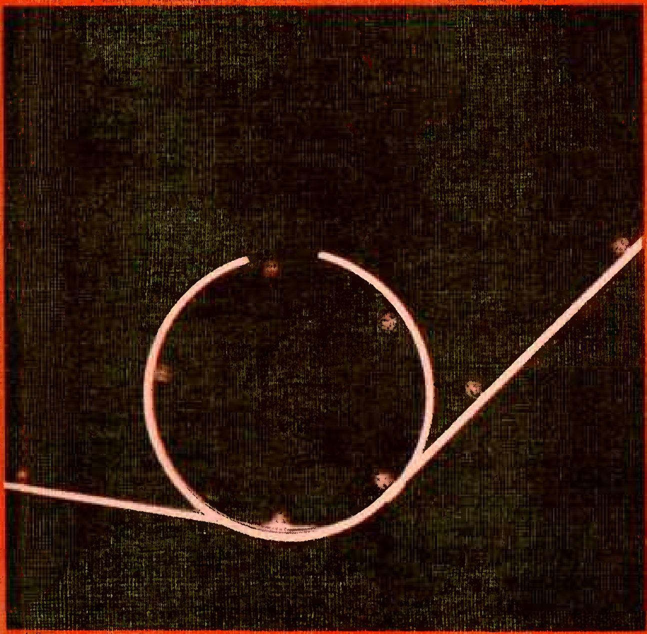
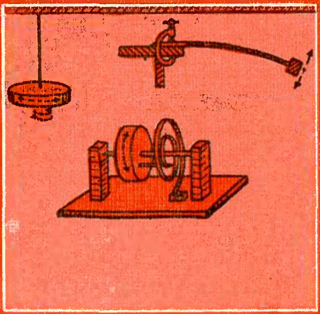
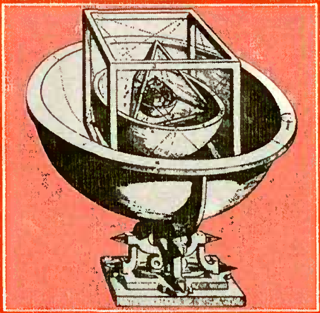
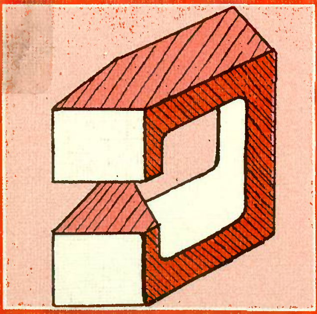
PHYSICS

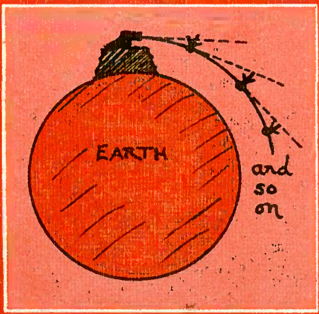
Teachers' guide V

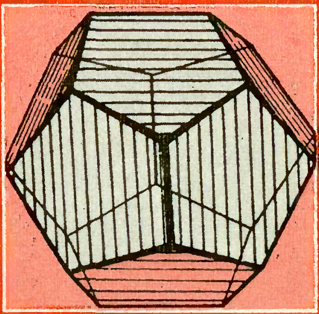












NUFFIELD PHYSICS TEACHERS' GUIDE V

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FOREWORD

This volume is one of the first to be produced by the Nuffield Science Teaching Project, whose work began early in 1962. At that time many individual schoolteachers and a number of organizations in Britain (among whom the Scottish Education Department and the Association for Science Education, as it now is, were conspicuous) had drawn attention to the need for a renewal of the science curriculum and for a wider study of imaginative ways of teaching scientific subjects. The Trustees of the Nuffield Foundation considered that there were great opportunities here. They therefore set up a science teaching project and allocated large resources to its work.

The first problems to be tackled were concerned with the teaching of O-Level physics, chemistry, and biology in secondary schools. The programme has since been extended to the teaching of science in sixth forms, in primary schools, and in secondary school classes which are not studying for O-Level examinations. In all these programmes the principal aim is to develop materials that will help teachers to present science in a lively, exciting, and intelligible way. Since the work has been done by teachers, this volume and its companions belong to the teaching profession as a whole.

The production of the materials would not have been possible without the wholehearted and unstinting collaboration of the team members (mostly teachers on secondment from schools); the consultative committees who helped to give the work direction and purpose; the teachers in the 170 schools who participated in the trials of these and other materials; the headmasters, local authorities, and boards of governors who agreed that their schools should accept extra burdens in order to further the work of the project; and the many other people and organizations that have contributed good advice, practical assistance, or generous gifts of material and money.

To the extent that this initiative in curriculum development is already the common property of the science teaching profession, it is important that the current volumes should be thought of as contributions to a *continuing* process. The revision and renewal that will be necessary in the future, will be greatly helped by the interest and the comments of those who use the full Nuffield programme and of those who follow only some of its suggestions. By their interest in the project, the trustees of the Nuffield Foundation have sought to demonstrate that the continuing renewal of the curriculum – in all subjects – should be a major educational objective.

Brian Young
Director of the Nuffield Foundation

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ESTIMATED ALLOCATION OF TIME

YEAR V

If it is assumed that a school year includes 30 weeks and that each week includes 3 physics periods, each of which lasts 40 minutes, then a very rough estimate of the number of periods suggested for each section of this Year would be:

Chapter 1	9
Chapter 2	15
Chapter 3	21
Chapter 4	12
Chapter 5	12
Chapter 6	12
Chapter 7	9
	90

Although these estimates are rough they will, nevertheless, provide some guidance as to weight to be placed on the various parts of the programme. It should be noted that the relative amounts of printing are not proportional to the teaching time required. Where subject matter is new and unfamiliar, it has been dealt with at length in order to help any teacher who may wish to experiment with it. On the other hand, more familiar subject matter has often been dealt with briefly.

KEY TO MARGIN REFERENCES

C = Class Experiment

D = Demonstration Experiment

T = Teaching of material (lectures, discussions with pupils, etc.)

F = Film

H = Suggestions for optional experiments at home

*
* = Commentary (notes on methods, aims, etc., offered to
*   teachers)

The experiments are numbered serially through the Year, irrespective of the classification C, D, F or H. The same numbers will be found for each experiment in the *Teachers' Guide to Experiments and Apparatus*. Where (a), (b) ... are added, these refer in some cases to separate parts of the same group of experiments, in other cases to alternative versions of an experiment.

PREFACE TO YEAR V

PLANS AND HOPES

Before describing the structure of this Year, let us take stock of our position.

In Year V most pupils will be preparing for a public examination and this will inevitably influence the work to some extent. But we hope that examinations will not dominate the teaching. For a large fraction of our pupils this will be the last year of formal instruction in science. About one half of those dropping the subject will be leaving school: the remainder will go on to pursue non-scientific disciplines. What sort of scientific background do these people need?

Consider the school leavers first. Apart from those who enter engineering apprenticeships, they will not make direct use of their scientific knowledge, so that facility in experimental techniques is not of high priority; but they should know what it is like to conduct an experiment and something of the difficulties of interpreting the results. The ability to solve numerical problems is not a skill which is likely to survive the passage of years (dearly though one would like to think that the future householder could make a quantitative assessment of say the relative costs of using gas or electricity to heat his home). But the realization that physics *is* a quantitative science, in which it is possible to compute correctly from known data what will happen in a hypothetical situation, is of first-rate importance. As a citizen in a scientific world, he should neither be afraid of science nor be overawed by it. He should realize that natural phenomena usually have a rational explanation and that scientific methods can be powerful tools in understanding and controlling man's environment. In other words, he should have an educated person's knowledge of what science is. This is the end to which our scheme is directed. The details of which topics are included are relatively unimportant – we teach a representative sample of physics, not the whole of it – but it is important to remember that at the age of fifteen, most pupils will grasp concrete examples more readily than abstract principles.

In addition, there are certain key ideas that are so important, both in physics and in the world as a whole, that they should become second nature to everyone. These are:

The conservation of energy and the dominant role that energy plays in scientific theory and in the economy.

Heat as a mode of molecular motion; and the statistical nature of thermal laws.

The properties of electric currents (conceived as a stream of electrons); 'electronics' as powerful practical knowledge.

The nature of light and the properties of the electromagnetic spectrum.

The atomic nature of matter and existence of fundamental particles.

The nature of radioactivity and nuclear changes and the possibilities and dangers of these.

The growth of atomic theory from early pictures to modern models.

Anyone with this background should be able to listen to a scientist talking in general terms and follow at least the gist of his argument. That is an ability which should make life more interesting and meaningful for the average citizen and we hope that some pupils, at least, will be impelled to find out more for themselves. In addition, many people – businessmen, lawyers, shop stewards, nurses – may have to carry on a discussion with technologists in the course of their careers, while parents and teachers will have to answer the questions of young children. A scientific background is, if not essential, at least highly desirable in every walk of life.

There is nothing in the discussion above that does not apply equally to the future arts student and the future scientist – good science for citizens is also good science for specialists – but the latter require something more. Nowadays, the student of almost any of the humanities needs some knowledge of science. The historian must be aware of the impact of scientific knowledge on the thought and economy of the period he studies. The archaeologist uses scientific tools in his work. The economist is concerned with science as an economic force – also he aims at using scientific tools and analogies. The philosopher is increasingly concerned with scientific matters. To cater for these, it is important to include something of the history of science, more especially since new and improved methods of teaching tend to obscure the original approach. It is probable, too, that the future arts student will appreciate a panoramic view of modern science (though the potential science specialist might be content with traditional treatment), and he will certainly want to look closely at the way in which physical arguments are justified and how they hang together. We hope that these things will be taken up again, at a greater level of maturity, in general sixth-form studies; but the basis should be laid now and the relevance of science to humanities made plain.

These aspects too will be valuable as part of the general education of the school leavers, although some of the topics are more academic and theoretical than would be chosen for that group alone.

Lastly, the course must make some provision for the needs of the future science specialists – the future physicists, chemists, mathematicians, engineers, doctors *et al.* In some ways this is the most important group, since the economic well-being of the Country will largely depend on their skill. It could be argued that the whole syllabus should be designed for specialists and the others left to make what they can of it. But that is not our policy. We believe that, up to O level, education should be general and not vocational and that the needs of ordinary people, as citizens and individuals, should predominate – even over strong economic demands.

Nevertheless, it is important that the syllabus provided lay a broad and firm foundation upon which later specialization may be built and that nothing be done to turn the potential scientist from his path. One hopes also to attract some of the waverers into a scientific career.

One hopes that the future scientist will be interested above all in the *ideas* of science – he will be a poor scientist if he is not. It must be recognized, however, that many will be more attracted by the power over the environment that science places in their hands. So our teaching should include topics catering for this and giving some facility in experimental techniques. And the proper place of formulae – as servants, not as masters – should be taught. The employers of school leavers entering engineering apprenticeships will expect such preparation. And for the very bright pupils, we need to include some quite difficult problems to stretch their intellect, to show that science is worthy of their mettle.

Thus our syllabus must cater for many needs: inevitably some compromise is necessary. It is hoped that examinations will allow the teacher to emphasize the aspects which are best suited to the interests of his class. Some aspects will be treated mainly in laboratory work or in homework problems, where the emphasis can be changed to suit the class.

We have made no explicit mention of applied science topics. This programme does not base its teaching on them directly. Yet pupils should be aware of the way in which physics interacts with engineering and we should show them something of the nature of the latter – one of the prime needs of the Country is that young people should not despise applied science. We therefore recommend that the topics in the syllabus be illustrated, wherever possible, by examples of their application.

THE WORK OF THIS YEAR

This is a Year of important experiments and ideas, in which we draw upon the work of previous Years but expect more imaginative thinking, more reasoning, and new experimenting. We want to develop some taste for theory and to explore further in 'atomic physics', in both experiment and theory.

Newton's Laws of Motion – so far treated as great principles and tested in simple class experiments – are now put to the use that Newton himself set forth: to form a grand theory of the planetary system. For that we must have a quantitative treatment of circular motion – to be done by an experimental approach if pupils find the geometrical discussion too hard.

Then, armed with some understanding of orbital motion, we can continue previous work on electron streams by bending their path with a magnetic field. To analyse measurements, pupils must use some knowledge of the force exerted by a magnetic field on a stream of charged particles. That is difficult, but we shall not evade it (thus losing our chance of clear knowledge of electrons) or spoil it by announcing an unexpected 'formula'. Instead we shall make a direct experimental approach and measure the strength of the magnetic field that we use by putting a simple current-balance in it. For pupils who find this work too hard, we might offer a shorter qualitative treatment that would leave more time for the other topics of this year.

Essentially, however, this is a programme of reasonable intellectual standards, for average O-level candidates. If one topic in this Year seems too hard, others are likely to appear hard too. If pupils find the topics too hard, the proper solution is a change to a different programme. If teachers consider the topics too hard, at a first glance, we hope they will try teaching it – as they are experimental scientists – *twice*: a first round to see its possibilities, a second round to see how their own version runs.

A simple study of waves and oscillations will be resumed from earlier years. That will lead, on the one hand, to a discussion of interference by waves – for use in building atomic models – and on the other hand to experiments with alternating currents – for use in ordinary life. And pupils will take a short, informal look at simple harmonic motion.

Then while simple atomic models are being discussed, experiments on radioactivity will be carried out. This work will open up new knowledge and help to encourage the imaginative thinking by which scientists formulate a 'model'.

As the discussion of atomic structure continues, films and demonstrations will carry pupils as far as the ability and knowledge of each class will allow.

Class Experiments

The class experiments that are necessary for the teaching of this Year will not take up all the available time. Some class experiments with a.c. should be postponed from Year IV till now so that pupils can enjoy working at them carefully: experiments with the electromagnetic kit; and experiments with slow a.c. – all with plenty of use of oscilloscopes. Now is the time for a few pupils to make a careful measurement of 'J'; which would have taken up too much time in Year IV – and might have been misinterpreted then.

Able pupils who have time and interest may want to do their own Millikan experiment now; or some may even want to measure the speed of light. Either of those experiments will take much time; but the experimenters would gain so much experience of experimental physics that they could well afford to miss other experiments.

Thus the class experiments this Year should have all pupils making an estimate of e/m: and some pupils measuring e or c (or perhaps h or even G). One such 'great experiment' can make a tremendous contribution to a young person's education. It need not make great demands on the teacher's time once the apparatus is provided – in fact it *should not* do so, since the point of the experiment is not to train the pupil in advanced experimenting but to give him the experience of independent work. Teachers with heavy timetables and crowded laboratories may think this an unrealistic dream; but we believe that pupils who have followed our programme in spirit as well as in content will be ready to undertake such work in a trustworthy and skilful and resourceful way that will make that dream come true.

Aim

All through, the important thing for teachers to keep in mind is the overall view that they are giving to pupils who will end physics now: the knowledge of physics that those young 'scientists for a day' are gaining, and their picture of nature, explored and well-understood up to a point, then bounded by new regions of unfinished knowledge. Here at the end, as in the earlier Years, we hope pupils will conclude that 'science makes sense'.

NOTES ON THE TEACHING OF THE ASTRONOMY SECTION

Minimum Programme

With some groups, teachers will feel that the time available for astronomy is short. So the treatment must be held to a minimum, though it must be full enough to reach the principal aim: to show pupils the development of Newton's theory.

We suggest the following programme as a minimum:

1. Brief description of observed facts: motions of stars, Sun, Moon, planets.

2. Brief description of early man's use of astronomy for clock, calendar, and compass. Mention astrology. Importance of heavenly events promoted speculation about gods or demons as 'explanations'.

3. Describe, chiefly by pictures, a few Greek geometrical schemes as reasonable machinery to explain heavenly motions. Suggested examples:

   Simple revolving sphere (Thales)

   Concentric spheres with round Earth at centre (Pythagoras)

   Many spheres, revolving about different axes to imitate observed motions closely (Eudoxus)

   Circles and sub-circles (with Earth short distance off the centre of main circle); and elaboration of that (Ptolemy)

   (If time permits and interest encourages, short descriptions of Greek methods of estimating size of Earth, distances of Moon and Sun)

4. Descriptions of Copernican system, demonstrating how it accounted for observed motion of planets in orbits with loops. Example of Copernicus's calculation of orbit sizes. (With faster group, Copernicus' simple story for precession.)

5. Mention of Tycho Brahe as fantastically precise observer.

6. Kepler's Laws described, possibly with brief account of his work in extracting them.

7. Mention of Galileo contributing to development of astronomy by teaching Copernican view clearly and by devising a telescope and using it, among other things, to show Jupiter's moons as a model solar system. (For our teaching of astronomy only a brief mention of these contributions is necessary. It is tempting to give a much fuller account of his life and work; but, although that is of great interest, it is not essential here.)

8. Description of Newton's theory and its fruits: assumptions; predictions or explanations of Kepler's Laws, motion of comets, shape of Earth, tides, precession of equinoxes and perturbations of planetary motion – which led to the discovery of Neptune. We hope teachers will be able to show this unrolling of great theory by pointing to its fruits on a large chart.

Warning about Models for Greek Schemes

Ingenious Models: misleading here. We shall show Greek schemes on the way to our target, Newtonian theory. It is very tempting to make mechanical models to illustrate the schemes; but showing models is likely to take too much time and to divert attention of both teacher and class from the main advance to the target. The teacher who finds himself busy devising models would be wise to pause and ask himself whether he is in danger of losing the point of this teaching. We offer the following comments to teachers who are considering mechanical models:

1. In this suggested programme of teaching astronomy for the development of theory, mechanical models of Greek schemes are *not* necessary. We recommend avoiding them, because they will divert attention from ideas to machinery, from intellectual grasp to interest in mechanical ingenuity.

2. Where a laboratory already has models, they might profitably be shown, *if* they can be introduced *lightly* and shown very briefly.

3. We positively advise schools *not* to buy any mechanical models however tempting the description.

4. Where a teacher has devised his own model, we should not discourage him: the delight of making one's own gadget to demonstrate a new idea will often shine through the dangers of delay and diversion and illuminate one's own teaching. (But that does not transfer to other teachers.) Even so, we offer him three warnings:

a. In making the great profusion of models in the past, inventors have found that devices which involve spins about several axes have to be more complicated than one would expect.

b. A 'partial' model, such as an umbrella, which shows only a patch of the picture, helps the teaching *quickly onward.* A 'complete' model which shows the whole picture is very likely to mislead pupils in the matter of ideas/gears. (We have seen 'Meccano' models which are testimonials to the ingenuity and skill of their makers; yet we should not use them here.)

c. Having made a model, one meets a further temptation: to put it on film. That will make the dangers worse.

However, we shall suggest a few very simple models.

Models of Greek Schemes for Slower Groups?

Teachers who have slower groups may feel specially tempted to substitute the making of some models for the studies of theoretical schemes which promise to be too highbrow. That might seem wise at the moment; but there the study of astronomy would end. Newtonian Theory, our real target, would be none the easier for the move into model-making. Instead, teachers faced with a real difficulty, arising from a slower group's different tastes and interests, should consider making a major change of programme.

Theories for Slower Groups?

As this course has proceeded from Year III to Year IV to Year V, the demands on intellectual skill and interest – of an academic kind – have grown, we hope, in consonance with general growth in these years. That has been intentional, in carrying out our plans for an O-level programme of teaching science for understanding. Where a slower group finds these later stages unfruitful or unsuitable in demands, we should want the teaching to seek our aims (or corresponding aims) in ways that *are* fruitful, and not to try to force a standard shoe on every foot. We should not advocate half-measures: keeping our 'syllabus' but just watering-down each topic to a simpler form; or just changing the target from thought-out knowledge to some more practical result; or just giving out the results without basis or explanation. Any of these will lead to poor science – neither confident understanding nor knowledge gained with delight.

Nor would a patchwork treatment be good: teachers who enjoy the sequence of topics in these later years may forgetfully take for granted the aims and connected scheme that underlie our teaching. They may be tempted to select a few topics to make a programme for a slower group. That *might* be a good programme; but it is unlikely to be, because the interconnections of our teaching will be lost in the selection process, and a new attempt to build-in corresponding aims will be needed. A fresh start would be far better; not saying, 'Which items are nice ones for a simpler course?', but asking, 'What are our aims in science teaching for this slower group? What items (from anywhere) could be chained together well to show how science makes sense? And what treatment of those will be most fruitful?' Those questions may lead to a programme with little in common, as regards syllabus or equipment, with our present one; yet, if it is fruitful, we shall be very glad.

Theories for Average Groups?

Returning to our present programme, for average O-level groups: we earnestly hope that teachers who feel doubtful whether an average group can follow our treatment of astronomy with fruitful enjoyment will give it a full trial. (Remember the question to the visiting explorer, 'But how do you know you won't like boiled missionary?')

This is a special topic and a special kind of teaching for teacher and class to explore together: yet it deals with one of the greatest intellectual developments in the scientific world. As A. N. Whitehead put it,

> '... The moral of the tale is the power of reason, its decisive influence on the life of humanity. The great conquerors, from Alexander to Caesar, and from Caesar to Napoleon, influenced profoundly the lives of subsequent generations. But the total effect of this influence shrinks to insignificance, if compared to the entire transformation of human habits and human mentality produced by the long line of men of thought from Thales to the present day, men individually powerless, but ultimately the rulers of the world.'‡

So, for an average group, we advocate neither half-measures nor patchwork treatment, but rather 'thin' treatment: quick, confident, rapid travel to the main target. With a slower group, the choice should be either (1) the same 'thin' treatment – but running a bit

‡ *Science and the Modern World* by Alfred North Whitehead. Cambridge University Press, 1926, pp. 299–300.

slower, or (2) omit this whole section. In the latter case, one should either treat the other topics of the Year extra carefully, or consider remaking the programme.

Much of the value of this part of the programme depends on our own approach in teaching. So we urge teachers to give this a confident trial, even if they have provisional doubts for their class. We venture to guarantee that a teacher's enthusiasm and skill will be greatly rewarded in this.

TOO FULL A YEAR?

Before they are half-way through the year, teachers will wonder whether the year is too full. Can they reach 'matter waves' and other exciting topics in modern physics in time? If Year III has prepared for Year IV and Year IV has had its full time, pupils and teachers *will* cover Year V, happily. Just at the middle of the year in any good teaching programme there is a stage of depression when teachers feel things are running too slowly. The early topics have proved more interesting or more difficult than one expected; and the later topics loom ahead too forbiddingly.

Suppose we stand and survey our course from the vantage point of the end of the first term:

Survey: Looking Backward and Looking Forward

In the suggested programme for this Year, we began with central acceleration for motion in a circle, to be used for making measurements on electron streams and used again to show how good theory is developed in Newton's explanation of the solar system. The measurement for electrons also remains as a useful background that we can refer to if we mention similar measurements for ions in a mass spectrograph, alpha particles and beta particles from radioactive material, etc.

We then looked at simple harmonic motion qualitatively, and continued the study of waves, started in Year III, on into interference effects with light, estimates of wavelength, and a look at gratings and spectra. That was intended to do three things:

1. Give factual knowledge of waves and interference, which is an important part of one's general knowledge of physics. (And it is a beginning for some A-level physics.)
2. Let pupils see for themselves why we think light consists of waves, and enable them to make their own estimate of the wavelength of light.

3. Provide a necessary background for introducing a topic of really modern physics: matter waves. If we have any time to mention this phenomenon and discuss it briefly and gently, we must prepare pupils beforehand by making them familiar with the behaviour of waves with gratings.

Teachers may feel tempted to continue from the discussion of interference and gratings to a further study of waves and spectra, theories of light, and the contrasting behaviour of quanta or photons of light. That lies ahead, and we hope pupils will hear some of it, because it is an essential part of our modern view. Yet, before we proceed to that we have two other things to consider:

(i) We must continue our building of atomic models, from the stage of hard, round molecules or atoms that sufficed in kinetic theory, to a picture of a Rutherford nuclear atom. We may feel tempted to go farther still, but progress after that is likely to be difficult for pupils at the present stage.

(ii) Pupils will need time for revision.

Revision will, of course, be a problem for each teacher to judge in terms of his class and their work. We certainly do not suggest that the Year should go right up to the examination without revision, just because the atomic physics now at the end is so important. Yet we do believe that many teachers will find, when they get to this Year, that the kind of examinations suggested to fit our programme do not need the same type of revision as the traditional ones.

True, our suggested examinations will dip back into the work of Year IV and Year III; but in doing so they will look for understanding, in the sense discussed in the General Introduction.

There we gave a general account of our aims in teaching for understanding, to let pupils learn by doing their own experiments, arguing things out (with help) and by answering problems and questions that ask for thinking. We suggested that taking more time for a topic to gain a sense of mastery might give lasting understanding.

However, such general descriptions of teaching are not very helpful when we are thinking about the actual examinations. The comparison that was offered in terms of the French verbs, *savoir*, *connaître*, *comprendre*, was again at best a helpful admonition. But we also gave a relevant and useful definition: we reminded readers that most of us say, at one time or another, 'I never really understood that part of physics until I came to teach it', and we suggested that in the same sense but on a much simpler scale, the test of a pupil's understanding can be whether he can teach it. We elicit his teaching by asking him to explain something to someone else – his younger brother or his non-scientist uncle, rather than to a mysterious, fierce examiner who requires the knowledge to take on a formal polish. We have been using that device for problems all through our programme, both for current teaching and as preparation for questions like those in examinations. If, as we hope, O-level examinations for our programme are slanted in the direction of asking the candidate to teach things to someone in his answer, they will have a good chance of testing understanding. Of course, such questions have always been used by good examiners: our suggestion here is that the questions should take a less formal style and that the answers should be read by examiners with this requirement of understanding still more actively in mind.

In marking the answers for that, examiners will find they have to make subjective judgments, since they are looking for the understanding that they see in the answer, and for the feeling of mastery, rather than memory of facts. In doing that, examiners will be doing great good on behalf of our teaching in particular and science education in general. They may find that marking schemes of very precise form are unsuitable for some questions; but they will be able to judge whether the pupil understands in much the same way that many of us judge in an interview whether the applicant understands the work he is to do. With such hopes in mind, we urge teachers towards careful teaching and good experimenting by pupils, and away from a great deal of revision of factual material which might not be so useful in examinations as pupils' demonstrations of understanding. As with so many things in our suggested programme, this is a matter where the first time of teaching will be difficult and uncertain, and teachers will find that they know far better what to do when they come to a second round.

Whatever revision seems necessary, in the view of both pupils and teachers, must of course be done. But we hope that there will be time to carry the teaching at least far enough to include the Rutherford atom, and perhaps as far as matter waves.

This Guide is Very Long. This guide is long and discursive. That is intentional, because these notes are offered to many different teachers with varied interests and experience, for guidance in following a new programme of teaching.

Where one teacher wants to know our reason for suggesting a topic, another may want to know why we advocate some crude apparatus instead of a modern machine; and, elsewhere, why we recommend a strange modern machine instead of simpler traditional apparatus.

Some teachers may welcome detailed instructions for running an experiment. Others in turn will be distressed by the lengthy discussions of details; and they will ask for a short list of topics, such as the following:

Motion in a circle: central acceleration
Measurement of e/m for electron streams
Planetary astronomy and gravitational theory
S.H.M.; waves, alternating currents
Interference of light: Young's fringes
Diffraction grating; spectra
Radioactivity – properties of rays with electroscope and counter
Alpha-particle scattering and Rutherford atom model
Photo-electric effect
Theories of light: waves and photons
Matter waves: particle and wave behaviour
Newer atomic models ... uncertainty? ...
Appendix on electromagnetic spectrum
Appendix discussing theories of light

Given like that in a dozen lines, our list can hardly satisfy any teacher planning a new programme with changes of aims and attitudes – in examinations as well as in teaching – such as we are suggesting. At most it tells an external critic our topics, without telling him our intentions.

Because our suggested programme is a new one and the format of treatment of this final year is unfamiliar, we shall enter into long discussions and give considerable details at some points.

We trust that teachers who would prefer a quicker summary will bear with that profusion and will extract whatever they need.

A Fable. We have tried to make our course include some of the modern physics of today. Rather than emphasize the atomic physics of half a century ago, we suggest bringing the teaching nearer to the present day, even with O-level pupils.

Imagine a Conference on the teaching of physics, convened in A.D. 1700. A resolution might well be passed to the effect that teaching of Aristotelian mechanics is in good order, and should continue; teachers in schools have good apparatus and are skilfully expounding the dynamical principle that motion requires a force proportional to velocity. The new ideas of Newton would be recommended for advanced seminars in universities.

Now imagine a Conference in the early 1800s: the teaching of Caloric would be endorsed and the unorthodox view of heat as connected with motion – with the new name energy about to appear – would be viewed with suspicion and restricted to graduate discussion.

Now shift our imaginary Conference on teaching physics to the early 1900s. Newtonian dynamics, energy and its conservation, atoms, molecules and kinetic theory, are all being taught clearly and well; but measurements of electron streams are regarded as very difficult to teach and the rumours of a quantum restriction are pushed away to professional studies.

The lag is natural enough: in each generation the older material seems to be secure knowledge and easy to teach well; and the newest material is not only strange but, as yet, difficult to teach. Of course that is partly due to the different way in which teachers have learned it. In many cases, the older material was taught them in their own student days with firm authority – and if they were given some of that material at a sufficiently early age by a strong capable expounder they may have accepted it quite uncritically. Whether we like it or not, we must accept that as one general characteristic of education – we who are teaching now must be giving strong dogmatic force to some of the physics we are teaching, without knowing it.

On the other hand material that a teacher did not learn in student days is apt to remain a little strange and not seem so strong a part of the syllabus. For example, many an older physicist today regards Relativity as somewhat uncomfortable – however well he now understands it and, perhaps, teaches it. When he first met the new ideas of Relativity they struck him as almost a misfortune: well-assured geometry was being attacked and could be shown to be 'wrong'. But, to the next generation of physicists, Relativity will be a commonplace, heard about at school, used as a normal part of student physics.

Thus, the lag is there and forgivable; and in past ages it has been harmless. There has been time for each generation to catch up. Now with science growing and changing so rapidly, and ideas travelling so fast around the world, is it any longer safe to let teaching lag in a comfortable way? Trying to make the teaching catch up and lessen the lag would be uncomfortable and even dangerous, if done carelessly. Yet when we move our imaginary Conference on teaching physics to the year 2000 we may feel uneasy about the prospect. Will so much of today's newest physics still seem too strange to teach?

With that question in mind, we offer suggestions of teaching some new physics in this Year.

The Newest Physics. In dealing with new, recent, physics – the physics-in-the-making of the last quarter century – we can only suggest topics and give some notes on teaching in this Guide. Many teachers would like to read fuller accounts of such topics. Yet when they look at books on modern physics they are disappointed. There are up-to-date advanced texts for university teaching or professional use; and there are some popular accounts of the latest physics, written for laymen. Many a book that gives the careful exposition of modern physics that one would like to have as background for O-level teaching seems to stop short at the state of physics fifty years ago, or at least treats later topics too briefly. With that need in mind, we suggest the following books which might be useful:

The New Age in Physics by Sir Harrie Massey (Harpers, 1960).
(This is a remarkable book, likely to be of great help in the present matter. The author largely neglects the physics that was 'new' fifty years ago – the first magnificent measuring of e/m for electrons, the early mass spectrograph with difficult geometry, laborious sorting out of radiations by absorption characteristics – and proceeds at once to the really new physics. The book is a popular account and we must not expect it to provide detailed training – yet it gives the right perspective.)

Turning Points in Physics by R. J. Blin-Stoyle and others (North-Holland Publishing Company, Amsterdam, 1959).
(Six very useful lectures, on Fields, Quanta, Probability, Relativity, Causality, and Elementary Particles.)

Knowledge and Wonder by Victor F. Weisskopf (paperback, Heinemann Science Study Series, 1964).

(A set of essays, which do not lose as much as most by being short, because the author is a very powerful modern scientist.)

Accelerators, Machines of Nuclear Physics by Robert R. Wilson and Raphael Littauer (paperback, Heinemann Science Study Series).

(This gives accounts of early machines, cyclotrons, linear accelerators, etc. It gives solid physics and yet is elementary. Without using mathematics, it nevertheless explains fully how a cyclotron works, discussing difficulties of focusing; extends the stories to synchrotrons, and even reaches the new story of clashing beams of electrons.)

The Nature of Solids by Alan Holden (Columbia University Press, 1965).

(This is an excellent, simple, introduction to solid-state physics and transistors.)

One, Two, Three ... Infinity by George Gamow (Macmillan, 1947).

(This is more light-hearted and scrappy but stimulating. Some pupils would enjoy reading it.)

In addition, among the flood of new paperback books a series called 'Momentum Books' is appearing. We urge teachers to watch for these because they are written by good physicists, with the aim of helping the teaching of serious modern physics.

'Scientific knowledge is *knowledge*, not fact – a gallery of pictures painted by men to portray in some simplified, comprehensible way the seemingly infinite complexity of nature. The pictures are put up and taken down, cleaned, replaced, and destroyed. Any account of scientific knowledge is therefore a "progress report" – an account of unfinished business.

'... Indeed, in the eyes of those who have made them, all these pictures are only fragments of a single picture. It is a picture of nature that is always incomplete, but must always hang together with the consistency contributed by the single palette used in painting it: the mind of man.'

Alan Holden
in the Foreword to *Conductors and Semiconductors*
Bell Telephone Laboratories, Inc., 1964

NOTE TO TEACHERS ON 'CENTRIFUGAL FORCE'

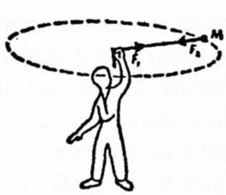
In elementary teaching we must make a clear decision between centripetal force and centrifugal force. A mixture of both is fatally muddling for beginners.

Centripetal force, used with Newton's second law will of course yield the right answers, and forces will always be in the right direction – strings will pull and never push: lorries rounding a corner will skid or fall outwards, . . . but the method will seem artificial to pupils, who have all heard of centrifugal force. The following discussion with an imaginary pupil may be helpful to teachers dealing with this question.

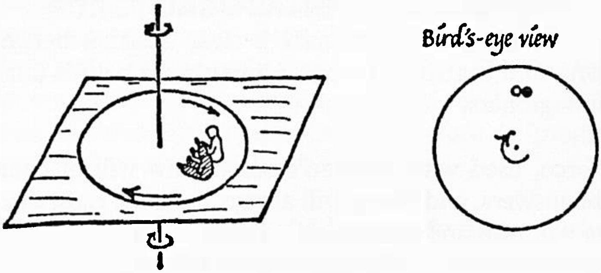
Motion in a circle needs a real inward force, provided by real external agents. This view of centri*petal* force will help you to deal with all real problems of circular motion. Then what is centri*fugal* force? You often hear of it, may find yourself speaking of it when you whirl something around, and will find books using it to explain things in physics. Here are a variety of opinions on it. You may choose according to your taste.

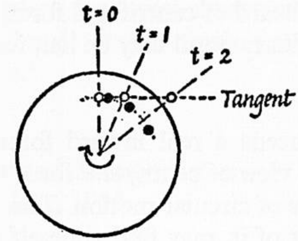
OPINION I: '*Centrifugal force is a phony force, imagined through a misinterpretation of evidence confusing agent and victim.*'

If you whirl a stone on a string, the string-tension pulls your hand outwards (just as it pulls the stone inwards). This is a real centrifugal force on your stationary hand, not on the whirling stone. You feel your hand being pulled outwards, so you say, 'I feel the stone and string pulling my hand outwards. That tells me the stone is being pulled outwards, by some centri*fugal* force, and the string is



just transmitting that force.' That is where you are mistaken. There is no outward force on the stone. Really the string, in a state of tension, pulls at both its ends. While it pulls your hand outwards it pulls the stone inwards. The only real force *on the stone* is inward, centri*petal.*





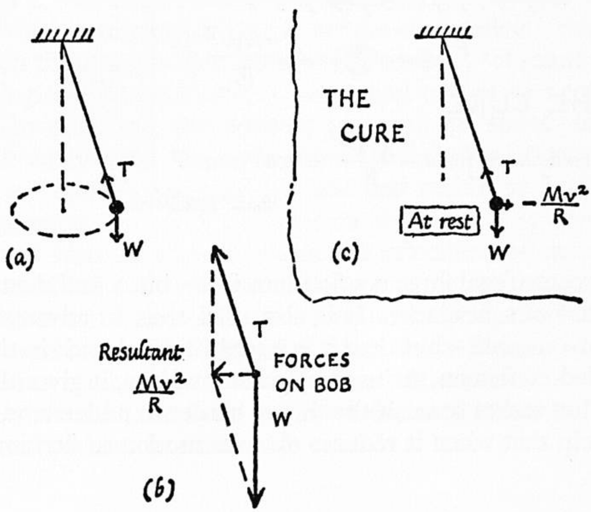
Again, suppose two boys, A and B, visit one of those amusements in which people sit on a floor that rotates. Suppose A and B enter the room while the floor is at rest, and sit on the polished floor. Knowing the trick of the performance, A glues himself to the floor. When the floor begins to spin A notes that a mysterious force seems to pull him outward; and, but for the glue, it would make A side slide out to the wall. B, without glue, slides out to the wall if A does not hold on to him, exerting an inward pull on him. Each feels he is struggling against 'centrifugal force'. But now let a stationary observer take a bird's-eye view from above. Seen from outside the spinning room, A and B are both moving in a circular orbit, and both need real *inward* forces to keep them in orbit. For B, the force is the inward pull A provides: for A it is the pull of the sticky floor on him. Once again, A merely imagined an outward force on B because he had to apply a real inward force to him. As the outsider sees, these inward forces are not neutralizing a mysterious outward force, they are *making an inward acceleration*; they are making A and B move in a curve. The outside observer offers a further comment. When A lets go B then continues along a tangent (if there is no friction). B's successive positions along that tangent are farther and farther out from the centre of the circle; so, as seen by A (spinning with the floor) B *seems* to be sliding out along a radius. But really B is just *continuing a straight* (*tangent*) *path, a simple example of Newton's First Law.*

OPINION II: *'Centrifugal force is a delusion arising from living in the rotating system and trying to forget it.'*

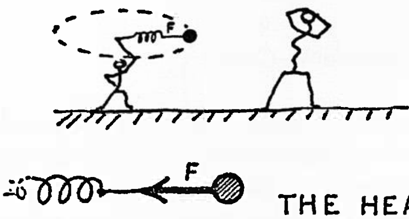
The rotating-floor discussion leads straight to this view. To people sitting on the table in a concealing fog – and ignoring its motion – there is an outward field of force, endowing every mass M with an outward force Mv^2/R. Unless some real agent applies an inward force to balance this, any object left alone will seem to slide outward with acceleration v^2/R. Preferring to take a sober view from outside, we say that both the outward field of force and the outward sliding are delusions due to living in a rotating framework and not allowing for its motion.

OPINION III: *The Novice's Headache-Cure*

Here is a good use for centrifugal force. Let us be rude and say, with some truth, that some beginners prefer 'Statics', the physics of things at rest (in equilibrium), to the physics of motion. Problems involving acceleration and rotation make his head ache; and the novice wishes they could be reduced to simple statics and problems that he is so good at – forces in bridges and cranes. And they can. Consider, for example, the problem of a pendulum whirling around in a conical motion. The two real forces acting on the bob are its weight and the string tension. These two real forces must add up to a resultant force Mv^2/R inward – otherwise the bob could not continue around the orbit. Here then are two forces W and T which have horizontal resultant Mv^2/R inward. Let us turn this into a statics problem with equilibrium (resultant zero) by

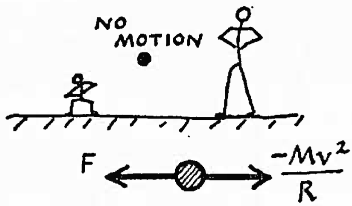


adding an extra fictitious force. *What fictitious force must we add to W and T to make zero?* The third force would have to be $-Mv^2/R$, or *Mv^2/R outward.* So some teachers say this to the novice: 'Yes, you can turn any problem with circular motion into a statics problem if you *take all the real forces acting on the moving body and ADD a fictitious centrifugal force, Mv^2/R outward, and then write an equation stating that these forces (including the fictitious one) have resultant zero.* Solving the equation will give you the same information as the method of making the real forces combine to produce inward acceleration v^2/R.'



THE HEADACHE

The spring (= agent) provides the real force, F, to make acceleration v^2/R



THE CURE

Imaginary force $-\frac{Mv^2}{R}$ + real force F make equilibrium

On this view, centrifugal force is a fictitious force, but a useful one, to cure the novice's headache. It is also used thus in advanced physics, to save trouble – but then it is a sophisticated trick in the hands of skilled craftsmen. As used by most students, it gives the right answer but makes some of the theory harder to understand – how can it help that when it reduces obvious motion to fictitious rest? The trustful user, with his right answer, is confused about the forces: he is not sure which are real or which way they pull. *If you value your understanding of physics, avoid this headache-cure at all costs.* Of course, a *mixture* of this centrifugal headache-cure with centripetal forces will produce utter confusion!

OPINION IV: *Relativity*

(This opinion sketches some comments from sophisticated relativity theory. Read it for amusement or for a good moral warning, but do not let it convert you to the headache-cure method for novices. This relativity-view is true, but only within the framework of definitions constructed for it.)

Can nothing better be said of centrifugal force? Returning to Opinion II, some scientists ask, 'Why is it so wicked to view things from a rotating framework? After all, we live on a spinning Earth. Are the 'centrifugal forces' that arise from our rotating-framework viewpoint really different from other forces, and less real? Who are we to say which is really rotating, ourselves or everything else?' (We are back to Copernicus vs. Ptolemy.) This last question is like the problem of testing Newton's laws in an accelerating railway train. By building a tilted room in the train we could still find the same laws, though we should find 'gravity' changed in size and direction. We suspect that we *cannot* distinguish between the effect of acceleration and a real change of gravity – Einstein built General Relativity theory on an elaboration of that 'cannot'.

Relativity theory starts with an axiomatic statement, that we cannot tell which is moving, ourselves or 'the other fellow', that there is no such thing as *absolute* motion. If that is so, 'absolute space' is meaningless; it should not be used, and cannot be needed, in science. In that case, the working geometry of 'space' must be such that we discover the same physics whether we think we are moving or 'the other fellow' is. And that makes us modify the simple geometry of space and motion that Euclid assumed and Galileo and Newton used. For constant velocities, we have many experimental failures to distinguish absolute motion even with the help of light-signals, so we feel justified in accepting the Relativity principle and its modified geometry. In practical life, the modifications are not noticeable, and they only affect experiments noticeably when very high speeds are involved, as they are in atomic physics and perhaps in astronomy.

Extending the Relativity attitude to accelerated motion we assume that a local observer will find the effects of acceleration indistinguishable from a local change of gravity; and thus we decide that gravitational fields can be treated as local changes of geometry in space-time. This is Einstein's Principle of Equivalence. Though the viewpoint is entirely new, its practical form shows only small deviations from Newton's law of gravitation.

Extending this idea to rotation, we suggest that a local observer cannot distinguish between the effects of a rotating framework and a local change of gravity, if he is moving with that frame. In that case centrifugal force tugging outward would be just as real to him on his spinning floor as an extra, horizontal pull of gravity. Then, to a tiny creature in a centrifuge, centrifugal force-fields should appear just like real gravitational fields, only some thousands of times as strong as ordinary gravity and gravity would take on a new direction – he would quite forget about its old direction. This General Relativity view has proved useful in co-ordinating thinking; and so far we have not observed anything inconsistent with it. In this way, centrifugal force has grown to be respectable. When we want to test the effects of large gravitational fields, unattainable on Earth, we think we may use a centrifuge instead.

The general principle of equivalence forbids us to call the motions of the Earth absolute. It therefore leads to a new mechanics and geometry that will predict the same effects whether the Earth spins and moves around the Sun, or the stars and Sun move around us. On General Relativity theory, a rotating universe would produce 'centrifugal forces' at a stationary Earth; so tests of a spinning Earth, with a Foucault pendulum or equatorial changes of 'g', could not distinguish between the two causes: Earth spinning or everything-else-spinning. Faced with the old question, 'Is Copernicus right and Ptolemy wrong?' we must demur at Galileo's cocksure insistence and say, 'Both views *may* well be equally true, though one is a simpler description for practical thinking and working'.

Opinion on the Four Opinions?

Make your own choice. However, for problems and experiments in this course, you are advised to use only centri*petal* force.

YEAR V

SYNOPSIS OF PROGRAMME FOR THE WHOLE YEAR

As explained in the Preface, this is a Year of putting physics to work to build stronger knowledge, principally in understanding of theory and in atomic physics.

We do not intend to provide new topics compactly taught for examinations or to spend a major part of the time revising old topics. The earlier Years will have taught many regions of physics on which examinations can draw with questions that ask for constructive thinking. This Year should give pupils practice in such thinking at a more mature level, but should not aim at packing in new content where that is solely of use as examination material. However, we shall survey a good deal of new atomic physics.

The attitude this Year should be: 'Now we can extend and use earlier knowledge to tackle great problems of the structure of the world.'

Essentially, this programme introduces six new tools and uses them together to develop five areas of physics:

1. We discuss motion in a circle and arrive at $a = v^2/R$ and $F = mv^2/R$.

2. We obtain from experiment a quantitative measure of the force exerted by a given magnetic field on a current in a wire; and we extend that, by argument, to a charged particle moving in a stream across a magnetic field.

3. The idea of an inverse-square law, introduced for gravitational fields, but applicable with the same geometry to electric fields, the spreading of light, etc.

4. Devices using ions to exhibit 'atomic' events: cloud chambers, geiger counters, scintillation counters, etc.

5. The use of alpha particles from radioactive substances (or protons or electrons from accelerators) as projectiles with which to explore atomic structure more deeply.

6. Studies of water ripples and light are combined to provide new criteria for waves.

With those tools, we develop:

A. Quantitative knowledge of electrons, positive ions, and nuclei as parts of atoms.

B. An example of physical theory – seen in stages of construction. We describe the history of man's knowledge of the stars, Sun, Moon and planets, from early observations through successive stages of building a 'theory', to the age of Kepler and Galileo, when man had a great body of empirical information, organized in rules that were verified with precision but still disconnected pieces of knowledge. Then we unroll Newton's great gravitational theory to show the use of good theory in science.

C. Knowledge of radioactivity.

D. The wave-particle idea. We touch briefly on the modern picture of both radiation and matter having particle aspects and wave aspects – the behaviour which we observe and measure being determined by our choice of experiment.

We cannot, with pupils at this age, pursue this duality far; but we should introduce our modern view, both for the sake of non-scientists who will read about such things later and to set the stage for further studies by physics specialists.

E. Atomic models. We develop successive models of atoms, from hard billiard balls of kinetic theory to a hollow Rutherford model. We may give a survey of later developments of atom models. We owe some modern knowledge to our pupils, but the experiments and reasoning that led to such knowledge (even if we show modern simplified forms) are too complex for our teaching. All we can offer at this stage is a survey of results, descriptions of models. However, in this region of modern developments we feel justified in breaking our resolution to offer supporting experiments so we suggest giving only short descriptions. We can give occasional support and elucidation by films, but there we must beware of two dangers:

1. A film which shows the real apparatus and working of a fundamental experiment may be merely confusing, owing to the profusion of auxiliary apparatus.

2. A film which describes either ideas or experiments by animation may be very misleading in another direction. It can sketch the

story we think or hope is true and fail to give real teaching of science. However tempting such a film looks as a clarifier, we should be unwise to show it.

Pupils should hear about:
A nuclear model with stable and unstable nuclei.
The photo-electric effect and its strong suggestion of quanta; the idea of photons of light and their behaviour, possibly a mention of specific heats and their suggestion of quanta; use of wave-particle views to sketch an atom model. Perhaps even a comment on uncertainty.

It is doubtful whether we can give more than a passing, brief description of any of these; though we hope that, with a fast group, teachers will be able to select some aspects of present-day physics for expansion.

Experiments

Links with Earlier Years: the following experiments are essential, if they have not been done fully in previous years:

Millikan's experiment: discussion and film (and possibly demonstration).

This should be done in two parts: first, a clear proof that electric charges come in multiples of a single universal basic charge; second, a measurement of the size of that charge. The first part is both more important (for our present teaching), and easier to show, though even that will have to be shown by film. The measurement of the value of *e* will have to be taught by assertion.

Young's Fringes by ripple tank (class experiment)

Young's Fringes for light (qualitative class experiment)

Young's Fringes for light, rough measurement (class experiment)

Cathode rays: demonstrations of properties (except effect of magnetic field, which will be studied this Year).

It will not be necessary to do experiments on Force, Mass and Motion, even if pupils missed them. However, pupils must not only know $F = ma$ and $Ft =$ change of (mv) but have an understanding of the nature of mass, force, weight, gravitational field strength and kinetic energy. They must know that $K.E. = \frac{1}{2}mv^2$.

Pupils need not do or see experiments with electric fields even *if* they missed them, provided they know the pattern of the field between parallel plates and are ready to accept the idea that field strength $X/?$ is given by P.D./distance between the plates.

Experiments this Year. The teaching of this Year involves some important experiments: a test of $F = mv^2/R$; measurement of wavelength of light with a grating; measurement of e/m for electrons; and some radioactive experiments. These should be class experiments as far as possible. Even so, they will not occupy the full amount of time the laboratory has and deserves.

We suggest four categories of experiment that might be offered at any suitable places:

a. Demonstrations and class experiments with alternating currents, including experiments with 'slow a.c.' (These should be class experiments for everybody.)

b. A transistor experiment (instead of in Year IV).

c. A careful measurement of 'J'. If pupils are at a stage where they can see that this is at the same time very important and necessarily inaccurate but yet worth doing, then they should do it.

Pupils embarking on this should take time to learn the ways of the apparatus and discuss its troubles.

They should work in small groups, pairs if possible. This is an experiment that should be done in an atmosphere of strong personal involvement, with the odds against the experimenters.

This should not be treated as a measurement to 'get the right answer'. There *is* no *accurate* J-apparatus for student use that can possibly yield the *right* answer except by a happy coincidence of cancelling errors. An experiment done carefully with a detailed series of cooling corrections can yield a result fairly close to the accepted value – but those corrections are tedious and would be puzzling to pupils at this stage: they would spoil the experiment.

If we look at the huge task of the experimenters who made the most trusted measurements of J we shall suspect many difficulties; and if we consider the nature of heat losses and the conditions of pupils' thermometry, difficulties come to the forefront.

With apparatus carefully devised to minimize errors, we shall still be giving pupils an inaccurate experiment. It is good for them to know that; and then it is good for them to do the experiment.

d. Difficult measurements for able groups or pupils with special interests:

Millikan's experiment done by a small team of pupils;

possibly a measurement of the speed of light;

measurement of *e*/*m* done by a small team.

These may seem to impose a great burden on organization of apparatus and teaching – we only suggest them for those cases where teachers find that they have a group that they can set to work on some individual experiments. Needless to say, such experiments should be given considerable time. For slower groups, some of the experiments suggested for this Year, such as work with electroscopes or investigations of pendulums, could be spread out into longer experiments done by pupils on their own.

Chapter 1

MOTION IN AN ORBIT

Central Acceleration and Satellites

Introduction to Circular Motion

Experiments and Questions about Motion in a Circle. We start with three demonstrations and a class experiment.

1. A carbon dioxide puck on a smooth table travels in a straight line at constant speed (a ring with cardboard lid and solid CO_2). D1

We ask whether this is natural motion and whether any forces are needed to keep it going.

2. We refer to the motion of the Moon round the Earth. We ask whether that is natural motion and whether any force is needed to keep it going – and leave the question unanswered. D2

3. The Leybold fine beam tube first without, then with, magnetic field. We ask the same questions. D3

4. Then, feeling our way towards the need for a force, we ask pupils to tie a small massive object (a ring or a hex nut) to a string and whirl it round their heads and decide which way the force must be on the object. C4

We ask pupils which way the force does act on them if they sit on a smooth seat in a car that rounds a sharp corner. Which way do they slide? Which side of the car then pushes on them? T

(We might discuss the banking of a bicycle rounding a corner, but this often leads to more confusion than help, because the problem is better discussed by taking moments than by considering a single force.)

'Flying Off at a Tangent.' Now or later the teacher should give a very important demonstration and discussion: the motion of an object released from its orbit. He whirls a light block of wood on a string, in a horizontal circle round his head. Each time the block is in front of him, nearest the class, he says 'Now'. He threatens to let go of the string at that stage, when he says 'Now'. He does that. Some pupils will flinch, because they expect centrifugal force to make the block rush out towards them. All will see that the block does not travel towards them but simply continues its motion out to one side of the room, along the tangent. The teacher should also point out that the block does not *fly off* along the tangent in an aggressive way, but just *continues along* the tangent – an example of Newton's First Law. D5

Centrifugal Force?: Centripetal Force? We find ourselves immersed at once in questions of centri*fugal* force versus centri*petal* force.‡ We must be sympathetic and firm: we ask the direction of the force on a stone being whirled round one's head, and remind the pupils that 'strings pull, never push'. T

We ask again which side of the car pushes on the passenger on the smooth seat. We may ask, quite unfairly, 'What do you think the Earth does to the Moon, repels it or attracts it?'

We agree with our critics that the same string which pulls the stone inward also pulls our hand outward; and that the sliding passenger in the car will push the side of the car and smash it outward if it is weak enough. And we agree that the Moon must pull the Earth outward, towards the Moon. That will not convince our critics that there is not a centrifugal force acting on the stone, the passenger, the Moon. And we ourselves would certainly infer the presence of such an outward force if we were in a rotating frame of reference, riding on the stone, or in the car, or on the Moon. (In case of a rider on the Moon, where gravity is the controlling force, the evidence for a centrifugal force would be suppressed, because the observer himself would be pulled by a force proportional to his mass.)

‡ Note to teachers on choice of policy. The choice between rival treatments – centrifugal and centripetal – has advocates on both sides.

In advanced physics, we ourselves call on centrifugal force. We reduce a problem of orbital motion to a statement of equilibrium by adding an outward, centrifugal force to the forces applied by strings, gravitation, etc. And when we explain the action of a centrifuge (or a merry-go-round) in detail we all want to resort to centrifugal force – though of course a logical centripetal explanation can be given.

For an elementary beginning, some teachers prefer to use centrifugal force because it draws on pupils' common belief. Others hold that this start will lead to difficulties. They maintain that, having started with the view that acceleration a needs force F in the same direction ($F = ma$), we should continue to take that view for the acceleration v^2/R, which is certainly centripetal, not centrifugal.

One thing is sure: in elementary teaching at any rate, a mixture of the two approaches is fatally confusing.

In planning our suggested course, the Nuffield Physics group *decided to set forth the teaching in terms of centripetal force.*

There are some comments on the matter in the next few pages; and a separate note discusses the choice at length.

We shall have to say that centrifugal force is one way of looking at the problem; but not our way.

Experiment to illustrate Centripetal Force. We give one more illustration of our centripetal view: we ask pupils to put a penny on a rotating gramophone table, and let it spin faster and faster until the penny cannot command enough friction to anchor it and slides off at a tangent. We ask pupils to watch very carefully what happens from the point of view of an outside observer at rest; and then to speculate what that would look like to an observer standing on the turntable. We ask where a drawing pin should be stuck in the turntable to keep the penny from sliding away. (Answer: Just beyond the outside edge of the penny.) D6 H6

We shall use Centripetal Force. We insist that we shall treat the problems ahead of us by the clumsy, unrealistic-looking method of saying that anything moving in a circle must be acted on by an inward, centripetal force that pulls it in from a straight-line path. A real *inward* pull is needed. 'No force, no orbit.' Before this has time to build up irritation or boredom, we proceed to a satellite. * * * * * *

Satellite

> 'Throw a cricket ball out horizontally. It falls to the ground some yards away. A rifle bullet, fired faster but also horizontally, reaches the ground after a mile or so. T
>
> 'Try a thought experiment: fire a bullet so fast that it covers an appreciable part of the Earth's circumference before it reaches the ground. What effect has the Earth's curvature on the bullet's fate? Fire it even faster and it "falls over the edge of the Earth". The Earth falls away from the buller's original direction just as fast as the bullet does.
>
> To the bullet all parts of the world are the same, and it soon forgets where it started from. Such a bullet with just the right speed will always be falling over the edge, and so it will go on and on round the world – keeping just above the ground – until it arrives back at the starting point and hits us from behind.'

In practice air resistance (heat barrier) absorbs energy, and down comes the bullet. So we must start outside the atmosphere.

Here is a simplified story for a typical satellite that we may give a class that is interested:

1. Rocket starts off nearly vertically. The exhaust gases exert an upward push, greater than the rocket's weight; so the rocket accelerates upward;

2. Fuel exhausted, motor cuts out and the first stage separates;

3. Parabolic (free fall) trajectory until the path is horizontal at maximum altitude;

4. Final stage ignites and accelerates its relatively small mass to high velocity before unlatching the satellite proper and leaving it in orbit;

5. Exhausted final stage is also in orbit, but, in the course of time, a small relative velocity puts a big distance between them;

6. There is some air resistance even at 100 miles up, so energy is C7
used up slowly and satellite descends. In the course of that, its time to circle the Earth grows smaller. Pupils should try letting a string carrying a whirling stone wind up round their finger. They will see it speed up.

Questions about satellites and rockets will flourish now, whether *
we want them yet or not. We should welcome them and perhaps *
use some of them to lead to the topics ahead. *

We ask about the energy necessary to raise a satellite 100 miles. T
Would it be better to burn fuel slowly, giving thrust for a long time, or burn it rapidly and then have a long rise time under free fall conditions? The answer lies in the cost of raising the first stage fuel load. If the fuel-burning continues during most of the rise, we have to raise a good deal of fuel. We have to compromise between saving that cost by a rapid initial acceleration‡ and the stresses on man and machine involved by acceleration being too great. At the final stage, we must provide $\frac{1}{2}mv^2$ energy for the motion in orbit. All the latter – in fact, all the energy released by the fuel – is dissipated as heat on re-entry.

Teachers will find a very useful discussion of rockets and satellites, *
with some data for satellites that have been fired, in a Penguin book *
by Michael Ovenden, *Satellites*. *

‡ Only if the acceleration is infinitely large can we avoid wasting fuel on raising fuel for the later stages of that acceleration. An infinite acceleration would bring the rocket to a suitable final speed in an infinitely short rise-distance. But that would be infinitely dangerous.

'Centrifugal Force' Again. Even now the question of centrifugal force will crop up with strong advocates. We make two new attacks on it: T

1. We ask pupils to think about a boy running along a straight path, with a larger, stronger boy running beside him and pushing him sideways. What would be the effect of that continuous sideways push? Suppose, as the smaller boy changes his path, the large boy continues to push sideways, perpendicular to the new path. What kind of a path would the smaller boy take? A circle seems reasonable. In this case the force is clearly inward.

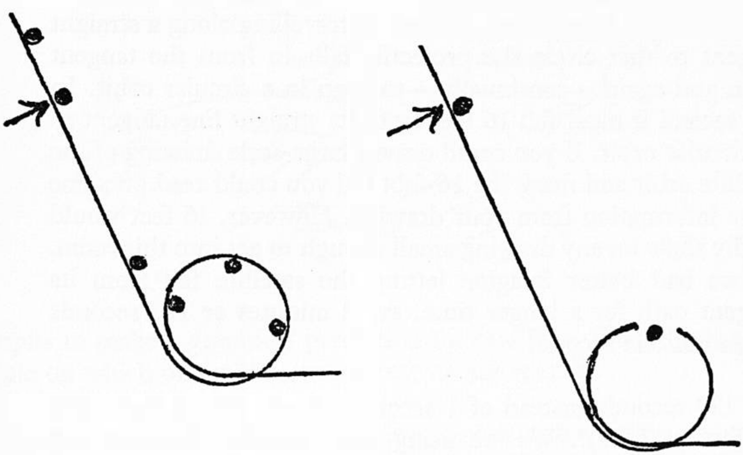
2. We point out that there is always a visible (or, if invisible, well-known) agent applying a force towards the centre; the string that
pulls inward, the pull of gravity on a satellite. (But if we like we *
may imagine there is an outward centrifugal force as well; and then *
acted on by those two forces, the visible inward one and the *
imaginary outward one, the moving object might be treated as in *
equilibrium – we forget its motion and imagine it remaining at *
rest. Then we have a case of balanced forces on an object at rest – *
a standard problem with engineers, who have long favoured *
building bridges at rest. On that view centrifugal force is a trick *
to reduce complicated problems in motion to problems that look *
simpler because they are concerned with equilibrium.) *

Demonstration with Loop-the-Loop. If the laboratory has a sloping runway for a steel ball, ending in a vertical circle, the teacher should use this for a valuable discussion. He releases the ball high enough up the slope to make it 'loop the loop'. Then he asks: D8

'What makes the ball go round a circle? What pushes or pulls the ball with a real force to make it do that? It must be some *inward* force towards the centre of the loop.

'What provides that force here, at the side, half-way up the loop? ... Yes, the rails push inward. (Does gravity also act on the ball when it is there? ... Yes, of course gravity always acts. But it pulls vertically; and its only effect is to make the ball slow down a bit.)

'What provides the inward force here, at the top of the loop? ... Yes, the rails may push downwards; but what other force helps? ... Yes, gravity helps – and what is more, it insists on helping fully, whether it is wanted or not! Look what happens when we have the ball moving more slowly, needing less force for its orbit. Gravity is too strong, and makes it fall away from the rails.'



The teacher shows the notion with various speeds, and finds the speed at which the ball just follows the loop, with the rails exerting no force at the top of the loop, because gravity suffices. Then he asks what would happen if the rail were cut out just at the top of the loop. If possible, the top section of rail should then be removed, and the experiment tried.

Though this experiment is worth showing just for fun, the main point of it is the discussion of inward forces that it facilitates.

Simple Treatment of Satellite Orbit

Before we embark on a formal treatment of satellite orbits, pupils should try the following simple, graphical approach.

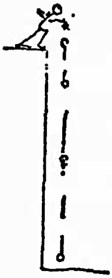
'We are going to make a scale drawing of an Earth satellite's orbit. See if we can use that to find how long a satellite takes to go round the Earth, if it is controlled by ordinary gravity. C9

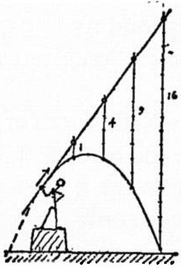
'Suppose we launch an Earth satellite 100 miles or so above the Earth. Then, since the radius of the Earth is some 4,000 miles, it is not really much farther away from the Earth than a stone or cricket ball thrown in the air. We know that for a falling stone the pull of the Earth produces an acceleration 32 ft/second per second. So a stone dropped from rest falls 16 feet in the first second. Any projectile does the same: instead of continuing along a straight line in the direction in which it is fired, it drops 16 feet from that straight line in the first second. (Remember the 'monkey and hunter' demonstration last Year).

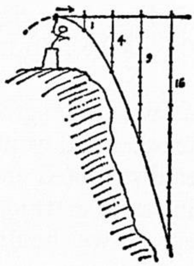
'Now think of an Earth satellite travelling round the Earth in a circle, about 100 miles up. Instead of travelling along a straight tangent to that circle the projectile falls in from the tangent again and again – continually – to keep in a circular orbit. In one second it must fall 16 feet from its straight line tangent to its circular orbit. If you could draw a large-scale drawing of the satellite orbit and mark the 16-foot fall you could read off some more information from your drawing. However, 16 feet would hardly show on any drawing small enough to get into this room. So we had better imagine letting the satellite fall from its tangent path for a longer time: say 2 minutes or 120 seconds instead of one second.

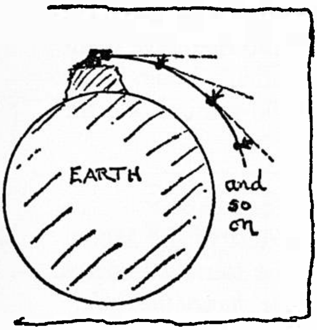
'In 120 seconds instead of 1 second a freely falling body falls 16 feet $\times$ $(120)^2$. We are using the "formula" that you met before: $s = \frac{1}{2}at^2$. Work out that fall in miles. Then we can put that on a large-scale drawing to find out how far the satellite goes in 2 minutes. And from that we can predict how long an Earth satellite will take to go round the whole Earth.'

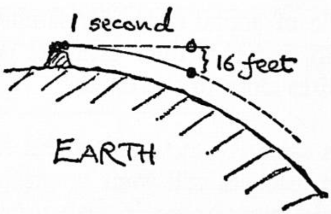
Then pupils calculate the fall, about 44 miles. The teacher should arrange to draw arcs of a circle on large sheets of brown paper for

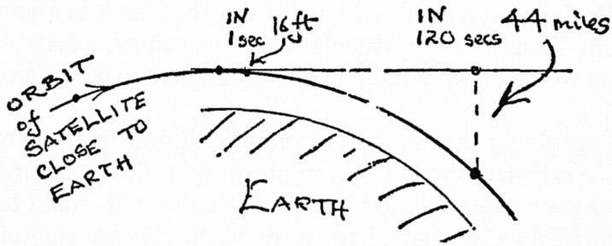












pupils to make a graphical prediction for satellites. We suggest a scale on which one millimetre represents one mile.‡

‡ Careful drawing, or calculation, shows that, in fact, such a satellite travels about 590 miles in 2 minutes. That would need an arc 59 cm long on the brown paper; but pupils' estimates will vary considerably, so the arc should be drawn at least 80 cm long.

In drawing the arc, the teacher should also mark a *radius near one end*, to help the pupil to draw a tangent there.

Better still: draw an arc twice as long, with the specimen radius near its mid-point. Then, although 'draw a tangent' is still the official instruction, pupils can draw a symmetrical chord 4·4 cm in from the circumference there. That is more

The radius of the circle corresponding to a satellite orbit of radius 4,100 (4,000+100) miles should therefore be 4·1 metres.

A thin wire of length 4·1 metres anchored at one end with a pencil at the other end would enable the teacher to draw arcs on sheets of paper distributed among pupils for a class experiment. An error of 3 inches in that radius will make only a 1 per cent error in the final answer, so we should aim at ease rather than accuracy.

Pupils draw a tangent and find where the distance of fall is 44 millimetres from the tangent to the circle. Then, given that the travel from the tangent point to the point they have found is a 2 minute trip for the satellite, pupils to work out the time taken for the satellite to go once round the Earth. We should then give them a table of actual times for satellites from Ovenden's book. They should find that their estimate is close to the 90-minute one for satellites close to the Earth.

Does Gravity extend to the Moon? (*Optional*)

Some teachers will want to extend the satellite test to the Moon, without mentioning inverse-square gravity. The same 4·1-metre arc will serve for a piece of the Moon's orbit *if we change the scale.* C/D 10

We tell pupils that the Moon's distance from the Earth is about 60 Earth-radii. Then, for the Moon's orbit, instead of a scale of one millimetre to a mile we now have one millimetre to 60 miles.

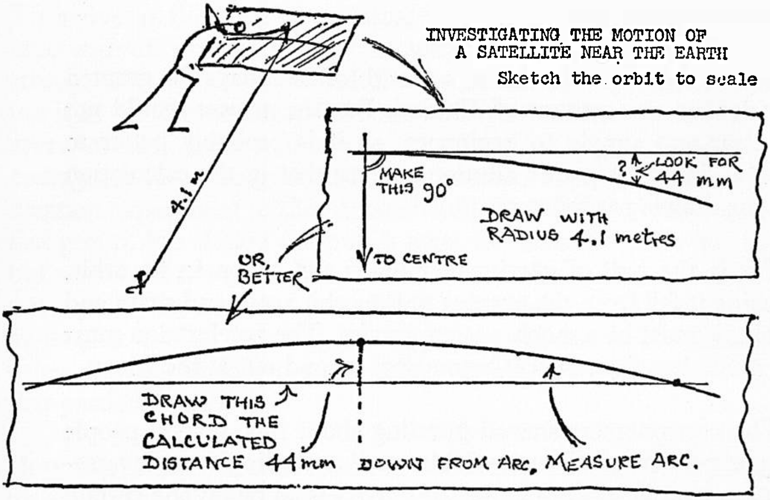
We first try imagining that gravity extends undiminished out to the Moon. We calculate the fall in 2 minutes: 44 miles as before. With our new scale, one millimetre to 60 miles, that fall would be only $\frac{44}{60}$ millimetres – too small to work with. So we suggest letting the Moon travel the same arc on the diagram as we found for the Earth satellite falling for 2 minutes. (That was somewhere between 40 and 80 centimetres, according to each pupil's success in making the drawing.)

But now the fall of 44 millimetres from the tangent to that place *on the diagram* no longer represents 44 miles. It represents a fall of 44×60 miles. How much time would a falling body need for that, under full gravity? Look at $s = \frac{1}{2}at^2$. If we make s 60 times as big,

easily done with some precision. For that, paper must be long enough to take an arc at least 150 cm long, but need not be wider than 20 or 30 cm. (There is no need for the centre of the circle to be on the paper.) So a long strip from a roll of wrapping paper will suffice.

It is unwise to change to a less simple scale in order to fit a smaller piece of paper. The arithmetic will bring enough troubles without that.

with the same a, t^2 must be 60 times as big. Then t must be $\sqrt{60}$ times as big: that is, $7\frac{3}{4}$ times as big.‡ Then the Moon would travel the arc in $7\frac{3}{4} \times 2$ minutes; and it would travel the whole circle in $7\frac{3}{4}$ × the 90 minutes that pupils obtained for the satellite; that is, between 11 and 12 hours.

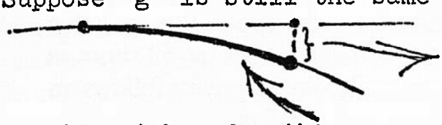


MOON'S ORBIT. Same arc on sketch, radius 4·1 metres.

R = $\boxed{60}$ Earth Radii

∴ new scale is 1 millimetre for $\boxed{60}$ miles.

Suppose "g" is still the same as near Earth's surface.



44 millimetres as before now represents 44 x $\boxed{60}$ miles

time taken for this arc must now be
2 minutes x $\boxed{\sqrt{60}}$
or 2 minutes x $7\frac{3}{4}$

And time for whole orbit would be 90 min x $7\frac{3}{4}$ which is much too small.

(because, in $s = \frac{1}{2}at^2$,
$44 \times \boxed{60} = (\text{same } g)t^2$
therefore $t^2 = \left[\frac{44 \times 2}{\text{same } g}\right] \times \boxed{60}$
therefore $t = 2 \text{ min} \times \boxed{\sqrt{60}}$

‡ Since this is a difficult discussion, teachers who propose to try it with their class are advised to make sure that a rough value for $\sqrt{60}$ is known beforehand. $7\frac{3}{4}$ will serve well.
$(7\frac{3}{4})^2 = (\frac{31}{4})^2 = \frac{961}{16}$ and $\frac{960}{16} = 60$ ∴ $\sqrt{60} = 7\frac{3}{4}$ within 0·06%.

Even if pupils' answers for the satellite varied far from 90 minutes, this new answer is clearly wrong for the Moon, which takes a month.

Therefore, if the Moon is constantly pulled from tangent to orbit by the Earth's gravity, there must be a greatly diluted strength of gravity out of the Moon.

Then we might take the Moon's month of 27·3 days for granted and calculate the amount of dilution. But the answer would not look clear and simple to beginners; so it is probably better to *suggest* an inverse-square‡ dilution and try that in the calculation from the drawing as follows:

'If it is the pull of gravity that holds the Moon in its orbit, making it fall from the tangent to the orbit again and again and again, it must be a much weaker gravity. The acceleration must be much less than 32 feet/second per second out at the Moon.

'When astronomers started puzzling about this, several people suggested that gravity may "thin out" according to an inverse-square law. According to that, if gravity is so much at a certain distance, it is $\frac{1}{4}$ at double distance, $\frac{1}{9}$ as much at treble distance ... $\frac{1}{100}$ as much at 10 times as far away from the attracting body.

'An apple near the Earth is pulled so strongly that it falls with acceleration 32 ft/sec per sec. The Earth attracts an apple as if all the Earth were concentrated at the centre, 4,000 miles below the surface, a whole Earth-radius from the apple. But we know that the Moon is about 60 Earth-radii away from us, 60 times as far from the Earth's centre as an apple. So, if gravity follows an inverse-square law, it must thin out by a factor $\frac{1}{60}^2$ when we change from apple to Moon. If so, free fall under gravity at the Moon would not have an acceleration 32 ft/sec per sec; but it would have acceleration

$32/60^2$ or $32/3{,}600$ ft/sec per sec.

'How would that change affect your calculation for the Moon? Look at $s = \frac{1}{2}at^2$. If we make a 3,600 times smaller, then, for the same s, t^2 must be 3,600 times bigger: and t must be 60 times longer. As a result of our diluting gravity, we should expect the Moon to take *60 times our previous estimate for the whole orbit.*

‡ See page 179 for suggestions concerning teaching the idea of an inverse-square law.

'That is $60 \times (7\frac{3}{4} \times 90)$ minutes or $60 \times (7\frac{3}{4} \times 90)/(60 \times 24)$ days: very close to 28 days.

'It looks as if the Moon may be "falling", to keep its orbit, with inverse-square-diluted gravity.'

To arrive at that result, we must use the inverse-square law – otherwise all we can say is that undiluted gravity is much too strong. We shall describe and discuss the inverse-square law and use it in our Newtonian prediction of Kepler's Third Law. However, to bring it in at this introductory stage may well be discouraging; so we suggest that this extension of the brown-paper diagram experiment to the Moon should be offered only to a very fast group. We should tell pupils what the inverse-square law is, but we should not give a long lesson on it; and we should certainly not say that the inverse-square law is right and that they ought to know that already. Instead, our pupils should follow Newton in using this discussion to see whether gravity *does* 'thin out' in that particular way.

How Big is the Force Needed to Maintain Orbit? We explain that to deal satisfactorily with satellites or electrons, we must know just how much force is needed to hold something in a circular orbit. We say the force is given by $F = ma$ as usual and we *state* that a point moving round a circle does have an inward acceleration. We ask pupils to feel the force. The higher the speed, v, the bigger the force needed to hold the objects in orbit, so the bigger the central acceleration must be. And, for the same speed, the smaller the radius, or the sharper the curve, the bigger the force, and therefore the bigger the acceleration must be. So we expect the central acceleration to go up with v and go down with increasing radius. In fact $a = v^2/R$. We shall offer a geometrical proof of this for those pupils who can learn it easily. And we shall ask all pupils to give it an experimental test. T

C/D 4 again

Motion in a Circle

Necessity for mv^2/R. If we restrict ourselves to a qualitative description of motion in a circle, and the forces it involves, we can talk generally about satellites but we cannot account for Kepler's Laws and we certainly cannot show Newton's great synthesis for the solar system in any clear light; we can describe what is shown by demonstrations with electron streams but we cannot make any measurements and so must stop short at a very general picture of atoms. Measurements of beta rays and the working of a mass spectrometer would remain equally vague.

So we must arrive at $a = v^2/R$ and $F = mv^2/R$ because we want that for several uses. If we proposed to show that simply as a piece of physics to be used for examinations, we should certainly find it difficult for many O-level pupils – and it would seem to them an odd piece of geometry rather than an essential piece of knowledge. As we use it here, it is an essential piece of knowledge and we must face the difficulty of providing it.

We suggest that teachers should try the geometrical derivation of $a = v^2/R$, with any group that does not find it too hard, starting by showing clearly why it is needed – introducing the problem through satellites or through the electron stream in a magnetic field – then spending plenty of time on the geometry and algebra so that the derivation becomes familiar by repetition. Then pupils should put it to such uses that it seems worth while in retrospect.

It is easy enough to propose that for specially able groups; but what about the average group for whom the derivation will remain puzzling? Even for them, we suggest that this is something to see, something to try once, or at least to see done.

Few of us intend to climb Mount Everest, but we can all appreciate an account of the expedition and join in it by reading or by seeing a film, at least to the extent of understanding some of its hardships and enjoying some of the successes. If, at the start, we remove the bogey of 'being examined' and assure pupils that this is something they should see, and even try doing for themselves, but not something that we propose to hammer into a compact shape that can be reproduced in examinations, our pupils should be old enough by now to appreciate this as valuable experience.

The Nuffield Physics group hopes that teachers will experiment with this approach – this method of talking about what one is going to teach and the aim and method of its teaching before one embarks on the teaching itself. In this, we are doing little more than following the good practice that any teacher adopts when he is explaining something to adults. He does not try to drive home every stage of his story until his adult listener could reproduce it; nor, on the other hand, does he pare away the essentials of the story so that the adult says it makes no sense. Like such adult listeners, our pupils should be able to say 'I have seen that. It was difficult but it was sensible and from now on I can take it on trust – trust vouched for by what I have seen myself.'

Then there are slower groups for whom teachers feel convinced any geometrical or algebraic derivation would prove much too puzzling. Even in such cases, we hope that teachers will first try the geometrical derivation to see whether, released from the examination bogey, the class can appreciate it after all. (We know no way of finding out whether that is feasible with a given class except by trying it.)

Where the derivation must be avoided, we have three choices: (1) give up most of the physics of satellites and planetary systems, and atomic particles; (2) treat those things, but take out all quantitative discussions, so that physics seems to lose its backbone; (3) justify $F = mv^2/R$ by experimental tests. We hope that teachers who have slow groups will experiment with the last method. We can provide apparatus and suggestions for its use – though we wonder whether a slow pupil will not have as much difficulty in following the argument of the test as in following through a carefully taught derivation.

Derivation of v^2/R

We suggest two methods below. Whatever method the teacher tries, he should preface it by considerable discussion of the general idea and a reassuring statement about watching and seeing it done, so that science is not a mystery. (In fact, if we ask pupils to watch this done and then later ask them to write it out for homework: then still later on ask them to watch it done again, we shall find that, given in repeated lighthearted doses, the story will both make sense and be remembered.)

There are several good ways of showing that an object moving round a circle of radius R at constant speed v has an inward acceleration v^2/R. Our choice should depend upon the skills and mathematical training of the class. Two versions are suggested below:

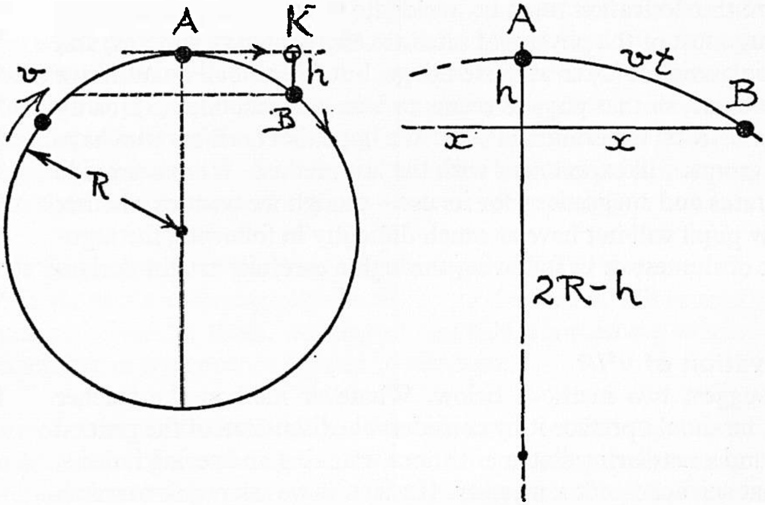
1. **The 'Crossed Chords' Method.** For pupils who know that when two chords of a circle intersect the products of the segments of the two chords are equal, this method is probably best. It follows almost directly from the brown-paper experiment. It is, in fact, Newton's own method.

We draw a circle to represent the orbit and suppose the object, moving with speed v, proceeds from A to B. It may be helpful to continue the previous story and call the moving object the Moon. We draw the tangent to the circle at A and show the fall of the

Moon from K to B, where K is where it would have got to if there had been no force (Newton's Law I). We add construction lines, etc., as in the sketch here, marking the fall KB, equal to h, and labelling the same distance h on the diameter that runs down from A. Then, appealing to the 'crossed chords' property, we say,

$$h(2R-h) = x^2,$$

then, $$h = x^2/(2R-h).$$



We persuade pupils to neglect h in $(2R-h)$. 'When you are weighing a haystack, don't worry about losing one needle. But when you have a needle all by itself and are trying to find how much it weighs, you must not throw the needle away. You must keep the h on the left; but you may throw away the h that is subtracted from $2R$.'

