had thought of the Moon as a perfect, shining sphere, free from any defect. He saw spots on the Sun — again a disturbing modification of the view that the heavens were perfect. He saw the planet Venus as a bright crescent, changing to other "phases" as it moved round the Sun -- a series of changes that would be very difficult to account for with the Ptolemaic machinery.

He looked at Jupiter, noting some small stars near it; and then, the next night, found the pattern changed.



Galileo's Observations of Jupiter's Moons

Jupiter seemed to have moved the wrong way relative to those stars. Waiting impatiently through a cloudy night, Galileo then saw the pattern change again. It was clear; the small stars were moons moving round Jupiter. Delighted with this, Galileo published a full description and claimed Jupiter and his moons as a model to support his strong arguments for the Solar System of the Copernican view. You can see the moons yourself with field glasses or a small telescope.

<u>The Development from Copernicus to Newton.</u>

Galileo brought the Copernican picture out into public knowledge. He devised telescopes, which were to bring astronomy to much higher levels of precision in future generations. And he developed and taught, in unfinished form, the new view of force and motion that Newton was to use in his theory. Kepler extracted precise, reliable laws of planetary motion; but he gave only vague suggestions for the mechanism of forces to produce such motion.

The educated world of philosophers, scientists, and other thinkers, was soon alive with questions about this Solar System of which people then knew so much but understood so little. It was in this climate of active discussion, with scientific societies being formed and growing here and there in Europe, that Newton grew up and, as a young man, took astronomy to his heart.

CHAPTER VIII. ISAAC NEWTON (1642-1727)

New Questions.

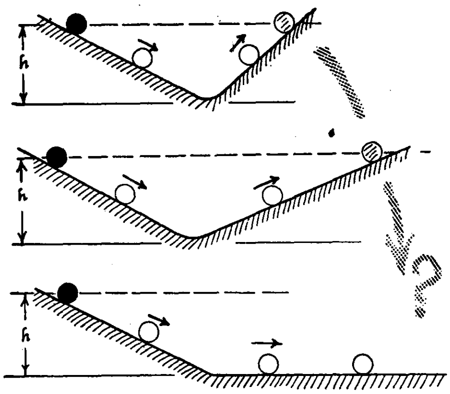
By the middle 1600's, scientific discussion and experimenting were popular and growing. Science had become respectable and interesting. Scientific societies met to show experiments and exchange views -- the young Royal Society of London, and academies in France and Italy. Many scientific instruments had been devised -- telescope, microscope, thermometer, barometer, ... -- so that experimenters could make good measurements as well as general observations.

There were new mathematical tools to help scientists extract knowledge or express it clearly. Descartes invented graph-plotting; and Galileo had put knowledge in mathematical form and set the stage for the invention of calculus.

Great astronomical questions were in the air: the time was ripe for a complete change of viewpoint. The old view had been that motion round a circle is the natural one in the heavens -- so people did not ask why a planet moves round an orbit. But they did ask why it continues to move; and the answer was "because a force continues to push it along".

Kepler thought about the force which must be needed to make a planet move round the elliptical orbit. He pictured an arm running out from the Sun to the planet and shoving the planet <u>along</u> its orbit. Where the planet moved faster, the arm must shove harder. That seemed to fit naturally with his Law II -- where the planet is far away from the Sun it moves slowly along its orbit, the arm sweeps out area slowly, and that is what one would expect if the arm's along-the orbit push became weaker at the greater distance from the Sun.

But Galileo by his brilliant thought-experiment, decided that <u>no</u> force is needed to keep an object moving with constant velocity; and Newton took that into his Law I.



GALILEO'S THOUGHT-EXPERIMENT

He felt sure that, without friction a ball rolling or sliding down a hill would run up to the SAME HEIGHT on an opposite hill. Suppose the second hill is horizontal..?

(There was doubt whether this argument predicted the body would continue for ever along the tangent or continue for ever parallel to the ground -- that is, round the Earth.)

So, when Newton attacked the problem of the planets, and met the question "What force pushes the planet along its orbit?", he was able to say "No force is needed; momentum just continues." -- that is why Newton's Law I is so important as a separate law and not just a special case of Law II: it was a clear, surprising reply to a tremendous astronomical problem.

But then Newton could add, "However, an inward force is needed for a curved orbit, pulling the planet away from its simple straight-line "tangent"; and he guessed that gravitation provided the needed force.

Appeal to Newton.

In the ferment of new science, Astronomy had an important place. Kepler's Laws were the "talk of the town" among scientific people. They were clear simple rules for the planets, that fitted the facts accurately; yet there was no "explanation" of them. There was no link between elliptical orbits in the sky and the mechanics of projectiles or of a horse and cart on Earth. "What is the story behind Kepler's Laws?". That was the question that led to much keen argument.

Several scientists saw that a body moving round a circular orbit should be regarded as having an acceleration towards the centre. The geometry of that motion was worked out by several people. Huygens (who devised a wave theory of light), and Hooke (a prominent member of the Royal Society), and others, knew that orbital motion has an inward acceleration v^2/R -- but ideas of force and motion were still not fully clear. Some of those people saw that Kepler's Law III (R^3/T^2 same for all planets) fitted with gravitation spreading straight out from the Sun. But there the story stopped, unfinished: Kepler's Laws I and II remained "unexplained"; and the whole business of motion in the sky was still a remote mystery to most people. It was then, about 1684, that the astronomer Halley, despairing of the discussions in London, journeyed to Cambridge to ask Isaac Newton whether an inverse-square law of attraction could account for elliptical orbits.

Who was Newton, and why consult him? He was a professor of mathematics living quietly in Cambridge, interested in problems of chemistry and thinking about his own scheme of religion. He was already known as a brilliant scientist and mathematician, presently to be recognized as one of the greatest scientific thinkers ever known. He not only connected Kepler's Laws with the laws of ordinary machines on Earth but expanded his idea of universal gravitation into one of the greatest theories in all physics.

Newton's Life.

As a schoolboy, going to the Grammar School in Grantham, he seems to have led an ordinary quiet life, making some mechanical toys but showing little sign of his great ability until he went to the university. At Trinity College, Cambridge, he raced through geometry, finding Euclid fairly easy, and Descartes harder.

He thought about astronomy for his own pleasure. He quietly speculated about the whole Solar System being controlled by universal gravitation; and he started by trying out the idea of inverse-square law gravity on the Moon's motion. He worked on problems of planetary motion, solving them and putting his notes away, just as you might wrestle with a crossword puzzle, with no interest in telling anyone else.

Angry arguments had developed when he described his work on light and colour; and, later, friends on both sides promoted a violent quarrel as to who had invented calculus, Newton or Leibnitz. Actually, calculus was needed, as Galileo foresaw half a century earlier -- the time was ripe -- and the two mathematicians invented it independently. But Newton felt shy and hurt, and in later years, he shrank from publishing his work.

Newton was in his early twenties when he invented calculus, for his own use; began to think about planetary motion and gravitation; and started his experiments on optics, which he described in a lecture to the Royal Society. His mathematics professor recognized Newton's genius when the young man brought him an extended solution to a difficult problem. And a few years later the professor retired in Newton's favour; at the age of 26 Newton was appointed to the most distinguished mathematical professorship in Europe. He worked with marvelous skill, twisting and turning as he used one mathematical device after another to find his way through his theory of the Solar System.

Gravity and Planets. Newton had been thinking about gravitation and Kepler's Laws for nearly 20 years when Halley brought him the problem from friends in London. Newton said, characteristically, that he had already solved that problem and knew that inverse-square law gravity would make a planet follow an ellipse. And he knew the interpretation of Kepler's IInd and IIIrd Laws also. He could not find his notes, but soon sent much fuller notes to London. Then friends pressed him to publish his work but he feared it would start more controversy. However, persuasion won and he put his work into a great book that set forth his whole gravitational theory: "The Mathematical Principles of Natural Philosophy," which we still call by its Latin name the "Principia." He wrote it in Latin, the international language of scholars; though he wrote a later book on optics in English.

The "Principia" did far more than just explain Kepler's Laws by gravitation. It set forth a theory that provided a whole family of explanations and predictions and gave people a general sense of many things in Nature being linked together by one consistent story. In fact, Newton cleared up our knowledge of earthly mechanics and heavenly motions at the same time and gave us language with which to talk about our knowledge.

The "Principia" was a work of genius and once it was published, Newton's fame spread abroad. There were translations into other languages and textbooks to describe its material more simply.

In England, Newton's fame was recognized by special honours; he was elected to Parliament, made Master of Mint, and knighted. As a member of the Royal Society, he was elected president -- one of the highest scientific honours in Europe -- and he held that post for 24 years to the end of his life.

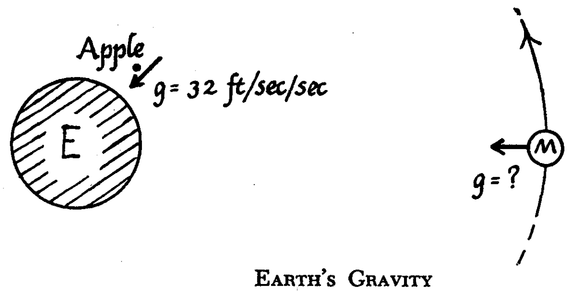
Motion round an orbit: centripetal acceleration

An object moving with constant velocity just continues along a straight line. An object moving round a circle at constant speed is changing the direction of its velocity. It has an acceleration although it is not moving any faster. And that acceleration needs a force. Whirl a stone on a string, in a circle round your head. To keep the stone in orbit you have to pull the string and the string pulls the stone, inward. There is an inward force on the stone, a centripetal force. The stone has an inward acceleration $(\text{speed})^2/(\text{radius of orbit})$, v^2/r .

This is obviously a very important formula for a modern theory of planetary orbits. Can we safely combine $a = v^2/r$ with $F = ma$ and say that inward force mv^2/R is needed? Such a force must have seemed strange at first -- a force "across" the motion, compared with the ordinary forces that make things move faster. But experiments show that this inward force is needed, and it must be provided by some real agent: no force, no orbit. Newton made one of the first tests of this strange force when he tried it on the Moon's motion.

Newton and His Theory

Newton, thinking about the planets (in Cambridge, or away at home when the Great Plague was raging) guessed at his law of gravitation by working backwards from Kepler's IIIrd Law. He asked, "What kind of force spreading out from the Sun would be needed to make (R^3/T^2) the same for all the planets?" He tested his answer on the motion of the Moon -- an experiment in a huge frictionless lab half a million miles across.



EARTH'S GRAVITY

Newton himself said (in a letter to a relative) that an apple falling from a tree started him thinking of gravity stretching out from the Earth to the Moon. Could the same Earth's gravity that pulls the apple also pull the Moon and hold the Moon in orbit? He saw that, to fit the facts, gravity must be much weaker out at the Moon.

We shall calculate the Moon's acceleration towards the Earth, v^2/R, as Newton did. Here are data:

Radius of Moon's orbit = 60 x (Earth's radius)

= 240,000 miles = 3.84×10^8 metres.

Time taken by the Moon to go round its orbit (= 1 month!) = 27.3 days *

To calculate the Moon's central acceleration, v^2/R, first calculate v as follows:

v = speed along the orbit = circumference/period = $2\pi R/T$

= $2\pi(3.84 \times 10^8$ metres$)/(27.3$ days x 24 hrs/day x 3600 secs/hr)

The result is: v = 1022 metres/second (about 2300 miles/hour).

Then we calculate the inward acceleration, v^2/R, that the Moon must have to continue round this orbit

$v^2/R = (1022$ metres/sec$)^2/3.84 \times 10^8$ metres = 0.00272 metres/sec^2

* 27.3 days looks surprisingly short for a month, but it is the true time taken by the Moon to make one complete orbit, judging its direction by the fixed stars. However, since the Earth is also moving (round the Sun in a year) in the same direction, the Moon has to travel farther to return to the same position relative to the Sun. That is why the Moon month in our calendar is about $29\frac{1}{2}$ days from full Moon to full Moon. For our calculations of the Moon's motion we need the true period, 27.3 days.

Then the Moon's actual acceleration towards the Earth is about 0.0027 metres/sec^2 -- far smaller than 9.8. If gravity does produce the Moon's acceleration, it must be much weaker at the Moon. What scheme could we try for diluting gravity, thinning it out at greater distances? The simplest scheme, halving gravity when the distance doubles, does not fit the result above. That would give 1/60 of our "g" at the Moon, or about 0.17 metres/sec^2.

Another simple scheme is an inverse-square-law. That is a good guess for any measurable quantity that spreads out in straight lines without getting lost. Light from a small lamp gives illumination that varies as 1/(distance)^2, in clear air (but not in a fog). *

For gravity Newton guessed that if an object of mass M_1 attracts another object of M_2 at distance d with a certain pull, then at twice the distance the pull is 1/4 as large; and 3 times the distance, only 1/9 as large. This was Newton's guess, that $F \propto 1/d^2$.

The Moon is 60 Earth's radii away. So, compared with an apple one radius away from the center of the Earth the Moon is 60 times away. If gravity does thin out with an inverse-square-law, the apple's acceleration 9.8 metres/sec^2 should decrease to a value at the Moon $9.8/60^2$ or 9.8/3600 metres/sec^2. If you work that out, you will find it very near to the actual acceleration that Newton calculated from $a = v^2/R$.