Then, $h = x^2/2R$.

x, which is equal to AK, is practically equal to the arc AB. If the Moon travels from A to B in time t with speed v, $x = vt$;

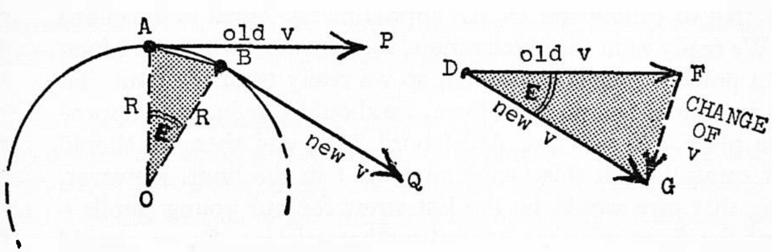
$\therefore\ h = (vt)^2/2R = \frac{1}{2}(v^2/R)t^2$

but h is the vertical fall (with no initial velocity in that direction), with some acceleration, in time t. We use $s = \frac{1}{2}at^2$, so $h = \frac{1}{2}at^2$.

Then, the Moon's acceleration, a, must be v^2/R.

The same holds for the motion at all places round the circle, but with *vertical* always taken to mean the direction from Moon to centre.

2. **The Similar—Triangles Method.** We draw a circular orbit with a Moon moving from A to B with speed v in time t. We draw a long vector AP to represent the velocity of the Moon at A. This must be along the tangent at A. We draw another vector of the same length, BQ, to represent the velocity at B (it is helpful to draw these much longer than the radius of the circle, to lessen confusion). We redraw these vectors in another place nearby, both starting from the same point D. (As experienced physicists, we are tempted to do this drawing on top of the main picture, producing the second tangent backward to cut the first and making that intersection the common point D. But, with beginners, that will make the story much more confusing. It is important to emphasize the distinction between lengths, such as R, and AB, and velocities such as v.



In the second diagram we draw two vectors from D, each of length v and label them

'Old velocity: velocity at A' 'New velocity: velocity at B'

We ask: 'What must be added as a vector to the old velocity to get the new velocity?' We draw that in (shown as a broken line in the sketch here) and label it 'change of velocity'. (We do not need to give a special lesson here on subtracting vectors. We simply ask, 'When the velocity changes direction, what must we *add* to this earlier velocity to get this later one?')

Then we join A and B to the centre of the circle and point out that we have two similar triangles because each velocity vector is perpendicular to the corresponding radius. Then we argue from the property of similar triangles as follows:

$$\frac{[\text{Change of velocity}]}{v} = \frac{AB}{R}$$

$$\therefore\ [\text{Change of velocity}] = \frac{AB \,.\, v}{R}$$

Suppose this kind of change of velocity, which is perpendicular to the actual motion, is related to an acceleration just like any other acceleration – a surprising supposition which must be tested. If so, we can calculate that acceleration as usual:

$$\frac{[\text{Change of velocity}]}{[\text{time taken, A to B}]} = \frac{\text{AB}\,.\,v}{R[\text{time A to B}]}$$

$$\text{Acceleration} = \frac{v}{R}\,\frac{\text{AB}}{\text{time taken}} = \frac{v}{R}\,.\,v = \frac{v^2}{R}$$

Proceeding to Limit. In either of the methods above, we have the problem of proceeding to the limit as B approaches A. If we do not proceed to the limit, we are left with an approximation, essentially that of calling the arc AB approximately equal to the chord AB. We really want the acceleration 'at an instant' when the Moon is at a point A and B combined; so we really want the limit. To keep our calculation in good form, we should put in, at an appropriate place, a factor (arc AB)/(chord AB); and then we should show carefully that this factor tends to 1 in the limit. However, taking that care would be the last straw for our young pupils – except for those who are natural mathematicians. So we should avoid labouring, or even referring to, the need to proceed to a limit, or the method of doing so. If a pupil objects, we should just point out that the jump we have made becomes more and more trivial as we move B closer to A.

Calculus. There are some quick methods that use calculus. In this case, such a method is likely to be too obscure, however quick. It should be avoided. We should also avoid methods that use trigonometry: either the sine that is used cancels out – and similar triangles would have been clearer – or it involves differentiation.

The similar triangle method is closely related to the hodograph method, but we do not advocate the latter because it seems more sophisticated to pupils.

Putting $a = v^2/R$ to use. As soon as we arrive at $a = v^2/R$ we should put it to use. The simplest use that looks real – and not artificial, like problems that ask for tension in the string of a whirling stone experiment – is the calculation of the orbital time of an Earth satellite. As in the graphical method, we point out that the acceleration of the satellite is much the same as that for a projectile slightly nearer the Earth's surface. So the acceleration is g. We write $v^2/R = g$ and take R just over 4,000 miles or 6·4 million metres, and ask pupils to calculate the time of going once round the Earth.

Here again we need a large poster of the periods of the satellites that have been launched. D11

Experimental Test of $F = mv^2/R$ as the Force needed for Motion in a Circle

We should give the new expression an experimental test. Even if they find the geometry and algebra easy, pupils do not feel quite happy about applying $F = ma$ to this motion where the acceleration is 'across the motion' never changing the speed but only the direction of motion. So we should make a test of the prediction that motion in a circle needs a force mv^2/R.

Comment to Teachers on Apparatus for Test. Ingenious physicists have devised many forms of apparatus for carrying out such a test. It is a tempting problem for all of us; there is a strong need for a test and there are intriguing opportunities for ingenuity and skill; so we devise apparatus for the test and then make it more and more complex by adding improvements. But when pupils try such apparatus the intrinsic difficulty of the essential idea at stake makes the complexity most unwelcome. So we should try to keep the apparatus as simple as possible, provided it can yield some kind of quantitative tests. We need a simple device in which a known mass is held in an orbit of measured radius with measured speed by an inward force which pupils can measure directly and compare with the calculated force $F = mv^2/R$.

Several simple forms of apparatus for this test are suggested below. Where a teacher has already devised and made good apparatus of his own for this test, he should certainly use it instead of the suggestions here. The enthusiasm and confident knowledge of the man who made the apparatus are very valuable in this case: they help to carry the pupil through the experiment with enjoyment. However, where the teacher decides to make use of one of the suggestions below, we urge him to resist the temptation to add improvements because this experiment is so easily obscured by its own machinery.

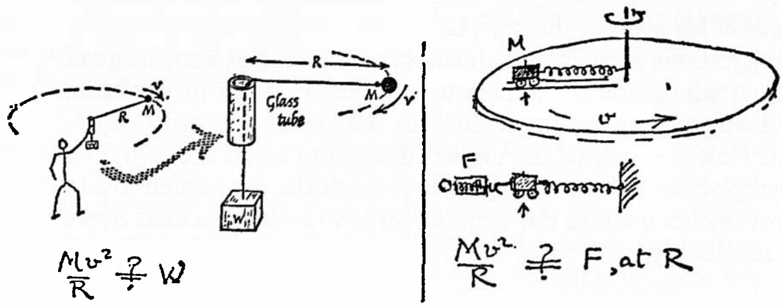
Even with simple apparatus, pupils are easily confused about the nature of the test. We have to explain clearly that the measured force (provided by some spring or weight) is the real force that T pulls the moving body inward. The calculated force mv^2/R is not the real force, but is the theoretical prediction, which is under test. To young pupils who have just been carried through the derivation, mv^2/R seems so important that they are apt to insist that it is the real force and the measured value only a blundering attempt to approximate to what they know is really true. Unless we can straighten out that confusion, gently but firmly, the experiment may do more harm than good.

Experimental Tests of $F = mv^2/R$

These serve as tests of the rule obtained by geometry, $a = v^2/R$, combined with $F = ma$, and applied to this strange form of acceleration perpendicular to the path.

These tests might take the form of empirical investigations to enable pupils to arrive at the rule $a = v^2/R$ if they do not do the geometry. However, we must be careful not to make the investigation too long or complicated. A single test, with the rule already given, is probably better for a slow group. If an empirical investigation is carried out, we should be careful to distinguish between the *form* of the rule (such as its containing v^2) and the *absolute value* of the force. It is much easier to demonstrate proportionality to v^2 than to show that F is actually equal to mv^2/R.

In choosing among these alternative methods, and in teaching them, we should remember that this is strange new territory for pupils; so that clear statements and simplicity of apparatus and emphasis on the nature of the test are far more important than ingenuity of design in devices to ensure accuracy. Complex designs are almost certain to succeed for mature physicists and to fail for these young pupils.



A. **Whirling Object, Pulled by Gravity Load.** The pupil whirls a small metal ball round his head in a horizontal circle by a string which passes through a glass tube held in his hand and carries a weight hung on its end below the tube. The weight can be replaced by a spring balance anchored to the floor and read by another pupil squatting beside it; that avoids bringing gravity into the discussion at this point. C12

Although this simple experiment needs several pupils to co-operate in making measurements, we should be careful to make sure that each pupil takes his turn in doing the actual swinging. To a young person, participation makes the test much more real. T

The 'team' (two to four pupils) measure the time for a counted number of revolutions. The radius of the orbit is measured from the 'Moon' to a mark on the string which was kept just at the mouth of the glass tube during swinging. The force actually needed to hold the 'Moon' in orbit is the *weight* of the load hung on the lower end of the string. That force is compared with the theoretical force mv^2/R.

A single set of measurements, repeated by different pupils and averaged, will give the 'formula' one overall test; and that might suffice for pupils who have done the geometry.

For pupils who are using this as an empirical approach to the *
formula, it is useful to try several different forces for the same *
radius of orbit, and look for a relationship between v and force. *
And they should try at least two different masses of 'Moon'. *
Then they might change to a different radius; but this is a more *
complicated test, since the radius is involved in deriving v from *
their measurements. *
*
With the encouragement of interest in satellites and electrons, *
slower pupils may get through this investigation with success; but *
we must prepare for it carefully and maintain interest throughout *
the experiment, restricting it to a single test if necessary. *

B. **Toy Railway on a Turntable.** This is perhaps the easiest of all models for pupils to understand. The turntable is a horizontal disc of hardboard two or three feet in diameter, driven by a small electric motor, or by hand if necessary. A section of model railway C/D13 line extends from the centre to the edge along a radius; and a small toy wagon runs on it. The wagon is pulled towards the centre by a spiral spring of steel wire. (It is important to make this spring of a wire that is easily available, because it will get damaged and will have to be replaced frequently.)

The turntable is kept rotating steadily and the position of the wagon is marked while the rotations are timed. A small flag will serve as marker for the wagon's position, or an electric contact can be arranged – but even that welcome help will make a distracting side issue for slower pupils.

Then, in a separate experiment with the turntable at rest, pupils measure the force which the spring applies when stretched to the mark in which the force is measured by a separate statical experiment. This is with a spring balance (or a load hung over a pulley).

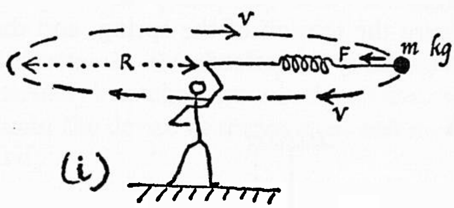
Pupils could make a series of measurements as in A. This arrangement is unfortunately sensitive to levelling of the turntable. Unless the turntable is held firmly level and the whole device well constructed it will afford only a rough test.

This turntable experiment should be shown as an additional demonstration if method A is used as a class experiment. But if this turntable experiment *replaces* A, it should be given as a class experiment.

There are many variants of B in which a moving wagon or sliding bob is held by a horizontal spring while the whole contraption is rotated faster and faster until some device shows that an agreed radius is reached. We fear that these do not look like an obvious Moon in orbit to the eye of a beginner. He sees them as ingenious devices for testing something; and he can follow instructions and carry out the test, but he does not understand clearly what that 'something' is. So we do not recommend them.

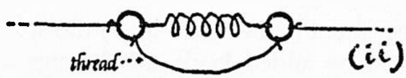
C. **Object Whirled on String, with Spring Inserted to Measure Force.** This is somewhat like A, but the string is attached to a ring held by the experimenter, instead of passing through a tube to a load. The tension in the string is measured by inserting a length of spiral spring of steel wire with a simple device to show when the spring is stretched by a standard amount. C14

The experimenter whirls a metal ball (mass 50 to 100 grams) round his head in a horizontal circle, as in method A. Part of the thread between hand and revolving ball is replaced by a stretched spiral spring of light steel wire (Fig. i).

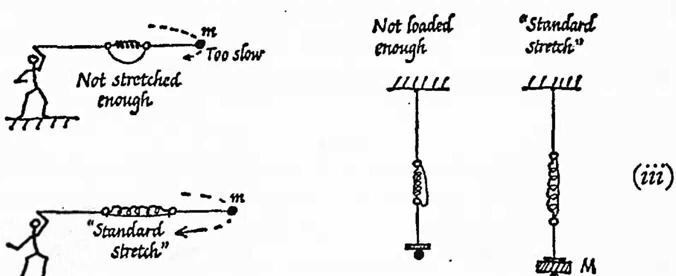


The spring serves both to produce a variable force – so that the motion is stable‡ – and to measure that force.

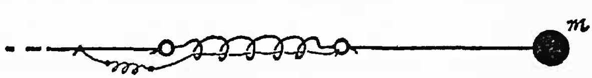
To prevent accidental overstretching of the spring, its ends are connected by a loose thread (Fig. ii).



In whirling, the experimenter tries to keep the spring stretched enough to pull the thread almost, but not quite, taut. We may call that the 'standard stretch'. (See Fig. iii.)



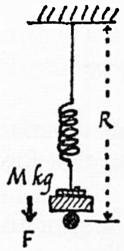
To prevent that thread from getting tangled, it is carried through the coils of the spring (Fig. iv).



(iv)

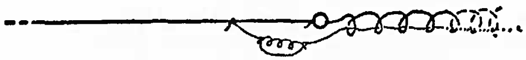
‡ If the revolving radius consisted of spring all the way out, and no thread, and if the spring had negligible length when unloaded, the motion would not be stable. The frequency of revolution would be independent of radius and the experiment would be confusing and perhaps impossible. But, with the proportions suggested, it works well.

As Fig. iv shows, the thread ends with a tiny, weak spring. That 'sub-spring' is only used as a signal. It does not contribute significantly to the central force. Some signal is needed when the device is being whirled to tell the experimenter how much the main spring is stretched. Then he can perform a subsidiary experiment to measure the tension of the spring, and the length of the orbit radius for exactly that stretch.

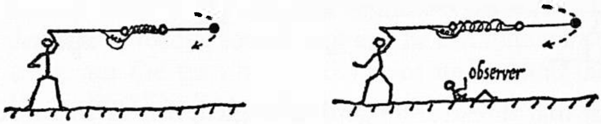


(v)

In the subsidiary experiment (shown in Fig. v) the spring is hung vertically and stretched by added loads till the signal (the sub-spring) shows the stretch is the same as in the main whirling experiment. Then the weight of the total load (including the ball if it is kept there) gives the actual force the spring must exert during whirling. And the length from the ball up to the ring that is held in whirling, gives the orbit radius.



(vi)



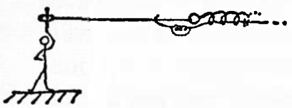
(vii)

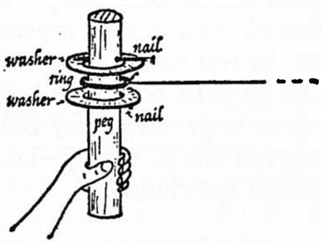
The sub-spring is a tiny, weak coil of very thin wire – steel from an old wire recorder, or very thin manganin from a high resistance, wound on a pencil. It is easily damaged and should be easy to replace. It is attached to the thread through the main spring, as in Fig. iv. Since that sub-spring would easily get overstretched, it too is given a loose safeguarding 'sub-thread' joining its ends, as in Fig. vi.

Then, when the device is whirled faster and faster the spring stretches more and more until it begins to pull its main safeguarding thread taut. The sub-spring remains unstretched, with its sub-thread loose, until the safeguarding thread is just taut. Then, at that speed, the sub-spring acts as a signal.

So the experimenter should speed up his whirling until he sees the sub-spring partially stretched, its sub-thread almost taut. Then he tries to maintain the device at that radius, and make his measurements (Fig. vii).

The device is attached to a ring which slips loosely on a wooden dowel held in the experimenter's hand (Fig. viii).





(*viii*)

During the whirling, the radius, from ring to ball, is not horizontal: it moves round a shallow cone. This would appear to introduce a cosine factor. However, (*a*) the cosine is very close to 1; and (*b*) further examination of the geometry will show it occurs at two places and cancels out!

This device sounds complicated. It is not as simple as A but some teachers consider it behaves more consistently. If actual force and theoretical force disagree by less than 2% we must thank good luck; between 2% and 7% skilful manipulation; above 7% careless experimenting or arithmetic can be blamed.

A number of physicists have devised modifications – ranging from a small piece of red tape at the sub-spring to a set of traffic lights – but the form described seems the simplest that works well. Even so it has the severe disadvantage of being a gadget, 'special' apparatus to test a general piece of natural behaviour; so it is not recommended strongly.

D. CO_2 Puck making circular Orbit on Glass Table. Use the device suggested for D59 (page 197) and pull the thread up with a spring balance to measure the force.

Comments on Tests of $F = mv^2/R$

What is being Tested? Again and again in every discussion with pupils, we need to emphasize the order of reasoning in the test, to avoid letting the logic be reversed:

> 'The *actual* force is the pull of the string or the pull of the spring, the pull that you measure. The force that you calculate by working out the value of mv^2/R is the *theoretical* force, the force that you hope will be a good prediction of the actual force. You are doing the experiment to find whether your hope is reliable. It is mv^2/R that is under question, not the actual pull which you know is true.'

A Plea for Simplicity. Whatever device is used, we hope that it will be simple and cheap. An expensive device would have only limited use as a class experiment and a complicated one will obscure the real nature of the test. Even if the simple cheap one is too rough in its behaviour to afford an accurate test, pupils are now at the stage when they can distinguish between an experimental result that is 'wrong'–i.e. contrary to expectation–and one that is right but clumsy.

In carrying out these tests as a class experiment and perhaps illustrating them by a further demonstration, we must be careful not to let the test grow too heavy so that it seems to pupils more important than the uses of mv^2/R to deal with electrons, satellites and the whole solar system. A systematic investigation in which the force is measured for a series of different orbit-radii, all at the same speed, then for different speeds, and so on, may seem tempting to well-trained physicists, but it would lead our young beginners into discouragement rather than keen understanding.

It may even be better to go ahead to uses of mv^2/R and fit the test in later when there is a good opportunity. Pupils should now be at an age when they understand that the intermediate stages between the general idea of an object moving round a circular orbit and the final result that the acceleration is v^2/R inwards are not mysterious pieces of abstruse science or mathematics but consist of ordinary geometry and algebra. Even if the whole story of that connection is kept in a black box, they should know that the box contains only the ordinary gears and levers of algebra and geometry that they have met before. If they have gone through some earlier derivations (like Galileo's geometrical derivation of $s = ut+\frac{1}{2}at^2$) those should serve as assurance to enable them to take the new derivation for granted. We should not have to assure pupils that we are honest; but we do have to assure them that they are not muddled.

Uses of mv^2/R

We shall use our ability to calculate the inward force (necessary to maintain an orbit) for electrons, etc., and for satellites and for our Newtonian study of the solar system. In each case, we need some new knowledge before we can put the central force expression to use: the force on charges moving across a magnetic field; the inverse-square law of gravitation; and some historical knowledge of planets, etc.

Since none of the uses can be shown at once without such extra knowledge, we choose electrons first because we can continue the work of Year IV with them.

Note to Teachers: Uses of Magnetic Fields in High-energy Physics. When we come to teach the effect of a magnetic field on a stream of electrons and use it to measure e/m for the particles in the stream, we may feel discouraged and say, 'Why do we bother to derive v^2/R just to make this difficult measurement? Why do we carry pupils through this business of using magnetic fields to bend the stream to an orbit, just to find e/m?' Then it may be wise for our own encouragement to keep in mind the great importance of magnetic field deflections in modern physics.

In high-energy physics, we accelerate particles such as protons to huge energies – thousands of millions of electron.volts – then direct them on to a target, then do experiments with the sub-atomic particles that splash out from the bombarded target. In such work, magnetic fields are used for four purposes.

a. **Holding** the original protons (or electrons) in orbit so that they can be accelerated again and again, each time they go round a circular accelerator

b. **Sorting out** the products of bombardment. Even if the products are all sub-atomic particles of the same mass, say mesons, they may have charge $+e$ or $-e$ or 0. In the stream from the target in a big machine to the experimental area, we find an electromagnet sorting the stream into three beams, positive, negative and uncharged, which fly on out through different portholes to different experiments *c.* **Measuring** a particle's momentum. Deflection by a magnetic field involves a particle's mv as well as its charge. Cloud-chambers, bubble-chambers and other apparatus are often crossed by a magnetic field to make momentum measurements that help distinguish particles – as in our own class experiment to measure e/m for electrons

d. **Focusing** a stream of particles. When a beam of charged particles emerges from a slit or gun muzzle, it is seldom a thin, threadlike stream (as it is in the fine beam tube): it is usually a fan of diverging streams. A specially shaped magnetic field can be used to focus these streams by bending them so that they do not diverge more, or even bending them so that they converge to a sharp spot on a target or recording film.

With all these uses, magnets in many forms are to be found in use everywhere in high-energy research laboratories.

These uses of magnets in research are not things to teach our pupils now. But it may be useful to remember them when we are teaching.

Chapter 2

ELECTRONS IN ORBITS

Electron Streams and Magnetic
Fields; Measurement of e/m;
Atom Models

Demonstrations of Electron Streams

We show the following demonstrations of electron streams (cathode rays) from hot cathode tubes:

Straight line streams (cathode rays through a slit splashed on to a screen that glows when bombarded) D15

Fine beam tube D16

Casting shadows (Maltese cross) D17

An 'Electron Gun'. If it is available, we show the large diode tube again, not as in Year IV for a study of its characteristic current-p.d. graph, but to point out that if we punch a hole in the anode (plate) we have an electron gun. D18

Discussion of Electron Gun

We ask what changes occur in the speed of electrons as they boil off the glowing cathode. They leave the filament with very small speeds, but we apply an electric field that accelerates them towards the plate. T

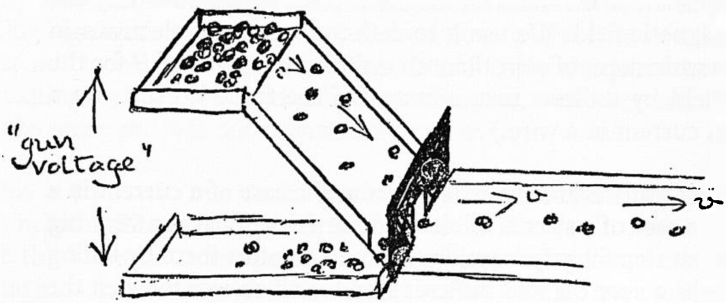
Suppose each electron has a charge e coulombs, and suppose a voltmeter connected between filament and plate shows we have applied a p.d. of V volts. Then the battery we have connected there gives V joules to each coulomb passing from filament to plate. (We must make it clear that this is nothing to do with the battery used to heat up the filament – we could use a Bunsen burner for that, but for practical difficulties.) Then each electron gains energy eV joules.

But, unlike electrons in a wire, these electrons have nothing to hit, nothing to give energy to, as they travel across towards the plate. All the energy they gain must be retained as K.E., until they hit the plate. Those that hit solid metal are brought to a stop, and share out their K.E. as heat among the atoms of the metal. (Any X-rays? At most, only a very tiny fraction of the collisions produce X-rays, even in the best of X-ray tubes.)

Those electrons that arrive at the hole we have drilled in the plate, the 'gun muzzle', keep their K.E. and go on through the hole. After that (unless we add a further battery) there is no more acceleration: they keep a constant speed until they hit some barrier.

Pupils will soon need to have a clear idea of such an electron gun, from which electrons emerge in a stream, and thereafter continue with unchanging speed. And they need to understand why we say the K.E. of each such electron is equal to its charge times the gun voltage: $\frac{1}{2}mv^2 = eV$.

We might then show a crude model of a herd of marbles running down a sloping board to crash into a wall – except for those that hit an opening in the wall and continue along the table. The slope corresponds to the electric field we apply inside the gun.



More Demonstrations of Electron Streams

Deflection by electric fields (fine beam tube or other e/m tube) D19

Effect of magnetic field: fine beam tube with magnetic field D20

The last demonstration shows a circular orbit and therefore tells us that the magnetic field must exert a force sideways on the stream of electrons. We therefore go back to the electro-magnetic kit and ask C21 pupils to repeat the experiment with a wire carrying a current placed in a magnetic field. However clearly they seem to remember this from earlier Years, they should certainly try it again at this stage (see *Guide to Experiments III*).

It is not necessary for pupils to continue from that to make a D22 model electric motor all over again; but we may profitably show a large motor to emphasize the fact that this force on a current is not trivial but is a very important one which can be very large and which forms the basis of much modern machinery.

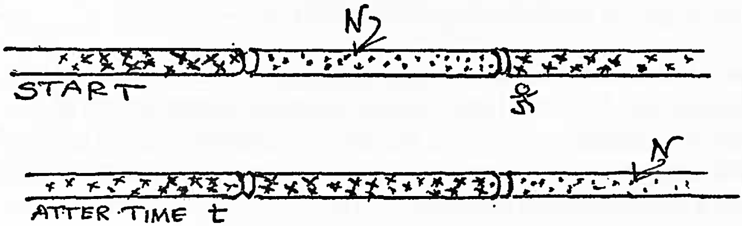
Discussion of Force on Moving Charged Particles

Pupils now know that there is a force on a wire carrying a current T across a magnetic field; that the force is proportional to the current, and, presumably, proportional to the length of wire. A moving-coil ammeter, with visible works, shows that force in action; and its uniform scale shows the force is proportional to current, if we trust Hooke's Law for the hairsprings.‡ We express this knowledge in the form:

‡ If one wishes to take the modern definition of current-measurement by magnetic effects, this proportionality of *force* and *current* is implied by the definition itself, and no experiment could be needed to 'prove' it. Even so, the ammeter illustrates it well.

force F = (B) (current, C) (length of wire, L), $F = B.C.L$ where B is a measure of the magnetic field (or rather induction) combined with a constant that is determined by our choice of units. (In our present course, we need not discuss the value of that constant or the units for magnetic field, because we only use one magnetic field. We use it to deflect a stream of electrons in our measurement of e/m; then we estimate the value of B for the *same* field by a direct measurement of the force it exerts on a known current in a wire.)

We need to transfer that knowledge from the case of a current in a wire to the case of a stream of charged particles. That is a very big and difficult step for pupils; and we should comfort them by telling them it was a very big and difficult step for all scientists when the well-developed theory of electric circuits and forces had to be extended to electrons, etc., late in the last century.



The Argument. We draw a section of wire AB of length L containing N electrons each of charge e and we suppose that when there is a current C, the electrons drift along with speed v from A to B. We post an imaginary observer at the 'outgoing' end B and ask him to count electrons like a small boy counting the cars as they go by. He starts counting when an electron emerges at B. The electron that is at A at that instant arrives at B some time, t, later having travelled distance L with speed v. In that time t, all the N electrons in the wire between A and B arrive at B. Therefore in time t the observer counts a total charge Ne and says the current is Ne/t. But length L is vt.

$\therefore\ CL = (Ne/t)\ (vt)$ or Nev

We assume that the force on current C through a length of wire L is BCL, where B is a constant, involving the strength of the magnetic field. Then the force is also $BNev$.

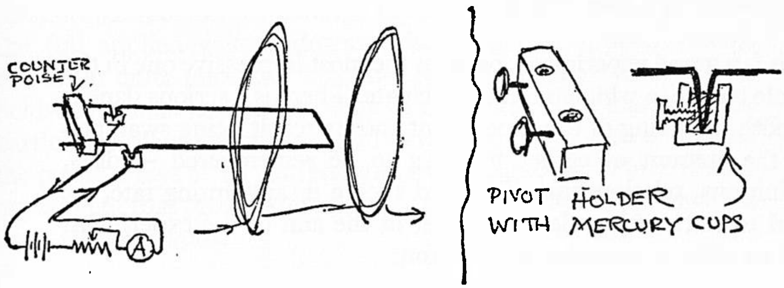
Therefore the force on a *single moving charge* is Bev

Like the derivation of v^2/R this is a difficult sophisticated argument which will not succeed with young pupils unless we preface it with two encouragements: (1) We show pupils that we and they need the result badly; (2) we assure them that this is not a rigmarole that they must learn for reproduction in examinations. This is something high up that they can only just reach to touch: not inaccessible, but not to be fully grasped. They should enjoy the privilege of touching it; but then need only remember that what they saw did make sense and was not mysterious nonsense.

This is a place in our teaching where a short animated film *would* be helpful. There is a danger that animated films will show what we think 'ought to happen' – in some simplified scheme that we imagine – rather than what does happen. But here we need to show an unfamiliar piece of 'geometrical' reasoning rather than an unfamiliar phenomenon. F

Current Balance to Measure Force due to Magnetic Field. In all this, the constant B remains unknown. It contains the strength of the magnetic field. Its value also depends, of course, on our choice of units and of 'system of units', which in electricity and magnetism often contains natural constants relating to the properties of materials and even the properties of vacuum. Here, we shall not go further with the nature or value of B but shall measure its value by a direct experiment on a known sample of current in whatever magnetic field we use for electrons, etc. For that we must use a simple current balance.

We suggest a simple design that will weigh the force on a short section of straight wire in the magnetic field of the Helmholtz coils used for the electron stream. D23

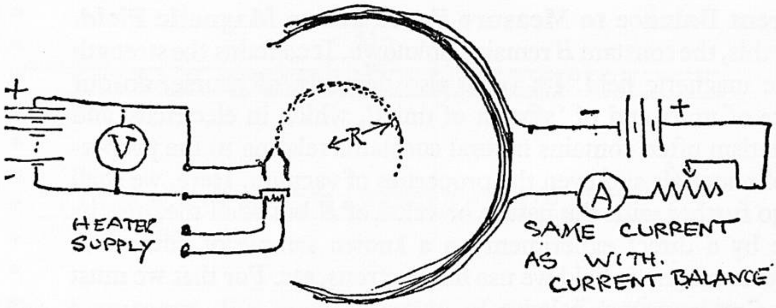


(For example, suppose the 'current balance' has a test wire of length 20 centimetres, 0·20 metre, that carries a current 20 amps. Suppose when placed in the magnetic field of the Helmholtz coils the test wire needs a load of 0·5 gram hung on it to restore balance. Then the force is 0·005 newton. Then B has the value 0·00125 for the Helmholtz coils carrying whatever current they are carrying. The force on a charge e coulombs moving with speed v metres/sec across *that* magnetic field is 0·00125 ev newton.)

That needs demonstration and careful explanation; and then it will be used for a measurement of e/m. *
*

Measurement of e/m

Pupils should use the fine beam tube, taking it in turns. This is a very important 'atomic' measurement and it should if possible be treated as a class experiment. C24



Although this should be a class experiment, the teacher may want to run a demonstration of it right through first. With any except a fast group, we should go through the calculation completely in that demonstration but ask the pupils not to make any record of it and then leave them to carry out their own calculation when they do the experiment as a class experiment.

This is a grand experiment, perhaps the most impressive one in the whole course in which pupils participate. There is a serious danger of both the doing of the experiment and its result being swamped by the amount of earlier teaching to be remembered – ideas, definitions, relationships. To avoid such a disappointing fate, we need to start with a clear reminder of the aim of the experiment and an offer of considerable revision. T

We should review the meanings of electric charge and p.d.; the definitions of newton, joule, coulomb, volt; the idea of electric field; the name 'electron-gun' and its meaning; the idea of electrons T accelerating in an electric field in a vacuum, and of their continuing with constant velocity after that. Of course, we should not go to the other extreme of over-tedious preparation. This is a matter for wise judgment in offering revision just when it is needed.

Electric Measurement: Gun Voltage. In a hot-cathode tube, such as the fine beam tube, all the electrons that emerge from the muzzle of the gun have the same kinetic energy, equal to the electron charge times the gun voltage. Therefore we can write T

$$eV = \tfrac{1}{2}mv^2 \ldots \text{(I)}$$

This provides one of the two pieces of experimental information that we need. The effect of a magnetic field provides the other.

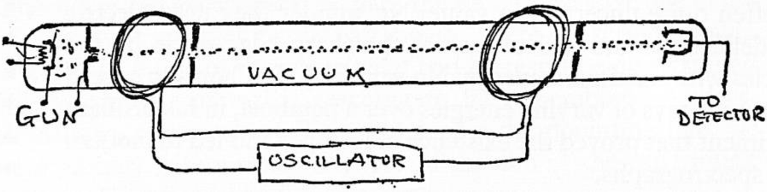
Note to Teachers: Schemes of Measurement for *e/m*. In making measurements on electron streams (or streams of any other charged particles, such as positive ions) there are two quantities that we do not know: e/m and v, the speed of the particles; so we need two separate measurements. Any measurement of the effect of an electric field yields e/mv^2. Any measurement of the effect of a magnetic field yields e/mv. If we make a measurement of each of those two kinds, we can extract e/m and v; but it is no good making two measurements of one kind instead. For example, if we measure the gun voltage used to accelerate the electrons (which gives them kinetic energy) and measure the deflection of the stream by an *electric* field, we have two measurements of the same kind; and each will tell us only e/mv^2.

The reason why early experimenters, such as J. J. Thomson, used electric field deflections instead of the gun voltage was because they could not command streams of particles which all had the same kinetic energy: their particles were manufactured in the plasma-like mess in a discharge tube and ranged in energy from the full applied voltage downwards. Even that applied voltage was often quite uncertain in value. No wonder they used electric field deflections and had to look at the sharp edge made by the particles with maximum energy. No wonder J. J. Thomson spread the positive rays of varying energies over a parabola, in his brilliant experiment that proved the existence of isotopes and led to modern mass spectrographs.

However, those are now historical methods, dating back more than half a century in the rapid history of atomic physics; and with our pupils we should not treat them as part of modern knowledge but should relegate them to special studies in the history of science. Moreover, they were conducted by master physicists to whom the geometry of electric field patterns and fringe effects were child's play, in contrast with our pupils for whom every simplification of mathematics adds greatly to the chances of understanding.

Even for sixth-form physics specialists, some critics wonder whether the historical methods deserve attention when newer methods and further knowledge are pressing for inclusion. If we could listen to colleagues in another science discussing great experiments in their history, such as a brilliant investigation of nitric oxide in a mixture with air – in an age when even names of gases were confusing – we may share those colleagues' doubts about use of historical teaching; and we should turn those doubts upon the teaching of our own modern science.

The early experimenters had one useful trick. While one of their measurements was made with a single applied field (usually magnetic) the other was made with electric and magnetic fields, applied simultaneously, adjusted to produce no deflection – and that gave v. That might seem to offer a very simple way of making measurements ourselves with a hot cathode tube containing large plates for electric field deflections. Then the forces exerted by the two fields must be equal and opposite; and we can state that as an equation without having to measure any deflection. But for any simple use of this method, the two fields must be 'co-terminous', they must extend over the same region of the path of the stream. We cannot secure that in any available apparatus. So we could only use this method for very rough measurements. And, having obtained v by this null method, we should still have to measure a deflection to obtain e/m as well.



Instead of making two measurements (one with the electric field, one with magnetic field), we could make one of those and do a separate direct experiment to measure v. The latter is done by

some form of 'chopper': the stream passes through two slits in *
screens far apart, with synchronized 'valves' to interrupt the stream *
just before each of the slits. The 'valves' are usually electromagnets *
carrying rapidly alternating current, whose field swings the stream *
off the slit, so that it can only pass through the slits in periodic *
bursts. If the succession of bursts gets through both slits we know *
the time of travel between one slit and the next in terms of the *
frequency of the oscillating deflecting field. Although this chopper *
method has been used for electrons (and for positive ions) it is *
much too difficult for a teaching demonstration. (We might show *
it by film.) *

Measurements. So, in our experiment, we must make measure- T
ments with an electric field and with a magnetic field. We suggest that the electric field measurement should be that of a gun voltage, used as in equation (I) above.

Magnetic Measurement. The magnetic field should bend the path of the stream into a circle whose diameter pupils can measure. We first remove the tube and place the current balance in the same region of the magnetic field and measure the force on a known current in a known length of wire.

Then we know B in $F = BCL = Bev$
Then, for orbit $Bev = mv^2/R$ . . . (II)
and R, or rather the diameter $2R$, must be measured.

Measuring the Orbit. The orbit is seen as a faintly glowing T
circle in a glass bulb in an almost dark room. How can the teacher, let alone the pupils, measure the orbit-radius quickly and easily? By holding a ruler up in front and making a guess at the orbit diameter. That will yield rough estimates of v and e/m; and, in our exploration of the micro-world of atoms, rough estimates are good science. We might expect to be correct within 10%. Such a guess at $2R$ should be correct within 10%, certainly within 20%. We ask pupils:

> 'Hold a ruler up in front of your neighbour's face and guess the distance between his ears. Yes, you may hold the ruler above his head if you prefer. Can you guarantee your estimate within 10%? 20%? A 20% error would be more than one inch.'

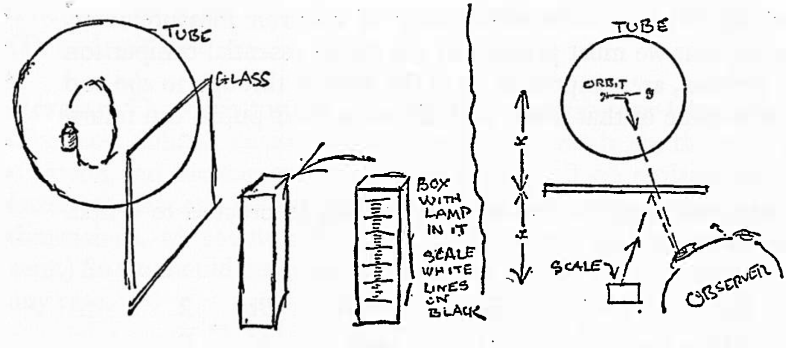
The fine beam tube has been used for teaching in many laboratories. *
The measurement of orbit diameter is difficult; and it has a compli- *
cated history of attempts to make it easier. Teachers have devised *
skilful schemes for measuring it with some precision – illuminated *

scales, double scales to avoid parallax, special callipers, special image projectors, observing telescopes, etc. These do facilitate fairly precise measurements; but they also make the experiment more complicated and difficult for pupils to see and remember. Even when handled by the teacher with considerable practice, any such special device adds weight to an experiment that is already almost too heavy for many a pupil.

Furthermore, if we emphasize measures to improve accuracy we are missing the point of the experiment: at this stage it is to bring pupils into contact with a real measurement of electrons. The importance lies in their seeing and doing, in the principle of the experiment and its general success. Accurate measurements can come at other times and places. For average pupils, a successful dive into the micro-world to make a real measurement should be a great achievement. Emphasis on accuracy might make the whole business too hard; or, just as bad, it might turn the experiment into an anxious game to 'get the right answer'. Even with very able pupils the idea that it is good science to make a rough estimate first we should emphasize and make a rough measurement. *Then* if those pupils want to devise refinements for accurate measurements, well and good: they should repeat the experiment as they wish. (Even then, we hope teachers will point out the contrast between making an estimate of a fundamental quantity and trying to get the right answer.)

Therefore we suggest that both teacher and pupils should estimate the orbit diameter simply by holding a ruler outside the tube. Since the room must be dark, it is easier to use an illuminated transparent ruler. We suggest a Perspex ruler with a small electric lamp taped on at one end, the bulb itself covered with black tape to cut off direct light. If the ruler graduations are not bright enough, stripes a centimetre wide should be painted on alternate centimetres with red nail polish.

The best modification so far produced forms a *virtual* image of an illuminated scale inside the tube, in the plane of the electron stream. A vertical sheet of clean plate glass is placed just in front of the tube. An illuminated scale is placed in front of the sheet at such a distance that the image of the scale, behind the sheet, is in the middle of the tube. This does make measurements easier; but we do not recommend adding this complication except with a very fast group.



Calculating Electrons' Speed and e/m from the Experiment. The orbit-diameter $2R$ is estimated and the gun voltage V is read. Keeping the magnetic field unchanged (same current through the coils), we put the current balance in the place of the tube and *measure* the force on a known length of wire carrying some measured current. Substituting in $F = BCL$ gives us B for use in $F = Bev$. (Since we are going to use the latter for F in $F = mv^2/R$, we must express the force measured with the current balance in newtons.) C24

Then we have all the measurements we need for use in

$$\text{(I)} \quad eV = \tfrac{1}{2}mv^2 \quad \text{and (II)} \quad Bev = mv^2/R$$

to find v, the electrons' speed, and e/m their charge/mass ratio.

But in this new elaborate experiment, even the algebra and arithmetic can become confusing. It is not that the algebra is too difficult for pupils, but it delays their seeing the physics. Therefore *we should ask for v first, not e/m.* There are several advantages:

a. The speed, v, is a simple, clear property, easily visualized. Pupils can have little doubt about the kind of thing they are working out. (On the other hand, e/m is a strange quantity, *not* necessarily appealing to beginners – though we ourselves know its importance in the historical development.)

b. v emerges directly when we divide equation (II) by equation (I). (To obtain e/m directly we must square one equation before dividing: a trivial extra burden, but just enough to lose some pupils.)

c. The huge value for v is astounding to beginners: the value of e/m is not.

Teachers, familiar with the history of electron measurements, knowing that we must proceed to e/m for an essential comparison with protons, are tempted to carry the algebra through to e/m and obtain a value of that first – and of course their pupils can follow that.

We urge teachers, for reasons given above, to proceed to v first. Then we argue thus:

$$\left.\begin{array}{ll}\text{(II)} & Bev = mv^2/R \\ \text{(I)} & eV = \frac{1}{2}mv^2\end{array}\right\} \quad \therefore \quad \frac{Bev}{eV} = \frac{mv^2/R}{\frac{1}{2}mv^2} \quad \therefore \quad \frac{Bv}{V} = \frac{2}{R}$$

$\therefore$ $v = 2V/BR$ and we calculate v from that.

Results. The estimate of v is affected by errors in the current balance experiment as well as those in the electron stream measurements. However, teachers may find it a help to keep some values of v in mind for use as rough checks. The following are correct within 1 per cent:

For gun voltage	100 volts electrons have speed	6×10^6 metres/sec
Then for	140 volts	7×10^6 metres/sec
	180 volts	8×10^6 metres/sec
	230 volts	9×10^6 metres/sec
	285 volts	10×10^6 metres/sec

We should pause there, and comment on the huge value of v for quite a small gun voltage, one or two hundred volts. Speaking in a sloppy, qualitative way, we might say that this means that e is enormous compared with m. (Of course, we cannot, as good scientists, compare the numerical values of two utterly different quantities like that. We mean that in comparison with the charges we can place on large masses, the electron's charge is enormously bigger than we would expect for something of its mass. More definitely still, when we know the constants in Coulomb's Law and the Law of Gravitation, we find that the electrical repulsion between two electrons is so enormously greater than the gravitational attraction between them, that the latter would be negligible.)

We may point out that a 'chopper' experiment somewhat like the one used to measure gas molecule speeds can be used to measure v directly. It gives values that agree with this less direct measurement.

Then we substitute the value we have calculated for v in Equation (I), and calculate e/m. C24

Results. The value for e/m is $1{\cdot}76 \times 10^{11}$ coulombs per kilogram. The most difficult measurements to make accurately are the orbit diameter and the current balance force. (Even if we replaced the latter by magnetic field strength calculated from current and coil dimensions, we should still expect an uncertainty of several per cent.) So we should not claim this as a very precise measurement in any case.

A Rough but Very Important Measurement. Here, with our rough measurements, errors up to 40% may well be expected. A large error *is* disappointing. To avoid spoiling the success of the experiment by that disappointment, teachers are urged to discuss the matter with the class *beforehand*.

> 'This will be a rough measurement. It will not be accurate because we cannot get inside the bulb and measure the circle precisely: and our current balance will make a rather rough measurement. But the result will be real knowledge of electrons. You will find out how fast they move from this gun, also the proportion of electric charge to mass for each single electron. The experiment will be rough, but worth doing.
>
> 'Suppose the speed for 100 volts on the gun is really 10 miles a second, and our rough measurements give 7 miles/second in one experiment, 13 in another, and even 20. You would still have a very useful idea of the electrons' speed. Do you want to try it, knowing it is rough?'

Importance of e/m Measurement. While the value of e/m is to us an intensely interesting piece of atomic information, it may seem to pupils a dull thing for us to work out unless we advertise its importance by pointing out two things: T

1. This is a piece of information about extremely small things, individual electrons, information that we obtained from large-scale measurements. We never applied a microscope to our experiments. We never counted some vast number of electrons or alpha particles or anything else. We made ordinary-sized measurements and obtained atomic information.

2. We can compare this measurement of e/m with e/M measurements for other 'atomic' particles: the ions that carry currents in solutions. (And we should assure pupils that ions in gases have similar values of e/M.)

If our group of pupils did not see water electrolysed and the products measured in Year IV we should at once do that experiment. An ammeter and stopclock tell us how many coulombs pass through the electrolysis apparatus. From the measured volume of hydrogen produced, we calculate its mass. (We have to use the density of hydrogen but even if we do *not* make a separate measurement, pupils should accept that as a piece of data similar to the result of the measurement for air which they have seen and done.)

We point out that it seems very likely that the current in 'water' (= water + acid) is carried by particles of hydrogen, each of them carrying the same size of + charge. In Year IV, we suggested a demonstration experiment, to show that the quantity of hydrogen liberated is directly proportional to current and to time and therefore to the total electric charge carried across. This does not prove that the hydrogen is travelling across as atoms all alike or that those atoms, if they exist, all carry electric charges of the same size; but those are the easiest assumptions to make. Measurements with hydrogen show that: One kilogram of hydrogen is liberated when 96 million coulombs pass across.‡ If this has not been shown in Year IV or in Chemistry, it should be done now. D/C25

(Pupils can see oxygen being liberated at the other electrode; and they have probably heard about negative ions travelling in electrolysis as well as positive ions, though in the opposite direction. So they may object to our saying that *all* the current is carried by positive hydrogen ions alone. Unfortunately that is both true and untrue, and the detailed story would divert attention from the essential discussion here. In the middle region between the electrodes, the current is carried by positive and negative ions moving in opposite directions with different speeds. But very near an electrode ions of one kind are driven away when electrolysis starts, and the current is then carried wholly by ions of the other kind, which therefore have to move faster in that region, just before they arrive at the electrode.)

‡ More precisely 96·5 million coulombs for 1·008 kg of hydrogen. This makes e/M 95·7 million coulombs for 1·000 kg of hydrogen.

If the carriers (ions) all have the same charge and mass, e/M for a hydrogen ion must be 96 million coulombs/kilogram. We compare that with the value obtained for the electrons in our experiment. T

about 10^8 coulombs/kg for H ions	about 2×10^{11} coulombs/kg for electrons

The value for electrons is nearly two thousand times greater. Electrons must have two thousand times bigger charge or two thousand times smaller mass, or some combination of those disproportions. We cannot give pupils clear experimental evidence that the charges *are* the same in size: but we can assure them that a number of different types of experiment converge to indicate the same size; and Millikan's experiment suggests that all charges on ions are one electron charge or a multiple of it. And we point out that if a hydrogen ion is made by knocking one electron off a neutral atom, the charges must be equal and opposite. All that is a mixture of reassurance and plausible assertion, which is well vouched for in our own experience but not in what we can show to pupils.

If we agree that the charges are the same size, electrons must be very much lighter than atoms. In fact the electron has a mass only 1/2000, or more accurately 1/1840, of the mass of a hydrogen atom.

Incidentally, at this point we should give the hydrogen atom which has lost an electron a name, a 'proton'.

Mass of Electron; Mass of Proton; Avogadro Number

Since pupils have heard about Millikan's experiment in Year IV and have been told the result, $e = 1{\cdot}60\times10^{-19}$ coulomb, they can now calculate the mass of a single electron. T

$$\text{Mass electron } m = (m/e).e = \frac{e}{e/m}$$

$$= \frac{1{\cdot}6\times10^{-19}\text{ coulomb}}{1{\cdot}8\times10^{-11}\text{ coulomb/kg}} = 9\times10^{-31}\text{ kilogram}$$

We can also work out the proton's mass though that may well have been done in Year IV, as soon as Millikan's experiment was done. T

Mass of proton (from Millikan's experiment and electrolysis)

$$= \frac{1{\cdot}6 \times 10^{-19} \text{ coulomb}}{9{\cdot}6 \times 10^{6} \text{ coulomb/kg}} = 1{\cdot}67 \times 10^{-27} \text{ kilogram}$$

And thence the 'Avogadro number', the number of molecules in a mole. The mass of a molecule of hydrogen, H_2, is T

$2 \times 1{\cdot}67 \times 10^{-27}$ kg or $2 \times 1{\cdot}67 \times 10^{-24}$ grams

So the number of molecules in 1 mole of hydrogen, 2 grams, is:

(2 grams)/($2 \times 1{\cdot}67 \times 10^{-24}$ grams) or 6×10^{23} molecules.

600000000000000000000000 molecules in a roomful of hydrogen, air, or any other gas.

We are beginning to find our bearings in our exploration of the micro-physical world.

Positive Rays: Atomic Ions

We should treat positive rays briefly, since we cannot give a *
convincing demonstration of their deflection by electric or magnetic *
fields, and we certainly cannot make any measurements. *

The production of positive rays in discharge tubes is a very complicated business in which bombardment by ions, electrons and X-rays all play a part, and we suggest that that should be left to the historians of science. A discharge tube should at most be shown as a pretty sight. Instead we should describe modern methods to pupils briefly. D26

We ask how we might obtain a 'positive ion', an atom or molecule that had lost one electron. T

> 'What would you use if you wanted to knock an electron off an atom?‡ What size of cannonball would be best if you wanted to

‡ The real story is much more complicated. An ion, of atomic mass, moves far slower than an electron *with the same K.E.* A slow-moving ion merely pushes the whole target out of shape temporarily, making an elastic collision. To be as effective as an electron the ion must have a comparable speed – consider what the target atom experiences: the electromagnetic field of a charged particle sweeping by. However since the moving ion is so much more massive than the moving electron, there are differences in the amount of recoil produced. All told: an ion must have comparable v and therefore much larger $\frac{1}{2}mv^2$ to be as effective in ionizing a target atom.

knock the head off a man? A ping-pong ball? An elephant? Put a large glass marble on the table and fire another marble at it. What happens if the missile marble is a very, very light one? The missile just bounces off. What happens if the missile marble is a huge heavy one? It pushes the target ahead.

'What size of missile marble do you want if you wish to have it come to a stop and give all its kinetic energy to the victim that it hits? What would you suggest hitting an atom with, if you wanted to knock a loose electron off? Yes, bombard it with electrons. You will see that being done in an experiment later on though the experiment is a complicated one to interpret.

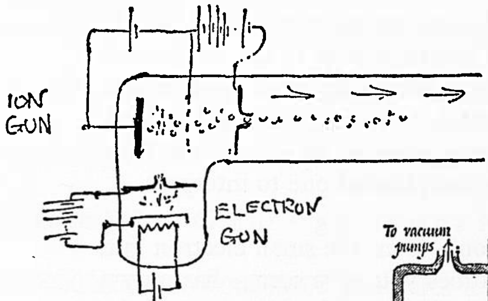
'When the bombarding electron – from the small electron gun like those in the cathode-ray tubes you have seen – has only a little energy, it makes elastic collisions. The electrons of the atom that it hits are too strongly attached to be knocked out of place by the missile, so the missile considers it is hitting a huge massive atom as a complete structure and it just bounces away like a tennis ball from a massive wall.

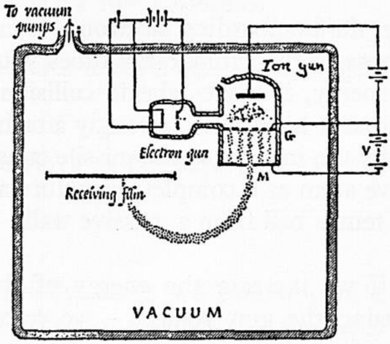
'But if we increase the energy of the missile enough – by increasing the gun voltage – we reach a stage in which the bombarding electrons are energetic enough to knock an electron off an atom or molecule of gas. Then we are left with a positively charged particle, a positive ion. That in turn can be accelerated by a voltage between a plate (positive) and a "muzzle", another plate with a hole in it (negative).

'Thus, we can make an "ion gun". A little gas is fed into the region between two plates in a vacuum. The gas is ionized by electron bombardment. A battery or power-pack connected to the two plates makes an electric field between them, which drives the positive ions to the negative plate. There is a hole in that plate – the "muzzle" of the ion gun – and ions arriving there pass through and emerge moving straight ahead. Those ions have various energies according to where in the region between the plates they were manufactured and started accelerating.

'However we can arrange to give *all* the ions that emerge from the gun muzzle the same K.E. If they all have the same mass, they all emerge with the same speed.

'If we apply a strong magnetic field (across the stream) it bends the stream into a circular orbit. We make measurements just like the ones you made for the electron stream, except that these, being positive ions of much greater mass, need a *far bigger* magnetic field.'

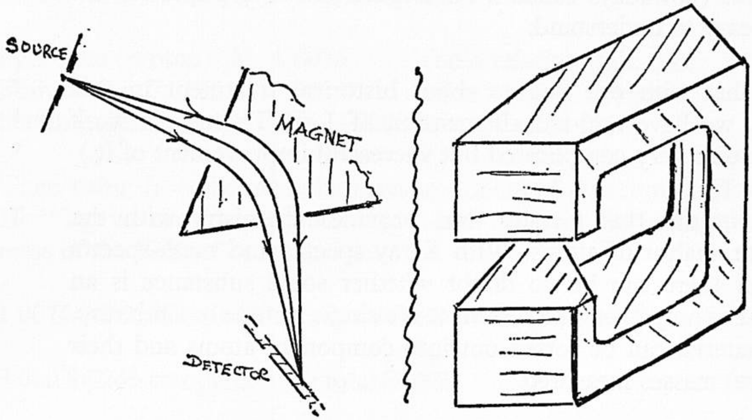




We should describe a simple modern mass spectrometer (Dempster T
or Nier model): A small sample of gas is fed into the 'ion gun' region of the apparatus – and excess gas is continually pumped away. The sample is bombarded by electrons from a small electron gun at one side, so that ions of the sample gas are manufactured. A weak electric field is applied to the region where the ions are made. That field drives them gently through a grid. They therefore arrive the other side of that grid with very little energy. There, however, they are accelerated through a much larger voltage, to emerge from the 'muzzle' of this 'ion gun' with kinetic energy which is essentially given by that main gun voltage. All the ions emerge with the same kinetic energy, so all ions of any one mass will all be bent into the same circular orbit by the strong uniform magnetic field applied perpendicular to the stream.

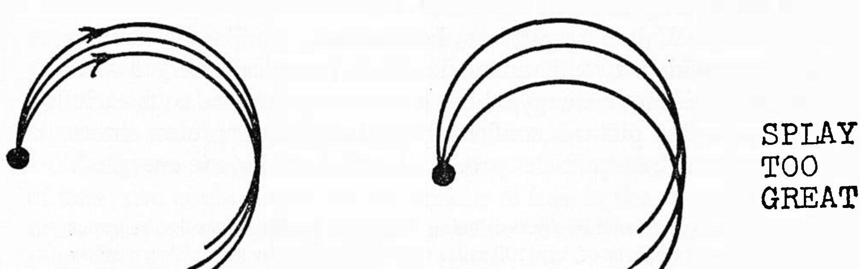
Even with a fine hole or slit at the muzzle, that emerging stream splays out through a small angle, but the circular orbits of that collection will focus sharply after a half circle, and a photographic film (or a collector for an amplifier and electrometer) will record a strong focused stream of ions of each mass at an appropriate place in that region.

At this stage we should not bother pupils with the refinement *
of focusing streams of ions – though that has always been a very *
important problem in designing machines, from earliest mass *
spectrographs to modern accelerators. *



The simpler modern instruments use '60° focusing' but '180° T
focusing' is still easier to see. So we might show 'semicircular focusing' to pupils who ask, because they can see it for themselves with a delightful pencil-and-paper experiment:

> 'Place a round object, such as a penny, on a sheet of paper. (Any round object will do. A larger one, such as a beaker or a saucer, is better.) Draw a circle by running a pencil or ball-point pen around the coin. Mark one point on the circle, to show the "starting point for a stream of ions". Shift the coin a little, making sure its edge still passes through that starting point, and draw another circle. Draw several more circles, all passing through the starting point, to show several streams of ions all splaying out from the starting point. If you have not taken too wide a splay, you will see the streams meeting again, focusing roughly, after travelling almost half a circle.'



SEMICIRCULAR FOCUSSING

In more modern forms, the magnet has a special shape and the deflection from source to focus is only 60° as in the sketch.

If we sketch a simplified picture of the general arrangement of this apparatus (nowadays called a Nier spectrometer)‡ pupils should find it easy to understand.

(Note that with our doubts about historical treatment in this matter, we have omitted all mention of J. J. Thomson's work and Aston's very complicated but successful improvement of it.) * * *

We point out that physics thus provides chemistry with the ultimate analyst of atoms. With X-ray spectra and mass-spectra together there can be no doubt whether some substance is an element or not, whether an element is a single isotope or a mixture. Any material can be sorted out into component atoms and their (relative) masses measured. T

(Yet, with electron bombardment, compounds need not be broken up into elements: we can obtain many kinds of compounds and semi-compounds (radicals) whose masses are measured by the mass spectrograph.) * * *

Furthermore, modern instruments offer such precision that we can measure the tiny *differences* of mass involved when nuclear changes convert one element into another. And from those differences we can predict the energy released or absorbed in those changes. In reverse, a study of those tiny mass-differences in cases where the energy-release has been measured enables us to confirm $E = mc^2$: a quantity E of energy, in any form, has mass m given by $E = mc^2$. T

As an example, we might quote one of the early measurements of nuclear reactions used to test $E = mc^2$:

Protons, hydrogen nuclei driven by a p.d. of 150,000 volts from a Cockcroft-Walton accelerator, bombarded a target of lithium. In some collisions two helium nuclei (alpha particles) emerged with considerable kinetic energy, about 8·5 million electron.volts each. Cloud-chamber pictures confirmed the description 'proton enters lithium: two alpha particles produced with huge kinetic energies'.

‡ Small Nier spectrometers are now being sent up in rockets to study the upper atmosphere. At a height of, say, 100 miles the lid of the apparatus is blown off by a small explosive charge and a sample of the local 'vacuum' is let in and analysed.

$$_1H_1 + {}_3Li_7 + \begin{bmatrix} 0{\cdot}15 \text{ million} \\ \text{electron.volts} \end{bmatrix} = {}_2He_4 + {}_2He_4 + \begin{bmatrix} 17 \text{ million} \\ \text{electron.volts} \end{bmatrix}$$

Mass spectrograph measurements gave the following masses for nuclei without accompanying electrons:

$$\left.\begin{array}{l} \text{hydrogen (proton) } {}_1H_1 \ 1{\cdot}0076 \\ \text{lithium } {}_3Li_7 \ 7{\cdot}0165 \\ \text{helium (alpha particle) } {}_2He_4 \ 4{\cdot}0028 \end{array}\right\} \begin{array}{l} \text{(on a relative scale that} \\ \text{takes mass of oxygen}^{16} \\ \text{isotope as } 16{\cdot}0000) \end{array}$$

Then using $E = mc^2$ and the measured mass of a proton ($1{\cdot}67 \times 10^{-27}$ kg) we can make up a balance sheet for mass, including the mass of the K.E. of each particle.

$$1{\cdot}0076 + 7{\cdot}0165 + (0{\cdot}0002) =?= 4{\cdot}0028 + 4{\cdot}0028 + (0{\cdot}0183)$$

Total 8·0243 compared with total 8·0239

Here there is a loss of *mass of matter* of 0·0185 and an estimated gain of *mass of K.E.* of 0·0181, accounting for 98 per cent of the loss of mass of matter. (The 2 per cent discrepancy is smaller than the admitted experimental error of estimates of the alpha-particles' K.E.)

Masses of Atoms: Isotopes

We can measure e/M for positive ions by using electric and magnetic fields to deflect a stream of them. We obtain just the same value for ions made from hydrogen gas that we found for hydrogen ions in electrolysis. For oxygen ions the value of e/M is 16 times smaller still, suggesting that the ions (oxygen atoms with one electron missing) are 16 times as massive as protons. Nowadays with delicate detectors we also find record of a few ions of which several have e/M 17 times smaller, telling us that there is a rare 'twin brother' of oxygen of mass 17 times the hydrogen atom mass. T

'What would you expect when chlorine gas is used? Look at its atomic mass, 35·45 compared with hydrogen. You might expect to find e/M for its ions $35\frac{1}{2}$ times smaller than the value for hydrogen ions because the atomic weight of chlorine, obtained in chemical measurements, is 35·45 compared with hydrogen 1·008. But no such value appears. Instead, we find two streams of ions; two circles made by the streams of ions in the magnetic field, one corresponding to e/M a value of 35 times smaller than hydrogen e/M the other 37 times. This tells us clearly that chlorine atoms come in two sizes, a lighter kind with atomic weight 35 and a heavier kind with atomic weight 37. But the "35" stream is nearly 5 times as plentiful as the "37" stream – that is why together they average 35·45. These, which we call isotopes, are inseparable chemically – to any chemical experiments they look as closely alike as identical twins – but one is 5 per cent more massive than the other.

'When it was discussed half a century ago, the fact that a chemical element does not have all its atoms exactly the same came as a great surprise. It was discovered by experiments like these, with positive ions; though the details of the apparatus were much more complicated in the early days of these atomic discoveries.

'And we now know that every element has two or three or many "isotopes". All the isotopes of an element have the same *chemical* properties – but in some cases one isotope shows quite different properties from another when we dig deep enough into the atom and try to make *nuclear* changes take place. In fact, that is why we do such complicated things to separate the isotopes of uranium: the lighter of the two common isotopes can split its‡ nucleus fairly easily into almost equal pieces – fission which releases an enormous store of energy – while the heavier isotope does not normally do that.

'Furthermore the heavier isotope threatens to get in the way of a fission chain reaction with a sample of the lighter isotope. So it is necessary to separate the lighter isotope from the heavier one before a fission bomb or a small-scale nuclear reactor can be made.

'That separation was mentioned when we discussed diffusion of gases in Year IV. Now you can invent another way of separating the light isotope of uranium from the heavy one. Make a guess: what could you do, if you were provided with any amount of

‡ Strictly speaking it is [the isotope + one neutron], i.e. an isotope one unit heavier, U^{236} not U^{235}, that shows fission easily.

A U^{235} nucleus absorbs a neutron easily (if it manages to come very close). When a neutron is absorbed, the energy of the new arrangement 'falling together' is sufficient to disrupt – unlatch – the U^{236} nucleus.

The more common isotope U^{238} absorbs neutrons much less easily; and when it captures a neutron it still needs a further supply of energy to bring about fission.

apparatus and a sample of mixed isotopes of uranium, to make arrangements to catch the heavy isotope in one small metal can and the light isotope in another small metal can? You could of course have anything you wanted, such as vacuum pumps, magnets, etc.'

Atomic Models

We explain to pupils that we always find the same value of e/m for T
electrons whatever source we use – hot filaments of one metal or another; photoelectric effect; bombardment of gases by other electrons; enormous electric fields tearing electrons out of cold metal and even radioactive nuclei emitting beta rays.

(This is not the moment to mention the smaller values of e/m that *
we obtain with electrons at very high speeds. That can come as a *
welcome modification that does not disturb the main story. There *
is every evidence that the electrons which have abnormally high *
mass when we see them moving very fast return to normal when *
we have slowed them down relative to the observer.) *

So we think of electrons as universal ingredients of matter, all alike, T
tiny chips of atoms, all of the same mass, and the same negative charge. Positive ions seem to be the 'rest of the atom' carrying most of its mass and having, therefore, different masses for atoms of different chemical elements. Mass spectrograph records of positive ions show a great array of different marks, for atoms of different elements and for isotopes of the same element; but electrons make a *single* mark, they are universally identical.

So we picture an atom as a round blob out of which an electron can be chipped. Therefore since matter is normally electrically neutral the rest of the blob is positive – whether a diffuse body of positive electricity like a pudding or made of knobs of positive electricity, we cannot yet tell. However we can knock more than one electron off an atom. Analysis of positive ions made by bombarding gases with electrons – 'positive rays' to use the antique word – shows that some ions have twice, three times, ... etc., the normal e/M suggesting they have multiples of the basic electron charge. Very early experiments in streams of positive ions showed that oxygen ions can have several charges and mercury ions as many as eight positive charges.

So scientists pictured atoms as a sort of pudding of positive electricity with negative electrons as plums in it. This picture was never intended to be a description of reality but just a way of remembering how atoms behave under electrical attack. An atomic model is more like a form of words, a part of our vocabulary for talking with other scientists about the behaviour we observe and the experiments we plan.

(Presently when pupils look at the fantastic scheme of spheres within spheres imagined by Greek geometers to describe the motions of the heavenly system, they may laugh at the silly ideas of medieval philosophers who thought that those 'crystal spheres' were so real that a comet passing through must smash them. If pupils laugh, we should laugh with them at the medieval philosophers who had tangled reality with their dogmatic arguments; but we should remind them that the Greeks who conceived 'theories' were exceedingly able, imaginative, thinkers who knew quite well that they were describing effective machinery – a scheme that could describe and predict successfully – and not an impossible reality. The same warning should be applied to atomic models today.)

Further Models: Nuclei? When pupils ask 'but what about the nucleus?' we should say clearly that nothing seen in experiments described so far conflicts with our picture of an atom as a pudding. As good scientists, we shall not build further details into our picture, such as the idea of a small massive nucleus, until new evidence forces us to do so.

New Theory by Necessity. That is a very important thing that we must teach all our pupils; non-scientists and scientists alike; that the great advances of theory, as in our pictures of atoms, are not just made by imaginative flights of fancy – the scientist's paint brush twirled at random – but are forced upon us by the growth of surrounding knowledge. True, our models always contain an imaginative element; but we try now – as scientists have tried for the last 300 years – to avoid unnecessary imaginative frills. Young people would like the frills; they would like to think of electrons crawling about metal surfaces like beetles – they would almost let us tell them how many legs those beetles have. They would like to think of electrons whirling round on sharply cut elliptical orbits in atoms. Young nuclear enthusiasts would like to say a neutron contains a proton and an electron inside it. They are not pleased when we express doubts and ask whether a half-crown contains two shillings and a sixpence inside. They do not welcome our scientific caution. In setting forth that caution, we should make it clear that we thereby aim at greater wisdom and fuller knowledge and are not just expressing an insecure agnosticism.

Programme

We shall return to atoms and atom-models when we have studied radioactivity. However, pupils can continue with the measurement of e/m for electrons in their practical work, and after that they may embark on preliminary work for the radioactivity experiments. (See instructions for Electrostatics and Radioactivity.)

We proceed to another use for $F = mv^2/R$*: Newton's development of planetary theory. If we just announced what Newton did, this topic would lose its main value as an example of the growth of theory; so we shall have to go back a long way in time and give some account of astronomical knowledge and its development. (Since this is not a usual part of an O-level syllabus, we shall provide a more detailed outline in a Pupils' Guide that will go into considerable detail.) The following account is only a brief summary, too brief to show the essential quality of developing knowledge.*

Chapter 3

THE GRAND THEORY

Planetary Astronomy and the Development of Theory

Each star makes an arc of $360° + 1°$ more in 24 hours of our solar time. More precisely, each star makes just one revolution more than the Sun in the course of one year.

We should illustrate that with a small celestial sphere if the laboratory has one. (It is not worth while to buy one specially since we shall not spend long with this aspect of astronomy.) We spin the sphere to show what is observed – and we avoid a spinning-Earth interpretation at this stage. An ordinary umbrella, with a few stars marked with chalk, is useful here as a simple celestial sphere. Needless to say, the real sky is best of all, however long one has to wait. D 27a

Pupils should look at the starry sky one evening, and then again later in the same evening. We should show a photo of the sky, taken with the camera shutter open for several hours. On that each star makes an arc of a circle, with the pole star almost at the centre. D 27b

Pupils who are interested should be encouraged to try making a photograph like that for themselves. They do not need an expensive camera with a large aperture. They must experiment to find the stop to use. Successful photographs should be placed on exhibit – if the picture includes the silhouette of the school building or of well-known trees nearby, it is much more impressive than a lantern slide of a photograph taken elsewhere. H 27b

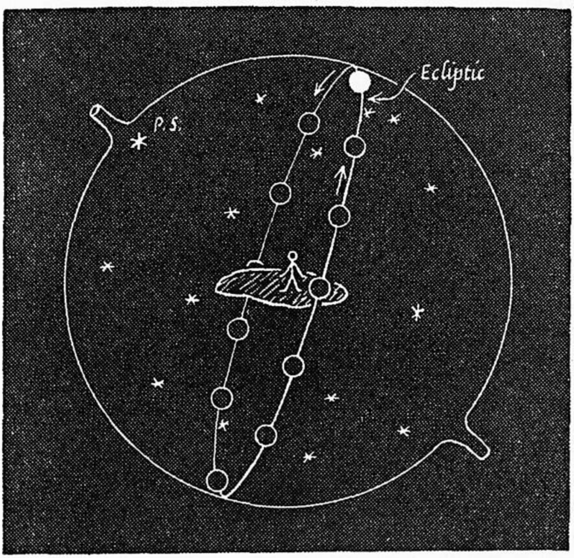
Moon. We ask pupils about the Moon: T

'On some nights you see the Moon among the stars. On other nights the stars are there but there is no Moon. The Moon must travel across the star pattern. Have you watched it do that?

'Look at the Moon one night when it is there and see where it is among the stars. Look again an hour later; and then later still. The Moon sweeps across the sky from east to west during the night with the stars, but not quite as fast as the stars. If you watch carefully, you will see the Moon lags behind the stars (like a lazy child on a walk), 90° in a week; all the way round in a month. You can see the full Moon one night (when the Sun is down below the Earth in just the opposite direction), then no Moon at all a fortnight later; and full Moon another fortnight later. Even in a single hour, the Moon moves by its own diameter, relative to the stars. H 28a

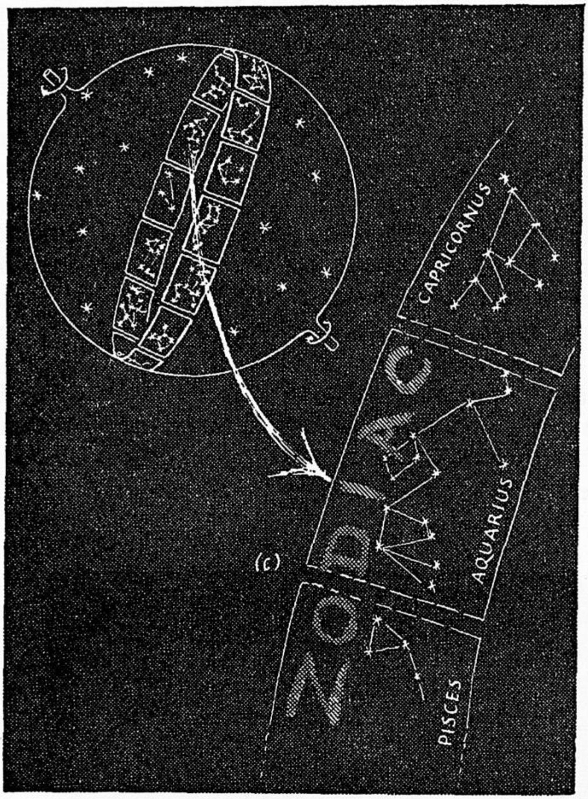
'If you watch the Moon carefully and mark its lagging path from night to night throughout the month, you will find that path is a slanting one. It does not just drift backward along the same direction as its east-to-west forward motion during the night. It drifts along a slanting line through the star patterns, close to a line that we call the "ecliptic".' H28b

Sun. At noon the Sun is always due south (or north). It makes one revolution from noon to noon (except for some minor deviations which are connected with the changing speeds of the Earth's orbital motion). But the stars make about 1° more than one revolution, so the Sun does not move quite so fast. Like the Moon, the Sun lags and does not quite keep up with the star pattern. The lagging motion of the Moon carries it right round the ecliptic circle through the star pattern in a month, but the lagging motion of the Sun is slower: 1° in a day, all the way round in a year. T



THE ECLIPTIC, the Sun's track through the star pattern in the course of a year. Here the daily motion is imagined 'frozen'.

Ecliptic. The Sun's lagging path through the star pattern is a slanting circle (not far from the Moon's path) and we call that circle the ecliptic.



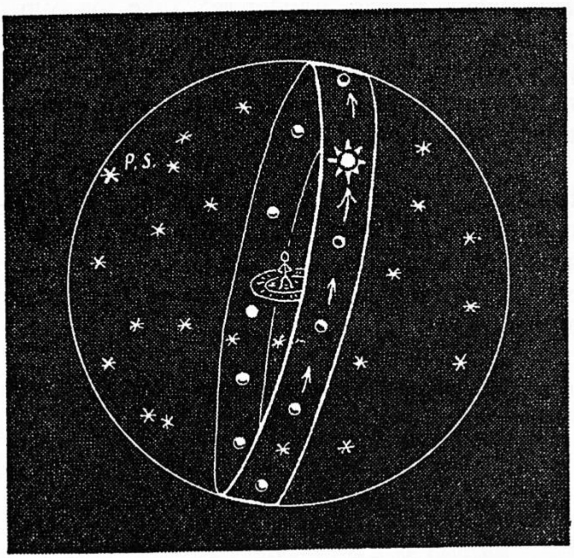
THE ZODIAC, a belt of the celestial sphere tilted $23\frac{1}{2}°$ from the equator. The Sun's yearly path (the ecliptic) runs along the middle line of this belt. The paths of Moon and planets lie within this belt. The Zodiac was divided into twelve sections named after prominent star groups or constellations. (Zodiac patterns after H. A. Rey, *The Stars*.)

Zodiac. In fact, Sun, Moon and planets (which will be important objects in our study) all follow lagging paths through the star pattern that are fairly close together. All those paths fall within a band about 15° wide, running round the whole sky with the ecliptic as its centre line. We call that band the 'Zodiac'.

Celestial Sphere. Pupils should see sketches of the celestial sphere; or, much better, an actual model if the school already has one. It is not worth while to buy one specially. Any ball, painted black, with a few lines chalked on it will suffice – since we are not trying to teach descriptive astronomy and locate constellations. The pole star should be marked and a few others added to show the general scheme – and those need not be real stars in real positions. The celestial equator should be marked. The line through the sphere from Earth (at the centre of the sphere) to pole star should D 29a be pointed out. To us, it is the axis round which the Earth spins. But to early astronomers it was just an important fixed direction from Earth to pole star, around which they saw all the stars, in fact the whole celestial sphere, revolving. Like Tycho Brahe in his youth, a keen pupil might make his own celestial sphere on a small ball, even an ordinary orange. H 29

The ecliptic path of the Sun's yearly lagging motion will be shown on that sphere; but we should add, by using tape or paint, a broader band to show the Zodiac. D 29b

'Freezing' the Daily Motion. Once we have seen the daily motion which carries every star in a circle round the pole star and found that it repeats regularly, night after night, with no change in the star pattern, it seems rather dull. The exciting things are the motions of Sun, Moon and planets; and the exciting part of the motion of each of those is its strange lagging, or wandering, through the star pattern rather than its rapid motion across the sky each day or night, trying to keep up with the rest of the stars. So, astronomers, at a very early stage of the science, started leaving out that daily motion, 'subtracting' it. In other words, they imagined the daily motion stopped, or 'frozen', and T D29c



ZODIAC BELT WITH POSITIONS OF MOON, IN VARIOUS PHASES, IN THE COURSE OF A MONTH
The daily motion of the celestial sphere is 'frozen' here.

catalogued the strange motion of Sun, Moon and planets through a stationary star pattern. That was an enormous step forward, a difficult intellectual jump, made by astronomers in early civilizations.

Sun's Yearly Motion along Ecliptic. With the daily motion frozen, we see the Sun crawling slowly backwards from west to east on a slanting circle through the star pattern, completing the circle in a year. That ecliptic circle is inclined at $23\frac{1}{2}°$ to the celestial equator: and in terms of present-day knowledge, it represents the Earth's orbit round the Sun. The Sun does not travel uniformly round the ecliptic. Its speed varies a little, so that the four seasons are not exactly equal in length. T

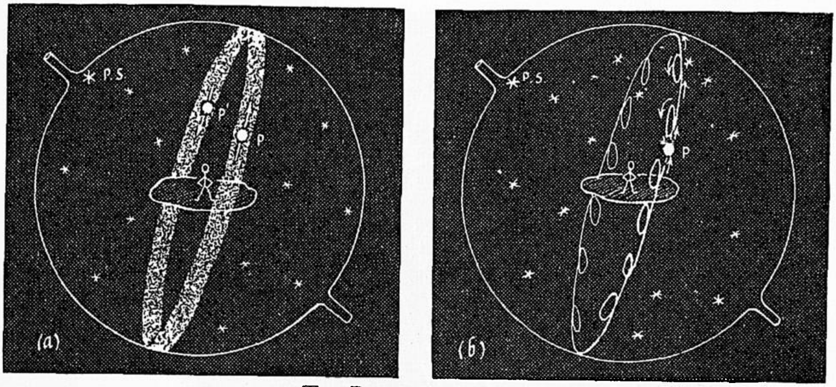
Pupils should watch the Sun and stars for themselves, and think about the changes in height of the Sun's daily arc.‡ H28c

These geometrical matters should not be laboured, or astronomy will take on a puzzling air before we are started. * *

Planets. Just a few 'stars' show entirely different behaviour. Those are the ones singled out by some of the earliest observers to be watched with care and awe. We call them 'planets', using the Greek name, which means 'wanderers'. Like the Sun, the planets sweep round with the star pattern in a daily motion. Freezing out that daily motion, we find that each planet slips slowly 'backward' from west to east through the star pattern in the course of years, along a path in the Zodiac belt. T

But, unlike the almost steady motion of the Sun round the ecliptic (or the Moon round its orbit in the Zodiac), each planet has a much more irregular motion through the star pattern. It slides backward for some time, comes to a stop, then moves forward, then continues backward again, and so on. T

‡ Pupils with a strong interest will find Lancelot Hogben's *Science for the Citizen* offers a good account.

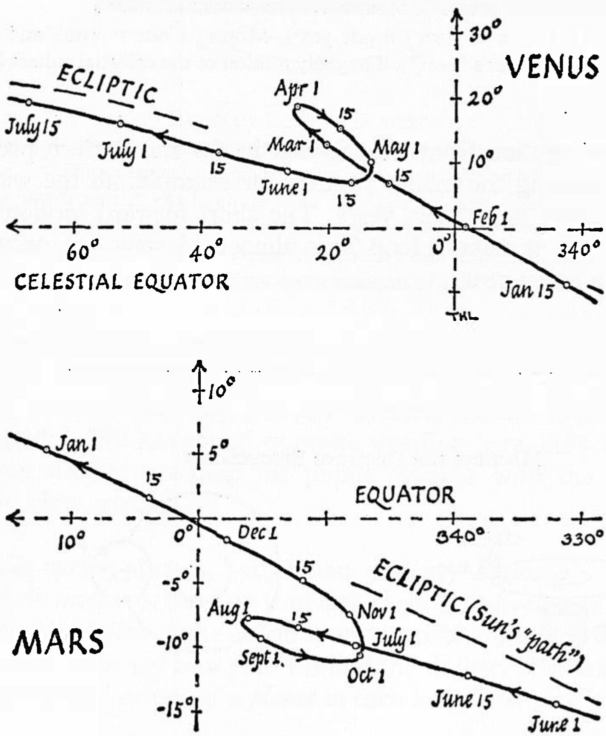


THE PATH OF A PLANET

All the planets wander through the star pattern in a belt near the ecliptic – the Zodiac belt.

a. General region of a planet's path – the Zodiac belt.

b. In detail, a planet's path has loops – an epicycloid seen almost edge-on.

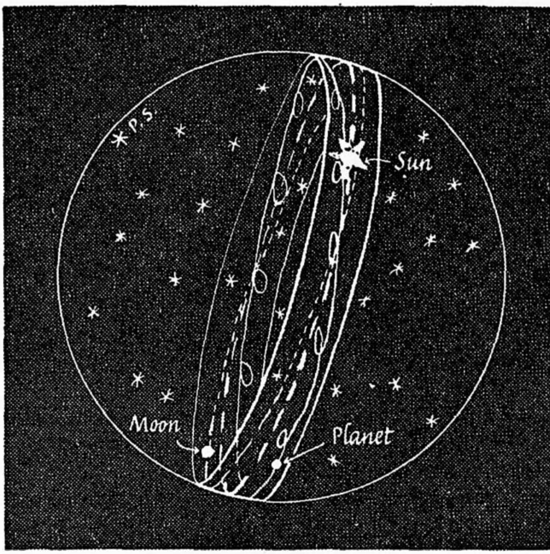


PATHS OF PLANETS THROUGH THE STAR PATTERN

a. Venus (January–July 1953)

b. Mars (June–December 1956)

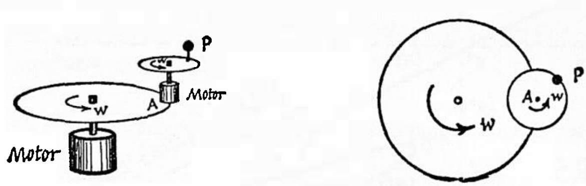
The ecliptic is the Sun's apparent path. The planets' orbits run close to the ecliptic, because the planes of those orbits are close to the plane of the Earth's orbit (or the Sun's apparent orbit, the ecliptic.)

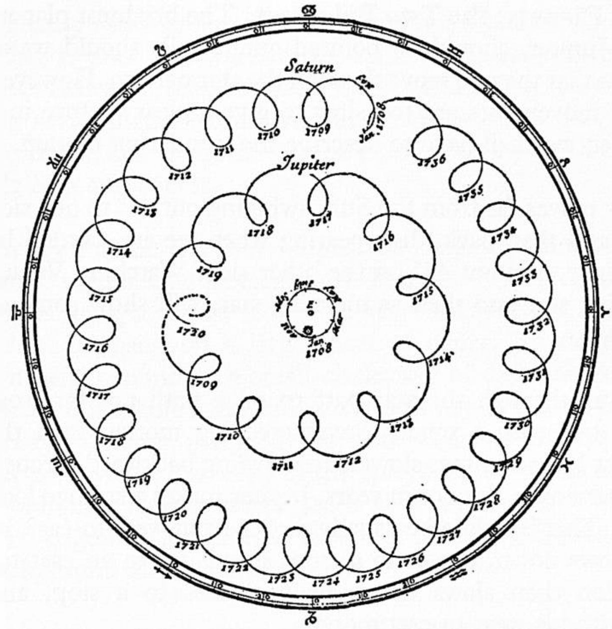


ZODIAC BELT WITH PATHS OF SUN (in one year), MOON (in one month), and a specimen PLANET (in planet's 'year'). The daily motion of the celestial sphere is 'frozen' here.

The backward motion from west to east in the star pattern predominates, carrying the planet Jupiter, for example, all the way round the Zodiac in a dozen years. The short forward motions, in which the planet makes a loop (seen almost sideways on), occur about once in every year.

MACHINE FOR DRAWING EPICYCLOIDS





PATHS OF PLANETS IN THE SKY

This sketch shows the apparent paths of Jupiter and Saturn, plotted for many years, as they would appear to an observer attached to the Earth but viewing them from *far* out from the Earth, so that the epicycles are seen face-on, without the foreshortening really observed. The apparent orbit of the Sun is also shown. The Earth is at the centre. When the astronomer Cassini constructed this diagram in 1709 he used Copernican measurements of orbit sizes.

Note to Teachers. Professional astronomers think of the general backward sliding of the planets round the Zodiac as the main, normal motion, and they call the reversed motion in the loops 'retrograde'. We have used opposite wording here, only because it seems simpler teaching for pupils familiar with the nightly motion. Here we call:

The daily motion of stars, Sun, Moon, planets, '*Forward*'
The yearly motion of the Sun round the ecliptic, '*Backward*'
The monthly motion of the Moon round its orbit, '*Backward*'
The general motion of each planet round the Zodiac, '*Backward*'
The 'retrograde' motion of a planet in each loop, '*Forward*'.

These 'wandering stars' – the planets – are the chief object of our present study. It was their motion that presented the greatest problem to astronomers who wanted to explain the heavens or 'save the phenomena', as the Greeks described their attempts to make theories that fitted the facts.

Looking at Planets: the Two Brightest. The brightest planets, Venus and Jupiter, should be pointed out. Pupils should watch them and see that they do move through the star pattern. However, the planets' movements are too slow to give a clear picture in a short time; so we shall have to describe the wandering motion. T H28d

Venus never moves far from the Sun, swinging out 46° to one side of the Sun and then back, disappearing when we are dazzled by the Sun nearby, and out 46° on the other side. Watching Venus, as the evening star and then as morning star, will show some of that story.

Jupiter moves through the star pattern on a path not far from the ecliptic but with a much slower creeping motion than the Sun's, in fact 11 or 12 times slower. In crawling backward through the star pattern once in a dozen years, Jupiter makes a strange loop once a year. That is, while he usually moves from west-to-east, he presently slows down, comes to a stop, speeds up in an east-to-west direction then slows down again, comes to a stop, and continues with his west-to-east motion.

Table of Planets. We want pupils to be familiar with the planets and their names. It would be good to post a large table, like this: D30

NAME	(*judged against a background of the stars*) TIME FOR COMPLETE ORBIT‡
Mercury	87 days
Venus	225 days
Mars	687 days
Jupiter	12 years
Saturn	30 years
Sun	365·3 days
Moon	27·3 days

‡ The period of a planet's motion, 'its time to get round its orbit', depends somewhat on our viewpoint. The value given here is the '*true*' period, or 'planetary year', as an observer on the Sun would see them. (contd. on page 97)

To the early astronomers the Earth was not a planet. They were sure that it does not wander but remains at the centre of the universe. However, they included the Sun and Moon as wanderers, making seven planets in all. In our table above, we left a space for the Earth, but it should not be placed there yet – we should include Sun and Moon. * * * * * *

Planetary Paths. To show pupils the shape of planetary path, we should sketch it on a blackboard. It is also helpful to show the pattern as an epicycloid, looked at very obliquely. We draw an epicycloid freehand on a large sheet of paper. To do that, we move a pencil round in a small circle, say of diameter 4 inches, and sweep our whole hand round more slowly in a large circle, of radius say 15 inches, using our forearm. Then we tear off a patch of the paper with a few loops of the epicycloid on it and hold that patch obliquely so that pupils see it almost edge-on. We explain that that edge-on view is what we see of planetary motion against the background of the starry pattern. D31a D31b

A toy electric motor carrying a lamp, placed near the edge of a *slow* record-player turntable, provides a useful model. D31c

As we now picture the solar system, we might draw a radius from the Sun to each planet and on out to the stars, to mark its position in its orbit. As that radius turns through 360° the planet goes once round the orbit and returns to the same place, as it would be by an observer on the Sun, against the star pattern.

However, in that 'true period' of the planet's motion round its orbit, the Earth moves to a different position, and an observer on the Earth would not see the planet back at the same place 'among the stars'. The planet's *apparent* period, recorded by an observer on the Earth, is a modified compound value derived from the planet's 'true' orbital motion and the Earth's orbital motion. If we were giving a proper account of the planetary picture seen by early observers, we should give the 'apparent' periods. Then we should have to disentangle the 'true' periods from them when we came to the Copernican picture of the solar system.

In our present teaching we want to give a simple, clear picture of the problems that led to theory rather than explore such special details. So, we suggest giving only the 'true' periods, as we have done in the table here.

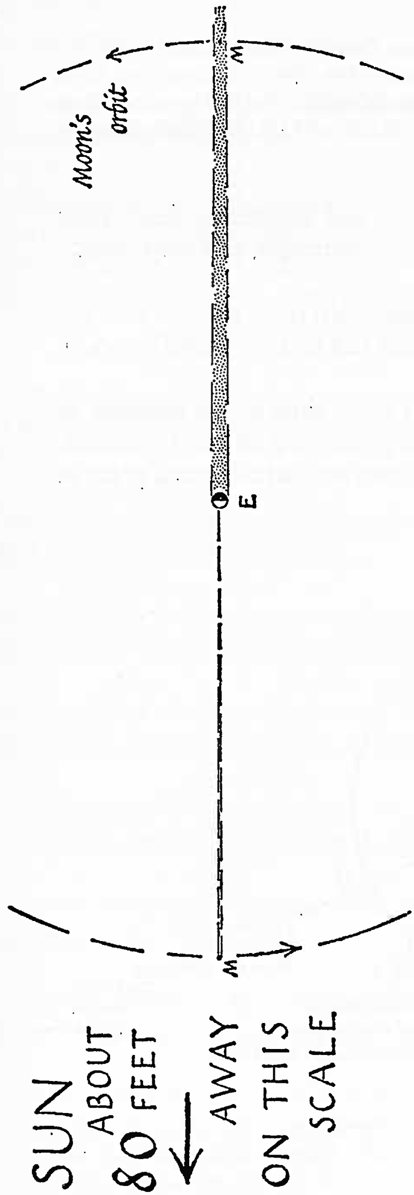
Pictures of Planets' Paths. We may show lantern slides or posters of the paths of one or two planets, plotted from tables of observations. However, those careful diagrams are likely to look confusing and dull to pupils who do not yet see any particular point in worrying about the motion of the planets. So we urge teachers to start by showing *simple* pictures, sketched by hand, as suggested above. D32

(If we were teaching Astronomy for its own sake, we could, of *
course, ask pupils to plot planetary motions in considerable detail. *
Here, however, we only wish to provide a simple, clear story as a *
basis for developing theory.) Again, if we were teaching Astronomy *
itself, from an experimental viewpoint, we should certainly give *
pupils the delightful experiment of plotting an orbit from photo- *
graphs of the Sun, taken at intervals through a year, all with the *
same camera. If the diameter of the Sun's picture is d, $1/d$ gives a *
measure of the distance of the Sun from the Earth. Given a set of *
photographs, pupils measure d, take the direction of the Sun *
among the stars from the date of each photograph – using tables or *
common sense – and plot a yearly orbit to scale. This is the Sun's *
orbit, as early astronomers considered it, or the Earth's orbit as we *
now think. However, the time and interest spent on this will lead *
the class away from our main target; so we do not suggest it as an *
experiment. However we mention it in case some have special *
interests. *

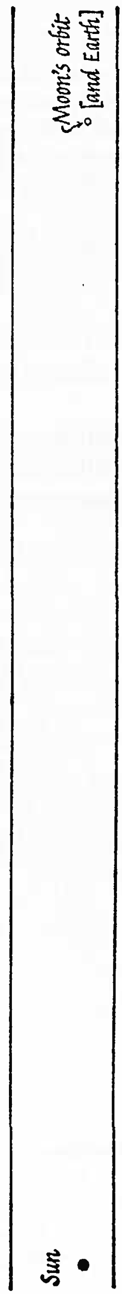
Photographs of Planets. At some stage, pupils should see good photographs of planets taken through a modern telescope. Some teachers prefer to show these straight away, especially if pupils are going out to look at planets with telescopes themselves. Others prefer to keep this close-up view of planets until they reach the story of Galileo and the telescope. D33

Eclipses. Eclipses deserve a brief mention, and explanation, but not with the usual diagrams of umbra and penumbra. Those diagrams give names to be learnt without being true enough to scale to give good knowledge.

How many capable scientists realize that the shadow of the Moon is a cone of angle only $\frac{1}{2}°$, whose very tip only just reaches the Earth? How many realize that the shadow of the Earth, which must itself have narrowed by one Moon-diameter out at the distance of the Moon, just covers $2\frac{1}{2}$ Moon-diameters as the Moon D34 passes through it in an eclipse? From that, and the $\frac{1}{2}°$ angle, subtended by the Moon, we can at once show that the Moon must be about 60 Earth-radii away, some 240,000 miles.

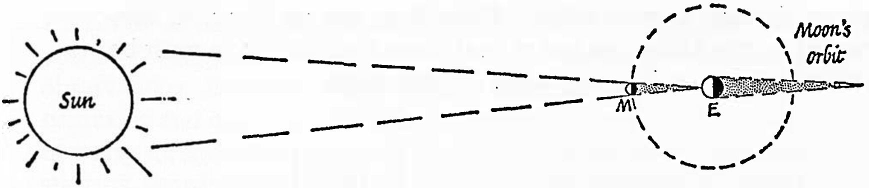


Sketch to scale showing the shadow cones of Moon and Earth.



Sun, Moon, Earth

Sketch *not* to scale. The Sun is shown much too near, and the Moon is too big and too near.

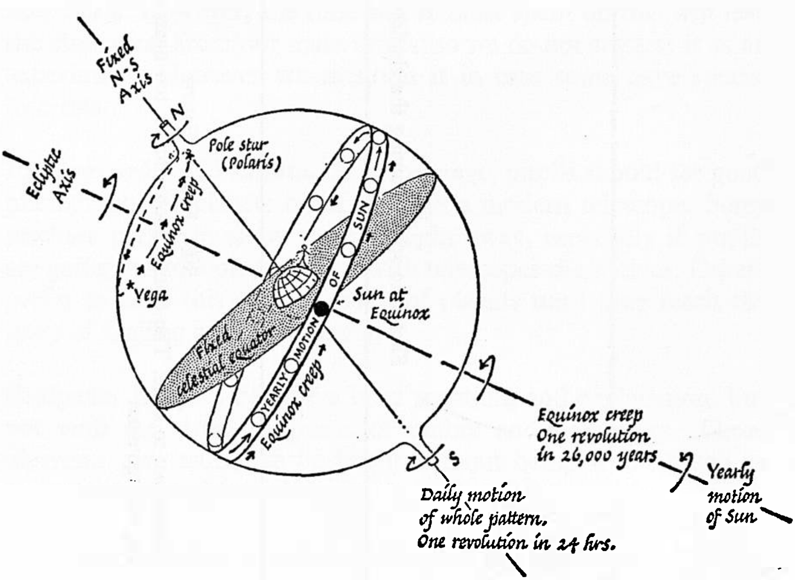


SKETCH TO SCALE SHOWING THE SHADOW CONES OF MOON AND EARTH.
Sketch to scale. Here the scale has been reduced so that Sun, Moon and Earth are in the picture. The small circle is the Moon's orbit. The Earth, at the centre of that circle, is too small to show. On this scale it is a dot 1/1,000 inch in diameter. The Moon is much too small to show.

Eclipses have always excited interest, and sometimes fear. They have helped to make astronomy seem interesting and important.

At a very early stage astronomers concluded from eclipses that the Moon shines only by reflected sunlight and that the Earth is round.

Precession of the Equinoxes. At some time in the teaching of our factual story, we must mention precession of the equinoxes. As described by early astronomers, from an Earth-centred point of



PRECESSION OF THE EQUINOXES
In addition to (*a*) the daily motion of the whole heavens around the N–S axis fixed in the fixed Earth and (*b*) the yearly motion of the Sun around its ecliptic path in the Zodiac band of stars, Hipparchus discovered (*c*) a slow rotation of the whole pattern of stars around a different axis, *the ecliptic axis* (perpendicular to the Zodiac belt).

view, that was a very obscure creeping motion of the whole system of stars around a special axis (the axis of the ecliptic); in that form it is much too difficult for pupils to understand. The sketch here is offered only in case teachers are interested in seeing the ancient description. As described by Copernicus, it is much simpler and we should describe it when we get to his model, so that Newton's 'explanation' may be enjoyed.

If the laboratory has a celestial sphere, this motion might be demonstrated with it, but it is not worth while buying a sphere for this. Use thumbs or small suction caps to establish an axis perpendicular to the ecliptic ($23\frac{1}{2}°$ from the axis through pole star). Ask a pupil to make the sphere revolve very slowly round that new axis. At the same time, the sphere must be imagined to be spinning ten million times faster about the pole star axis.

Summary

To appreciate the story we are going to unroll, pupils must know that: T

There is an unchanging pattern of stars, revolving daily round an axis through the pole star.

Sun, Moon and planets share that daily motion, except that they drift slowly 'backward' through the star pattern.

The paths of all those 'backward' motions fall in a narrow band of the star pattern, called the Zodiac.

'Freezing out' the daily motion, we find the Sun travels round the ecliptic, the central line of the Zodiac, in a year.

The Moon travels round an orbit in the Zodiac (tilted at some 5° to the ecliptic) in a month.

The planets travel round orbits in the Zodiac, making reverse loops (one for each of our years) as they do so.

Jupiter completes an orbit in a dozen years, Saturn in 30 years, Venus in a fraction of our year.

Models, Planetarium

In describing the heavenly system, teachers will want to show some kind of model.

Planetarium. A visit to a planetarium, if that can be arranged, provides a very good model, the second best in all the world. (The best, of course, would be watching the real sky for months and years.) That will show the motions as they are observed. D36a

As a rule, planetarium authorities like to show a sample picture of the heavens for some particular date, with Moon and planets making only small motions during the performance. They hesitate to run their planetary machinery very fast and show several years' motion in a short time. That will necessitate stopping ('freezing out') the daily motion, and it asks for unusual speed. Therefore they hesitate because they know how easily an audience forms misconceptions. However, if the teacher explains to the planetarium authorities the special need for an important part of our teaching programme, to show planetary motions through the stars, he should find that such a demonstration can be arranged.

A visit to a planetarium makes a strong contribution to our teaching. We hope that teachers will explain to school authorities that such a visit would not be just an amusing trip but a very important part of our physics teaching.

Toy Planetarium. Instead, or in addition, we might show a toy planetarium. If that is just a spherical shell, with holes pricked in it for stars and a lamp shining through it to make star spots on the ceiling, it will show only the simple daily motion. (Arranging other spots of light to imitate the motion of a planet round the ecliptic with yearly loops is a delightful problem for special ingenuity. But we do not recommend spending time or money on that. It is too easy to let such a model take charge and divert interest from our main objective. Sketches on the blackboard, and perhaps lantern slides of plotted paths, should suffice.) D36b

Orrery. The teacher will also want to show a model of the solar system, often called an orrery. However, that represents a jump ahead in our story, to an entirely different pattern – Copernicus' scheme, with the Sun at the centre. Showing an orrery at this stage would add confusion and raise the objection, 'But we know the Earth is not at the centre ...', all the more strongly. So, although teachers naturally think of an orrery as an easy illustration when they are first talking about planets, we urge them to keep it hidden and postpone showing it until they get to Copernicus. D36c

EARLY HISTORY

A Quick Survey. We should not spend very much time on the *
early history of astronomy; and yet if we pass it over pupils will *
miss the sense of primitive wonder and fear, and the important *
practical needs, that were driving forces behind the priest astrono- *
mers and made astronomy the first physical science. We should *
point out that astronomy provided three things that were needed *
for man's progress from savage life to village life and from village *
life to urban civilizations: calendar, clock and compass. *

Earliest Man. Very early man, living by hunting and chance T
cropping, may have looked at the stars and wondered. He may have used stars as guides at night. He may have welcomed the Moon's light for hunting. We do not know. He probably used Sun and stars unconsciously as rough clocks; but it is very unlikely that earliest man used Moon or stars for reckoning days or weeks, or even the Sun for reckoning hours; because he lived at a simple level and such things were not needed.

The First Revolution: Food Producing. When man gathered T
into villages and developed agriculture and domesticated animals – the first great revolution, from a food-collecting life to a food-producing culture – a calendar became very important. Man needed some way of predicting seasons, both for sowing wheat, one of the earliest crops, and for the seasonal breeding of sheep – or at least village life profited from a calendar to organize the work in advance.

The calendar made the priests who administered it powerful and T
important; and, whether they liked it or not, made it easy for their knowledge to become invested with mystery. Things that happened in the sky assumed obvious importance. Even to simplest man, the Sun seemed very important for warmth and growing life; a worship of the Sun developed. The Moon and stars had magical values for hunting and journeys by night. No wonder primitive people speculated about these lamps in the sky; and no wonder they worried about the few lamps which wandered, the planets.

(Teachers will find some interesting discussion of early man and *
astronomy in Lancelot Hogben's *Science for the Citizen* (4th edn., *
Allen and Unwin, 1956), and some more romantic comments in *
H. G. Wells's *Outline of History* (1920). It is both easy and tempting *
to speculate on the way in which primitive man thought about *
things as he developed. But anthropologists warn us that such *
commonsense speculation is very risky. Nor should we try to infer *

things about primitive man from observations of savages today, *
because almost all savage communities have lived close to civiliza- *
tion for a long time. Contemporary savages may be primitive in *
technology, yet maintain a complex of customs or religion de- *
veloped over millennia. It would be safer to base surmises on the *
behaviour of children today.) *

Early Civilizations. By the time the early civilizations, such as T
those in Babylon and Egypt, had grown up systematic rules had been extracted from watching and astronomical observations. The calendar priests could predict the motions of Sun and Moon, predict the seasons and even Moon eclipses, quite accurately.

In the later ages of those civilizations men had a good knowledge of the lengths of the seasons, the length of the year, etc., good weights and measures, knowledge of algebra and geometry (including Pythagoras' theorem long before his name was put to it).

Their astronomers had schemes for predicting the slightly irregular motions of the Sun and Moon along their paths through the stars. Those schemes, in the hands of the Babylonians, amounted to zig-zag empirical graphs that were used for calendar making. The astronomers who used them seemed to have no idea of giving any reason for the patterns or imagining any mechanism responsible for them. They were just working graphs, such as an engineer might sketch for the detailed running of a piece of machinery.

Astrology. With this astronomical knowlege there grew up a T
body of superstitious belief in astrology. People thought that the positions of Sun and planets at the time of someone's birth could determine his character and fate. That belief, still alive today, provided a driving force for much astronomical observing – and a source of financing for astronomers for many centuries.

Superstition; and our Duty in Teaching. We might ask our T
pupils, 'What is superstition? Can you explain in a short sentence what that word means?' There seems to be no compact answer to that question, except some dogmatic statements that lead to hot arguments.

Some teachers find this question and its discussion helps a dis- *
cussion of theory. Others consider it an unfortunate diversion. *
One should be guided by taste. *

Even today many educated people look to astrology for some guidance, if only with half-hearted belief. Why should non-scientists prefer a physicist's Newtonian treatment of the planetary system to the more romantic view embodied in astrology? If, as scientists, we believe the preference should go towards Newtonian theory, this throws light on our duty in the present suggestion of teaching of astronomy.

GREEK THEORIES AS AN INTRODUCTION TO FORM AND USE OF THEORY

It is easy enough to tell pupils what we see happening in the sky and encourage them to check some of those observations themselves. They accept the factual picture. Most pupils accept a factual picture in great detail and like to collect more facts because that gives them a sense of growing knowledge. While we should not discourage that, we should encourage a growing interest in imagining patterns, machinery, models, schemes to hold the knowledge together. That is our present objective – to look at the development of theory.

In teaching this history of astronomy as basis for Newtonian theory, teachers should describe some samples of Greek theories. And we hope they will describe them with love and admiration. Those theories were works of genius that need careful study to master the details of the machinery so that both teachers and pupils appreciate their success. However, teachers may meet a difficulty: 'But, sir, we know this isn't true. We know that the Earth ...'

But, sir . . . When we start speculating on schemes, or describing schemes that have been suggested, we meet that difficulty. Pupils have been told, much earlier, some things about the Earth and heavens. They know that the Earth moves, and not the Sun. They know quite well that the Earth spins and travels round the Sun in a year. Many of them also know that the planets go round the Sun in a circular orbit, and do not swoop back and forth in loops as they travel round the Zodiac. The teacher needs to anticipate the objections that run: 'But, sir, we know the Earth goes round the Sun.'

So a very clear explanation of our programme must be given at the outset – and it may have to be repeated – that this is a tricky problem in thinking and understanding, to see how people could build a picture that would fit the facts, and then go farther to link the facts with other knowledge, then even farther and predict some new things to look for. In a sense, we have to say to pupils:

> 'If you really want to understand, you must try making explanations yourselves; you must see how some simple explanations can be made, and then how those can be improved. So you need to be very clever and imagine that you have taken a jump backward in time to the days of the early astronomers. You see what they saw; but imagine that you have not been told explanations made up by other people since then. Think about the stories that you could tell – say, to your younger brother – if both you and he knew nothing but the things you see.'

In some such way the teacher needs to emphasize repeatedly the cleverness of imagining good schemes and not the stupidity of going back to something of which pupils say, 'We know that's wrong.' This advertising of a teaching aim can be done: its success depends on the skill the teacher brings to it and on his own enjoyment as a constructive explorer in the making of science.

In describing successive attempts at making a model of the heavens (i.e. a theory), the teacher should select examples to suit his own taste and try to set them forth quickly as well as attractively, without giving a discouraging wealth of detail. A mature physicist exploring the history of science soon becomes involved with great interest in the details of the development. It is satisfying to him to see how skilfully some theory was devised and then modified to a better one. But pupils involved in a much more rapid survey are not likely to have time or interest for the details. So teachers setting forth this development of theory in astronomy are urged to describe the stages clearly but briefly, choosing their treatment to fit their own enjoyment but not allowing the latter to prolong discussion beyond pupils' interests.

The Main Target of this Teaching. Our most important advice to teachers at this stage is, 'Keep your eye on the main target: *Newton's gravitational theory and its rich variety of explanations.*' The sooner we get there, the better, as long as pupils have on the way seen enough of the earlier stages to be ready to welcome a general theory.

The Growth of Greek Theory. As Greek civilization grew up, philosophers gathered astronomical knowledge from Egypt and from their own observations, and constructed an entirely different treatment. Instead of just stating rules for practical calendar-making and superstitious power, the Greeks imagined simple machinery that would make the whole system of the heavens seem reasonable – seem to behave sensibly, and not to be controlled by demons or spirits. These simplifying schemes, dreams of wise philosophers, were in a way the first scientific theories. T

The Greeks linked heavenly motions to pictures of earthly wheels. The early Greeks did not worry much about fitting their theory exactly to the facts; they sought the general satisfaction of having a scheme to fit the facts reasonably – to make nature seem reasonable. (They said their aim was 'to save the phenomena', and that meant 'to fit the facts'.)

Teaching Greek Theories. The notes that follow are suggestions to teachers. Some teachers will wish to go into greater detail – though there is a risk of that taking too much time. We suggest as a minimum description of the models made by:
Thales,
Pythagoras and his school,
Eudoxus,
and an amalgamation of the simpler eccentric schemes of Hipparchus and others with the scheme devised by Ptolemy.
(We are aiming for the moment at Ptolemy's scheme to show it as successful but complicated, as a prelude to Copernicus's simplification.)

Note on Models to Illustrate Greek Schemes

It is tempting to devise ingenious models to illustrate Greek schemes. If one thinks of those schemes as machinery, it seems natural to make mechanical models to show them. But our whole aim here is to show Greek schemes as clever geometry – intellectual machinery – and go quickly on towards our main target of Newtonian theory.

Showing models would take time and divert attention from ideas to machinery, from intellectual grasp to interest in mechanical ingenuity. For a very fast group, models would probably be harmless, though unnecessary. For average groups, any but a few simple models would mislead and do considerable harm by diversion. To advocate models would be to miss the point of this teaching entirely. (See the Note, in the Preface to this Year, which gives a warning about models.)

On no account should a school buy any models. We do not advise schools to construct any but the very simple models described at appropriate places below.

Until the stage of Greek astronomy, positions of planets, *etc.* had been recorded and predicted simply as marks on the pattern of stars. There were no measurements, or even guesses, of distances. By the time the Greeks had established their great university at Alexandria, astronomers were making estimates of the size of the Earth, the distance of the Moon, and even the distance of the Sun from the Earth. The planets were thought of as being at intermediate distances between the Earth and the stars; but that idea was suggested by the theoretical machinery they chose to adopt and not by any measurements. It will do serious harm to our teaching scheme, with Newtonian theory as its main target, if we do not teach something about the Greek measurements. Yet the methods used by those early astronomers were fairly simple, and a short description of them may delight many a young scientist. So we shall give an account of them, in case teachers wish to use them. Incidentally, they offer some good examination material.

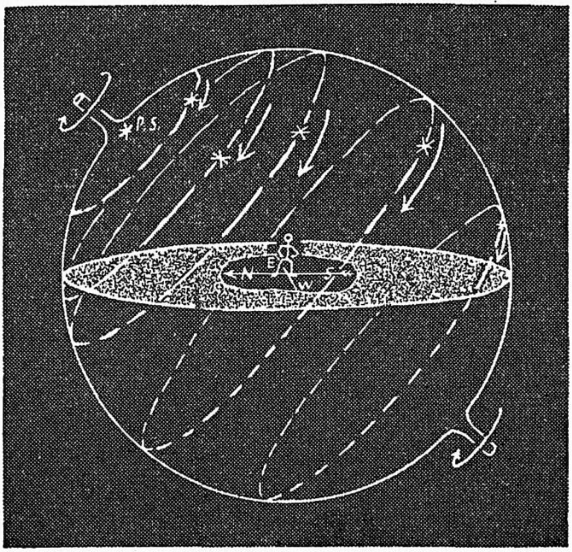
In describing theories, we should show lantern slides or blackboard sketches: or pupils should see diagrams in a guide for their own reading. D37

We should show a few very simple models. Complicated models, which have to be bought, or take time and ingenuity to construct, are not recommended – see the Note, in the Preface to this Year, which gives a warning about models.

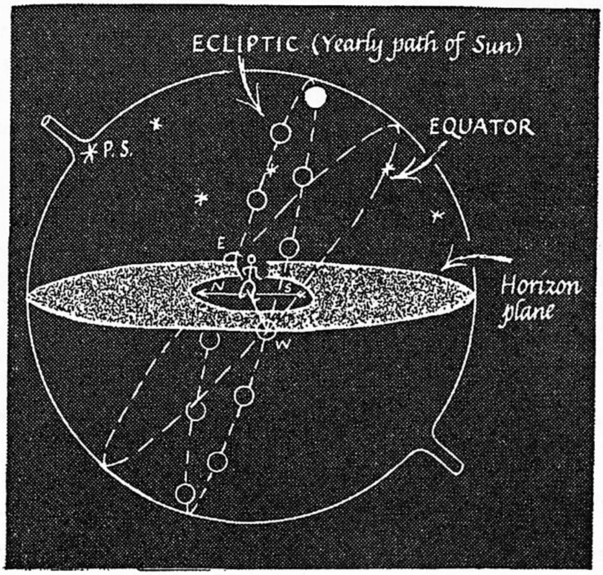
Earliest Greek Theory. Thales (600 B.C.) described a simple model: a small, flat Earth, surrounded by a great sheet of water, with a vast sphere carrying the stars and revolving daily round an axis through the pole star‡ that did account for the daily motion. T

‡ In Thales' day there was no real pole star such as we see today. Look at a star map marked to show the precession motion of the Earth's spin axis. Nowadays, the Earth's spin axis meets the celestial sphere of the star pattern very close to a bright star which we call our pole star. About 3000 B.C. the Earth's axis cut the celestial sphere very close to another bright star, alpha in the Dragon (contd. on page 110)

It left the extra motions of Sun, Moon and planets with no explanation except that they must crawl backward on the inner surface of that sphere.



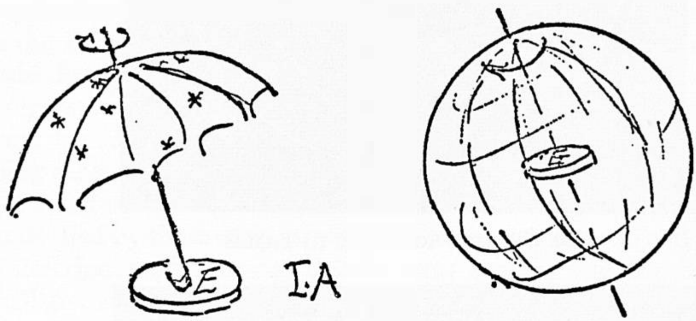
THE UNIVERSE ACCORDING TO THALES



EARLY GREEK VIEW

The Sun's yearly path through the star patterns was mapped. This is the tilted band called the ecliptic. The Sun is shown in one position (near midsummer) and other positions are sketched. Here the celestial sphere is not spinning, but 'frozen' with one star pattern overhead.

Simple Models of Early Greek Scheme. It is important to show the general idea with a concrete model; a fixed pattern of stars carried round on a sphere. An ordinary umbrella, preferably with stars chalked on it, represents the sphere of the heavens. A saucer may be held near the crook of the handle to represent a flat Earth. (For a later model a small Earth-globe might be held there – but by the round-Earth stage Greek theories had become more elaborate.) This use of an old umbrella has two advantages over a fuller model that has to be bought: (1) it avoids giving the impression that science always has to be done with special devices to prove a point or illustrate a scientific idea; (2) it enables pupils to repeat the story at home, where we hope that they too will emphasize ideas rather than gadgets.



EARLIEST GREEK SCHEME
(A) Umbrella model.
(B) Luxury model. Not worth buying.

constellation. But at intermediate times, such as the period of the great growth of Greek astronomy, there was no bright star to serve as pole star near the place where the Earth's axis cuts the celestial sphere. In teaching beginners, it is easier to describe the heavens as we see things today, with a real pole star very close to the right place. Then, in describing early theories, we have these choices:

Speak glibly of the 'pole star' (meaning a fictitious one but not saying it is fictitious) to make the picture simple.

Speak of the 'pole star' and encounter some confusion.

Use a longer description involving the Earth's axis – then some pupils will lose track.

Explain very carefully that there was no real pole star in those days, and say that when we speak of 'the pole star' we mean a fictitious one.

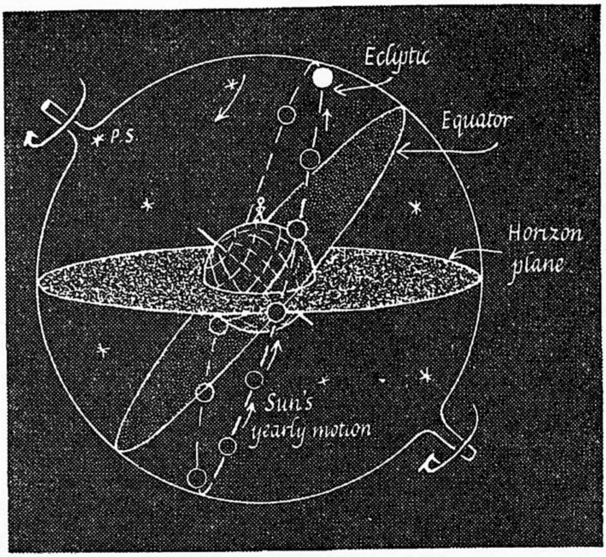
In the notes given in this Guide we have chosen the first of those methods.

A fuller model that uses a round-bottomed flask should also be shown. The flask is half-filled with water and closed with a cork that carries a knitting needle which extends through the flask inside. The flask is supported on a ring-stand with its neck slanting downward. A loose ring or washer of wood threaded on the knitting needle floats on the water inside, representing a flat Earth at the centre. Stars are marked with a chinagraph pencil on the outside of the flask. The neck is turned by hand to simulate the daily motion. After a first demonstration, the ecliptic should be marked on the outside. The Sun, represented by a small bright sticky label, should be placed on the ecliptic. Turning the neck will show the Sun's daily motion. Then the Sun is moved to successive positions on the ecliptic, the daily motion being shown for each. D38b

(A large transparent spherical shell with stars marked on it and a *
flat Earth at the centre might be shown if the school already has *
one. We do not recommend buying one, since it would teach no *
more than the simpler models, and it might divert attention from *
ideas to ingenious machinery. See the Note, in the Preface to this *
Year, which gives a warning about models.) *
*

(A sphere with an internal lamp and pinpricks for stars is *not* a *
model of this scheme: it is a planetarium to illustrate what we *
observe rather than show the Greek idea. Nor is an opaque celestial *
sphere or a celestial frame ('astrolabe') suitable here. They are not *
so much models of schemes as mapping devices for recording or *
teaching what we see.) *

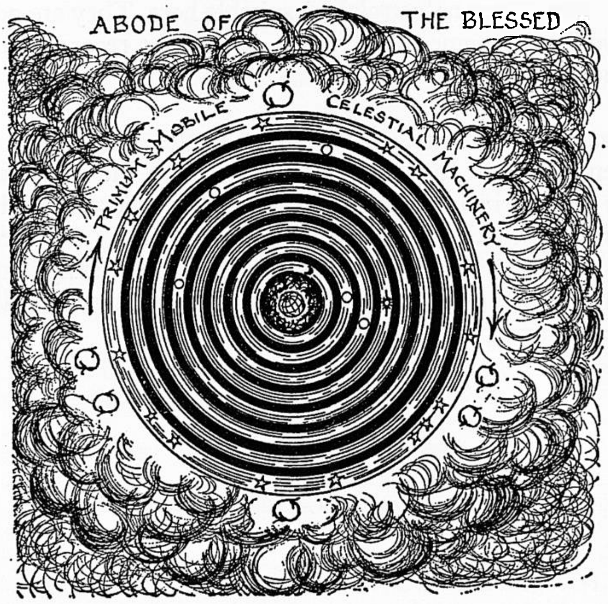
A Beginning in Natural Philosophy. Thales also made a general statement about the nature of the universe. He said that water is the 'first principle', a basic material of everything. He assumed that the whole universe could be explained by ordinary knowledge and reasoning; so we should not laugh at simplicity of his heavenly model or of his general principle – they were bold beginnings in natural philosophy. T



PYTHAGOREAN VIEW

The Pythagorean school adopted spherical Earth; and separated the general daily motion of stars, Sun, Moon, and planets, from the slow, backward motion of Sun, etc., through the star pattern.

Pythagoras and his School. Pythagoras (about 530 B.C.), and others who followed him, imagined a scheme of concentric spheres

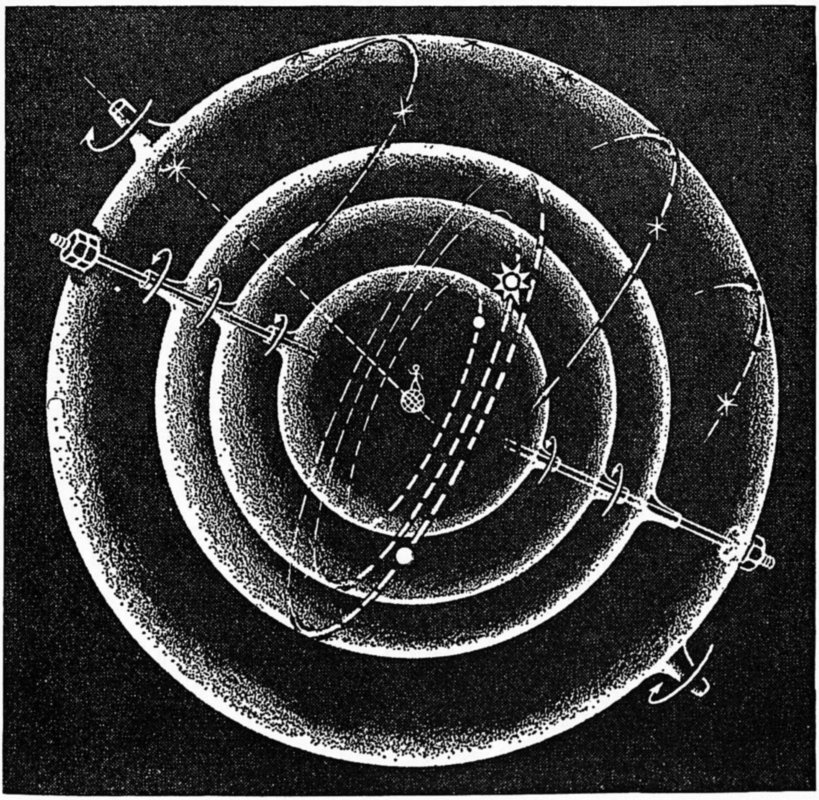


EARLY GREEK SYSTEM OF CRYSTAL SPHERES

A 'section' of the whole system in the ecliptic plane.

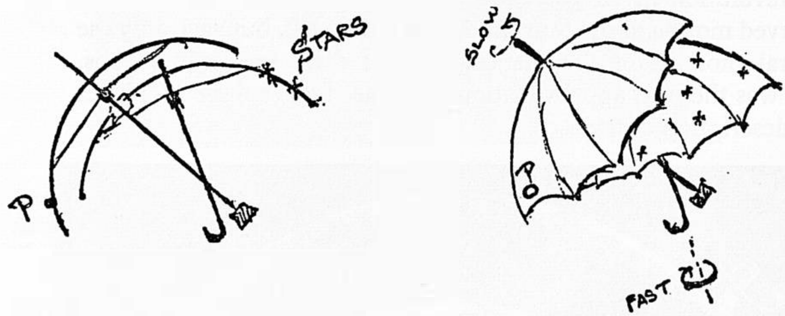
like shells of an onion. The outermost spheres carried the stars with the daily motion. Inside that were other spheres, each carrying a planet. (Remember, the Greeks counted the Sun and Moon as planets as well as Mercury, Mars, Jupiter, Venus and Saturn.)

In later stages this model of celestial spheres had all the inner spheres attached to the outermost one which carried them round with the 24-hour motion. Then the Sun's sphere revolved backward, once round in a year, about an axis perpendicular to the ecliptic ($23\frac{1}{2}°$ from the pole star axis). The spheres for Moon and other planets all revolved slowly backward about the same axis, with appropriate speeds: one revolution in a month for the Moon, one revolution in 12 years for Jupiter. This model imitated the observed motion of the Sun and Moon fairly well, but gave only the general motions of the planets without their retrograde loops. This was the next approximation after the single sphere of Thales to a description of the facts.



EARLY GREEK SYSTEM OF CRYSTAL SPHERES (Pythagoras)
Part of the system, showing the rotating spheres of the Sun and two planets, carried around by the outer sphere of stars which spins daily.

Simple Models of Pythagorean Scheme. The umbrella model D39a
should be shown again, with a second umbrella added, to carry the Sun in a yearly motion backward relative to the daily spin. One umbrella is held inside the other with its axis (handle) tilted to show the $23\frac{1}{2}°$ difference. The inner one carries the Sun (or one planet) on its rim, revolving slowly from west to east round the ecliptic. The outer umbrella may have its handle transferred from inside to outside where it will continue the spike. It carries the stars and carries the inner umbrella with it in its daily spin. (By the time this point is made clear, the model becomes impossibly difficult to manipulate – so much the better: thinking then takes charge.)



PYTHAGOREAN SCHEME
Illustration with two umbrellas. The inner umbrella has its spike cut off short, and a hole cut in its fabric for the handle of the outer one, which carries a planet.

The flask model suggested for the simple Greek scheme should D39b
also be shown now, with the flat wooden washer replaced by a bead or ball to represent a round Earth. As before, one position of the Sun in the ecliptic is marked by affixing a small bright sticky label and the daily motion shown by turning the neck of the flask. Then the Sun is moved to a neighbouring position in the ecliptic and the daily motion shown again; and so on.

(Models with two or more hoops instead of spheres fail to make the *
motions clearer: they should not be shown.) *

(More professional models are, of course, possible, such as a large *
transparent sphere (stars), with a smaller sphere inside having its *
axle embedded in the large sphere, at $23\frac{1}{2}°$ to the axle of the latter, *
with a small spherical Earth at the centre. This requires an *
electric motor or gears or both. If the school already has such a *

model it should be shown. But we recommend strongly against buying one or constructing one. See the Note, in the Preface to this Year, which gives a warning about models.)

The Crystal Spheres. We should not laugh at this scheme of crystal spheres. It was far from childish or stupid; it was a brave attempt to give a simple scheme, to 'explain' the heavens to ordinary people. It was over-simple, in that it failed to show the retrograde loops in the planets' motion and it failed to show the irregularities in the Sun's motion from season to season.

This scheme of spheres was like a tale for children, saying, 'It is all reasonable; there is machinery which carries the stars and planets round; it fits together and runs with simple rules; there is nothing to fear.' But the scheme gave no hint of the way in which the motions were started or maintained.

Outside the outermost starry sphere was the Primum Mobile, the celestial powerhouse, which was also said to be the 'abode of the blessed'. The original astronomers may have just imagined the outermost starry sphere as a theoretical device for describing the motion; but it soon took on an air of solid reality. Heaven for departed souls was clearly beyond that sphere. This picture of heaven became so well established that the Copernican view, which did not need a sphere for the stars, but placed them at all kinds of distances in remote space, was met with violent dismay and opposition.

Constancies: the Essence of Scientific Description. The Greeks insisted on spheres for their machinery and made those spheres revolve at constant speed. Those were not whimsical assumptions made for artistic delight. Some such assumptions are *essential* for a scientific description, which is what the Greeks were aiming at.

When we want to describe some behaviour in nature in the compact way that scientists like, we have to extract some constancies. Each scientific law that we state (usually derived from experiment) is really a statement about something that remains constant, independent of some other changes or details. Thus, in building science, we try to single out things that are constant. (Pressure) times (volume) is constant in Boyle's Law. (Stress)/(strain) is constant in Hooke's Law. If we could not make use of such constancies in our descriptions, we should go mad with the profusion of irregular details. One might almost claim that every natural law can be stated with the word 'constant' in it somehow.

The Greeks had to express their knowledge of heavenly motions in statements that contain some constant elements – otherwise they might just as well have ascribed the motions to wayward gods or demons. A sphere has *constant radius*, the same in all directions; and that gave it a great advantage in the Greek view, as part of the machinery. And each sphere was given a *constant speed* of spinning – again, without that constancy, the description would hardly make nature seem reasonable or easy to understand. As the descriptions were elaborated to fit the facts more closely, the Greeks would add more spheres within spheres, each with its own constant motion, rather than lose that essential characteristic. Later, when the profusion of spheres itself lost the attractiveness of simplicity, Greek astronomers modified their insistence on constant speed but installed other constancies instead.

Pythagoras and the Round Earth. Pythagoras' pupils, if not the great man himself, knew that the Earth is round. The time was ripe for the idea of a round Earth. Travellers' tales of ships and stars suggested a curved Earth to an enquiring mind.

Aristotle, two centuries later, supported schemes of concentric spinning spheres with a dogmatic reason: 'The sphere is the perfect solid shape.' By the same token, the Sun, Moon and planets must be spherical in form; so that the heavens are regions of perfection, or unchangeable order among spheres moving with constant motions. To Aristotle, the space between Moon and Earth was unsettled and changeable, with downward fall the natural motion.

Aristotle made a strong case for the Earth itself being round. He gave theoretical reasons:

1. Symmetry: a sphere is symmetrical, perfect.

2. Pressure: the Earth's component pieces, falling naturally towards the centre, would press into a round form.

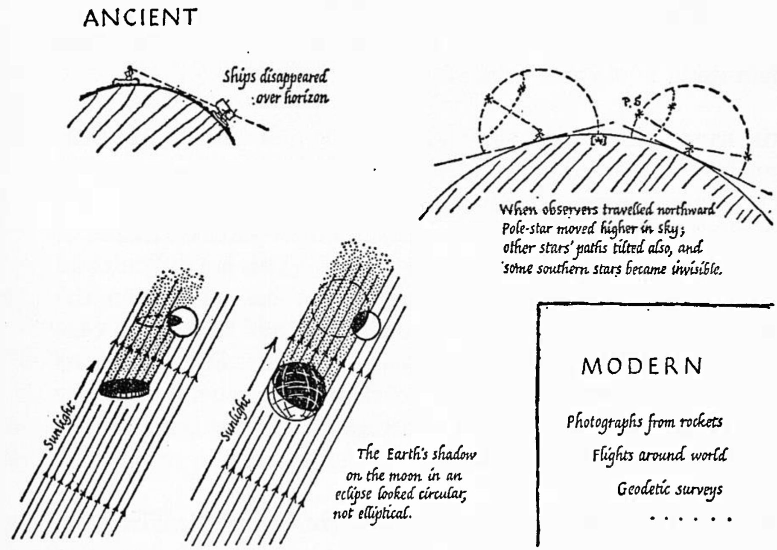
and experimental reasons:

3. Shadow: in an eclipse of the Moon, the Earth's shadow is always circular: a flat disc could cast an oval shadow.

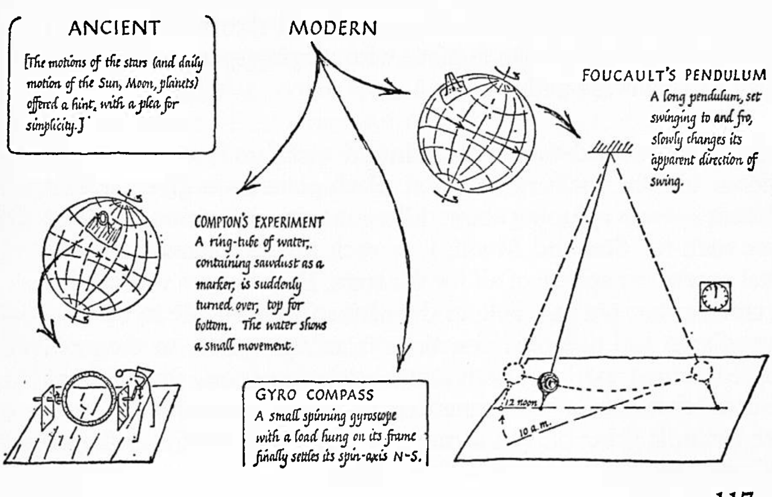
4. Star heights: even in short travels northward or southward, one sees a change in the elevation of the star pattern.

This mixture of dogmatic 'reasons' and experimental common sense was typical of him, and he did much to set science on its feet. His whole teaching was a remarkable life work of vast range and enormous influence. At one extreme he catalogued scientific information and listed stimulating questions; at the other extreme he emphasized the basic problems of scientific philosophy, distinguishing between the true physical causes of things and imaginary schemes to save the phenomena.

EVIDENCES FOR ROUND EARTH



EVIDENCES FOR SPINNING EARTH



Teachers may like to post up a large chart showing evidence for a round Earth, like the diagram here. Soon, they may like to post up a companion chart showing evidence for a spinning Earth. These may promote discussion since they deal with things that pupils take for granted. D 40a D 40b

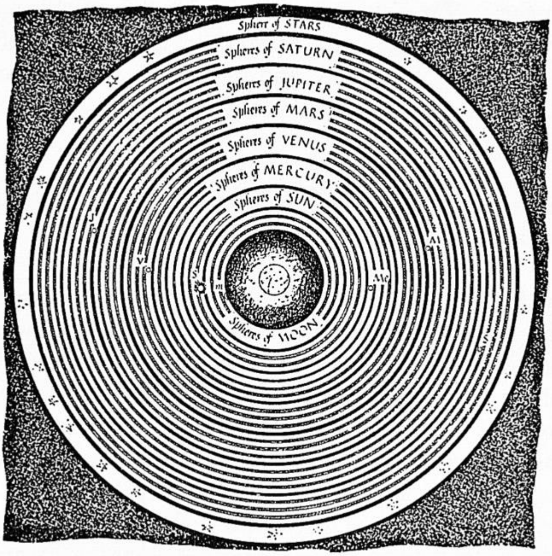
It is not necessary to discuss the flat Earth–round Earth controversy in teaching towards our main target; but that does offer a good topic for questioning pupils' assurance. We ask:

> 'How do you know the Earth is round? How would you convince your younger brother, or a savage, that the Earth is round, other than using quotations from some book?'

In the hands of a teacher who likes running a discussion like that, it can be of considerable use. A much harder discussion begins, 'How do you know the Earth spins?' In other words, 'How do you know the Copernican scheme is right, and these Greek schemes of spheres round a stationary Earth are wrong?' (This is a difficult and dangerous question. Copernicus argued for his scheme on the basis of simplicity. Scientists in the eighteenth and nineteenth centuries argued on the basis of Newtonian mechanics. Some scientists in the twentieth century are apt to claim that general relativity would expect the same effects whether the Earth spins or the stars whirl round the Earth.)

Eudoxus' Scheme of many Concentric Spheres. The great Greek mathematician, Eudoxus, devised a tremendous system of spheres to match the facts very closely. The simple system of a few spheres, one for each moving body, was obviously inadequate. A planet does not move steadily along a circle among the stars. It moves faster and slower, and even stops and moves backward at intervals. The Sun and Moon move with varying speeds along their yearly and monthly paths. T

Eudoxus elaborated that scheme into a vast family of concentric spheres, like the shells of an onion. Each planet was given several adjacent spheres spinning about different axes, one within the next: three each for Sun and Moon, four each for the planets; and the usual outermost sphere of all for the stars. Each sphere was carried on an axle that ran in a hole in the next sphere outside it, and the axes of spin had different directions from one sphere to the next. The combined motions, with suitably chosen spins, imitated the observed facts. Here was a system that was simple in form (spheres) with a simple principle (uniform spins), adjustable to fit the facts – by introducing more spheres if necessary.



EUDOXUS' SCHEME OF MANY CONCENTRIC SPHERES
Each body, Sun, Moon or planet, had several spheres spinning steadily around different axes. The combination of these motions succeeded in imitating the actual motions of Sun, Moon and even planets across the star pattern.

Models of Eudoxus' Scheme? No physical models should be shown – except perhaps an onion to illustrate the structure. D41

Refer to the shells of an onion, and use pictures. Proceed quickly, since appreciation of cleverness and complexity is sought rather than knowledge of details.

In explaining the details of Eudoxus' machinery (described in a later paragraph below) anything more than a sketch of the four spheres for one planet would defeat its own ends. The B.B.C. constructed a brilliant model with four hoops for its 'How and Why' programme. That even shows the inner pair of a planet's quartet of spheres producing the loops in the path. Teachers with special interests might borrow the B.B.C. model, or a 20-second film of it in action. We do not recommend constructing one; and certainly not buying one. * * * * * * * * *

Simplicity in Theory. To make a good theory, we must have basic principles or assumptions that are simple; and we must be able to derive from them a scheme that fits the facts reasonably closely. Both the usefulness of a theory and our aesthetic delight in it depend on the simplicity of the principles as well as on the * * * * * close fitting to facts. We also expect fruitfulness in making predictions, but that often comes with these two virtues of simplicity and accuracy. To the Greek mind, and to many a scientific mind today, a good theory is a simple one that can save all the phenomena with precision.

Nowadays we also expect a good theory to provide *language* to facilitate interchange and growth of understanding.

Is the Theory True? Questions to ask, in judging a good theory, are, 'Is it as simple as possible?' and 'Does it save the phenomena as closely as possible?' If we also ask, 'Is it true?' that is not quite the right requirement. We could give a remarkably true story of a planet's motion by just reciting its locations from day to day through the last 100 years; our account would be true, but so far from simple, and so spineless, that we should call it just a list, not a theory.‡ The earlier Greek pictures with real crystal spheres had been like myths or tales for children – simple teaching from wise men for simple people. But Eudoxus tried to devise a successful machine that would express the actual motions and predict their future. He probably considered his spheres geometrical constructions, not real globes, so he had no difficulty in imagining several dozen of them spinning smoothly within each other. He gave no mechanism for maintaining the spins – one might picture them as driven by gods or merely imagined by mathematicians.

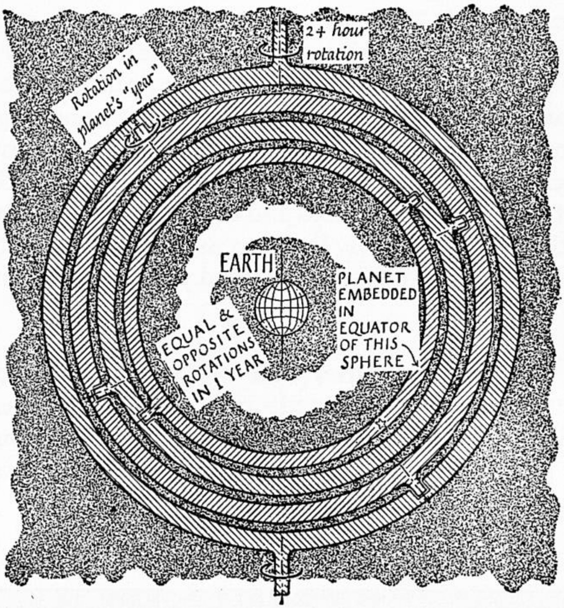
Unless teachers or pupils have special interests, we do not suggest giving a detailed account of Eudoxus' machinery, because it would take some time and is quite difficult. For those who would like to know about it, here is a short account.

Details of Eudoxus' Scheme. Here is how Eudoxus accounted for the motion of a planet, with four spheres. The planet itself is carried by the innermost, embedded at some place on the equator.

The outermost of the four spins round a north–south axle once in 24 hours, to account for the planet's daily motion in common with the stars.

The next inner sphere spins with its axle pivoted in the outermost sphere and tilted $23\frac{1}{2}°$ from the N–S direction, so that its equator is the ecliptic path of the Sun and planets. This sphere revolves in the planet's own 'year' (the time the planet takes to travel round

‡ Young scientists are urged, nowadays, not to be satisfied with just collecting specimens, or facts or formulae, lest they get stuck at the pre-Greek stage.



PART OF EUDOXUS' SCHEME: FOUR SPHERES TO IMITATE THE MOTION OF A PLANET
The sketch shows machinery for one planet. The outermost sphere spins once in twenty-four hours. The next inner sphere rotates once in the planet's 'year'. The two innermost spheres spin with equal and opposite motions, once in our year, to produce the planet's epicycloid loops.

the Zodiac), so its motion accounts for the planet's general motion through the star pattern.‡ These two spheres are equivalent to two spheres of the simple system, the outermost sphere of stars that carried all the inner ones with it, and the planet's own sphere. * * * * *

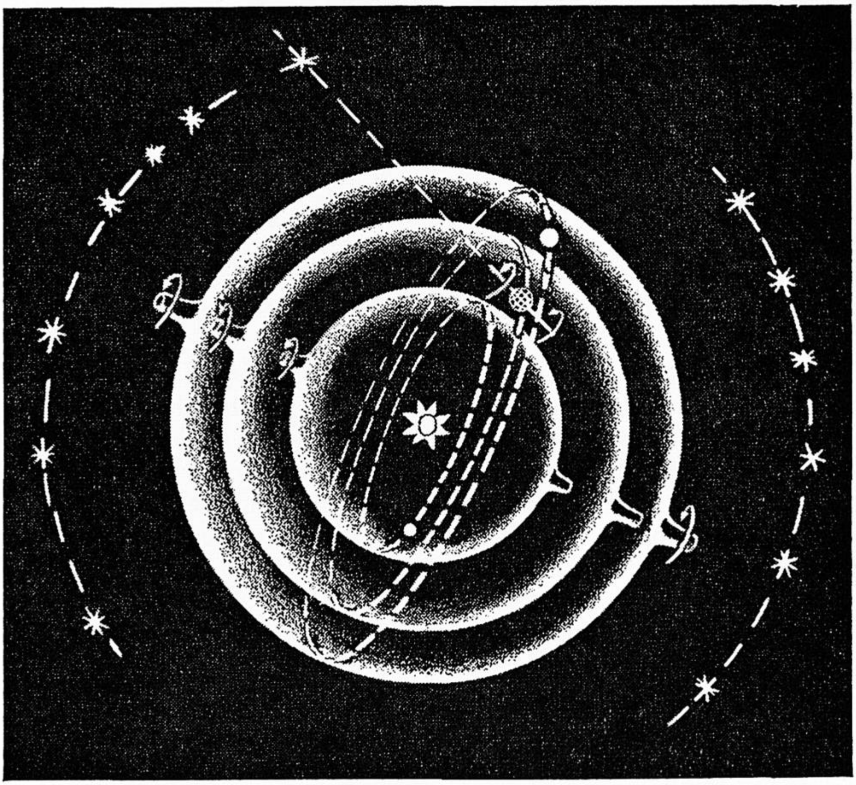
The third and fourth spheres have equal and opposite spins about axes inclined at a small angle to each other. The third sphere has its axle pivoted in the Zodiac of the second, and the fourth carries the planet itself embedded in the equator. Their motions combine to add the irregular motion of stopping and backing to make the planet follow a looped path. The complete picture of this three-dimensional motion is difficult to visualize.‡ * * * * * * *

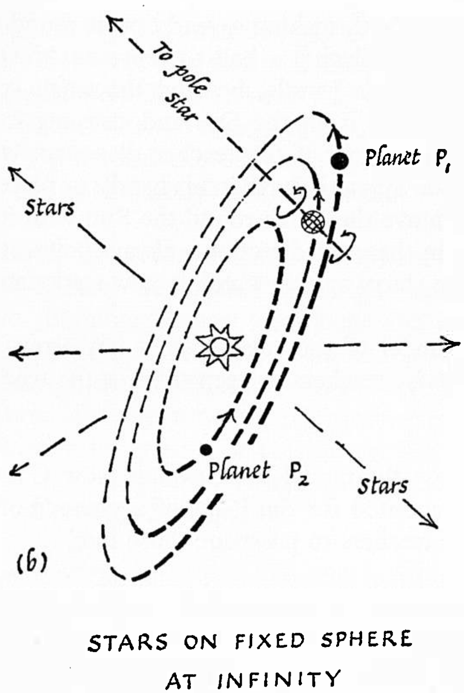
‡ In terms of our view today, the spin of the outermost sphere corresponds to the Earth's daily rotation; the spin of the next sphere corresponds to the planet's own motion along its orbit round the Sun; the spins of the other two spheres combine to show the effect of viewing from the Earth which moves yearly around the Sun.

‡ Pupils keen on horses may be amused to hear the Greeks' description of the motion given to the planet by the innermost pair of spheres: the figure-of-eight motion of a pony weaving a 'bending' exercise round two posts.

A Good Theory. With 27 spheres in all, Eudoxus had a system that imitated the observed motions quite well: he could save the phenomena. The basis of his scheme was simple: perfect spheres, all with the same centre at the Earth, spinning with unchanging speeds. The mathematical work was far from simple: a masterpiece of geometry to work out the effect of four spinning motions for each planet and choose the speeds and axes so that the resultant motion fitted the facts. In a sense, Eudoxus used harmonic analysis – in a three-dimensional form! – two thousand years before Fourier. It was a good theory. T

An Entirely Different Scheme: Sun-in-Centre. Aristarchus and a few other astronomers suggested a radically different scheme – a spinning Earth to account for the daily motion, and a stationary Sun, with the planets and the Earth travelling round it in circular orbits. T





ARISTARCHUS' SCHEME

Only two specimen planets are shown. P_1 might be Mars, Jupiter or Saturn. P_2 might be Mercury or Venus. (*a*) View of spheres. (*b*) Skeleton scheme showing planetary orbits.

Models of Aristarchus' Scheme. The contrast may be shown by a simple model with an umbrella marked with a few stars. A tiny Earth-globe is held near the crook of the umbrella's handle, at the centre of that star sphere. Spinning the umbrella, with the Earth held still, shows the earlier scheme. Spinning the Earth instead shows one aspect of Aristarchus' scheme: the daily motion made by a spinning Earth. D42

ARISTARCHUS' SCHEME

Umbrella with stars to show a spinning Earth equivalent to spinning star sphere.

The other aspect – the Earth making a yearly orbit round a fixed Sun – may be shown by placing a ball to represent the Sun at the crook of the umbrella's handle, holding the small spinning Earth a small distance out from the Sun and carrying it slowly round the Sun. That is easier if the teacher dispenses with the umbrella and just holds Sun and Earth in his hands, or places them on a table. He must move the Earth round the Sun with its spin-axis always pointing in the same direction – always pointing to the same pole star on the starry sphere which is now stationary.

(A more elaborate model of the solar system (an orrery) could be shown; but we advise teachers to keep that for the teaching of Copernicus.)

(We could show, now, the models that explain how Copernicus (and Aristarchus) accounted for the loops of a planet's observed motion; but we urge teachers to postpone those too.)

(A small planetarium could be constructed – a W.C. float with a small lamp inside and holes pricked for stars; or a small projection lantern with a slide of stars, on a turntable or with a rotating mirror. That could be used to show the contrast; but there is danger of confusion since the planetarium simply shows what we see, the same whatever the explanatory scheme.)

Aristarchus' Scheme Unpopular. This simple scheme, which T
we now teach as true, failed to attract support. To the Greeks it was unacceptable because:

1. It would displace the Earth from its obvious, important position, Man at the centre of the Universe.

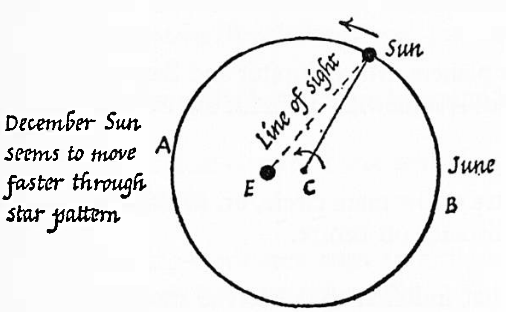
2. It involved motions that seemed impossible. Objects would be flung off a spinning Earth, or left behind – when the clouds and everything else obviously stay with the Earth. An Earth hurtling along a vast orbit round the Sun would certainly leave things behind, in hopeless contradiction to the observed facts.

3. The Earth moving round such a yearly orbit would travel nearer to some stars and then farther away in the course of a year; so the starry patterns should change their apparent proportions and shapes by foreshortening, or by parallax motions. The D43
teacher should illustrate this by walking across the room, looking at the seated class as he goes, and commenting on the changes of the pattern of pupils that he sees as he moves nearer or farther

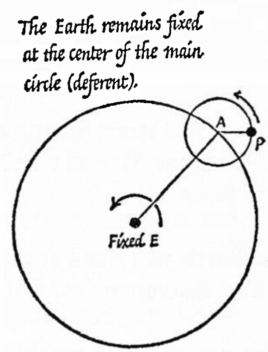
away from a group. No such changes in the starry patterns were observed. (Nor were any observed until the last century when tiny parallaxes were measured and told us that stars though not infinitely far away are so remote in comparison with the size of the Earth's orbit that the Greeks had no chance of observing anything but an 'infinitely' remote heaven.) * * * * *

Simpler Earth-in-Centre Schemes: 'Wheels within Wheels' and an Off-centre Viewpoint. Although Eudoxus' model was successful in summing up past knowledge and predicting future positions fairly well, it had developed the simple set of spheres into a complex arrangement which lacked the full appeal of simplicity as an explanation. By the time of Hipparchus (about 140 B.C.), astronomers were using simpler machinery. We shall describe it with circles instead of spheres, though the Greek delight in spheres survived in the original descriptions. T

For the motion of the stars, there was the usual outermost sphere revolving daily. For the yearly motion of the Sun, a radial arm carried the Sun round a circular orbit at constant speed. To imitate the slight variations in the Sun's motion, which make our seasons uneven, the Earth was placed a little way off centre so that observers on the Earth are nearer the Sun in our winter and therefore see it moving faster along its ecliptic path than in summer. This eccentric placing of the Earth was chosen to fit the facts. A similar scheme did fairly well for the Moon, though the eccentric shift for the Moon's circle was not in the same direction as the shift for the Sun's circle.

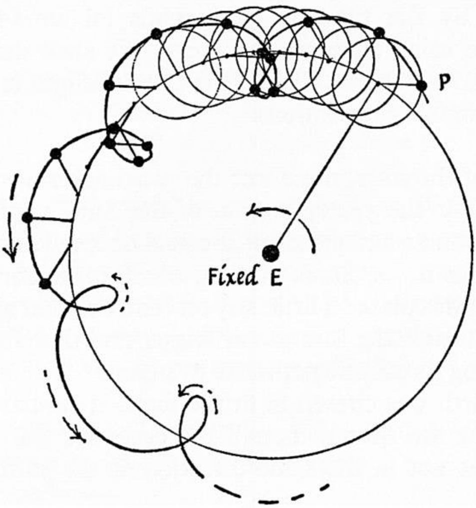


THE ECCENTRIC SCHEME FOR THE SUN
The Sun is carried around a circular path by a radius that rotates at constant speed, as in the simplest system of spheres. The observer, on the Earth, is off-center, so that he sees the Sun move unevenly—as it does—faster in December, slower in June.



EPICYCLE SCHEME

For a planet the machinery consisted of a big circle and a small circle (epicycle) carried round the circumference of the big circle. A radius of the big circle revolved at constant speed, making one revolution in the planet's own 'year', e.g. twelve of our years for Jupiter. The end of that radius carried the small sub-circle, whose radius revolved once in each of our years and carried the planet at its end. Combining these two motions we obtain a looped epicycloid. An observer looking at that motion from the Earth near the centre of the main circle would see the epicycloid very obliquely and the scheme would imitate the observed motion of a planet very well.



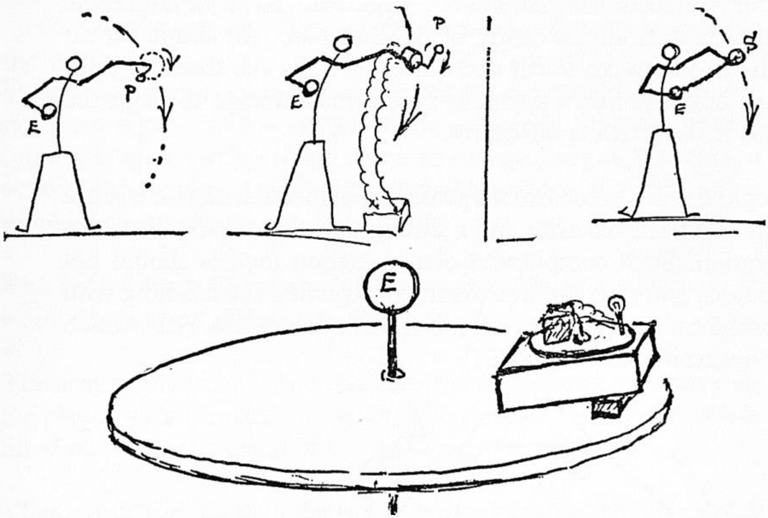
MAKING THE PATH OF A PLANET BY THE EPICYCLE SCHEME
This sketch shows how the two circular motions combine to produce the epicycloid pattern that is observed for a planet.

(That is the story for the outer planets, Mars, Jupiter and Saturn. The story for Mercury and Venus is somewhat different but has the same essence.)

The Earth was fixed at the centre of the main circle, or, to explain further discrepancies, a short distance off centre.

Here was simpler machinery that imitated the observed motions more closely, and preserved some essential constancies: circles of constant radius, arms rotating at constant speed.

Simple Models for Epicycle and Eccentric Scheme. This *is* simpler machinery and should be demonstrated simply. D44



Simple model for epicycle scheme. 'Planet' held in hand moves fast round small circle, while hand sweeps slowly round large arc whose centre is fixed 'Earth'.
a. Hand simply revolves round wrist.
b. Hand carries electric motor, or clockwork, to make planet revolve.
Simple model for eccentric scheme for 'Sun'. Hand carries 'Sun' round large arc. 'Earth' held fixed, a small distance off centre of arc. Elastic thread joining E and S shows speed changing with seasons.
Model of epicycloid scheme using a ball on a record-player on large turntable. The ball may be replaced by a small lamp lit by battery.

The teacher uses his own arms. He holds a large ball in one hand to represent the fixed Earth. In the other hand he holds a small ball, representing a planet, and sweeps his outstretched arm slowly round an arc whose centre is at the fixed Earth. The small, fast, circular motion (for the 'epicycle') is made by:

a. revolving the hand, flexibly, round the wrist, or

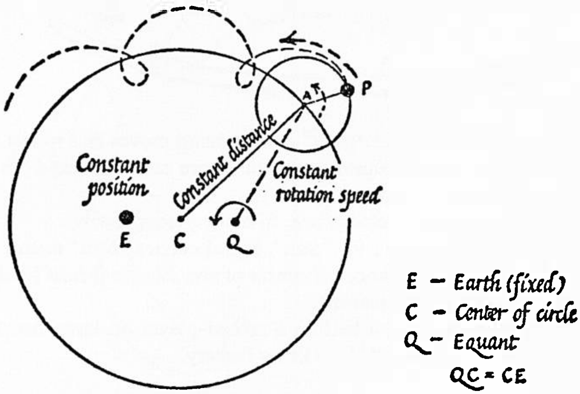
b. carrying an electric motor with the planet on an arm attached to its axle, or

c. carrying clockwork with an arm, as in (*b*) (e.g. an old clock with the escapement removed).

Or the scheme for a planet's motion can be shown by a ball (or lamp) on a record-player placed on a large turntable. The record-player should be tilted a little so that its axis makes a small angle with the axis of the turntable.

The motion of the Sun, as seen from an eccentric Earth, is demonstrated by the teacher sweeping a ball representing the Sun round with outstretched arm. He holds a large ball (Earth) a noticeable distance away from the centre of the Sun's arc. An elastic thread may be tied between Earth and Sun. Watching the thread, pupils can see how the Sun's apparent speed must change through the seasons if the Earth is off-centre. D45

(It would be easy to construct a professional model with two electric motors and balanced arms and a sliding sight-line – an unwelcome elaboration. Such complicated demonstration models should not be bought; and they are not worth constructing for teaching with our present aims. See the Note, in the Preface to this Year, which gives a warning about models.) * * * * * *



THE PTOLEMAIC SCHEME
This system imitated the motions of Sun, Moon, and planets very accurately.

Ptolemy's Successful Machinery. Ptolemy, about A.D. 120, modified the scheme of circles and sub-circles into a tremendously successful scheme, a brilliant mathematical machine. His great book, the *Almagest*, remained the authority for describing and predicting the motions of Sun, Moon and planets for fourteen centuries. T

If teachers or pupils are amused by the extra shifts that Ptolemy had to make to produce his very successful machine, a short description should be given. Otherwise, we should either leave the machinery at the stage described above, or simply state very briefly what Ptolemy did, saying that he devised his scheme to make it fit. But in giving any description of Ptolemy's scheme, however short, we must emphasize his constancies. Like any good scientific theory, it had essential constancies.

Ptolemy used a main circle with the Earth fixed near its centre, but he did not make the radius of that circle revolve at constant rate. Instead, he made another arm, the arm from an 'equant point', revolve at constant rate. The Earth was fixed a short distance from the centre of the circle on one side. The equant point, Q, was placed an equal distance off-centre on the opposite side. An arm ran from equant to the centre of the sub-circle that was carried round the main circle. (Since that arm changes length, we must, if we wish to imagine detailed machinery, think of it as sliding through some knob out there.)

The radius from the centre of the main circle was still there to maintain constant distance to the circumference.

The arm of the small sub-circle revolved at constant rate carrying the planet, as in the earlier scheme. The plane of the sub-circle was tilted out from the plane of the main circle.

Thus, to fit the facts, Ptolemy had many variables that he could choose: the ratio of radii sub-circle to main circle; the constant speed at which the arm from the equant revolved; the constant speed at which the radius of the sub-circle revolved; the tilt of the sub-circle's plane; the eccentric distance of the Earth (equal to the eccentric distance of Q); and the direction of that shift of the Earth out from the centre. Ptolemy had to choose the ratio of radii rather than two separate radii, since his machine only predicted the direction of a planet's position seen among the stars. For that, his machinery was remarkably successful.

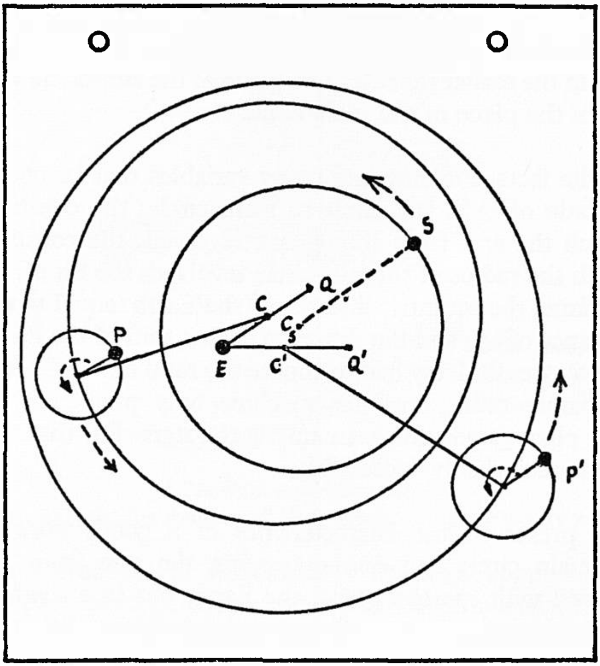
The scheme preserved the characteristics of a good, working T
theory; the main circle had *constant radius*, the arm from the equant revolved with *constant speed*, the Earth was in a *constant position* and *a constant distance off-centre*; the radius of the sub-circle revolved with *constant speed*.

No 'ultimate cause' was given for this machinery or its motions. Planetary motions were presumably started by gods and perhaps maintained by gods; and there was no link between them and the motions observed on Earth.

The planets were just bright stars moving in the starry pattern. Their real distances were unknown and no one knew whether they were much nearer to us than the stars, or even which ones were nearer than others. Ptolemy's system could place Jupiter nearer than Mars, or Mars nearer than Jupiter, equally easily.

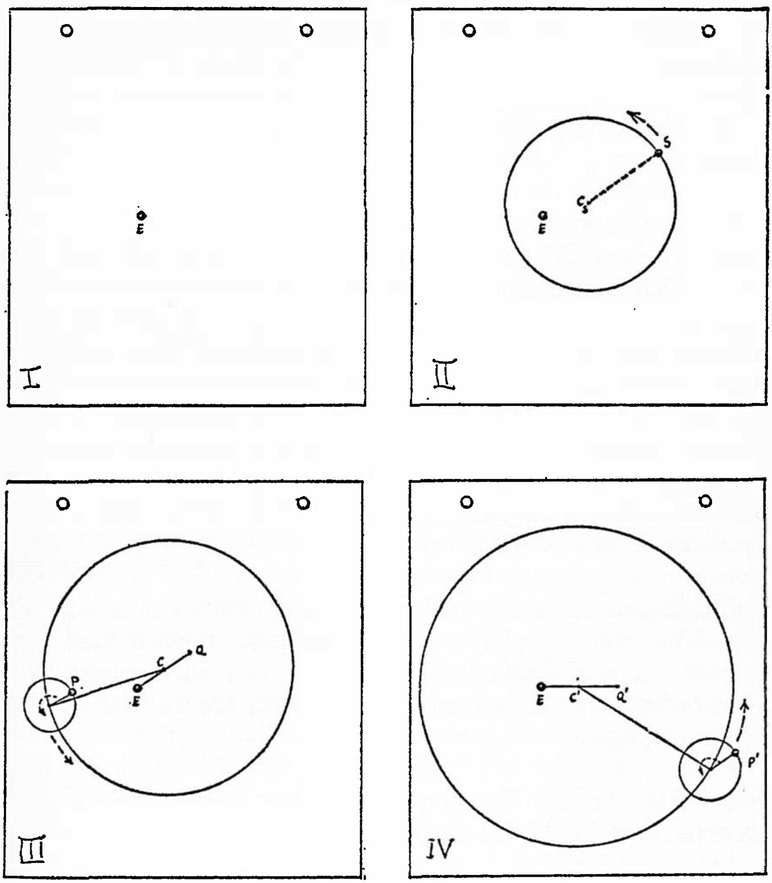
However, since Jupiter moves backward through the star pattern so much slower than Mars, astronomers guessed that Jupiter is much farther away, just as we guess relative distances of cows, trees, etc., seen from a moving train. In fact the order of planetary distances guessed at by the Greeks agreed with what we know today; but they had neither an experimental reason for it nor any idea of the proportions within that order.

If teachers like to form a composite picture with overlays of transparent sheets, they can easily convince pupils that even the clear, clever Ptolemaic system was complicated enough to make a headache. D46



THE PTOLEMAIC SCHEME for the Sun, S, and two planets, P and P′.
THIS IS THE COMPOSITE PICTURE THAT THE SEPARATE SHEETS (SHOWN OPPOSITE) WILL PRODUCE WHEN SUPERPOSED.
(The two small rings at the top show the locating holes in each transparent sheet.)

(Apart from that flat picture, with its special purpose, Ptolemy's refinements are too difficult to show by a simple model. If pupils have understood the previous stage – the epicycle and eccentric scheme – a blackboard drawing will do best for the new modification. A sound mechanical model could be manufactured, with two electric motors and arms that revolve and slide – though there are difficulties in arranging the supports so that they do not obstruct each other. Such a model would be expensive and fascinating; but it would *not* be a good aid in our present teaching.)