Therefore, young Newton had a satisfying test of his guess of inverse-square gravity. It also assured him that the strange inward acceleration v^2/R -- where the body travelling round a circle does not change its speed

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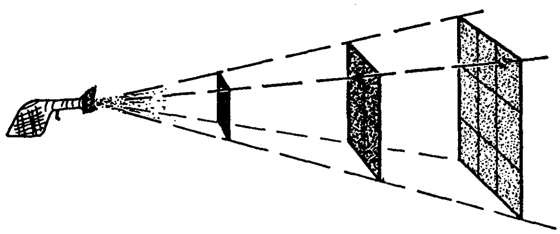
*** Gamma Rays, very-short-wavelength X-rays from radioactive material travel straight out with little loss in air. So if the source is a large dangerous one, you are four times as safe at twice the distance, a hundred times as safe at ten times the distance away from the source.**

Imaginary Illustration of Inverse-square Law: You may get a clearer picture of inverse-square thinning out from this fable: imagine the owner of a restaurant invents a gadget to butter many slices of toast efficiently. It is a small motor-driven pump that squirts out a fine spray of specks of melted butter from its muzzle in straight lines, in a wide square cone.

Suppose the cone of spray just covers one slice of toast held one foot from the muzzle. The toast is held there for a standard time, say one second. The toast receives rich buttering. Now place the toast 2 feet away instead. At that double distance the cone is twice as high and twice as wide. The spray now covers 4 slices of toast, and each of them receives medium buttering in one second. Now place the toast 3 feet away; and the same cone of spray covers 9 slices: economy treatment.



Therefore, on specimen slices of toast placed 1 foot, 2 ft, 3 ft, ... from the muzzle, the thickness of buttering will be in the proportions $1 : \frac{1}{4} : \frac{1}{9} :, \ldots$.

This is the inverse-square law of buttering.

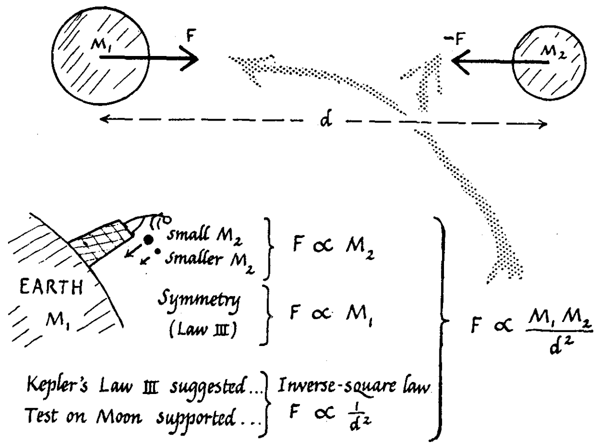
and never moves closer to the center -- <u>does</u> require a pull according to the F = Ma that we use for a force accelerating a falling apple or a speeding car.

Then, with an inverse-square-law for gravity guessed at and tested on the Moon's motion, Newton tried universal gravitation for the Solar System: acting on Earth, Sun, planets, Jupiter's moons, comets, ... all bodies.

Universal Gravitation

Newton assumed universal gravitation: each piece of matter attracts each other piece with an inverse-square force, and the attractions are proportional to the mass.

UNIVERSAL GRAVITATION



If we release a large stone and a small stone together, they fall with the same acceleration, "g". They have different masses but the same acceleration. Therefore, applying F = Ma, we are sure that the Earth must pull the stones with forces proportional to the masses. If one stone is 10 times as massive as the other the Earth pulls it with 10 times the force, making the acceleration the same for both. Newton guessed at his law of gravitation in the form

making the acceleration the same for both.

So Newton guessed at his law of gravitation in the form $F = G\dfrac{M_1 M_2}{d^2}$ where G is a universal constant. *

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* G is quite different from "g". G is a constant for all bodies in the universe; but g is the local acceleration due to Earth's gravity, usually near the surface of the Earth. g is also the field-strength of the Earth's gravitational field there, measured in newtons/kilogram. For an Earth of mass M and an apple of mass m,
(m kilograms x g newtons/kg) = G M m/r^2, where r is the Earth's radius
$\therefore$ g = G(mass of Earth)/(radius of Earth)^2.
We know the value of g. So did Newton, of course, but he did not know the value of G or the mass of the Earth.

Newton's great general theory: his assumptions and his deductions

Then, to build his theory, Newton started afresh with simple clear assumptions; from which he deduced his many predictions. You can see that he did not just pick his assumptions out of the blue sky; he chose them with a careful eye on experimental knowledge of the real world. He assumed:

LAW I Any object remains at rest, or continues to move with constant speed in a straight line, if it is left alone; that is, if there is no RESULTANT force acting on it.

LAW II A force acting on an object makes it accelerate, in the direction of the force; and FORCE = MASS x ACCELERATION

or FORCE x TIME = change of MOMENTUM (mv)

LAW III Action equals reaction. Whenever one object pushes or pulls another, the other exerts an equal and opposite push or pull -- whether the objects are at rest, or moving with constant velocity, or accelerating in any way whatever.

LAW OF UNIVERSAL GRAVITATION

Every object in the universe attracts every other object with a force which is proportional to each of the masses concerned, and inversely proportional to the square of the distance between them.

$$F = G\ M_1 M_1 / d^2$$

From those as a starting point, taking them for granted as "true" summaries of nature, Newton argued out many a prediction: he derived from his assumptions some things already known -- thus "explaining them" -- and he predicted others. This was the work he put into the Principia.

Here is a list of outcomes of Newton's theory.

Results. From those as a starting point, taking them for granted as true, Newton argued out many a prediction. He derived or "explained" or predicted the following:

(1) The Moon's motion round the Earth is controlled by the Earth's inverse-square law gravity (Newton's original test).

(2) Kepler's Law III. $(\text{Orbit radius})^3/(\text{Planet's year})^2$ same for all planets of the Solar System.

(3) Kepler's Law I. Planets' orbits must be ellipses with Sun in one focus.

(4) Kepler's Law II. Arm from Sun to planet sweeps out equal areas in equal times: shown to be necessary for any "central" force.

(5) A planet's moons: same rule, R^3/T^2 = constant, applies to all the satellites of a planet, but with different value of constant for a different "owner" of satellites. (E.g., all Jupiter's moons; and now all Earth satellites).

(6) Relative masses of Earth and Sun, Earth and Jupiter, etc., estimated through Kepler's Law III. (Estimate can be made for any two bodies which own satellites).

(7) Comets, until then lawless and mysterious, follow elliptical orbits according to Kepler's Law I, as members of Solar System. Times of comets' returns predicted successfully.

(8) Shape of Earth must be oblate spheroid (bulge at equator): amount of bulge estimated. Surveys soon after confirmed this unexpected prediction.

(9) Small differences of "g" predicted, due to shape of Earth and due to Earth's spin: both make measured "g" slightly smaller at equator than at poles.

(10) Ocean tides, due to differences of Moon's attraction. (Two tides in 24 hours predicted).
Similar tides due to Sun are smaller. Added to Moon's tides, these make the large "spring" tides, twice a month; subtracted, they make "neap" tides. Relation with phases of Moon also predicted.

(11) Mass of Moon estimated by treating our ocean tide as a satellite of the Moon.

(12) Precession of the Equinoxes. Shown to be a gyroscopic motion due to the gravitational pulls of Sun and Moon acting on the equatorial bulge of the spinning Earth. The 26 000-year period predicted roughly.

(13) Irregularities of the Moon's motion. The elliptical orbit of the Moon shows several small changes in the course of time, all of them now explained as due to small differences of the Sun's gravitational pull as the Moon goes nearer and farther in the course of the month. Newton predicted several, tested some.

(14) Perturbation of planetary orbits. Each planet is affected slightly by the gravitational pulls of other planets. Newton started the prediction of these small perturbations.

(15) Discovery of Neptune. Long after Newton's death, when the planet Uranus had been discovered, it showed small residual perturbations from its expected orbit (in addition to the effects of known planets.) From these, Adams and Leverrier predicted the location and mass and orbit of an unknown planet that could produce these tiny perturbations by inverse-square law gravitation. Then the planet was seen: a triumph of Newtonian theory.

Detailed Discussion of Outcomes

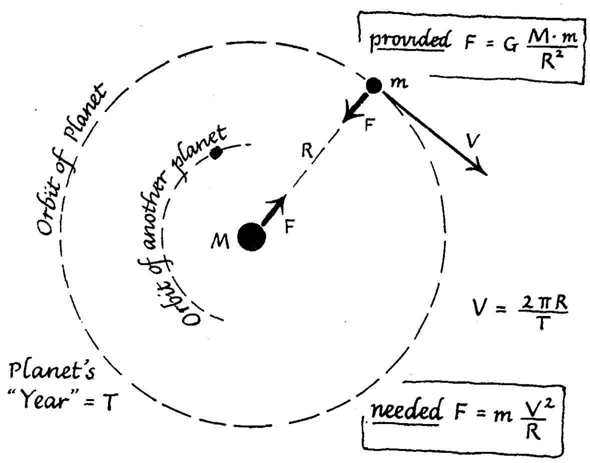
It is good to look at such a summary of Newton's achievements first. But just making a list is not very good science -- and learning a list by heart is certainly not a very valuable part of learning science. A scientist wants more details. So if, as we hope, you are reading this as a scientist, you should see how some of these predictions were made. In the following pages each prediction listed above is described in more detail. Some of the descriptions will seem difficult; and we expect different readers will choose different items in these following pages. There is no need to try to learn them all. What we hope is you will enjoy seeing, in some of them, how Newton used his theory.

(1) Moon's motion is accounted for by inverse-square gravity, which at the Moon's distance of 60 Earth's radii is predicted by theory to be 3600 times weaker. That agrees with the actual acceleration, v^2/R, calculated from the Moon's distance and time-to-go-round-the-orbit. *

Now for Kepler's Laws. We take Law III first -- the law that Newton used to get his preliminary hint.

(2) Kepler's Law III. For any circular orbit, the force that is needed is m.a ; that is, mv^2/R inward. Some real agent -- string, road, rails, or gravitation -- must provide that force: otherwise, no orbit.

The force provided by the Sun's gravitational attraction is $G\frac{Mm}{R^2}$



* This was Newton's early test of inverse-square-law gravity and of v^2/R needing a force. When we draw conclusions from a theory it is rather like a balance-sheet for accounts. We place each assumption in the expenses (debit) column and each prediction on the income (credit) side. The more credits, the more we value the theory; and it is quite fair, having "paid" the assumptions, to claim all the credits we can, even old long-established facts.

So we can start by claiming this test of the Moon's motion as one of the achievements of Newton's theory -- because we are watching Newton make a fresh start, taking his assumptions for granted, and deducing predictions, the more the better.

For a circular orbit suppose a planet of mass m moves with speed v in a circle of radius R round a Sun of mass M. This motion needs an inward resultant force on the planet, mv^2/R, to produce its inward acceleration, v^2/R .

If gravitational attraction between Sun and planet just provides this needed force, then

$G\frac{Mm}{d^2}$ must $= \frac{mv^2}{R}$, where distance d between m and M is the orbit-radius, R.

$$\therefore \quad G\frac{Mm}{R^2} = \frac{mv^2}{R}$$

But $v = \dfrac{\text{circumference}}{\text{time of revolution}} = \dfrac{2\pi R}{T}$ where T is the time of one revolution.

$$\therefore \quad G\frac{Mm}{R^2} = m\frac{(2\pi R/T)^2}{R} \qquad \therefore \quad G\frac{Mm}{R^2} = \frac{4\pi^2 mR^2}{T^2 R}$$

To look for Kepler's Law III, collect all R's and T's on one side; move everything else to the other.

$$\therefore \quad \frac{R^3}{T^2} = \frac{GM}{4\pi^2}$$

What does that tell you about R^3/T^2 ? It is equal to the gravitation constant G multiplied by the Sun's mass M divided by a constant number $4\pi^2$ What will happen if we calculate R^3/R^2 for another planet also owned by the Sun?

That is Newton's derivation of Kepler's Law III from his simple assumptions. (Look back at the table of measurements and values of R^3/T^2 in the chapter on Kepler). With any other law of force than the inverse-square-law, R^3/T^2 would not be the same for all planets. An inverse-cube law, for example, would make R^4/T^2 the same for all; then values of R^3/T^2 would be proportional to $(1/R)$, and would not be the same for all planets. In fact, as Kepler found, they are all the same. The inverse-square law is the "right" one.

Orbit Independent of Planet's Mass. The planet's mass m cancels out. Several planets of different masses could all pursue the same orbit with the same motion. (You might have foreseen that -- it is another form of the simple experiment of dropping a large stone and a small stone together. Their vertical fall is motion along a sort of orbit, straight to the Earth. Try putting two stones or two bits of chalk, a large one and a small one, in orbit together by throwing them.)

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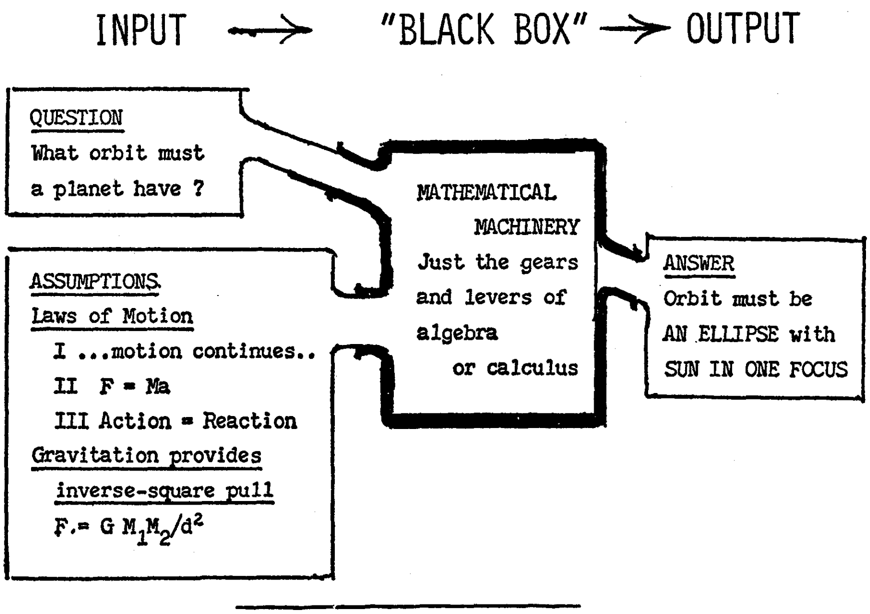
(3) Kepler's Law I. It was one of Newton's great achievements to show that with inverse-square-law gravitation from the Sun, a planet's orbit would be an ellipse with the Sun in one focus. Unfortunately, his mathematics is too difficult to be of use here -- there is a choice between heavy algebra and a shorter method using calculus -- so, to our regret, we must ask you to take Newton's proof on trust. But we have put a sketch here to remind you that the missing proof, which we have put in a "black box", is only a piece of clever grinding-round of mathematical machinery, and not itself a new piece of scientific knowledge.



(4) Kepler's Law II.

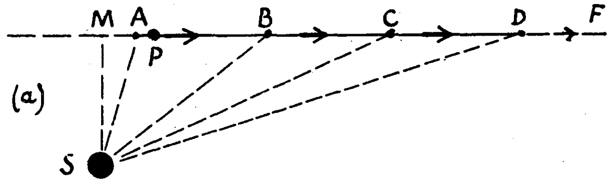
Newton showed that Kepler's law of "equal areas in equal times" will hold for any type of planetary motion providing the force on the planet is always directed straight towards the Sun. It does not have to be an inverse-square law force: any "central" force will do.

Here is Newton's own method of proof. You need not learn this in detail; but if you want to see how clever Newton was in using his knowledge of mechanics and geometry, you may find it interesting to work through the proof yourself. (The first time you read it, you are likely to find it obscure and difficult. But, if you think about it carefully and master it, you will have seen into Newton's mind. Some of the diagrams will be much easier to understand if you sketch them for yourself with several colours.)

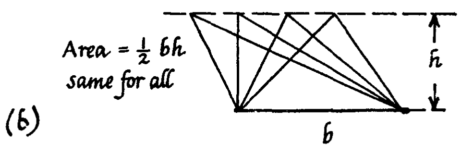
We use Newton's Law II: CHANGE OF MOMENTUM = Force x time. Then changes of mv are vectors, along the direction of F.

First suppose we have a planet that is pulled with no force at all. That is a queer kind of force-law; but it is a clear statement, and it certainly is possible - a star or a gas molecule out in space, far from any other matter, experiences no force and moves with constant velocity.

The planet, P continues to move with fixed speed in a straight line, AF (Newton's Law I). Mark the distances travelled by the planet in equal intervals of time (1 week, the next week, the next...), AB, BC, CD, ... etc. Since the speed is constant, AB = BC = ..., etc.



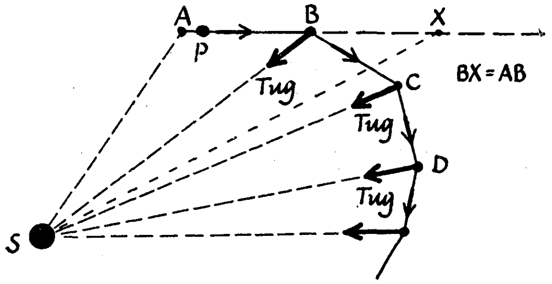
(a) Motion of a Planet with No Attraction. Planet P moves in a straight line with constant speed. SP sweeps out equal areas in equal times.



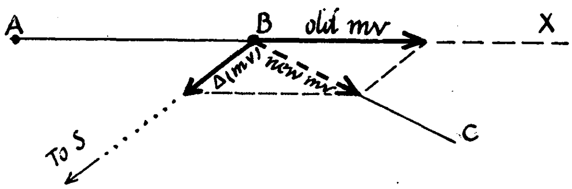
(b) The Property of Triangles Used Here. All triangles on the same base and with the same height have the same area. Another version: If triangles have the same base and their vertices lie on a line parallel to the base, their areas are equal.

We can still draw a radius SP to our planet, from the non-attracting Sun, even though there is no force. Consider the areas swept out by the radius SP as the planet moves from A to B, B to C, etc., in equal times. How do the following triangles compare, SAB, SBC, SCD? All these triangles have the same height, SM, and equal bases, AB, BC, CD. Therefore all their areas are equal: the spoke from S sweeps out equal areas in equal times. This simple motion does agree with Kepler's Law II.

Now suppose the planet P moves in an orbit because the Sun pulls it inward along the radius PS. But, to simplify the geometry, suppose the attraction only acts in sudden big tugs, for very short times, leaving the planet free to travel in a straight line betweenwhiles. Then it will follow a path such as that shown below. Suppose it travels AB, BC, CD, etc., in equal times, the inward tugs occurring abruptly at B, at C, at D, etc.



Motion of a Planet with Tugs of Attraction
Without tug at B, P would move on to X.



Enlarged Sketch of Momentum-Change at B

The planet moves steadily along AB; then, acted on by a brief tug at B, along BS, it changes its velocity abruptly and moves (with new speed) along BC. Except for the tug at B the planet would have continued straight on, as in the simple case discussed above. On this continuation, mark the point X an equal distance ahead, making AB and BX equal. Without the tug at B, the planet would have travelled AB and BX in equal times, and the radius from S would have swept out equal triangles, SAB and SBX. But in fact the planet reaches C instead of X. Does this change spoil the equality of areas? If the planet travels to C, the two areas are SAB and SBC. Are these equal?

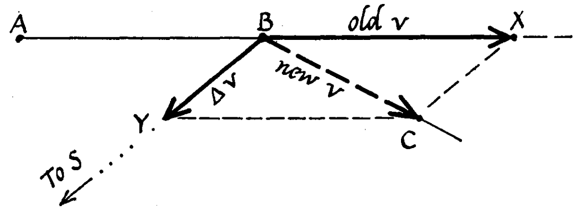
To change the motion from along AB to along BC, the tug at B pulls straight towards the Sun, along BS. This tug gives the planet some inward momentum along BS, which must combine with the planet's previous momentum to make the planet move along BC. The planet's previous momentum was along AB.

$$\text{Therefore,}\quad \begin{bmatrix}\text{original}\\ \text{momentum}\\ \text{along AB}\end{bmatrix} + \begin{bmatrix}\text{gain of}\\ \text{momentum}\\ \text{inward}\\ \text{along BS}\end{bmatrix} \text{ must } = \begin{bmatrix}\text{new}\\ \text{momentum}\\ \text{along BC.}\end{bmatrix}$$

Newton's Law II reminds us that momentum is a vector. So the adding must be done by vector addition. (See sketch). As the planet's mass is constant, we may cancel it all through and use velocities thus:

$$\begin{bmatrix}\text{velocity}\\ \\ \text{along AB}\end{bmatrix} + \begin{bmatrix}\text{gain of velocity}\\ \\ \text{along BS}\end{bmatrix} \text{ must } = \begin{bmatrix}\text{velocity}\\ \\ \text{along BC}\end{bmatrix}$$

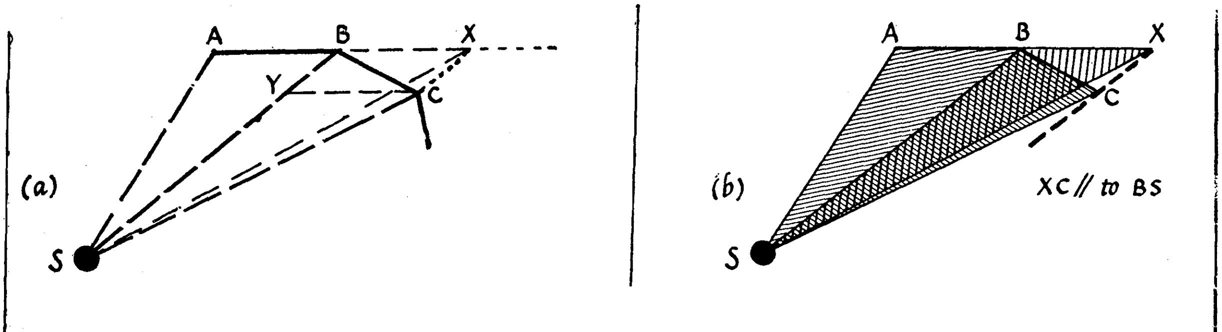
Let us use the actual distance AB to represent the planet's velocity along AB. Then, BX must also represent this velocity and BC must represent the planet's new velocity along BC (since all these are distances travelled in equal times). Using this scale, we make a vector diagram (see sketch) ex-



VECTORS TO SHOW VELOCITIES AT B
Scale has been chosen so that AB or BX represents original velocity along AB, before tug acts at B.

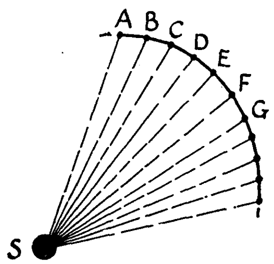
pressing the equation above. Use BX (= AB) for the original velocity before the tug. Use BC for the velocity after. The change of velocity must be shown by some vector BY along BS <u>straight towards S</u>. Complete the parallelogram, with BC the diagonal giving the resultant. Because this is a parallelogram the side XC is parallel to BY, so <u>C lies on a line parallel to BS</u>.

Now look at the triangles SBC and SBX, in sketch below. They have the same base, BS, and lie between the same parallels, BS and XC, so they have equal areas. Therefore, area of SBC = area SBX, which = area SBA. There-



(a) Earlier sketch, redrawn with C in its proper place on XC parallel to BY or BS. (b) Sketch A redrawn with equal-area triangles shaded.

fore, the triangles SBA and SBC have equal areas. By a similar argument, the triangles SBC and SCD have equal areas, so all the triangle areas are equal, and Kepler's second law does hold for this motion. This argument only holds <u>if all the tugs come from the same point S</u>. If we now make the tugs more frequent (but correspondingly smaller) we have an orbit nearer to a smooth curve, and Kepler's law still holds provided the tugs are directed straight from planet to Sun. If we make the tugs still more frequent, we approach the limit of a continual force, with an orbit that is a smooth curve. The argument extends to this limit, so Kepler's Law II holds for a smooth curved orbit.

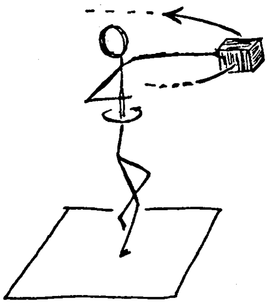
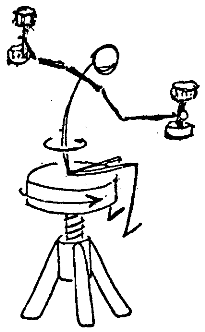


The equal time-intervals from A to B to C . . . are much shorter. Orbit is nearer to a smooth curve. With a smooth-curve orbit, the segments swept out in equal times may each be regarded as a bunch of small triangles like those here. So the segments must have equal areas.

<u>Experiments to Illustrate Kepler's Law II.</u> Illustrate the law yourself, by pretending you are the Sun, pulling a whirling planet in towards you:

(i) Use a small stone or a chunk of wood as the planet; tie a piece of string to it and whirl it in a circle round your head. Then let the string wind up round one finger of the hand that is holding it, so that the planet is pulled inward. It moves faster when it is at shorter radius, as you would expect from Kepler's Law II -- because your inward pull is practically central.

(ii) Hold a book or some other massive object at arm's length while you spin on your heels. If you can do so without falling over, pull the book in to your chest while you are spinning, and notice how your spin speeds up. Or sit on a revolving chair or stool that can spin. Hold a pair of dumb-

bells or massive books in your outstretched hands and pull them in while you are spinning.

(iii) Watch a high diver, a skater, or a ballet dancer, change from slow spin to fast spin by drawing in arms and legs.
All these are illustrations of Kepler's Law II.

In more advanced physics we can describe this law in another way: we say it is the Law of Conservation of Spin-Momentum (or Angular Momentum, to use the poorly chosen technical name). When planet or book or skater's leg is pulled in, spin-inertia (the equivalent, for rotational motion, of mass) is decreased; so the velocity increases keeping the product, spin-momentum, constant).

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(5) Moons.

For another system, such as Jupiter's moons, M will be different, (this time, the mass of Jupiter). Therefore R^3/T^2 will have a different value but it will be the same for all those moons. Here are actual measurements for Jupiter's four largest moons. Does Newton's prediction hold?

JUPITER'S SATELLITES AND KEPLER'S THIRD LAW

Name of satellite	*Distance from Jupiter in miles* (R)	*Time of revolution in hours* (T)	*Calculations for test of Law III* R^3 (*miles*)^3	T^2 (*hours*)^2	$\frac{R^3}{T^2}$
Io	262,220	42·36	$1{\cdot}803\times10^{16}$	1,802·8	TRY
Europa	417,190	85·23	$7{\cdot}261\times10^{16}$	7,264	THIS
Ganymede	665,490	171·71	$29{\cdot}473\times10^{16}$	29,484	*
Callisto	1,170,700	400·54	$160{\cdot}440\times10^{16}$	160,430	

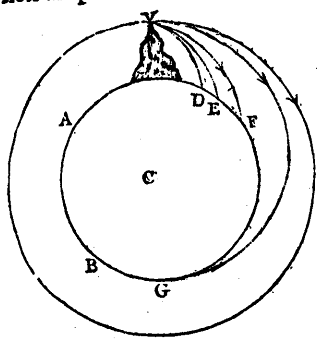
* The test is made easy by a lucky chance arising from the choice of units, miles and hours. Look at the numbers.

Thus Newton justified Galileo's use of Jupiter and his moons as a model of the Solar System.