These are successive diagrams, to be sketched on transparent sheets and hung one in front of another on a translucent screen illuminated from behind. As these stages are added, they build up the picture shown on the opposite page. The two holes at the top fit on pegs in the frame of the screen, so that each stage is located correctly on the rest: I, I+II, I+II+III, I+II+III+IV.

Thus, Ptolemy had a magnificent scheme of circles, arms, subcircles and arms, which could reproduce the heavenly motions so accurately that it could be trusted to predict the positions of Sun, Moon and planets for century after century. In fact it continued in use, with occasional corrections, for more than a dozen centuries. And for practical purposes, navigators and astronomers would use a scheme of that form today. It was neither stupid nor clumsy, it was very clever and accurate, a successful machine.

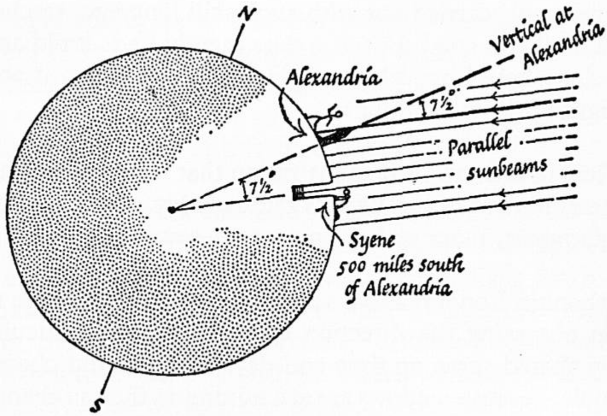
The only things we could say against it now are that it did not offer to link the heavenly phenomena with anything else we know in science; and that as machinery it seemed quite complicated. Pupils learning about the Ptolemaic system should not be taught to despise it but if they develop a headache over its complexity we might consider that a good preparation for the simplification by Copernicus.

Greek Measurements

Meanwhile, before Hipparchus and Ptolemy perfected the machinery of rotating arms on eccentric circles, Greek astronomers at the university at Alexandria made a great advance in scientific knowledge by making real measurements of distances. They estimated the size of the Earth – which was known in those days to be a sphere. They estimated the distance, and therefore the size, of the Moon by an ingenious method based on eclipse shadows. And they attempted a rough estimate of the distance of the Sun. Such measurements brought the purely pictorial mathematics of the sky into the realm of measured science.

Those Greek measurements remained in use by astronomers for centuries. They were simple; and we should teach them if time permits, not just to add information about methods of measurement, but because they show how mankind learnt the real size of local space long ago.

The Size of the Earth. The first measurement to be made was the size of the Earth itself, and the other measurements emerged in terms of the Earth's radius. T



HOW ERATOSTHENES ESTIMATED THE SIZE OF THE EARTH

Eratosthenes (about 240 B.C.) made one of the early estimates. He T
compared the direction of the local vertical with parallel beams of sunlight at two stations a measured distance apart. He assumed that the Sun is so remote that all sunbeams reaching the Earth at any instant are practically parallel.

He needed simultaneous observations at two stations far apart. Good clocks that could be compared and transported were not available. So he obtained simultaneity by choosing noon (highest Sun) on the same day at stations in the same longitude. He used observations at Alexandria, where he worked, and at Syene,‡ 500 miles farther south. The essential observation at Syene was this: at noon on midsummer day, 22 June, sunbeams falling on a deep well there reach the water and are reflected up again. Eratosthenes knew this from library information. Therefore the noonday Sun must be vertically overhead at Syene on that day. At noon on the same day of the year, he measured the shadow of a tall obelisk at Alexandria and found that the Sun's rays made $7\frac{1}{2}°$ with the vertical. He assumed that all sunbeams reaching the Earth are parallel. So it was the vertical (the Earth's radius) that had different directions. Therefore the Earth's radii to Alexandria and Syene make $7\frac{1}{2}°$ at the centre. Then, if 500 miles of Earth's circumference subtended $7\frac{1}{2}°$, what length would subtend 360°?

Measuring the 500 miles separation was hard – probably a military measurement done by professional pacers. There is doubt about the units he used, but some say his error was less than 5 per cent – a remarkable success for this early simple attempt. He also guessed at the distances of Sun and Moon.

‡ Modern name: Aswan, where the great dam has been built on the Nile.

If that measurement, carried out with such skill long ago, catches pupils' fancy, well and good. But, if a class merely finds it old and dull, the teacher should not labour it. He should mention it and then go straight on to other things. * * * *

A School Measurement. A demonstration that would bring this to life is a measurement of the Earth's radius today, conducted by two schools far apart, more or less on a north–south line. D 46

Suppose a school in London and a school in Newcastle arrange to co-operate in observing the direction of sunlight at a particular instant. They should agree on time and day. They should choose noon, if possible, because shadows are at a minimum then so change little in the course of a few minutes. (Also, with the Sun highest in the heavens the subsequent calculations or drawings feel more comfortable.) Each school sets up a pole of known height, say 10 feet. The pole must be vertical, as shown by a plumb line; or at least the point on the ground vertically below must be clearly marked. The shadow of that pole is measured. It is not necessary to have the same height of pole at both stations. The shadow of a building could be used instead, provided the height is known and the shadow falls on horizontal ground. Then, having prearranged the day, the schools communicate by telephone at noon:

> 'Have you got bright sunlight there? Is the shadow sharp? ... Is it on horizontal ground? ...
>
> 'How tall is your pole? ... (the same questions in the other direction) ...
>
> 'It is nearly noon. Are you ready? ...
>
> 'The shadow of our 10-foot pole is 12 feet long. What's yours?
>
> 'Oh, *our* 10-foot pole casts a shadow 11 feet long.'

Of course, that can be done by postcard correspondence, but a telephone call is much more romantic and may be well worth the cost, because it emphasizes the necessary condition of simultaneous observations. (On S.T.D. 3*d* will suffice to find out if the Sun is shining at both stations.)

Then the class must know the north–south distance between the stations, measured along the surface of the Earth. If we asked professional surveyors to find the distance between two stations thousands of miles apart, we might find them nowadays reversing Eratosthenes' measurement and estimating it by Sun or star heights and the known size of the Earth – and that could be unfortunate for the logic of this experiment.

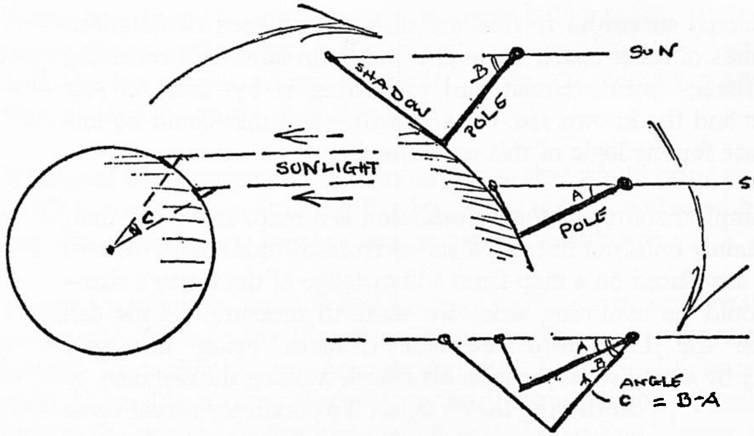
The simplest source of this information is a map. If we use that, we certainly must not find the distance from latitude angles from it. Those are placed on a map from a knowledge of the Earth's size – we should be assuming what we want to measure! Pupils can imagine the London-to-Newcastle distance being measured directly by a road surveyor with his wheel, walking the distance, or less precisely, a car driving the distance. To obtain the actual value for use, they may take the distance in miles (but not in degrees) from a map; but they will probably feel that it is more realistic to take the distance from the ABC timetable, which gives the rail distance and the fares.

In our (fictitious) example above, pupils and teacher would then use trigonometry or a graphical method to find the radius of the Earth.

To use trigonometry, we sketch the Earth as if we already knew its size, then continue the line of the vertical pole at each station down into the Earth, as a radius. On the sketch we see those radii meeting at the centre of the Earth, making a small angle. The slanting lines of sunlight are parallel (assuming the Sun infinitely distant). We complete the observation-triangles on the sketch, making a right angle at the base of the pole at each station. Then pupils who are competent at geometry will find that the angle at the centre of the Earth between the radii is equal to the difference of the angles at the two stations between base line and sunlight. They calculate those angles by saying,

tan [base angle] = [pole-height]/[shadow length]

They calculate the tangents, the angles, the difference of angle.



Unless pupils are confident and clear with that method, it will be better to do it by drawing two triangles to scale. We first show a sketch of the Earth with the triangles at the two stations. We draw those triangles larger. Then, pointing out that the Sun's rays have the same direction at both we place one triangle on the other, with the tops of the poles coincident and the rays of sunlight through them coincident. Pupils draw a scale picture of the two superposed triangles and measure the angle between the two lines that represent the vertical poles.

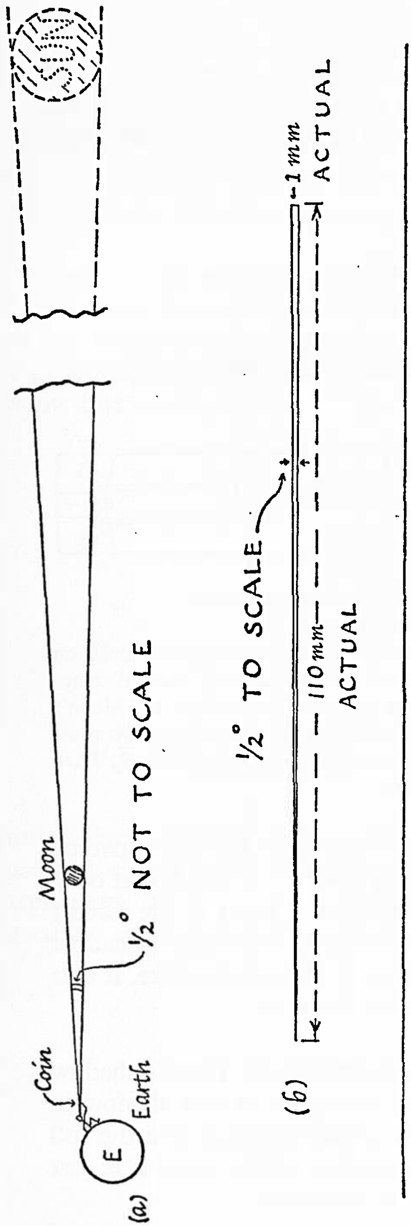
The angle at the centre of the Earth will be small. For example, for stations in London and Newcastle it will be about $3\frac{1}{2}°$. Then we argue that the measured distance, say 250 miles, corresponds to that angle, say $3\frac{1}{2}°$; and we ask the value of the circumference of the Earth corresponding to 360°. Thence, we estimate the Earth's radius.

This is not an essential demonstration. If time is short, or pupils *
do not show much interest, it should be omitted. But we hope that *
many classes will find it exciting. *

Size and Distance for Moon or Sun. We ask pupils how they C47
could tell the size of the Moon if they knew its distance. We suggest each pupil should try holding a coin at arm's length and finding the distance at which it just blots out the full Moon. The answer will be about 110 coin distances. Therefore, the Moon's distance is about 110 Moon-diameters.

A similar estimate can be made for the Sun (with care to avoid hurting eyes). The answer is almost exactly the same. That is why the Moon can only just make total eclipses of the Sun.

We use this proportion, 110 to 1, in our estimate of the Moon's distance.



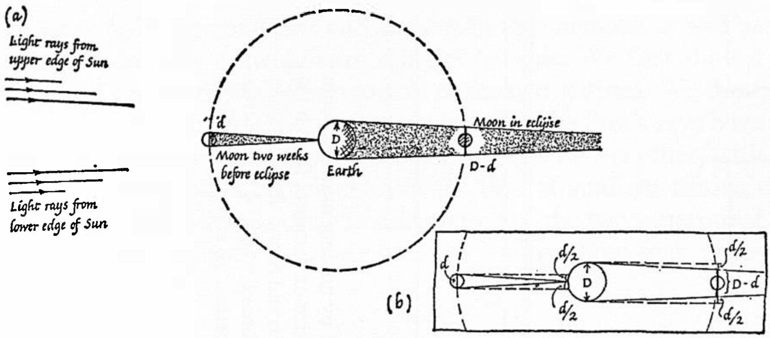
RELATION BETWEEN SIZE AND DISTANCE can be found by holding measured coin at measured distance. This does not tell us absolute size or distance. Sketch (*a*) is *not* to scale. Sketch (*b*) shows the 'angular size' of Sun and Moon *drawn to scale*. Measurements show the Sun and Moon each subtend about ½° at Earth. Measurements or trigonometry tables give a proportion of about 1 : 110 for base : height.

Distance of Moon. This is not a measurement that pupils can make, but we should tell them briefly how the Greeks carried it out. * * *

*

Later measurements have been made by standard surveying methods, with telescopes sighting a point on the Moon from stations a large distance apart; but the Greeks made an early estimate by means of eclipses. * * * *

We show a diagram of the Earth's shadow in an eclipse of the Moon. The usual sketches in books exaggerate the size of Earth and Moon and fail to show how unlikely an eclipse is. (See sketches on page 99.) T



Early Greek Measurement of Size of the Moon
(and therefore Its Distance)

Observations of eclipses showed that the width of the Earth's shadow at the Moon is 2·5 Moon-diameters. However, the Earth's shadow narrows as its distance from Earth increases because the Sun is not a point-source. Since the Moon's shadow almost dies out in the Moon-Earth distance, the Earth's shadow must narrow by the same amount – one Moon-diameter – in the same distance. Then Earth-diameter must be 3·5 Moon-diameters.

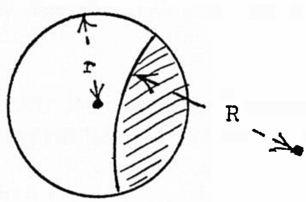
We point out that, since the Sun and Moon each have an apparent diameter of about $\frac{1}{2}$ a degree, the full shadow of the Moon only just reaches the Earth. It tapers almost to a point at the Earth; so we only just see total eclipses of the Sun. The Earth's shadow tapers with the same angle from Earth to Moon; therefore, it also narrows by one Moon-diameter in that distance. T

By watching the passage of the Moon through the Earth's shadow in an eclipse, astronomers estimated the width of that shadow in Moon-diameters: about $2\frac{1}{2}$. However, that does not give the full diameter of the Earth because the shadow tapers, since it is cast by the large Sun which is about $\frac{1}{2}°$ in diameter.

So the shadow of the Earth must be $2\frac{1}{2}+1$, or $3\frac{1}{2}$ Moon-diameters wide. Therefore, the Earth's diameter is $3\frac{1}{2}$ times the Moon's diameter. Therefore, the Moon's diameter is $\frac{2}{7}$ of the Earth's diameter, and the Moon's distance is $110\times\frac{2}{7}$ Earth-diameters, or just over 60 Earth's radii, some 240,000 miles.

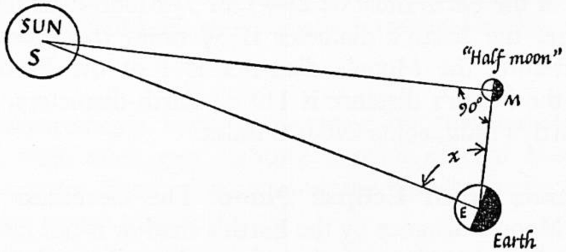
Pupils' Estimate from Eclipse Photo. The cleverness of *
estimating the Moon's distance by the Earth's shadow is not likely *
to appeal to pupils unless they can do it themselves. From time to *
time, there are announcements in the papers of an eclipse of the *
Moon which give the time it starts and ends. But those will not *
yield a good estimate unless they are the stages of a total eclipse. *
(Also, the predicted times are calculated from a knowledge of the *
distance we are trying to measure.) *

However, a rough estimate could be made from a photograph of a D48
partial eclipse. Best of all, pupils take a photo themselves. Second
best, we supply a printed photograph and ask pupils to estimate H48
the proportion between the Moon's radius and the radius of the
shadow-bite on the Moon.



$$\frac{r}{R}=\frac{\text{Moon-diameter}}{\text{Earth-diameter}-\text{one Moon-diameter}}$$

Sun's Distance. The earliest guesses placed the Sun absurdly T
close to the Earth and therefore supposed it quite small. Greek astronomers made a clever attempt, as follows, and its very inaccurate result remained in use for a long time.



SUN'S DISTANCE

Early Greek estimate of the Sun's distance from the Earth, in terms of the Moon's known distance. Greek astronomers tried to measure the angle x (or SEM), which is itself nearly 90°.

When the Moon seems to be exactly at half-moon to an observer on the Earth, the directions from the Moon to Sun and to Earth must make 90°. If observers know the directions of the Sun and Moon at that instant, they have data for a vast right-angled triangle with a right angle at the Moon and an angle almost 90° at the Earth. T

It is very difficult to fix that exact instant, so the Greek estimate of 87° was far from right. It yielded, by simple geometry, the result that the Sun is 20 times as far away as the Moon. We now know the ratio is about 400.

That was a valiant attempt. We shall meet the method again in Copernicus' estimate of the size of Venus' orbit.

'Dark Ages'

For a dozen centuries astronomy was taught with the authority of books and careful adherence to the Ptolemaic system. It was part of the general teaching of mathematics that was given to church scholars; and it was kept alive by the need to train navigators and by strong superstitious interest in astrology. The intellectual world was occupied with other matters, and astronomy was largely taught without question. From time to time the Ptolemaic machinery was endowed with new radii and modified periods of rotation to bring it into still better gear with observations.

THE COPERNICAN REVOLUTION: A SIMPLER SCHEME

Copernicus (1473-1543) was brought up by his uncle to be a church administrator; but at an early stage he developed an intense interest in bringing the heavenly system into a simpler scheme, which he thought would be to the greater glory of God – the Ptolemaic system with its artificial equants seemed to him too clumsy to be God's best choice. He believed that the planetary system, spheres and all, was a divine creation; but he believed God's arrangement would be a simple one, all the more splendid for great simplicity. He collected together observations of the planets in more reliable tables than had so far been available; and in thinking about the planetary motions he was struck by the simplicity that would come from changing to a system with the Sun at the centre of the universe. This model had been suggested by a few Greek philosophers but it was unpopular and soon forgotten. Copernicus' development was far more powerful because he built into it a system of measurements. As a careful, quiet, contemplative monk, with a great love of truth, he spent his lifetime perfecting his scheme and was not willing to publish it – apart from talks with visiting pupils – until near the end of his life. He believed that a simpler scheme, such as he suggested, was all to the glory of God; and he fervently believed that his scheme was true.

Copernicus assumed that the Earth spins daily, and that accounted for the daily motion of stars, Sun, Moon and planets. He assumed that the Earth travels round a central fixed Sun in a yearly orbit. In making the change from the Ptolemaic system he moved the Earth out of its grand central position and made it an *ordinary* planet like the rest. That was a tremendous change of viewpoint which horrified people when, later on, they came to understand it.

Copernicus pictured all the planets moving in circular orbits around a fixed Sun. He made the Earth travel once around the Sun in a year, spinning once in 24 hours as it goes. The 'fixed stars' and the Sun could then remain at rest in the sky.

This scheme replaced Ptolemy's epicycles and equants with simpler circular motions. The daily motion of the stars, carrying Sun, Moon, and planets as well, could obviously be replaced by a daily spinning Earth. That alternative had often been discussed, but had been turned down because the critics did not understand the mechanics of motion. (They claimed that there would be a howling wind of air left behind, and that the ground would outstrip a stone dropped from a high tower. On the other hand, the stars, etc., could well be carried around by Ptolemy's spheres because spheres and rotations were 'natural' in the heavenly region.)

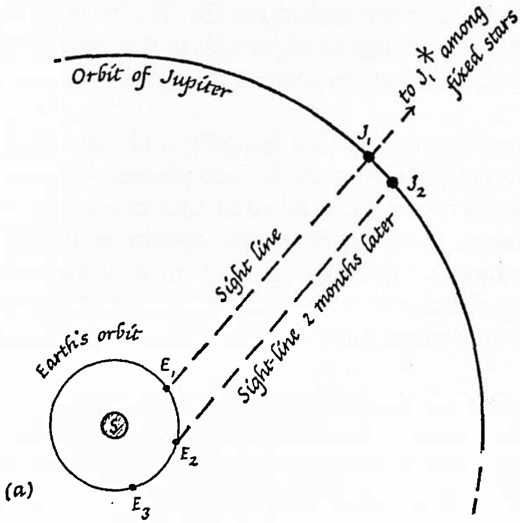
The slower, irregular motions of Sun and planets through the star pattern were simplified by a scheme of circular motions around the Sun.

This was Copernicus' main contribution: to stop the Sun and place it at the centre of the planetary system. Then the Sun's yearly motion around the ecliptic was only an apparent one due to the Earth's yearly motion around the Sun.

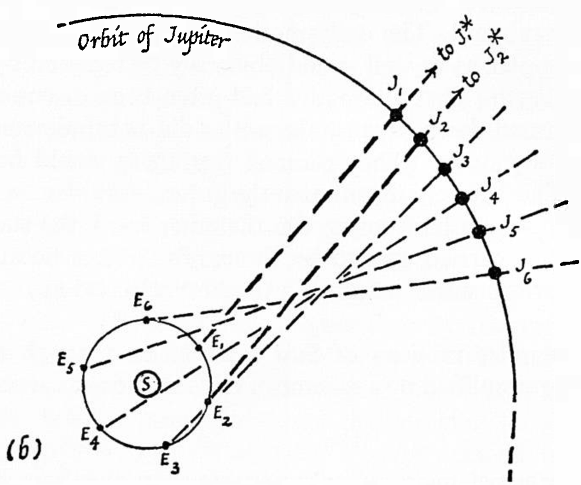
The complex epicycloid of a planet was simply a compound of the planet's own motion around a circle and the Earth's yearly motion. (On this view, the epicycloid picture is making us pay for ignoring the Earth's motion.)

Copernicus' Explanation of Planetary Epicycloids

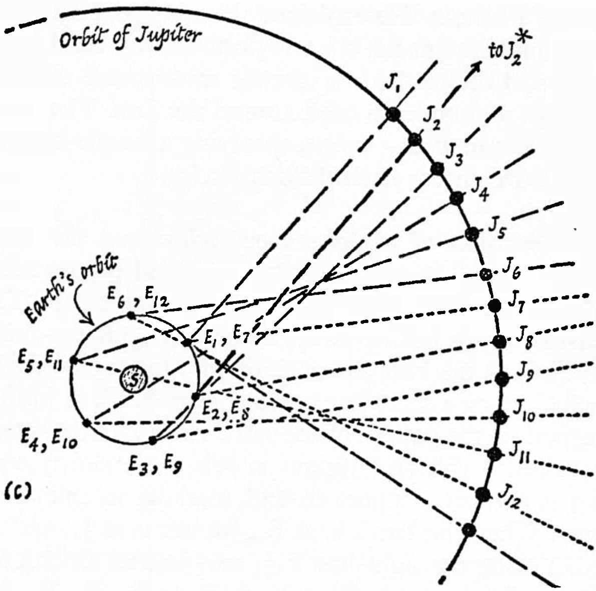
The lines E_1J_1, E_2J_2, etc., are sight-lines from positions of the Earth every two months through Jupiter's position towards the stars.



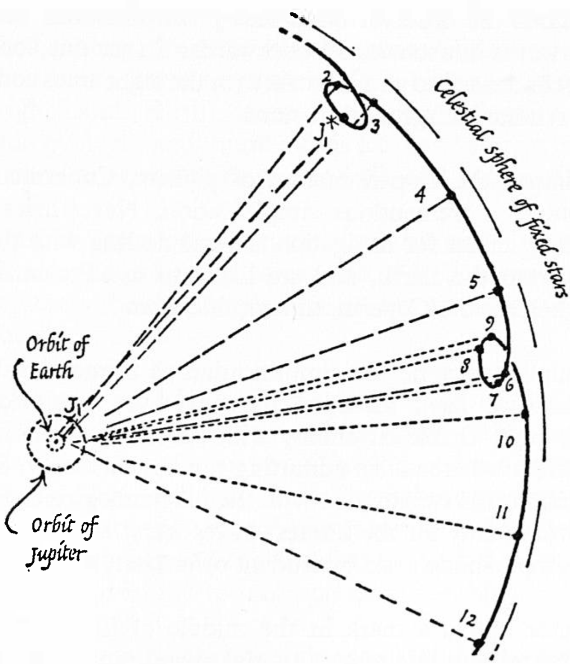
(*a*) Two stages sketched.



(*b*) More stages sketched.



(*c*) Many stages sketched. The sight-line EJ wags up and down in a complicated way.



Copernicus' Explanation

The apparent positions of Jupiter in the background of fixed stars. This shows Fig. (*c*) redrawn on a much more condensed scale with the sight-lines from Earth to Jupiter continued on out to the stars (e.g. the line to J_1* here is continuation of EJ_1). The specimen sight-lines are drawn parallel to the corresponding ones in Fig. (*c*).

Looped Paths of Planets. He explained that the looped pattern of planetary motion through the stars is produced by combining the simple motion of the planet in a circular orbit round the Sun with Earth's simple motion in its orbit around the Sun. The loops are due to the Earth's motion – we are observing a simple circular motion from an Earth that is itself making circles. T

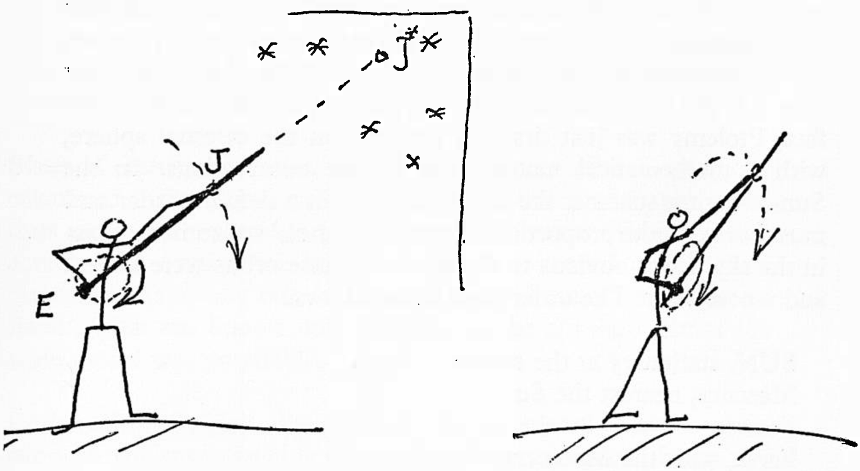
His detailed explanation of a planet's epicycloid ran like this. Suppose the Earth travels around a circular orbit and Jupiter more slowly around a bigger orbit, both with the Sun at the centre. The fixed stars must be much farther away, because no parallax-shifts are observed. Then in marking the position of Jupiter among the fixed stars we look along a sight-line running from Earth to Jupiter and on, far beyond, to the pattern of the stars. As the Earth sweeps round and round its orbit and Jupiter crawls more slowly, this sight-line wags to and fro as it goes around, marking an epicycloid among the stars. When the Earth is at E_1, Jupiter is at J_1, and an observer looking along the sight-line E_1J_1 sees Jupiter among the stars at J_1^*. As the Earth travels from E_1 to E_2 to E_3, E_4, E_5, E_6, etc., Jupiter travels steadily but slowly forward from J_1 to J_2 to J_3, J_4, J_5, J_6, etc. Then the observer on E sees J^* in directions that swing mostly forwards but sometimes backwards. To see this, look at the sketch (*d*) condensed to a small scale with the sight-lines continued out to a remote background of stars.

In thus 'explaining' the looped motion of planets, Copernicus offered astronomers a tremendous simplification. Nevertheless, people constructing tables for navigation are still dealing with the planets as seen from the Earth, and are likely to use Ptolemaic machinery and neglect the Copernican simplification.

Nor will our pupils welcome the simplification as tremendously important, unless we have managed to convey the historical development through Greek astronomy with special enthusiasm. They will just think we are at last admitting the obvious story. So we should not labour the explanation with the diagrams given here – these are provided only for the interest of teachers. Instead, we should show a simple model, run by student or teacher, as follows. * * * * * * *

The demonstrator makes a mark in the middle of his chest to represent the central Sun. He moves his *right* hand rapidly round that 'Sun' in a small circle of radius 8 or 10 inches. That hand represents the Earth. D49

He stretches his *left* arm out fully, to represent the orbit radius of another planet, say Jupiter. He moves his left hand, representing Jupiter, more slowly round the larger circle with the Sun at centre.



To show where Jupiter will be seen among the stars which are much farther away, the demonstrator carries a light wooden pole as 'sight-line' from Earth to Jupiter. He holds one end of the pole in his right hand (Earth), letting the pole run loosely through a ring made by finger and thumb of his left hand and on out beyond to the 'stars' imagined to be on the walls and ceiling of the room.

As the 'Earth' goes quickly round its orbit and 'Jupiter' moves more slowly, the pole wags to and fro as well as making general progress across the sky. Pupils will see how the epicycloid pattern is produced.

(More elaborate models with an electric torch on the pole have been tried; but they are apt to add confusion rather than clarity. This is a demonstration in which intelligent imagination should play a part. More elaborate models can be constructed with a large turntable carrying a record-player turntable, but these require considerable planning and construction to avoid their showing the wrong thing. We do not recommend them for this.)

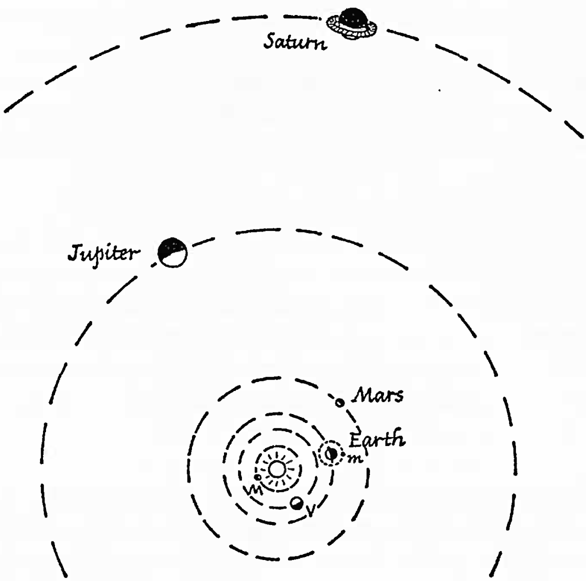
Copernicus accounted for the epicycloids of Mars, Jupiter, and Saturn by making them move around large circular orbits outside the Earth's orbit. He made Venus and Mercury move around smaller orbits, nearer the Sun than the Earth's. This accounted for their observed behaviour – they keep close to the Sun and swing

to and fro each side of it. Thus the same scheme served for both the *
'inner' planets and the 'outer' ones. *

Copernicus did not just offer an alternative that looked simpler; T
he extracted new information from his scheme: the order and sizes of the planetary orbits, a remarkable advance contributed by theory. In the Ptolemaic scheme the main circles could be chosen with any sizes – it did not even matter which planet was put outermost. In fact, Ptolemy was just drawing patterns on the celestial sphere, with a mathematical machine, to fit the observations. In the Sun-in-centre scheme, the orbits must be in a definite order and must have definite proportions. From the planets' apparent motions in the sky it was obvious to Copernicus whose orbits were largest and whose least. The order *must* be as follows.

SUN, stationary at the centre
Mercury, nearest the Sun
Venus
Earth, with the Moon travelling round it
Mars
Jupiter
Saturn, farthest of the planets then known.

A chart to be posted up would be useful. D50

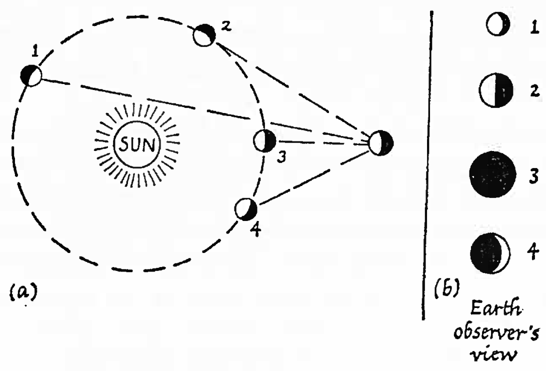


Copernicus' Planetary System

Orrery? If the school has a mechanical model of the solar system, an orrery, it should certainly be shown. But one should not be bought. They are expensive toys that do not teach as much as one expects. But making a simple one is profitable. If pupils or teacher like to construct a model it will be worth while. Ingenious gears are not necessary: here again it is the *idea* of the scheme that we are trying to show. An informal model in which each planet is carried round by a pupil can be a great success. D36c OPT.

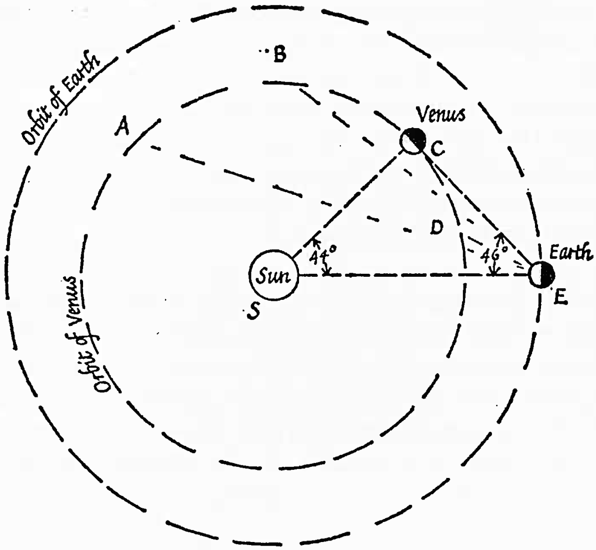
Sizes of Orbits. Treating the orbits as simple circles, Copernicus calculated their relative radii from available observations; he could thus plot a fairly accurate scale map of the system. To obtain the actual radii from these relative values, he needed an absolute measurement of any one of them, say the distance from Sun to Earth. This was known only roughly, so the absolute size of his scale model was unreliable. T

Estimating Orbits. To see how he calculated relative radii, suppose you are attacking the problem for an inner planet, say Venus. Venus, nearer the Sun than the Earth, travels in a small orbit round the Sun. This circle is seen practically edge-on from the Earth; so Venus seems to swing to and fro in front of the Sun or behind it, travelling only a small way each side of the Sun before it turns back. Thus it is seen only near the Sun as a morning or evening 'star'. T



PHASES OF VENUS, AS SEEN FROM THE EARTH

When Venus seems farthest to one side of the Sun, just about to turn back, it must be at a point such as C lying on a tangent from the Earth to its orbit. In positions A, B, D, ... etc., it would seem nearer the Sun. This tangent is perpendicular to the radius, SC, of the orbit. So the triangle ECS has a right angle at C and an angle at E that can be measured by sighting from the Earth.



Estimating Relative Radii of Orbits
Venus is shown farthest from the sun.

Copernicus knew the angle SEC from observations, about 46°. Then he knew the proportion

$$\frac{SC}{SE} = \frac{\text{Radius of Venus' orbit}}{\text{Radius of Earth's orbit}} \quad \text{was sin } 46°\text{, about } 72/100.$$

Pupils could be given the argument and the measured angle, 46°, *
and asked to find that ratio by trigonometry or by drawing to scale. *

Copernicus had measurements which gave him this angle, and he T
performed this calculation for Venus and Mercury. For the outer planets the argument and the geometry are rather more complicated, but Copernicus calculated the relative sizes of their orbits in much the same way. He could draw a scale diagram of the solar system, placing the planets in the right order at the right relative distances. In that way, Copernicus' theory gave much fuller knowledge of the heavenly system than the Ptolemaic machine, which offered no such details. However, the new knowledge was only yielded by the new theory *in terms of the theory's own assumptions*. The same holds in modern models of atoms; the information we extract is only evolved in terms of the pattern we have chosen; and we should remember that.

Copernicus could draw a scale map of the orbits and place the T
planets correctly in them at some chosen starting time. To predict

their positions at other times he needed to know each planet's 'year', the time it takes to travel round its orbit. These 'years', or times of revolution, he found from recorded observations. Essentially, he found how long the planet took to get back to the same place among the stars.

Using recorded data, Copernicus placed the planets on his scale map and predicted their positions at other times, past and future. He could check the past ones, and thus test his 'picture', or 'theory' as we should now call it. These tests were encouraging, but there were some disagreements which led, through long careful calculations, to modifications of the simple picture.

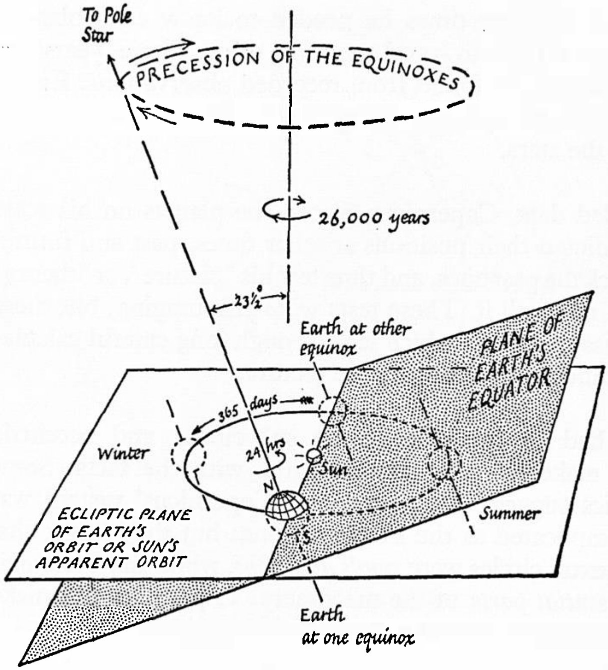
Copernicus had to introduce some sub-circles and eccentric positions to make his solar system agree with the facts. Some modern critics suggest that in the end Copernicus' system was almost as complicated as the Ptolemaic one: but they forget that Copernicus' extra circles were *small additions*, while the Ptolemaic ones were *essential parts* of the machinery. Copernicus did make a great simplification of thinking.

Copernicus gave other points in support of his theory: he showed that the changes of brightness of planets and the placing of the loops in their motion agreed with his model.

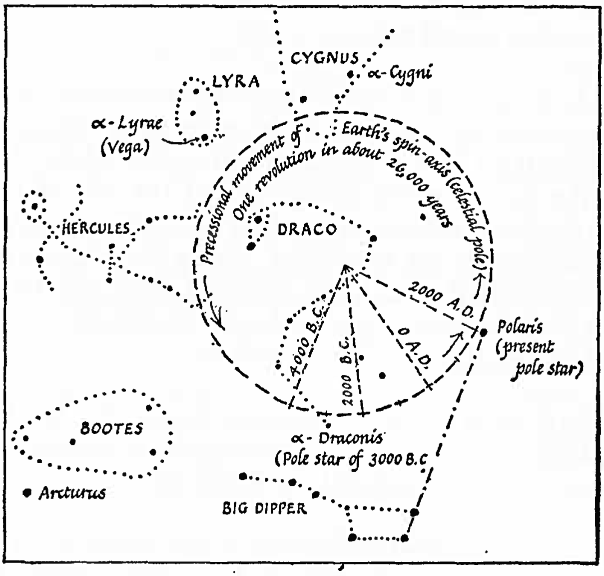
Precession. As a crowning virtue of simplicity, Copernicus gave a new interpretation of the precession of the equinoxes. Precession, as discovered by the Greeks, was described as the whole star system (and the Sun) crawling slowly around the axis of the ecliptic, while the Earth and its equator plane and N–S axis stayed still. Copernicus reversed the description, saying the Sun and its ecliptic plane stay fixed; that is, the plane of the Earth's orbit stays fixed. And the Earth's equator-plane (and celestial equator) swings slowly around, always tilted $23\frac{1}{2}°$ to the ecliptic.

Then Copernicus could describe precession simply: the Earth's spin-axis has a slow conical movement; carrying the equator-plane, it gyrates around a cone of angle $23\frac{1}{2}°$ in 26,000 years.

Though Copernicus gave this clear picture of what happens in the precession of the equinoxes, he had no idea what 'caused' it. He gave no reason for that motion any more than he gave a reason for the motions of the planets which he described so simply. That problem had to wait for Newton, who showed that, like so many astronomical phenomena, it is a result of universal gravitation.



SKETCH SHOWING MOTION CALLED THE PRECESSION OF THE EQUINOXES



THE PRECESSION OF THE EQUINOXES

Sketch of a large patch of Northern sky (about 90° by 90°), showing the slow movement of the celestial North Pole among the stars. The point where the Earth's spin-axis cuts the pattern of the stars moves slowly around a roughly circular path making one revolution in about 26,000 years. (After Sir Robert Ball.)

We should show this picture of precession with the help of an ordinary Earth globe. We place the globe on the table, with its spin axis making about $23\frac{1}{2}°$ with the vertical. Keeping the globe spinning, we move it round a large circular orbit on the table, with a fixed Sun in the centre, keeping the spin axis pointing in a constant direction.

Then, having established that picture of the Earth's yearly motion, we show precession by making the spin axis revolve very slowly round an axis perpendicular to the Earth's orbit. To do that, we simply turn the whole globe, stand and all, round a vertical axis, by hand – and, doing that slowly, we keep the globe spinning fast and move it at medium speed round the yearly orbit on the table. That conical motion takes, in fact, 26,000 years.

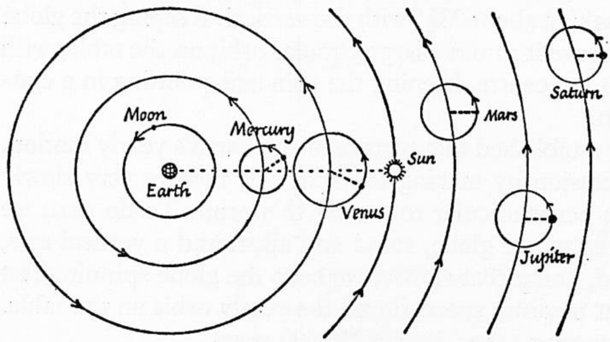
Copernicus was at last persuaded to write his scheme in a great book, and the book was published at the very end of his life.

(This is one of the few great books in the history of science of *
which we have the original manuscript. A photographic copy of *
Copernicus' original writing, with ink blots and corrections, has *
been published recently.) *

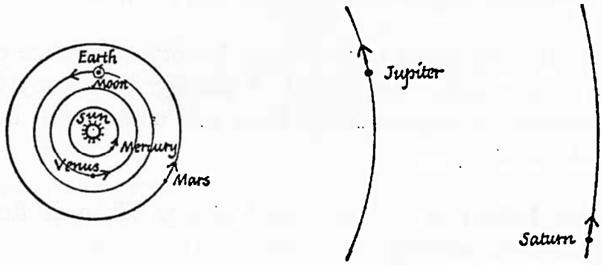
The Explosive Effect of Copernicus' Book. When it first *
appeared, the book was read by astronomers but in its formal Latin *
it was not read by educated people in general, so the new scheme *
did not have its full impact for some time. Galileo, born some years *
after Copernicus' death, expounded the scheme and put forth *
winning arguments for it in popular, rolling Italian. That was a *
bombshell, because educated readers far and wide enjoyed it, *
understood it, and realized that the Copernican system had made *
the Earth common and ordinary, 'just a planet', and had left the *
stars fixed in space, at any assortment of great distances, with no *
place for Heaven. *

That was disturbing, both to man's picture of Heaven and to *
the teaching of Church authorities. No wonder Galileo got into *
trouble for insisting so loudly and clearly that the Copernican *
system is true. *

COMPARISON OF SIMPLE PTOLEMAIC SCHEME AND SIMPLE COPERNICAN SCHEME



PTOLEMAIC SYSTEM, sketched without eccentricity or equants. Order and proportions of orbits not determinate. Epicycle radii not 'to scale'.



COPERNICAN SYSTEM, sketched without eccentricity or minor epicycles. Orbit proportions, which *are* determinate, are roughly to scale. (Moon's orbit out of scale.)

TYCHO BRAHE, THE AMAZING OBSERVER

Tycho Brahe (1546-1601) was a Danish nobleman who, in his early school days, developed a passion for accurate astronomical observation. He was delighted by the way in which astronomers could predict an eclipse. But then he was disappointed at the inaccuracy of predictions for the close 'conjunction' of Jupiter and Saturn – an important astrological event. He determined to spend his life making such accurate and systematic observations that astronomers could not only make good predictions but would also know which model of the heavens fitted best. In that, he was to provide the precise measurements of planetary motions which were essential to Kepler.

At an early stage Tycho realized that the old practice of collecting and using chance observations did not suffice. Systematic observations and records were essential. He also realized that, to obtain great precision, he must make robust instruments and then calibrate them, making tables of their errors, rather than strive for an instrument that was 'perfectly accurate'.

Using royal endowments and his own fortune, he built a magnificent palace as an observatory on an island off Copenhagen. He spent 20 years there, building a tremendous record of accurate observations, all made with naked eyes (telescopes had not been invented), using instruments which he designed and constructed in the workshops of his palace. Students came from far across Europe to work with him; and he drove them hard to make observations and reduce them to records, cross-checking between one observer and another and one instrument and another.

When he died Tycho left a magnificent record of the motions of the planets and of Sun and Moon, recorded against the star pattern. He was a magnificent observer but not a strong theorist. He left to his pupil, Kepler, the making and testing of theories with those records.

In our present teaching we need not go into the history of Tycho *
Brahe and his work; but pupils should know that there was a great *
observer who provided such good records that Kepler could *
disentangle his laws of motion for the planets. Since the planetary *
orbits are almost circles, and the motions of planets along them *
almost uniform, Kepler's achievement depended on very precise *
measurements which he could trust. *

KEPLER, 'THE LAW-GIVER OF THE HEAVENS'

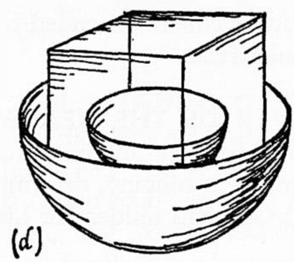
Kepler (1571-1630) was a brilliant mathematical speculator, T
fascinated with the problems of astronomy, determined to extract the laws which he believed God had hidden for him to discover.

Kepler and Tycho form a strange contrast. Tycho, 'rich, noble, vigorous, passionate, strong in mechanical ingenuity and experimental skill, but not above the average in theoretical power and mathematical skill'; and Kepler, 'poor, sickly, devoid of experimental gifts, and unfitted by nature for accurate observation, but strong almost beyond competition in speculative subtlety and innate mathematical perception'. Tycho's work was well supported by royalty, at one time magnificently endowed; Kepler's material life was largely one of poverty and misfortune. They had in common a profound interest in astronomy and a consuming determination in pursuing that interest.

The story of Kepler's life is interesting, showing a poor boy with poor health, struggling as a Protestant in a largely Roman Catholic world, emerging from the university with philosophy and religion his chief interests. He was offered a post to teach mathematics and astronomy and accepted it unwillingly, saying he should be provided for in some more brilliant profession. Once started, however, he threw himself into a study of the planets and, as he said, 'brooded with the whole energy of his mind on the subject'.

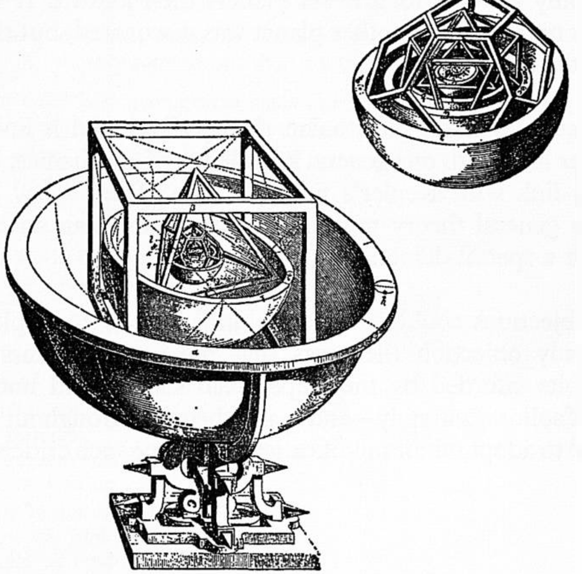
We do not need to give pupils even that much of an account of *
Kepler's life: they only need to know the laws he extracted and the *
assurance that he had that the laws were true. Some teachers, *
however, may want to describe Kepler's life and work in detail. *
The notes that follow are too short for that, but there are good *
biographies. *

Kepler's Work. Kepler's mind burned with questions: Why are T
there only six planets? Why do their orbits have just the proportions and sizes they do? Are the times of the planets' 'years' related to their orbit sizes? The first question, 'Why just six?' is characteristic of Kepler's times – nowadays we should just hunt for a seventh. But then there was a finality in facts and a magic in numbers. The Ptolemaic system counted seven planets (including Sun and Moon, excluding the Earth) and even had arguments to prove seven must be right.



This shows the basis of Kepler's final scheme. He chose the order of regular solids that gave the best agreement with the known proportions of planetary orbits.

He tried to find geometrical schemes that would 'predict' the T
relative proportions of planetary orbits which he obtained from Copernicus. Geometry was the fashion, so fitting a square between two circles (or spheres) seemed a promising way of predicting two orbits. Such schemes, however, failed to fit; and Kepler tells us he



KEPLER'S SCHEME OF REGULAR SOLIDS, FROM HIS BOOK
The relative sizes of planetary orbits were shown by bowls separating one solid from the next. The bowls were not thin shells but were just thick enough to accommodate the *eccentric* orbits of the planets.

was suddenly inspired to use regular solids to separate spheres instead of squares and other plane figures. Greek mathematicians had shown that only five regular solids are possible. Therefore, interposing one of each between spherical shells whose sizes represented planetary orbits would provide six shells, thus accounting for the existence of six planets; and Kepler found he could juggle the arrangement of solids to predict the known orbit proportions fairly well.

As a young astronomer, he was fired with enthusiasm by this discovery and determined to wrest other secrets from the great record of observations he inherited from Tycho.

It is customary to laugh at Kepler's mystical scheme of solids and pass quickly on to his discovery of the three laws of planetary motion that now bear his name. Yet, to Kepler, that 'five solids' rule was a wonderful discovery and good theory. It was a piece of mathematical mysticism in keeping with the spirit of the times; and, in a way, it was not unlike some rules that we now assume as fundamental in modern atomic theory. We should not laugh at it but we do now discredit it because:

1. It could only provide for the six planets then known. It was two centuries more before another planet was discovered, but then that spoiled the scheme.

2. When, a century later, Newtonian theory developed it linked Kepler's other laws with our general knowledge of mechanics, but it offered no link with Kepler's five-solid law. Then a law left isolated from general theory must seem little more than an empirical rule or a special demon.

Those two objections could not operate until long after Kepler's time. The only objection then was that the measurements of planetary orbits afforded by the Copernican scheme did not fit the pattern of solids accurately – and many theories throughout the ages have had to adopt minor modifications to escape such criticism.

THE REGULAR SOLIDS. A geometrical intelligence test

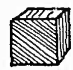
How many different shapes of regular solid are possible? To find out, follow argument (a); then try (b).

A regular solid is a geometrical solid with identical regular plane faces; that is, a solid that has:

all its edges the same length
all its face angles the same
all its corners the same
and all its faces the same shape.
(See opposite for shapes that do not meet the requirements.)

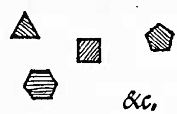
Fig. 18-2.

For example, a cube is a regular solid.



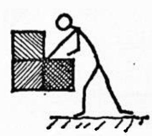
The faces of a regular solid might be:

all equilateral triangles
or all squares
or all regular pentagons
or . . . and so on . . .



(a) Here is the argument for square faces. Try to make a corner of a regular solid by having several corners of squares meeting there.

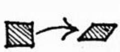
We already know that in a cube each corner has three square faces meeting there. Take three squares of cardboard and place them on the table like this, then try to pick up the place where three corners of squares meet. The squares will fold to make a cube corner.



Therefore we can make a regular solid with three square faces meeting at each of the solid's corners. (We need three more squares to make the rest of the faces and complete the cube.)

Could we make another regular solid, with only one, or two, or four square faces meeting at a corner?

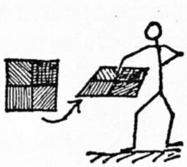
With *one square,* we cannot make a solid corner.



With *two squares,* we can only make a flat sandwich.

With *three squares,* we make a cubical corner, leading to a cube.

With *four squares* meeting at a corner, they make a flat sheet there, and cannot fold to make a corner for a closed solid.



Thus, SQUARES CAN MAKE ONLY *ONE* KIND OF REGULAR SOLID, A CUBE.

(b) Now try for yourself with regular pentagons, and ask how many regular solids can be made with such faces.

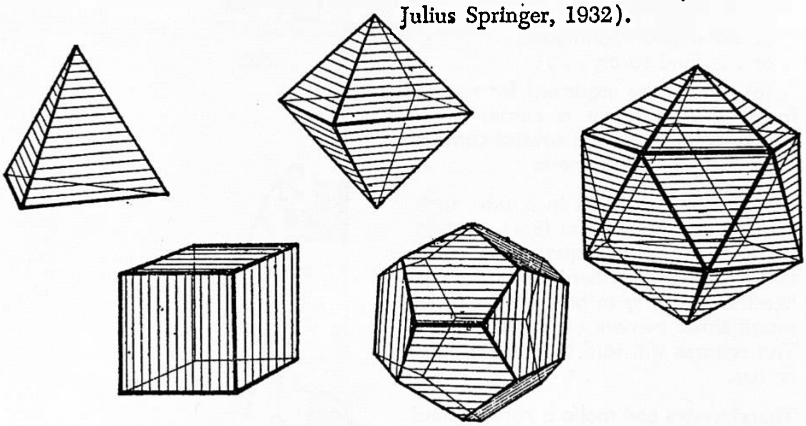
Then try hexagons, and other polygons.

Then return to triangles and carry out similar arguments with triangular faces.

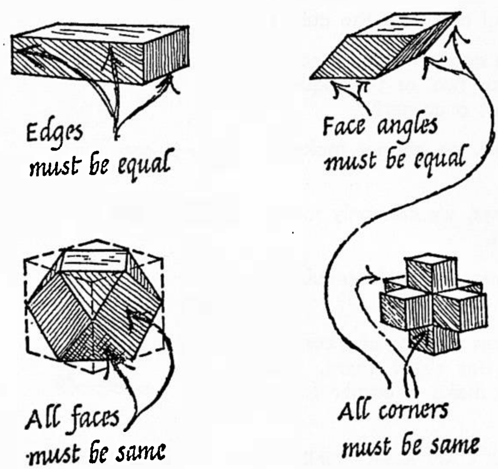
THE RESULT: Only FIVE varieties are possible in our 3-dimensional world. (Fig. 18-3)

(NOTE that these arguments need pencil sketches but can be carried out in your head without cardboard models.)

THE REGULAR SOLIDS drawn after D. Hilbert and S. Cohn-Vossen in *Anschauliche Geometrie* (Berlin: Julius Springer, 1932).



THE SOLIDS BELOW ARE <u>NOT</u> REGULAR SOLIDS



So teachers may want to describe the five-solid theory, just for fun, *
but without making fun of it, so that pupils can see a theory change *
from being acceptable to being quite unacceptable as knowledge *
grows. Nowadays we think of a scheme of spheres and solids as *
artificial, unscientific, silly. But that is only because our taste has *
been educated by Newton and his successors to expect astronomical *
theories to link up with knowledge of the Physics we meet on *
Earth. In Kepler's day, theory was expected to describe reasonable *
machinery, as the Greek theories did, and geometry was the *
essence of such machinery. *

A century after Kepler, the five-solid rule found no place in *
Newton's theory: it was threatened with exile because it was not *
a respectable member of the club. Then a law left isolated from *
general theory must seem little more than an empirical rule or a *
special demon, but it always offers strong critical doubt. If teachers *
discuss this rule with pupils and comment on it again when *
teaching Newton's theory, we hope they will emphasize this aspect *
of critical exile – because it illustrates a great value of theory *
though also a great danger. They should, also, point out that a *
century later still, one more planet was discovered (Uranus); and *
that broke up Kepler's rule in a matter-of-fact way. *

Kepler's Three Great Laws

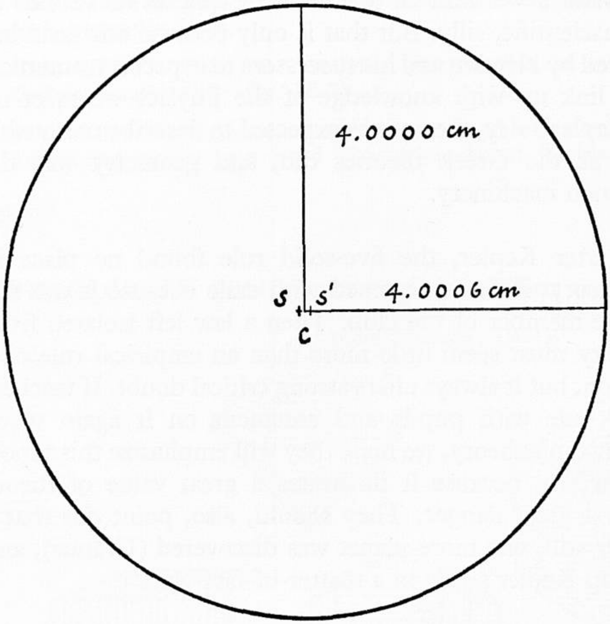
In the course of his lifetime, Kepler extracted the three great T
planetary laws which we now call by his name.

I. The orbit of each planet is an ellipse with the Sun in one focus.

II. The arm from the Sun to planet sweeps out equal areas in equal periods of time. If we mark the position of a planet once a month on its elliptical orbit, and draw radii from the Sun to those points, the areas of sectors between those radii are all equal.

III. If for each planet we take the average orbit radius, R, and the time, T, the planet takes to go once round its orbit (its 'year') then R^3/T^2 is the same for all the planets. This third law which binds the whole planetary system together mathematically Kepler discovered, with tremendous delight, quite late in life.

Drawing an Ellipse. Pupils should try drawing ellipses with two pins and a loop of thread. C52



ELLIPSE: THE EARTH'S ORBIT DRAWN TO SCALE

The actual eccentricity of planetary orbits is very small. The orbits are almost circles, yet Tycho's observations enabled Kepler to show that they are not circles but ellipses. The sketch above shows the Earth's orbit drawn to scale. If a 4·0000 centimetre line is used, as here, to represent the minimum radius, which is really some 93,000,000 miles, the maximum radius needs a line 4·0006 centimetres long. The eccentricity of Mars' orbit is over thirty times as big, but even then the ratio of radii is only 1·0043 to 1·0000. Mercury is the only planet with a much greater eccentricity of orbit, with radii in proportion 1·022 to 1·000. Even this eccentricity of orbit seems small, but it is sufficient to involve Mercury in such speed changes around the orbit that Relativity mechanics predicts a very slow slewing around of the orbit – a precession of only $\frac{1}{80}$ of a degree per *century*, discovered and measured long before the Relativity prediction!

Mars, the Difficult Planet. As a young man, Kepler travelled across Europe to join Tycho Brahe. Together they worked on the orbit of Mars, 'the difficult planet'. We now know that the orbit of Mars is an ellipse; but it is very close to a circle: the ratio of maximum radius to minimum is only 1·0043 : 1·000. Yet the observed motion of Mars differed enough from simple motion round a circle with constant speed to show up clearly in the observations and make Mars 'the difficult planet'.

Kepler was sure that the Copernican solar system would turn out to be the true model. Under the influence of the Greek tradition, he tried circular orbits with the Sun a short distance off-centre and the planet carried round by a constant-speed arm from another point a small distance off-centre. He made dozens of trials with different directions and amounts of eccentric placing. In each trial he used some of Tycho's observations to determine the circle, then continued the motion of his theoretical planet and predicted its position at some other date; and checked that with Tycho's observations.

Scheme after scheme failed to fit and had to be thrown away. When, after many trials, Kepler found an eccentric-circle scheme that fitted well, he made one more test and found his predicted position for Mars differed from the observed position by 8 minutes of angle.

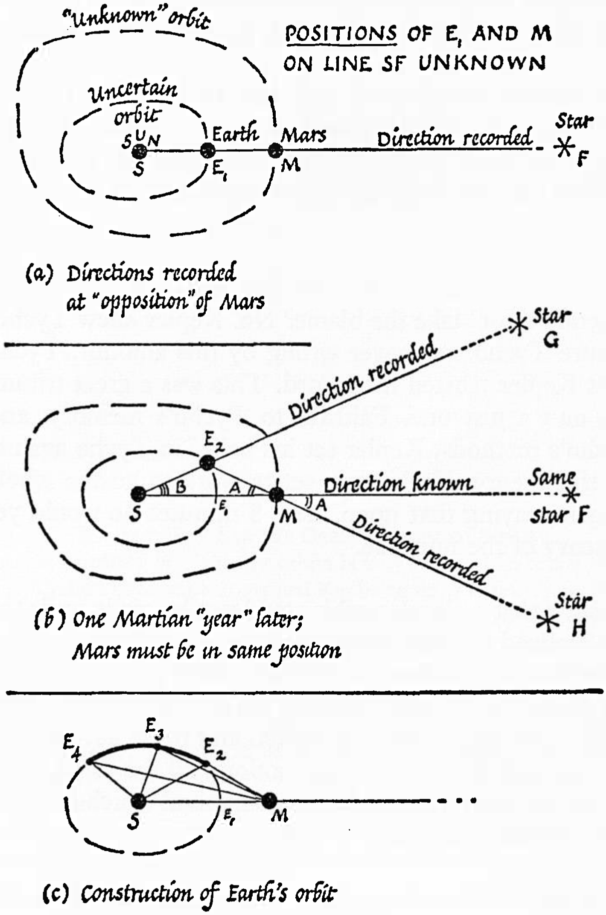
Might not the observations be wrong by this small amount? Would not 'experimental error' take the blame? No. Kepler knew Tycho, and he was sure Tycho was never wrong by this amount. Tycho was dead, but Kepler trusted his record. This was a great tribute to his friend and a just one. Faithful to Tycho's memory, and knowing Tycho's methods, Kepler set his belief in Tycho against his own hopeful theory. He bravely set to work to go the whole weary way again, saying that upon these 8 minutes he would yet build up a theory of the universe.

Then it was clear that a circular orbit could not be made to fit the facts. Kepler realized he must obtain an accurate picture of Mars' real orbit from the observations – not so easy, since we only observe the apparent path of Mars from a moving Earth. The true distances were unknown; only angles were measured and those gave a foreshortened compound of Mars' orbital motion and the Earth's. So Kepler attacked the Earth's orbit first, by a method which Einstein once said was Kepler's real mark of genius.

The following description shows how Kepler did that. It is not offered for teaching – except perhaps to a group with special interests – but is given here for the teacher's own interest.

Mapping the Earth's Orbit in Space and Time. To map the Earth's orbit around the Sun on a scale diagram, we need many sets of measurements, each set giving the Earth's bearings from two fixed points. Kepler took the fixed Sun for one of these, and for the other he took Mars at a series of times when it was in the same position in its orbit. He proceeded thus: he marked the 'position'

of Mars in the star pattern at one opposition (opposite the Sun, *
overhead at midnight). That gave him the direction of a base line *
Sun-(Earth)-Mars, SE_1M. Then he turned the pages of Tycho's *
records to a time exactly one Martian year later. (That time of *
Mars' motion around its orbit was known accurately, from records *
over centuries.) *



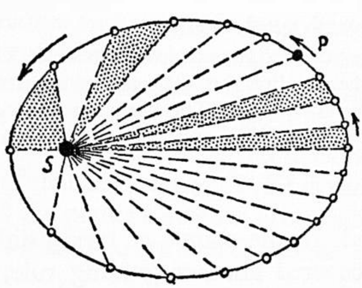
KEPLER'S SCHEME TO PLOT THE EARTH'S ORBIT

Then Kepler knew that Mars was in the same position, M, so that *
SM had the same direction. By now, the Earth had moved on to E_2 *
in its orbit. Tycho's record of the position of Mars in the star *
pattern gave him the new apparent direction of Mars, E_2M; and *
the Sun's position gave him the direction E_2S. Then he could *
calculate the angles of the triangle SE_2M from the record, thus: *

since he knew the directions E_1M and E_2M (marked on the celestial *
sphere of stars) he could calculate the angle A between them. *
Since he knew the directions E_1S and E_2S, he could calculate the *
angle B between them. Then on a scale diagram he could choose *
two points to represent S and M and locate the Earth's position, *
E_2, as follows: at the ends of the fixed base line SM, draw lines *
making angles A and B and mark their intersection E_2. One *
Martian year later still, he could find the directions E_3M and E_3S *
from the records, and mark E_3 on his diagram. Thus Kepler could *
start with the points S and M and locate E_2, E_3, E_4, ... enough *
points to show the orbit's shape. *
*

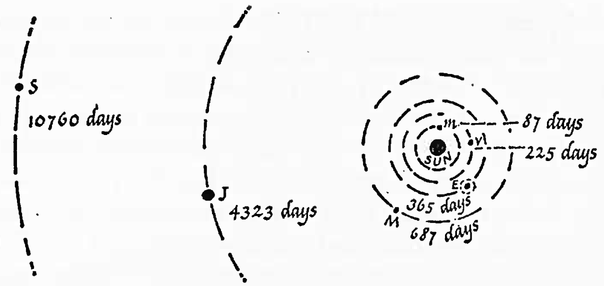
Then, knowing the Earth's true orbit, he could invert the investiga- *
tion, and plot the shape of Mars' orbit. He found he could treat the *
Earth's orbit either as an eccentric circle or as slightly oval; but *
Mars' orbit was far from circular: it was definitely oval or, as he *
thought, egg-shaped, but he still could not find its mathematical *
form. *

Law I. Then, wrestling with a plotted shape for Mars' orbit, Kepler suddenly found it must be an ellipse, with the Sun in one focus. He found that the same rule must hold for the other planets. T

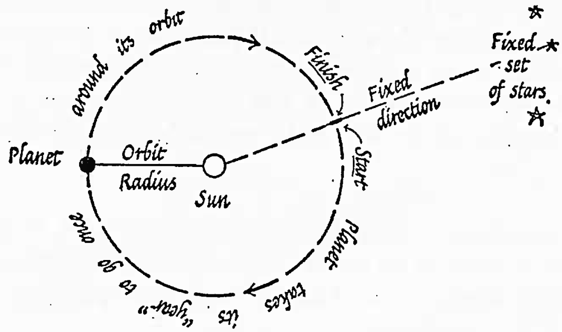


KEPLER'S DISCOVERIES FOR MARS

An ellipse with the Sun in one focus fits the orbit of Mars. The spoke from Sun to Planet sweeps out equal areas in equal times. The positions marked here show planet's positions at equal intervals of time, $\frac{1}{20}$ of its 'year' apart. The planet moves with such speeds that all the sectors marked here – a few of them shaded – have equal areas.



RELATIONSHIP BETWEEN *RADIUS* AND '*YEAR*' FOR PLANETARY ORBITS?
(Planetary orbits roughly to scale.)



PLANET'S 'YEAR'

The planet's year is the time it takes to go once around its orbit. This is the time-interval from the moment when its direction hits some standard mark in the star pattern until it returns to the same mark. (The Earth moves too. An allowance for the Earth's motion must be made when extracting the planet's true year from observations.)

Law II. Variable speed. In the course of trying different orbits for Mars, Kepler discovered another guiding rule, now called his second law. He could not arrange a spoke from any eccentric point to sweep round with constant speed and carry the planet along the orbit, in agreement with the facts. But, instead, he found that an imaginary spoke, running straight from the Sun to the planet, does sweep out area at a constant rate. As for Ptolemy and the Greeks before him, some constancy had to be ascribed to the machinery, or it was useless as a scientific theory. Kepler replaced constant rate of revolution by constant rate of sweeping out area.

PLANETARY DATA – TEST OF KEPLER'S THIRD LAW
(These are modern data, more accurate than Kepler's)

Planet	*Radius of planet's orbit* R *(miles)*	*Time of revolution (planet's 'year')* T *(days)*	R^3 *(miles)*^{3}	T^2 *(days)*^{2}	$\frac{R^3}{T^2}$ $\frac{(miles)^3}{(days)^2}$
Mercury	$3{\cdot}596 \times 10^7$	87·97	$46{\cdot}49 \times 10^{21}$	7,738	$6{\cdot}008 \times 10^{18}$
Venus	$6{\cdot}716 \times 10^7$	224·7	$303{\cdot}3 \times 10^{21}$	50,490	$6{\cdot}008 \times 10^{18}$
Earth	$9{\cdot}290 \times 10^7$	365·3	$801{\cdot}7 \times 10^{21}$	133,500	$6{\cdot}009 \times 10^{18}$
Mars	$14{\cdot}16 \times 10^7$	687·1	$2{,}836 \times 10^{21}$	472,100	$6{\cdot}008 \times 10^{18}$
Jupiter	$48{\cdot}33 \times 10^7$	4,333	$112{,}900 \times 10^{21}$	18,780,000	$6{\cdot}012 \times 10^{18}$
Saturn	$88{\cdot}61 \times 10^7$	10,760	$695{,}800 \times 10^{21}$	115,800,000	$6{\cdot}011 \times 10^{18}$

The test of Kepler's guess is shown in the last column.

Law III. Connecting the motions of all the planets, Kepler had then extracted two great 'laws' from Tycho's tables, by his fearless thinking and untiring work. He continued to brood on one of his early questions: what connection is there between the sizes of the planets' orbits and the times of their 'years'? He now knew the average radii of the orbits; the times of revolution ('years') had long been known. (As the Greeks surmised, the planets with the longest 'years' have the largest orbits.) He felt sure there was some relation between radius and time. He must have made and tried many a guess, some of them sterile ones like his early scheme of the five regular solids or wild mystical ones like his speculation of musical chords for the planets. Fortunately there is a connection between radii and times, and Kepler lived to experience the joy of finding it. He found that the fraction R^3/T^2 is the same for all the planets, where R is the planet's average orbit radius, and T is the planet's 'year', measured in days. See the above Table.

In teaching this, we must remember that our pupils are not waiting eagerly for the answer to a great, urgent question, as Kepler was. They do not even feel the strong need to find constancies in nature, which is one of the main drives in science. Our pupils have grown from the stage of seeing things, collecting facts, of making fuller and fuller acquaintance with nature; and they have watched us extract some rules, codify our acquaintance. Yet we should have been careful not to emphasize those rules in a way that would make them formulae to be learnt by heart for use in answering examination questions. Therefore, rules and relationships are likely to be interesting to pupils, but not yet recognized as the essence of our knowledge.

Teachers will need to preface their discussion of Kepler's Law III by asking the questions that were in Kepler's mind, by sketching the relative sizes of planetary orbits on the blackboard, and writing the planets' orbital periods ('years') opposite them. T

The questions need to be quite clear:

> 'Is there any simple relationship connecting these?'

> 'Is there something which you could work out for Jupiter's orbit size and orbit time, for Mars' orbit size and orbit time, for Earth's orbit size and orbit time, and so on ... and get the same answer for each planet?'

We need to explain that Kepler, not knowing the right answer – not even sure that there was one – had to try many combinations such as R/T, R^2/T, working out the value each time to see if it came to the same answer for all the planets. At last he found that R^3/T^2 did. Some teachers may wish to publish part of the table of data, just the values of R and T, a week or more before the discussion of Kepler's Law III, and pose it as a prize problem to see if any pupils can discover that R^3/T^2 is the same. Given a hint by the teacher demonstrating *with quick arithmetic*, some false starts, such as R/T^2, some pupils may succeed.

Kepler was a mathematical speculator. He looked for many kinds of connections among planetary data and found some that he considered successful. His three great Laws were clear, simple, and powerful. We still hold them as descriptions that fit the facts very accurately. If all the planets were controlled by the Sun alone and exerted no disturbing effect on each other, we should expect inverse-square-law gravity to hold them in elliptical orbits, fitting Kepler's Laws perfectly.

GALILEO, THE POWERFUL ADVOCATE

Galileo (1564-1642) was a tremendous teacher who did much to prepare physics for the Newtonian development. He insisted on making theory realistic by tying it to experiment, and on putting laws of physics in mathematical form as far as possible. Contrary to the popular myth, he was a mathematical codifier and arguer rather than a precise experimenter. He did no experiments, but quoted them in rough form to support his arguments about Nature.

In sorting out mechanics from mystical speculation, Galileo arrived at Newton's first Law – a moving body continues to move if left alone – by an ingenious argument. He knew that a constant force, such as the weight of a body, produces constant acceleration, constant rate-of-change of speed. (A number of scientists had stated that much earlier, but it was Galileo who expounded it clearly.) And he laid the foundation, by arguments and experiments, for the Newtonian relationships between force, mass, and motion.

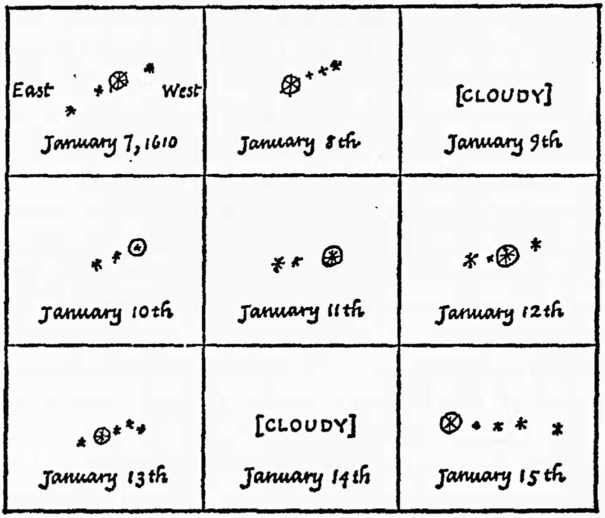
Teaching the Copernican System. While Kepler was at work on the planets, Galileo, who corresponded with him, was preaching and teaching the Copernican system with great enthusiasm. As a teacher of tremendous power he was able to put the case for Copernicus in such clear and compelling form that for the first time readers far and wide understood it and saw its full import. He taught it as true and gave strong reasons for believing it. And when he had built his telescope he showed the system of Jupiter's moons as a scale model of the Copernican system itself. This was a violent and disturbing attack on established thinking; on the central position given by all humanist scholars to the Earth and Man; and on the teaching of the Ptolemaic system by the Church. When Galileo insisted the Copernican system was *true*, he was attacking the general authority of the Church. This brought Galileo, himself a devout member of the Roman Catholic Church, into grave trouble with the Church authorities. We look back on such troubles as resulting from Galileo's argumentative temper and his fanatical insistence that the Copernican system was not just theory but true. They were signs of the times, part of the picture of relations between Church and State and Science in that age.

Telescope. In the middle of his teaching life, Galileo heard rumours of an optical instrument that would make distant things look closer. He designed a telescope himself and made a small, weak one. Then he made larger and stronger telescopes, always grinding the lenses himself.

When he turned his telescope to the heavens, he saw many surprising things, some of them very disturbing to the traditional view that was being taught by Church authorities and other astronomers. He saw the Milky Way resolved into a cloud of tiny stars. He saw that the Moon is rocky, with mountains and craters – that was a shock to astronomers, who thought of the Moon as a perfect, shining sphere, free from any defect. Galileo saw the Moon as earthy, rocky and ordinary. He saw spots on the Sun, again a disturbing modification of the view that the heavens were perfect.

He saw the planet Venus in crescent shape, changing to other 'phases' as it moved round the Sun – difficult to reconcile with the Ptolemaic machinery.

He looked at Jupiter, noting some small stars near it; and then, next night, found the pattern changed.



GALILEO'S OBSERVATIONS OF JUPITER'S MOONS
These sketches are copied from Galileo's handwritten record. (The orbits of the moons are nearly in planes containing our line of sight from Earth to Jupiter; so the moons are often in front of Jupiter or behind, and they are often eclipsed by moving into Jupiter's shadow. They move quickly around their orbits. That is why the pattern changes so quickly and why, often, less than four moons are visible.) (For a copy of Galileo's written record, see *Galileo* by J. J. Fahie.)

Jupiter seemed to have moved the wrong way relative to those stars. Waiting impatiently through a cloudy night, Galileo then saw the pattern change again. It was clear the small stars were moons moving around Jupiter. Delighted with this, Galileo published a full description and claimed Jupiter and his moons as a model to support his strong arguments for the solar system of the Copernican view.

The Development from Copernicus to Newton

Galileo brought the Copernican picture out into public knowledge. He devised telescopes, which were to bring astronomy to much higher levels of precision in future generations. And he developed and taught, in unfinished form, the new view of force and motion that Newton was to use in his theory. Kepler extracted precise, reliable laws of planetary motion; but he gave only vague suggestions for the mechanism of forces to produce such motion. The educated world, philosophers, scientists, and other thinkers, were soon alive with questions about this solar system of which people then knew so much but understood so little. It was this climate of active discussion, with scientific societies being formed and growing here and there in Europe, that Newton grew up as a young man and took astronomy to his heart.

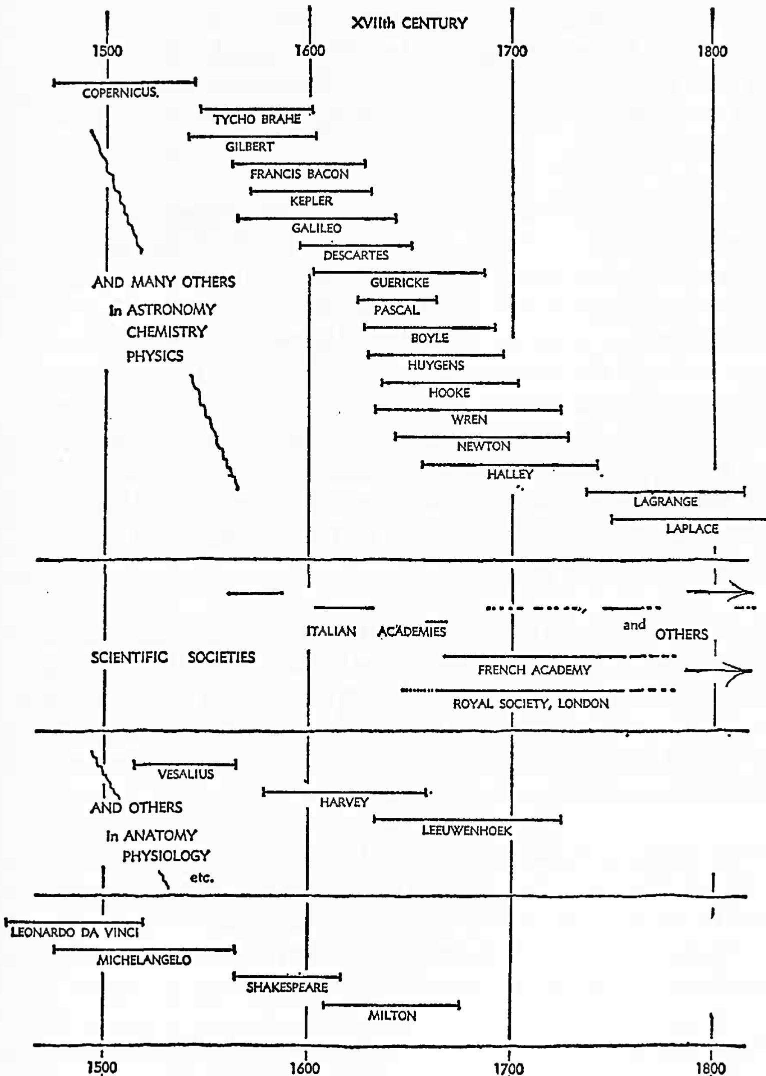
1500–1700. Two Centuries of Development

In 1500 the Ptolemaic scheme reigned unquestioned – Man and the all-important Earth at the centre of a universe of revolving spheres and sub-spheres. Copernicus was a young man, just beginning to think out his new system of the world. In 1700 Newton's *Principia* had been published and was being discussed far and wide: the picture of the heavens had swung over completely to a solar system described in accurate laws and now explained in terms of universal gravitation.

This change of view, and its full acceptance, went far beyond a mere replacing of one celestial scheme by another. It influenced man's view of his own position in the universe; and his attitude to nature, natural laws, and scientific explanations. It threatened to modify his choice of philosophic outlook on divine power, free-will and determinism.