Earth Satellites. A satellite is a moon. Any object that pursues an orbit round a massive body, Sun or planet -- held in orbit by gravitational attraction -- is called a satellite. The planets are all satellites of the Sun; Jupiter's moons are his satellites; the Moon is an Earth satellite. A cricket ball, after it leaves the bat, is an Earth satellite, for a limited time. And we now have "artificial" Earth satellites -- high up, practically free from air friction -- placed in orbit by man. Newton saw the possibility of an artificial satellite, though he thought we could never achieve it. He pictured a gun on a mountain top firing projectiles out horizontally, faster and faster until one could go right round the Earth and hit the gun from behind.

DE MUNDI SYSTEMATE.

non amplius in terram caderet.



Deſignet AFB ſuperficiem Terræ; c centrum ejus; & VD, VE, VF, lineas curvas, quas projectile de montis præalti vertice v, ſecundum lineas horizonti parallelas, auctis cum velocitatis gradibus, ſucceſſivè emiſſum deſcribat. Et ne aëris reſiſtentia, quâ motus cœleſtes vix retardantur, in computum veniat, fingamus hunc omnem tolli, vel ſaltem nil reſiſtere. Et eâdem ra-

The picture shows Newton's sketch that he put in a later edition of his "Principia". Even apart from the difficulty of getting enough fuel to put a satellite in orbit, he was sure that air resistance would soon destroy its motion -- he could not of course, foresee our vast supplies of fuel that can now put satellites far above the atmosphere.

Newton's sketch provides an easy way of explaining orbital motion to someone who finds it difficult. A slow projectile falls to the ground nearby; a faster one goes farther before it reaches the ground. Each of them falls towards the ground from the horizontal tangent that it would pursue without gravity. The path is sharply curved for the slow projectile, not so sharply for a faster one; and if we fired one sufficiently fast it could fall away from the tangent in just the same way as the surface of the Earth curves away. Then that projectile would continue round the Earth, always at the same height above the ground, until it returned to its starting place.

You can calculate the satellite's period -- the time it takes to travel once round its orbit -- if you know its distance from the Earth. That distance will tell you how strong the pull of gravity must be, out at the satellite. And that pull of gravity is the force that gives the satellite its inward acceleration v^2/R which it must have if it moves in orbit.

If the satellite is near the Earth, only a few hundred miles up $v^2/R = g = 9.8$ metres/sec^2, where R is the radius of the orbit, a little more than 4000 miles or 6 400 000 metres. Remembering that the circumference of the orbit is $2\pi R$, you can calculate the period. You will find your answer close to the 90-minute period of real satellites that keep close to the Earth.

If the satellite is farther away, you can still calculate its period T from its orbit-radius R, or vice versa, by going through a calculation just like Newton's test when he compared the Moon and the apple. Or you can use that calculation in a ready-made form by appealing to Kepler's Law III. The satellite you wish to know about is an earth-satellite; so is the Moon. Therefore R^3/T^2 must be the same for both, and you know R and T for the Moon. *

Problem. Suppose an Earth satellite like Telstar is to hover over one place on Earth, to act as a relay station for radio. It must have a circular orbit with a suitable orientation and it must have exactly the right period. What period must it have? The Moon takes 27.3 days to go once round the Earth. Use Kepler's Law III to find how far out from the Earth such a hovering satellite must be.

---

(6) Masses of Planets.

Newton could estimate the relative masses of the Sun and a planet or of any two planets, whenever each body concerned owns a satellite.

The satellite, through Kepler's Law III, provides a value for the mass of its controlling body. Although Newton could not check those relative masses, this was a typical case of theory providing new numerical knowledge in terms of its own assumptions.

For example, Newton could calculate the mass of the Sun in terms of the Earth's mass, because each owns at least one satellite. (The absolute mass of the Earth itself was not known and could not be estimated without some terrestrial measurements like those of Cavendish).

The calculations can be carried out as follows:

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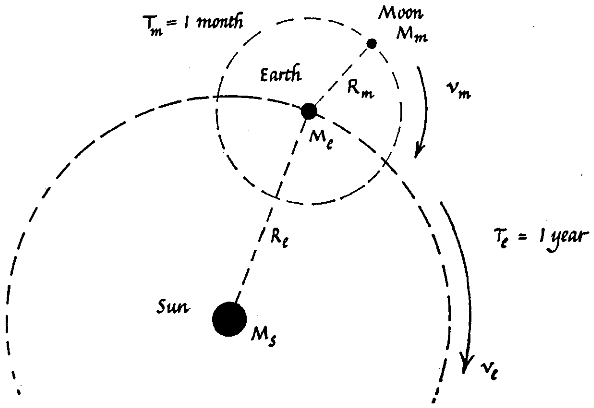
* Many of our artificial Earth satellites have orbits which are ellipses, and are not nearly circular. In that case, as Newton showed by further mathematics, R in Kepler's Law III must be the average of the ellipse's largest and smallest radii. Those are the distances from the focus (where the controlling attracting body is) to the nearest nose of the ellipse and to the farthest nose. The distance from nose to nose is called the "major axis," R must be half the major axis of the ellipse.



Subscripts $_s$ and $_e$ and $_m$ refer to Sun and Earth and Moon

<u>Earth as Satellite of Sun.</u> For the Earth's motion around the Sun in its yearly orbit,

$$G\frac{M_s M_e}{R_e^2} = M_e\frac{V_e^2}{R_e} = M_e\frac{4\pi^2 R_e^2}{R_e T_e^2} \qquad \therefore\ M_s = \frac{4\pi^2}{G}\left[\frac{R_e^3}{T_e^2}\right]$$

Note that the Earth's mass, M_e, cancels

<u>Moon as Satellite of Earth.</u> For the Moon's motion round the Earth in its monthly orbit,

$$G\frac{M_e M_m}{R_m^2} = M_m\frac{v_m^2}{R_m} = M_m\frac{4\pi^2 R_m^2}{R_m T_m^2} \qquad \therefore\ M_e = \frac{4\pi^2}{G}\left[\frac{R_m^3}{T_m^2}\right]$$

Again, the Moon's mass, M_m, cancels

Therefore, dividing one equation by the other

$$\frac{M_s}{M_e} = \left[\frac{R_e^3/T_e^2}{R_m^3/T_m^2}\right] = \frac{R_e^3}{R_m^3}\,\frac{T_m^2}{T_e^2} = \left[\frac{\text{DISTANCE OF SUN}}{\text{DISTANCE OF MOON}}\right]^3\left[\frac{1\text{ month}}{1\text{ year}}\right]^2$$

With the known values of these times and orbit radii, the ratio of the Sun's mass M_s to the Earth's mass M_e can be calculated.

Of course Newton would not end his calculation with an algebraic formula: he would then put in numbers; and so should you. It would be sad if you did not want to know, just roughly, how the Sun compares with the Earth. But it would also be sad if you felt compelled to calculate that out with terrific precision. A good scientist's motto is:

> "Rough estimates for good general knowledge: great precision when there is a special scientific need."

Taking orbit radii in miles we have 93.6 million miles for the Sun and 240 000 miles for the Moon, making a ratio about 400 to 1. There are 13 moon-months in a year. Now make your own estimate of the ratio (mass of Sun)/(mass of Earth). You will see why the Earth is unlikely to upset the orbits of other planets noticeably. *

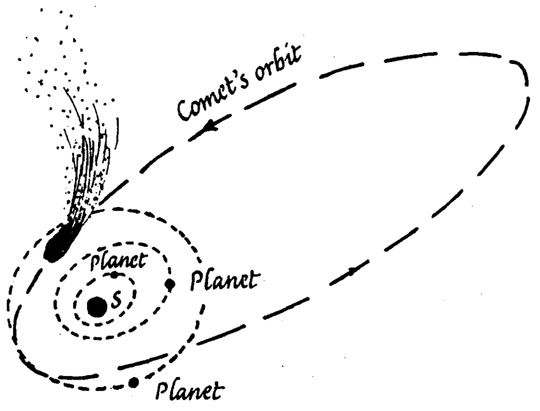
Newton could make similar estimates for other mass ratios, such as Jupiter's mass to Earth's mass or Jupiter's mass to Sun's mass, for any two bodies each of which owns a satellite. Even great Jupiter, by far the biggest of the planets, has only 1/1000 of the Sun's mass -- and will **not** disturb other planets' orbits much.

Where a body has no satellite, the estimate could not be made. You might think Newton was unable to estimate our own Moon's mass as a fraction of the mass of Earth or Sun, since the Moon had no satellite moon of its own and yet he succeeded in doing that! See later.

<u>(7) Comets.</u>

Occasionally, a bright object quite different from a star or planet, appears in the sky and moves through the star pattern in the course of months, looking like a great brush of bright shining material. That is a comet; and when a large comet appears near us it is a very impressive sight, a splash of bright hazy stuff stretching across a large part of the sky.

When big comets appear, people cannot fail to see them and be impressed. It is not surprising that such comets excited awe and even fear as well as



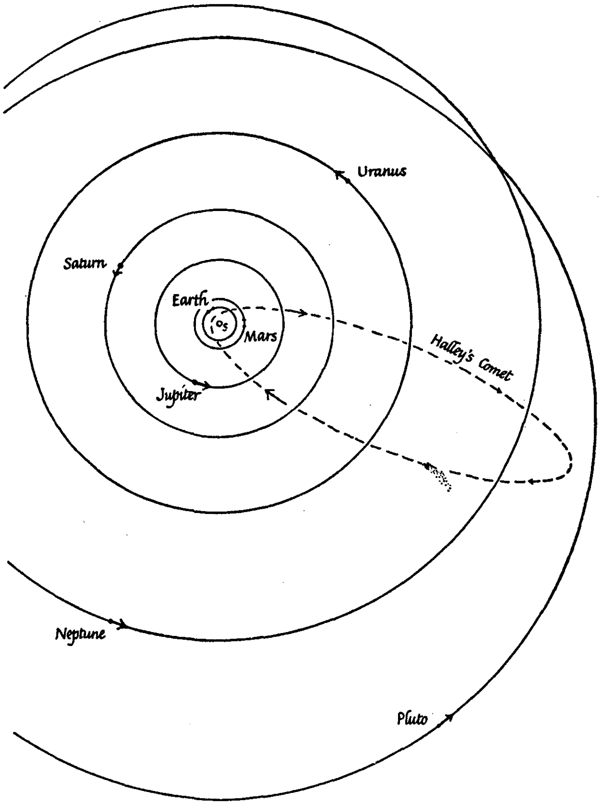
COMET, MOVING IN AN ELLIPTICAL ORBIT WITH THE SUN IN ONE FOCUS, PASSES THROUGH SOLAR SYSTEM

* This is one benefit of a good theory: it yields estimates of sizes, in this case masses -- estimates to be trusted so far as we trust the assumptions that went into the theory. Copernicus' calculation of relative orbit-sizes is one example of a theory yielding measurements. Another example: our kinetic theory calculation of the speed of air molecules from pressure and density.

wonder. To the superstitious they were more strange and queer than planets. When measurements showed that some comets pass quite close to the Earth, there were added worries: the practical fear that the comet might hurt us -- and, in earlier times, the theoretical disaster that the comet had smashed through the crystal spheres.

In past ages the arrival of a comet was regarded as a very serious event that boded ill or good for mankind.' Up till Newton's day comets were thought to be very mysterious; but Newton used observations of several comets to show that each moves in a long ellipse that fits with Kepler's laws. They are members of the Solar System that come visiting from very far out, probably from some reservoir of material outside the outermost planets.

So Newton welcomed comets into the Solar System.



Sketch of the Solar System, with Halley's Comet Shown
The most recently discovered planet, Pluto, is very small and pursues an elliptical orbit extending from within Neptune's to a much greater distance. (Mercury and Venus are not shown)

Small comets are observed by telescopes every year; but a very large one has not arrived in recent times. Here are notes of two comets seen and studied by Newton.

Halley's Comet. Seen by Halley in 1682. Orbit deduced with period 76 years. Returned within a month of prediction, in 1759. Returned twice since then: 1682 1759 1835 1911 (expected 1987)

Newton's Comet. Seen 1680. Orbit deduced: very long ellipse. Expected to return about 2255. If estimate is correct it is the comet seen in 1106, and the comet in 531, and the comet in B.C. 44 (Caesar's death).

We now believe that comets are mostly made of fine dust and gas, shining by reflected sunlight, so that we see them when they are near the Sun.

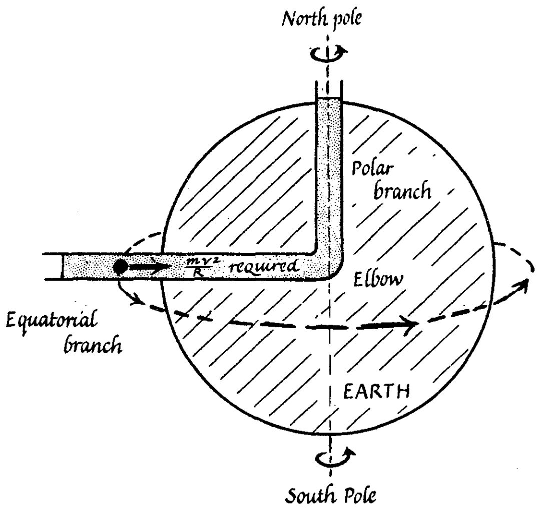
Occasionally, a comet has failed to arrive back on time but has appeared late, or even broken into two parts. Calculating back along its orbit, we find it had a close encounter with a planet, say Jupiter. Then Jupiter changed the comet's orbit; but the comet did not change Jupiter's orbit noticeably. That is how we know that comets are not at all massive.

---

(8) Shape of the Earth: an oblate spheroid.

In Newton's day the Earth was thought to be a perfect sphere. Newton predicted it must be a spheroid, flattened at the poles, bulging at the equator. Surveys, not long after Newton's time, showed that the Earth does have the kind of shape Newton predicted and more careful surveys have confirmed that, with some modifications.

Newton realised that a spinning sphere with mobile oceans on it would maintain a tremendous equatorial bulge of water. Although we should not observe huge tides on account of that bulge, (because it would all be carried round with the Earth's daily spin), it seemed unlikely, and we now know there are not those great ocean depths. It seemed more likely that the Earth, in some early pasty state of its formation, had itself taken on a shape with an equatorial bulge.



To Estimate the Bulge of a Spinning Earth imagine a pipe of water running from North Pole to center and out to equator. Calculate the extra height of water in equatorial branch needed to provide mv^2/R forces for spin. This gives extra radius of bulge for a pasty Earth congealing while spinning.

Therefore, Newton thought about the shape the Earth must have if, long ago, in pasty form, it took the "equilibrium shape" such a spinning body required. His argument ran somewhat like this:

Consider a pipe of water running through a spherical Earth from the North Pole to the centre and out to the Equator. If this were filled with water, just to the Earth's surface at the North Pole, where would the water surface be in the equatorial branch of the pipe?

At the centre of the Earth the water pressure at the bottom of the polar pipe is due to the weight of the water in that pipe; and this pressure pushes round the elbow at the bottom and out along the other branch, trying to push the column of water up that branch. The weight of water in that branch pulls it down. But these two forces on water in the equatorial branch must be unequal. They must differ by enough to provide an inward centripetal force to act on the water in that pipe, which is being carried around with the spinning Earth. The weight (inward) of the water in that branch must exceed the outward push from the water at the elbow by the amount needed for mv^2/R forces.

(We might say, more carelessly, some of the pull of gravity on the water in the equatorial pipe is "used" to keep the various portions of that water moving round in a circle, and only what is left over makes the pressure at the bottom of the pipe.)

Therefore, the water column in this pipe must be taller than that in the polar pipe. The equatorial pipe must extend out beyond the Earth's surface to carry the extra height of water.

Newton calculated the extra height and found that 14 miles would be required. He argued that the Earth at an early pasty stage would bulge out about this distance. A short time later, measurements of the Earth confirmed the prediction.

Problem. Look at Jupiter, in photographs or through a telescope with fairly large magnification. Although Jupiter is covered with clouds, astronomers know he is spinning and can even estimate his rate of spin. How?

Make your own model of a spinning Earth. Take a strip of very thin paper about 8 inches by 1 inch and make it into a round loop by joining the ends with sellotape or paste. Make some small cuts through the join with a knife, so that you can push a pencil through it. At the other end of the diameter, cut a hole so that a pencil can pass through it freely. Install this loop on a pencil, so that it looks like this ϕ and try rolling the pencil very quickly to-and-fro between the palms of your hands.

(9) Differences of "g".

Both the equatorial bulge and the spinning of the Earth have slight effects on the apparent value of "g". Newton predicted that the acceleration of free fall would be slightly smaller at the equator than at the poles. At the equator we are a little farther from the centre of the Earth so gravity should be a little weaker, by about 0.2%. And at the equator a falling object is orbiting round the Earth's axis with the rest of the lab, so a little of the Earth's pull on it is used to give it the necessary centripetal acceleration. That makes gravity seem slightly weaker, about 0.3% less.

These again were quite unexpected things but later surveys have confirmed the prediction roughly; though there are additional local differences due to mountain masses, etc., and some of those are of great interest to prospectors for oil or metal ores.

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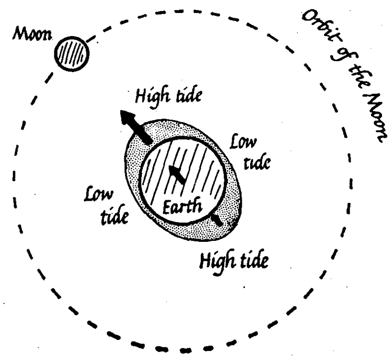
(10) Tides.

Newton showed that tides can be explained as due to differences of gravitational pull on the ocean, exerted by the Moon. This was one of the greatest achievements of his theory because it linked well known important events on Earth with the story he was building up concerning gravitation and the heavens.

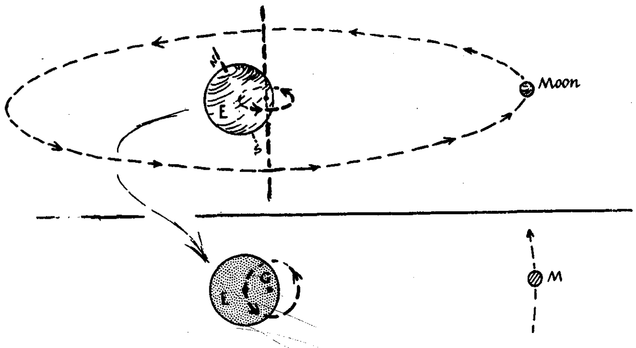
OCEAN TIDES ARE CAUSED BY DIFFERENCES OF MOON'S ATTRACTION
The extra-large pull on ocean nearest the Moon raises one high tide. The extra-small pull on ocean farthest from the Moon lets it flow away into another high tide.



You probably heard long ago that the Moon's pull makes the tides; but in Newton's day it was a surprising suggestion.

The Earth and the Moon pull each other. The pull of the Earth holds the Moon in its orbit. That gravitational pull provides the force mv^2/R which each piece of the Moon of mass m needs to keep it in oribt.

But the Moon also pulls the Earth with an equal and opposite pull. That force keeps the whole Earth moving in a small monthly orbit round a "centre of gravity" point (centre of mass of Earth + Moon) 3000 miles out from the centre of the Earth towards the Moon.



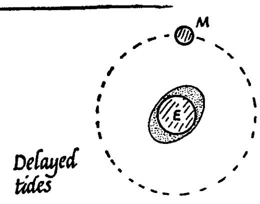
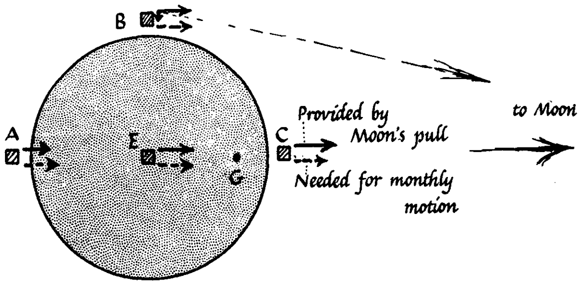


Diagram for footnote on opposite page

The centre of the Earth moves round that centre of gravity point in a circle of radius r = 3000 miles. And all the other parts of the Earth make similar circles at the same time -- the whole Earth moves like a cloth in a man's hand, cleaning a window with a circular motion. Every piece of mass m needs the same force mv^2/r towards the Moon. (r = 3000 miles).

Tide-producing forces on different parts of the Earth.



The Moon's gravitational pull produces just that needed force on a mass m at the <u>centre</u> of the Earth. But the pull thins out as distance from the Moon increases. At places on the Earth farthest from the Moon the pull is a little weaker. And at places on the Earth nearest the Moon the pull is a little stronger. These differences of Moon-pull make the water of the oceans pile up into two humps: a hump farthest from the Moon where the water is not pulled "inward" quite enough; and a hump nearest the Moon where the water is pulled "outward" too much.

(These small differences of Moon-pull are greatest at the places farthest from the Moon and nearest; and one would expect them to provide the chief forces driving the ocean up into humps. A careful study shows that the strongest tide-generating effects occur in regions at 45° to the Earth-Moon direction because their gravity-field has a big mass of water to pull towards the hump. However, the humps are where we suggested above).

Those humps <u>are</u> the ocean tides. They are only a few feet high, (not nearly as big as the 14-mile bulge of the Earth's equator, because this monthly motion is much slower than the Earth's daily spin).

The Earth also has its 24-hour daily spin; and that carries the land masses round to meet those humps in turn. Then those humps of water go sloshing up every shore in turn, and back again, making two high tides in 24 hours.

If you live near the sea, you know that high tide does not occur at the same hour each day. That is because the Moon travels round the Earth in the same direction as the Earth's daily spin, so you must wait a little more than 24 hours to meet the same hump of water that is opposite the Moon. *

Spring Tides and Neap Tides. Like the tides due to the Moon, there are also two humps of ocean due to differences of the Sun's pull on water farthest from the Sun and nearest to the Sun. These are smaller than the humps due to the Moon, because the distance across the Earth's diameter does not make such an important difference in Sun's gravity. The gravitational field due to the Sun is much stronger than that due to the Moon because the Sun has a very much bigger mass; but the differences of Sun's pull are smaller than the differences of Moon's pull.

The humps of tide due to the Sun will fall on top of the humps due to the Moon when the Sun is in the same direction as the Moon, at new Moon. They will also fall on top of each other at full Moon, when the Sun is just opposite the Moon. At each of those two times there will be extra-large tides, Moon-tide + Sun-tide together. Those are the large "spring tides".

Half-way between spring tides, when the directions of Sun and Moon are at 90° (half Moon), the smaller humps due to the Sun will fall on the low-tide troughs between the humps due to the Moon. Then there will be smaller tides, "neap tides", which are Moon's tide - Sun's tide.

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(11) Mass of the Moon.

We promised to show how Newton found a "satellite" for the Moon, and could therefore estimate the Moon's mass. He estimated the Moon's mass from the tides that the Moon causes. In other words, the Moon had a satellite after all: the two humps of water that we call tides.

Large tides (spring tides) occur every fortnight, when the Sun's tide and the Moon's tide agree; small tides (neap tides) occur a week later when the Sun and Moon are in directions at right angles.

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* There are considerable delays and peculiarities in the motion of tides, which have been sorted out by studies since Newton's day. These are due to friction and inertia, combined with the patterns of land and ocean boundaries. So tides on the seashore are larger than one would expect in some places and smaller in others; and the high tide may not arrive at the time one would expect from simple theory.

From measurements of spring tides and neap tides, in open ocean, Newton could separate out the tide due to the Sun from the tide due to the Moon:

Spring tide = Effect of Moon + Sun

Neap tide = Effect of Moon - Sun

Simple algebra - adding two equations, also subtracting them, gives the two separate effects. Knowing the size of ocean humps due to differences of the Moon's attraction, Newton could treat those as a satellite of the Moon and estimate the Moon's mass roughly.

Nowadays, we can give the Moon a satellite, a spaceship flying round it; and by watching that we can measure the Moon's mass very well.

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(12) Precession of the Equinoxes.

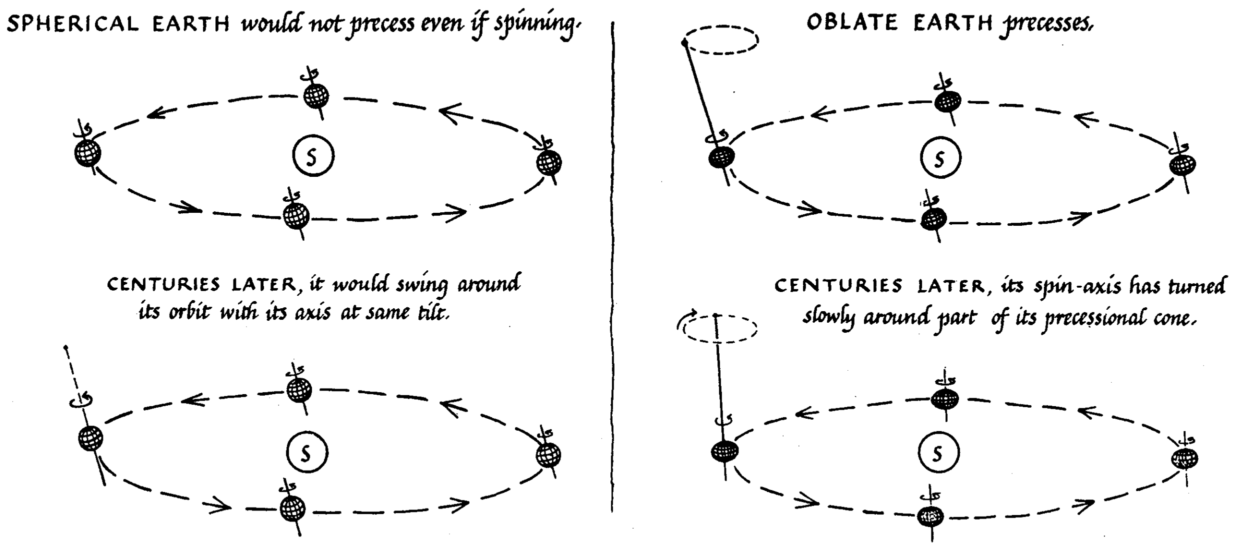
Newton gave a clear reason for the strange motion known as precession of the equinoxes, and he even predicted its period successfully.