Such developments in astronomy were part of a tremendous change of intellectual interests, a change of the fashionable flavour of mind, a move away from the full power of traditional authority towards questioning, experimenting, new scientific knowledge. It was not that people had been stupid in earlier days. They had been just as able as in any generation, but had other interests; and the political and economic climate had not favoured new science. But by 1600 scientific enquiry was becoming respectable, almost a matter for general intellectual interest; and through the seventeenth century science grew in respectability and popular interest, and power. Scientific societies were formed to discuss experiments and exchange news – science became public as well as respectable. Mathematical tools developed: algebra came into use to supplement geometry, coordinate geometry was devised, and (because the time was ripe for it) calculus was invented – all bringing great help to scientific discovery. Instruments were devised – telescope, microscope, vacuum pump, barometer, thermometer and pendulum clock were all invented or developed in the seventeenth century. And there was a new attitude to scientific knowledge: experiment joined with speculative theory to promote great developments.

Teachers may wish to show a time-chart starting with Copernicus, running through the great band of scientists in the seventeenth century and on to the mathematicians who continued Newton's work in the century beyond. That should show the development of the scientific societies; and it might include great names outside science. A specimen of such a chart is shown opposite.


XVIIth CENTURY
1500
1600
1700
1800
COPERNICUS.
TYCHO BRAHE
GILBERT
FRANCIS BACON
KEPLER
GALILEO
DESCARTES
GUERICKE
PASCAL
BOYLE
HUYGENS
HOOKE
WREN
NEWTON
HALLEY
LAGRANGE
LAPLACE
AND MANY OTHERS
In ASTRONOMY
CHEMISTRY
PHYSICS
ITALIAN ACADEMIES
and OTHERS
SCIENTIFIC SOCIETIES
FRENCH ACADEMY
ROYAL SOCIETY, LONDON
VESALIUS
HARVEY
LEEUWENHOEK
AND OTHERS
In ANATOMY
PHYSIOLOGY
etc.
LEONARDO DA VINCI
MICHELANGELO
SHAKESPEARE
MILTON
1500
1600
1700
1800

NEWTON'S THEORY AND ITS FRUITS

The history of Newton's life and work and accounts of his person- *
ality are given in several good biographies. (More's is now available *
in paperback; so is the *Principia* in English translation.) *

Newton (1642-1726) and Astronomy. As a young man, soon T
after taking his degree at Cambridge, Newton thought about astronomy for his own pleasure. He quietly speculated about the whole solar system being controlled by universal gravitation; and he started by trying out the idea of inverse-square-law gravity on the Moon's motion.

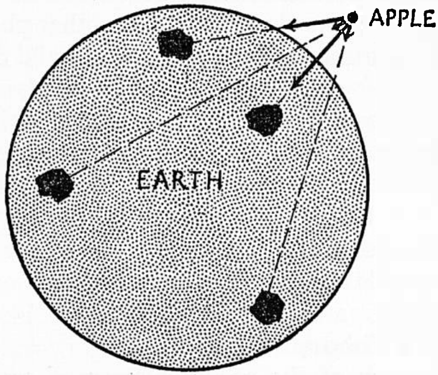
Great astronomical questions were in the air: the time was ripe for a complete change of viewpoint. The old view had been that motion round a circle is the natural one – in the heavens – so people did not ask why a planet moves round an orbit. But they did ask why it continues to move; and the answer was 'because a force continues to push it along'. On the new view, there was no need to ask about that force: no force is needed. It was already clear from the writings of Galileo that the Greek idea of motion needing a force to maintain it was wrong – at least for motion on Earth. Left alone, a moving object would continue in a straight line with constant speed; so the revolving spoke imagined by Kepler and others to push a planet along its orbit was unnecessary.

That was a revolutionary idea: no force needed *along* the orbit. On the other hand it was fairly clear from Galileo's work that *changes of motion* (changes from constant speed in one straight line) *do* need forces. So the new question was: 'What forces shape the planetary orbits? What force or other mechanism could account for the new, precise laws of orbits?'

Kepler's Laws were the talk of the day in the scientific world. Several scientists saw that a body moving round a circular orbit should be regarded as having an acceleration towards the centre. Huygens, Hooke, Newton, and probably others too, worked out the expression v^2/R for that inward central acceleration.

Newton arrived at his own scheme for deriving $a = v^2/R$ and tried it on the Moon's motion, to test inverse-square gravity. He was discouraged and put away the calculation without saying anything – in his characteristic way of avoiding controversy.

There is a rumour among amateur historians that Newton's test *
failed because he did not have an accurate value for the radius of *
the Earth. That is probably untrue. It is much more likely that *
Newton stopped because he saw the grave difficulty of calculating *
the attraction of a large body like the Earth on an object nearby. *
The attraction of the Earth on the Moon is easily dealt with as a *



Newton's Difficulty. Assuming inverse-square-law forces, how does a solid Earth attract a small object nearby? Consider equal samples of rock A, B, C, D. A would exert a big pull, and D a very small pull; and B and C pull in slanting directions. What is the resultant for the whole Earth? How far from the apple should we imagine the whole mass of the Earth concentrated to give the same resultant attraction? (Surprising answer: one radius away; at Earth's centre.)

force between point-objects. But Newton had to compare that *
force with the attraction of the whole Earth on, say, an apple. In *
the latter case the adding up of inverse-square attractions from all *
the pieces of a great round globe seemed impossible until calculus *
had been invented. When Newton solved this problem, he was *
surprised and delighted at the unexpectedly simple result; that *
the Earth attracts as if its whole mass were concentrated at its *
centre. (This is not a matter to be taught, when we are aiming at *
Newton's whole theory; but it might be mentioned to a fast group.) *

As discussions of Kepler's Laws spread, there was growing interest T
in them in the young Royal Society in London, and there were attempts to show that something like inverse-square-law gravity would account for them. Several people could show that it would account for Kepler's Law III; but elliptical orbits and the area law remained too difficult. An appeal to Newton in Cambridge produced the characteristic reply that he had already tried the problem, solved it, and shelved it.

After much encouragement, Newton gathered his work together and expanded it into a great book, the *Principia*. There he set forth some definitions and general rules or laws for motion, and then applied them to the whole heavenly system. His own account of his purpose was:

'from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; … the motions of the planets, the comets, the moon and the sea …'

If we are to teach our young pupils in a way that will give them a useful picture of the growth of theory, we must not at this point pour out dramatic praise in sermons about Newton's greatness; we must show them that this theory *was* great by carrying them through its achievements. We must not make statements about the theory, we and our pupils must *do* it.

Teaching Newton's Theory

Pupils could read much of the earlier history of astronomical knowledge and theory on their own, but at this stage it seems essential for the teacher to immerse himself in the unrolling of Newtonian gravitation and give pupils a sense of his own enjoyment of it.

We suggest teachers should set the stage by describing inverse-square-law forces and going through Newton's test on the Moon's motion. So we give below a suggested outline of that teaching in some detail.

When teachers come to the unrolling of Newton's theory – the target of all our teaching of astronomy – there will be wide variations both in their choice of treatment and in their needs for detailed information. Therefore we shall offer first a summary of the story we hope will be taught. Then we follow with an account giving considerable details, to provide a background of guidance and suggestions. There it is difficult to distinguish clearly between commentary to teachers (* * * …) and direct suggestions for teaching (T …). We hope that each teacher will read and think about the development of Newtonian theory and then teach it in his own way.

A Tall Chart. As a useful guide in that teaching, we offer a tall chart of Development of Astronomy, which notes facts and ideas D54
for the development of theory. The later part of that gives the

main items or achievements of Newtonian theory. Teachers will find that an enlarged copy of the chart, pasted together to make one tall column and exhibited in class, is a help as a 'prompt copy' in teaching. If it can be kept posted in the classroom, pupils will see the development and may feel the tremendous power of Newton's theory when they see its results written there like a series of blows from an insistent hammer.

A large version of the chart could be mounted as an endless loop like a roller-towel, on a pair of roller-blind rollers. Or it could be arranged to roll up on the upper or lower roller, so that only a short section is visible at one time. The longer the visible section, the better, of course.

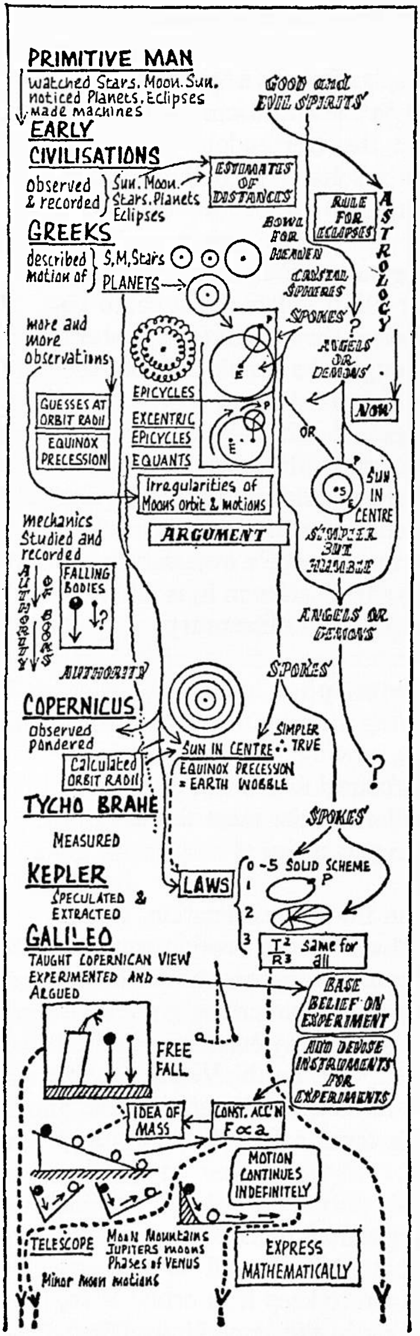
We should *not* encourage pupils to copy the chart; though a miniature version of it in a Pupils' Guide would be useful. Copying it is likely to achieve the wrong flavour: neat drudgery instead of admiration for growing power. But making one's own version of the chart, bringing careful thought to a fresh start on it, would be a very valuable activity which some pupils might like to try.

Newton's Idea of Universal Gravitation

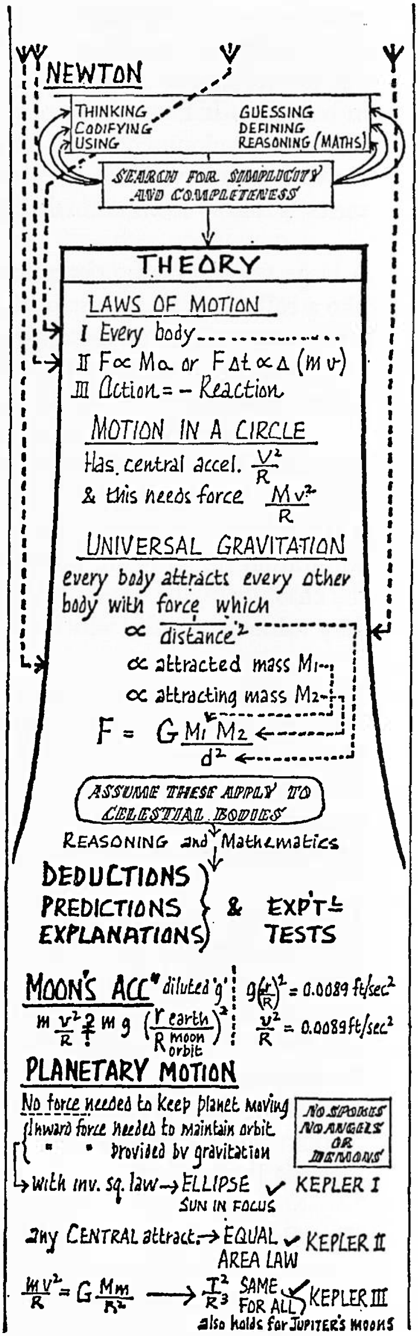
Newton saw that no force is needed *along* the orbit to keep a planet, or the Moon (or any other satellite), moving; but that an *inward* force is needed to change motion continuing along a straight tangent to motion round an orbit. The satellite must fall inward, from tangent to orbit, again and again and again.

That falling could be described as an inward acceleration, which Newton showed should have the value v^2/R for motion round a circle. If that kind of acceleration does need a force in the same way as acceleration along a path (in which the moving thing goes faster and faster) there must be an inward pull acting on every satellite. Something must pull the Moon towards the Earth; Venus, Earth, Jupiter, and all the other planets towards the Sun; and Jupiter's moons (discovered and seen with telescopes before Newton's day) towards Jupiter.

The Earth does exert an inward pull on objects nearby: the common pull of gravity. Could that kind of pull extend out farther and provide a pull on the Moon to keep it in orbit? If so, could the same kind of pull, spreading in the same kind of way, extend from the Sun to the planets; and from Jupiter to Jupiter's moons? Newton tried out that idea first on the Moon's motion.



PRIMITIVE MAN
watched Stars. Moon. Sun.
noticed Planets. Eclipses
made machines
EARLY CIVILISATIONS
observed & recorded
Sun. Moon. Stars. Planets Eclipses
ESTIMATES OF DISTANCES
GOOD and EVIL SPIRITS
ASTROLOGY
RULE FOR ECLIPSES
BOWL FOR HEAVEN
CRYSTAL SPHERES
SPOKES
GREEKS
described motion of
S, M, Stars
PLANETS
more and more observations
GUESSES AT ORBIT RADII
EQUINOX PRECESSION
EPICYCLES
EXCENTRIC EPICYCLES
EQUANTS
ANGELS OR DEMONS
NOW
OR
SUN IN CENTRE
SIMPLER BUT HUMBLE
Irregularities of Moons orbit & motions
ARGUMENT
mechanics studied and recorded
AUTHORITY OF BOOKS
FALLING BODIES
AUTHORITY
ANGELS OR DEMONS
SPOKES
COPERNICUS
observed pondered
Calculated ORBIT RADII
SUN IN CENTRE ∴ SIMPLER ∴ TRUE
EQUINOX PRECESSION = EARTH WOBBLE
?
TYCHO BRAHE
MEASURED
SPOKES
KEPLER
SPECULATED & EXTRACTED
LAWS
0 - 5 SOLID SCHEME
1
2
3
T^2/R^3 same for all
GALILEO
TAUGHT COPERNICAN VIEW EXPERIMENTED AND ARGUED
BASE BELIEF ON EXPERIMENT
AND DEVISE INSTRUMENTS FOR EXPERIMENTS
FREE FALL
IDEA OF MASS
CONST. ACC'N $F \propto a$
MOTION CONTINUES INDEFINITELY
TELESCOPE
Moon Mountains JUPITERS moons Phases of VENUS
Minor Moon motions
EXPRESS MATHEMATICALLY



NEWTON
THINKING CODIFYING USING
GUESSING DEFINING REASONING (MATHS)
SEARCH FOR SIMPLICITY AND COMPLETENESS
THEORY
LAWS OF MOTION
I Every body........
II $F \propto Ma$ or $F\Delta t \propto \Delta(mv)$
III Action = − Reaction
MOTION IN A CIRCLE
Has central accel. $\frac{V^2}{R}$
& this needs force $\frac{Mv^2}{R}$
UNIVERSAL GRAVITATION
every body attracts every other body with force which
$\propto \frac{1}{\text{distance}^2}$
$\propto$ attracted mass M_1
$\propto$ attracting mass M_2
$F = G\frac{M_1M_2}{d^2}$
ASSUME THESE APPLY TO CELESTIAL BODIES
REASONING and Mathematics
DEDUCTIONS PREDICTIONS EXPLANATIONS & EXPT'L TESTS
MOON'S ACC" diluted 'g'
$m\frac{v^2}{R} \overset{?}{=} mg\left(\frac{r_{earth}}{R_{moon\ orbit}}\right)^2$
$g\left(\frac{r}{R}\right)^2 = 0.0089$ ft/sec²
$\frac{v^2}{R} = 0.0089$ ft/sec²
PLANETARY MOTION
No force needed to keep planet moving
Inward force needed to maintain orbit
" " provided by gravitation
with inv. sq. law → ELLIPSE SUN IN FOCUS ✓ KEPLER I
any CENTRAL attract. → EQUAL AREA LAW ✓ KEPLER II
$\frac{mv^2}{R} = G\frac{Mm}{R^2} \longrightarrow \frac{T^2}{R^3}$ SAME FOR ALL ✓ KEPLER III
also holds for JUPITER'S moons
NO SPOKES NO ANGELS OR DEMONS

MASSES OF SUN, JUPITER

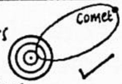
& any planet having satellite can be calculated in terms of Earth's mass.

? No check yet
No alternative method of estimating ?

COMETS – Solar system visitors

(elliptical orbits).

return on time.



TIDES due to differences of Moon's attraction on ocean.

Connection with Moon long suspected (Galileo laughed)

∴ 2 tides per 24 hrs. ✓

Sun also causes (smaller) tides

spring tides due to M+S   neap tides due to M−S

MASS OF MOON

Estimate by comparing its tide to Sun's tide and using Sun's mass and distance

No check yet but density reasonable

TIDES CAN NOW BE PREDICTED YEARS AHEAD

MOON'S MOTION has many IRREGULARITIES

{ orbit changes, eccentricity, tilt etc. speed changes

Moon's speed in orbit changes during month and during year. Orbit precesses, changes its eccentricity, its tilt etc. Newton showed these changes are due to differences of Sun's pull. He predicted a number of these succesfully.

BULGE OF EARTH

Assuming earlier pasty earth Newton calculated shape

{Pole Vs equator}

Too small to measure then LATER GEODESY VERIFIED

and effect orig' of bulge and centrifugal force.

PRECESSION OF EQUINOXES

Due to pulls of Sun and Moon on Earth's extra bulges making it wobble like a spinning top.

Newton calculated time of wobble – 26,000 yrs ✓

agreed with astronomical estimates

PERTURBATIONS OF PLANETS.

Planet not held in orbit by Sun alone but also pulled slightly by other planets. These small extra pulls change orbit slightly.

[Not directed to Sun ∴ Kepler II xx]

Newton started this investigation

Lagrange
Laplace
{ Continued it and showed it does account for observed changes of orbit

Adams
Leverrier
used $F = G\frac{M_1 M_2}{d^2}$ to discover NEPTUNE from its tiny effects on orbit of Uranus } ✓

BUT

A minute residual change of Mercury's orbit was not explained until EINSTEIN modified NEWTON'S gravitation law slightly

Gravity Pulling the Moon. As pupils will have found if they tried the brown-paper drawing experiment for the Moon, plain gravity at full strength provides much too big a force to account for the Moon's motion.

Since most pupils will not have tried that extension of the brown-paper work, the teacher should make a test now by algebra and arithmetic. The following data are needed:

Radius of Moon's orbit $= 60 \times$ [Earth's radius]

$= 240{,}000$ miles $= 3{\cdot}84 \times 10^8$ metres.

Time taken by the Moon to go once round its orbit (=1 month!) = 27·3 days‡

We first calculate the Moon's actual, central acceleration, v^2/R.

(Teachers used to dealing with such problems at a more advanced level will be tempted to change v^2/R to $R\omega^2$, where ω is the angular velocity of the Earth–Moon radius. However, we shall not make enough use of this kind of calculation to justify introducing ω here. That would only add a new, confusing quantity; so we do *not* recommend that change, and we hope that teachers who are tempted to make it will first try teaching this material in the way suggested below.)

We first calculate v as follows:

v = speed along the orbit = circumference/period $= 2\pi R/T$

$$= \frac{2\pi[3{\cdot}84 \times 10^8 \text{ metres}]}{[27{\cdot}3 \text{ days} \times 24 \text{ hrs/day} \times 3{,}600 \text{ secs/hr}]}$$

With a fast group, we might leave that in factors and continue with the next stage of the arithmetic. An average or slow group will

‡ 27·3 days looks surprisingly short for a month, but it is the true time taken by the Moon to make one complete orbit, judging its direction by the fixed stars. However, since the Earth is also moving (round the Sun in a year) in the same direction, the Moon has to travel farther to return to the same position *relative to the Sun.* That is why the Moon month in our calendar is about 29½ days from full Moon to full Moon. For our calculations of the Moon's motion we need the true period, 27·3 days.

find it more comfortable to work out the Moon's speed along the orbit now. The result is:

$v = 1{,}022$ metres/second (about 2,300 miles/hour)

Then we calculate the inward acceleration that the Moon must have to continue round this orbit v^2/R:

$$\frac{v^2}{R} = \frac{[1{,}022 \text{ metres/sec}]^2}{[3{\cdot}84 \times 10^8 \text{ metres}]}$$

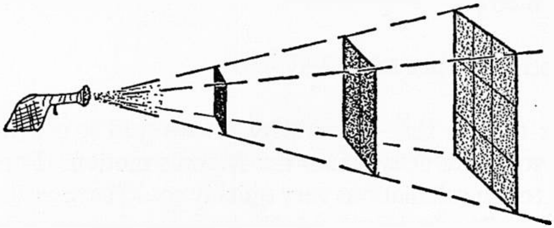
$$= 0{\cdot}00272 \text{ metres/sec per sec.}$$

One look at that tells us that full gravity, as we find it near the Earth, is far too strong to account for the Moon's motion. (Pupils who enjoy doing rough calculations very quickly could have assured themselves of that by condensing the work above to rough estimates – and that would be a good scientific move.) 'Well then, what gravity *is* that – if it is gravity at all?'

Inverse-square Law. At this point we explain that Newton, like the other scientists who were worrying about 'explaining' Kepler's Laws by gravity, had to try some scheme of diluting gravity, thinning it out at greater distances.

> 'The simplest scheme, halving gravity when the distance doubles, does not fit the result above (that would give $\frac{1}{60}$ of our g at the Moon, or about 0·17 metres/sec²).
>
> 'Another simple scheme is an inverse-square law. That would be a sensible one to try because it is the way in which anything thins out if it sprouts in straight lines from a small source and *continues out in straight lines without getting lost*. Light from a small lamp does that. At double distance the same light, spreading out through clear air without being absorbed, falls on 4 times the area at double distance, giving only $\frac{1}{4}$ of the illumination, and so on for other distances.
>
> 'Water, spreading from a hidden spring in a lake or ocean, would flow out in straight lines, and the speed of flow would have to be $\frac{1}{4}$ as great at twice the distance from the spring – otherwise water would be disappearing or being created.'

We, as physicists, are familiar with an inverse-square law for such things, and for the electric field spreading out from a point-charge and the magnetic field from a compact 'pole', so we are apt to announce an inverse-square law for gravity as an obvious easy guess. But to our pupils it is strange and new, as it was to Newton's contemporaries. We need to give pupils a clear idea of what it means, and to emphasize its nature as the essential characteristic of anything that spreads out in straight lines without getting lost. A crude example which helps in teaching is the story of the 'butter gun':



Inverse-square law: 'Butter Gun'

'Suppose the owner of a restaurant invents a gadget to butter many slices of toast efficiently. It is a small, motor-driven sprayer that squirts out a fine spray of specks of melted butter from its muzzle, in straight lines, in a wide cone. T

'Suppose the cone of spray just covers one slice of toast held one foot from the muzzle. The toast is held there for a standard time, say one second. The toast receives rich buttering. Now place toast 2 feet away instead. At that double distance the cone is twice as high and twice as wide. The spray now covers 4 slices of toast, and each of them receives medium buttering in one second. Now place the toast 3 feet away; and the same cone of spray covers 9 slices: economy treatment.

'Therefore on specimen slices of toast, placed 1 foot, 2 feet, 3 feet, ... from the muzzle the *thickness of buttering* will be in the proportion $1 : \frac{1}{4} : \frac{1}{9} :$, ...

'This is the inverse-square law of buttering.'

We may also illustrate the inverse-square law by showing it holds for illumination at various distances from a small lamp in a darkened room. To show that by measurements with a traditional photometer would be tedious and probably confusing because of the way the inverse-square law is used in photometry. But simple, D 53 direct-reading light-meters for photography are now so common that many pupils accept them like stopwatches. So we advise teachers to borrow one and use it here. Hold the light-meter 1 foot, 2 feet, 3 feet, ..., from a small lamp and show its readings.

(Of course, a photo-diode could be used, with a demonstration meter; but that is likely to take too much time and attention when we are anxious to proceed.)

(Remember, when using examples of illumination or of sound out of doors, that the human eye and ear do not judge intensity on a linear scale.)

Newton's Test. When pupils are familiar with the meaning of an inverse-square law, we return to Newton's problem and try diluting gravity. Except with an unusually fast group, we do not worry over Newton's difficult question: How should the attractions of all the parts of the Earth on an apple be added up? That asks: What is the 'distance' from Earth to apple which we must use when diluting g for the Moon? We simply take Newton's result for granted and say:

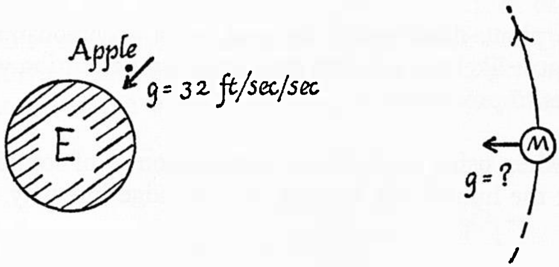
'Newton himself said that the idea of gravity extending out to the Moon and controlling the Moon's motion occurred to him when an apple fell on his head as he sat in an orchard. (Most such stories of great men are fables. We know that this one is true; because we have a letter in which Newton related it.) He tried making gravity "thin out" with an inverse-square law.

'An apple near the Earth falls with an acceleration 9·8 metres/sec^2. We count the apple as being 1 Earth's radius from the centre of the Earth; and we know the Moon is 60 Earth-radii from the centre of the Earth. (We know that from measurements like the one the Greeks made with eclipse shadows.) Then, the Moon is 60 times as far away as the apple. If an inverse-square law *does* apply to gravity, g out at the Moon would be, not 60 times smaller, but 60^2 times smaller.

'g would be $9{\cdot}8/60^2$ or 0·00272 metres/sec^2.

'Now look back at the inward acceleration we said the Moon must actually have if it is to get round its orbit in 27·3 days. Newton looked at the two accelerations and was very pleased with what he saw.'

That was Newton's test on the Moon. He was trying out the idea of inverse-square law of gravity extending from the Earth to the Moon.

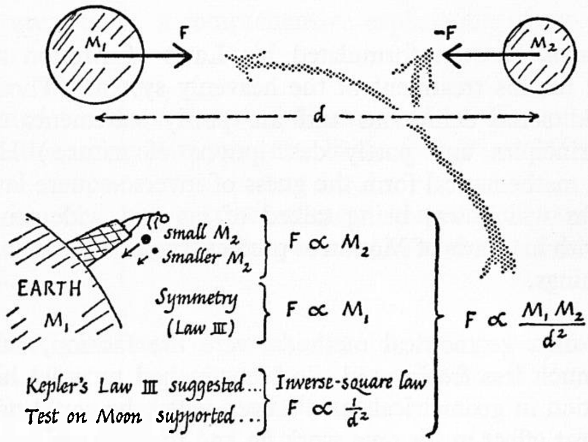


Earth's Gravity

(Newton was also trying out some other things too concerning motion round an orbit: the use of v^2/R for central acceleration; and the idea that a force produces such an acceleration, just as forces in our earthly laboratory produce the more familiar kind of acceleration in which trolleys, etc., move faster and faster. In other words, Newton made several assumptions in his prediction for the Moon; inverse-square law for gravitational attraction, $a = v^2/R$ and $F = ma$. The success of his test might vouch for them all, or it might mean that two or more of those were wrong in ways that compensated. But in our teaching we should take the simpler view that the successful test showed that Newton was entirely on the right lines. Thus, we say that he not only felt sure of inverse-square law attraction from the Earth but also had made a successful test of the idea that acceleration v^2/R 'across the motion' does need a force like any other acceleration.)

(Newton did not have to guess an inverse-square law 'out of the blue', trying it merely because it seemed a simple idea. He himself said that he got a clear hint by working backwards from Kepler's Law III and finding that an inverse-square law of attractive forces from the Sun would fit with that.)

Universal Gravitation. With an inverse-square law for gravity guessed at and tested on the Moon's motion, Newton tried universal gravitation for the solar system.



We know that the Earth's attractions on different objects nearby are directly proportional to the masses of those objects. We know that from the fact that the pulls all produce the same acceleration, g, when different masses are allowed to fall freely. Newton argued by symmetry that the attractive force should also be proportional to the mass of the attracting body – Earth, Sun, Jupiter, etc. Then with two masses, M_1 and M_2, Newton could describe the gravitational attraction of each on the other by

$F = G\dfrac{M_1 M_2}{d^2}$ where G is a *universal* constant.

We shall use this Law of Universal Gravitation in describing Newton's explanation of Kepler's Law III. So this is a point at which teachers will need to discuss that equation quite carefully, pointing out that G is a universal constant.

G is quite different from g. For the Earth of mass M and an apple of mass m, we have:

m kilograms $\times g$ newtons/kg $= GMm/r^2$, where r is the Earth's radius

$\therefore g = G$[mass of Earth]/[radius of Earth]^2.

(Teachers who like thinking in terms of field strength of electric fields will see that g here is the field strength of the Earth's gravitational field.)

Newton's Laws. Newton formulated his Laws of Motion as working rules for his treatment of the heavenly system. (Those laws contained some definitions and are partly statements of accounting principles and partly descriptions of nature.) He formulated in mathematical form the guess of inverse-square law and gravitation which was being talked of far and wide, and combined it with his Laws of Motion to predict Kepler's Laws and many other things.

In Newton's day, geometrical methods were the fashion, and algebra was much less freely used. So Newton had to write his public exposition in geometrical form – even where he could use calculus to great effect in his own work he had to reinforce it by geometrical proofs that would appeal to mathematicians who still regarded calculus as strange.

Developing Theory. Having guessed at inverse-square law gravitation (as he said, by working backwards from Kepler's Law III) and tested in it on the Moon's motion, Newton made a fresh start and constructed a *deductive* theory. That is a structure that starts with assumptions – general principles or laws assumed to hold – and draws from them predictions and explanations of nature by logic. Logic, in this matter, includes the reasoning that we call mathematics: geometry, algebra and the calculus that Newton and Leibnitz developed.

Newton chose his assumptions with a careful eye on the behaviour of nature. They were in fact his Laws of Motion and his Law of Universal Gravitation, together with rules for treating accelerations, forces and momenta as vectors which could be added by the parallelogram method. Granted those few assumptions, Newton could lead from them to many predictions, some of them things already known, such as Kepler's Laws; and other things yet to be discovered – all of them now known and found to fit.

Our main work in teaching is now to describe the great list of things Newton extracted from his theory, aiming more at piling up a great record of successes than teaching the details of each item fully.

Here is the short summary of that teaching: longer notes follow.

Newton's Theory and its Predictions (summary of teaching)

Newton made a fresh start. He assumed four laws and unrolled, in his great book, a comprehensive explanation of the heavenly system. T

He stated his three Laws of Motion and his Law of Universal Gravitation, which we now write in the form $F = GM_1M_2/d^2$

Every object in the universe attracts every other object with an inverse-square-law force, proportional to each of the masses concerned.

Results. From those as a starting point, taking them for granted as true, Newton argued out many a prediction. He 'derived' or 'explained' or predicted the following:

1. **The Moon's motion** round the Earth, controlled by inverse-square-law gravity (Newton's original test).

2. **Kepler's Law I.** Planets' orbits are ellipses with Sun in one focus.

3. **Kepler's Law II.** Arm from Sun to planet sweeps out equal areas in equal times (shown to be necessary for *any* 'central' force).

4. **Kepler's Law III.** [Orbit radius]^3/[Planet's year]^2 same for all planets of the solar system.

5. **Planet's moons:** same rule applies to all the satellites of a planet, but with different value of constant (e.g. Jupiter's moons; and now Earth satellites).

6. **Comets,** until then lawless and mysterious, follow elliptical orbits according to Kepler's Law I, as members of solar system. Times of comets' returns predicted successfully.

7. **Relative masses** of Earth and Sun, Earth and Jupiter, etc., estimated through Kepler's Law III (estimate can be made for any two bodies which own satellites).

8. **Shape of Earth** must be oblate spheroid: proportion of radii estimated.

9. **Small differences of *g*** predicted: due to shape of Earth and due to Earth's spin: both make measured g slightly smaller at equator.

10. **Ocean tides,** due to differences of Moon's attraction. (Two tides in 24 hours predicted.)

Similar tides due to Sun are smaller; added to Moon's tides, they make spring tides, subtracted they make neap tides. Relation with phases of Moon also predicted.

11. **Mass of Moon** estimated by treating our ocean tide as a satellite of the Moon.

12. **Precession of the Equinoxes.** Shown to be consequence of gravitational pulls of Sun and Moon acting on the equatorial bulge of the spinning Earth. The 26,000-year period predicted roughly.

13. **Irregularities of the Moon's Motion.** The elliptical orbit changes eccentricity and moves round in its own plane; the plane of the orbit slews round slowly; and the Moon shows small extra monthly and yearly accelerations. All are symptoms of small differences of Sun's gravitational pull. Newton predicted several, tested some of them.

14. **Perturbation of Planetary Orbits.** Each planet is affected slightly by the gravitational pulls of other planets. Newton started the prediction of these small perturbations.

Discovery of Neptune. Long after Newton's death, when the planet Uranus had been discovered, it showed small residual perturbations from its expected orbit, in addition to the effects of known planets. From these, Adams and Leverrier predicted the location and size and orbit of an unknown planet that could produce these tiny perturbations by inverse-square-law gravitation. Then the planet was seen: a triumph of Newtonian theory.

(Still later, in this century, a tiny residual motion of the planet Mercury remained unexplained. The major axis of its ellipse precesses at a rate of 43 seconds of angle per century. This probably points to the need for a modification of the law of gravitation, now explained by relativity.)

Newton's Theory and its Predictions (more detailed account)

As a basis for his deductive theory, Newton described his views of space and time and force and motion. He defined mass as 'quantity of matter', and seemed to consider density an inherently obvious concept that could be used in describing quantity of matter.

He stated his three Laws of Motion. We might give them in less formal wording:

I. Any object remains at rest, or continues to move with constant speed in a straight line, if it is left alone; that is, if there is no (resultant) force acting on it.

II. A force acting on an object makes it accelerate in the direction of the force. A constant force gives constant acceleration.

[Force] = [Mass] × [Acceleration]

or [Force] × [time] = Change of [Momentum, mv].

While each of those forms tells us something about force and motion, each really amounts, in *our* teaching, to a description of mass.‡ In some more advanced teaching mass is taken as self-evident (!), and Newton's Law II is used to define force. * * * *

III. Action = Reaction. If one object pushes or pulls another, the other always exerts an equal and opposite push or pull – whether the objects are at rest, moving with constant velocity or accelerating in any way whatever. T

Newton also assumed Universal Gravitation, $F = GM_1M_2/d^2$.

Every object in the universe attracts every other object with an inverse-square-law force, which is proportional to each of the masses concerned.

‡ Statements of Newton's Laws of Motion contain some knowledge of nature and some definitions and assumptions. We have an open choice between taking force as known by experiment and developing a concept of mass, and taking mass as known by experiment, or assumed to have certain properties, and developing a concept of force to be measured by mass × acceleration. Either scheme works well in a discussion of natural knowledge. But, of course, a mixture of both schemes together will lead to confusion. We should certainly avoid that.

Results. From those as starting point, taking them for granted as true, Newton argued out many a prediction. Here are fuller descriptions of the things that he 'derived' or 'explained' or predicted; with some details of the methods he used.

In discussing Newton's explanations with pupils we should say clearly that scientific explanations like these do not 'explain' in the sense of giving the first cause, the underlying truth, the 'really true' reason. They 'explain' in the sense of linking a new or less familiar phenomenon with another that is already known or accepted. Thus, explaining in Science is economical; it simplifies our knowledge by pointing out connections, thus reducing the number of separate bits of knowledge to be piled up in our records. And those links or connections facilitate thinking and enable us to make predictions. Some philosophers say that explaining a phenomenon in science means nothing more than predicting it from other phenomena (or from rules generalized from other phenomena).

This view of scientific explanations is often disappointing at first, but reaches full importance when we see its power and feel the satisfaction of building connected knowledge. But for our delight in economy and the feeling of fuller understanding that we gain from connected knowledge, we might explain all nature by a horde of demons, each constructed to produce an observed phenomenon: the demons would be arbitrary, disorganized and multitudinous in number and variety. We prefer scientific explanations as more economical, more satisfying because they form a framework, and more fruitful in new predictions. Yet we can hardly say they tell us the ultimate truth, or give basic reasons – Newton himself said clearly he had no idea of the cause of Gravitation: he merely used its inverse-square behaviour to link together the solar system.

(The numbers in the accounts that follow match those of the items in the Summary given earlier.)

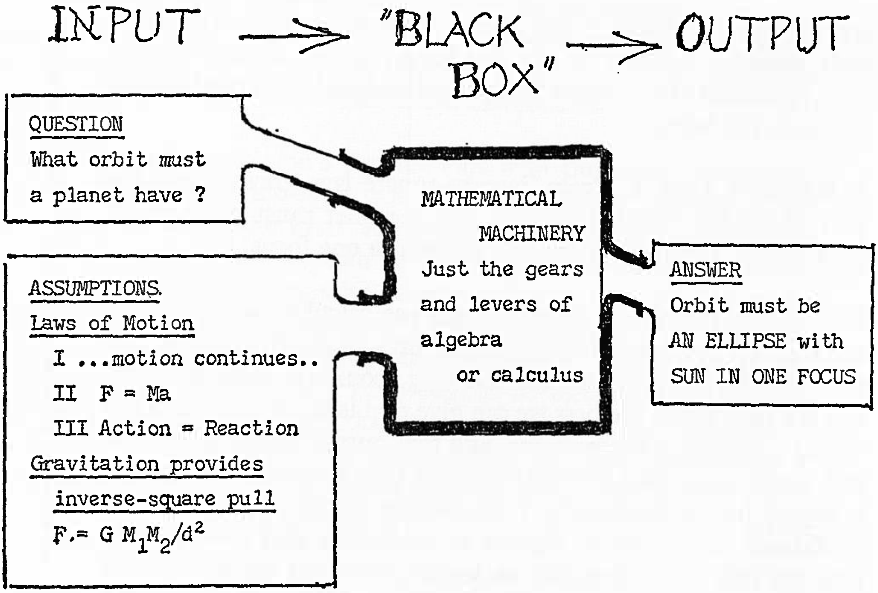
1. **Moon's Motion** is accounted for by inverse-square gravity, T
which at the Moon's distance of 60 Earth's radii is predicted by
theory to be 3,600 times weaker. (That agrees with the actual *
acceleration, v^2/R, calculated from the Moon's distance and time- *
to-go-round-the-orbit. That was Newton's early test of inverse- *
square-law gravity and of v^2/R needing a force.) But that does not *

prevent us from listing it now as one of the achievements of Newton's theory – because we are watching Newton making a fresh start, taking his assumptions for granted, and deducing predictions, the more the better.

2. **Kepler's Law I.** From inverse-square-law gravity and his laws of motion Newton showed that a planet must move in an orbit that is a conic section with the sun in one focus.

Even today, the easiest proofs, which use calculus, seem clumsy; and they are much too difficult for our present pupils. Newton had to offer even clumsier alternatives using geometry. Unfortunately, this is a case where the best we can give pupils is a statement of the 'input', Newton's assumptions, and the 'output', elliptical orbits, and assure them that the mathematical path from input to output is merely logical machinery. We hope that pupils will have gained confidence in the use of algebra as machinery and some insight into the role of mathematics as logical, obedient machinery that leads from input to output. The problem of the man throwing a stone up at a bird in a tree, which leads to a quadratic with two answers, gave an opportunity for pupils to see mathematics acting as an obedient servant, offering both answers because both are consistent with the form of problem (input) supplied to the algebra. We told a story about the bird being hit, but we forgot to instruct the algebra to make sure of contact, or to make sure that the stone was still moving upwards when it reached the bird. We only asked the algebra, 'At what time will the stone be at the bird's height above the ground?' The algebra truthfully gave two answers, one for the time on the way up, the other for the time on the return fall downwards. (This was the problem in Chapter 1 of the Year IV Teachers' Guide, discussing 'Mathematics, the honest servant'.)

So in this case we treat the mathematics as 'machinery in a black box'. We show pupils the 'input' to the box and the 'output' from the box – each of them quite simple and easy to understand – and we assure pupils that the machinery inside the box is only the gears and levers of mathematics.



(There is a short film made by P.S.S.C. which gives a partial connection between an elliptical orbit and inverse-square law gravitation. Teachers will find this film interesting, but we are doubtful whether it will help very much in actual teaching.)

Unfortunately, there is no simple demonstration which shows a force that is known to follow an inverse-square-law making an object move in an ellipse.‡ The only simple demonstrations are just illustrations:

a. A steel ball rolling round in a slanting path in a glass funnel. D 55a

b. A steel ball rolling on a thin rubber sheet (dentist's rubber dam). The sheet is supported in a horizontal frame and depressed at its centre by a vertical rod held in a clamp. The depressed sheet curves to match the potential well of an inverse-square-law force reasonably closely. However, friction takes its toll as the rolling ball deforms the rubber locally. So this is not recommended. D 55b

‡ Ingenious schemes have been devised to make a CO_2 puck move in an ellipse, by using a complex arrangement of magnets. Another scheme holds a puck in orbit by a thread which runs over a pulley and holds a ping-pong ball hung in a tapering, conical box in which there is a strong air blast maintained by a vacuum cleaner – the arrangement is extremely expensive and very troublesome to set up; and the demonstration takes considerable time because the force exerted by the ping-pong ball must be shown to be an inverse-square one.

Neither of these demonstrations shows a very convincing ellipse. *
The orbit precesses and shows the effect of friction. In neither *
case do pupils know that the force law must be an inverse-square *
one. So these are the best qualitative exhibits of a real particle *
following an oval orbit. Nevertheless, teachers may find that one *
of them helps to make things clearer. *

Note to Teachers. It is easier teaching to proceed to Kepler's *
Law III now, because that can be predicted fully with easy algebra, *
and then return to Kepler's Law II where the prediction uses more *
difficult geometry. However, the Laws are taken in the present *
summary in their traditional order. *

3. **Kepler's Law II.** Newton showed that Kepler's Law of 'equal T
areas in equal times' does not require an inverse-square-law force.
Any force *directed straight from planet to Sun* will make the planet
move in an orbit for which that law holds.

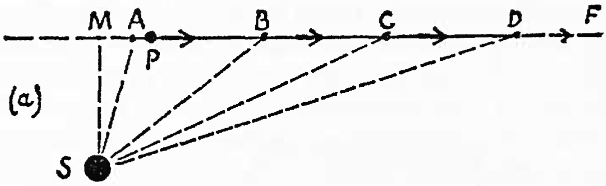
Newton gave a geometrical proof, using changes of momentum *
as vectors. This proof, in simplified form, should be offered to *
pupils in average and faster groups. Most of them would not be *
able to reproduce it, and may even remember the showing of it with *
something of a headache; yet it is a derivation they should see done, *
as part of their contact with a great piece of work. *

Here is an outline of Newton's proof. Seeing it for the first time anyone, pupil or teacher will find it dull and difficult if it is given in words with black-and-white diagrams. Shown on a blackboard with coloured chalks it can appear clear and compelling—and give delight.

We use Newton's Law II, Change of momentum = Force $\times$ time. T

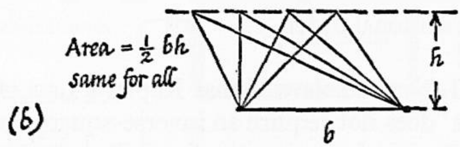
Then changes of mv are vectors, along the direction of F, proportional to F.

First suppose we have a planet that is pulled with no force at all. That is a queer kind of force law; but it is a clear statement and it certainly is possible – a star or a gas molecule out in space, far from any other matter, experiences no force and does move with constant velocity.



a. Motion of a Planet with No Attraction

Planet P moves in a straight line with constant speed. SP sweeps out equal areas in equal times.



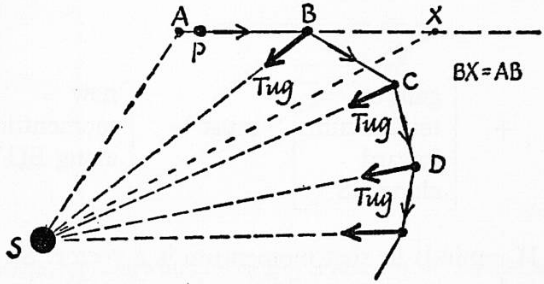
b. The Property of Triangles Used Here

All triangles on the same base and with the same height have the same area. Another version: If triangles have the same base and their vertices lie on a line parallel to the base, their areas are equal.

The planet, P, continues to move with fixed speed in a straight line, AF (Newton Law I). Mark the distances travelled by the planet in equal intervals of time: AB, BC, CD, ... etc. Since the speed is constant, AB = BC = ..., etc.

We can still draw a radius SP to our planet, from the non-attracting Sun, even though there is no force. Consider the areas swept out by the radius SP as the planet moves from A to B, B to C, etc., in equal times. How do the following triangles compare, SAB, SBC, SCD?

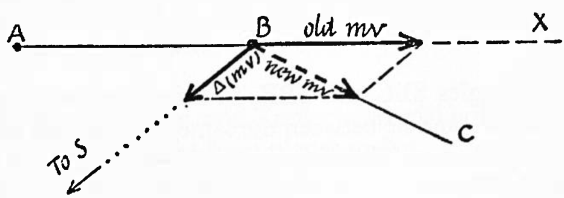
All these triangles have the same height, SM, and equal bases, AB, BC, CD. Therefore all their areas are equal: the spoke from S sweeps out equal areas in equal times. This simple motion does agree with Kepler's Law II.



MOTION OF A PLANET WITH TUGS OF ATTRACTION
Without tug at B, P would move on to X.

Now suppose the planet P moves in an orbit, because the Sun pulls it inward along the radius PS. But, to simplify the geometry, suppose the attraction only acts in sudden big tugs, for very short times, leaving the planet free to travel in a straight line betweenwhile. Then it will follow a path such as that shown below. Suppose it travels AB, BC, CD, etc., in equal times, the inward tugs occurring abruptly at B, at C, at D, etc.

The planet moves steadily along AB; then, acted on by a brief tug at B, along BS, it changes its velocity abruptly and moves (with new speed) along BC. Except for the tug at B the planet would have continued straight on, as in the simple case discussed above. On this continuation, mark the point X an equal distance ahead, making AB and BX equal. Without the tug at B, the planet would have travelled AB and BX in equal times, and the radius from S would have swept out equal triangles, SAB and SBX. But in fact the planet reaches C instead of X. Does this change spoil the equality of areas? If the planet travels to C, the two areas are SAB and SBC. Are these equal? To change the motion from along AB to along BC, the tug at B pulls straight towards the Sun, along BS. This tug gives the planet some inward momentum along BS, which must combine with the planet's previous momentum to make the planet move along BC. The planet's previous momentum was along AB.



ENLARGED SKETCH OF MOMENTUM-CHANGE AT B

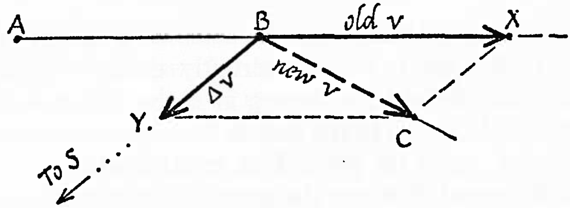
Therefore,

$$\begin{bmatrix}\text{original}\\ \text{momentum}\\ \text{along AB}\end{bmatrix} + \begin{bmatrix}\text{gain of}\\ \text{momentum}\\ \text{inward}\\ \text{along BS}\end{bmatrix} \text{ must } = \begin{bmatrix}\text{new}\\ \text{momentum}\\ \text{along BC}\end{bmatrix}$$

Newton's Law II reminds us that momentum is a vector. So the adding must be done by vector addition (see sketch). As the planet's mass is constant, we may cancel it all through and use velocities thus:

$$\begin{bmatrix}\text{velocity}\\ \text{along AB}\end{bmatrix} + \begin{bmatrix}\text{gain of velocity}\\ \text{along BS}\end{bmatrix} \text{ must } = \begin{bmatrix}\text{velocity}\\ \text{along BC}\end{bmatrix}$$

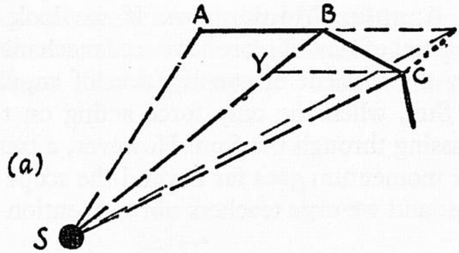
Let us use the actual distance AB to represent the planet's velocity along AB. Then, BX must also represent this velocity and BC must represent the planet's new velocity along BC (since all these are



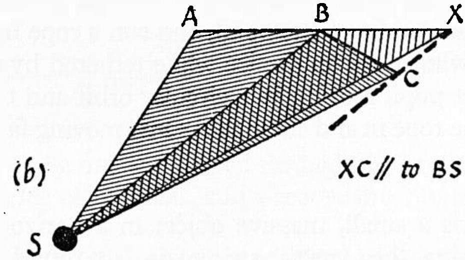
VECTORS TO SHOW VELOCITIES AT B
Scale has been chosen so that AB or BX represents original velocity along AB, before tug acts at B.

distances travelled in equal times). Using this scale, we make a vector diagram (see sketch) expressing the equation above. Use BX (= AB) for the original velocity before the tug. Use BC for the velocity after. The change of velocity must be shown by some vector BY along BS straight towards S. Complete the parallelogram, with BC the diagonal giving the resultant. Because this is a parallelogram the side XC is parallel to BY, so C lies on a line parallel to BS.

Now look at the triangles SBC and SBX, in sketch below. They have the same base, BS, and lie between the same parallels, BS and XC, so they have equal areas. Therefore, area of SBC = area SBX, which = area SBA. Therefore, the triangles SBA and SBC have equal areas. By a similar argument, the triangles SBC and SCD

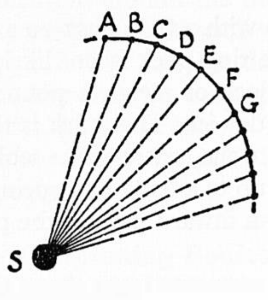


(a) Earlier sketch, redrawn with C in its proper place on XC parallel to BY or BS.



(b) Sketch A redrawn with equal-area triangles shaded.

have equal areas, so all the triangle areas are equal, and Kepler's second law does hold for this motion. This argument only holds if all the tugs come from the same point S. If we now make the tugs more frequent (but correspondingly smaller) we have an orbit,



The equal time intervals from A to B to C ... are much shorter. Orbit is nearer to a smooth curve. With a smooth-curve orbit, the segments swept out in equal times may each be regarded as a bunch of small triangles like those here. So the segments must have equal areas.

nearer to a smooth curve, and Kepler's law still holds, provided the tugs are directed straight from planet to Sun. If we make the tugs still more frequent, we approach the limit of a continual force, with an orbit that is a smooth curve. The argument extends to this limit, so Kepler's Law II holds for a smooth curved orbit.

Kepler's Law II and Angular Momentum. If we look at Kepler's Law II from the point of view of more advanced mechanics, we find that it is simply a statement of conservation of angular momentum around the Sun, when the only force acting on the planet is a *central* one passing through the Sun. However, a treatment in terms of angular momentum goes far beyond the scope of our teaching at this stage: and we urge teachers not to mention it. * * * * * * *

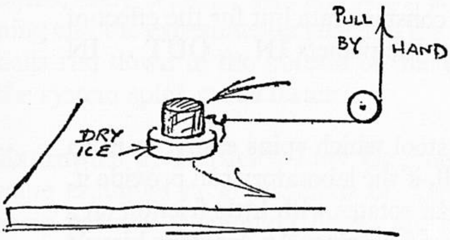
Demonstrations of Kepler's Law II. We can give a number of illustrations. If a safe, frozen pond is available, we install a person as Sun, firmly fixed in the middle of the pond, and run a rope from him to a (light) pupil who slides on the ice while tethered by the rope. We get the planet pupil moving in a circular orbit and then ask the 'Sun' to pull the rope in and show the planet moving faster when closer. D56

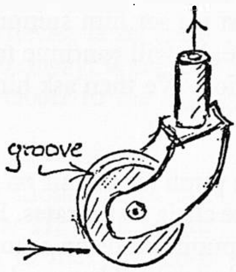
Teacher or pupil whirls a small, massive object in a horizontal circle at the end of a string, then lets the string wind up round one finger; the 'planet' moves round faster and faster as the string shortens. D/C57

As an interesting demonstration, start a small steel ball rolling round in a circle inside a conical glass funnel. Pupils watch the ball as friction takes its toll. The orbit moves lower and lower in the funnel and the time taken for each trip grows shorter and shorter. D58

Optional Demonstration with a CO_2 puck on a glass table. For this particular experiment a ring puck is not likely to be massive enough. Instead, a large block of metal, a pound or so, should be placed on a thin slab of dry ice. This puck is then pulled by a horizontal thread that runs to the centre of the table, round a light pulley wheel and vertically up to a hand or a spring or a load over another pulley, to provide an inward pull on the puck. D59 OPT.

(The *main* pulley has to be supported just above the centre of the table by some clamping arrangement that hangs from above, so that the planetary motion is not obstructed. The pulling thread must pass over the pulley and vertically up; and the frame of the pulley must be able to rotate about the vertical thread as axis. If this device is to work well, it must have two very good sets of ball bearings. Whenever this has been tried, the same difficulty has always appeared: that the pulley must be a very good one and it must be free to swivel round – otherwise the system cannot maintain constant angular momentum. A pulley that is really good would be very expensive and not justified. Therefore, we do not recommend a precision-made pulley for this demonstration, but we do suggest a rough form. The rough form can be made from a swivelling chair



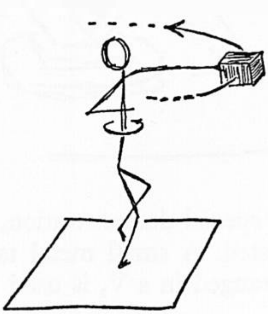


castor. A hole must be drilled through the shank that is intended to fit in the leg of the chair, and a groove must be cut in the wheel.)

The simple class experiment of whirling an object on a thread that passes through a glass tube held in the pupil's hand (C 12) can be modified to show this, if the pupil replaces the load hung on the thread by a pull exerted by his other hand. C 60

Each of the demonstrations suggested above is a case of shortening the radius from 'Sun' to 'planet' by pulling the planet in with whatever (central) force is necessary. Although such a force cannot change angular momentum around the Sun, friction can carry a large amount away to the Earth and spoil the demonstration. The experiments suggested below can be done quickly, with less trouble from friction, but they do not appeal to young pupils as such clear models of a planet's motion. They are the standard demonstrations of conservation of angular momentum in a system free from external torque.

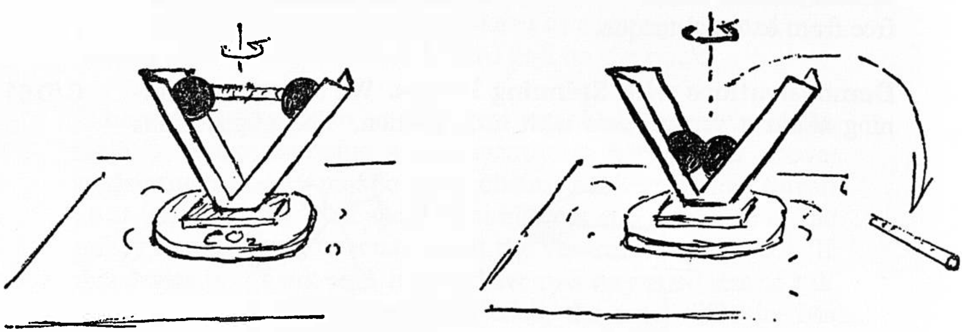
Demonstrations with Spinning Bodies. We set a pupil spinning about a vertical axis with little friction. The pupil stands C/D 61



upright, stretches his arms sideways, and holds a massive object, a book or a dumbbell, in each hand. We tell him he represents the Sun and the books are two planets. We ask him to pull the planets TO his chest, then move them OUT again with arms extended. Then we set him spinning about a vertical axis. If we leave him alone, he will continue to spin at constant rate but for the effect of friction. We then ask him to move the planets IN ... OUT ... IN ... OUT ...

The pupil should sit on a music stool which spins easily, or in an office chair that rotates. Better still, if the laboratory can provide it, the pupil stands on a turntable that rotates with little friction on a vertical axle. That needs a very good bearing. An ordinary bicycle wheel, with its axle vertical, is not strong enough; but a motor-bicycle wheel covered with a plywood disc to make the platform and supported on a massive base with its axle vertical makes an excellent turntable. Unless the school already has this, we do not advise buying or making it. The expense is not justifiable at this level.

As a simple version, which every pupil should be encouraged to try himself, the pupil simply stands on the floor, starts spinning by a sudden twist, and spins on one foot for a short time. With practice, it is possible to make two or more revolutions before the other foot has come to the rescue. (During this, one foot must remain off the ground and the pupil must try to remain as upright as possible. Some pupils find it best to raise the toe of the pivot foot and spin on the heel as pivot. Some pupils find it easier to spin on the toe as pivot.) The pupil carries books with his outstretched arms and pulls the books in close to him during his free spin. C62

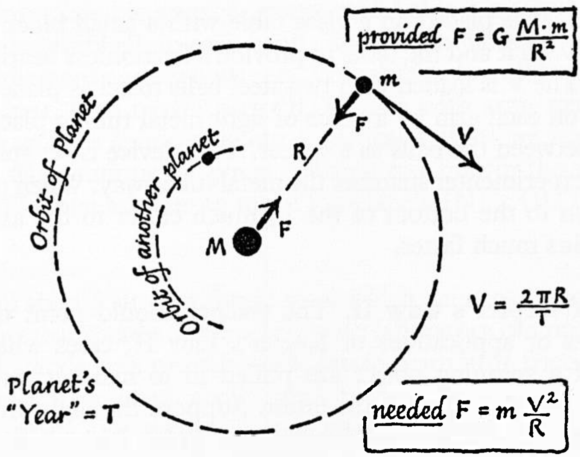


Where the teacher likes to construct a special demonstration, the arrangement sketched below is suggested. A small metal table, carrying two pieces of vee-channel, arranged in a V, is used as a D63 rotating frame. It is placed on a glass table with a small block of solid CO_2 between it and the table to provide a frictionless bearing for spinning. The V is loaded with two steel balls to act as planets, held high up on each arm by a piece of light metal tubing placed horizontally between the balls as a spacer. This device is set spinning and the experimenter snatches the metal tube away. When the balls run down to the bottom of the V, much closer to the axis, the system spins much faster.

Examples of Kepler's Law II. The teacher should point out some examples or applications of Kepler's Law II, cases where some parts of a spinning object are pulled in to make it more compact and then the object spins faster. Suppose the high-diver doing a somersault discovers half-way down in his flight that he is going to hit the water feet first, when that is not his plan. He doubles up, knees against chest, and spins faster for a carefully judged time. Or, if he wants to spin slower, he spreads out his arms in an even grander pose than usual. The skater starts spinning on one toe with arms stretched out and body tilted, then pulls himself upright with arms folded and spins very fast. T

(Some teachers like to mention the marvellous way in which a *
cat can turn over and land on its feet even if it fails to acquire any *
angular momentum before it starts to fall. This is complicated, and *
likely to be confusing, or at least to seem irrelevant here. Further- *
more, the usual explanation of the changes made by the cat, *
flinging legs and tail in and out, is only part of the story: the cat *
also makes important contributions by arching its back and *
twisting – much like the effect of twisting a bicycle pump's rubber *
connector while one holds it bent into a U shape.) *

4. **Kepler's Law III.** If we combine $F = ma$ and $a = v^2/R$ with inverse-square-law gravitation, simple algebra produces Kepler's Law III.



PLANETARY MOTION

For a circular orbit suppose a planet of mass m moves with speed v in a circle of radius R around a Sun of mass M. This motion requires an inward resultant force on the planet, mv^2/R, to produce its centripetal acceleration v^2/R.

Assume that gravitational attraction between Sun and planet just provides this needed force. Then

$$G \, . \frac{Mm}{d^2} \text{ must } = \frac{mv^2}{R}$$

and distance d between m and M = orbit radius, R.

$$\text{But } v = \frac{\text{circumference}}{\text{time of revolution}} = \frac{2\pi R}{T}$$

where T is the time of one revolution

$$\therefore \; G.\frac{Mm}{R^2} = m.\frac{(2\pi R/T)^2}{R} \qquad \therefore \; G.\frac{Mm}{R^2} = \frac{4\pi^2 mR^2}{T^2 R}$$

To look for Kepler's Law III, collect all R's and T's on one side; move everything else to the other.

$$\therefore \frac{R^3}{T^2} = \frac{GM}{4\pi^2}$$

Now change to another planet, with different orbit radius R' and time of revolution T', then the new value of $(R')^3/(T')^2$ will again be $GM/4\pi^2$; and this has the same value for all such planets. That is because G is a universal constant and M is the mass of the Sun, which is the same whatever the planet. Thus R^3/T^2 should be the same for all planets owned by the Sun, in agreement with Kepler's Law III.

For another system, such as Jupiter's moons, M will be different (this time the mass of Jupiter) and R^3/T^2 will have a different value, the same for all the moons.

The planet's mass, m, cancels out. Several planets of different masses could all pursue the same orbit with the same motion.

> 'You might have foreseen that – it is the Leaning Tower experiment on a celestial scale.'

With any other law of force than the inverse-square law, R^3/T^2 would not be the same for all planets. An inverse-cube law, for example, would make R^4/T^2 the same for all; then values of R^3/T^2 would be proportional to $(1/R)$, and not the same for all planets. In fact, as Kepler found, they are all the same. The inverse-square law is the right one.

Calculus predicts Law III for elliptical orbits too, where R is now the planet's greatest distance from the Sun, the major semi-axis.

The only experiment we can offer for Kepler's Law III was produced by Kepler himself: it is the table of values of R^3/T^2. Although pupils saw that Table when we described Kepler's work, and although they must have it in mind when we are trying to derive the law from Newton's assumptions, we must certainly show it to them again now. We should exhibit the full table of measurements of R and T for the planets and show the results of calculating R^3/T^2, and the final testing value, R^3/T^2.

T

D64 (=D50)

With a fast group we might show a table for the four largest of Jupiter's moons. D65

Name of satellite	*Distance from Jupiter* in *Jovian*‡ *diameters*	*Distance from Jupiter* in *miles* (R)	*Time of revolution* in *hours* (T)	*Calculations for test of Law III* R^3 (*miles*)^3	T^2 (*hours*)^2	$\frac{R^3}{T^2}$
Io	3·02	262,220	42·36	$1{\cdot}803 \times 10^{16}$	1,802·8	TRY
Europa	4·80	417,190	85·23	$7{\cdot}261 \times 10^{16}$	7,264	THIS‡‡
Ganymede	7·66	665,490	171·71	$29{\cdot}473 \times 10^{16}$	29,484	
Callisto	13·48	1,170,700	400·54	$160{\cdot}440 \times 10^{16}$	160,430	

Pupils may ask whether we could make a similar table nowadays *
for the Earth's family of satellites. We have the Moon moving in *
a circular orbit and some of the artificial satellites have orbits near *
enough to a circle to provide a simple test. Data can be obtained *
from Ovenden's book. (A physicist, needing such data in a hurry, *
would be tempted to work out a satellite's distance from its period *
of revolution. In that case, he would be assuming Kepler's Law III *
for its basis. However, satellite distances can be measured, by *
radar, and even by direct telescopic observations; so we *do* have *
data for genuine tests.) *

Newton himself imagined Earth satellites fired horizontally from a T
gun on a high mountain. With sufficient muzzle velocity, the projectile would fall just enough from its initial tangent to match the falling away of the round Earth. It would continue like that always falling to match the Earth's curve, until it arrived back at the gun and hit it from behind.

This is a very useful idea in teaching. Pupils can imagine the pro- T
jectile falling with a perpetual inward acceleration as it pursues an orbit round the Earth. Thought of like that, motion in a circle obviously *does* have an inward acceleration, and pupils find it easier to accept the paradox of the object never getting any nearer the centre in spite of accelerating towards it.

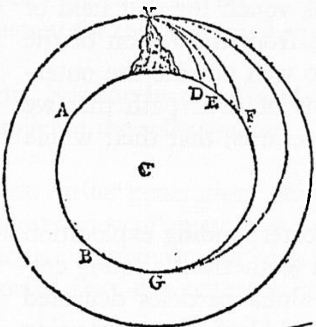
‡ It is simplest to measure the moons' orbits in terms of Jupiter's diameter. The radii could remain in those units for a test of Kepler's Law III; but, if these data are to be used in gravitational theory (e.g., to compare Jupiter's mass with the Sun's), then the same units, e.g. miles, must be used on both sides of the comparison.

‡‡ The test is made easy by a lucky chance arising from the choice of units, miles and hours. Look at the numbers.

The picture in the box shows Newton's own diagram in his non- *
mathematical part of the System of the World which he added to *
the third edition of the *Principia* in 1726. The English translation *
was probably done by Motte a year or so later. *

DE MUNDI SYSTEMATE.

non amplius in terram caderet. Deſignet AFB ſuperficiem Terræ; C centrum ejus; & VD, VE, VF, lineas curvas, quas projectile de montis præalti vertice V, ſecundum lineas horizonti parallelas, auctis cum velocitatis gradibus, ſucceſſivè emiſſum deſcribat. Et ne aëris reſiſtentia, quâ motus cœleſtes vix retardantur, in computum veniat, fingamus hunc omnem tolli, vel ſaltem nil reſiſtere. Et eâdem ra- ... velocitate minore



NEWTON'S SYSTEM OF THE WORLD

[3.] *The action of centripetal forces.*

That by means of centripetal forces the planets may be retained in certain orbits, we may easily understand, if we consider the motions of projectiles (pp. 2–4); for a stone that is projected is by the pressure of its own weight forced out of the rectilinear path, which by the initial projection alone it should have pursued, and made to describe a curved line in the air; and through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth. . . .

Let AFB represent the surface of the earth, C its centre, VD, VE, VF the curved lines which a body would describe, if projected in an horizontal direction from the top of an high mountain successively with more and more velocity . . . . . let us suppose either that there is no air about the earth, or at least that it is endowed with little or no power of resisting; and for the same reason that the body projected with a less velocity describes the lesser arc VD, and with a greater velocity the greater arc VE, and, augmenting the velocity, it goes farther and farther to F and G, if the velocity was still more and more augmented, it would reach at last quite beyond the circumference of the earth, and return to the mountain from which it was projected.

Note to Teachers on the Value of Kepler's Law III. As we now see this law through Newton's eyes, it offers a survey of the field of force in the solar system. Ranging all the way from the orbit of Mercury to the orbit of Saturn, the farthest then known, Kepler's Law III vouches for an inverse-square-law field of attraction: a tremendous exploration, over a vast range, by the planets as 'investigating missiles'.

(A bright pupil may ask whether comets vouch for that field of force over a still wider range. They do, from the region of the innermost planets and perhaps nearer, to well outside the outermost orbits. We know from the portions of their path that we observe, combined with their punctual returns, that their whole orbit is an ellipse.)

Presently we shall describe to pupils a corresponding exploration of the field of force *inside an atom*. When Rutherford's young colleagues, Geiger and Marsden, counted alpha particles deflected from their path as they met a thin wall of gold leaf, they were using alpha particles as investigating missiles. They found large-angle deflections so rare that it seemed certain that those were the result of a single encounter with a strong deflecting field (repulsive) – the numbers counted did not fit with the idea of many encounters in this thin gold leaf building up a large deflection.

Therefore, each alpha particle travelling to the gold and out along a deflected path was like a planet in the solar system telling us about the field of force controlling its orbit. Measurements of numbers of alpha particles scattered showed clearly that over a tremendous range of distances, deep inside the atom of gold, the alpha particles encountered an inverse-square-law field of force.

We shall want to describe that to pupils and quote it as the evidence that suggested and supported our picture of atoms as 'hollow', with a tiny massive nucleus carrying a large electric charge. When they come to that, it will not seem very convincing if that is their first glimpse of 'investigation by orbits', since the events inside the gold atom are far too small to see and the geometry that leads us from macroscopic observations to our microscopic picture is too difficult to give in detail at this stage.

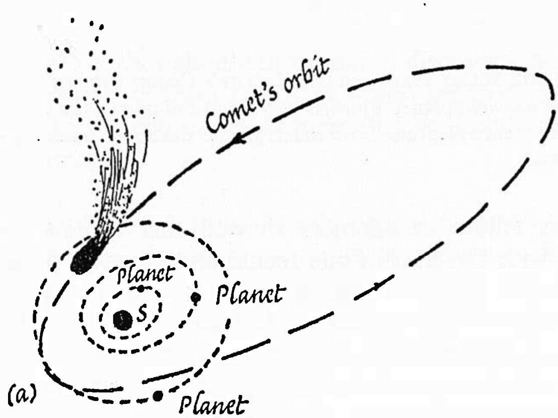
Therefore, if Newton's prediction of Kepler's Law III catches pupils' fancy now, teachers may want to take this opportunity to point out that we now take Kepler's Law III as evidence of a vast region of inverse-square-law field, and even give a hint that we shall meet a similar story inside atoms, though with electrostatic repulsion instead of gravitational attraction. If that forward look creates enthusiasm, now is the time for it. But if Kepler's Law III seems 'just algebra', and rather difficult at that, for a slower group, it may be better to postpone any mention of hollow atoms.

5. **Moons.** Kepler's Law III should also apply to a sub-family – any planet and its satellites. As Kepler and Galileo had shown, it applies to Jupiter's moons though, of course, with a different constant for that system, $4\pi^2$[mass of Jupiter]/G. T

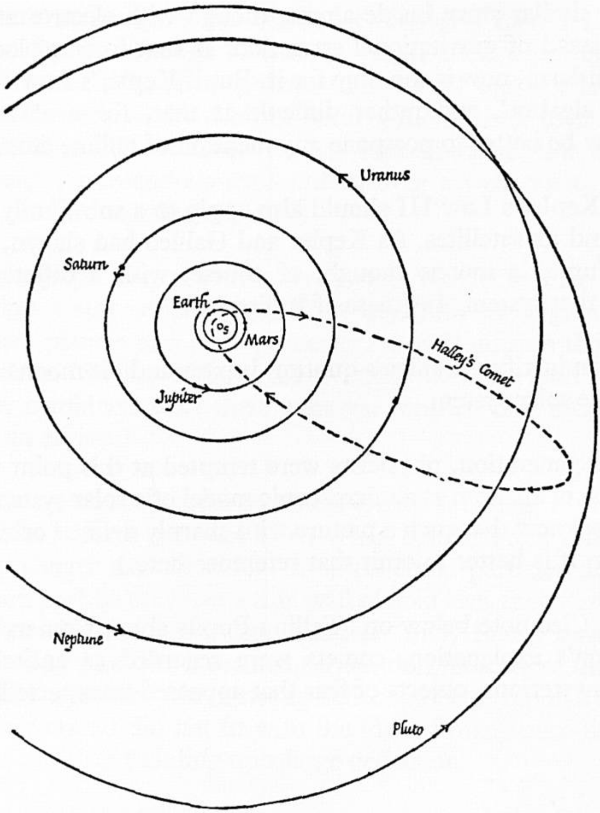
Thus, Newton justified Galileo's quoting Jupiter and his moons as a model of the solar system.

(In an earlier generation, physicists were tempted at this point to suggest a view of an atom as a microscopic model of a solar system. Since we now know that such a picture with sharply defined orbits is misleading, it is better to omit that reference here.) * * * *

6. **Comets.** (See note below on 'Telling Pupils about Comets'.) Until Newton's explanation, comets were regarded as entirely lawless and mysterious, objects of fear that appeared unexpectedly in the sky. T



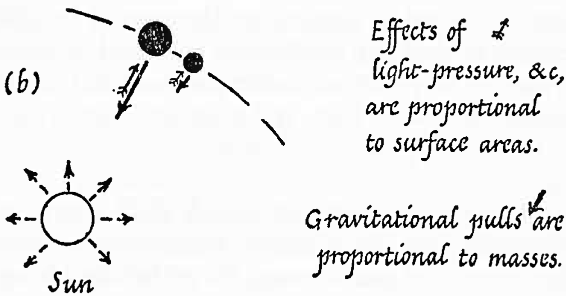
COMET, MOVING IN AN ELLIPTICAL ORBIT WITH SUN IN ONE FOCUS, PASSES THROUGH SOLAR SYSTEM



SKETCH OF THE SOLAR SYSTEM, WITH HALLEY'S COMET SHOWN
The most recently discovered planet, Pluto, is very small and pursues an elliptical orbit extending from within Neptune's to a much greater distance. (Mercury and Venus are not shown.)

Newton and his fellow astronomers showed that comets have elliptical orbits with the Sun in one focus; and Newton pointed out that we should therefore regard them as satellites in the general solar system. Their ellipses are members of the family of planetary orbits, except that they happen to be much more eccentric. Thus, Newton took comets out of the realm of superstition and mystery in which they had long resided (and still reside for some people today).

Different parts of a comet, if it is a collection of small pieces, *
should all travel together along the same orbit – the acceleration *
of the falling body, or the orbit of a projectile, is independent of its *
mass. When we observe the tails of comets trailing behind and *
pushed out sideways, away from the Sun, we infer that other *



FORCES ON PARTICLES IN A COMET'S TAIL

If one particle of the comet has 10 times the diameter of another, its mass, for the same density, is $10 \times 10 \times 10$, or 1,000 times as great as the other's. Gravitational forces on it will be 1,000 times as big. But surface forces, e.g. light pressure from sunlight, will be only 10×10, or 100, times as great. Thus surface pressure matters proportionally more for small particles; less for large ones. It can push the very tiny particles in the comet's tail away.

forces not proportional to mass are acting upon them. For many years, astronomers thought those forces were due to the pressure of light from the Sun. It now seems more likely that the forces are chiefly due to bombardment by proton streams from the Sun. That is one matter which is waiting for confirmation when the next big comet returns. * * * * * * *

(We should not divert our teaching to a long sidetrack over that 'scaling' argument, which might strike pupils as obscure and unrewarding. Yet they should meet that in discussing heat losses – from calorimeters or from buildings – and neutron escape from reactors; and they will certainly meet that in discussing strength and size of animals.) * * * * * *

Note to Teachers: Telling Pupils about Comets

The older we are, the more we think of comets as interesting, special things that appear in the sky. We see pictures in astronomy books, and in newspapers, and we may even remember seeing a large comet with our own eyes. But young pupils have merely heard the name; they are unlikely to have seen one because most comets visible in recent years have required a telescope. (Professional astronomers are still waiting for the next return of a big comet to try out their present views.) * * * * * * * *

Before we say that Newton brought comets into his theory of the solar system, we should show photographs of comets and give examples of their regular returns. For example: T

Halley's Comet. Appeared as a great, brilliant comet in 1682. Halley (and Newton) worked out its elliptical orbit and predicted its return in 76 years. It did return soon after the predicted date, in 1759. It has returned twice since then, and is expected again about 1984.

Working backwards with the estimated period of 76 years, and allowing for some calculated disturbances, we find several dates that fit with earlier records of great comets. So we believe Halley's comet has been seen in the following years:

1066 — — — — 1456 1531 1607 *1682* 1758 1835 1910

The earliest in that list is the great comet that was thought to herald disaster in the Battle of Hastings, the *stella* over King Harold in the Bayeux tapestry.

We may also want to point out that the big comets, such as Halley's, appear as such a great bright brush of light across the sky that people cannot fail to see them and be impressed. It is not surprising that, as well as wonder, they excited awe and even fear. To the superstitious they were more strange and queer than the planets. When observations of comets showed that they pass quite near to us, there were added troubles: not only the practical fear that the comet might hurt us, but also the theoretical disaster that the comet had smashed through the crystal spheres.

If pupils ask, 'Where do comets really come from?', we should say that astronomers now believe there is a large reservoir of planetary material circulating out beyond the orbits of large visible planets. In interactions between a clump of such material and a planet, the clump's orbit is changed and occasionally becomes a narrow ellipse that brings that clump in close to the Sun, where it is illuminated by bright sunlight, and we see it as an impressive visitor.

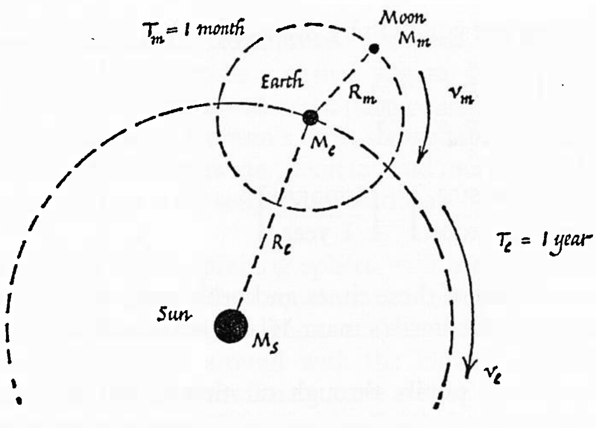
Some comets seem to have a dense head, as if they are quite massive. But we know their mass cannot be big on a planetary scale, because on the rare occasions when a comet has a close encounter with a planet such as Jupiter, the comet's orbit is usually changed, but the massive planet seems to suffer no noticeable effect.

7. Masses of Planets. Newton could estimate the *relative* masses of the Sun and planets, whenever the body concerned owns a satellite. T

The satellite, through Kepler's Law III, provides a value for the mass of its controlling body. Although Newton could not check those relative masses, this was a typical case of theory providing new numerical knowledge in terms of its own assumptions.

For example, Newton could calculate the mass of the Sun in terms of the Earth's mass, because each owns at least one satellite. (The absolute mass of the Earth itself was not known and could not be estimated without some terrestrial measurements like those of Cavendish.)

Newton's calculation can be carried out as follows: *
Subscripts $_s$ and $_e$ and $_m$ refer to Sun and Earth and Moon. *



CALCULATING THE RATIO OF SUN'S MASS TO EARTH'S M_s/M_e, by using the motion of the Moon. The Moon's mass M_m cancels out. For each motion, Earth-around-Sun and Moon-around-Earth, just write an equation stating that the needed force Mv^2/R is provided by gravitation.

EARTH AS SATELLITE OF SUN. For the Earth's motion around the Sun in its yearly orbit,

$$G.\frac{M_s M_e}{R_e^{\ 2}} = M_e\frac{V_e^{\ 2}}{R_e} = M_e\frac{4\pi^2 R_e^{\ 2}}{R_e T_e^{\ 2}}$$

$$\therefore\ M_s = \frac{4\pi^2}{G}\left[\frac{R_e^{\ 3}}{T_e^{\ 2}}\right] \quad \text{Note that the Earth's mass, } M_e\text{, cancels}$$

MOON AS SATELLITE OF EARTH. For the Moon's motion around the Earth in its monthly orbit,

$$G\frac{M_e M_m}{R_m^{\ 2}} = M_m\frac{v_m^{\ 2}}{R_m} = M_m\frac{4\pi^2 R_m^{\ 2}}{R_m T_m^{\ 2}}$$

$$\therefore\ M_e = \frac{4\pi^2}{G}\left[\frac{R_m^{\ 3}}{T_m^{\ 2}}\right] \quad \text{Again, the Moon's mass, } M_m\text{, cancels}$$

Therefore, dividing one equation by the other

$$\frac{M_s}{M_e} = \frac{[\,R_e^{\ 3}/T_e^{\ 2}\,]}{[R_m^{\ 3}/T_m^{\ 2}]} = \frac{R_e^{\ 3}}{R_m^{\ 3}}\ \frac{T_m^{\ 2}}{T_e^{\ 2}}$$

$$= \left[\frac{\text{DISTANCE OF SUN}}{\text{DISTANCE OF MOON}}\right]^3 \left[\frac{1\text{ month}}{1\text{ year}}\right]^2$$

With the known values of these times and orbit radii, the ratio of the Sun's mass M_s to the Earth's mass M_e can be calculated.

(We should not drag pupils through all that unless they are interested in comparing the various masses of the solar system *and* are capable people with algebra so that they follow the working comfortably. But, if we *do* start, we should not leave the result unfinished. Using quick arithmetic, we must work out the value of that proportion roughly. Taking well-known values in miles, we have 93·6 million miles for the Sun and 240,000 miles for the Moon, making a ratio about 400 to 1. There are 13 Moon-months in a year.

$$\text{Therefore, } \frac{\text{Mass of Sun}}{\text{Mass of Earth}} = [400]^3\left[\frac{1}{13}\right]^2 = 64\times10^6/169$$

which will come to somewhere between 200,000 and 400,000 according to our choice of rough arithmetic. Careful arithmetic with good data gives 330,000, but attempts to reach an accurate result miss the point. Pupils should *not* want to know the proportion of masses of Sun and Earth with terrific precision. But they should be interested in knowing that the Sun is several hundred thousand times as massive as the Earth; because then they will know that the Earth is unlikely to upset the orbits of other planets, such as Mars, noticeably. Here is a case where a rough estimate is very useful.)

Newton could make similar estimates for other mass ratios, such as Jupiter's mass to Earth's mass, or Jupiter's mass to Sun's mass, for any two bodies which each own a satellite.

Where a body has no satellite, the estimate could not be made. It looks as if Newton could not estimate our own Moon's mass as a fraction of the mass of the Earth or Sun, since the Moon has no satellite. And yet he succeeded in doing that – he found the moon *does* have a 'satellite'! (We promise to explain this soon.)

8. **Shape of the Earth: an Oblate Spheroid.** In Newton's day T
the Earth was thought to be a perfect sphere. Newton predicted it must be a spheroid, flattened at the poles, bulging at the equator. Surveys, not long after Newton's time, showed that the Earth does have the kind of shape Newton predicted and more careful surveys have confirmed that with some modifications.

Newton realized that a spinning sphere with mobile oceans on it would maintain a tremendous equatorial bulge of water. Although we should not observe huge tides on account of that bulge (because it would all be carried around with the Earth's daily spin), it seemed unlikely, and we now know there are not those great ocean depths. It seemed more likely that the Earth, in some early pasty state of its formation, would have itself taken on a shape with an equatorial bulge.

Therefore, Newton thought about the shape the Earth must have if, long ago, in pasty form, it took the equilibrium shape such a spinning body required.

We can show a simple demonstration of a spinning sphere taking an D 68
oblate shape. We spin a large rubber ball – preferably the hollow kind sold in toyshops for small children – with an electric motor or a hand-drill. Or a sponge-rubber ball will show the effect if it is spun fast enough. A hole is drilled through the ball from north

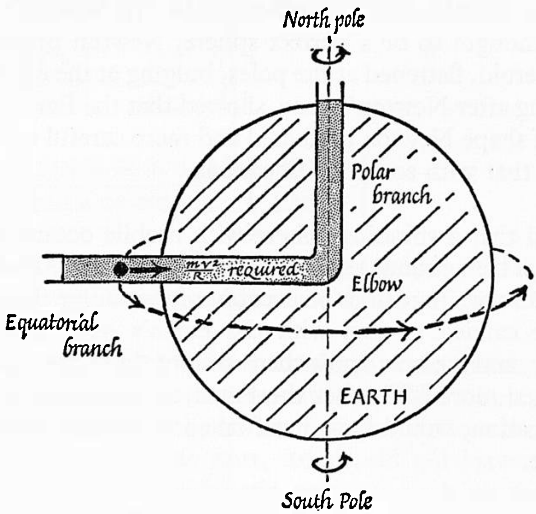
pole to south. A thin rod is passed through the holes and held in a chuck attached to the motor. (An electrically driven hand-drill serves well and has the chuck already attached.)

The ball must be free to slide on the rod at one hole, so that it can change its shape; but it must stick to the rod at the other hole so that the rod can spin it. There must be a stop at the outer end of the rod.

(This homemade demonstration is more convincing, and more fun, than the traditional model made with flexible metal strips, spun by a special motor; so we do not advise schools to buy the latter.)

The general method of Newton's calculation is simple and should be described to faster pupils.

His argument ran as follows: T



TO ESTIMATE THE BULGE OF A SPINNING EARTH, imagine a pipe of water running from North Pole to centre and out to equator. Calculate the extra height of water in equatorial branch needed to provide mv^2/R forces for spin. This gives extra radius of bulge for a pasty Earth congealing while spinning.

Consider a pipe of water running through a spherical Earth from the North Pole to the centre and out to the equator. If this were filled with water, just to the Earth's surface at the North Pole, where would the water surface be in the equatorial branch of the pipe? At the centre of the Earth the water pressure at the bottom of the polar pipe is due to the weight of the water in that pipe; and this pressure pushes around the elbow at the bottom and out along the equatorial branch, trying to push that column of water outward. The weight of water in that branch pulls it in. But these two forces on water in the equatorial branch must be *unequal.* They must differ by enough to provide an inward centripetal force to act on the water in that pipe, which is being carried around with the spinning Earth. The weight of the water in that branch must exceed the outward push from the water at the elbow by the amount needed for mv^2/R forces. More colloquially, 'Some of the pull of gravity on the water in the equatorial pipe is used to keep the various portions of that water moving round in a circle, and only what is left over makes the pressure at the bottom of the pipe.'

Therefore, the water column in this pipe must be longer than that in the polar pipe. The equatorial pipe must extend out beyond the Earth's surface to carry the extra head of water.

Newton calculated the extra height and found that 14 miles would be required. He argued that the Earth at an early pasty stage would bulge out about this distance. A short time after Newton's day, measurements of the Earth confirmed the prediction.

If pupils are interested, particularly if they have looked at Jupiter with a good telescope, we might point out that Jupiter shows a more marked elliptical shape. Therefore, although Jupiter is covered by clouds so that we never see the solid surface, we can make a good estimate of the rate at which Jupiter as a whole is spinning.

The most valuable way of presenting this for good teaching would not be to point it out as one more example of a spinning planet having a flattened spheroid shape, but to raise two questions at different stages:

(i) When Jupiter is looked at with a telescope, ask, 'What shape does Jupiter seem to have?' And then, if pupils notice the oval shape, say, 'I wonder what that means?'

(ii) Now when we come to this point in Newtonian theory, or any time after this, we say, 'Jupiter is well covered by clouds, and yet astronomers know quite well how fast Jupiter as a whole is spinning. How can they tell that, without any rocket to go there and find out?'

9. **Differences of *g*.** Both that equatorial bulge and the spinning T
of the Earth have slight effects on the apparent value of g. We expect to find the acceleration of free fall smaller at the equator than at the poles.

We should hardly worry pupils with the two small effects that *
Newton predicted. In more advanced teaching in the past, g has *
assumed great importance in both experiment and theory, as part *
of our training in mechanics. Nowadays we may leave that to *
specialists. Precise knowledge of g is still important in geophysics; *
but for our pupils it should be only an interesting local acceleration, *
which yields a useful field-strength. However, pupils who enjoy *
seeing the ways in which Newton's theory offered unexpected *
knowledge may want to hear of these two effects: *

(i) Since an object near the Earth's surface at the equator is T
farther from the centre of the Earth than one at the poles, we predict g will be slightly smaller at the equator.

(ii) Also, since an object at the equator is revolving in a circular orbit with the spinning Earth, it must have inward resultant force to keep it in orbit. Therefore, when we hold an object at rest with a spring balance, the upward pull of the balance on the object must be slightly less than the object's weight (the pull of the Earth downwards on it). Therefore, the balance reads a little less than the object's weight. Or, if we let the object fall freely and measure the acceleration, relative to the surface of the spinning Earth, the acceleration seems less than we should expect because we are trying to measure it against a frame of reference which itself has an inward acceleration. Both those stories are difficult; and we should be wiser to say colloquially, 'At the equator some of the pull of the Earth is used up to keep the object moving in orbit; so it seems to weigh less.' The effects are small:

(i) makes g 0·2% smaller;

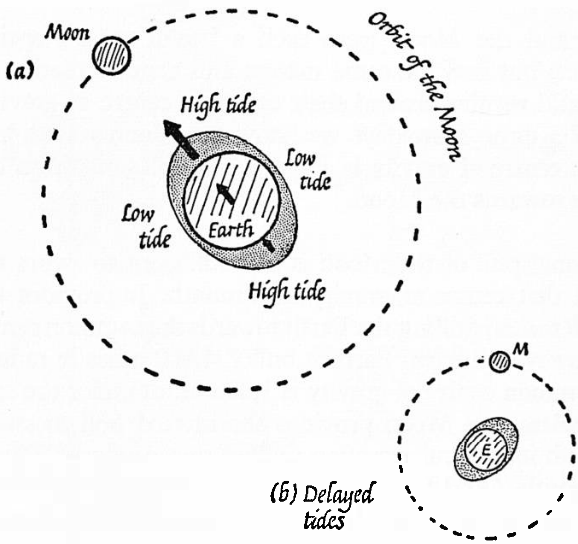
(ii) makes g seem 0·3% smaller.

Surveys confirm the prediction roughly, but there are additional local differences due to mountain masses and other irregularities.

We might point out to pupils that geologists hunting for deposits of *
dense minerals, oil, etc., use surveys of g – often with apparatus *

that gives only *differences* of g from the general average for the region. The measurements are delicate and their interpretation rather complicated.

10. Tides. Newton showed that tides can be explained as due to differences of gravitational pull on the ocean exerted by the Moon, and exerted by the Sun. This was one of the greatest achievements of his theory because it linked a well-known, important phenomenon to the story that he was building up from common gravity. We have all of us heard of this explanation of tides when we were children, and it seems a natural suggestion to us and to many of our pupils. In Newton's day it was an astounding suggestion; a burst of light upon an important phenomenon that had been crying for 'explanation', and receiving strange answers.



Ocean Tides are Caused by Differences of Moon's Attraction

(*a*) The extra-large pull on ocean nearest the Moon raises one high tide. The extra-small pull on ocean farthest from the Moon lets it flow away into another high tide.

(*b*) Delayed tides. Actually the high-tide humps are delayed by inertia, tidal friction, and effects of rotation. As the Earth spins, they are not opposite the Moon. In most places they arrive about $\frac{1}{4}$ cycle (6 hours) late.

Galileo himself, probably in an attempt to please the Pope when he sought permission to publish his book, suggested that tides were due to a breathing motion of the Earth! The Mediterranean tides are very small (only about 6 inches); but scientists seeing tides on the open Atlantic must have suspected some connection with the Moon. Yet a simple connection through differences of gravity was too big a jump in thinking. Essentially, the idea of gravitational fields, weakening as they spread farther, producing differences of force, was an easy one to Newton, but an unlikely guess for his contemporaries.

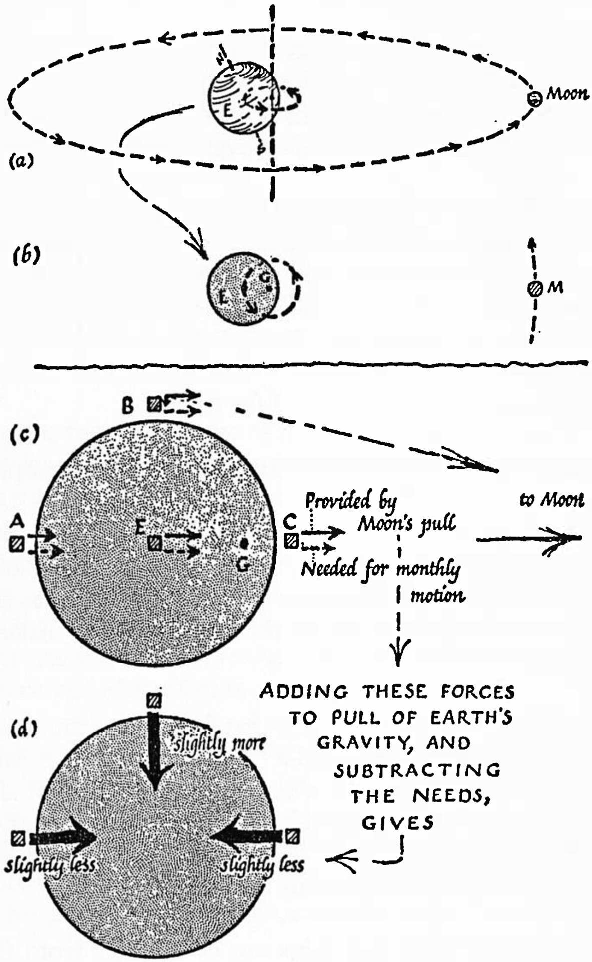
(Although it is a simple idea that gravitational pull produces tides, the detailed explanation is often put in a complicated way. We suggest giving pupils only a very simple account, which will be described later. Before that, however, here is some general discussion for teachers who want to consider the background. Some texts and encyclopaedias give a very complex treatment. (G. H. Darwin's book, *The Tides and Other Phenomena*, 1901, reprinted in paperback, W. H. Freeman, 1952, is recommended.)

If the Earth and the Moon were each a 'point mass', a tiny compact object, but had the same masses and general motion as now, they would revolve around their common centre of gravity, once every 27·3 days. Nowadays we know the Moon's mass and know that the centre of gravity is about 3,000 miles out from the Earth's centre towards the Moon.

The gravitational pull of the Moon is just sufficient to swing the Earth around that centre of gravity in a month. It provides the needed Mv^2/R forces, pulling the Earth towards the common centre of gravity. However, the real Earth is bulky, 4,000 miles in radius, so that the common centre of gravity is 1,000 miles under the surface. Nevertheless, the Moon provides the inward pull to swing the whole Earth in a circular motion around that centre of gravity once every month. (And the equal and opposite attraction of the Earth on the Moon keeps the Moon in its orbit, swinging around that common centre of gravity once in a month.)

That monthly motion of the whole Earth is *not* one in which the Earth spins as well as going round its orbit: the Earth does not – so far as that motion goes – point the same part of its surface always towards the Moon, though the Moon does behave like that towards the Earth. Instead, this motion of the Earth is like that of a window cleaner's hand, sweeping round a circle with his cloth on the window. Thus, *all parts of the Earth go round circles of radius 3,000 miles, all in the same phase.* All need the same 'inward' force on every kilogram of matter to keep them in their monthly orbit. All such 'needed forces' have the same direction, the direction

from Earth's centre to common centre of gravity of Earth and *
Moon – therefore in the 'Earth-Moon' direction. *



THE TIDE-PRODUCING FORCE

Material near the centre of the Earth itself is pulled with just the *
right force to provide the needed Mv^2/R. But out on the surface of *
the Earth farthest from the Moon, the gravitational pull of the *
Moon is a little weaker: it is not big enough to provide the same *
needed Mv^2/R. Therefore, in that region, some of the Earth's own *
common gravity must be used to provide that acceleration, and *
material there will seem abnormally light. Oceans there will exert *
abnormally low pressure. So, oceans elsewhere, exerting full *
pressure, push the ocean up into a hump in the region farthest *
from the Moon. And material at the Earth's surface nearest the *

Moon is pulled outward by the Moon with slightly too strong a gravitational attraction. The force on it is a little more than the needed Mv^2/R. So the oceans there, too, are pulled up into a hump.

Thus, differences of the Moon's gravitational pull lead to two humps of ocean, one farthest from the Moon, one nearest. As the Earth spins daily, different portions of land meet these humps and ocean tides go sloshing up and down every shore.

That is the proper story of the cause of tides, in the simple form that does not take account of modifications by land shapes and other geographical conditions. Yet even that preliminary story is a difficult one for young pupils, likely to give them a feeling that the explanation of tides is complicated instead of showing it as a delightful achievement. Therefore, the discussion above is only offered as a reminder of the background for teachers.

Teaching about Tides. For pupils, we suggest at most a simpler story such as this: T

'The Earth and the Moon pull each other. The pull of the Earth keeps the Moon in its orbit. That gravitational pull provides the force Mv^2/R which each mass, M, on the Moon needs to make it pursue its orbit.

'But the Moon also pulls the Earth and that force keeps the whole Earth moving in a small orbit round a "centre of gravity point" 3,000 miles out from the centre of the Earth. The pull of the Moon gives just the right force, Mv^2/R, for a central mass of Earth in its monthly swinging-round. It must do so, or the Earth and the Moon would not conduct this monthly dance.

'The Moon's gravitational pull thins out as distance from the Moon increases. At places on the Earth farthest from the Moon the pull is a little weaker. And at places on the Earth nearest the Moon the pull is a little stronger. These differences of Moon-pull make the water of oceans pile up into two humps: a hump farthest from the Moon where the water is not pulled "inward" quite enough; and a hump nearest the Moon where the water is pulled "outward" too much.

'Those humps are only a few feet high (not nearly as big as the 14-mile bulge of the Earth's equator, because this motion is much slower than the Earth's daily spin). Those humps *are* the ocean tides. The Earth also has its 24-hour daily spin; and that carries the land-masses round to meet those humps in turn. Then those humps of water go sloshing up every shore in turn, and back again; the two humps make two high tides in 24 hours.'

(There are, however, considerable delays and peculiarities in tidal motion, which have been sorted out by studies since Newton's day. They are due to friction and inertia, combined with the patterns of Earth's masses of land and ocean as boundary conditions.)

The Sun also produces tides. Since the tide-generating forces are differences of gravitational pulls, the Sun's tides are smaller. Because the Sun's distance is much greater than the Moon's and the Earth-diameter from nearest surface to farthest surface is a smaller fraction of the Sun's distance.

'Like the tides due to the Moon's pull, there are also two humps of ocean due to *differences* of the Sun's pull on water farthest from the Sun and nearest to the Sun. These are smaller than the humps due to the Moon, because the distance across the Earth's diameter does not make such an important difference in Sun's gravity. The gravitational field due to the Sun is much stronger than that due to the Moon because the Sun has a very much bigger mass; but the *differences* of Sun's pull are smaller than the *differences* of Moon's pull.

'The humps of tide due to the Sun will fall on top of the humps due to the Moon when the Sun is in the same direction as the Moon, at new Moon. They will also fall on top of each other at full Moon, when the Sun is just opposite the Moon. At each of those two times there will be extra-large tides, Moon-tide plus Sun-tide together. Those are the large "spring tides".

'Half-way between spring tides, when the directions of Sun and Moon are at 90° (half Moon), the smaller humps due to the Sun will fall in some of the low-tide troughs between the humps due to the Moon. Then there will be smaller tides, "neap tides", which are Moon's tide minus Sun's tide.'

Where pupils live at the seaside or are familiar with the details of tides, they will raise questions about this explanation:

Question: Why does high tide not occur at the same time day after day?

Answer: Because the Moon travels round the Earth in the same direction as the Earth's daily spin, so you must wait a little more than 24 hours later to meet the same hump of water that is opposite the Moon.

Question: Why is the high tide not exactly 'under the Moon'? Why do tides differ so much in timing from one port to another?

Answer: Because the response of humps of water is delayed by friction and inertia; and the motion of the humps is further changed by shapes of shore, estuaries, etc.

Unless pupils want to know about such details, we certainly should *
not bring them in here: we are trying to show the richness of *
Newtonian theory; we are not training navigators. *

11. **Mass of the Moon.** We return to our promise that we would T
show how Newton found a 'satellite' for the Moon, and could therefore estimate the Moon's mass.

'Newton estimated the Moon's mass from the Moon's tides. In other words, the Moon *has* a satellite after all: the pair of humps of water that we call tides.'

Large tides (spring tides) occur every fortnight, when the Sun's *
tide and the Moon's tide agree; small tides (neap tides) occur a *
week later when the Sun and Moon are in directions at right *
angles. *

'From measurements of spring tides and neap tides, in open T
ocean, Newton could separate out the tide due to the Sun from the tide due to the Moon:

Spring tide = Effect of Moon + Sun

Neap tide = Effect of Moon − Sun

'Simple algebra – adding two equations, also subtracting them, *
gives the two separate effects. Knowing the size of ocean humps *
due to differences of the Moon's attraction, Newton could treat *
those as a satellite of the Moon and estimate the Moon's mass *
roughly.' *
*

We certainly should not labour this matter of estimating the *
Moon's mass in our account to pupils. But we may mention it as *

one more product of Newton's theory, perhaps praising it with a *
tribute to his perception and skill in extracting knowledge on a *
basis of his theory. *

12. **Precession of the Equinoxes.** Newton gave a clear reason T
for the strange motion known as precession of the equinoxes, and he showed that its period should be about 26,000 years, the known value.

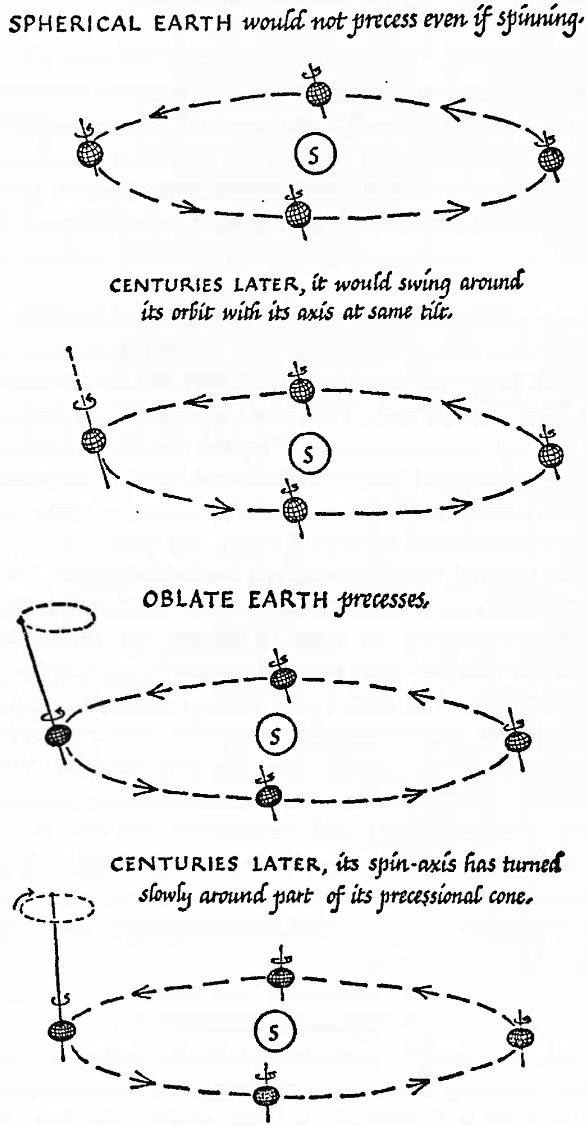
This was one of the most unexpected outcomes of Newton's *
theory. Instead of needing special assumptions, or appearing as *
a minor variation of motion due to practical boundary conditions, *
precession appeared as a direct, necessary consequence of gravi- *
tational pulls by Sun and Moon on the equatorial bulge of the *
spinning Earth.

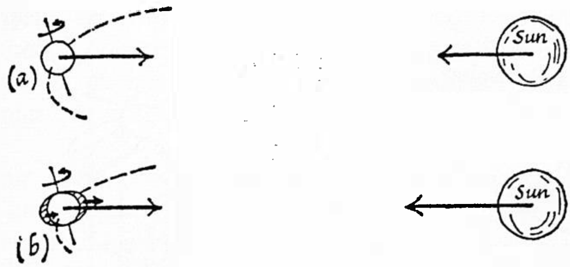
Precession is our name for a slow, conical motion of the Earth's T
spin-axis around the axis of the ecliptic – that is, around a line through the Sun perpendicular to the Earth's orbit, an axis at $23\frac{1}{2}°$ from the Earth's polar axis. That was Copernicus' description of the slow, creeping motion discovered by the Greek astronomers and described by them in a more complicated way. That motion carries the Earth's axis round in about 26,000 years. So the place where it meets the celestial sphere of stars, the place for a pole star, shifts round a path that is roughly a circle, in 26,000 years. Our present pole star, which almost falls on the Earth's spin-axis, is thus a happy accident for our stage in history. At some earlier stages there was no marked pole star, where the Earth's spin-axis met the pattern of stars. Still earlier, e.g. around 3000 B.C., another prominent star served as pole star.

After its discovery by Hipparchus, astronomers made allowance for this motion in their records and predictions, but they had no explanation beyond providing an extra arm or sphere in the machinery to imitate the motion. Copernicus described the motion more simply, but offered no explanation connecting it with other motions in the sky or on Earth.

Newton showed this strange motion is a necessary consequence of gravitation and the Earth's spin. A spherical, spinning Earth would continue spinning with its axis pointing in a constant direction among the stars as it pursued its orbit around the Sun. (We should call that, now, an example of conservation of angular momentum.) But an Earth with an equatorial bulge will suffer slight *extra* gravitational pulls exerted by the Sun and by the Moon on the parts of that bulge. Since the Earth's spin axis is tilted and not perpendicular to the Earth's orbit, those extra pulls make a rocking force, which tends to change the tilt of the Earth's spin axis.

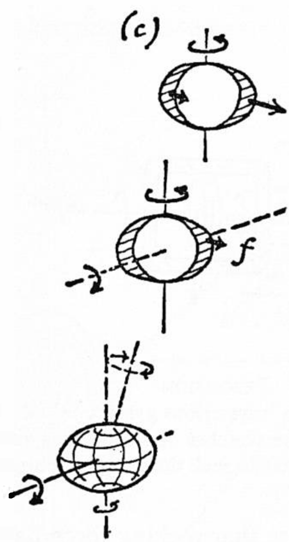
PRECESSION OF THE EQUINOXES



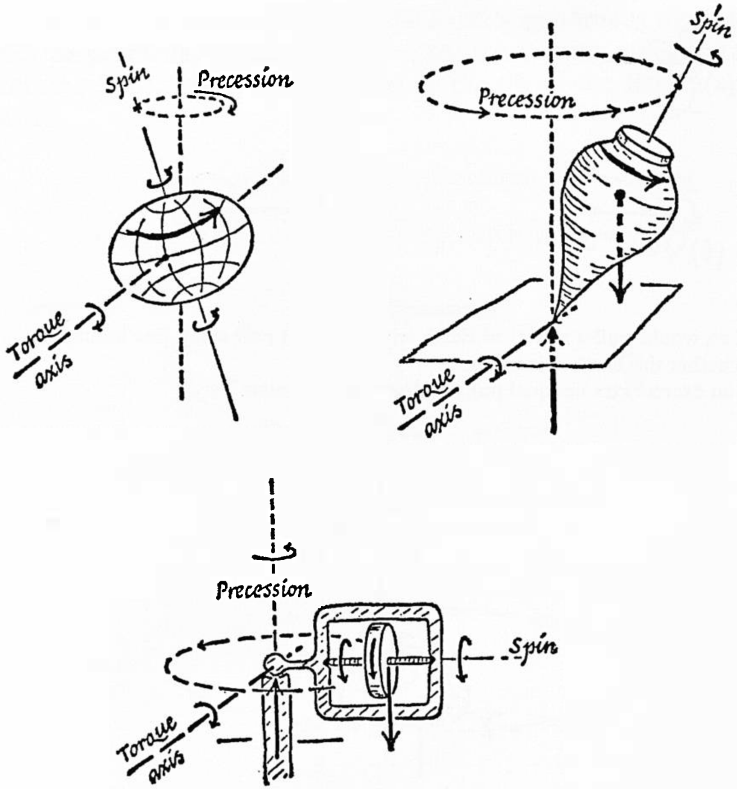


PRECESSION

(*a*) The Sun would pull a spherical Earth with a central pull along line joining centres, whether the Earth spins or not.
(*b*) The Sun exerts extra unequal pulls on the bulge of oblate Earth.



(*c*) The Sun pulls the nearer part of the bulge harder than it pulls the remoter part.
These small extra pulls are equivalent to some extra pull along the line of centres *and* a small residual force, *f*, which tries to rock the Earth's axis.
As with any spinning body, the effect of any force, *f*, tending to tilt the spin-axis is NOT to tilt the axis but to make it 'precess' instead around another axis.



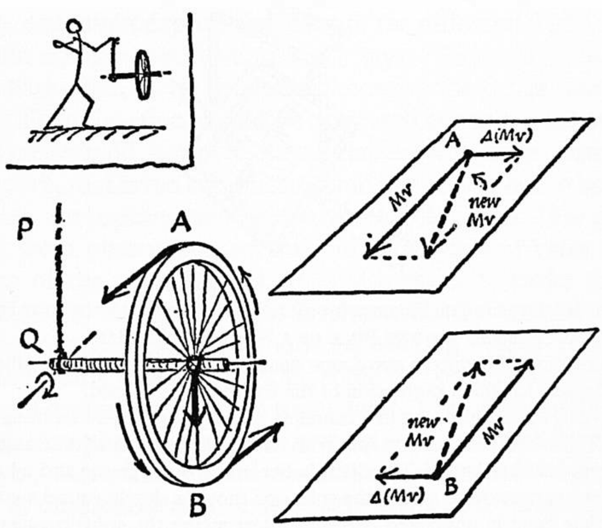
PRECESSION

The Earth, a spinning top, and a 'mysterious gyroscope' all precess in the same way, for the same reason. In the sketches above 'torque axis' means the axis around which the tilting force tries to rock the spinning object.

Since the Earth is spinning, that rocking force has the effect that such a force has on any spinning object, such as a gyroscope: the spinning Earth is made to precess. The rocking force does not succeed in changing the tilt of the Earth's spin-axis; instead, the spin-axis slews round the ecliptic axis with a conical motion, taking some 26,000 years. Thus, precession, which the Greeks discovered and Copernicus described in a simple form with no hint of explanation, now becomes part of the Earth's behaviour expected on the basis of universal gravitation.

The details of Newton's calculation fall quite outside the scope of *
our present teaching. But we should give pupils a glimpse of the *
general idea. That glimpse makes no sense unless it includes *
some knowledge of the way in which a gyroscope behaves when a *
torque is applied. All of us who have tried treating gyroscopes in *

any physics teaching know what a disturbing topic that is. The *
experimental demonstrations are fascinating and paradoxical; but *
attempts to explain them as part of ordinary knowledge of *
mechanics comes to grief easily. *
*
Treating angular momentum as a vector is unconvincing until *
pupils have reached a much more advanced stage. Other explana- *
tions are apt to have a similar *ad hoc* flavour. Here we shall meet the *
same difficulty: delightful experiments in a confusing background *
that lacks any linking explanation. We suggest teachers should not *
attempt explanations of gyroscopes. They should give only two *
demonstrations – and should not expand much into the delights of *
tightrope walkers and paradoxical toys. (Small gyroscopes are *
available as toys and we hope pupils will play with them and enjoy *
them, but we suggest keeping that separate.) *

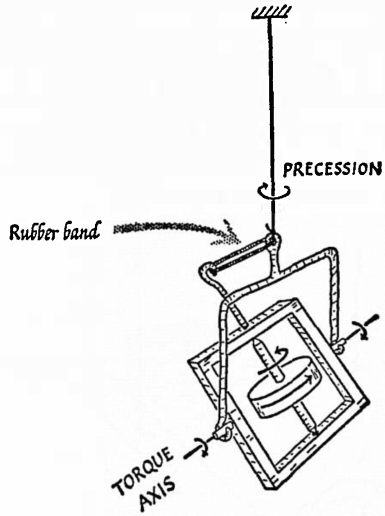


Demonstration of Precession with a Bicycle Wheel, with Extended Axle Hung Horizontally on a Rope

Crude explanation with Newton's Law II, considering only the top and bottom of rim, A and B. If wheel rocks ever so little in response to torque, A gains momentum Δmv to the right; B gains Δmv to the left. Combining these with previous momenta, we find wheel must be precessing.

One demonstration should be the simplest available gyroscope – D69
just to introduce its paradoxical behaviour and show that a rocking
force applied to a spinning body does not succeed in rocking the
body, but produces precession instead. The other demonstration is D70
a special one that imitates the case of the Earth's precession.
A simple flywheel – more massive than the usual toy gyroscope – is

held in a simple frame suspended by a long thread. A stretched rubber band is installed to rock the frame; but, if the flywheel is spinning, the frame precesses instead of rocking. This model is worth the trouble of home-manufacture – but not the high cost of a commercial gyroscope – because its geometry resembles that of the real situation closely enough for it to be very helpful in teaching. Its value is not obvious from a sketch; but teachers who construct it will find its message clear and useful.



EXPERIMENT TO ILLUSTRATE THE PRECESSION OF THE EARTH, WITH SPINNING FLY-WHEEL 'ROCKED' BY THE PULL OF A TAUT RUBBER BAND

This is a modification of ordinary gyroscope demonstrations, specially arranged for teaching the way in which precession of the equinoxes is caused.

A small massive flywheel, spinning in a frame, is held by another frame hung by a long thread. The wheel continues to spin with its axis pointing in an unchanging direction. When a rubber band is installed, between the outer frame and a hook on the inner frame, precession starts, the spin axis moving slowly round a cone. When the rubber band is unhooked (without interrupting the spin) precession stops.

The rubber band's pull represents the Sun's net pull on the Earth's equatorial bulge.

13. **Irregularities of the Moon's Motion.** The Moon's orbit is roughly a circle in a plane tilted about 5° from the Earth's orbit plane. But careful observations show many minor deviations and changes as time goes on. T

The orbit is actually an ellipse, and that would fit with Kepler's Law I just as well as a circle; but the ellipse changes. Its eccentricity varies periodically. Its major axis swings round in the plane of the orbit; and that plane itself slews round slowly. On * * * * top of that, the Moon's motion in its ellipse shows extra accelerations and retardations with periods of one month and one year. Some of those modifications from simple motion at constant speed round a circle were well known when Newton showed that his theory predicted them. But Newton predicted more.

Newton worked out predictions of these effects on a rough scale, explaining them by differences of the Sun's attraction as the Moon travels closer and farther from the Sun in its orbit round the Earth.

(Of course, *if* the Moon stayed at exactly the Earth's distance from the Sun, it would follow exactly the same orbit as the Earth – a satellite orbit does not depend on the satellite's mass, and, therefore, two satellites started on the same orbit will continue together whether they have the same mass or different ones.)

Ideally, one might express the effect of the differential gravitational causes in a single specification which says: 'This is the description of the true orbit, as we predict it, knowing the forces, etc., in full detail.' Then the effect would be expressed as one overall modification, which would, however, have a rather complicated description. In practice, that is too complicated and even turns out to be clumsy when we are making comparison with observation. It is easier to extract from observation several different types of extra motion, as parts of the overall effect. It is also easier to make the same separation in the theoretical development. So we see Newton working out separate deviations, specifying them, recognizing some as already known, checking some by contemporary observations, and leaving others to be tested later. The details of deviations are very complicated and were not fully worked out by Newton. However, those that he did describe and estimate – all of them effects of differences of inverse-square-law pull from the Sun – are now confirmed by measurement.

In teaching pupils, we should not describe the separation of the whole modification into several deviations. We should merely point out that the Moon does move nearer and farther from the Sun, and as a result experiences *differences* of gravity – rather like tides on the Earth on a much wider scale – which make some changes in the Moon's motion.

If any pupils ask, as a result of their own reading, why there are several kinds of change, we should point out that those are symptoms of the effects of one main modification. They resemble the symptoms of measles, all arising from the same infection, but separated out by the patient into sore throat, itching spots, troubled eyes, and a high temperature. The doctor may use all these in diagnosing a single ailment.

In one case – the slewing round of the Moon's orbit in its own plane – Newton's prediction disagreed with observation. He predicted $1\frac{1}{2}°$ per month, and observation clearly showed 3°. Mathematicians in a later generation attempted to explain the difference by changing from an inverse-square law of gravity; but then they found that Newton's own treatment had neglected a term which would bring the prediction up to 3°. Much later still, a paper was found in the archives of Newton's work that showed he had found his own mistake and corrected it, then just put his notes away as usual.

The only important contribution to our present teaching is a brief comment that Newton extended his theory to predict small disturbances of the Moon's motion. He did not need to make any new assumptions for this and his predictions proved correct so far as their rough form went. And in finer detail that work of predicting and checking is continued today.

14. **Perturbation of Planets.** Extending his thoughts of universal gravitation to the effects of one planet on another, Newton saw that the larger planets must pull neighbouring planets out of their simple Kepler orbits noticeably. He knew it would be only a very small perturbation because he knew the masses of the planets are very small compared with that of the Sun. Jupiter, the most massive of the planets, has a considerable effect on the others. T

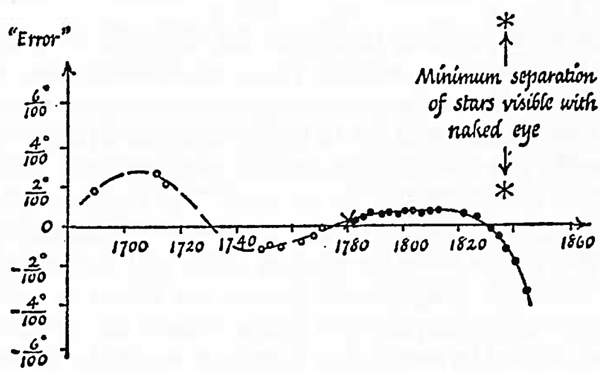
Newton showed how to estimate such disturbances. Since the pull of a neighbouring planet does not act along the radius from the Sun, Kepler's Law II will not be obeyed exactly, nor will the orbit be exactly an ellipse following Law I. Newton made a beginning on methods of working out deviations from those laws, still holding only his original assumptions. We have continued his work to the present time, in increasing detail, and so far the predictions have fitted the observed disturbances or deviations very closely.

(Teachers must decide for themselves whether this is the moment to mention the motion of the perihelion of the planet Mercury. After all the perturbations by other planets have been allowed for, there is still a slow precession, or slewing round, of Mercury's orbit in its own plane. Observations, after all those allowances, show a motion of 43 minutes of angle per century. That residual discrepancy is generally considered to be a successful test of the modification of inverse-square-law gravitation suggested by general relativity. However, there are still doubts in the air. Decision hangs on a very small motion; and other causes, such as a concealed inhomogeneity in the Sun may account for the deviation instead.)

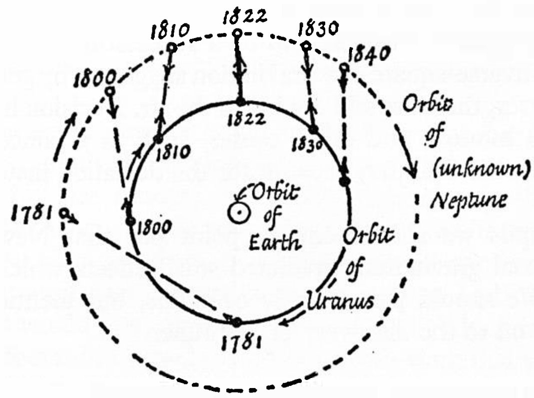
In teaching pupils we should simply point out that Newton, assuming universal gravitation, predicted small effects which we have verified. We should pass quickly over this, but mention it because it leads on to the discovery of Neptune.

Outcomes: Explanations and Connections. All these outcomes of Newton's theory were linked with our observation of accelerated fall of any object near the Earth. Thus, Newton linked the heavenly system to direct earthly knowledge. However, an even greater virtue of his theory is the wide variety of effects that he showed to be connected by his simple rules of mechanics and universal gravitation.

15. **Discovery of Neptune.** A century after Newton's time, another planet, Uranus, had been discovered by observation. Presently that planet showed some small deviations from its Kepler orbit, beyond those that could be accounted for by the perturbing attractions of neighbouring planets. These residual discrepancies were small but demanded explanation more and more insistently.



RESIDUAL 'UNEXPLAINED' PERTURBATIONS OF URANUS (A.D. 1650–1850)
The 'error' is the difference between the observed position of Uranus and the expected position (for a Kepler orbit) after known perturbations had been subtracted. The point X marks the discovery of Uranus by Herschel. Working back to its orbit in earlier times, astronomers found that Uranus had been observed and recorded as a star in several instances. These earlier records are marked by ° on the graph. (After O. Lodge, *Pioneers of Science*.)



PERTURBING FORCES ON URANUS, DUE TO NEPTUNE

The sketch shows positions of the planets in the years marked. Before 1822 Neptune's pull made Uranus move faster along its orbit so that it reached positions ahead of expectation. After 1822 Neptune's pull retarded Uranus. (After O. Lodge, *Pioneers of Science*.)

Modifications of the inverse-square law of gravitation were tried without much success. Then two astronomers, Adams in England and Leverrier in France, took the suggestion of an unknown planet as the cause of the perturbations and attacked the problem with great courage and skill. How they succeeded, how the planet Neptune was discovered through pure theory, is a romantic story that all pupils should hear and understand as a crown to Newton's work.

It was a stupendous problem, to account for the small observed perturbation by locating an *unknown* planet of *unknown mass*, at an *unknown distance out*, in an *unknown direction*, moving round its orbit in a time which would not be known until the distance out had been found. The two mathematicians, working quite independently, were indeed groping in the dark. The observed perturbations did not even 'point' in the direction of the disturbing planet. Nor did they indicate the amount of its pull at the time Uranus was observed. They merely showed the effects of accumulated changes of motion produced by the pulls of the unknown planet. Adams decided he knew where the planet was in the sky and asked the Astronomer Royal to look for it in the direction he suggested. Many a crank sends suggestions to the Astronomer Royal; so no search was started. The Astronomer Royal wrote back to Adams, asking a question about a further detail. He was surprised to receive Leverrier's suggestion of an unknown planet in almost the same position; and he did then start a leisurely search. However, Leverrier wrote to the Berlin Observatory, where a new star map made a quick search for a tiny planet much easier. The planet was seen. Neptune was discovered.

Theory

After describing Newton's work we should look back at it and comment on its nature as a theory. Newton started with a few assumptions – chosen with good knowledge of nature – and from them built a great scheme which predicted some things already known, other things later to be discovered, and it gave a sense of connected knowledge which scientists feel is a powerful possession. * * * * * *

Evidence for Universal Gravitation

We should show pupils a table of measurement of the gravitation constant. In Newton's day there was nothing but a wild guess, made with surprising success by Newton himself. Then came a long series of experiments, starting with work of Cavendish to measure the tiny attraction between objects of known masses. T

MEASUREMENTS OF G

Date (approx.)	Experimenter	Attracting Mass		Attracted Mass		Distance apart meters	Result G newton • m.²/kg²
		Description	Mass kg	Mass kg	Description		
A							
1740	Bouger	Mountain	many millions of millions of kg	pendulum mass: a few tenths of a kg to a few kg	pendulum	several thousand meters	12 $\times 10^{-11}$
1774	Maskelyne	Mountain			pendulum		7 to 8 " "
1821	Carlini	Mountain			pendulum		8 " "
1854	Airy	Outer shell of Earth	3×10^{24}		pendulum	6,000,000 meters	5.7 " "
1854	James	Mountain	many millions of millions of kg		pendulum	several thousand meters	7 " "
1880	Mendenhall	Mountain			pendulum		6.4 " "
1887	Preston	Mountain			pendulum		6.6 " "
B							
1798	Cavendish	lead ball	167	0.8	lead ball	0.2	6.75 $\times 10^{-11}$
1842	Baily	lead ball	175	0.1 to 1.5	balls of: lead, zinc, platinum, glass, brass	0.3	6.5 to 6.6 " "
1881	von Jolly	lead ball	45,000	5	metal ball	0.5	6.46 " "
1891	Poynting	lead ball	160	23	lead ball	0.3	6.70 " "
1895	Boys	lead ball	7	0.0012	gold ball	0.08	6.658 " "
1896	Braun	brass ball iron ball	5 9	0.05	brass ball	0.08	6.66 " "
1898	Richarz and Krigar-Menzel	lead cube	100,000	1	copper ball	1.1	6.68 " "
1930 1942	Heyl and Chrzanowski	steel cylinder	66	0.05	platinum, glass, gold	0.1	6.673 " "

There were outdoor tests too, using a measured mountain as attracting mass. Pupils who see a table of dimensions and masses and results will be impressed by the great range of those experiments, so that the inverse-square law of gravitation seems well vouched for by earthly tests – heavenly tests already vouched for it, by Kepler's Law III.

We look forward to a similar test of the field of force inside an atom, when the scattering of alpha particles follows exactly the prediction for an inverse-square law of electrostatic repulsion. * * *

Chapter 4

OSCILLATIONS AND WAVES

Simple Harmonic Motion
Alternating Currents
Waves

PROGRAMME

In the suggested work on oscillations and waves, interference of light waves is the most important section, from the point of view of our development of atomic physics this year. Pupils will see Young's fringes, diffraction phenomena, and grating spectra, as evidence that light has properties of waves.

The most important class experiments will be the interference measurements mentioned above and the continuing series of experiments with alternating currents.

Although we comment on other matters at length – such as simple harmonic motion, algebra of alternating currents, and observations of many different spectra – we do not advise spending much time on those.

OSCILLATIONS AND WAVES

Programme

The work of this part of the year will depend considerably on the amount pupils already know about waves and about alternating currents from studies in Years III and IV.

In Year III we suggested that experiments with the ripple tank should extend into a first look at interference. We even suggested a rough measurement of wavelength of light by Young's fringes; but we expect many pupils will have missed that.

We suggested in Year IV that some pupils should continue with the electromagnetic kit into experiments with alternating currents; but we expect that many will have done no more than see the wave-form of a.c. from the bicycle dynamo. Pupils who have missed such experiments should do them now as class experiments. They should be given enough time to continue into experiments with very slow alternating currents.

This Year's work with electron streams in magnetic fields will have taken considerable time but the history of astronomy will have needed practically no time for class experiments. And the work in radioactivity, though very important, can be done in fairly short demonstration or class experiments. So pupils should have a considerable time to work with waves and a.c.

Alternatives: Special Experiments (*Buffer options*)

Those who wish to try Millikan's experiment themselves will need to save considerable time for it; but the experience of doing it will be so good that we shall not grudge them economies elsewhere.

Those who try an accurate measurement of 'J' *should be given plenty of time – not to get rid of errors, but to learn how inescapably the errors are there. (However, since this experiment is so easily treated as a measurement 'to get the right answer' – which would damage our programme's teaching – we do not advocate it unless the teacher is anxious to give considerable guidance to pupils working on it. We do* not *advise schools to buy special apparatus for this. Only where a school already has the apparatus and wishes to use it carefully should this experiment be included.)*

SIMPLE HARMONIC MOTION

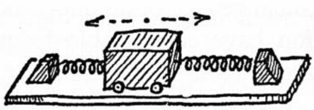
Looking at S.H.M.

We start by giving pupils three examples of S.H.M. to be investigated as class experiments, without any warning or explanation of the nature of the motion: C71

a. a simple pendulum C71a

b. a load hung on a spring C71b

c. a trolley, on a smooth horizontal table, tethered between end-walls by stretched springs. C71c



The last is in some ways the best example, because gravity is not involved and the two essential factors that control S.H.M. in all its forms – the spring-factor and the inertia-factor – are visible and variable.

We have been insisting in astronomy that constancy is the essence of scientific descriptions and laws; so it should not seem a strange question when we now ask, 'What can you find that is constant in the behaviour of each of these devices?'

We suggest that pupils should move round among the three forms fairly rapidly: this is only a preliminary glance at a new type of motion so it can be brief – but it should *not* be replaced by a demonstration, or pupils will miss the sense of involvement that they need to carry them through further experiments.

Give, say, one class period (certainly not less) for pupils to find out what they can. Then at the beginning of the next class, ask what each found out. Some will have discovered things that we do not normally list as essential characteristics of S.H.M. (Examples: the dying-down of amplitude; the loaded spring's exchange of motion between vertical oscillations and pendulum swings.) We should not condemn or even disregard these, but should accept them and even encourage further investigation. For example: is the dying-down of amplitude exponential? It would be easy enough to find out: and the question itself can be reworded in a much simpler form for a young experimenter. Or the observation can provoke the question: 'How could you make that worse?', leading to suggestions of paper sails, immersion in water, or even to a discussion of mass. T

Again, the loaded spring's strange interchange of motions may lead to questions:

1. 'Where have you seen something like that before?' (And we might award a scholarship to the pupil who says 'in the coupled pendulums in Year I or II'.)

2. 'How could you discourage that strange change of the vertical bouncing into sideways swings?' That is too hard; so we give a hint by saying, 'That exchange of motion happens easily because the two types of motion have comparable frequencies. What could you do to make one motion much slower or faster without changing the other?'

If that is not enough, we offer a practical hint: a length of string.

Note to Teachers. A load hung on a spring has two quite different types of possible *motion*: bouncing up and down and swinging sideways like a simple pendulum. The spring acts as a coupling agent, connecting these two motions together. As in other 'coupled systems', energy is carried from one motion into the other, and then back to the first. The closer the two frequencies are to each other, the easier it is for the load, when moving with one motion, to excite the other motion.

If we work out the period of vertical bouncing of a load on a spring, we find it is equal to the period of a simple pendulum whose length is the stretch (the extension of the spring) when that load is applied to the unloaded spring. Therefore, if the spring has a very small unloaded length, and most of the loaded length is stretch, the period of vertical motion is only a little shorter than the period of pendulum-motion for the loaded spring. Then, the vertical motion soon transfers its energy to pendulum-motion; then back again to vertical motion, and so on.

If an experimenter is trying to measure the period of the vertical motion, this is a very irritating phenomenon. The cure is to insert a considerable length of string between the lower end of the spring and the load. This does not change the forces involved in vertical bouncing, so the period of that is the same as before. But the period of pendulum-motion is now much longer, and transfer to that motion will happen much more slowly.

Outcomes. Most pupils will have found the isochronous property, or at least a strong hint of it; and some may have found some of the mass-relationships.

S.H.M. Important. We explain that this is a very important type of motion because: T

'It is very common. You will see many examples of it. And it is the motion in musical instruments when they make a pure musical note.' So we call it simple harmonic motion.

We explain that it is also important in more advanced physics because (later on, A-level) we can calculate its frequency; and because (still later) we can analyse ANY repeating motion – tides, sound waves, motion of moon, electron-waves in atoms, and many more – into a whole set of S.H.M's. (If we ask for an earlier example, the pupil who says, 'Yes, Eudoxus', should get a scholarship at once.)

Then the teacher should give as many demonstrations of S.H.M. as possible. Here are some examples: D72

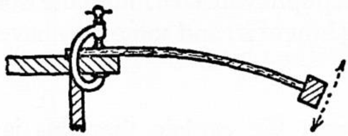
a. a simple pendulum side by side with one twice as long and one four times as long, without comment; D72a

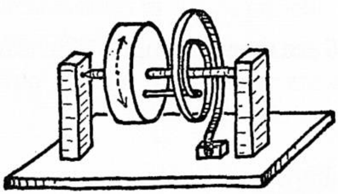
b. a torsion pendulum; D72b

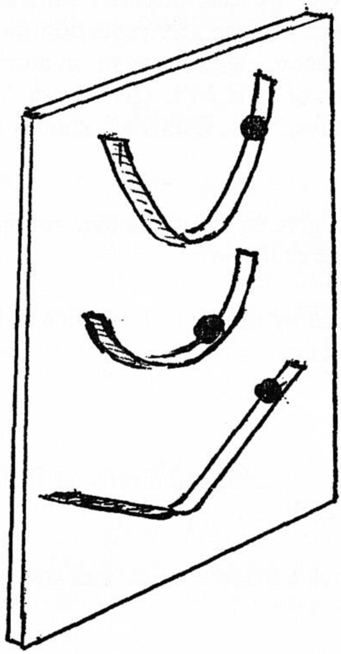
c. a horizontal lath of wood anchored firmly at one end, with a massive load on the free end; D72c

d. a huge model of a watch's balance wheel and spring; D72d *e*. a U-tube with water in it (ask whether the period would be the same with mercury); D72e

f. a ball rolling in a bowl ('Listen to the sound: what can you say about the motion?'); D72f

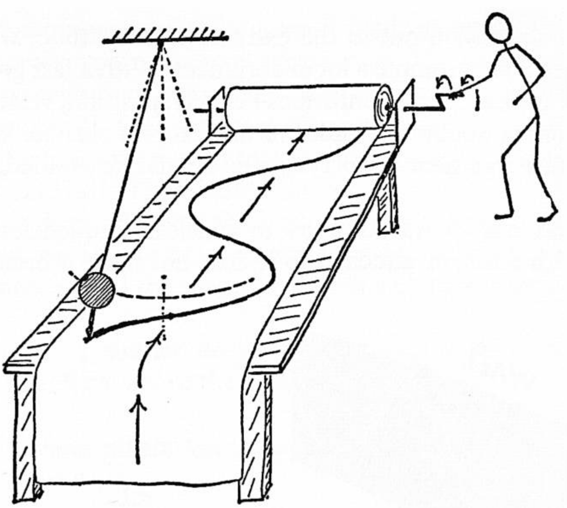






g. If available: an almost vertical board carrying three metal 'shelves' on which a ball can roll to and fro as in a bowl; one shelf a circular arc – a short arc, not a whole semicircle; one a parabolic arc; one a V shape of straight hills making an obtuse angle joined by a short curve at the bottom. Pupils listen. The first will give what sounds like isochronous motion; the second, a frequency that changes while a big amplitude dies down, but isochronous for small oscillations; the third, not isochronous at all as the ball rolls up and down the shallow opposing hills; D72g

h. the 'wig-wag' that was used in Year IV to compare masses. D72h



Time-graph of S.H.M. Then show that the time-graph of S.H.M. is a sine curve. This may be shown by a massive pendulum that carries a paintbrush on its bob, tracing a time-graph on paper that is moved steadily. Or a pendulum with a funnel of sand as bob can trace its motion on a moving sheet of paper. Or if we have a blackboard that can be moved steadily up or down we can write on it with a toy paintbrush fed with water from a small reservoir carried by a tall vertical lath that sways to and fro. D73

This should be followed by a demonstration (again) of the wave-form of the a.c. voltage, with a C.R.O. D74

Descriptions of S.H.M. Note that we have not defined S.H.M. or even described it precisely – that belongs to A-level. We have simply shown many examples, and shown the time-graph of one or more. * * * *

We should now ask one 'theoretical question': 'Is the acceleration *constant*?' *
*

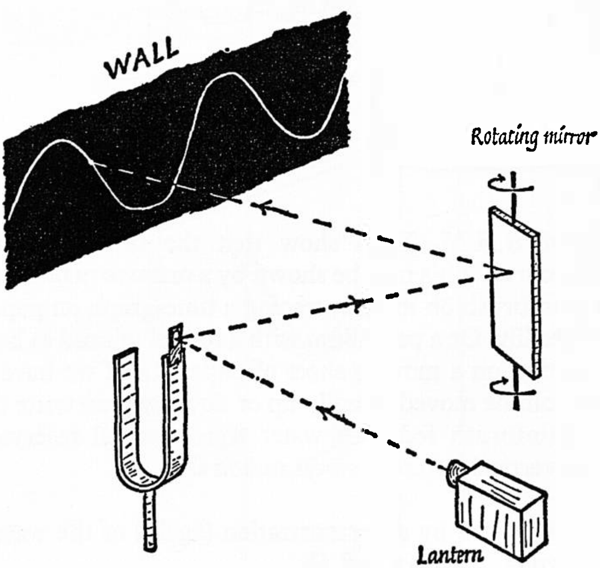
With a very long pendulum we could even make measurements of velocity with the scaler and millisecond pulses using a photo-diode. D75 OPT.

We can illuminate the matter by argument. T

> 'Where is the bob moving *fastest*? ... Yes, at the centre. If it is moving fast*est* there, can it move any fast*er* just beyond the centre? Can it be accelerating just there? ... Well, if it is, it would have to ...'

Extending the discussion out to the extreme of amplitude, when the bob is at rest, will promote a lot of argument. With a fast group this can reveal and cure some confusions between position, velocity, and acceleration, or doubts over maxima and rates of change. With a slow group this argument is worrying and should be avoided.

We can extend our demonstrations to musical frequencies. A tuning fork with a mirror attached to it does not make a beam of D76



light oscillate enough to show clearly; the motion will be made visible if the mirror is attached to a small strip of mica stuck to the tuning fork arm. The mirror on the mica will be driven by the fork with much the same motion but larger and it will deflect a small beam of light with considerable motion which can be compared with the trace given by an oscilloscope with a.c.

Let a microphone listen to the tuning fork and show the waveform on an oscilloscope. D77

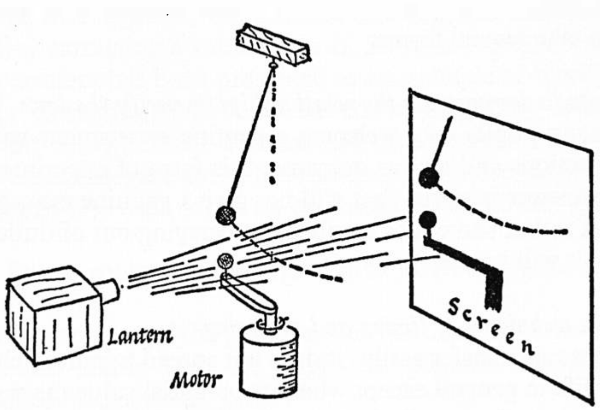
S.H.M. as Projection of Circular Motion

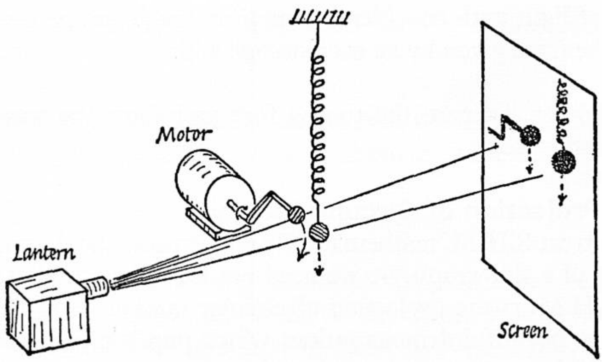
We shall not treat S.H.M. mathematically and express the description in terms of a sine graph. So we need not labour discussion to show that S.H.M. is the projection of circular motion. However, there are some beautiful demonstrations which pupils might see if they are available. * * * * *

We attach a ping-pong ball or a small lump of cotton wool to the end of a revolving arm on an electric motor. The arm should make one or two revolutions per second. We shadow this circular motion on a distant wall. The compact-filament lamp should be in the plane of the ball's motion so that the ball's shadow moves up and down with S.H.M. Then beside the rotating arm we hang a small load on a spring and adjust the load to have the same period as the arm. Then we start a true vertical S.H.M. side by side with a circular motion, and the shadows of both will move together. D78a OPT.

We can make a similar demonstration with a short pendulum swinging to and fro above the rotating arm. D78b OPT.

We should of course show a train of simple harmonic waves starting out along a rope. D79





We may give pupils pendulums to investigate but we should not ask them to make detailed measurements with pendulums to find g unless they are so able that we can go through the full derivation of the formula. (If we give pupils the formula, with little idea of its derivation, and expect them to make measurements of g with its help, we may well give science a poor reputation. Those pupils who will proceed to A-level physics will certainly have an opportunity there to use the formula to measure g with full understanding. Anyway, detailed precise measurements of g do not seem so important in this course as in some other programmes of physics.)

For the teachers who wish to spend considerable time on pendulums, here are comments and suggestions.

Comment to Teachers: Pendulum Experiments
Class experiments with pendulums in different teaching programmes can take several forms:

a. Experiments to demonstrate the relationships or verify the laws
(Although many pupils may welcome a routine experiment with definite instructions and a clear outcome, this form of experiment in which the answer is stated first will not give a genuine example of science but rather show it as an obedient carrying out of duties. In retrospect it will be dull.)

b. Training in techniques of timing and observing
(Training does not transfer easily; it does not spread to other fields of science or life in general except when its potential value makes a strong impression. For pupils in general, this training with pendulums will be wasted.)

c. A scientific investigation
(Even though pupils know that we are well aware of the 'answer', that we have some 'formula' that tells us how pendulums behave, they can do genuine scientific work if they regard the experiment as an investigation. The experiment itself may be almost the same as 'verifying the formula' but the instructions need an essential twist: we must put it to pupils that they are asked to find something out and not to show that what we have already said is true. To set the stage for such experiments, we *do* say what should be measured (e.g. many swings should be timed) or what relationships should be investigated; but we certainly *do not* say what results are to be expected.)

Large amplitudes should be tried as well as small, even if they necessitate rough timing, to show that the clockmakers' hope is not exactly fulfilled. Different masses of bob should be tried, but that should not be laboured – instead we should follow the experiment with a question about expecting it with its results from earlier knowledge.

Timings of period for various lengths, which can be so tedious, should be reduced to a quick anxious competition among pupils to contribute to a communal graph and table of results for the whole class.

Given the right spirit, this could be one of the best experiments of the Year.

An Accurate Measurement of *g* (*Optional for very fast groups*). (After an investigation, the formula can be given and able pupils will enjoy extracting a value for *g*. If, however, the general part of the experiment has been presented as an example of 'verifying the formula' extracting *g* will be the last straw. To pupils *g* is not a very important measurement and instead of welcoming this indirect measurement they are apt to feel that it is an unnecessary excursion. We suggest (*c*), with (*d*) as an extension for some fast groups.)

Class Experiments: Measurements with Pendulum
[Treatment (*c*)]
We give the period of the pendulum as the thing to be investigated and discuss briefly with the pupils the physical conditions that might affect the period. C80 OPT.

We ask for suggestions. Pupils know that a pendulum of greater length takes a longer time to swing to and fro, so they are likely to suggest *length* as one factor affecting period. We encourage further suggestions and hope to hear: *amplitude* (which we define as the angle of swing from the vertical, giving a sketch); *mass of bob*; perhaps *g*.

If pupils start to investigate, say, mass as a factor, and make timings for a long pendulum with a small bob and then with a large bob, they can keep the length factor constant – as they should while they investigate something else – but unfortunately they cannot keep the amplitude constant. They will soon come back with the complaint that the swings are dying down in amplitude, although they are not trying to investigate that change. Therefore, we have to suggest that the first factor to be changed should be the amplitude, because that will die down in any case. Then, if after that we know what to do about changes in amplitude, we can go on to investigate other factors such as mass of bob.

This is a difficult discussion of planning, too strange and difficult for most pupils to do on their own. So, with most class groups, the teacher should suggest trying different amplitudes first and simply mention our special reason for that choice. With a very fast group, putting the question to the class, 'Which factor must be tried first?' could stimulate good thinking, and we hope teachers will try it.

a. Period for various amplitudes. We suggest very rough measure- C80a
ments with big amplitudes, which can be plotted on a graph; and then more careful measurements when it becomes apparent that the period does not change rapidly at small amplitudes.

Pupils should plot a quick graph, hand to mouth, as they proceed, plotting period upwards and amplitude in degrees along. The rough measurements will show that the period does change somewhat when the amplitude is decreased from 80° to 60° to 40° to 20°. Therefore, in any attempt to investigate the effect of other factors (such as mass or length) on the period of the pendulum, it will be essential to choose some amplitude region other than the large amplitudes, since there changes of amplitude would also affect the period as the motion died down.

With a fast group, we raise the question in a careful discussion:

> 'As the amplitude drops down from large values, the period does change a certain amount; so you could not expect to make a precise investigation of the effect of different lengths on periods if

you have changes of period due to the amplitude dying down at the same time.

'Does the curve continue like that to small amplitudes, or is there any chance that, at a still smaller amplitude than you have tried, the graph of period against amplitude will be a bowl with a minimum-region at which you could make your measurements; or flatten out to a bowl at zero amplitude? You would have to do some careful measurements to find out which of these is true. If you cannot find some way of dealing with decreasing amplitudes you may just as well give up the experiment.'

With encouragement and care, and clear exposition of the nature of the problem which threatens to stop the investigation, pupils will find the period changes very little at smaller and smaller amplitudes.

b. Period for various masses of bob. Then pupils should try two different bobs on a long pendulum. The timing of these will show that period is independent of mass; and, although it is not the simplest way of showing this, it is probably the most impressive. (The simplest is to start two pendulums side by side and watch them swing together!) C80b

When pupils come back with the surprising result that the period with a heavy bob is the same as the period with the light one, we tease them saying: 'Could you have predicted that from something you already knew?' If they cannot guess, we finally point out that this is a case of falling bodies of different masses. The string of the pendulum does not help the acceleration of the bob along its arc, so we should expect this to be a diluted case of the leaning-tower experiment which Galileo did not do.

c. Period for various lengths. Making careful measurements for pendulums of different lengths and plotting graphs can easily become a tedious business – and for pupils at this stage a rather pointless one. Instead of that, we suggest that each pair of pupils should make one or two measurements, repeated once or twice, to obtain fairly reliable values of period for a measured length. C80c

To provide a useful range of results, the teacher must assign different lengths to different pupils beforehand.

Then a communal graph should be plotted by the teacher – and perhaps pupils should take home measurements of the whole class to plot their graphs. If the initials of the pupils who made the measurements are pencilled against each plotted point, some very useful (and heated) discussions ensue.

Graph of Measurements; and Graph to give Straight Line. C80c
The graph of experimental results, T versus L, plotted directly, will be a curve. We should not say it is a parabola, though a fast group could profitably discover that if we asked a few leading questions.

We then ask what should be plotted if we want a straight-line graph. (See below for discussion of straight-line graphs, their advantages and interpretation.) We elicit the suggestion: plot (period)^2 against length, T^2 versus L. With an average group, the teacher may have to suggest that. With a fast group, the teacher should ask pupils to look at the graph of T against L and find on it what happens to T when L is doubled (Answer: not an obvious factor) and then what happens to L for double T (Answer: 4 times as long, if pupils remember to include the radius of the bob).

Each pupil should then plot a graph of T^2 versus L for the results of the whole class. Or, in a slower group, the teacher should plot the graph on a large board. If the points seem to lie close to some straight line, pupils should use a taut black thread to look for the 'best straight line' and draw it. If they think the line passes near to the origin, they may prefer to force their chosen straight line to pass through the origin. If they make the latter choice, their straight line will ask the question, 'How close to the simple law $T^2 \propto L$ is the behaviour of the pendulums in our lab?'

Note to Teachers, on Arguments about the Meaning of a Straight-line Graph. We remind pupils that the advantages of a straight-line graph are:

1. It is easily drawn with a ruler.

2. In choosing the best straight line, we take a 'weighted average' of our measurements, giving less weight to points that seem out of line with the rest. Though that may be dangerous, it is often good scientific procedure.

3. *If* we draw a straight line through the *origin*, that represents direct proportionality between the two things plotted. And if our plotted points lie close to such a line, we can say that our measurements show the behaviour of our experiment is close to that proportionality.

With a fast group, we should clear up the logic of this fully. Pupils should see that the straight line we draw expresses perfect proportionality. Our points express the facts of our experimental observations. When we compare the points with the line, we are comparing the facts with a simple proportionality law.

There was an important example in earlier years when we measured the motion of a trolley running down a hill, or a trolley pulled by a constant force. We hoped to find this was a case of constant acceleration. If we plotted distance, s, against t^2, where t was the total time of travel from rest, we hoped to get a straight line. A straight line drawn through the origin on that graph has the equation

$$s = (\text{constant})\ t^2.$$

We may draw such a line whenever we know that s is directly proportional to t^2.

In fact, we do know that s is proportional to t^2 for any case of *constant acceleration from rest.* We know that through irrefutable logic, simple, reliable mathematics, leading from the statement

$$\frac{\text{change of velocity}}{\text{time taken}} = \text{constant, } a$$

to the result $s = \frac{1}{2}at^2$. No experiment is necessary to show that IF a is constant, THEN $s = \frac{1}{2}at^2$, because logic does that. No experiment is necessary to show that the graph of s versus t^2 is a straight line through the origin for constant acceleration from rest. Then why do we plot the graph? What are we doing when we draw the straight line among our plotted points? *We are trying to find out whether our trolley moved at constant acceleration.* We know the line is true for constant acceleration; so, by seeing how close our points come to the line, we are seeing how close our trolley's measured motion comes to constant acceleration.

In many experiments we can find or construct some functions of our measurements which, when plotted, will bring the points near a straight line; but the 'best straight line' may fail to go through the origin. In that case, drawing a straight line and looking to see how close the points are to it is not asking whether we have a case of direct proportionality, $y = kx$. But we are still asking whether there is a simple linear relationship $y = kx+c$. To us as physicists the latter is almost as simple and interesting as simple proportionality, but pupils either find it less clear and simple or think it is just the same as proportionality; and we need to teach the difference as far as we find pupils appreciate it easily.

In some experiments, all our measurements of one quantity are wrong by a constant amount. (For example, in a pendulum investigation of T versus L all the lengths may be too small because we forgot to add the radius of the bob.) Then, we should choose our functions for plotting so that the measurement with the constant defect is left untouched.

(In our example, plotting T^2 against L will still give a straight line, if every value of L is too short by the radius, but plotting T against $\sqrt{L}$ will not give a straight line.) Then, if our graph in the ideal case would be a straight line through the origin, the intersect where the best line fails to pass through the origin may give us valuable information. (In our example, it simply tells us the radius of the bob, probably rather inaccurately.) One of the most far-reaching examples is the graph of pressure of gas in a flask (constant volume) against temperature. The intersect on the temperature-axis gives an estimate for absolute zero.

Discussion of Errors in Pendulum Measurements. If our pupils, in a fast group, understood the random walk argument in Year IV, we might mention it again here. We point out that if we make a measurement a large number of times and take the average we are really dealing with the result of a large number of errors like steps in a random walk. Therefore by taking ten times as many sets of measurements and averaging them we do not reduce our likely error by a factor of 10 but only a factor of the square root of 10. T

A Value for *g*. If the teacher wishes, he may help pupils to arrive at a value of g from the graph that they have drawn. That should not be a graph of T against the length but of T^2 against the length. T

Our overall aim should be to give pupils experience in real experiments and a general feeling for the nature of pendulum motion rather than skill in making pendulum measurements.

ALTERNATING CURRENTS

Dynamos: D.C. and A.C.: Meters and Oscilloscopes

We now return to the electromagnetic kit, resuming the study of the simple dynamo. Pupils should try for themselves d.c. and a.c. dynamos on a simple moving-coil meter; then on an oscilloscope. C81

Then we provide the bicycle dynamo and ask pupils to try that on a meter at various speeds; and on an oscilloscope. C/D 82a, b

Then they try a sample of the mains supply on an oscilloscope. That should be a class experiment. It is not dangerous, since an isolation transformer can provide a small safe alternating voltage. C 83

Characteristics of A.C. It would be easy to leave a.c. as a slightly *
mysterious version of the direct currents that we deal with in *
simple d.c. experiments, and to suggest that detailed studies belong *
to later work in engineering. But a.c. is our standard form of supply, *
far more economical in distribution because of the efficiency and *
simplicity of transformers. *

*

Pupils are likely to be interested in its characteristics: the obvious *
ones, such as giving the same heating effect as a direct current, and *
failing to move a d.c. ammeter visibly; and the surprising ones *
involving phase-differences. *

*

Therefore, we should give a little teaching of a.c. to all classes, *
expanding it for faster groups, and extending it still farther for *
groups with special interests. We should point out the difference *
between peak values and average values, explaining how we use *
root-mean-square averages. *

Oscilloscope. We show the wave-form, or time-graph, of an alternating voltage on an oscilloscope. Then we can define some useful names: peak voltage; average voltage, which is 0. D 84
We point out that there is a voltage nearly all the time, and ask for suggestions of a way to specify something useful instead of the plain average. With a fast group we should suggest the idea of squaring the voltage, V, finding the average value of V^2, and taking the square root of that. We might even point out that such an average, the R.M.S. value, is what we really calculated in Year IV for the speed of air molecules.

For a slower group, we should just say that we can take an average of the upper half of a cycle which is positive, and arrange to take a similar positive average for the lower half. It is helpful to make an oscilloscope show an alternating wave-form and then sketch such an average value on its face, with a china-marking pencil.

Meters. We explain that there are ammeters and voltmeters designed to measure R.M.S. average values. As a voltmeter, a gold- T

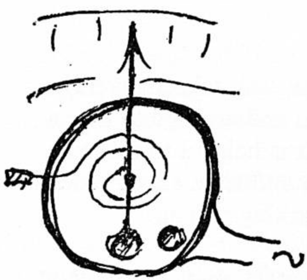
leaf electroscope does that because, whenever the voltage drives charges on to the leaf, the leaf is repelled from its neighbouring rod whether those charges are all positive or all negative. We may show D85
that with + and − electronic charger, then with a high a.c. voltage.

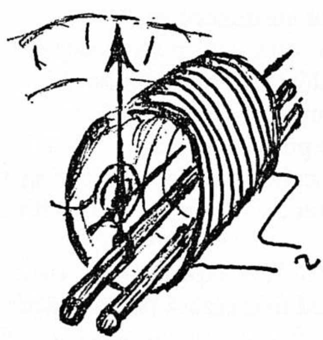
For an ammeter, we might mention the old-fashioned 'hot wire T
ammeter', in which the current heats a wire which expands and sags. The meter measures the sag. Pupils know from earlier experiments that, whichever way the current goes, there is heating. If pupils understand that the heating due to a current, C, in a wire of resistance R, varies as C^2R, they will appreciate the possibility of measuring R.M.S. values of current by heating – if the scale for the sag of the wire is suitably marked. We might show a working D86
model. OPT.

Modern moving-iron ammeters use either the attraction on a small T
piece of iron sucked into a solenoid carrying the current, or the repulsion between two pieces of soft iron side by side in a solenoid carrying the current.

A large, crude model of the latter form is well worth showing. Two D87
iron bars (large nails will do) are placed in a hollow coil to which we OPT.
apply first a d.c. supply, then an a.c. one. When current flows round the coil, the iron bars push each other apart.

Teachers may like to convert this into a rough working model by holding one bar fixed, installing an axle and pointer for the other bar with a hairspring against which the repulsion pushes the other bar around. A small model is apt to be confusing. A large, crude one is easily put together; but we do not advise any school to buy one specially made.





Algebra for A.C. (*Fast groups*)
For pupils who are familiar with the graph of a sine function, we can express the voltage thus: T

$V = V_0 \sin (2\pi nt)$ where V_0 is the peak voltage.

To find the R.M.S. average value, we need the average value of $\sin^2$ as time runs on and on. We can help pupils to find that without calculus. We point out that the graph of $(\sin t)$ and the graph of $(\cos t)$ look the same except for a shift of origin. They are the same pattern. So $(\sin^2 t)$ and $(\cos^2 t)$ have the same average as time goes on. But $\sin^2 t + \cos^2 t = 1$, all the time. Therefore, the *average* values of $(\sin^2 t)$ and $(\cos^2 t)$ must each be $\frac{1}{2}$. Therefore, the *root* mean square value of $V_0 \sin (2\pi nt)$ must be $(1/\sqrt{2})V_0$.

The R.M.S. value is 0·707 times the peak value. And the peak value is 1·41 times the value the voltmeter shows.

Warning: the peak value for '240-volt a.c.' is 40 per cent higher, about 340 volts.

Current and Voltage for Resistor (*Fast groups*)
We could ask pupils what current a steady voltage V_0 will drive through a resistance R ohms, and then what current an alternating voltage, $V_0 \sin (2\pi nt)$, would drive through R ohms. That current will change in time, in phase with the voltage (since electrons are such quickly obedient fellows) and we shall have a current: T

$$C = C_0 (\sin 2\pi nt), \text{ where } C_0 = V_0/R.$$

And the R.M.S. average current will be $C_0/\sqrt{2}$.

Therefore, if we use R.M.S. meters for voltage and current we shall find their readings give a constant ratio R, as for d.c.

'Ohm's Law' with A.C. As a quick demonstration (or class experiment if there are enough meters) we let an alternating current from a low-voltage supply pass through a resistor and make enough measurements of p.d. and current to show whether the ratio of the (R.M.S.) meter readings is constant. C/D 88

Current and Voltage for Capacitor and Inductance. Teachers with a fast group may feel tempted to go farther and show phase changes with an oscilloscope for voltages applied to a capacitor or an inductance. But that will prove a much harder matter to *
*
*
*

understand than one expects. And some special device is needed to produce two traces to show the phase difference. It is much wiser to add this study, if it is tackled at all, to the work with very slow a.c.; because then pupils can follow the story in detail by watching the moving pointers of meters. * * * * *

Slow A.C.

Although mature physicists can easily see peak values, and discuss the meaning of root-mean-square values and even phase differences with mains a.c. and an oscilloscope, these things are much easier for young pupils if they see them with very slow a.c. and watch the moving pointers of meters. * * * * * *

Demonstrations with slow a.c. have long been a tradition, and the A.S.E. has published a booklet giving details of generators and experiments. We hope that everyone teaching alternating currents at this stage will not only have demonstrations of slow a.c. but will have enough equipment to put some of these experiments in the hands of pupils as class experiments. A number of separate groups of pupils can be fed by the same slow a.c. generator. * * * * * * *

Bicycle Dynamo to Introduce Slow A.C. In passing from the mains frequency to very slow a.c., produced by a special generator, we should show the simple bicycle generator again, running at various speeds, making a sinusoidal trace on the oscilloscope. Running it slower gives lower frequency and smaller voltage. D/C89

Very Slow A.C.: Generators. The generators of slow a.c. suggested in the A.S.E. booklet are not the simplest ones. We need a generator whose mechanism is obvious. In this modern age, simple transistor oscillators can easily be made to give an alternating supply of frequency one or two cycles per second. But for our purpose, something more obvious is necessary. The following device (due to Pohl) seems best: * * * D90 OPT.

A continuous bond of resistance alloy wire is installed round the circumference of a wooden disc. It may be a single wire stretched as a tyre round the rim, or it may be a close-wound coil, like a cheap spring curtain rod. Leads are attached to this loop at two points at opposite ends of a diameter. These run to an axle attached to the disc and out through slip rings to a small d.c. supply, such as a 2-volt battery. That maintains a steady current through each semicircle of the loop. The disc is kept revolving slowly by a motor, and two brushes are brought up to touch the loop at opposite ends of a D91 diameter. The voltage thus provided between the brushes alternates with the frequency of the revolving disc and shows a roughly sinusoidal wave-form.

Very Slow A.C.: Meters. With very slow a.c., pupils can see the pointers of meters wagging to and fro to show the sequence of instantaneous values. D 91

Although ordinary voltmeters and ammeters with centre zeros will show this, we need instruments which will follow with the same phase for both voltage and current. For that, it is better to use small moving-coil galvanometers with a taut suspension and a mirror, with a large swamping resistance in series. Then, shunting the combination for an ammeter and adding series resistance for a voltmeter should give instruments with the same phase response.

Very Slow A.C. with Resistor. With such meters, we show current and voltage for a resistor: the meters wag to and fro in phase. (Obviously, if the meters available in the laboratory are suitable, this should be a class experiment.) D 91

Very Slow A.C. with Inductance or Capacitor. Then, with a fast group, we may show the same thing with the resistor replaced by an electromagnet, or a capacitor. D 92 OPT.

Further Work (*Buffer options*). At that point some pupils will want to continue on their own this very interesting study with obvious importance in electrical engineering. For example, pupils might return to a.c. of mains frequency with an oscilloscope and some form of 'electronic switch' that will provide two traces so that current and voltage can be compared. That can be a thrilling investigation, but we do not suggest that buying an expensive electronic switch is justifiable.

For other pupils, this is the beginning of a mystery for which there will not be time or interest for further exploration. The most we suggest for general teaching beyond this point is a quick look at power.

Power with A.C. (*Buffer option*)
With a fast group, we sketch graphs of voltage against time and current against time for a resistor. We point out that both V and C are positive at the same time, both are negative at the same time. The power at any instant, VC, is always positive. T

Good mathematicians might want to work out what the graph will be. Using $\sin^2 t = (\frac{1}{2})(1 - \cos 2t)$, they sketch a cosine curve wholly above the axis and may even see that the average power is given by (R.M.S. voltage) (R.M.S. current).

Then we draw similar graphs for an inductance or capacitor for the phase difference shown clearly. That will lead to the surprising suggestion that in extreme cases the power may be 0. We say, 'Yes, the energy just surges in and out of the magnetic field' or, 'Charges pile up on the capacitor plates and store energy, and then it comes pouring back again.'

We point out that electric motors have large electromagnets in them, and those will take more current for the same power and may therefore make wasteful demands on power lines.