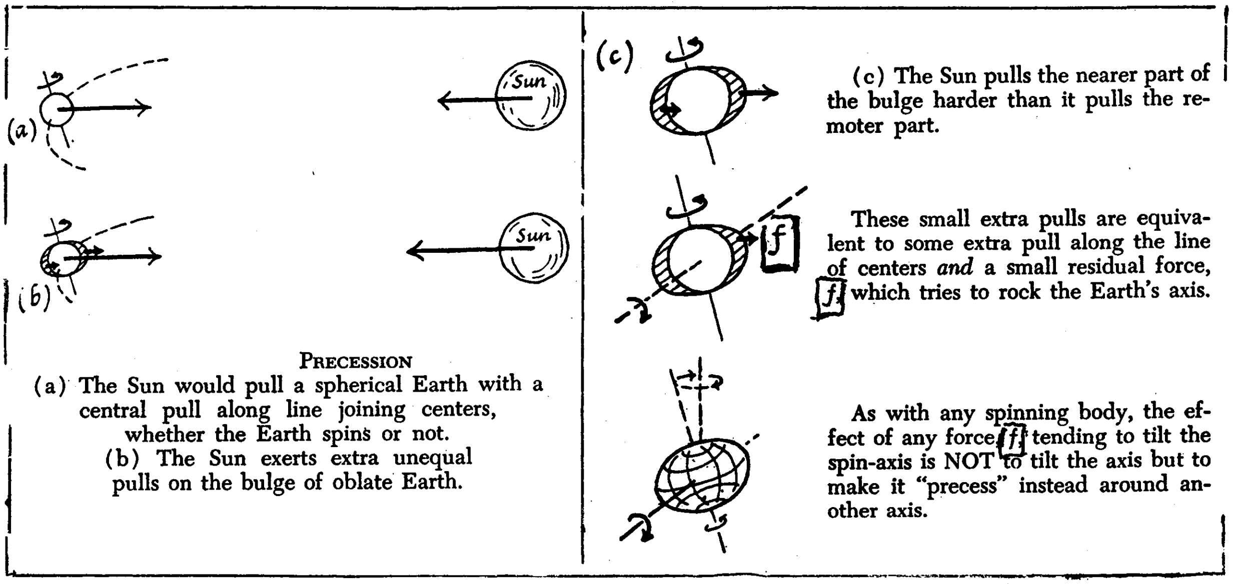
This was one of the most unexpected outcomes of Newton's theory. Instead of needing special assumptions, precession appeared as a direct, necessary consequence of gravitational pulls by Sun and Moon on the equatorial bulge on the spinning Earth.

Precession is the slow, conical motion of the Earth's spin-axis round the axis of the ecliptic. That was Copernicus' description of the slow, creeping motion discovered by the Greek astronomers and described by them in a more complicated way. That motion carries the Earth's axis round in about 26 000 years, making a cone of half-angle $23\frac{1}{2}^{\circ}$

After its discovery by Hipparchus, astronomers made allowance for this motion in their records and predictions, but they had no explanation beyond providing an extra sphere or arm in their imaginary machinery to imitate the motion. Copernicus described the motion more simply, but offered no explanation connecting it with other motions in the sky or on Earth.

Newton showed this strange motion is a necessary consequence of gravitation, and the Earth's spin. A <u>spherical</u> Earth whether spinning or not, would keep its axis pointing in a constant direction among the stars as it pursued its orbit round the Sun. But an Earth with an equatorial bulge will suffer slight <u>extra</u> gravitational pulls exerted by the Sun (and by the Moon) on the parts of that bulge. Since the Earth's spin-axis is tilted and not perpendicular to the Earth's orbit, those extra pulls make a rocking force $\boxed{f}$, which tries to change the tilt of the Earth's spin-axis.





PRECESSION
(a) The Sun would pull a spherical Earth with a central pull along line joining centers, whether the Earth spins or not.
(b) The Sun exerts extra unequal pulls on the bulge of oblate Earth.

(c) The Sun pulls the nearer part of the bulge harder than it pulls the remoter part.

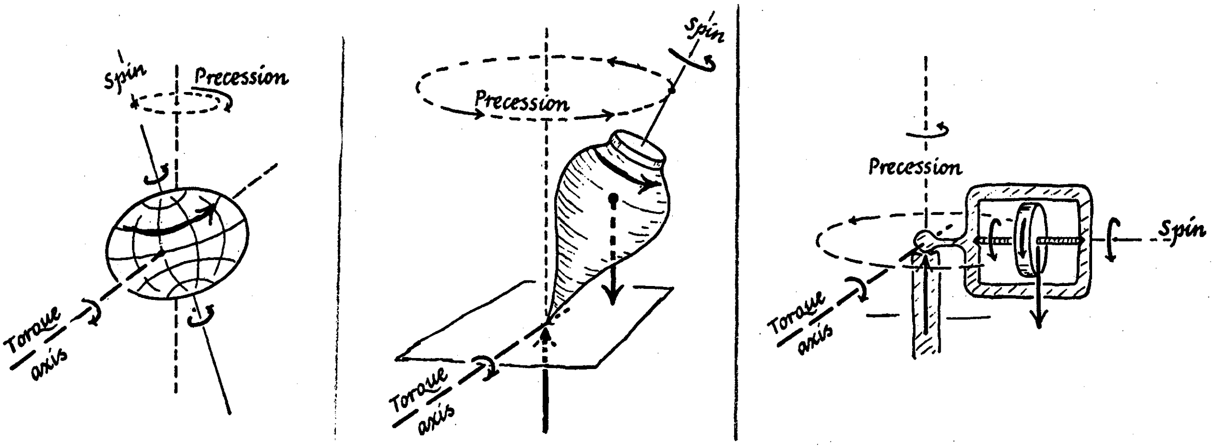
These small extra pulls are equivalent to some extra pull along the line of centers *and* a small residual force, $\boxed{f}$ which tries to rock the Earth's axis.

As with any spinning body, the effect of any force $\boxed{f}$ tending to tilt the spin-axis is NOT to tilt the axis but to make it "precess" instead around another axis.

But the Earth is spinning, so the rocking force does not change the tilt of the Earth's spin-axis as you would expect: instead, it makes the spin-axis slew round with a conical motion. You can see that with an ordinary spinning-top, placed on the floor leaning over, not quite upright. If it is spinning, it does not fall over but moves round with a conical motion. The details of Newton's explanation and calculation are too difficult to give here. You need the advanced mechanics that deals with spinning motion and predicts the behaviour of spinning tops and gyroscopes -- nevertheless, the basic physics in that advanced mechanics is still simply Newton's laws of motion -- it is the geometry that is difficult.

PRECESSION

The Earth, a spinning top, and a "mysterious gyroscope" all precess in the same way, for the same reason. In the sketches above "torque axis" means the axis around which the tilting force tries to rock the spinning object.



Newton could make a rough calculation of the time taken for a complete cycle of precession and found 26 000 years in agreement with astronomical measurements.

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(13) Irregularities of the Moon's Motion.

The Moon's elliptical orbit round the Earth changes its eccentricity slightly as time goes on and it moves slowly round in its own plane; the plane of the orbit slews round slowly; and the Moon shows small extra monthly and yearly accelerations. All these small changes of the simple orbit are symptons due to the same cause: small differences of the Sun's gravitational pull. The Sun holds the Moon, as well as the Earth, in a yearly orbit; but as the Moon goes round the Earth it is sometimes a little nearer to the Sun and feels a little extra pull from the Sun. And a fortnight later it is a little farther from the Sun than the Earth, and feels a slightly smaller pull from the Sun. Those differences of pull make the changes mentioned above. Newton predicted several of them and was able to test some of his predictions.

Why worry about these small changes of the Moon's orbit? For two reasons:

(i) They are part of our testing of Newton's theory or universal gravitation; and we want to back up the theory completely and let no tiny difference slip through our hands unexplained.

(ii) In Newton's day, and after, navigators desperately needed a clock that could keep time during a rough voyage. They could find their latitude by observing the pole-star; but for longitude they needed a clock -- to compare local noon-time with noon-time at their original port. The need was so great that Parliament had offered a huge prize; and two schemes were competing for it:

(a) timekeeping by precise observations of the Moon's position among the stars;
(b) a seaworthy clock, that could carry Greenwich time on a long voyage.

Both schemes proved workable -- thanks to extensions of Newton's work on the Moon's motion and Harrison's invention of a chronometer with a good escapement for its hairspring and balance-wheel. The two successful schemes shared the prize.

Problem. Suppose an 18th-Century mariner, setting out from England and sailing westward, reaches an island after several weeks of cloudy stormy weather. There, he sees the pole-star in a direction that makes 76° with the vertical. And when the Sun is at its noon-time highest, his chronometer, (which was set correctly in London and has kept going throughout the voyage), says 4 minutes past 4 P.M. What is his latitude? What is his longitude? (And what real island has he reached?)

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(14) Perturbations of Planetary Orbits.

Universal gravitation meant that, in addition to the Sun's holding pull, each planet must also be pulled by neighbouring planets with tiny disturbing forces. The larger planets must pull neighbouring planets out of their simple Kepler orbits noticeably. The forces must be very small. Newton could calculate that the Sun is 300 000 times as massive as the Earth and 1000 times as massive as Jupiter. So, for comparable distances, the Sun's great gravitational pull far outweighs the deflecting forces due to other planets. Yet those forces are there; they do have small noticeable effects, which we call perturbations.

Since each of those attractions is directed towards the perturbing planet, they are not "central forces" straight to the Sun: therefore Kepler's Law II will not hold exactly. Nor do those forces when added to the Sun's pull by vectors make a simple inverse-square-law resultant; therefore Kepler's Law III will not hold exactly. So, although actual orbits are almost exactly ellipses, those ellipses show slight slow changes.

Newton started the calculation of perturbations, particularly the effect of Jupiter's attraction on Saturn's orbit. Later mathematicians continued the work and we can now make full allowance for all inter-planetary attractions -- and we find close agreement with observation. *

Why worry about those tiny modifications of Kepler orbits? Because as well as adding their testimony for the general theory, they led to one of the finest triumphs of Newton's work -- described in the next section.

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(15) The Discovery of Neptune.

Planetary perturbations were not just a matter of trivial corrections which when verified gave the theory a little additional support. A century after Newton's death, they led to the discovery of an entirely unknown planet -- by pure Newtonian theory.

One new planet had been found by direct observation. That would have saddened Kepler. He had been happy with his five-solid scheme. Even though he had to thicken the bowls to fit the facts, it gave him a clear reason for the Sun having only six planets. A seventh planet would make the scheme impossible. In Kepler's day no more planets were found; nor in Newton's day. But in 1781 the first planet beyond the six known to Copernicus was discovered, by telescopic observation. The astronomer Herschel noticed a "star" that looked larger than its neighbors, and he found that it moved. This proved to be a planet, soon named Uranus. Measurements showed it was twice as far from the Sun as Saturn; and its orbit fitted with Kepler's Law III.

As time went on, 1781... 1800... 1820, precise observations of the new planet Uranus showed that it was not quite following a Kepler ellipse. The perturbing forces due to Jupiter and Saturn were carefully calculated and allowed for. There were still deviations of Uranus left over, unexplained. Though these were small, they were too big to blame on "errors of observation" -- anyway, they did not change irregularly from year to year like chance errors -- and they demanded explanation more and more insistently.

Here was a new challenge: why did Uranus seem to disobey Newton's theory, even by a little? Some astronomers questioned whether gravitation followed the inverse-square law exactly; others wondered about another unknown planet pulling on Uranus. That was ingenious, but it posed an almost impossible problem. It is hard enough to work out the effect of one known planet on another. Here was the reverse problem, with one of the participants quite unknown: blind-man's-buff reaching out into space a thousand million miles and more to locate an unseen planet, of unknown mass, at un-

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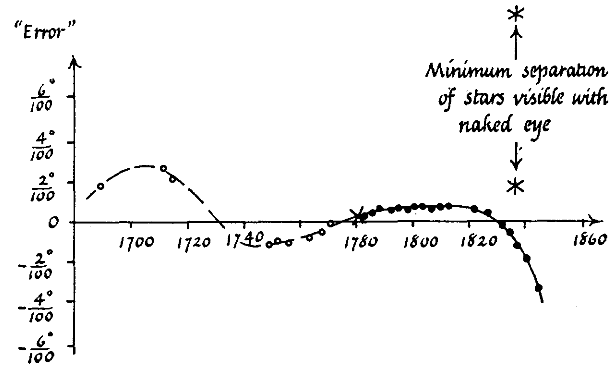
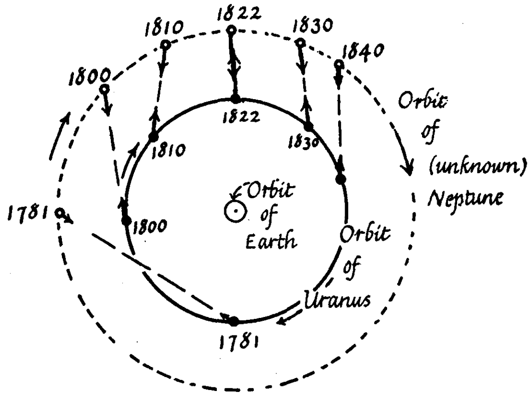
* Yet, to this day, a tiny part of the motion of the planet Mercury remains unexplained by Newton's gravitational theory. The major axis of its ellipse slews round very slowly, only 1° in 9000 years. This probably points to the need for a modification of the Law of Gravitation, now suggested by Relativity Theory.



Residual "Unexplained" Perturbations of Uranus (A.D. 1650-1850)

The "error" is the difference between the observed position of Uranus and the expected position (for a Kepler orbit) after known perturbations had been subtracted. The point X marks the discovery of Uranus by Herschel. Working back to its orbit in earlier times, astronomers found that Uranus had been observed and recorded as a star in several instances. These earlier records are marked by ° on the graph.
(After O. Lodge, *Pioneers of Science*)



Perturbing Forces on Uranus, Due to Neptune

The sketch shows positions of the planets in the years marked. Before 1822 Neptune's pull made Uranus move faster along its orbit so that it reached positions ahead of expectation. After 1822 Neptune's pull retarded Uranus. (After O. Lodge, *Pioneers of Science*)

known distance out, with only rough hints of its direction. *

Two young astronomers, Adams in England and Leverrier in France, took the suggestion of an unknown planet as the cause of the perturbations and attacked the problem with great courage and skill.

Adams started on the problem as soon as he finished his degree at Cambridge. He tried to calculate where the unknown disturbing planet must be. Two years later he wrote to the Astronomer Royal telling him where to look for a new planet. Actually, Adams was within 2° of the right place; but the Astronomer Royal did nothing beyond writing to Adams to ask for more information. (Then, as now, professional scientists were besieged with letters from cranky enthusiasts and often had to ignore them.)