All this is just a suggested line of quick demonstrations and brief commentary, leaving some questions for future work where pupils are interested. * * *

WAVES

Revision? Wave motion should have been studied qualitatively early in Year III. In extending those studies now we should remember that many of the pupils will not proceed to further physics after this Year. So we should *not* make an extensive revision of the earlier work. We should only do what seems interesting and relevant. * * * * * *

Wave Models

We should demonstrate several forms of wave model. Wave models are expensive and often seem to pupils rather 'special': that is, they are gadgets specially designed to show waves. Yet waves are general phenomena. So we suggest it is more important to show some simple examples of waves in common media than to show complicated models. Of course, whatever models the laboratory has already bought or the teacher likes to devise should be shown. They will be valuable. However, we do not suggest that laboratories should buy models specially for this; instead we suggest the following: D 93

a. Waves along a rope, and along a rubber tube. D 93a

b. Waves along a slinky, and any other longitudinal model that is available. D 93b

c. (*Optional.*) Water waves in a narrow, transparent tank observed from the side. (This was shown in Year III, Experiment D 3. As suggested there, slower waves can be seen at the interface between D 93c OPT.

paraffin and water if the tank is filled half full of water, with a deep layer of paraffin poured on top.) The motion of a particle in the surface is a vertical circle. A little sawdust in the water may enable this motion to be observed; but it is rapid and not easily seen, so only some pupils will notice it.

d. (*Optional.*) A long line of trolleys connected by springs (such as those used for class experiments in Year I) makes a good model for longitudinal waves. Turned sideways, they make a slow transverse model. D93d

e. (*Optional.*) Ring-magnet dry ice pucks placed in a line spaced short equal distances apart, make a very interesting model in which pupils can see the wave being transmitted from each 'particle' to the next by magnetic fields. The line of pucks must be kept in line by hedges which limit them to a narrow runway. D93e

Some waves, and of course all pictures of waves, move without carrying any energy or momentum. We should try to show examples of waves that do carry energy and momentum.

Many teachers will wish to spend some time on sound waves, and on instruments that produce sound; and on sound and music. This is an admirable part of physics for good clear teaching with some fine demonstrations. It is very interesting to some pupils, but dull and irritating for some of the less musical ones. So we do *not* suggest treating Sound in this programme, except for a brief mention of sound waves, simply because we feel the time is needed for some other parts of physics, especially atomic physics. There are already good books for pupils with a special interest in this field.

Stationary Waves

We should show some standing waves, but we should *not* go into the difficult idea of their production by two trains of moving waves. (That is an artificial way of treating what seems clearly to beginners a simple vibration-pattern, not a wave – if we insist on the wave-synthesis story, we make the simple seem complex and artificial.) Suggested illustrations: T

a. **Rope.** The teacher ties one end of a long rope to the wall and holds the rope taut, pulling at the other end. By moving his pulling hand up and down he excites transverse waves; and he feels for resonance, changing frequency or tension till he builds up a stationary wave pattern of several loops. The rope should be flexible, like soft flexible clothes line. D94a

When a steady standing wave pattern is maintained, energy is dissipated to the air as fast as the driving hand feeds it in. For efficient driving, the hand should be very close to a node, at a point where there is little motion. That is contrary to one's commonsense feeling that it is best to hold the rope at an antinode and move it a lot; but if one tries driving at a node, one feels one's hand making a good impedance-match with the rope and driving it hard. To make it easier to give this demonstration, mark the rope with ink or a ribbon at, say $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ of its length, tie it, taut, to a wall at each end; and drive at one of those marks. Make a ring with finger and thumb round the rope there. Move the hand up and down with the rope loose in that ring, and change the frequency until the 5-loop motion builds up.

b. **Slinky.** Build up a longitudinal standing wave. D94b

c. **Water in a Tank.** Show standing waves in water in a transparent tank observed from the side. D94c OPT.

The corresponding experiment at home with a bath half full of water is valuable as well as messy. Pupils need to gain a strong physical picture of standing waves if they are to use the concept in atom models. H94c

d. **A Ring of Standing Waves** (*Optional extra*). In the early *
stages of developing a wave-mechanical atom model, electron orbits *
were replaced by wave-electrons maintaining circular standing *
waves. Only if the circumference of the ring, $2\pi r$, contained a whole *
number of wavelengths, n (quantum constant h/momentum of *
electron mv), could that be a stable state – corresponding to a stable *
Bohr orbit. That new model agreed with the Bohr model in its *
simple predictions. Nowadays our models remain in mathematical *
form rather than painting realistic patterns. Yet the standing waves *
are there, defining the stable states; and it would be good to show *
pupils the circular standing waves that served as models in a great *
advance of theory. *

We fill a large shallow round glass trough half full of water and excite ripples near the edge. We can find a (high) frequency that will maintain a pattern of standing waves round the edge, with therefore a whole number of wavelengths in the circumference. D95 OPT.

e. **Monochord** (*Optional*). The teacher should pluck the wire near one end, asking pupils to listen. He should pluck it again, this time placing a finger lightly at the mid-point. That finger should D96 OPT.

touch the wire very lightly because its function is only to 'suggest to the wire that it should vibrate into loops' – the finger is not intended to force half the wire out of action and hold the vibrating part at half length. Repeat with a finger lightly touching at $\frac{1}{3}$, $\frac{1}{4}$, ... of the length from the end. Pupils may be able to see the wire vibrating in several loops, but that is difficult at a distance. (If a number of monochords are available, this might be a class experiment.)

Then, to show that the vibration does occur in several loops, the teacher places small paper 'riders' on the wire, one at each place where he expects a node of no motion, and others where he expects loops. Then he produces that motion, touching the wire at one node and plucking it gently at the loop nearby: the riders at loops jump off, while those at nodes remain. This needs practice. It is easier if the wire is excited by bowing instead of plucking. It is easier still if we use resonance: the teacher adjusts the wire carefully before class so that it vibrates in, say, three loops with exactly the frequency of a certain tuning fork. Then, in class, he places the riders on the wire, excites the tuning fork and places its shank on the bridge at one end of the wire.

f. **Melde's Experiment** (*Optional*). A light, flexible string, such as a piece of embroidery thread, is tied at one end to a vibrator and the other end runs over a pulley to a load. The length and tension are changed until the vibrator makes the thread vibrate with standing waves in several loops. Then that is observed stroboscopically by pupils with a hand stroboscope or by illumination from a compact lamp whose image is formed on the slits of a motor-driven strobe disc. Pupils who have seen this are more likely to see the motion of the monochord wire if they return to it. D97 OPT.

Demonstrations with Sound Waves (*Buffer option*)

If the laboratory has an oscillator that will drive a loudspeaker at high frequency, say several thousand cycles per second, and a microphone that will receive this frequency and give a good trace on an oscilloscope, some interesting demonstrations with sound are possible. However, we do *not* advise buying special equipment for this. Time will be short. D98 OPT.

a. The loudspeaker is driven by the oscillator and the microphone listens to it. Pupils watch the oscilloscope as the microphone is moved away. This will show amplitude decreasing with distance, though reflection from the walls may be very troublesome. If the oscilloscope's synchronizing device can be driven by an *external* signal, the pattern can be locked to the loudspeaker by connecting a small voltage from the loudspeaker's supply to that 'external synch'. Then, as the microphone is moved away, pupils will see the 'waves' of the sound it receives changing in phase as well as decreasing in amplitude.

b. The loudspeaker, driven by oscillator, is set up in front of a large reflecting wall. The microphone is moved about in the space between speaker and wall, and shows standing waves.

c. It is tempting to try two small loudspeakers driven in series from the same oscillator, in the hope of finding Young's fringes with the microphone.

This experiment is usually spoiled by waves reflected from the walls of the room. It succeeds out of doors. If the members of the class are asked to find positions where they hear a loud sound (or to find minimum positions instead) they can mark out hyperbolas of the Young's fringes pattern on a vast scale. They may find that easier if they block one ear with a finger.

Demonstrations with 'Centimetre' Wireless Waves (*De luxe buffer option*)

Very good equipment is now available for making and receiving D99a
electromagnetic waves of wavelength a few centimetres. With this, OPT.
one can give delightful demonstrations of these short waves travelling in straight lines, being reflected by a big prism of wax, interfering to form Young's fringes, showing amazing diffraction with 'half-period zones', and even performing in a working model of an interferometer with partially reflecting plates.

The source is a special compact oscillator, which includes a modulator so that the amplitude of the electromagnetic waves varies sinusoidally with an audible frequency, say 1,000 cycles per second. When the waves are received, they are rectified by a simple diode and the output current passing through a loudspeaker produces the audio note. Alternatively, the original wave is not modulated, but an audible note is produced at the receiver by mixing the incoming oscillations with oscillations of a slightly different frequency to make an audible 'beat note'. (A recent device uses a small cavity-oscillator, in a waveguide, as source; and that produces strong enough waves for a simple rectifying receiver to run a milliammeter directly.)

The apparatus is easy to set up and run; it is versatile; and the demonstrations it gives are delightful. However, it is expensive in both money and time. Where the laboratory does not already have this equipment, our suggestion is to try teaching this Year without it the first time, and then consider buying it.

In any case, this equipment should find an important place in A-level teaching, and teachers may prefer to postpone its use until then. A film has been prepared, for the guidance of teachers, to show some uses of this apparatus in detail. That film will prove very helpful when teachers are trying those experiments for the first time. It is intended solely for guidance of teachers – it would be an unfortunate use of the film to let it replace the real apparatus in class teaching. Celluloid microwaves are not very convincing.

Oscillations (*De luxe buffer option*). If this equipment is used, teachers may want to give a series of demonstrations of electrical oscillations, ranging from very low frequency (mechanically excited) right up to the very high frequency of the centimetre-wave generator. Again, a film for teachers is available to show the demonstrations in this series. However, most teachers will consider that these demonstrations belong in A-level – where detailed treatment is expected. D99b OPT.

Wave Trains and 'Stationary Waves': *v, n,* wavelength

We define wavelength and frequency and arrive at the relation between them and speed.‡ We may return to measurements with the ripple tank, or we may merely refer to them. T

In talking of wave speed, we should show simple examples in which pupils see a wave form moving along at definite velocity. C/D100 OPT.

‡ The relation $v = nL$ is, to many a physicist, automatically true, like the relation $s = vt$ which connects distance travelled with speed and time. The latter seems to us a definition of v, and we hardly think it fair to try to verify $s = vt$ by experimental measurements.

On the other hand, if we have a speedometer that is alleged to measure v directly – and instantaneous v at that – we may well want to test the speedometer's reading against measurements of s and t. In doing that, we shall be testing an arbitrary instrument (unless we look inside and decide we are testing a law of electromagnetic induction!). But *we are also giving the beginner some help* in growing familiar with the idea of speed, defined as s/t or ds/dt.

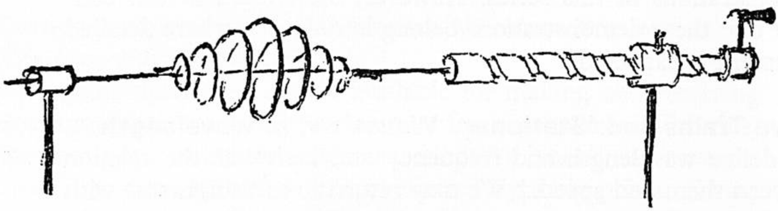
Similarly, separate measurements of v, n and L for continuous waves help the beginner to understand those terms and their relationship, even if the measurements seem to us to be testing a tautology. In this case we do not have a separate
contd. on page 260

Waves along a rope or rubber tube travel at a speed that is independent of wavelength and so do sound waves in air. Water waves do not; so any detailed discussion of $v = nL$ with water waves may prove to be unwise.

Group Velocity

With very fast pupils we can ask whether the pattern of ripples on a pond from a stone (or in a ripple tank) moves as a whole with constant speed. We do not tell pupils that the group speed is different from the wave speed but leave them to find this out, if they are interested, for themselves. If they do find out, we should comment on it – since it is so important in advanced discussions of waves and electrons. T

A Wave-group Model (*Luxury advanced option*). Teachers may like to construct a mechanical shadow model that illustrates the contrast between group velocity and wave velocity clearly and cleverly. This is not available commercially, and it would be expensive to make in a robust form. But a simple home-made form will prove satisfying. Here is a brief description: D101 OPT.



instrument like the speedometer which might be under suspicion. Pupils make a *direct* measurement of v for low-frequency ripples by timing the sweep of a ripple across the tank. They make a direct measurement of wavelength by measuring the distance from crest to crest on a photograph taken with a single flash; or by 'freezing' the progressing ripples with a stroboscope. If n were small enough, pupils might make a direct measurement by counting oscillations in a measured time; but at even the lowest frequency for the ripple tank n is too high for that; so pupils should use a stroboscope to estimate the rate at which crests pass some marking or the rate at which the generating vibrator runs.

Teachers may feel that there is a hint of arguing in a circle – or at least of proving nothing – if the stroboscope is used both in measuring L and in measuring n. But this is a case of the same device serving two separate functions. For L, the stroboscope is just an auxiliary device to freeze the pattern while we make a measurement. For n, however, it is the essential measuring instrument and a new piece of data has to be obtained from it – we have to know its speed of revolution and count the number of slits to argue out the frequency.

For L, the stroboscope could be replaced by a single-flash-photo camera; for n it could not. The two uses are quite independent.

Stiff wire is bent into a spiral, shaped so that its shadow will imitate waves in a *limited pulse*, as in the sketch. This spiral is not observed directly, but its shadow is cast on a translucent screen by a compact light source. The spiral is attached to a rod as axis. To show a wave-pulse progressing, the rod is made to revolve and at the same time move along. That is done by having a screw-thread of very large pitch cut on the rod and threading that portion of the rod through a suitable stationary nut. Then, as the rod is cranked by hand, it also moves along. (For a screw like that one can use a long spiral auger bit.)

Chapter 5

LIGHT WAVES

Diffraction
Young's fringes
Gratings and estimates of wavelength
Spectra

WAVES: INTERFERENCE AND DIFFRACTION

Ripple Tank: Young's Fringes

We then get the ripple tank out again, and ask pupils to look at the wave pattern from two small sources vibrating in phase. They see the patterns that lead to Young's fringes. C102

We discuss interference, showing the geometrical demonstration described in Year III. Pupils who missed those should see them now. T

a. Two long slats of transparent plastic, each marked with a wavy line of many wavelengths, represent waves starting from two sources in phase. The left-hand ends of the slats are anchored, one vertically above the other, several wavelengths apart. The teacher holds the right-hand ends, with the slats crossing near those ends, and shows how the two waves would agree or disagree at various places on an imaginary screen near him. This demonstration, which sounds too obvious to be worth constructing, is in fact very helpful to many pupils. We must remember that the ideas of constructive and destructive interference of waves are not self-evident; they are new and puzzling to many people. D103a

b. Pupils who wish to experiment with similar model strips on a tiny scale may be encouraged to make them at home by cutting thin slices from corrugated cardboard. These are anchored with drawing pins at one end, pulled taut and made to overlap near the other ends. (For details, see Year III.) Long strips like this may even be used to test the relation that we employ in estimating wavelengths. H103b

If pupils made a measurement with Young's fringes for light in an earlier year that may suffice now if it is clearly remembered. But in general now is the time for a measurement of the wavelength of light. So, after pupils have seen Young's fringes with their own ripple tank we do experiments with light waves, as follows.

Diffraction

We first show diffraction, because pupils need to know about that when considering Young's fringes.

Demonstrations with Light

We show light from a very bright, very small point source‡ casting shadows of various objects. We let pupils look at the shadows in detail. D103

‡ The little tungsten and iodine lamp does well, if distances are large. The most compact of the 'Wotan' high pressure mercury lamps is excellent but expensive.

Seeing Diffraction Easily. For pupils to see diffraction easily, we must provide translucent screens, to be observed from behind. Even then, the pictures will be faint because, for the small wavelength of light, we must have a large distance from the object to the screen; and, since the source is not a point, a fairly large distance from source to object.

Therefore, the room should be fairly dark. However, if we try to show diffraction in a completely blacked-out room, we defeat our own aim in many cases: pupils cannot see what is happening and there may be difficulties with discipline.

In these experiments, where pupils need time for their eyes to adjust, we must remember that darkening the room cuts off fresh air as well as light; and as the atmosphere grows warmer and wetter, pupils feel less comfortable and discipline is likely to become more difficult for that simple physiological reason. So we do not recommend complete blackout, but only half or three-quarters darkening of the room and then it is essential to have a very bright source, such as the little tungsten and iodine bulb, and to use *translucent* screens for viewing.

The source, well-housed and shielded, is placed a yard or two from the objects and the viewing screens are placed as far as possible beyond the objects, say 3 yards or more away. Pupils look at the screens from behind – and, as with the pinhole camera, we need to remind them to hold their heads back at a reasonable distance.

The translucent screen must be made of material which scatters transmitted light over a small angle so that the observer directly behind it gets a large share of the light. (Therefore, architects' tracing linen, which is so good for a 'translux' background because it scatters over a wide angle, is quite unsuitable here.) Ground glass would be good (the acid-etched form being ideal), but for problems of storage and breakage. Fortunately, there are now good translucent plastic materials that imitate ground glass. Failing those, greaseproof kitchen paper does very well indeed as a screen. (Or thin white paper soaked in melted paraffin wax will make a good screen.)

In showing diffraction (or interference) like this it is very important to avoid glare from the table-top reaching the screen. If some light from the lamp reaches the table at oblique incidence it is reflected only too well and adds unwanted illumination on the screen. The lamp must be carefully shielded, or black cloth must be spread on the table.

As objects to cast shadows, there should be some familiar things such as a pin and a needle, and perhaps a hair. There should also be a metal plate with holes of various sizes or, failing that, a sheet of paper with punched holes. The cases where diffraction through the holes produces a black spot at the centre of the bright round patch will be very surprising; but we should let pupils see those, and tell them those are the result of wave contributions adding up. With the tungsten-iodine lamp there is enough light to show the white spot in the shadow of a small disc or ball. A pin with a large black-glass head (like a small hatpin), or a steel ball $\frac{1}{4}$ inch to $\frac{1}{2}$ inch diameter, stuck with wax on a sheet of plate glass, will cast a shadow to show that surprising thing.

Teachers concerned with demonstrating diffraction at a more advanced stage will be tempted to try a V-shaped slit as object. Unless pupils look at the slit carefully before observing the shadow, they will be confused by that. So the V-slit should be avoided.

This is an experiment that takes a good deal of time if pupils are to take turns looking at the diffraction pattern on a screen. However, it is a fundamental observation, which once seen is never forgotten.

The Paradox. We hope teachers will include the shadow of a small disc or ball, without warning pupils what to expect. It is a famous experiment. When Fresnel as a young man submitted his paper on wave theory of light to the French Academy, Poisson as judge reported a severe objection that, if applied to the shadow of a disc, the theory would predict a white spot at the centre. Fortunately, the spot was seen, later photographed, and is now easy to observe with our special lamp.

Teachers who wish to show this to a large audience should make an array of black-headed pins at a distance of a yard or more from an open, compact light source and distribute pieces of greaseproof paper among the audience. Here, however, we hope that teachers will arrange things so that each pupil can look for himself, first at the object that casts the shadow and then at the shadow itself.

Some teachers arrange these demonstrations with a small lens through which pupils observe the diffraction pattern. That enables the experiment to be done in a room which is only half dark: it does not require such a bright source of light or such large distances. However, it seems to us to make the whole experiment very much more vague. Pupils so easily think that the peculiarities which they observe are put there by the lens itself.‡ So we urge teachers to avoid that.

The diffraction demonstration is there to start a question: does light consist of bullets or waves? T

We then show an interference demonstration, Young's fringes. It is probably best to give this first as a demonstration; but then there should be a class experiment in which each pair of pupils have their own double slit and observe the interference pattern on a translucent screen, viewing it from behind.

Young's Fringes

The simplest form uses a line-filament lamp instead of the first slit and a piece of translucent plastic as the observing screen, without any lens at all.

The next simplest form uses one lens to form an image of the original source on a distant screen. Then a double slit is placed just after that lens; and again there is a plain screen, without any eyepiece, for pupils to observe the fringes.

(Adding an eyepiece for the observer makes the demonstration seem more complicated and less direct, though it makes it easier to do the experiment. We hope teachers will avoid that.)

Demonstration of Young's Fringes

The teacher should show pupils a demonstration of Young's fringes by the simplest method, so that they know what to look for. D104

Then pupils should do a class experiment, setting up the arrangement, perhaps even ruling the double slits themselves, and looking at the fringes. They should try interposing a piece of green glass and then a piece of red glass, to see whether the fringes show different spacings for different colours. C105a

The source should be a 48-watt, 12-volt lamp run at excess voltage to give extra brightness at the expense of a shorter life. The lamp should be housed in a shield to minimize the stray light spreading over the laboratory. The lamp is best placed high on the wall at one

‡ Ask a physicist who wears spectacles what *he* thinks of the patterns he sees when raindrops settle on his spectacle lenses.

end of the laboratory, (with a duplicate lamp at the other end if there is a large class, so that some have to point their experiment one way while others point it the opposite way). The lamp should be one or two metres from the slits, which are carefully adjusted to be parallel to the filament.

The screen should be as far beyond the slits as possible, say 3 metres. As in the case of the diffraction demonstration, the screens used by pupils should be made of translucent material that scatters light through a *small* angle – not architects' tracing linen which scatters through a large angle.

Pupils observe the pattern from behind the screen. But, in a well-darkened room, they may also see it on white paper, viewed from the front – once they have seen, with the translucent screen, what to expect.

Then the teacher should discuss the essential meaning of those fringes as evidence for light waves. T

Pupils should make rough measurements of the spacing of the fringes. That is easily done by catching the fringes on a piece of translucent plastic and making marks with a pencil on the rough side of the plastic. They should measure the other distances that they will need for an estimate of wavelength. C105b

The distance between the slits is the difficult measurement because it is likely to be only half a millimetre, at most. Pupils may try using a magnifying glass and a millimetre scale (or a half-millimetre scale from the Year I oil film experiment) for a very rough estimate. If microscopes are available, pupils should use them to look at the slits and the scale, to make a better estimate. Failing that, pupils might hold their double slit in a small projection lantern, measure its image on a distant screen, and compare that with the image of a transparent millimetre scale.

We show pupils the geometry that enables them to work out the wavelength from measurements. That ends with a formula, but we must *not* produce that formula without showing how it is arrived at; and we must *not* encourage pupils to learn the formula by heart – instead we should assure them it will be printed on any examination paper. T

The geometry itself should not be too difficult, but some pupils will find the argument unreal or evasive – because there *is* an approximation in it. We can lessen that difficulty by sketching a realistic diagram with the two slits *very* close together and the screen extremely far away; then the approximation will seem much easier to swallow.

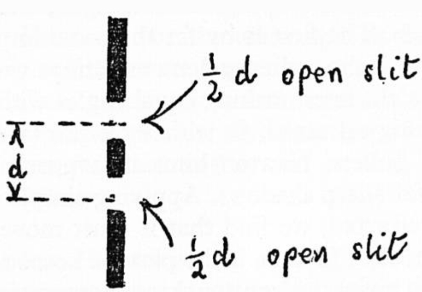
Pupils who find the geometry too difficult should see it carried through – for the good reputation of science – but the teacher should then tell them they need not try to learn it.

Since the making of the slits and choice of lamp, etc., will make all the difference between success and failure for this class experiment we discuss those arrangements in detail below.

Size of Slits. Both for the demonstrations and for pupils' class experiments the two slits need to be very close together, so that the fringes are widely spaced; BUT each slit must itself be wide enough to let through sufficient light to make the fringes adequately bright. These two requirements obviously conflict if pushed too far. When the two slits are widened a great deal they merge to make a single wide slit!

Also, when we widen each of the individual slits we see fewer fringes brightly illuminated on the screen. The spacing of fringes in that pattern is determined by the distance, *d*, between the centres of the slits: it is proportional to $1/d$. For a given value of *d* that fringe spacing is determined, and it does not change if the individual slits are widened. But the illuminated patch in which the fringes are visible is much smaller for wider slits – that is because that patch is determined by the diffraction pattern of each individual slit.

Suppose we have slits with distance, *d*, between their centres and each slit is itself an opening of width $\frac{1}{2}d$, as in the sketch, then



the pattern on the screen will only show a central bright fringe and one bright fringe to each side of it. (The next fringe beyond will be in a region where the contribution from each individual slit just falls to 0 in its own diffraction pattern; so the next bright fringe each side will be invisible and fringes beyond that too poorly illuminated to be seen.) Making the individual slits still wider would make the pattern brighter but would soon jeopardize the visible fringe each side of centre.

On the other hand, making each slit a little narrower than that would lead to one more fringe showing up on each side, a good pattern of 5 bright fringes in all. To give a reasonable fringe spacing in a room of ordinary size, the slit separation, d, should be about $\frac{1}{2}$ millimetre. At 3 metres from the double slit, the spacing from bright fringe to bright fringe will then be about 3 millimetres.

So we recommend ruling slits about $\frac{1}{2}$ millimetre apart, centre to centre, and slightly less than $\frac{1}{4}$ millimetre wide. In other words, if we look at the slits when they have been ruled, the opaque strip between the slits should be only a little wider than each of the slits themselves.

Ruling Slits. We now have an opaque material that is easy for ruling: a coating of graphite on glass, applied by painting 'Aquadag' on a microscope slide. When the coating is dry, slits are ruled with a blunt needle or a ballpoint pen, run along the edge of a ruler.

It is better to rule slits quickly by hand, making several trials and selecting the best, than to complicate matters with a special ruling device. A number of ingenious gadgets have been devised to enable teachers or pupils to rule double slits, but ruling by hand proves simpler and better.

Discussion of Young's Fringes. Young's fringes do two things for us: provide evidence of the wave nature of light; and yield an estimate of wavelength. The first is by far the more important in our present teaching. All the ordinary demonstrations with rays of light travelling in straight lines, making equal angles with a mirror on reflection, and being refracted, fit with a picture (a theory) of light as a stream of bullets. Newton himself favoured that idea because it accounts for sharp shadows. Applying simple dynamics to a particle being refracted, we find that it must move faster in glass or water than in air. (That is, if the particle keeps a constant mass. If we assumed it maintains constant kinetic energy instead, we should find ourselves led to the opposite prediction for speeds!)

In Newton's day, Huygens developed a wave theory of light, predicting reflection and refraction and showing that on his assumptions light must move slower in glass or water than in air. Long after Newton's day, Foucault measured the speed of light in water and found it smaller than the speed in air, thus lending strong support to the wave theory. But the really convincing evidence for wave behaviour of light comes from interference and diffraction.

About 1800, Young allowed light to pass through two pinholes and fall on a common patch on a distant wall. There he saw light and dark bands. We ask pupils to see for themselves the bright and dark alternating stripes.

In a bright fringe: LIGHT+LIGHT makes LIGHT.

But in a dark fringe: LIGHT+LIGHT makes DARKNESS.

In the latter case, two contributions cancel out. Waves can do that, but we cannot imagine particles cancelling.

The name 'interference' for this formation of fringes is unfortunate. It does not aid our teaching because it suggests that one lot of light upsets another: far from that, two contributions of light from the same source add their motions algebraically. One does not frighten the other away or modify the other: we merely get the proper mechanical sum of both.

In teaching, we need to emphasize this amazing behaviour, and its simple explanation on a wave basis. Only if light behaves as waves can we predict or explain the interference pattern of Young's fringes.

Other patterns made by diffraction of waves passing various objects are also predictable and explainable as wave effects. Neither the black spot in the centre of the bright patch made by light through a hole at some distances, nor the white spot at the centre of the shadow of a disc at all distances, makes much sense if we treat light as a stream of bullets. Nor do those make easy sense on a wave basis without considerable geometrical discussion; so if we show them, we can only assure pupils that they are, in fact, effects that we should expect of waves.

Attempts to discuss Fresnel zones, etc., lead to confusion at this stage and are, in fact, rather questionable optical teaching at later stages. All the more reason for giving great importance to Young's fringes which pupils can see with light, compare with water ripples, and understand fully geometrically.

The idea of two wave contributions that arrive in opposite phase adding up to 0 is sufficiently strange and new to pupils to need illustration, however obvious it may be to us. In addition to the two slats with wave patterns which are used as a model for the formation of Young's fringes, we should give two simple stories, however childish they sound, of the actual addition.

1. 'Watch my hands. This hand shows what happens where one lot of light arrives. It makes forces which wag up and down, up and down, like my hand. The other lot of light makes forces that wag up and down, up and down, like my other hand. Where both lots of light arrive in phase, this happens.' [Teacher pumps both hands up and down, in phase.] 'But, where the two lots arrive out of phase, this happens.' [The teacher pumps his hands up and down in opposite phases.]

2. 'Suppose each lot of light makes something – say an electron – move up and down, flip-flap, flip-flap. Then, where the two lots of light arrive in phase, we have

flip-flap, flip-flap ... + flip-flap ... = FLIP-FLAP, FLIP-FLAP ...

'But where they arrive in opposite phase, we have

flip-flap, flip-flap ... + flap-flip, flap-flip ... = 0.'

Coherent Sources. In such descriptions we assume that the two sources, or the light waves emerging from the two slits, are 'coherent'. In the early studies of interference, that was a very important condition: ingenious schemes had to be used to derive both lots of light from the same original source. And in teaching, one emphasized that and pointed out that two electric light filaments, strung close together, would never produce Young's fringes because the sources of light in the two filaments would oscillate quite independently with arbitrary changes of phase. However, that should not be emphasized here – probably not even mentioned unless a pupil asks – because we are discussing a basic demonstration of wave behaviour rather than examining the more advanced details.

Furthermore, we now know how to make atoms in some light sources gang together and co-operate to produce all their light waves in the same phase. Then we have tremendously strong sources of coherent light. These devices, called lasers, can pour light through two slits and form very bright, sharp fringes on a screen. However, the large ones are dangerous, because they produce such intense beams of light; and they are expensive. Their use in teaching lies in the future (they are already doing very valuable things in research). We do *not* suggest acquiring a laser for the present experiments because the essence of our aim is to show the wave nature of light *very simply*, and lasers will always look complicated even if they are not. * * * * * * * * * * * *

Estimate of Wavelength

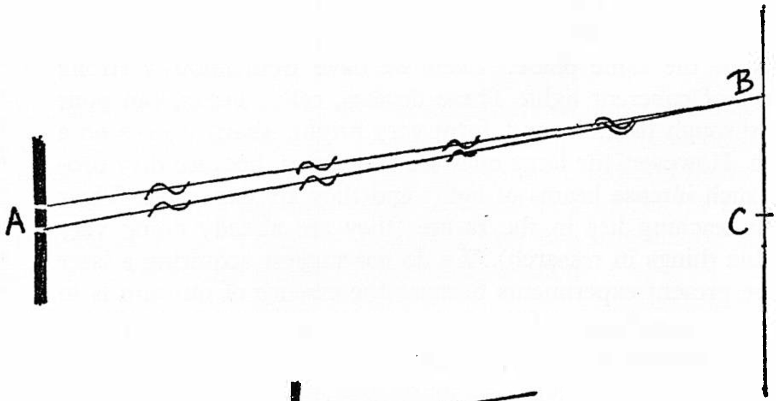
A fast group should certainly extract an estimate of the wavelength of light from measurements of Young's fringes. When they see how small that is, they may understand why light seems to cast sharp shadows. In a ripple tank, waves seem to pass straight ahead through a gateway many wavelengths wide, but they spread out in all directions on passing through a narrow gateway a fraction of an inch wide. Similarly, all effects of diffraction round edges show up only on a scale comparable with a wavelength. T

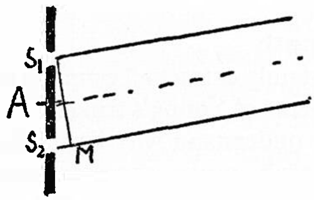
To estimate the wavelength of light from Young's fringes, pupils must go through some geometry. A really fast group should make the geometry their own, being shown it and then learning it so well that they could teach it to others.

An average group might be shown the geometry and then be expected only to say, 'We have seen that and are satisfied that it is sensible' – and then they could use the result.

The geometry is essentially similar to that for a diffraction grating. The latter is a Young's fringes arrangement with many slits, making its fringe pattern at infinity or the equivalent. Teachers familiar with the use of simple diffraction gratings will feel that the grating geometry is simpler; and they will plead for a grating instead of Young's double slit. Here, however, where we want to get light fringes directly, experimentally, convincingly, we urge teachers to deal with Young's fringes first.

To make the geometry simple, and to prepare for the grating, we draw a more realistic diagram than the usual one. We show the two slits a small distance apart and place the screen an enormous distance away (see sketch).





Then it is obvious that the wave paths from the slits to a place on the screen are almost parallel. And pupils will be able to see that when the waves arrive in phase at the screen, the extra path is one or more whole wavelengths. In such a diagram the extra path shows up clearly just outside the slits. Farther away, we see the two 'rays' of waves travelling along almost parallel, looking practically in phase, to arrive in phase at the screen. The extra path can be marked by drawing a line S_1M perpendicular to the ray from slit S_2.

If pupils find this difficult to picture, they can gain some help by trying the home experiment with thin ribbons of corrugated cardboard if they use very long ribbons. H106

Or the teacher can demonstrate the geometry with two long pieces of chain – such as the kind with light figure-of-eight links used for dog chains. In fact, with a slow group this may be the only useful approach to the 'formula'. The ends of the two chains are anchored at two points several 'wavelengths' apart at one end of the table – a 'wavelength' being one link. The teacher pulls the chains taut as they lie on the table, brings their free ends together and marks the junction point. Then he increases one chain by one link, two links, ... by adding links at the anchorage, and he again marks the junction of the free ends when he pulls them taut. In this way he marks points on the table that lie on the loci of 'bright fringes'. He sketches those loci; then makes simple measurements and calculations, to test the 'formula'. The advantage of this demonstration is that pupils participate and soon understand the changes of path-difference and the general idea so well that they can transfer it to interference fringes.

Then, with our geometry, we have two similar triangles, S_1S_2M and ABC, where A is midway between the slits, C is at the central bright fringe on the screen, and B is the place where we are adding up the two waves. T

[the extra path, S_2M] / [the slit separation S_1S_2 or d]

is equal to BC/AB

In the real experiment, the fringes are so close together that BC itself is small compared with AB, so AB is almost equal to AC.

Then, for the first bright fringe out from the centre, the extra path S_2M is one wavelength, L. So,

$$\frac{\text{Wavelength, } L}{\text{Slit separation, } d} \approx \frac{\text{Fringe separation, } CB_1}{\text{Distance to screen, AC}}$$

Example. Suppose the fringe spacing is estimated to be 3 mm (being the same from fringe to fringe as we go on out from the centre), when the screen is 3 metres away from a pair of slits $\frac{1}{2}$ millimetre apart. Then:

$$\frac{\text{Wavelength}}{\frac{1}{2}\text{ millimetre}} = \frac{3 \text{ millimetres}}{3 \text{ metres}}$$

This gives a rough measure of wavelength

$(\frac{1}{2} \times 10^{-3})(3 \times 10^{-3})/3$ or 5×10^{-7} metres or 5,000 Ångström units.

That is a good average value for white light, true for green light to which our eyes are most sensitive.

A Valuable Rough Estimate

The point of making this estimate is not precise measurement. This is an important case of desperate measures in desperate circumstances, where any rough estimate is very valuable because it tells us which county we are in. * * * *

And that is doubly valuable if the pupil makes the estimate himself with his own measurements. *
*
*

On that view, any value between 2,000 A.U. and 10,000 A.U. is still in the right order of magnitude and well worth having – probably much more valuable at this stage than a precise measurement done as a demonstration. In fact, the wavelength of light is something to take home as a precious demonstration. We hope that pupils will literally take it home with pride, and we hope that schools will encourage them to take home a pair of double slits, or the materials for making them, and a lamp and show people they can measure the wavelength of light themselves.‡ For home use, where complete darkness can be arranged, perhaps in a long corridor or cellar, an ordinary 6-volt or 12-volt lamp will suffice (preferably run at extra voltage to make it still brighter). We hope that schools will lend the necessary transformer for such a lamp. * * * * H105

Young's Fringes with Sound Waves. We can demonstrate interference with two small loudspeakers driven in phase by an audio-oscillator. A sinusoidal signal from the oscillator‡ is amplified (by D107 OPT.

‡ This linking between school physics and other people at home can do so much for the development of an understanding of science – and thence for the promotion of good teaching – that we plead strongly for this kind of home experiment. But now we should let it change from simple, practical things like making crystals to a glimpse of a great experiment that is bound up with theory. While we consider home experiments very important, we recognize the difficulties that school authorities may encounter over lending out apparatus which they feel may get lost or broken. Since we regard home experiments as so important, we offer at present to underwrite such home lending through a special fund. If teachers lend lenses, lamps, transformers, etc., and find that they cannot get them back or the apparatus comes back damaged or broken, they should apply to:

The J. Willmer Home Experiments Endowment
c/o A.S.E.
52 Bateman Street
Cambridge

The General Secretary, administering this fund, will only ask whether the apparatus went on loan with permission, whether the class is following a complete Year of our Nuffield Physics programme, what was damaged, and the cost to be met. He will not want to know the name of the pupil and he will not want the usual formal details of a report of damage. The cost will be reimbursed most happily.

‡ The scaler recommended for use in our Programme has a built-in pulse generator; but that produces sharp pulses rather than a simple harmonic signal, so loudspeakers driven by the pulse generator will radiate a mixture of harmonics and the interference pattern will be too complicated. We need a pure tone here.

any small amplifier) and sent through two loudspeakers in series. For a frequency of three or four thousand cycles/second, the wavelength of sound is of the order of four inches. The loudspeakers should be placed several wavelengths apart. Then pupils walk about, many wavelengths from the loudspeakers, and listen for loud and soft 'fringes'.

If we then try muffling one loudspeaker, we can show pupils that there are places where we hear two pigs under a gate make less noise than one pig.

It is practically essential to do this out-of-doors: otherwise waves reflecting from the walls of the room complicate the interference pattern hopelessly.

We suggest this as an optional demonstration to be done where oscillator, amplifier and loudspeakers are already available; but we do not recommend buying equipment specially for it.

DIFFRACTION GRATINGS

To us as physicists, the diffraction grating is an instrument of enormous importance: it has enabled us to show that light, X-rays, infra-red radiation, electrons, neutrons, etc., all have wave properties; and it has enabled us to measure their wavelengths. It enabled us to measure the wavelengths of light in line spectra from atoms and thus lend support to the quantum theory in its early growth, as it provided data and tests for the Bohr atom model. Then that model and its successors revealed a wealth of information from spectra, concerning energy levels and atomic structure. In another field, measurements of spectra enable us to estimate the speed with which stars are approaching us or receding. When we observe spectra of stars, we find tiny changes of characteristic wavelengths. These changes – measured with prisms but referred to grating standards – lead to an estimate of the star's speed. We even guess that the same Döppler effect is responsible for the great shifts towards the red in the spectra of distant nebulae – and we infer an expanding universe.

All these advances in knowledge have come essentially from measurements of wavelengths with gratings. We wish we could treat diffraction gratings fully and do justice to those great developments. But, at this O-level stage, we cannot. Take, for example, the passage from line spectra to energy-levels in a Bohr atom. We would have to build up the problem: show our young pupils the background of knowledge and the need for new answers; describe early ideas of atomic structure; lead up to the need for a quantum restriction; show the decoding of spectral series into term-formulae; and then would come the connecting together and fruitful conclusion. To pile that on the basic geometry of grating measurements – itself difficult for some pupils – makes too heavy and long a task now. Only for a few very fast pupils should we contemplate such an excursion.

On the other hand, we should teach the qualitative use of gratings to demonstrate waves; because we shall talk of wave behaviour extending throughout the whole of nature. We shall talk about the wave properties of light; mention the wave nature of X-rays; and show a film of 'matter waves' which itself will link optical grating spectra with electron wave demonstrations.

Therefore, pupils should do some simple experiments with gratings and try making a measurement. But we should *not* extend our experiments into a series of measurements of line spectra, since we shall not put them to use.

The diffraction grating is a topic to treat quickly, encouraging a feeling of success by giving help whenever it is needed.

Pupils should do class experiments with a coarse diffraction grating, to see many spectra; then with finer gratings which enable them to measure the wavelength of visible light.

Class Experiments with Gratings. We give each pair of pupils C108a
a small piece of *coarse* grating, and ask them to look at a distant lamp with a straight-line filament.

In the usual diagrams, showing plane waves (or a parallel beam of rays) falling on a grating and giving rise to parallel beams in various directions that form the spectra, we imagine a screen for the spectra at infinity; or we insert a lens to form the spectra on a screen in its principal focal plane. When a pupil holds the grating close to his eye, he is using his eye instead of that lens and his retina as a screen. In other words, his eye sorts out various parallel beams of light (or groups of contributions that make up plane waves) and brings each beam to a focus on his retina. Although that seems to us, as physicists, optically similar to an open demonstration with a glass lens, it will seem puzzling to many pupils – however delightful to look at.

(The real wave-story of optical images – in contrast with the ray- *
story, which is incomplete and only roughly true – is that all con- *
tributions from an object point must reach the image *in the same* *
phase. If we remember that, we can see that where the wavelets *
from successive rulings of the grating arrive *in phase* there will be a *
bright 'image' of the source. On that view, we need not try to *
claim that those separate wavelets mysteriously coagulate to form *
perfect plane wavefronts in several different directions for the *
pattern of several spectral orders.) *

So, as soon as pupils have tried this for themselves, we give a demonstration in which white light from a bright vertical line-filament passes through a lens to form practically plane waves, then through a piece of coarse grating and on to a distant screen. To give enough light, the lamp must have a very bright line filament. A 48-watt, 12-volt lamp, overrun for a short life, may suffice. For larger demonstrations, a projection bulb with a filament restricted to a tall narrow region will do well. D109a

The lens is arranged to form an image of the filament on the remote screen; and the light arriving at the grating or prism is not quite parallel. We adjust a spectrometer to make and receive 'parallel light', so that precise measurements can be made. But that arrangement makes too small a spectrum for a good demonstration. In demonstrations with a grating or a prism we do not aim at precise measurements, so we do not mind if different rays of light from a point on the slit meet the grating or prism at slightly different angles. Therefore, we use one lens (instead of two, the collimator and the telescope objective) and arrange it to form a real image of the slit at a large distance. We place the grating just beyond that lens. (Interposing a second lens after the grating brings a sharp pattern of spectra to a screen placed much nearer – but the pattern is much smaller.)

When pupils have seen that demonstration – with pieces of red glass and green glass interposed to ask a question about wave-length and colour – the teacher should exchange the coarse grating for a much finer one. D109b

Looking at Various Spectra with a Grating. Then pupils return to their own experiment with a piece of fine grating held close to the eye. After looking at a white-hot filament, they should C108b observe a neon tube, if possible a hydrogen tube arranged to give the atomic spectrum lines, a slit with a bright sodium flame‡ behind it, and then again at a white-hot filament, this time with a piece of red or green glass (or gelatine) held in front of it.

The neon tube and the hydrogen tube should, if possible, be capillary tubes so that each serves as a line source, like the lamp filament. Such a neon tube requires a much higher voltage than the ordinary neon lamp bulb, but a small cheap transformer will provide that. The tube with its transformer should be placed high up on an end wall of the room so that pupils can look at it from far away. The advantage of neon is that it gives a spectrum of many bright lines easily.

Hydrogen tubes show only three or four lines, and they are not very bright. Yet we hope that faster groups will look at the hydrogen lines. Then we can tell them that the series continues and its wavelengths have been coded in a famous formula. Unfortunately, simple hydrogen tubes change their behaviour in use and tend to give the molecular spectrum which is of no use for our purpose. T

Explanation of Action of Grating

Some pupils will take the grating as an extended form of Young's slits, and the pattern of spectra as a brighter form of Young's fringes. That view comes easily from the use of coarse gratings. On that view, the formula can be adapted to gratings. Instead of taking y to be the displacement of a bright fringe from the centre of the pattern, it is now the displacement of a bright line or region in a spectrum from the zero order, and instead of [y]/[distance from slits to fringe] we have $\sin\theta$, for viewing at infinity. T

However, physics teachers are accustomed to treating the grating more carefully, as a case of many wave contributions adding constructively on a screen at infinity. With a fast group, the teacher should give that description, pointing out the use of the lens placed after the grating to bring the picture in from infinity to a screen at its principal focus. However, that does not change the use of $\sin\theta$ in the formula.

We describe plane waves arriving at the grating and giving rise to a small (cylindrical) ripple from each 'slit' or ruling of the grating.

‡ A Bunsen flame with a piece of asbestos soaked in sodium chloride or bicarbonate solution is quite bright enough. A sodium lamp is an expensive, unnecessary luxury – and it may show an unwelcome pressure-broadening of the lines.

All of us who have taught the study of gratings know the next step: we draw successive, circular ripples from every slit, and sketch a slanting tangent, touching ripples of radii 1, 2, 3, ... wavelengths from successive slits; and then another slanting tangent that touches circles of radii, 2, 4, 6, ... wavelengths from successive slits. We assert that these tangents show a resultant wave front where the contributions from slits arrive in phase and add up to make something like a plane wave. (See the comment above on the wave-story for images.)

That story seem familiar and obvious to us from long practice. It is far from obvious to many pupils, and even seems silly: it pays attention to those bits of the cylindrical ripples which touch the tangent line, and it insists that those bits will make up a plane wave; it neglects all the rest of the ripples where they cross each other obliquely. In other words, we trust Huygens' principle and neglect its difficulties. This is the place where pupils should have careful, honest teaching if we are to treat the topic at all. The use of Huygens' principle in teaching has been called in considerable doubt.‡ Even if it can be justified by full mathematical analysis, the version we give in ordinary teaching is more than we should ask pupils to swallow. We should say quite honestly that near the grating we see round ripples emerging, but that farther away we *do* see those portions of ripples that touch the tangent line travelling on in agreement, making something like a plane wave; and that in other directions the round ripples *seem* to offer such a variety of contributions that they cancel out to practically nothing. This is just an assertion of the view we wish to use, referring to the physical observations. It is the same as the traditional use of Huygens' principle, except that it does not start by stating that Huygens is right, or continue by wrangling about Huygens' full geometrical story.

We must not, however, leave it as a simple assertion. We must at D110
once give a demonstration: plane waves in a ripple tank passing through a 'grating' of many slits (or splaying from a rake acting as a row of sources). The teacher should demonstrate the general idea of this with the ordinary ripple tank; but the picture is unlikely to

‡ See the Nuffield memorandum of 'Waves' by E. Mendoza, published in *Contemporary Physics*, February 1965.

show the formation of 'spectra' clearly; so that may be followed by photographs or films‡ showing the full story of water ripples. F111

Then, if pupils agree with the idea of the grating producing plane waves in various directions for the different orders of spectra, we can show them simple geometry. We draw 'rays' from successive slits to a diffracted plane wave-front and we point out that the extra path when we change from one slit to the next is one wavelength (or 2, 3, ... wavelengths for higher orders). Then the geometry is clear. T

$$\frac{\text{one wavelength, } L}{\text{slit separation, } d} = \sin\theta$$

where θ is the angle of deviation of the spectrum from the original beam.

In all this, we have spoken of the grating as made of slits. In a real grating, each ruling is a furrow of complex profile, but it is easier for beginners to think of a grating as an assemblage of narrow, parallel slits, spaced regularly a small distance, d, apart, each slit so narrow that light waves spread out widely from it by diffraction and thus contribute to several orders of spectra. (It may even help if we hold up a comb from the Year III ray-streaks kit to illustrate this.) * * * * * * * *

Manufacture of Gratings. It may help if we give a crude description of ruling a grating on glass: a diamond point ploughs a T

‡ In preliminary trials of Year III, some teachers tried summing up the observations pupils made with the ripple tank by showing films. There are excellent 4-minute films, made by P.S.S.C.; and other excellent ripple tank films are also available. It is very tempting to use such good films; but we cannot deprecate too strongly any such use of films in Year III.

We feel that teachers who want to show films then – when pupils have spent several weeks exploring the behaviour of waves on their own – have missed the whole point of our series of experiments. The ripple tank experiments are *not* intended to provide necessary factual knowledge in completely correct form. They are intended to give pupils an opportunity to work at physics on their own, for the experience of experimenting.

Some pupils will emerge with definite rules of wave behaviour; others will emerge with only a general memory of having done their own experiments. But in the case of the ripple tank, that range of yields will not matter. To show films 'to put things right' or 'for revision' would threaten to upset the sense of pride in depending on what one has done on one's own. Therefore, we very much hope that no teacher trying out our programme will use ripple tank films in Year III.

However, when pupils meet interference of waves now in Year V, teachers may wish to bring in films, for the first time, for revision.

furrow, spoiling the surface of a flat sheet of glass. It ploughs furrow after furrow, leaving a small strip of unspoiled glass between adjacent furrows. Then pupils can picture light pouring through those unspoiled strips.

(In fact, of course, the furrows also let light through, in various phases that compound to the same definite contribution from each successive ruling.)

Modern reflection gratings are ruled on soft metal, the diamond ploughing the metal up to make slanting ridges which provide preferential reflection in some particular direction. A grating that is 'blazed' like that throws most light into the spectrum in that direction.

Cheap gratings for teaching use are made by casting a film of plastic on a ruled grating – often a reflection one – and then peeling it off. If the casting and peeling are done carefully, the replica has the same grating-space, d, as the metal original. And the value of that is obtained from the maker, who counts the number of furrows ruled. Therefore, in telling pupils the grating-space for use in their measurements of wavelength, we are not arguing in a circle. (However, some makers of cheap replicas arrive at the statement of grating-space that they supply with the grating by a measurement with sodium light of known wavelength.)

From now on, we should change from saying 'slits' to saying 'rulings'. And instead of calling d the 'slit separation' we should call it the 'grating space'.

Estimate of Wavelength

This should be a class experiment. It means much more to a pupil C112
to conduct his own measurement of this tiny, inaccessible distance from crest to crest of a light wave than to see someone else do it and carry out a quick calculation – the latter leads only too easily to a formula-dominated examination question. And we hope that some pupils will be able to take a piece of grating home‡ and use it as a H112
sequel to their home experiment with Young's fringes.

The source should be a line-filament lamp with, say, a green filter in C112
front of it. If a mercury lamp with a visible capillary tube can be used, all the better. The lamp is placed at one end of the room and

‡ See footnote in section dealing with Young's fringes.

pupils work as far away as possible. For a rough estimate, the pupil holds a piece of fine grating close to his eye, at the near end of a metre rule pointed straight at the lamp. His partner moves a pencil along another metre rule, placed perpendicular to the first at the far end, so that the two rules form a T. The pencil is moved until it just obscures a green patch or a green line in the first order spectrum. From their measurements, pupils calculate tan θ and thence sin θ. They have to be given the grating space d. Then they estimate the wavelength.

If we just announced the value of d, that would be almost as bad as just announcing the value of the wavelength – it would make the experiment seem obedient rather than searching. In supplying the value of d, we must explain where it came from and make it clear that a mechanical counting during manufacture can supply it. If suitable microscopes are available, pupils should use them to look at their piece of grating and at the graduations on a finely divided ruler. (The $\frac{1}{2}$-millimetre scales provided for the oil-film experiment in Year I might do well.) Although they may be unable to measure the grating-space, they will certainly see that a direct measurement is feasible.

If this estimate of the wavelength of light will be a burden of strange geometry and unsure measurements, it would be better to omit it. If it will give a sense of delight and insight, it can be one of the most powerful experiments of the year.

(As an amusing, informal experiment, some pupils may like to look at the grating spectra formed by reflection from a gramophone record. Unfortunately, the rulings are too coarse to be of much use at direct incidence. The observer must take an oblique view as we do for gamma rays with crystals. To find the grating space of that grating, play the record, counting the turns, and using a ruler for the radial length of the recording.) H113

Note to Teachers on Spectra and our Programme

Spectra are beautiful things to see. The full spectrum of white light blazing on a screen as a large demonstration is a surprise and a delight to pupils, even though they have seen a dilute version often enough in a rainbow, and a small version with their ray streaks. Line spectra are interesting things to see; but they are not as surprising to pupils as to us; and they certainly do not present themselves to pupils as keys to atomic structure – that needs a modern Pythagoras. * * * * * * * *

Absorption spectra, such as that given by green glass held in a beam of white light, help pupils to understand the physics of colour.

The 'single line' spectrum of a salted flame is then a surprise: instead of a broad band of yellow, pupils see a very narrow band of pure yellow. With a fast group, the absorption spectrum of sodium vapour then raises an interesting question.

So, spectra are things that pupils should see anyway. Having made a very small spectrum with ray streaks in Year III, they should see some larger demonstrations; and if apparatus is available, they should look at spectra individually. But that look at spectra for delight and general knowledge should take very little time. It would not be wise to spend more time looking at spectra in detail and measuring wavelengths of spectral lines – unless our theoretical teaching during this Year is going to make use of that work in a discussion of atom models. There will not be time for that, except possibly for very able pupils; so the following section on line spectra and energy levels is only a background note for teachers.

Line Spectra and Energy Levels

Historically, line spectra were among the earliest phenomena to give hints of energy levels and a quantum behaviour in atoms, but the hint was not taken with serious profit by physicists until other phenomena pointed towards quanta. *We* think of line spectra as offering clear knowledge of energy levels; but we are using hindsight. To our pupils, the argument from frequency differences through the quantum idea to energy levels would be new and far too difficult. We should not try to set it forth at this stage. Colleagues in Chemistry press strongly for some teaching of energy levels in atoms. They want pupils to know – even before Year V – that atoms have well-defined, discrete, energy levels. And we wish we could show the amazing story of stability that goes with that: atoms and molecules are completely elastic in collisions, up to a certain energy – above which they can store or release energy in discrete jumps. But the experiments and reasoning which led to that knowledge are difficult; and they are not directly relevant to its use in chemistry. So we expect that this teaching in chemistry will have to rest on simple assertion.

Physicists study the stability of energy levels in atoms by experiments in which they bombard atoms of vapour or gas with electrons of known energy. Up to a certain kinetic energy, the bombarding electrons bounce off the target atom elastically. They give no energy to the atom, beyond the tiny share characteristic of the momentum exchange in an elastic collision. They do not change the *internal* energy of the target atom at all. But above a certain minimum kinetic energy, bombarding electrons make an *inelastic* collision, giving a sharply defined amount of their kinetic energy to the target atom, which is changed to a higher 'state' or energy level. That is shown by the Franck-Hertz experiment, in which, originally, electrons bombarded mercury atoms in warm mercury vapour. Nowadays, a similar experiment can be done with electrons bombarding an inert gas such as helium.

After inelastic collisions, the atoms of the target gas soon return to their ground state, emitting light as a spectral line. That would link up well with a full study of line spectra. Unfortunately the demonstration looks too complicated and too 'special'. The target material has to be adjusted to the right density to show the effects of inelastic collisions; the measurements are easily upset by stray potential differences; and the interpretation is not easy for beginners. This is now a standard lecture demonstration in some university physics courses. It may well find a place in A-level physics. But after looking at the experimental arrangements carefully, we consider it is not suitable for our O-level course. (With a tube of helium carefully chosen to give the right answer, one can give a dramatic demonstration; but even the most mature O-level pupils are apt to miss the essence of the experiment and its implication.) At most, we might offer a very fast group the P.S.S.C. film of *The Franck-Hertz Experiment.*

(When *photons* bombard atoms we again meet contrasting cases of elastic and inelastic collisions – plain scattering of light and Compton effect; and the various forms of Raman effect. These too tell us of discrete energy levels in atoms. They also remind us that radiation, from visible light to X-rays, carries its energy in quanta.)

Study of Spectra should remain Qualitative. Except where the teacher has special interests and the school already has special apparatus, we suggest that the study of spectra should remain a qualitative one. With a fast group it might extend into quick measurements of the atomic hydrogen spectrum with a grating. That spectrum, the Balmer series, has only four lines in the visible region. So pupils will not realize that the lines are part of a great series, nor could they decode the measurements into a general formula. Therefore, to supplement such measurements we would have to show a photograph of the Balmer series extending on into the ultra-violet. And we might even give those who like arithmetical

puzzles the Balmer formula and ask them to see whether their *
measurements fit it. If they then ask what we now think that *
formula tells us about atoms, it seems more honest to say that the *
rest of the story is very long than to assert briefly that the formula *
shows us there are energy levels. *

Looking at Spectra formed by a Prism

As demonstrations, we show pupils a large spectrum, formed by a D114
prism of high-dispersion glass on a distant white screen. Such a spectrum is a series of images of a slit, in a progression of colours side by side. The wider the slit, the more overlapping there is; the less 'pure' the spectrum, but the brighter it is.

For a white light source we need a lamp that sends as much light as possible through that original slit and on to the lens beyond it. Traditionally, a condenser lens, used to gather light from arc or lamp filament and 'concentrate it on the slit', formed a rough image of the source somewhere near the slit. In practice, that use of the condenser does not give as much advantage as one would expect. It may be better to make the light source itself take the place of the slit. If the source is a compact one – a small arc, a tungsten-iodine lamp, a projection lamp with compact filament – the spectrum will be practically a line of colours instead of a tall band.‡ To obtain a tall band, one should use a lamp with a vertical line filament in place of the original slit. On a small scale, a 12-volt lamp, overrun for a short life, may suffice. For larger demonstrations, a projection bulb with a filament restricted to a tall narrow region will do well.

As in the grating demonstration, we set up a lens to form a real image of the slit or line filament on a distant screen. We show pupils that. Then we place the glass prism just beyond that lens. Pupils see that the prism swings all rays round to a new direction so that the screen must be moved to a new place, at about the same distance. Furthermore, blue light is swung round through a bigger angle than red; so the image of the slit now becomes a band of coloured images that we call a spectrum.

In a spectrometer, the prism is usually turned to minimum devia- *
tion to ensure still greater precision – if the lenses are not quite *
correctly arranged for parallel light, the prism will still give all rays *

‡ That can be avoided by inserting an extra, cylindrical lens. Teachers may find it interesting to experiment in modifying a spectrum with lenses from the Year III ray-streaks apparatus; but in general this adds complications and takes time, so we do not recommend it as a standard arrangement.

the same deviation if it is set for a minimum. In our demonstration, the prism need not be turned to the minimum deviation position. As it is turned away from that, it gives a longer spectrum though not so pure. * * * *

We show the spectrum, then we try interposing colour filters such as green glass or gelatine. It is better still to give the pupils themselves pieces of coloured gelatine through which to view the projected spectrum. Remember that most gelatine filters transmit the red whatever else they are intended to transmit. It is wise to tell pupils that beforehand; then they expect and accept the unwelcome patch of red, instead of arguing about it. D114

We should try holding a piece of coloured material in the various parts of the spectrum. Scarlet cloth and green Christmas-decoration paper do well. Pupils should also look at those in pure yellow light from a sodium flame.

Colour Mixing (*Optional*). If teacher or pupil has a special interest in 'colour mixing', each pupil should be given six colour filters: red, green, 'true blue'; and cyan (blue-green, which is minus red), magenta (purple, which is 'minus green') and common yellow (red+green, which is 'minus blue'). Pupils can then see the effects of colour filters and try for themselves subtractive colour printing with the second trio of filters. For details of that teaching, and demonstrations of colour mixing with small projection lanterns, teachers are referred to other literature. C115 OPT.

This is a fascinating topic that may be useful as a source of enthusiasm and enjoyment for some groups; but we hesitate to suggest it in this very busy year for O-level candidates. (Theories of colour vision have been under discussion for many years. The practice of colorimetry has long seemed well understood. However, recent experiments by Land have raised new questions. Land has projected colour pictures when using only two spectral colours; and although interpretations of his work are still under discussion, those of us who have seen his demonstrations are convinced that his method succeeds. It may be well to read recent accounts before embarking on the teaching of colour.)

Another Look at Spectra with Gratings. This may be a good time for pupils to take a second look at spectra with a grating. They have just seen white light spread into a single spectrum by a glass prism; so we can remind them that the several spectra due to a grating are formed by interference – path differences of various C116 whole numbers of wavelengths – the multiplicity is not a property of white light itself.

A Simple Spectrum. If they have not tried it in an earlier Year, C117
pupils might now try the 'poor man's spectrum' of sunlight. Each pupil holds a cheap glass prism close to his eye and looks at a bright sewing needle held at arm's length, in sunlight. The needle should be parallel to the refracting edge of the prism. For each colour, the prism forms a line image of the Sun's line image in the needle. It is said that pupils can even see the Fraunhofer lines with this.‡

Spectrometers? Setting up spectrometers to show spectra seems unwise at this stage. The elaborate apparatus obscures the simple story. Nor do we recommend using spectrometers to make measurements with gratings – that precision work belongs at A level.

Sunlight. However, a spectrometer does make it easier to show D118
the Fraunhofer lines in the spectrum of sunlight. That is so surprising and important a sight that it is worth while to set up a spectrometer for it, even though pupils have to take turns in looking. A spectrometer with a good glass prism is adjusted and the slit is made very narrow. Sunlight is directed into the slit with a mirror and a small lens is used to form an image of the sun near the slit, so that there is enough light for a bright spectrum. We do not tell pupils to look for absorption lines, but simply say, 'Look at the spectrum. Is it brighter than usual at the blue end?' Then, when pupils see the thin black lines, we must give some explanation.

That will practically necessitate a demonstration to show the way in which such absorption lines can be produced. Though this may seem an advanced extension of our work that hardly fits here, we suggest giving it because of its great importance in modern physics. Such an absorption spectrum tells us the composition of the outer layers of incandescent stars, like the Sun. Slight shifts in the position of those lines tell us the speed with which stars are approaching or receding.

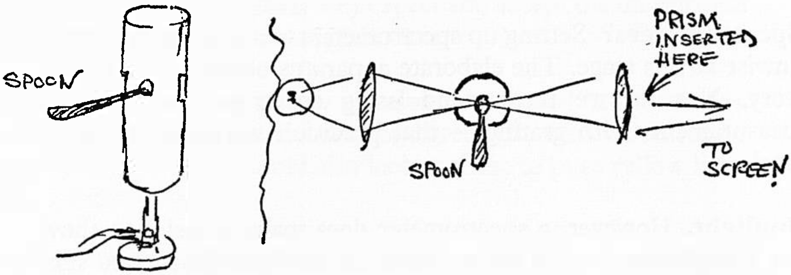
So we should show the absorption spectrum in white light that passes through (cooler) sodium vapour.

Two methods that work well are described below. In each of them, a lens is used to form an image of the white-light source in the

‡ For detailed instructions by M. E. Y. Gheury de Bray see *The Science Masters' Book*, Series II, Part I, p. 155 (John Murray, 1936.)

middle of the warm sodium vapour. Since the image is there, *all* rays of white light from the source go through that point – shades of our teaching in Year III. * * *

Sodium Absorption Spectrum (*Optional extra*). We can show the absorption spectrum of sodium vapour when white light from a hotter source passes through it. This is a troublesome demonstration to set up but well worth while for pupils who have seen the Sun's spectrum. D119 OPT.



a. Small demonstration with spectroscope. A prism spectroscope is set up and adjusted with a narrow slit. White light from a lamp filament is directed on to the slit. A sorted Bunsen flame is placed in the path of the light. Pupils are unlikely to see the black absorption lines in the yellow region unless we make sure that all the while light passes through the region of the flame that is rich in sodium atoms. To ensure that, we arrange a lens to form a real image of the lamp filament in the middle of the flame, and then another lens to form an image of that image on the slit of the spectroscope.‡ D119a OPT.

b. Large demonstration with vapour from sodium. We arrange a large demonstration spectrum with white light from an arc or an overrun lamp filament. We interpose in the path of the light a vertical chimney inside which metallic sodium is burned. The chimney is a piece of iron pipe about 2 inches diameter by 10 inches high. Wide slits are cut in it near the centre to allow the light to pass through it. A piece of metallic sodium, about $\frac{1}{2}$ cubic centimetre, is placed in a small metal spoon which is poked into the chimney through a hole in the side of the tube level with the slits. The spoon with sodium in it is heated by a Bunsen burner pushed up the tube from underneath. The sodium melts, then catches fire (and the bunsen is turned out). The burning sodium produces a D119b OPT.

‡ See the description by W. C. Badcock in *The Science Masters' Book*, Series II, Part I, p. 162 (John Murray, 1936.)

lot of excited sodium vapour inside the tube in the region where the light passes through the slits. Clouds of sodium oxide from the burning emerge from the top of the chimney, but there is not enough to be a danger. During the burning pupils see a black band appear in the yellow region of the spectrum; but if the white light is cut off they see a bright yellow band at the same place from the burning sodium.

To make sure that all the white light passes through a region rich in sodium vapour, we insert a lens to form an image of the filament in the middle of the chimney and that image serves as the original slit for the white light spectrum.

Chapter 6

RADIOACTIVITY

Experimental Study
Rutherford Atom Model

PROGRAMME

At this point we could continue with grating spectra in two dimensions and three dimensions, to throw some light on X-rays and crystals, and to enable us to discuss 'matter waves'.

Pupils are also ready to appreciate one aspect of a discussion of theories of light: evidence that light consists of waves. *The other aspect, the* particle behaviour of light, *which we emphasize when we speak of photons, should wait until pupils have seen the photo-electric effect, both in 'wholesale' form (demonstration), and in 'retail' form with photons arriving one by one (film). And that had better wait until pupils have received some explanation of the working Geiger counters, because we shall use a counter to demonstrate photons of ultra-violet light and perhaps of X-rays.*

Therefore, since pupils would probably enjoy a change from interference experiments, we suggest treating radioactivity now, followed by a study of the photo-electric effect which will lead us back to theories of light and the contrast between the two aspects of light, light waves and photons, and then on to matter waves.

We shall also suggest a short description of the general electromagnetic spectrum in that later discussion of light. (Appendix.)

RADIOACTIVITY AND ATOM MODELS

This should be an experimental study taught by real experiments and films of real experiments. How much is done by class experiments must depend on safety rules and equipment; but the whole treatment should be by class experiments and demonstrations combined, and not solely by giving pupils descriptions or asking them to read a book. We suggest the emphasis should be on the effects of radiations in making ions and on exponential decay and nuclear changes. It has been customary to emphasize the properties of alpha, beta and gamma rays by studies of their absorption, etc., but those are now only useful information for technical workers and are no longer essential tools of investigation – so we should treat them very lightly with our pupils. Experimental work on those properties is interesting and pupils enjoy it; but it leads them, as in the older teaching of chemistry, to a collection of information without any strong contribution to understanding.

Counters are important and pupils should be shown how they work so that these common instruments are not invested with mystery.

In our present teaching, cloud-chambers are even more important because they provide the nearest thing to direct evidence of nuclear collisions and the scattering phenomenon that leads to the idea of a nuclear atom. For the latter purpose, alpha-particle tracks are essential, and we should not only let pupils see real tracks but show a large exhibit of photographs, which should remain on view for weeks. (The Nuffield Physics Group is planning to collect a set of such photographs, in the form of large prints or transparencies, for use in schools.) Lantern slides run too quickly, and leave too shallow a memory for this very important use of cloud-chamber pictures.

Since radioactivity is a topic of considerable interest, with a number of aspects which can be described fairly simply, teachers may feel tempted to spend a long time on this section. But since we hope there will be time to proceed to the photo-electric effect, theories of light, and 'matter waves', to carry forward our building of atomic models, we suggest the teaching of radioactivity should not be prolonged. On the other hand, pupils who are especially interested will find there are good books, both on the early history of radioactivity‡ and on modern developments.

Atom Models So Far. In describing solids, liquids and gases, and in developing kinetic theory, we treated molecules and atoms as round knobs with no internal structure. We described them as exerting attractions on each other at short ranges of approach and, necessarily, repulsions at very short range. We assumed that those forces are 'the same on the way in as on the way out' when one atom or molecule approaches another; so that collisions and other interactions are elastic.

Then in Year IV, we met inelastic effects, when electrons are torn off atoms by violent electric fields, or knocked off atoms in violent collisions. The idea of ions in gases as well as in solutions was essential for an understanding of Millikan's experiment. We illustrated that idea by showing currents being driven through air by an electric field when a candle flame was placed in it, when a lighted match or small Bunsen flame provided ions, and – if possible – when a radioactive source was held nearby. Those currents were too small to demonstrate with a micro-ammeter, so we used an electroscope and showed the leaf rising as charge was driven across to it, or falling as charge leaked away without any driving battery. Such ions can only be produced at the expense of some supply of

‡ An excellent account of the early history is given in *The Restless Atom* by A. Romer (paperback, Heinemann Science Study Series).

energy to drag electrons off uncharged atoms. We did not say that then, but we should say it now.

In Year IV, we showed some properties of electron streams released by hot filaments in a vacuum. Now in Year V we have measured e/m for the particles in such streams; and, comparing the result with the value for hydrogen ions in solution, we came to picture electrons as very small chips of atoms.

Thus, pupils should have by now a picture of atoms as containing electrons (fairly easily detached) and some positive material, probably holding most of the mass of the atom, the whole being held together in some unknown way. We have gone one stage further in making pictures of atomic structure, in devising models to help our thinking.

Ions in Air Carry Current. We remind pupils of our tentative atom model – electrons embedded in positive electricity. We ask what ways they know of removing electrons, making ions. We show again a demonstration of a flame providing ions which can carry a current: T

a. A candle flame, projected by shadowing, in a strong electric field between two vertical plates. D120

b. A high voltage (E.H.T.) arranged to drive ions across an air gap between two vertical plates to an electroscope. And the same with the high voltage removed, showing ions carrying charge away from the charged electroscope. We provide ions for that by a small flame. A lighted match does well. D121

(This is not the time to show that effect with more complicated apparatus, such as a d.c. amplifier. If the school has that special device *and pupils are familiar with its use*, it could be shown here. But at this stage, where we have many new things to show, simplicity is best.)

Now pupils are ready to meet radioactivity with more understanding. We should first show the ionizing effect of the radiations from radioactive substances – the effects by which they were discovered, and by which they are still often measured.

Radioactivity

Radioactivity was discovered by the ionizing effect of certain substances; and at first that was its chief property: making ions T in air, in photographic emulsion, and in flesh. Nowadays we are even more interested in the nuclear changes that release the particles which produce that ionization; but pupils should start by seeing the original property.

We charge up an electroscope and hold a small alpha-particle D122a
source near it (one microcurie of pure alpha source will suffice). Holding a hand near a charged electroscope will change its local capacity and make the leaf move. But if the teacher then holds his hand still, further movement of the leaf simply shows ionization.

For comparison, hold a lighted match near the electroscope; that D122b
produces more ionization. To show that the match produces something conducting in the air, try holding the match some distance away from the electroscope and blowing across the top of its flame towards the electroscope. If we had a large enough radioactive source, we should see the same thing with that.

Then make a more stable arrangement: attach the radioactive D122c
source to the rod of the electroscope, or fix it on a stand nearby, and install a grounded plate above the electroscope's rod so that the source is in a region of electric field when the electroscope is charged. Charge up the electroscope and watch it.

Now set up a more formal arrangement for charging the electro- D122d
scope by ions driven by a battery across an air space between two metal plates. Try placing the radioactive source in or near the air space; and compare that with the effect of a flame.

Since large sources are not available or suitable for our teaching *
work, it is tempting to change from a common electroscope to a *
more sensitive instrument of the oscillating leaf or fibre type. *
However, we strongly urge teachers to avoid those. In those *
instruments, charge is driven to the electroscope as before, but *
when the leaf has moved only a short distance, it hits a metal plate *
and discharges. The rate at which charge is arriving – in fact the *
current flowing to the electroscope – is measured by the frequency *
of pulsing of the leaf. However, there is serious danger of the *
pulses of the leaf's motion being mistaken for pulses due to indivi- *
dual particles. Just here, where we are about to deal with pulses of *
charge produced by individual alpha or beta particles as evidence *
of random events – the radioactive disintegration of atomic nuclei *
– would be the worst possible place to let that confusion arise. We *
shall be content with an ordinary electroscope, which will show *

ionization by alpha particles but will not be sensitive enough to *
show ionization by beta particles unless special arrangements are *
made; and there will be no chance of seeing the effect of gamma rays. *
*
Then we proceed to a scaler with Geiger tubes where we shall *
see individual particles. *

Preparation for Understanding Geiger Counters

Pupils will understand our demonstrations of radioactivity much *
better if the Geiger counter is not presented suddenly as a mys- *
terious instrument. They may have seen it used without much *
explanation in Year I; and they have seen the scaler used as a *
clock in Year IV; but we should now show some experiments that *
will help pupils to feel that a Geiger-Müller tube makes sense. *
If the experiments described below seem to teachers rather *
childish, we hope they will try the treatment once, because we *
find it does help to introduce the counting of ionizing particles. *

1. *Salt into water.* We set up a circuit consisting of a mains lamp, D123a
two copper wires in a large empty beaker, a switch, and the a.c. or d.c. mains supply.

We switch on: the lamp does not light – ordinary air does not conduct a current.

We make the two wires in the beaker touch: the lamp lights – copper conducts a current. (The lighted lamp shows one of the effects of current, which are all we know an electric current by.)

We separate the wires again and fill the beaker with *distilled* water: the lamp does not light – water does not conduct enough.

We take a large handful of table salt and throw it into the beaker: the lamp lights – we have provided ions to carry the current. We then explain:

> 'There is a p.d. of 200 volts or more between the wires. That makes an electric field across the beaker, through the water. That field is ready to tug electric charges. It will drag a positive charge one way and a negative charge the opposite way. If there are charges there and they move under the action of the field, that will be a current. I must have put in some charged things – carriers, ions as we call them.

> '(We believe those ions are charged atoms of sodium and

chlorine. In fact we believe the sodium and chlorine atoms in the crystals of salt were already there as ions, not as neutral atoms, in the salt crystals.‡ The water has crawled in between the ions in the crystal and lessened the forces between them, so the crystal has fallen apart into separate ions.) Now those ions are carrying a current through the salt solution, and electrons in the copper wires take over the job and keep the lamp alight.

'You think this is an experiment to show electrolysis. It is not! *This is a salt counter.* It is a scheme for counting handfuls of salt. *I* throw the salt in: *you* count how many times the lamp lights. Of course you will have to wash out the beaker and refill it after each count.'

2. *A match and a spark.* We arrange to produce a large, fat spark across a gap between two metal balls. Before the class the teacher should try out the arrangement, find the maximum separation between the balls for a spark – that distance depending on the voltage of the E.H.T. – and then move the balls a little farther apart, so that the supply just fails to produce a spark. To make the spark a fat, noisy, one when it does occur, we connect a large capacitor across the spark gap. D123b

Then, in class, we explain that we are going to make a very strong electric field between the two balls, and that we shall store up positive and negative charges not only on the balls but also on the plates of the big capacitor, so that there are big charges waiting to be driven across the gap. However, the electric field between the balls is not quite big enough to start a spark.

'A spark will not start until the electric field is strong enough. We do not have quite enough voltage here to start a spark. The electric field is very strong: the camel's back is loaded very strongly. It will only take one or two straws to break the camel's back; it will only take a few ions in the air in the gap to start a spark. Now watch.'

When the capacitor and balls have been charged up to maximum voltage by temporarily bringing wires to them from the E.H.T. supply, we hold a match flame underneath the gap. There is a loud, visible, spark. We try that again.

‡ We might tell a fast group that we have evidence of that in the scattering of X-rays by the layers of atoms in a crystal of salt. But we could hardly give Bragg's ingenious argument by which he showed that his X-ray measurements lead to the conclusion that the atoms in the crystal are already ions, Na^+ and Cl^-.

'In that spark, we were not just driving across a few ions put there by the flame. They were only the starters, the pacers for the full team running the race. As each ion is driven by the electric field in the gap, it accelerates, it gathers speed, gains kinetic energy, until it collides with an air molecule. There it shares and loses its gains; and then makes a fresh start with accelerated motion. If the electric field is not very strong, the ion makes an elastic collision when it hits an air molecule; the two just bounce away from each other, with some sharing of energy.

'But if the electric field is strong enough, the moving ion has gained so much kinetic energy that it makes quite a different kind of collision: it knocks an electron off the atom it hits. It cannot do that unless it has a lot of kinetic energy: much more than the ordinary energy of an air molecule. If the moving ion knocks an electron off an atom or molecule of air, it thereby makes a fresh pair of ions. The positively charged remainder of the target is driven by the electric field one way, as a positive ion, and the electron that has been torn off is driven in the opposite direction. The electron may continue for some distance accelerating very fast because of its small mass, or it may join a neutral atom and become a more massive negative ion.

'Anyway, the collision produces two new ions, and as they are driven by the electric field they accelerate and have good chances of producing more ions when they make a collision. In this case, when the electric field gives an ion enough energy, as it travels one free path, to knock an electron off in the ensuing collision we have a multiplication of ions: a chain reaction, in which one original ion produces two more, and they produce more, and they produce more, and so on. Then there is an avalanche of charged particles, driven across to the metal ball at each end of the gap. That is a spark.‡

‡ Of course, the events in a spark are much more complicated than this simple story of an avalanche of electrons built up in ionization by collision. And the events in a Geiger counter are different and certainly complicated. Yet this gives beginners a sensible idea of the general process.

A useful, simple model of an avalanche can be made as follows: place a wooden plank (say 4 ft. long by 6 in. wide), on the table, raised at one end so that it slopes gently. Encircle the plank with a rubber band every 4 inches, so that each band makes a small horizontal ridge on the plank. Place a row of marbles just above each ridge. The marbles represent air molecules, regimented in rows one mean free path apart. The slope of the plank represents the strength of electric field applied to an ion. With a small slope one marble released at the top of the plank (to represent an initial ion) produces a small 'leakage current'. But if the plank is then raised to a greater slope – 'all ready for a few straws to break the camel's back' – a marble released at the top starts a dramatic avalanche.

'The spark that you see and hear is an after-effect of that avalanche. Atoms that have lost an electron give out visible light as they recover an electron and return to normal state. (Or perhaps some atoms have only been excited in some collisions; then they too give out visible light as their electron radiates the extra energy it had received.) The noise of the spark is simply due to the heat developed in that narrow region of the avalanche current: the sound wave comes from the high pressure, overheated air there.

'So we think of the spark as an avalanche of electrons and negative ions driven one way, positive ions driven the other way, all increased to tremendous numbers by a chain reaction of successive collisions, which starts from a few ions placed in the gap originally. Then in an electric field which is not quite strong enough to start a spark of its own accord, a few ions supplied from outside can start a spark.

'Now watch the experiment again.'

We charge up the system again and start a spark with a match flame. Then we say:

'That is not what you think. That is not a demonstration of a spark: *that is a match counter. I* light the matches, *you* count the sparks. Of course we have to charge up the capacitor again each time, but we can arrange to do that automatically, and it could be done quite quickly.'

3. *Alpha particles and sparks.* Then, still without any explanation of our line of teaching, we show a spark counter for alpha particles. D123c
That consists of a very thin wire, held taut a short distance away from a metal plate or gauze. We use the E.H.T. supply to produce an electric field between wire and plate a bit weaker than the field in which a spark starts of its own accord. Then we bring an alpha-particle source near, so that the region between wire and plate is within the range of the alpha particles. The great flock of ions left in the air by each alpha particle is more than enough to start a spark. So pupils see spark after spark flash between the wire and the plate at random places and with random spacing in time.

We explain that we are using a radioactive source of alpha particles, which pupils know can produce ions which carry away charge from an electroscope. We explain that we have arranged a high voltage between wire and plate, just short of the sparking voltage, so that the electric field is ready to drive an avalanche, if we provide some ions to start it. The most agile agents in the avalanche will be electrons. They will move fast, and often knock electrons off other atoms in turn before they themselves have time to be collected by some atom to form a lumbering negative ion. We want to make an avalanche of electrons, and collect them. We use the extra strong electric field round the thin wire – strong just as the field near a charged point is – so we make sure that the wire is positive. Then it will collect the electron avalanche.

A simple form of spark counter has a single wire, protected by a metal gauze, which acts as the other electrode. Pupils can see the sparks through the gauze, as well as hearing them, when an alpha source is brought near to it. A clearer form of counter dispenses with the gauze and has a grid of fine wires in front of the other electrode, which is a small metal plate. Then pupils can see sparks easily as they jump from various places on the grid, when an alpha source is held in a fixed position nearby. That makes the random nature of the phenomenon clearer. This form of spark counter is just as safe. The grid of wires can be connected to earth, and the plate, which is connected to the high voltage, can be protected in a plastic frame.