Meanwhile Leverrier was working on the problem quite independently. He, too, decided on an unknown planet and finally managed to predict its position. His result was near to Adams'. The Astronomer Royal surprised to receive Leverrier's suggestion of a planet in almost the same place, did then start a leisurely search. However, Leverrier was impatient and wrote to the Berlin Observatory, where a new star-map made a quick search for a tiny planet much easier. The planet was seen. The discovery raced round the world and was soon confirmed in every observatory.

This new planet, discovered by pure theory, was named Neptune.

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* The observed perturbations would not even "point" in the direction of the disturbing planet. Nor would they indicate the amount of its pull at the time Uranus' position was being reported. They would merely show the effects of accumulated changes of motion, produced by the pulls of the unknown planet during the preceding weeks and months.

Exploring the Field of Force.

Newton and Kepler were using the planets to explore the Sun's gravitational field of force, to show that it is an inverse-square-law force all the way from the innermost planet Mercury to the outermost known planet -- and still farther out, using the testimony of comets. Two centuries later Rutherford and his colleagues used alpha particles to explore the field of force inside atoms. Instead of planets, already moving in the heavens, they use alpha particles fired from radium. And, in a way that corresponds to Kepler's Law III, the alpha particles told them that atoms are almost entirely hollow with a tiny central nucleus exerting an inverse-square electrical force.

Newton's Insight.

"Hypotheses non fingo" -- "I will not feign hypotheses" -- Newton wrote fiercely at one time. He meant he would not invent unnecessary details in his description of nature, or pretend to explanations that could never be tested. Yet, in later writings, he offered many a keen guess: at the nature of light, at the properties of atoms,....

He reached his great successes by his extraordinary gift of concentrating continually on a problem in reasoning -- "his muscles of intuition being the strongest and most enduring with which a man has ever been gifted." His concentration on his work, and his power of bringing every piece of information and every mathematical tool (whether old or newly invented) to bear on it, enabled him to make quite surprising guesses about things in nature -- he guessed right more often than mere chance would account for. That was not good luck, but intense thinking. He guessed correctly at universal gravitation. He made a guess, on scanty evidence, at the mass of the Earth itself -- a guess which could not be tested then but had to wait for Cavendish's experiment. *

Again, Newton devised a theory of light: tiny bullets travelling fast in straight lines to make sharp shadows; but he added waves to guide them into interference patterns. For many years, scientists laughed at Newton's queer mixed scheme. Now, two centuries later, we have clear evidence that light does behave both as waves and as bullets. Once again, wide-awake Newton made a wise guess.

* He argued that the solid ground of the Earth must be denser than water, or it would float up into many more mountains than we now have. On the contrary, regions near the centre of the Earth must be much more dense than the outer rocks, on account of the intense pressure there.* He knew something of the effect of pressure, because he knew the size and mass, and thence the density, of Jupiter, whose huge gravity would make great internal pressures. So he guessed that the average density of the whole Earth must be between 5 and 6 times the density of water. (We now know that it is $5\frac{1}{2}$ times!)

A century after Newton the actual mass of the Earth could be estimated from the results of Cavendish's experiment; and we now know that the Earth's mass is about 6.7×10^{21} tons.

By using Newton's method of comparison we then find the Sun's mass must be 2×10^{27} tons. The Sun continually radiates light; and the energy of that radiation itself has mass, which we can calculate by using $E = mc^2$. The Sun loses mass at a rate of about 4 million tons per second; yet that is so small a fraction of the Sun's total mass that we expect it to make little difference for a very long time.

Is the Theory True?

The factual information that goes into the building of a theory comes from experiment and is presumably a true summary of actual behaviour. But scientists feel uncomfortable when they are asked if the rest of the structure is true: the assumptions, the pictures, the models, the imaginative ideas which all form part of the machinery that we call a great theory. They would rather say that they value the theory because it is fruitful and useful and because it gives a sense of connected knowledge. As our knowledge grows, we often have to change our theory, often modifying it, sometimes remaking it altogether. But at any given time our theory serves as a map of our knowledge, to guide our work and to help us in discussing it with others.

When people first learn science they expect science to explain many things, to give scientific reasons for things that happen. And most people continue all their lives to expect scientists to give the true explanations of things. Yet, in fact, science cannot give the final reason, the true story right underneath everything. Science can only show that some new, unfamiliar event or behaviour is another form of something we already know. We link the unknown with the familiar known things. For example, we say a lightning flash is a large electric spark; or we say the force holding the Moon in orbit is like the force of gravity on a cricket ball. And sometimes science can collect together many different things and give one story to cover them all. In Newton's great theory he did both those things.

Newton himself knew that inverse-square field of force would account for Kepler's laws, and many things besides; but he said clearly that he did not know the cause of gravitation. He suggested that it must be some kind of influence that spreads out from every piece of matter, and penetrates matter freely, but that was only a description of observed properties. He insisted he did not know its ultimate cause. Modern scientists agree. Science is not able to push its explanations so deep.

Newton's Theory and Its Explanations.

Newton used mathematics to deduce many things from a few laws; but his treatment was essentially different from the deductive methods of the Greeks and their followers. Newton did not just sketch imaginary machinery. He devised his theory with the help of guesses from experiments; then he drew from that theory many predictions; and then he tested as many of those predictions as he could by experiment. (Some of them, of course, were things already known, so they passed the test at once.)

Thus, Newton's theory was a framework of thought and knowledge, tied to reality by experiment and clear definitions, able to make predictions which in turn were tested by experiment.

A theory, as Newton used it "explained" a variety of mysteries by referring them to a few familiar pieces of physics that you and I can work with in a lab on Earth.

Scientists value a theory when it is fruitful, that is, when it makes many predictions. We expect a good theory to make many predictions from few assumptions. If we needed to add one or two extra assumptions for each further prediction we extract from a theory, we should be disappointed: we might just as well have everything in nature run by demons, a whirling demon with a loopy mind for each planet, an invisible demon with pounding fists for gas pressure, a demon to pull each falling stone downward, and another to push a rising balloon upward,.... By endowing each demon with suitable behaviour, we could "explain" everything that we observe in nature. And that is just how things were explained (with pleasanter or more mysterious words instead of demons) in the pre-scientific age, some centuries ago. The essential advantage of scientific theories and the explanations they give is that they are economical. They help us to organize our knowledge and simplify it, and put it to use.

Run through again all the things that Newton's theory predicted or explained, to see its full power.

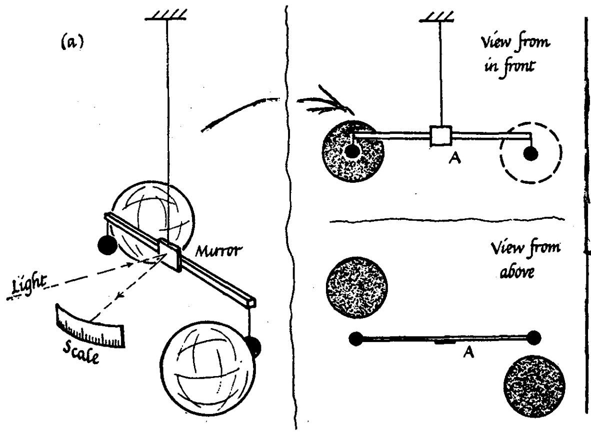
As a summary, to show how fruitful, and economical, that theory is, we have sketched a long chart of the history of astronomy from early man to Newton. It is printed on a separate page at the end, so that it can be cut out and pasted together to make one long list. (You are not advised to copy it. It is only offered as a "prompter in the wings" when you are running through the story or revising. Copying it out would be wasteful, and even misleading; but if you had time to start afresh and make your own version of such a chart, you would probably find that very valuable.)

CHAPTER IX. MEASURING G

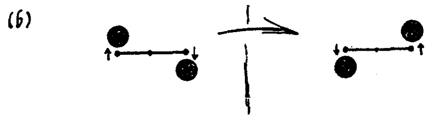
Newton knew that the gravitational attractions between man-sized objects in a lab on Earth must be extremely small -- perhaps too small for man ever to measure. But a century later those tiny forces were being measured -- a mountain pulling a pendulum ever so little out of the vertical, a large lead ball pulling a small ball on a very delicate balance.

The results built up further proof of universal inverse-square-law attractions. We shall not go into details of the apparatus. But the sketch shows the delicate "torsion balance" used by Cavendish to make the first measurement of gravitational attraction inside a laboratory. The table shows the results of many, varied, experiments to measure G, approaching a constant value as accuracy improved. Look at the table to see the variety of materials, distances and masses: then you can see how the agreement of the results assures us of Newton's Law of Gravitation on Earth -- while planets and comets carry the assurance out through the whole Solar System.

The Cavendish Apparatus



(a) The trapeze carrying the small lead ball was hung on a very fine twistable fibre. When the big balls were brought into position, their attraction made the trapeze twist the fibre slightly. This minute twist was shown by rays of light reflected by a small mirror, A, on the trapeze.
(b) To double the measured twist, the big balls were then moved across to the "opposite" positions, so that they pulled the trapeze round the other way.

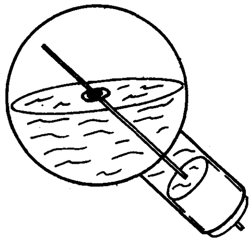


MEASUREMENTS OF G

Date (approx.)	*Experimenter*	*Attracting Mass* Description	*Attracting Mass* Mass kg	*Attracted Mass* Mass kg	*Attracted Mass* Description	*Distance apart* metres	*Result* G newton • m.²/kg²
1740	Bouger	Mountain	many millions of millions of kg	pendulum mass: a few tenths of a kg to a few kg	pendulum	several thousand metres	12×10^{-11}
1774	Maskelyne	Mountain			pendulum		7 to 8 " "
1821	Carlini	Mountain			pendulum		8 " "
1854	Airy	Outer shell of Earth	3×10^{20}		pendulum	6,000,000 meters	5.7 " "
1854	James	Mountain	many millions of millions of kg		pendulum	several thousand metres	7 " "
1880	Mendenhall	Mountain			pendulum		6.4 " "
1887	Preston	Mountain			pendulum		6.6 " "
1798	Cavendish	lead ball	167	0.8	lead ball	0.2	6.75×10^{-11}
1842	Baily	lead ball	175	0.1 to 1.5	balls of: lead, zinc, platinum, glass, brass	0.3	6.5 to 6.6 " "
1881	von Jolly	lead ball	45,000	5	metal ball	0.5	6.46 " "
1891	Poynting	lead ball	160	23	lead ball	0.3	6.70 " "
1895	Boys	lead ball	7	0.0012	gold ball	0.08	6.658 " "
1896	Braun	brass ball iron ball	5 9	0.05	brass ball	0.08	6.66 " "
1898	Richarz and Krigar-Menzel	lead cube	100,000	1	copper ball	1.1	6.68 " "
1930 1942	Heyl and Chrzanowski	steel cylinder	66	0.05	platinum, glass, gold	0.1	6.673 " "

Making models of the Solar System, and of earlier Greek schemes, can be good fun; but here, where our aim is to build good theory, there is a danger that models may divert too much attention from ideas to machinery. So we advise you to look at flat pictures like those in this book and to use your imagination. Yet you may find that one of the simple models described below gives you a helpful start.

(a) You can illustrate the sphere of stars with an umbrella. Place a saucer on the table to represent a flat Earth on a great ocean of water, and hold an umbrella, slanting, with its crook on the saucer. Mark stars on the umbrella by putting small pieces of sticky label on it. Then spin the umbrella slowly by twisting the handle. You might mark the ecliptic as a slanting circle, asking $23\frac{1}{2}^\circ$ with the edge of the umbrella. Then the Sun can be represented by a small yellow label stuck on the ecliptic, and moved along it from month to month. For each sun-position, turn the umbrella for the daily motion.



(b) You can make a more useful model with a round-bottomed flask half-full of water, supported on a ring-stand, with its neck slanting downward. Keep the flask closed by a cork, with a long knitting needle that runs right through the inside of the flask, to represent the Earth's north-south axis and point to the pole-star.

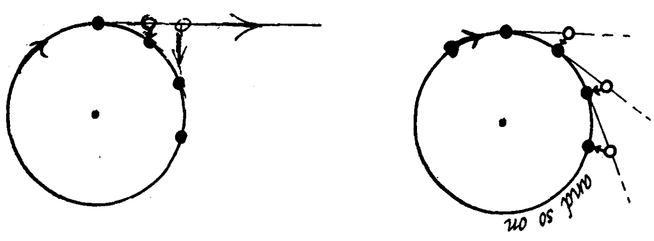
For Thales' model, place a small loose ring of wood inside the flask, threaded on the knitting needle. That ring will float on the water inside and represent a flat Earth. Mark stars on the outside of the flask with a greasy pencil, or with small sticky labels. Turn the flask, by twisting the neck, to imitate the daily motion of this celestial sphere. The water-line marks the horizon. Sketch the celestial equator on the flask, a great circle perpendicular to the knitting needle. Then sketch another great circle at $23\frac{1}{2}^\circ$ to the equator, to represent the Sun's path, the ecliptic. For the Sun, stick a small coloured sticky label on some point of the ecliptic and watch the Sun go round with the rest of the stars as you twist the neck of the flask. Then move the Sun "one month" (30°) along the ecliptic and again look at the Sun's daily path. Try moving the Sun to "mid-summer", "equinox", "mid-winter", always keeping the neck of the flask tilted at the same angle.

For Pythagoras' scheme you can replace the flat Earth by a small wooden ball, to represent a round Earth; but it is too difficult to construct a model with the other spheres for Sun, Moon, and planets.

Before you start work on satellites and gravitation, you may find it interesting to predict the time an Earth satellite must take to go round the Earth, by treating it as an ordinary projectile with gravity fall. For that, you should make a scale drawing of part of an Earth satellite's circular orbit, to illustrate the following story:

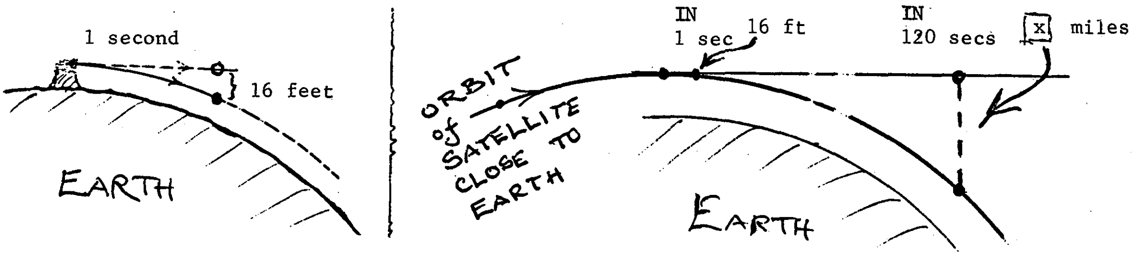
Suppose we launch an Earth satellite 100 miles or so above the Earth. Then, since the radius of the Earth is some 4000 miles, it is not really much farther away from the Earth than a stone or cricket ball thrown in the air. You know that for a falling stone the pull of the Earth produces an acceleration 32 ft/second per second. So a stone dropped from rest falls 16 feet in the first second. Any projectile does the same: instead of continuing along a straight line in the direction in which it is fired, it drops 16 feet from that straight line in the first second.
(You may have seen the 'monkey and hunter' experiment).

Now think of that Earth satellite travelling round the Earth in a circle, about 100 miles up. Instead of travelling along a straight tangent to that circle the projectile falls in from the tangent again and again -- really falling continually -- to keep in a circular orbit. In



one second it must fall 16 feet from its straight line tangent to its circular orbit. If you could draw a large scale-drawing of the satellite orbit and mark the 16 foot fall you could read off some more information from your drawing. However, 16 feet would hardly show on any drawing small enough to fit indoors. So we had better think of the satellite falling from its tangent path for a longer time: say 2 minutes or 120 seconds instead of one second.

In 120 seconds instead of 1 second a freely-falling body falls 16 feet x $(120)^2$. We are using the formula $s = \frac{1}{2}at^2$. Work out that fall in miles. Then put that on a large scale-drawing to find out how far the satellite goes in 2 minutes. And from that you can predict how long an Earth satellite will take to go round the whole Earth.



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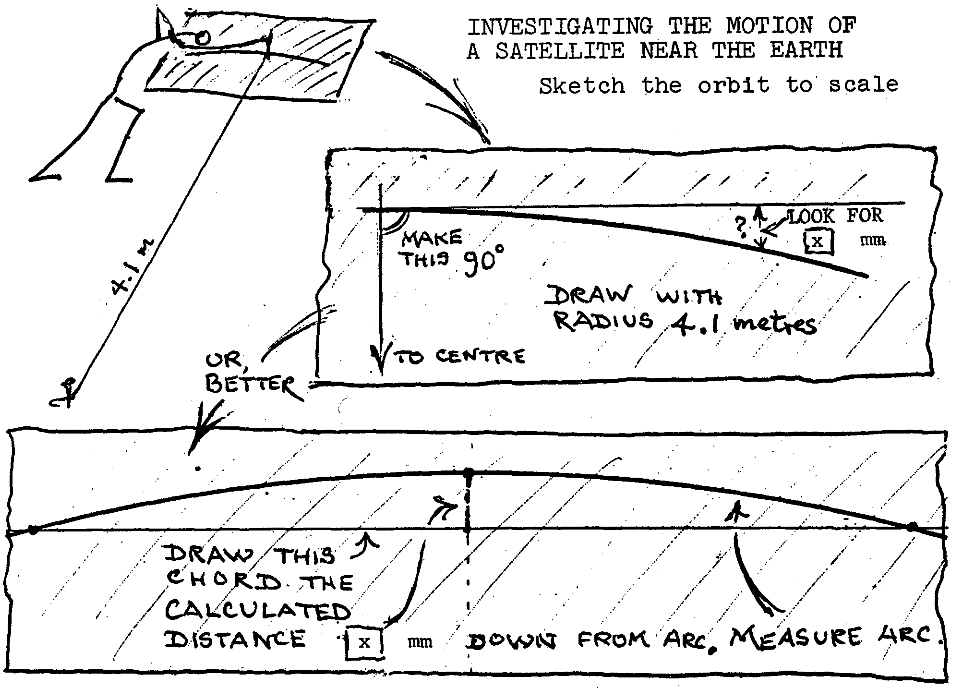
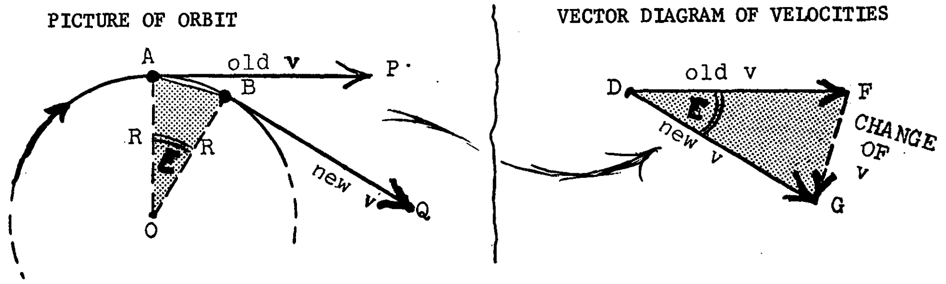
Here are instructions for your drawing. Since we are waiting for the result of your calculation of the fall in 2 minutes, we shall call it here $\boxed{x}$ miles, but you should use your actual result, of course. Use a scale of 1 millimetre to represent one mile. For the orbit, whose real radius is 4000 miles + 100 miles, you should use a circle of radius 4100 millimetres or 4.1 metres. Take a thin wire 4.1 metres long; anchor one end on the floor, stretch the wire taut across the floor and attach a pencil to the other end. Keeping the wire taut draw an arc on a long sheet of paper. You will need a piece of brown paper (or shelf paper or newspaper) about 5 feet long to take the length of arc; but it need not be more than 1 foot or even 6 inches wide.

The sketches here show how to treat your drawing. It is easier to make a double drawing, as in the final sketch; because then you can drop down your calculated fall $\boxed{x}$ from the middle of the arc.

On your drawing, mark the distance $\boxed{x}$, or rather your calculated value; then measure the distance AB, travelled in 120 seconds. Convert that to miles for the real orbit. You know the distance round the whole orbit: it is the circumference $2\pi \times 4100$ miles. Calculate the time for the whole orbit and compare your result with the published times for satellites near the Earth.



An object (or a point) moving round a circle with constant speed v has an acceleration v^2/R towards the centre. This is an important expression for many motions in physics. Since you need to use it for planetary orbits, you should see how it is arrived at, and not just accept it as a mysterious "formula". However, you should not have to learn the proof by heart for examinations.



We draw a circular orbit, with an object moving from A to B in time t. The object's speed, along the curved path, is v. At any instant, its velocity is v, along the tangent. We draw a vector AP to represent the object's velocity at A. That is along the tangent at A. We draw another vector, BQ, of the same length, to represent the object's velocity at B.

We re-draw those vectors in another place nearby, both starting from the same point D. There we we have two vectors, each of length v, which we label:

"OLD VELOCITY" (= velocity at A) and "NEW VELOCITY" (= velocity at B)

Since we want to find an acceleration, we need to know the change of velocity. So we ask: "What must be added, as a vector, to the OLD VELOCITY" to get the NEW VELOCITY?" It is the vector FG, in the sketch.

We re-draw those vectors in another place nearby, both starting from the same point D. There we have two vectors, each of length v, which we label:

We re-draw those vectors in another place nearby,.both starting from the same point D. There we have two vectors, each of length v, which we label:

Join A and B, and draw radii OA,OB. Then we have two similar triangles, AOB and FDG, because each velocity-vector is perpendicular to the corresponding radius -- so the angles at O and D are equal. Then, from the property of similar triangles:

$$\frac{\text{(change of velocity)}}{\text{(velocity, v)}} = \frac{\text{(chord AB)}}{\text{(radius R)}} \qquad \therefore \text{(Change of velocity)} = \frac{AB \cdot v}{R}$$

Suppose this kind of change-of-velocity, which is perpendicular to the actual motion, is related to an acceleration just like any other acceleration -- a surprising supposition, which must be tested. If so, we can calculate that acceleration as usual:

$$\text{ACCELERATION} = \frac{\text{(Change of velocity)}}{\text{(time taken, A to B)}} = \frac{AB \cdot v/R}{\text{(time A to B)}}$$

$$\text{ACCELERATION} = \frac{v}{R} \cdot \frac{AB}{\text{time taken}} = \frac{v}{R} \cdot v \quad \text{because} \left[\frac{AB}{\text{time}}\right] \underline{\text{is}} \text{ speed } v.$$

$$\therefore \text{ACCELERATION} = \frac{v^2}{R} \quad \text{and is directed towards the centre.}$$

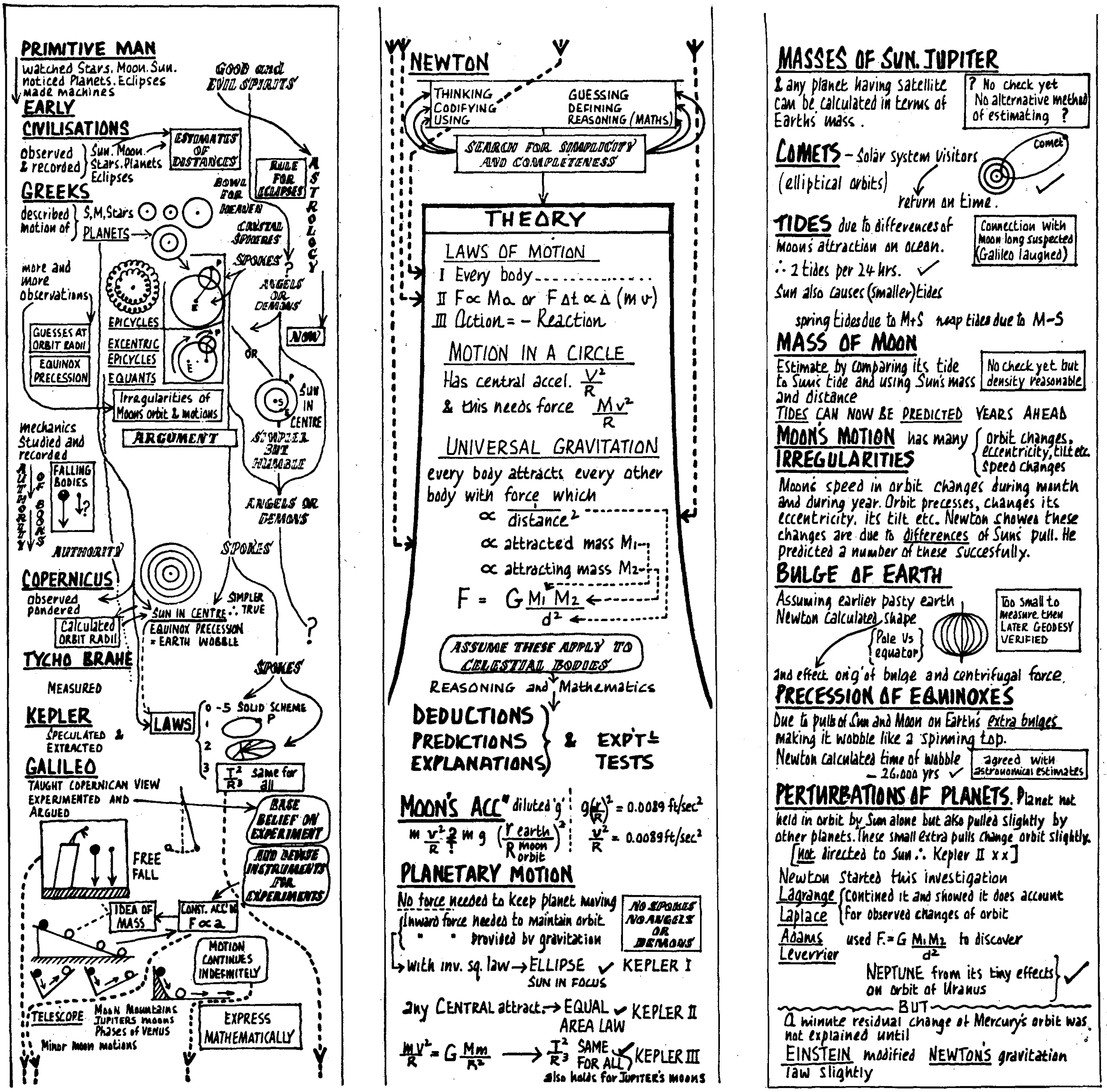
(NOTE: In dealing with the similar triangles, we took AB to be the chord straight across from A to B ; but when we said AB/time = v, we took AB to be the distance along the curved arc from A to B. Strictly speaking, we should allow for that switch by inserting a correcting factor (AB chord)/(AB arc); but in the limit as B is moved nearer and nearer to A, that factor tends to the limit 1. And, to find the acceleration "at an instant of time" we must move B up to A, and look at the limit).

On the other side of this sheet there is the chart that was mentioned at the end of Chapter VIII (Newton). This chart is offered in case it is of use in looking at the whole development.

The three columns are intended to be pasted together to make one tall column.

You are advised not to copy this chart. It is only offered as a "prompter in the wings" when you are running through the story (or revising). Copying it out would be wasteful and even misleading -- because this is only one person's way of sketching the development.

But, if you had time to start afresh and make your own version of such a chart, you would probably find that very valuable.



THESE THREE COLUMNS ARE INTENDED TO BE PASTED TO MAKE ONE TALL CHART