It should be clear to pupils that now they are seeing an alpha-particle counter. But we should complete the story and point out:

> 'Now we have an alpha-particle counter. The *radioactive material* here emits the alpha particle, *you* count the sparks. A spark starts when an alpha particle moving at fantastic speed through the air near the wire makes enough ions to "break the camel's back" and let the electric field make more ions by collision and so start a spark.
>
> 'You have seen a salt counter in which a handful of salt provides the ions and you can count the number of times the lamp lights; a match counter in which a lighted match provides ions to start a spark in a strong electric field, and you count the sparks; and now an alpha-particle counter in which again you count the sparks.
>
> 'Each alpha particle that passes through the right region starts a spark. The alpha particle itself has flown through at terrific speed long before the spark gets going; but it leaves ions in its wake and they start the spark. So when you count the sparks you are counting single alpha particles. Each is a small piece of

an atomic nucleus flung out with enormous energy when that nucleus "explodes" in a radioactive change. You are counting single atomic events.

'This is the essential property of radioactive atoms. They do not just stay there as atoms of ordinary copper or carbon do; they are unstable, they suddenly break up, flinging out a particle such as an alpha particle, becoming an atom of a different chemical element. By counting how many atoms of some stock of radioactive material break up in a given time, we can estimate the size of the whole stockpile. So counting alpha particles with a Geiger counter like this is very useful.

'The counting need not be done by watching sparks: it can be done automatically. That is what a scaler does.'

Geiger-Müller Tube and Scaler

We explain that the wire of the spark counter is placed inside a metal shield, which acts as the other electrode, and the high voltage supply is incorporated in the scaler which does the counting of the pulses of charge delivered by the electron avalanches to the central wire. T

The actual phenomena inside a tube are much more complicated than the simple story of ionization by collision producing an avalanche of electrons. We led up to that story with the intention of making Geiger counters seem reasonable to pupils, so that they could then appreciate what the counters tell us about radioactivity, feeling that they are using a familiar instrument. Inside the tube X-rays probably play an important part as well as colliding electrons and ions and the detailed picture is extremely complex. We do not apply sufficient voltage to an ordinary Geiger-Müller tube to produce a roaring spark when an energetic particle passes through.

Over a considerable range of high voltage, the plateau, a pulse of charge of almost constant size arrives at the central wire, whatever the ionization contributed by the initiating particle. Thus we count the number of particles by the number of pulses but we do not distinguish between one kind of particle and another or between a more energetic particle and a less energetic one. Other forms of counter, arranged to measure a more gentle level of ionization, can distinguish between one particle and another. For our teaching, we use tubes which just give pulses of a standard size, working on their flat plateau.

We show radioactive sources sending alpha, beta, gamma rays through appropriate counter tubes. Although a tube that will respond to alpha particles is more expensive, we should have one and start with it. It must have a very thin 'window' so that the alpha particles can get into the gas in the tube and make ions, without being stopped by the wall of the tube itself. If the window is made of transparent mica, we can also use it to demonstrate photons, by lighting a match in front of it. It will count some of the few ultra-violet photons that come from the flame. We shall make that very important use of it later. D124

We show that alpha particles have a very short range. D125a

We should at once show a cloud chamber in action for comparison. That will underline the short range of alpha particles and show the intense ionization they produce along their track.

We show how easily alpha particles are stopped: a piece of ordinary paper is too thick for them to get through; but a piece of cigarette paper will just let them through. D125b

If the laboratory already has sufficiently sensitive electroscopes for class experiments, pupils can watch a uranium oxide source discharging a charged electroscope. C126

We can show some quick experiments to illustrate absorption of beta rays and gamma rays by sheets of aluminium and lead; but we should not labour that study. Measurements of absorption coefficients were a very early way of distinguishing between one kind of radiation and another and of making rough estimates of energy. Although we still need to know about such matters, their importance lies in early history and they do not deserve much attention now. If we were to devote time and trouble to careful teaching of absorption properties, we should be giving pupils the kind of routine science that characterized chemistry in an earlier generation, when the learning of properties seemed to be of prime importance.

When counting gamma rays, we might show a fast group a demonstration suggesting an inverse-square law for the counting rate at various distances from a small source. Since they already know what an inverse-square law is like, from the study of gravitation, they should appreciate a real example. The experiment has an D127 OPT.

important moral: in safeguarding oneself from gamma radiation the best thing is to move farther away. When at ten times the distance one should feel 100 times as safe.

We should not overstress the dangers of radioactivity: from general talk and reading the public press pupils are likely to take such dangers all too seriously. We should say, however, that the way in which the radiations harm us is chiefly by making ions in our flesh and thereby upsetting or killing cells. (In the case of neutrons, some damage is done by knocking protons forward so that they make ions; but some damage is also done by the neutrons making nuclear changes which lead to radioactive material which itself provides particles that do damage.) * * * * * * * * *

Cloud-chamber

At the same time as seeing a scaler counting alpha particles, beta particles, gamma rays, pupils should see an expansion cloud chamber showing alpha-particle tracks. However many times they have seen this demonstration apparatus at work in earlier years, they should see it again now, because they are studying alpha particles seriously now, whereas earlier they must have looked at them as strange wonders of atomic physics. D128a

If the simple diffusion cloud-chambers suggested for Year I are available, pupils might try them again. They will see tracks of alpha particles from thoron. If they watch a carefully balanced diffusion cloud-chamber long enough they will see the tracks of high energy electrons from cosmic rays. C128b

As soon as the cloud-chamber is shown, teachers should post up a collection of enlarged cloud-chamber pictures, and possibly some pictures from bubble-chambers. These now serve as prime evidence of the properties and behaviour of the high-energy particles flung out by radioactive nuclei when they decay. The Nuffield Physics Group are making a collection of cloud-chamber pictures for this use. The pictures, when posted, should be accompanied by short printed descriptions, large enough to be read by pupils at a distance. D129

Pupils should see photographs of: a sheaf of alpha rays from a small radioactive source, showing their short range; alpha rays with one of them making a collision with a massive nucleus, such as nitrogen or oxygen in the air in the cloud-chamber; alpha rays with one of them making a collision with a hydrogen nucleus, perhaps from the water in the cloud-chamber; beta rays wandering through the wet air with an irregular path; making collisions less frequently, a beam of gamma rays or X-rays, shown by the tracks of the electrons that they eject with their powerful photo-electric effect.

There should be pictures showing alpha-ray tracks bent by a strong magnetic field and beta ray tracks bent far more by a weaker field.

Identifying Alpha Particles. There should be a special picture D129
showing the tracks of alpha rays in wet helium, with one making a nuclear collision.

The angle of that fork should be compared with the angle in an *elastic* collision between two large visible equal masses, one of
them originally at rest. That may be shown with two very long D130
pendulums carrying equal steel balls, or with equal ring magnets coasting on carbon dioxide on a level sheet of glass. The comparison suggests strongly that alpha particles have the same mass as helium nuclei. And since the two tracks of the fork in the cloud-chamber picture look alike we feel sure that alpha particles *are* helium nuclei, moving at high speed.

In the early studies of radioactivity the identity of spent alpha rays *
and helium atoms was tested by an important experiment, in *
which a spectrum gave the final verdict. That experiment by *
Rutherford and Royds was important at the time and is interesting *
now as an experiment that clinched a decision; but since we *
cannot do the actual experiment, it is better to show the right- *
angle fork instead and relegate the spectrum experiment to A-level – *
or to historical studies. *

If teachers wish, they can show pupils a theoretical attack on that T
right-angled fork; or they could ask a fast group to try to work it out for themselves. If a particle of mass m moving with speed v hits another particle of the same mass at rest, what must be the angle between their paths afterwards? We draw a triangle to show the two lots of momentum afterwards adding up to the same vector as the original momentum of the incoming particle. Since the masses are equal, each side of the triangle is a velocity multiplied by the same mass: so we may divide by that mass and take the sides of the triangle to represent velocities. We assume that the collision is elastic; that is, we assume that no nuclear energy is released or absorbed, and no electromagnetic radiation is emitted. Then kinetic energy is conserved in the collision. Since the masses are equal, we may cancel $\frac{1}{2}m$ throughout and say that (speed)^2

beforehand must equal the sum of the two lots of $(\text{speed})^2$ afterwards. We quote Pythagoras, and claim that the momentum triangle must be right-angled.

Scintillations. In early investigations, the tiny flashes of light made by a single alpha particle hitting a screen of zinc sulphide provided a very important tool. Counting 'scintillations' was tedious and tiring, but it provided the essential measurements of alpha-particle scattering that led to the nuclear atom-model. Then Geiger counters (and proportional counters) superseded that method completely for counting individual particles.

Then, decades later, scintillation counters came back into use, with more efficient sensitive materials and photomultiplier tubes (amplifying by millions) to take the place of human eyes.

(Warning: the spinthariscope, a device from the early days of radioactive studies, may well show only 'pile-ups' of several scintillations together, rather than separate flashes from individual alpha particles. Nevertheless it does show nuclear events occuring with random spacing in time.)

Now, solid state counters are coming into use, in which the impinging particle makes a similar disturbance – without a flash of light playing a part. It pushes some electrons into upper levels, leaving 'holes' that act as movable positive charges, in the solid semiconductor. Both those electrons and the holes are driven out by a strong electric field, making a pulse of current that can be counted. Thus, the mechanism closely resembles that of a gas ionization (proportional) counter.

Exponential Decay

One of the most important characteristics of radioactivity is that the rate at which a stock of radioactive material seems to die down exponentially, falling to half value again and again in the same time – or falling to $\frac{1}{10}$ value in equally regular, longer, intervals. That tells us that the chance of an atom disintegrating is constant in time. We are looking at a series of many chance events, all with a standard unalterable chance – or at least it appeared unalterable until we were able to bombard nuclei with streams of particles having such high energy that they can interrupt the course of normal radioactive life by effecting more immediate changes. T

So the rate at which we count disintegrations is proportional to the total number of *unchanged* radioactive atoms at the moment. Both that rate and the stockpile itself die away exponentially with the same characteristic half-life.

We should not attempt to teach the mathematical meaning of exponential decay at this stage – that will form an excellent study in A level – but pupils should, if possible, watch a counter recording the decay of a real substance. That may well be possible with the co-operation of the chemistry department.

The restrictions on the availability and handling of radioactive materials in schools do not apply to the 'natural' radioactive element uranium and its salts. Uranium itself decays with an extremely long half-life, emitting a low-energy alpha particle. Its daughter has a half-life of about 24 days. In a classical experiment D 131
in the history of radioactivity that daughter was separated out OPT.
chemically and its exponential decay measured. That would be too slow to be convincing here. But that daughter has a daughter in turn with a half-life just over 1 minute, emitting beta particles. The latter element, the granddaughter of uranium, can be separated from uranium by extracting with an organic solvent.‡ The half-life is short but a counter needs only a very small quantity to give an appreciable counting rate. Allowing for the time taken for the chemical separation we shall still obtain enough from a small quantity of ordinary uranium salt (where it will be 'in equilibrium') to enable the counter to give a clear story.

‡ The chemical separation is *not* difficult. A few grams of uranyl nitrate treated quickly will yield about a microcurie of the short-life granddaughter (protoactinium) enough to provide easy counting with a scaler.

The uranyl nitrate is dissolved in water acidified with HCl (hydrofluoric acid is the traditional agent, but hydrochloric will suffice). The solution is shaken for a few minutes with organic solvent: either amyl acetate or di-isopropylketone. The solvent extracts the granddaughter element, leaving the uranium and its daughter in the aqueous solution. The solvent is placed in a small plastic bottle near the beta-counter tube. (The 2·3 Mev beta rays get through the plastic wall easily.)

To separate the solvent for that, one may use a separating funnel, in which case it should be 'washed' by shaking with a fresh lot of acidulated water and separated a second time. Or one may keep solvent and aqueous solution together in a small plastic bottle (preferably inverted) and place the counter tube *above* the bottle to catch beta rays from the solvent. The latter method is easier, and can be repeated just by shaking the bottle again for a minute, but the background of radiation from the solution may be serious, making a much poorer signal-to-noise ratio than the former method's almost pure solution of the granddaughter.)

If we show exponential decay in a real demonstration with radioactive material, we should also show a model to help to make it clear – as simple a model as possible. We suggest the following, which sounds childish but preaches the moral so firmly that it proves worth while.

Give each member of the class a penny and ask the whole class to stand in line, each member shaking the penny in his closed hands or tossing it and catching it. Every quarter minute the teacher gives a signal and each pupil looks at his penny to see whether it is 'heads' or 'tails'. Pupils with 'heads' stay in the line, pupils with 'tails' fall out. This sounds like a long experiment, but we should reflect that it cannot last more than about 2 minutes! And towards the end it will raise the very interesting question 'What happens about the last atom?'

Changes from Element to Element

The changes when an unstable element emits a nuclear particle T
should be described very briefly.

(Even at A-level, we should not let this expand into a modern form *
of rote-memory chemistry in which nuclear reactions are learned *
by heart. In selecting examples to illustrate various types of change, *
we should survey the great variety known today and choose *
examples that seem simplest rather than adhere to the original *
cases of the historical development. In this matter, we might look *
at a similar choice in the teaching of chemistry, between choosing *
clear modern examples and following more difficult ones that made *
the early history.) *

We, as mature physicists, take radioactive changes from one ele- T
ment to another in our stride, but we should present them to pupils as an amazing break with the whole tradition of nineteenth-century chemistry, a change of the 'ultimate' atoms, a transmutation of immutable elements. We should describe the complete change of chemical properties between parent and daughter elements.

At this stage, we can only mention transmutation, and perhaps illustrate its meaning by the change from radium, a dense metal, to radon, an inert gas (+ the helium that emerges as an alpha particle).

Teachers might want to carry the discussion on further with *
specially keen pupils. The notes below are only suggestions for a *
possible *buffer extension.* *

Unstable Nuclei and Radioactive Changes from Element to Element (*Notes for a buffer-extension topic*)

We take the break-up and change of radioactive atoms for granted; but most pupils have been taught to regard atoms as the ultimate chemical particles. True, electrons can be chipped off an atom and possibly all an atom's electrons stripped off to leave a bare nucleus; but yet that nucleus remains impregnable – according to the simple story.

Although we no longer picture the electrons of an atom neatly pursuing sharply defined orbits, we do think of them as arranged in energy levels, and we even picture those in lowest energy levels as most likely to be found in the outer regions of an atom, some distance out from the nucleus. We think of each energy level as being clearly defined by the whole arrangement of nucleus and electrons: it is specified by a definite energy, the energy needed to remove that electron to infinity, or the energy which would be dissipated if that electron came into its place from infinity. The bigger that energy, the more strongly we consider the electron 'bound' to the system.

The outermost electrons of a neutral atom determine its chemistry; chemical changes involve exchanging or sharing electrons from the outermost groups, that is, those which have lowest 'binding energy' – the smallest energy needed to remove the electron.

Thus, the structure of the atom is best described by its whole series of energy levels: those in which electrons normally reside, and further levels to which some electrons can be moved when the atom is excited.

The arrangement of energy levels and their values depend on the number of electrons in the whole system and the electric field in which they find themselves. That field is determined by the charge of the nucleus (and by the charges of the other electrons in the atom). For a neutral atom the total number of electrons is just equal to the number of positive electron charges of the nucleus. For a neutral atom of atomic number Z there are Z electrons and a nuclear charge $+Ze$. Thus the energy levels of that atom are determined by Z, because that atomic number determines the number of electrons and the fields in which they find themselves. The energy levels for 'outermost electrons' determine chemical behaviour, and therefore we may say that chemical behaviour is determined by the atomic number Z. However, chemical changes do not affect the state of affairs of inner regions near the nucleus, except in the lightest elements so, in a way, the relation between Z and chemical properties is indirect, via the electron arrangement that the nuclear field engenders.

Atoms and Energy Levels for Chemistry. In order to interpret chemical changes in terms of electron behaviour and energy levels, we need to use the rules of quantum mechanics, which modify our simple, commonsense expectations very seriously. Therefore, there is no hope of our simple atomic model assisting in teaching Chemistry at this stage, unless one is willing to accept crude, qualitative descriptions – plain assertions to be taken on trust.

All we expect here is that pupils will learn that the nuclear charge Ze determines the number of electrons; and that their arrangement and selection of energy levels are determined by complicated rules which would require much more physics; but that those 'outermost' electrons which have small binding energy are exchanged and shared between atoms in the making of chemical compounds.

Therefore to change one element into another, the alchemist's dream of lead into gold, would require a change of nuclear charge, Ze. At first sight that seems impossible, as we picture the nucleus deep in the atom bound together by tremendous nuclear forces. But it does happen in the radioactive elements – the heaviest known, with nuclei too big to hold together permanently.

When first discovered, radioactivity was known by the ionization that appeared in air, etc., all around the material; then the actual radiations that caused the ionization were identified and studied: alpha, beta and gamma rays. And at the same time, over half a century ago now, transmutation was discovered. When a radioactive atom ejects an alpha particle, or an electron as a beta ray, it changes its whole chemistry. We are sure that the alpha or beta particle comes out from the nucleus, carrying away not only some 'nuclear energy' but also part of the nuclear charge.

Radioactive atoms are unstable: they have a definite, constant, half-life which tells us something about their probability of breaking up. For radium, for example, the chance is 50-50 whether an atom breaks up or not in the next 1,620 years, and that chance remains constant. However long we keep a sample of radium, those atoms which have not yet disintegrated are still just the same kind of radium atoms with just the same kind of chance of disintegration in the future, 50-50 for lasting less than another 1,620 years or more. The instability appears to be something inherent in the nuclear structure. Nowadays, taking a wave view of the behaviour of nuclear particles, we picture a stationary wave pattern defining the life of an alpha particle inside the nucleus. But that wave is not completely confined; it 'leaks' through the potential-wall barrier round the nucleus and runs on as a faint wave outside. We interpret that wave as telling us probabilities of locations – it is *not* a mechanical wave carrying energy and momentum – so while we expect to find the alpha particle *inside* the nucleus, we see there is a chance of finding it one day outside, despite what would seem an insurmountable potential wall. That chance of the alpha particle being outside – being emitted – is definite and constant, a part of the defining wave pattern, as long as the nucleus lasts.

That view is fruitful in helping us to understand nuclear structure and make predictions – for example, it tells us to expect high energy of alpha particles to go with short half-life of the parent nucleus.

All this goes far outside what we can teach O-level pupils; but it is relevant background to our own thinking when we are deciding whether to show a film of 'matter waves' and give the wave : particle idea. That is not just a strange little item decorating modern physics: it is of the essence in our view and understanding of the micro-physical world.

The alpha particle which emerges from a radium atom comes out with such huge energy that we are sure it came from the nucleus. And that is confirmed when we find that the 'daughter' atoms – the atoms that were radium until they ejected an alpha particle – are entirely different chemically. They form a dense inert gas in strong contrast with the properties of radium as a heavy metal. Placing radium in the calcium-barium family of the periodic table, we find we must move two columns downhill to place the product, radon gas, in the column for helium, neon, etc. An alpha particle carrying away charge $+2e$ would produce just that shift if the atomic numbers of elements in the chemical series *are* the nuclear charge numbers.

Again, radium has long been plentiful enough for good determinations of its atomic weight, close to 226; and the density of radon gas has been measured and found to agree well with the predicted atomic weight, 222. The story of this and all the other radioactive changes agrees with our view that we have transmutation from element to element.

When we have a mixture of parent element and daughter element, *
which have different chemical properties, we can separate them by *
ordinary chemical methods. We seldom have, or wish to handle, *
large enough quantities for visible chemistry, so we add stable, *
non-radioactive material of the same atomic number – an inert *
isotope – and then have enough for ordinary chemistry. Yet, we *
can trace the fate of the radioactive isotope with a Geiger counter. *
When we do that, we find that the radioactive material moves *
everywhere with the inert material of the same element – the *
chemistry of the atoms' outer electronic regions is essentially *
identical. On the other hand, when we use chemistry to separate *
out some pure parent stock from daughter element we find, if we *
wait some time, the daughter element appearing among what was *
pure parent stock. In other words, we see the effects of trans- *
mutation. *
*

This is not something that we can demonstrate directly to pupils *
because it means handling radioactive materials, mixing them with *
other materials, carrying out of chemical separations; and still *
making sure that nothing radioactive gets thrown down a drain or *
used in any dangerous manner. We could only *tell* our pupils about *
radioactive changes and chemistry. *

Manufacturing Unstable Isotopes: Tracers. Half a century T
ago, radioactivity was known as a peculiarity of a few heavy elements: the last dozen at the end of the chemical series are unstable. But now that we can bombard samples with high-speed, high-energy protons or neutrons – provided directly or indirectly by an accelerator such as a cyclotron or a linear accelerator – we can make unstable isotopes of every element in the whole chemical series. This has opened up a tremendous new field of 'nuclear chemistry'. Though it is fascinating and useful, we do not suggest that this should turn into a new field for learning-things-by-heart in our science teaching. But pupils should learn why radioactive isotopes that we now make are so useful: they serve as tags, luggage labels, or tracers, that enable us to follow any element we wish through processes such as digestion and circulation in a living body, and commercial manufacture.

We add a small quantity of radioactive sodium to a large sample of common sodium in salt, and can then follow with a Geiger counter the progress of sodium through any system, even a human body. Pupils should, perhaps, see a short film of a radioactive tracer being put to use.

Deflections by Fields

We should tell pupils that alpha, beta and gamma rays can be distinguished by applying electric and magnetic fields to their paths. We should avoid the traditional diagram which suggests that a stream of alpha rays and a stream of beta rays would be bent into arcs of the same curvature though in opposite directions. With a magnetic field, beta rays are bent enormously more than alpha rays – as pupils can see for themselves from the cloud-chamber pictures. In an electric field, the deflections are comparable, the paths of alpha rays being bent somewhat more because of their double charge, for particles of the same energy. T

For alpha rays, pupils must be content with seeing the bending by a magnetic field in a cloud-chamber picture. D129

For beta rays, we can set up a small source (a scrap of uranium salt would do) and a counter, with a sheet of lead hung between them as a barrier. We place a large horseshoe magnet under the lead, in such a position that the field between its arms will bend beta ray paths round under the barrier and into the counter-tube. Without the magnet, the counter shows very little – only a general background. With the magnet, the counter will count quite fast. With the magnet reversed the counter goes back to its small background. D133

If we had a source of positrons, instead of negative beta rays, the magnet would tell us because we should have to turn it the 'wrong way round'. However, with such a source there would also be a larger background, because positrons and electrons meet and annihilate, emitting gamma rays, some of which will affect the counter. * * * * * * *

With gamma rays we might be able to show pupils the straight line path, but that would necessitate heavy lead screens. * *

RUTHERFORD MODEL

Pupils have seen signs of electrons and positive ions so that a pudding model of atoms may well seem reasonable; but if we proceed to a description of a nuclear model without showing compelling evidence, we shall find ourselves in a hopeless field of assertion. * * * * *

Evidence for a Hollow-atom Model. Our first line of evidence for pupils is the almost direct one of cloud-chamber pictures. T

Pupils looking at a cloud chamber in action with alpha particles will see straight tracks and straight tracks and straight tracks, again and again, and never a nuclear collision. We emphasize the inference from that; that while an alpha particle makes 100,000 minor collisions, it almost never makes a major one in which it bounces away on a path in a new direction. In those minor collisions, the alpha particle tugs an electron off an air molecule as it hurtles by, and thus provides a positive ion and, soon, a negative one, on which drops of water can condense to make the track visible. Since the track of the alpha particle is not noticeably bent by these collisions, we infer that it must have hit something of trivial mass – in fact an electron some 7,000 times less massive than the alpha particle itself.

In fact, when cloud chambers first came into use a good deal was already known about the bombarding bullets, whose tracks they made visible. Alpha particles were known to be helium atoms with a double charge, He^{++}. Measurements with electric fields and big magnetic fields gave the charge/mass ratio, as in measuring e/m for electrons: it seemed to be half the value for hydrogen ions, H^+. Counting alpha particles in a stream and then catching the stream in a metal cup to measure its charge showed the charge on each to be $+2e$. That suggested a mass 4 times a hydrogen atom's mass, probably helium.

However, if we take a great many photographs of cloud-chamber tracks we find occasional examples of a fork in which the alpha particle's path is deflected. The enormous number of straight tracks is evidence for a 'hollow' atom, with detachable electrons somewhere in its outer region; and both the rareness and the big angles of the forks are evidence for a very small 'nucleus' in which most of the mass of the atom is concentrated.

It must be a massive nucleus, because it is able to hurl an occasional alpha particle backward, in some very close approach. In nuclear collisions with hydrogen, the alpha particle and the hydrogen nucleus that is struck both continue forward. However, in collisions with nitrogen, and other heavy atoms, the alpha particle sometimes rebounds in a backward direction. From that we conclude that the alpha particle must have a mass intermediate between hydrogen and nitrogen. In collisions with helium nuclei the fork shows a 90° angle, which indicates an elastic collision with an object of equal mass.

(See earlier discussion of this '90° fork'. We suggested a demonstration test with steel balls on pendulums or with pucks. And we suggested a fast group should try a theoretical attack with momentum, K.E. and geometry.)

Models of Scattering. Although the rare back scattering was a surprise to Rutherford, he knew quite well what the bullets were that he was firing; and he could see how the results would force him to make a new picture or model for atoms. But our pupils approach this evidence with no such preparation. We need to help them with illustrations. T

We ask them whether one could find out if cannon balls are concealed in a truss of hay, by firing rifle bullets at the hay. We ask them to think of rolling marbles along a slightly sloping table with a few spikes projecting up, as in a pin-ball machine. What would happen to the marbles?

We hang a long bar magnet (preferably an electromagnet) on a long pole – so that it forms a stiff pendulum – and place a strong electromagnet a short distance below it. The pendulum is free to swing, and if its magnet has a north pole at the end, above a north pole of the fixed magnet, pupils will see repulsion. Then we pull the pendulum aside and let it swing past the fixed magnet. D134

We suspend a metal-coated ping-pong ball by a very long nylon thread. We bring the main sphere of a small Van de Graaff machine near the ball. The ball gathers a charge and is pushed away. We then pull the ball out to one side and let it swing past the main sphere. D135

The electrostatic model gives a fairer illustration of the path of an alpha particle under nuclear repulsion; but the magnet model was Rutherford's own model, which he used in lectures with obvious delight.

THE GREAT SCATTERING EXPERIMENT

To use alpha particles as exploring bullets more precisely and still more fruitfully, one needs statistics of far more collisions than the few that cloud-chambers show. One must count alpha particles deflected ('scattered') from a copious stream by single collisions with an atom in a thin sheet of metal such as gold leaf, in a vacuum. In fact that was the first experiment to yield compelling evidence – the long series of cloud-chamber photographs came later. T

Counting scattered alpha particles by the faint scintillations they made on a screen, Rutherford was amazed to find a few bouncing back. Then he encouraged Geiger and Marsden to make a great series of measurements of deflections through small, medium, and large angles when a narrow beam of alpha particles hits gold leaf in a vacuum. Their measurements served to test Rutherford's new theory, or model, for atoms: a very small massive nucleus, with electrons so far out (and so light) that the alpha particles which were seriously deflected met the full force of a bare nucleus. He assumed that that nucleus carried a charge $+Ze$, where Z is the *serial number in the chemical series.* On such a theory the force between alpha particle (itself a helium nucleus) and the target nucleus is the inverse-square-law 'Coulomb force' of electrostatic repulsion.

Alpha particles making a very close approach to another nucleus are deflected through a large angle, while those missing the target widely are deflected through a small angle, so measurements over a big range of angles serve to investigate the field of force inside the scattering atoms over a large range of distances from the centre. We should post up a table of actual measurements of scattered alpha particles for various angles and show how the numbers counted fit the predictions for an inverse-square law of force.

We should not let pupils think that this is just one more measurement in atomic physics: we should describe the experiment, show the results, and even explain the general idea of the theory, so that they see it as one of the great turning points in physics. It changed our picture of atoms permanently. To do it justice in our teaching we must show pupils some of the real story and not just make assertions – otherwise we shall seem to be teaching witchcraft or telling fairy stories. We cannot show the actual experiment. So many conditions have to be arranged and so much apparatus explained that the experiment is obscured. Instead we must provide a film. That may show diagrams of the experiment to explain its general arrangement; but it *must* also show real apparatus and measurements being made.

Show P.S.S.C. film, *Rutherford Atom.* F136

Originally the scintillations were counted by eye: trained observers counted for a short time in a dark room. However when we teach pupils today there is no need to go decades back to that in history. We can show in films the observing being done by modern

scintillators with photomultiplier tubes (treated as a black box, no *
more complex than human eye+brain); and the counting done by *
a scaler. *

Then we ask pupils to compare theory and experiment.

On the theoretical side, we merely say that Rutherford assumed an T
inverse-square-law repulsion between the big electric charge on the massive nucleus of the gold atom and the smaller electric charge on the alpha particle flying past it. That is equivalent to Newton's assumption of an inverse-square attraction between the massive Sun and a planet. But instead of the simple circular orbits which serve approximately for planets, the change to a repulsive force predicts a different shape: hyperbolas. The alpha particle sails in, bends around a corner and sails out again on another almost straight track in a new direction. The simple calculation with circular orbits that predicts Kepler's Law III becomes too complicated.

Furthermore instead of a few individual planets, each with measured orbit and period, Rutherford had to use hordes of little alpha particles to give him a statistical test. He made his theory predict the number of particles that an observer would count on a receiving screen in various directions, in some standard time. In calculating that prediction he simply used an inverse-square law of repulsive force and Newton's laws of motion.

We ask pupils to take our word for that and we join them in comparing theory and experiment. We post up a table of observations, with the numbers predicted – for inverse-square electric repulsion – beside them. We discuss the comparison carefully, emphasizing its meaning for our knowledge of atoms. The table could be taken from Geiger and Marsden's original paper; or, better, it should be taken from a modern experiment shown in our film.

We should remind pupils that this was *not* an investigation directly concerned with radioactivity. Alpha particles were simply used as investigating projectiles. Nowadays we use particles from accelerators in continuing the investigation of nuclear force fields. With those investigating bullets we are exploring the field of force inside an atom in a way that corresponds closely with the exploration of the Sun's gravitational field by planets. Kepler's Law III is a summary of the latter exploration: it shows that an inverse-square-law field of force extends throughout the region from the orbit of Mercury to the orbit of the farthest planet – and comets in elliptical orbits extend that exploration to a still wider range. Here we use alpha particles to test the inverse-square-law field inside an atom over a range of distances of 10,000 to 1.

When, instead of alpha particles, we use protons that have been accelerated by a modern machine such as a cyclotron or a Van de Graaff, we have a much bigger choice of energies for the bombarding particles; and we can show that the inverse-square law breaks down at close approaches.

(If we used the scattering of protons from an accelerator throughout our discussion, that would remove the unnecessary association of this story with radioactivity; and we should be bringing our pupils closer to modern practice.)

The Nuclear Charge. The scattering of alpha particles not only provides us with clear evidence for a nuclear atom, but enables us to measure the nuclear charge. Chadwick used thin sheets of copper, silver and platinum instead of gold and measured the scattering of alpha particles by each. From his counts he calculated, with the help of Rutherford's theory, the charge on the nucleus of each of those three kinds of scattering nucleus. His results were: copper 29·3 electron charges, silver 46·3 electron charges, platinum 77·4 electron charges, with expected errors about 1 per cent. The serial numbers of those elements, arranged in order of atomic weights and placed in the chemical periodic table, are: 29, 47, 78.

Nowadays, we *define* the atomic number as the nuclear charge (measured in electron charges), but originally it was merely the serial number of the element. So Chadwick's measurements showed that the nuclear charge *is* the serial number, or the atomic number.

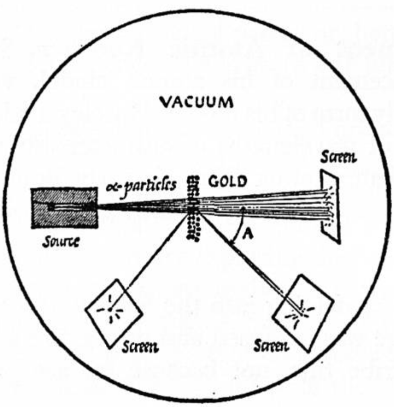
Another Measurement of Atomic Number. Soon after Rutherford's announcement of his atomic model, when Bohr was describing the early form of his model, Moseley made measurements, with crystals, of wavelengths of characteristic X-ray lines of several elements. Interpreting them on early Bohr theory he obtained numbers for nuclear charges which agreed with Rutherford's suggestion.

All this may seem to be delving into the history of a stage when our modern knowledge was confused and vague. We suggest that teachers should describe this, not because we are preoccupied with history, but because we are anxious to avoid our atom model seeming to be described by pure assertion. In this matter of making models, it is important to help pupils to see that the models and their changes have been suggested in part by experimental knowledge. Then pupils should feel some confidence in models and yet they should respect our warning that models may change.

SCATTERING OF ALPHA-PARTICLES BY GOLD
(Experimental Test by Geiger and Marsden)

EXPERIMENTAL MEASUREMENTS		TEST OF THEORETICAL PREDICTION	
Angle of Deflection[a] $A°$	*Experimental Count*† N	*Value of* $\frac{1}{(\sin \frac{1}{2}A)^4}$ from tables	*Test* $\frac{N}{1/(\sin \frac{1}{2}A)^4}$
150	33	1.15	29
135	43	1.38	31
120	52	1.79	29
105	69.5	2.53	28
75	211	7.25	29
60	477	16.0	30
45	1,435	46.6	31
30	7,800	233.3	33
15	120,570	3,445	35
10	502,570	17,330	29
5	8,289,000	276,300	30

[a] Of path of alpha-particles.



Chapter 7

WAVES AND PARTICLES

Photo-electric effect
X-rays and crystals
New atom models
Photons
Matter waves

Photons – A Note to Teachers

We have no sooner shown that light has a wave property and measured the wavelength than we upset the story with further demonstrations – by film – that light has a particle property: it packages its energy in small quanta.

Pupils should not be asked to wait a long time, preserving a wave view and looking on the quantum properties of light as a sort of unfortunate turn in the wrong direction. Instead they should see both properties as nearly as possible side by side. Although that makes us lose some of the delight of building up a triumphant wave theory, it enables us to present the modern view more wisely.

For our present teaching, the photo-electric effect gives much the clearest evidence for 'packets of energy', quanta, in light. Pupils will have to take on trust the general rule for those packets:

$$\text{energy in one quantum} = (\text{constant}, h).(\text{frequency of light})$$

Early Evidence and Origin of Quantum Theory. Several phenomena pointed towards a strange restriction on interchanges between radiation and atoms, in the early part of this century.

The idea that radiation energy (or at least its interchanges with energy of matter) comes in packages of amount proportional to frequency first arose in Planck's mind and he used it successfully in fitting a theoretical prediction to the experimental curve for the distribution of energy in the spectrum of a perfect radiator. Classical theory completely failed to predict the experimental curves until the quantum restriction was imposed in addition. That is far too difficult an avenue into quantum theory to carry any conviction with young pupils.

Then specific heats, both of solid elements and of gases, showed strange changes with temperature which were not predicted by classical mechanics (equipartition) but could be accounted for successfully by imposing the quantum rule on rotational motion and vibrations of molecules.

Then the photo-electric effect pointed in the same direction – or rather was found to be pointing when Einstein applied his clear vision to it.

Series in spectral lines, measured and decoded, waited for some explanation: and they too fitted into the quantum scheme when Bohr thought out his model for atoms.

X-rays and radioactivity added signposts pointing to quanta too.

It is clear now that all electromagnetic radiation carries its energy in quanta of size h. frequency; and that periodic motions of molecules (spins and vibrations) also have their energy in one or more quanta. The quantum-constant is another of the atomic constants in the universe.

THE PHOTO-ELECTRIC EFFECT

We should now turn to another phenomenon that is very fruitful in atomic knowledge; and easier for pupils to understand. They have seen photocells at work – in applications where light releases a horde of electrons from a sensitive surface in a vacuum, and the horde acts as a current to do jobs for us. That might be called the 'wholesale' photo-electric effect. Now we look at it in detail, the 'retail' effect; and we see light flicking electrons out of a metal, ultra-violet light tearing them out with the crack of a whip, X-rays hurling them out. This strange interchange between radiation and electrons throws much light on the microphysical world.

We start by a direct demonstration of the 'wholesale' photo-electric D137
effect. We connect a plate of freshly cleaned zinc to the leaf of an ordinary electroscope and shine light on the zinc. Since it is ultra-violet light which produces the effect in this case, we should use an open arc for this. (The tiny pencil-lead arc suggested in Year IV will do well.)

We charge the electroscope and watch the leaf. It may be necessary to install an earth plate or grid near the zinc. We show: (1) that a metal plate illuminated by suitable light can lose negative charge; (2) that an electric field driving negatively charged particles back into the plate stops this loss while an electric field in the other direction allows it; (3) that while a carbon arc will provide light that ejects electrons from a clean sheet of zinc, the effect is stopped by interposing a piece of glass.

This experiment suggests some of the photo-electric effect story, but it does *not* show that the negative electricity is coming out in particles, 'electrons'; it does *not* show that the light is arriving in bundles of energy, 'quanta'. It only suggests that there is some connection between the wavelength of the light and its efficacy in ejecting negative change. It seems essential to go much farther than this demonstration and show pupils photons arriving one by one; and we must show that, with light of a given wavelength, all the electrons ejected have the same energy (or rather the maximum energy is the same whatever the intensity of light). These experiments are too difficult to show directly: we offer them by film.

An understanding of the photo-electric effect plays an essential part in learning atomic physics. The 'wholesale' effect is fun to see, and it can be put to some good practical uses, but the essential story is masked. Pupils need to know something of the 'retail' effect – the behaviour of individual quanta of light in ejecting individual electrons. The standard experiments are difficult and confusing: however clear they seem to a trained physicist they strike beginners as inconclusive. They lead teachers perilously near to special pleading (see the warning in the next paragraph). Therefore we do not suggest any demonstration of the 'retail' effect, but we suggest teachers should show *two* films:

The Photo-electric Effect (P.S.S.C.) F138

Photons (P.S.S.C.) F139

Both are produced by P.S.S.C. for pupils of sixteen. Both show real experiments and give very good teaching commentary. They provide just the serious teaching that is needed here. (In contrast, we warn teachers against using animated films which merely show the picture we wish to assert.)

(*The advanced apparatus.* For more advanced teaching, there are several forms of photocell which can be used for a series of measurements with various wavelengths of light. Light of a chosen wavelength falls on a clean metal surface in a vacuum and ejects electrons. The electrons fly across to a collecting grid; but an opposing field is applied and increased until the electrons fail to reach the grid. At first sight this looks as if one could measure the kinetic energy with which the electrons emerge by measuring the applied p.d. Unfortunately a contact p.d. appears as well; so in practice the experiment is more difficult to run. And the cut-off voltage at which the electrons are just stopped is not as sharp as one would like. Those who have studied both the internal working of these tubes and the teaching of this experiment consider it an *extremely* difficult experiment to teach honestly and clearly.

The interpretation of the measurements with several wavelengths is a delight to a mature physicist, who sees a measurement of the quantum constant emerge, in the form of h/e. But to pupils it is

likely to seem interesting and complicated but not to give a clear *
demonstration of the essential photo-electric fact.) *

Detailed Knowledge. Our present picture of photo-electric *
effect emerged slowly from a variety of observations and sugges- *
tions, including Einstein's contribution which did so much to *
clear up early thinking. From the vantage point of our own *
mature modern knowledge it is clear that our picture of random *
quanta ejecting electrons is correct; but, to our pupils, each aspect *
of the phenomenon is strange and new and needs either an experi- *
mental demonstration or a clear statement from us that it is a new *
piece of knowledge. We should teach our pupils that: *

1. The particles ejected are electrons, with the usual value of e/m. T
(Once again a universal ingredient.)

2. The electrons emerge, for a given illumination, with a variety of speeds, the slower ones having probably lost energy by travelling through outer layers of the metal.

3. The maximum speed of electrons is determined by the wavelength of light used and not by the intensity. Brighter light only produces *more* electrons – to everyone's surprise, in the early days – and not *faster* electrons.

4. The maximum energy of ejected electrons appears, after an allowance has been made for energy to escape, to be proportional to the frequency of the light. This is the basis of Einstein's equation.

5. When the light is first turned on, there is no delay in production of electrons, as one would expect if a continuous stream of light had to build up enough energy in the metal to eject each electron in turn. Experiments with very weak light tell an impressive story. Sometimes an electron is ejected very early, almost at once when the light is switched on; sometimes no electron emerges till very late, after the weak light has been shining for some time; in general, a random distribution of timing. If the weak light is turned off again after so short a time that we could not expect its total energy to eject a single electron, we still see an electron ejected, during the illumination, *sometimes*. We are forced to picture the energy of the light arriving in small 'quanta' with random spacing in time.

All these are aspects of the photo-electric effect which are new and strange to pupils. And we must rely on descriptions and films.

There is no harm in using that mixture provided we are alive to its dangers. It is better to use such indirect methods of teaching than to puzzle pupils by supporting the new knowledge with experiments which do not really demonstrate it. The simple demonstration of negative charge being ejected from zinc fails like that. (Reflect on problems of teaching another topic: atoms. To mature physicists, crystals suggest atoms strongly; the thermal expansion of solids can be pictured comfortingly in terms of vibrating atoms and inter-atomic forces. But, to pupils, showing crystals as clear evidence of atoms does not seem convincing, and a demonstration of the expansion of metals ought to seem irrelevant – cart before the horse.)

Photo-electric Effect with G-M Tubes. A Geiger-Müller tube responding to gamma rays *is* demonstrating the photo-electric effect of those very energetic photons. But the random counting is due to the random instability of the parent radioactive nuclei: there one is not seeing or hearing the effect of photons arriving at random in a steady beam of radiation. But we can put a tube to the latter use.

A tube with a mica window, intended for counting alpha particles, D140
is likely to be sensitive to ultra-violet photons. We connect the tube to the scaler and test it with an alpha source. Then we try lighting matches in front of it at various distances. It will show random counts. A sheet of glass will prove that visible light is not the active agent: we suspect u-v. Perhaps the counts are due to individual photons or perhaps they only occur when there is a pile-up of several photons. Although a tube with a proper plateau gives the same pulse for particles of different energies, it is a far cry from an alpha particle with a million e.v to a photon with a dozen e.v.

A tube arranged to count gamma rays will respond to X-rays. If we D141
run a modern, hot cathode, X-ray tube with its filament so little OPT.
heated that there are very few electrons to make X-rays, a G-M tube at the other side of the room will count X-ray photons one by one, showing us a random stream. Even if the X-ray tube is run with alternating voltage between cathode and target, the large pulses of electrons, 50 pulses per second, will not spoil the random story when the filament is cool enough.

An X-ray tube, with a hot filament, running on 20 kilovolts a.c. is a dangerous thing. A tube with a filament that is barely warm and on a lower voltage might still emit enough X-ray photons to be counted one by one – and by that token it would be quite safe!

These are good demonstrations to give after pupils have been taught by the two films. They are not sufficient to do the initial teaching; but they make good sense to pupils who know what they are looking for.

X-RAYS

X-rays. We do not advise schools to buy an X-ray demonstration. It has to be hedged with safeguards; it is expensive; and when it is run it does not show much. Pupils learn that X-rays can cast shadows of bones in flesh – but they know that from general reading; and they can see pictures of X-ray photos. They might learn that X-rays ionize air or blacken photo films; but they can see ultraviolet light do that.

The only very important use of an X-ray tube now would be to show photons being counted one by one, as described above.

Production of X-rays. Yet we need to give pupils a short description of the production of X-rays: electrons from a hot filament accelerated by a large p.d. gain huge K.E.; on reaching the target they lose their K.E., nearly always converting it to heat in the target – much as a lead bullet would. Just a few electrons, far less than 1 per cent of the stream, convert their K.E. to a photon of radiation as they come to rest.

'What are X-Rays?' If pupils see the demonstration of X-ray photons being counted before the photon film, they may ask 'Are these particles or waves? How do you know?' In any case we should say that X-rays can be thrown into spectra – or a diffraction pattern – by the regular layers of atoms in a crystal.‡ If so, we infer (1) that they are waves, (2) that their wavelength is very small – nearer 1 A.U. than the 5,000 A.U. pupils measured for green light.

X-Rays and Crystals

Grating Spectra in Two Dimensions. Some teachers like to introduce diffraction gratings by asking pupils to look at a distant lamp through umbrella fabric, or a silk handkerchief or any other coarse two-dimensional grating. As a link with ordinary life that is an excellent beginning; but the complex array of spectra may make it more difficult.

‡ X-rays can be diffracted by ordinary ruled ('optical') gratings but only at very oblique incidence – somewhat like the use of a gramophone record for light.

Now, if not before, pupils should certainly try that. A small compact source of light is placed at the far end of the room, high up on the wall, and pupils look at it through some finely woven fabric. After seeing the behaviour of an ordinary grating, they should find this new observation easy to understand and interpret. Some pupils may be confused by having used ordinary gratings with line filament sources. We should give them the coarse gratings again and ask them to observe the compact light source with those gratings first and then use the stretched piece of cloth. C142a

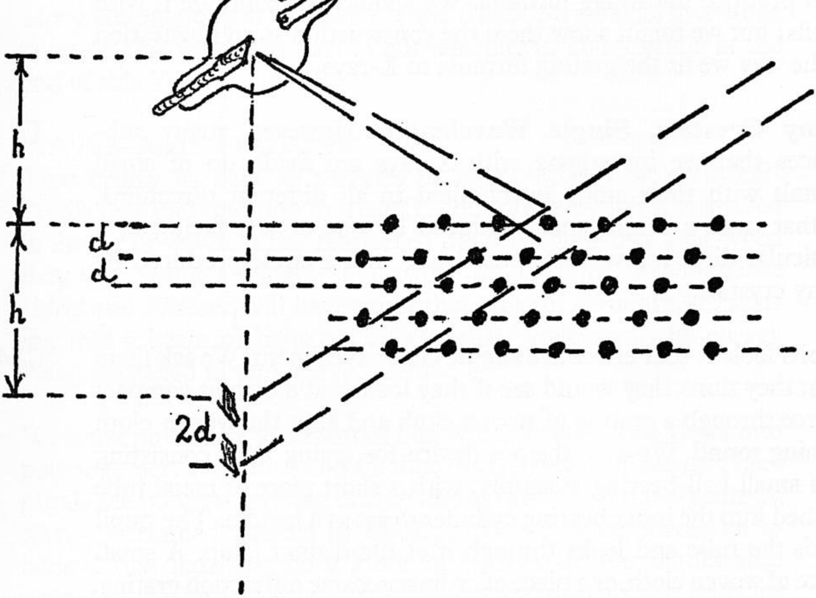
Gratings from Nuffield Chemistry Programme. If the school is following the Nuffield chemistry programme, they will have available the special two-dimensional gratings made up of black dots on transparent ground, arranged in various patterns. These are suggested in the chemistry programme as models with which to interpret X-ray diffraction pictures. C143

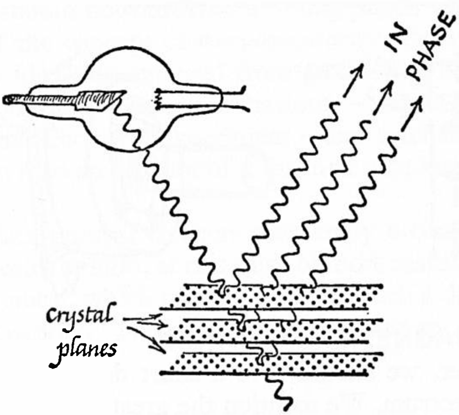
Teachers should borrow those and ask pupils to try looking at the lamp through them.

X-ray Diffraction: Single Crystal, Many Wavelengths. In any case, we should give some description and explanation of X-ray spectra at this point. We ask pupils to think of atoms arranged in regular layers in a crystal. Waves reflected from layer after layer would bounce out to an observer after travelling longer and longer paths from deeper layers. If the extra path required for each successive layer is one or more complete wavelengths, the reflected waves would all add up to an intense resultant. That is what happens when we try reflecting X-rays with a crystal. Only in some directions, and only with some wavelengths, do we get intense reflections – hence the pattern of spots that we see in a Laue photograph made by X-rays of many wavelengths diffracted by a single crystal. T

Single Wavelength, Single Crystal: Bragg Formula. Although most pupils will have proved the formula for an ordinary diffraction grating, we should not burden them with details of its extension to a formula for X-rays reflected from crystals. * * * *

Yet there is a very easy way of arriving at the Bragg formula. We sketch a section of crystal with horizontal layers of atoms, with an X-ray tube above, a short distance to one side. Its target is a vertical height h above the top layer of atoms. Then X-rays starting from the target of the tube and being reflected by the top layer of atoms (T)



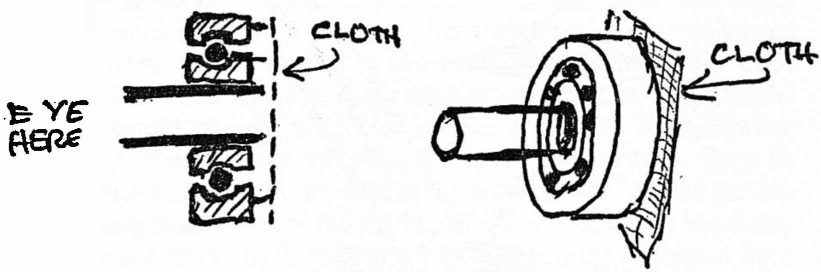


X-RAY DIFFRACTION BY CRYSTAL

will seem after reflection to come from an *image of the target* a distance h below the top layer of atoms. There we are using pupils' Year III knowledge of the position of the virtual image of an object formed by a plane mirror. X-rays reflected from successively deeper layers of atoms will seem to come from similar virtual images – all in a vertical line below the first one, spaced apart by $2d$, twice the 'grating-space'. Continuing the geometry from there will soon produce the Bragg formula. We should not continue it with pupils; but we might show them the construction as an illustration of the way we fit the grating formula to X-rays.

Many Crystals, Single Wavelength. However, many substances that we investigate with X-rays are made up of small crystals with their atom layers tilted in all different directions. In that case, a bright spot produced by one crystal facing in a particular direction will be spun round into a circle produced by many crystals. T

Pupils look at that effect in a simple class experiment. We ask them what they think they would see if they looked at a distant compact source through a grating of woven cloth and kept the woven cloth turning round. We give them a device for trying that, consisting of a small ball-bearing assembly, with a short piece of metal tube pushed into the inner bearing cylinder to act as a handle. The pupil holds the tube and looks through it at the distant lamp. A small piece of woven cloth, or a piece of ordinary coarse diffraction grating, is placed across the outer cylinder of the bearing. As the pupil looks through the tube at the remote light, he spins the outer cylinder so that he sees the light through a revolving grating. C142b



THE ELECTROMAGNETIC SPECTRUM

As suggested earlier, we should give a short description of the electromagnetic spectrum. We mention the great variety of waves in its compass, methods of production and detection, and common properties. We suggest that at O level, this account should be very brief. Its chief landmark should be a long chart; not the traditional poster crowded with information and sketches in many colours, but a simple strip showing ranges of frequencies in octaves, with names and wavelengths marked. T D144

Each teacher should decide the placing and size of this discussion, to suit his own tastes and those of his class. Although this is a * *

suitable place in the programme for it, we do not want to interpose *
a long commentary at this point in the *Guide*. So we shall give some *
notes on this particular part of our teaching in an appendix at the *
end of this Year. *

WAVES AND PHOTONS

That brings us back to theories of light: 'Waves or particles?' T
Pupils met that question in Year III, but probably left it answered in favour of waves. They reinforced the wave view this Year when they saw Young's fringes. The two P.S.S.C. films, *Photo-electric Effect* and *Photons*, will have upset that sense of certainty, suggesting that a beam of light has an essential 'graininess', its energy being packaged in photons or quanta.

We must now show an essential film which brings both views into
play together. It is the P.S.S.C. film *Interference of Photons*. There F145
pupils see photons arriving one by one at random in the organized, wave-determined pattern of Young's fringes. That forces us to believe that light has both kinds of behaviour. We should stop trying to decide *which*, and start trying to learn to use both, or rather *either* according to the kind of experiment we are doing.

We certainly should not burden our young pupils with a puzzling *
description of the concept of *complementarity*. Yet that is a very *
powerful new idea that emerged from our having to accept both *
wave behaviour and particle behaviour – mutually exclusive, *
according to our choice of experiment – and teachers may find it *
worth while to read an account of it for their own interest.‡ *
*

With some class groups, teachers may enjoy a considerable dis- *
cussion of theories of light, at this stage of more mature knowledge. *
Some general notes, which may be useful in such a discussion, are *
given in an appendix, 'Theories of Light', at the end of this Year. *

MATTER WAVES

Now at last our pupils are ready to learn about matter waves: they *
have enough knowledge, so that the revolutionary idea will make *
sense although it feels so strange. *

If they have not done the class experiments with a two-dimensional C146
diffraction grating of cloth, first held at rest, then revolving, they
must try those now. Then they should see the P.S.S.C. film *Matter* F147
Waves. It is a very good teaching film, showing real experiments

‡ See H. Massey, *The New Age in Physics* (Harper, 1960).

with a physicist's commentary, made for school pupils of sixteen to eighteen.

It shows how a revolving grating, or a random collection of small patches of grating, can produce an interference pattern of circles – as pupils have just seen in their own class experiment. Then it shows just such a pattern being produced by a stream of particles – electrons which we *know* are particles with definite mass and charge. We know electrons carry energy – and, we suppose, momentum – compactly when they are moving. Yet moving electrons seem to be guided to an interference pattern just like waves of light: or, rather, just like photons of light.

There are the two stories, of electrons and of photons. Each has particle properties in some circumstances; and each has wave properties in other circumstances. We have to learn to live with both views for light, and for electrons; and for hydrogen nuclei, neutrons, carbon atoms ... all moving particles, ... cricket balls, ... all moving objects. T

In the microphysical world, at the level of atoms and parts of atoms, wave-behaviour of a moving particle is very important: wave patterns guide the particles to interference patterns.

In the macroscopic world, large 'particles' like men, cricket balls, and aeroplanes have such *extremely* minute wavelengths, at any noticeable speed, that we never expect to see cricket balls making an interference pattern, or men diffracting round a corner.

As optional aids in setting forth this dual view, we may show two sets of pictures:

1. To help pupils to understand that more than one model may be kept in fruitful use, to persuade them not to ask too strongly 'Which model is *true*; which is *right*?' we show them two maps of London. One is the usual stylized Underground map in several colours. The other is a map of streets with the Underground lines marked on it too. We ask which is the right one; and we are likely to receive 'It depends on what you want it for.' D148 OPT.

2. To reinforce the rather indirect evidence of ring patterns for electron diffraction, we may show pupils two pictures of 'Young's fringes' – actually photographs of biprism fringes – one taken with yellow light, the other taken with a stream of electrons. The electron stream is split into two streams by a fine wire across their path. D149 OPT.

A positive charge on the wire produces deflecting electric fields that bring the two streams towards each other so that they overlap where they fall on a distant screen. In that overlapping region, the photo shows dark and bright fringes, patches of many electrons arriving alternating with patches of few electrons arriving. The latter experiment is far too difficult to do in any teaching laboratory, and even a film of it would be overburdened by technical details; yet the resulting photograph and its optical companion form a remarkably convincing pair.

ATOM MODELS

Bohr Model – Note to Teachers

Not long after Rutherford set forth his nuclear model for atoms, Niels Bohr made a brilliant move to bring that model into accord with some experimental knowledge of quanta; and he produced a very fruitful theory.

We should not try to describe the Bohr model in detail to our pupils, still less should we try to carry them through any calculations. For one thing it requires too large a background of new knowledge; and for another thing it is out of date. It does not agree with our present view, though it has played an essential part in leading physicists towards the present view.

Some notes on the development of Bohr's model are given in an appendix at the end of this Year. They are offered to teachers simply for their own interest, as background for teaching or for possible use if a pupil asks a question.

Wave Patterns in Atom Models

We turn from a running electron wave – which experiments forced us to imagine – to the idea of stationary electron waves in an atom. That is how the new idea of guiding waves carried the Bohr atom model into a great modern development.

In writing down rules that seemed *necessary* for a Rutherford atom model, Bohr had to state several novel rules, such as the requirement that an electron has a choice of definite, stable orbits, and the rule that in switching from one orbit to another the electron emits the difference of energy as a single quantum of amount (constant, h).(frequency). The latter rule simply insisted on the quantum relation that was already known. But the definition of stable orbits seemed rather arbitrary in Bohr's hands. There must be some such restriction – line spectra tell us that – but could one give a sensible rule for defining the particular orbits? Bohr did that with a rule for 'quantizing' angular momentum, which was at best plausible.

With the concept of guiding waves for moving electrons, it was easy to give a much more appealing rule for choosing orbits: the circumference of the orbit must contain a whole number of wavelengths, so that the electron's wave pattern is a stationary wave. That 'explained' why the Bohr orbits were stable, and told us we could not expect to locate an electron precisely at a point on a sharp orbit; and it led to Bohr's predictions of orbit sizes and energies. But when that wave model was developed further it no longer agreed with Bohr's model; but it has proved fruitful in successful predictions or explanations.

Now at this concluding stage of our programme, we should leave our pupils looking forward, not equipped with final knowledge – such as 'modern, correct, true model of the atom at last' – but keen to see how knowledge and understanding grow as experiments continue and theories change.

We should give pupils a glimpse of our present atom-model: a nuclear atom with a fuzzy distribution of electrons instead of sharp orbits – fuzzy in position but definite in energy-levels. The locations (and motions) of the electrons are described by their 'matter waves'. Those wave-patterns – which we write as equations when they are too difficult to sketch – tell us the *probability* of finding an electron in a given region of the atom. They tell us the betting, never a certainty. Yet the betting is useful: it tells us definite energy-levels; it explains chemical bonding by electrons; and it not only explains the known random laws of radioactivity but also predicts new nuclear particles.

This *is* disturbing new knowledge; and we should not be sorry to leave our pupils at this point – they will remember science as combining experiments with a continuing changing series of models in our thinking that we call theory.

Appendix A

THEORIES OF LIGHT

Discussing Theories

In Year III, pupils looked at the behaviour of ripples on water. The ripples proceed as 'wave fronts', lines of crests and troughs that travel out from a source, are reflected by a solid boundary, and can be refracted where there is a change of wave-speed as they pass into shallower water.

Pupils could sketch a snapshot of successive wave fronts. That also shows successive positions of the same wave front after 1, 2, 3, ... periods. On those sketches of wave progress, they could add guide lines along the direction of travel, perpendicular to the wave fronts. We hope teachers encouraged pupils to do that and even suggested the name 'rays' for those lines. Then pupils could see rays emerging from a small source along radii; and rays forming a 'parallel beam' for the progress of plane waves. We may certainly draw such guide lines if we please; but calling them rays is a suggestive move almost amounting to special pleading, which would be bad teaching. So we hope that the word 'rays' was introduced with caution and not used to suggest that the rays of light *must be guide lines of waves*.

In looking at the properties of light, pupils saw reflection and refraction qualitatively. We now remind them of those properties. We ask if a stream of bullets would behave like light: travel in straight lines, be reflected making equal angles at the wall, be refracted at a boundary between two media. The answers are: yes, if they are left alone as they travel (Newton's Law I); yes, if they bounce elastically from the wall; yes, *if* the bullets are attracted when they approach the boundary and then travel *faster* in a denser medium.

Then we remind pupils of several experiments with ripples and ask whether *waves* travel in straight lines, and are reflected and refracted. Could light consist of waves? The answers for straight-line travel, reflection and refraction are: yes; yes; yes, *if* the waves travel *slower* in a denser medium. That offers a clear test, a 'crucial experiment' to discriminate between our two theories of light. Does light travel faster in glass or water than in air (as it should for bullets), or slower (as it should for waves)? Foucault's experiment showed that light travels slower in water than in air. We accept the wave view with enthusiasm and proceed to certainty when we add the evidence of interference.

(Yet to an able group we should express a warning as a question: can a crucial experiment ever be a complete yes-or-no test? It only tests between one form of the first theory and one form of the second theory. If we are clever enough, we can usually modify the rejected theory to fit after all – as we can in this case by assuming the bullets retain the same kinetic energy rather than the same mass.

Even when we cannot find a saving modification, we can still assert an extra 'rule' such as, 'Well, these bullets just do travel faster in water than in air: that is our new discovery.' If that seems high-handed and unscientific, we should reflect that it is just what we have had to do in proceeding from classical atomic models to Bohr theory and on to quantum mechanics – we have had to assume extra rules for the microphysical world in order to succeed in describing it in terms of things large enough to observe.)

When pupils see interference effects such as Young's fringes for light, they should certainly support the wave view of light. (For a century after Newton, both theories had their supporters, but in the early 1800s the wave theory emerged as completely right because interference proved that. Yet, in the early 1900s it became clear that light has some bullet-like properties as well. The idea of quanta was suggested and grew stronger.

Visible light can eject electrons from suitable metal surfaces; ultraviolet light ejects electrons from any metal surface; X-rays eject electrons from any matter. When this photo-electric effect is studied, it points towards a picture of light arriving in packets of energy – the shorter the wavelength the bigger the packet. We call each packet a quantum; but as this picture of quanta grew clearer and more important, physicists coined a new name to be used for light whenever we are concentrating attention on its bullet-like behaviour: photons. A photon means a quantum of light, a single 'bullet' carrying energy and momentum with the speed of light, *c*.

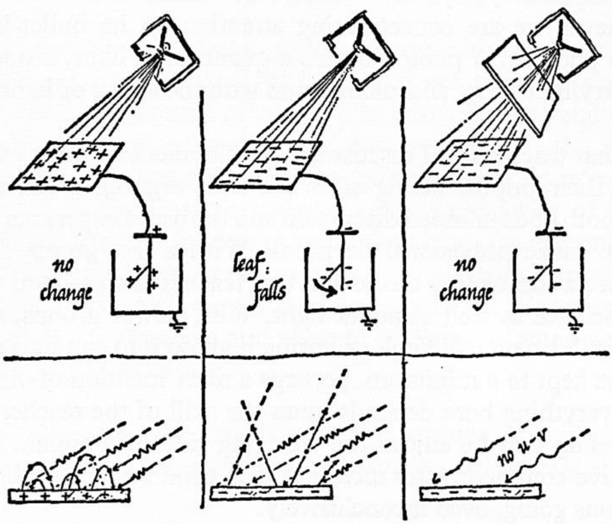
We hope that teachers will discuss the development of theories of light with their pupils, taking sides perhaps, arguing a bit, encouraging both open-minded discussion and the use of experimental evidence to make provisional decisions. With a fast group, this should prove a stimulating discussion that teaches things about the nature of science as well as about light. With slower groups, the discussion will become difficult or boring if allowed to run long; so it should be kept to a minimum, perhaps a mere mention of views of light. Everything here depends upon the skill of the teacher in giving a feeling that he enjoys the changing tides of opinion. He needs to give confident assurance that it is good science to keep rival opinions going, even inconclusively.

Pupils who will do no more physics deserve to hear of photons even if the concept appears only as a surprising complication in our views of light.

Teaching the Photoelectric Effect

After bringing interference in to support the idea of light waves, we turn away to show the photo-electric effect. The gross effect is easily demonstrated: light falling on a metal surface produces a current of negative charge that comes from the surface and can be driven by an electric field to a collector and on round a circuit. We may interpret this as a stream of electrons ejected from the surface by the light; but, if we teach that, it is pure assertion at this stage because the demonstration gives no hint of individual electrons being swept out one by one. It shows a steady stream which could equally well be pictured as a smooth stream of negative electric 'juice' being driven out of the metal. So far, only Millikan's experiment has given any evidence of electric charge being 'atomic', suggesting that electrons are definite, basic units. Nevertheless, we must start by showing a demonstration of the wholesale effect.

The best demonstration is a simple one in which a plate of freshly cleaned zinc is illuminated with ultra-violet light. The zinc is connected to a charged electroscope so that we can see if it loses charge. If the zinc is negatively charged (say by induction from a positively charged rod), the electroscope will show the charge leaking away when ultra-violet light is used. For that, it is essential to clean any oxide film off the zinc with steel wool just before the experiment. But if the zinc is positively charged originally, ultra-violet light



seems to have no effect. This supports a picture of the light ejecting negative charges. The easiest source of light to use is a naked carbon arc. If one is available the arc light is shone on the zinc. Then a sheet of glass is interposed; and although the visible light still shines on the zinc there is no effect, suggesting that ultra-violet light may be the active agent.

For this demonstration, a very simple arc will suffice: two rods of carbon held in clamps, connected to a main supply with 30 or 40 ohms in series. With a d.c. supply, the arc is easily struck by making the carbons touch momentarily. With an a.c. supply, the arc strikes less easily and requires a little practice. This open arc must be shielded from pupils by an opaque screen. However, they can see what is happening at the tips of the carbons if we use an ordinary converging lens to form an image of the arc on a distant wall.

Lacking an arc, we must use metals that respond to visible light. These are alkali metals, which must be enclosed in a glass bulb. The bulb is usually evacuated, making a photocell which can be used for measurements and for controlling apparatus. If a commercial photocell is used, we shine visible light on the sensitive electrode, connect the positive of a power-pack to the other electrode and run from the negative of the power-pack through some form of meter to the sensitive electrode. We see a current which is larger for more light.

Special forms of photocell have been developed for teaching. These are intended to be used with colour filters or other arrangements for shining light of known wavelength on the sensitive surface. A repulsive electric field is applied to prevent the electrons that are ejected by the light from reaching the collecting electrode. We do *not* recommend such a tube for the present simple, qualitative demonstration; but it is essential for a further demonstration described below.

How are pupils to see that the light ejects electrons, particles of charge, and that more light (of a given wavelength) only ejects more electrons and not faster ones? How are pupils to learn that the photons which perform this all have the same energy for a given frequency of light, and that that energy is directly proportional to the frequency? And finally, how are pupils to learn that the photons arrive at random in the beam of light, spaced along the beam by mere chance? That is roughly the historical order of questions about the photo-electric effect; and it gives the order of difficulty in demonstration. But yet the characteristics mentioned last seemed the most interesting and surprising of all, the most likely to excite wonder.

For that reason, we consider that pupils must see a film that demonstrates photons arriving one by one. There is a very good film, produced by P.S.S.C., and we hope that teachers will show it to average and faster groups and discuss it with them. This is the P.S.S.C. film *Photons*. A slow group may have to bring their Year V studies to a stop just before this.

We hope that no apology is necessary for suggesting American films. In our effort to bring our teaching into the realm of present-day physics, we have now reached topics where we cannot give direct demonstrations in class. Apparatus is available in some cases, but it is expensive and often needs considerable apprenticeship. The demonstration is then complicated by the profusion of apparatus so that we have little hope of getting the essential point over, even to a fast group. It seems better, instead, to use modern facilities of movie cameras to give pupils a simpler, clearer view. In film, the camera can move from a general view to a close-up of some apparatus, across to a blackboard for a sketch or a short piece of explanation, to a model that illustrates the general idea and then back to the experiment itself. Space can be magnified or condensed, time can be compressed. To make films that give good, clear teaching, at the same time keeping a strict eye for good physics, is very expensive: it seems to necessitate professional equipment and professional film-makers. Amateur films are delightful in much of our teaching, but here the teaching itself is so difficult that professional help is needed. The Nuffield Physics Group could have made such films, but since very good ones have already been made by physicists, at enormous expense and with great care, we suggest teachers in our Programme should make use of the existing films.

To understand the P.S.S.C. film, *Photons*, pupils need to know some of the things we listed in our earlier questions above. We should prepare for that film by showing another film, *Photoelectric Effect*, produced by P.S.S.C. That will show light of different colours ejecting electrons with different amounts of kinetic energy. Instead of the latter, preparatory, film, as a poorer, though direct alternative the teacher might give a demonstration with a special tube in series with battery and meter, illuminated by light through colour filters. Applying an opposing voltage between the collector and the sensitive surface from which light ejects electrons, he can show the voltage needed to stop the *fastest* electrons getting across to the collector. Although in the simple theoretical picture light of a given colour ejects electrons all of the same K.E., in practice most of the electrons have to struggle out through extra electric fields, because they come from just below the surface of the metal. So the demonstration shows the stream of ejected electrons being reduced more and more as the opposing voltage is increased, until it just reaches 0 at some critical voltage. The latter voltage can be read, *with considerable uncertainty*, from a graph. If that is done for light of several colours, an overall graph can be plotted showing critical opposing voltage (to represent maximum K.E. of electrons) against some measure of the light's frequency (say, 1/wavelength), to represent frequency.

With very careful work and some apprenticeship to the apparatus, the plotted points of the overall graph will be likely to close to a straight line. The best straight line will not pass through the origin because the opposing voltage we measure is not the full voltage that operates inside the tube to drive electrons back. There are other 'contact' potential differences which must be taken into account. To explain them, we should have to describe electrons accumulating more in one metal than another and establishing a potential wall at the boundary. That would be quite unsuitable here: it would certainly spoil the teaching of the simple, important ideas of the photo-electric effect. Therefore, we do not recommend this demonstration except where the teacher has a strong interest in carrying it out – in which case his explanation will gather force from the sense of personal experience that will invest his teaching. (And therefore we do *not* recommend schools to buy equipment for this.)

Pupils must also learn from this film that doubling the intensity of light does not change the maximum intensity with which electrons are ejected. Instead, it only leads to a more copious flow of electrons. This is, in a way, strong evidence for the existence of photons, bullets of light energy, each able to eject an electron – so that more light merely means more bullets, and therefore more electrons ejected.

Where there is time to spare and the film is available, teachers should wish to show one more film, *Interference of Photons*. This follows the film *Photons*, using similar apparatus, and shows photons arriving in Young's fringes: many in a bright band, few in a dark band. It shows that an individual photon seems to pick its path by pure chance and arrive at random anywhere on the screen. But the chances are determined by the wave behaviour that predicts the pattern of fringes, so that the probability of photons arriving in a bright fringe is much greater than that for a dark fringe. It is as if the wave behaviour guided the bulk distribution of photons into the pattern of fringes, while the fate of an individual photon remained quite random.

Thus, the teaching we suggest is:

1. All pupils should see the gross effect of ultra-violet light, or visible light, making a current of negative electricity stream away from a suitable metal surface.

2. All pupils should see a film of the photo-electric effect, showing its dependence on the intensity of the light and on the frequency.

3. All pupils should see a film showing photons arriving one by one at random.

4. A fast group should see a film that shows photons 'painting' an interference pattern.

Matter Waves

We go straight on from this new picture of light having both wave properties and particle properties to mention a similar picture for electrons, neutrons, atoms, all pieces of matter. That was the fantastic suggestion made by de Broglie in 1926. It changed the whole face of physics. It provided a means of developing a very successful quantum theory of atoms out of Bohr's beginning, which was only partly successful. It altered our whole view of chance and certainty in nature.

The physics that we use today, and modern chemistry too, are founded on quantum mechanics that arose from the new view. To give any sensible teaching of the new ideas and their use goes too easily beyond A-level into university physics, where the necessary use of differential equations is possible. Nevertheless, we suggest that pupils should hear of the new view. So, we suggest that teachers should show a very fine film, *Matter Waves*, produced by P.S.S.C.

The ideas of photons with light waves and matter waves with moving particles are well-expressed in those suggested films. Therefore, we shall not give further commentary here.

Appendix B

THE ELECTROMAGNETIC SPECTRUM

We all think of 'the electromagnetic spectrum', ranging from longest wireless waves to minute gamma rays, as a very important topic which should be taught as part of a general knowledge of physics. We feel such a grand sweep of radiations bound by simple common characteristics should be easy to expound. But with our conscience sensitive about teaching by assertion – or, worse still, by special pleading – we soon develop doubts. What parts of the spectrum can we show? What common characteristics can we demonstrate experimentally? Maxwell, Hertz, Tyndall and many others have built a tremendous picture in which we have complete confidence: waves consisting of combined electric and magnetic fields, all travelling with the same rate speed; with measured wave-lengths covering such a wide range that their calculated frequencies cover more than forty octaves. All carry energy and momentum and exert pressure, according to the same rule, on an absorbing or reflecting surface. All carry their energy in definite packages whose size is proportional to the frequency – or so we believe: we know that from direct measurements in the range from visible light to gamma rays; but in infra-red and wireless regions the quanta are very small and we can only infer their presence indirectly. All turn their (electromagnetic) energy into heat when they are stopped by an absorbing surface, except for the small fraction of quanta, only in the higher frequency region, which spend their energy on photo-electric effects. And they have properties in common with other kinds of waves: reflection, refraction, diffraction and interference. But these radiations differ greatly in their reactions with matter from one region of the spectrum to another; and in practice we have to use quite different methods of production and detection for different regions.

The things we can demonstrate make a very patchy story. The three binding characteristics – the velocity, the electromagnetic nature of the disturbance, and the quantum packaging – could only be demonstrated directly over limited regions. In ordinary teaching we have to omit the demonstrations. At most we describe them briefly.

We can make measurements of wavelength over a considerable range of frequency; but even then when we look into details we find that we have *calculated* the frequencies from electrical data, or assumed that the marks on an oscilloscope's sweep system are correct – so all pupils have seen is some different wavelengths which we state belong to different frequencies of something that we state is the same kind of waves.

In practice, our teaching here must be largely by assertion, with some pieces of description to support it. And that is no worse than the similar treatment we have adopted for atoms from the earliest year of our programme. As long as we know we are teaching largely by assertion and do not let too much of our teaching take that form, we shall make good progress in giving a perfectly fair picture of the physical world. But the moral here is that we should not spend long on the electromagnetic spectrum, since we are only going to describe the properties of some portions of it. We should give a quick survey and post up a large chart for the rest of the year. The notes that follow offer some suggestions and commentary.

TEACHING OF ELECTROMAGNETIC SPECTRUM

Our Eyes are Limited in Range

Whenever pupils are shown a spectrum they should be reminded that it probably extends far outside the visible region, at each end. Teachers who have long been familiar with this idea may find it difficult to understand why it is not obvious to pupils. The following 'thought experiment' is helpful: T

> 'Suppose your eyes were covered with a sheet of green glass – permanent green spectacles. What would you see? ... Well, what does happen to green light, red light, etc., on hitting green glass?'

(If necessary, the teacher should throw a demonstration spectrum on the screen and show the effect of interposing a green filter, then a red filter.)

> 'If you send white light through green glass, only the green part of the spectrum gets through.
>
> 'Suppose you shine white light on a piece of white paper and let the scattered white light go through green glass to your eye. You will receive only green light and the paper will look green. ... Well, yes, after a time you will get so used to seeing nothing but green that you will think that piece of paper white after all – you may begin to call all bright green things "white".
>
> 'If you shine white light at a piece of *green* paper, it will scatter only green light out of that white light, and it will look as bright as white paper.'

If pupils are surprised at that, the teacher should again throw a spectrum on the screen and show a piece of green paper in various parts of the spectrum.

(The crinkly green paper used for Christmas decorations can be obtained in a good green. Otherwise, it is advisable to take red and green filters to a draper's and look at samples of ribbon and cloth through them until one finds material which looks very bright through a green filter and practically black through a red filter. That will not be very convincing unless the green material is dyed a fairly pure green.)

It will help if we show green paper or cloth in a patch of light from a lantern with a filter in front of its projection lens. With a red filter, green material will look black; but with a green filter it will look bright.

'If you wore green spectacles permanently, what would you see if you looked at red things, green things, blue things?

'What would you see if you looked at the whole spectrum falling on white paper? *What would the range of the visible spectrum be in that case?*'

(The teacher should sketch a spectrum on the blackboard, colouring it roughly with coloured chalks, and then ask for the similar picture if everyone wore green spectacles.)

'Now take those green spectacles off. And instead of seeing just a narrow "visible spectrum" ranging from yellowish-green light to bluish-green light, you see what we call the full spectrum from red through green to blue to violet.

'How do you know that you have not still got some spectacles on, but this time spectacles that let through a broader band, all the way from what we call red to what we call violet? How do you know there is no "invisible light" arriving on the screen beyond the red and beyond the violet?

'In fact, there is; but the reason why we do not see it is not that we have limiting spectacles in front. It is the retina at the back of our eye that is limited, and unable to detect or interpret the light in the infra-red or ultra-violet.

'What else does light do, in addition to giving our eyes a sense of brightness?

'All light, streaming along and arriving at some surface, warms up that surface *if it stops there*. Which surface will stop light better, black paper or white? If you shine strong light on a piece of white paper and on a piece of black paper, you will in fact find the black paper getting a little warmer.'

(Unfortunately, a simple experiment, like the class experiment with radiation suggested for Year II, will show that common white paper is *black* in the infra-red region. If one holds the back of one's hand near a glowing electric heater and feels the warmth as the radiation is absorbed, one finds that pink skin, skin blackened with soot, skin coated with white paper pasted on, all record much warming, showing they are absorbing a lot of the radiation they receive. A coating of aluminium leaf reflects the radiation and very little warming is felt.

If we could repeat that with an intense beam of, say, green light, we should find a black surface being heated, but white paper would just scatter the green light and remain almost as cool as a bright metal surface. This experiment is not possible as a demonstration because such a small fraction of the total energy-flow from an ordinary light-source falls in the visible green.)

'The heating when light is stopped gives us a way of detecting light even if it is "invisible light". We let the light fall on a small heating-detector, which we paint black. We form a spectrum with white light from a very bright lamp filament, and we place the detector in, say, the green of the spectrum. It shows a little warming; and as we move it along the spectrum from blue to green to red, the warming increases.

'That does *not* mean that red light has a special heating property. It only means that there is more red light than green light in the white light from this particular lamp.

'Now, when we move on beyond the red into the infra-red, we find still more heating. That suggests there is some kind of "light" arriving there, though our eyes are unable to detect it. Furthermore, we know there is a lot of it because we find there is a lot of heating.'

At this point teachers are likely to meet the idea of special 'heat radiation' all over again – it is a misconception that arises very easily in early teaching. There is, of course, no such thing as a special heat radiation that is better at heating a black surface than, say, green light.

To say that there is would be rather like saying that there is a specially efficient kind of head-damager which is round and covered with red leather – irregularly shaped rocks being less effective. Families living at the edge of a cricket ground might well evolve such a view simply because the only stray missiles that hit their children's heads were round red balls. Families living on a mountain slope would blame irregular bits of rock instead. The choice of description for head-damagers would be based on what is most frequent rather than what is most damaging.

We may use the old riddle, 'Why do white sheep give more wool than black sheep?' The answer 'Because there are more of them', applies to infra-red radiation. The near infra-red has a much bigger energy-flow (in watts per square centimetre) than even quite a wide band of green, in the spectrum of light from a white-hot tungsten lamp filament.

In light from the Sun, however, we should find the energy-flow larger in the green. (These statements are scientifically careless because we cannot really specify energy-flow until we specify the width of band we shall consider, such as a certain range of wave-length or of frequency.)

In pursuing the spectrum out into the infra-red, we find the energy-flow increasing until, when we get far out, there is a sudden drop to practically nothing. That drop is due to the 'cut-off' of the glass used in lenses and prism (and in the bulb of the lamp itself). Beyond that cut-off, glass itself is black; so no wonder we see no further spectrum.

At the ultra-violet end, our blackened detector of energy-flow will still tell us the truth. But, just as there is much smaller energy-flow in green than in infra-red, there is still smaller in ultra-violet; in fact, too little for a demonstration detector to show, with any ordinary source. Therefore, we must use another type of detector which is specially sensitive to ultra-violet light. That is, photo-graphic film or fluorescent material or a photoelectric cell. Each of those make use of individual quanta of ultra-violet radiation to eject an electron from some atom (or at least move it to an excited level) and the large quanta of ultra-violet light are able to make much more striking changes of that kind than the smaller quanta in the visible spectrum. We are no longer measuring energy-flow proportionally: we are magnifying the violet and ultra-violet region enormously.

We might illustrate this change of detector by asking pupils to put on their green spectacles again and then imagine one of them has red spectacles. What would he see instead? Would the fact that he disagreed with the rest mean that he was wrong? Would it mean that there is no light in the green, as he would claim? And so on.

Using fluorescent material, we can show pupils that something is arriving in the ultra-violet region.

If there are facilities for developing photographic film, we should place that there also and show that something reaches it in the ultra-violet region.

The Full Electromagnetic Spectrum

Then the teacher should sketch the full electromagnetic spectrum, labelling infra-red and ultra-violet and letting those extend on outward. In the far infra-red there is an overlap with wireless waves. In the extreme ultra-violet there is an overlap with X-rays which in turn overlap with gamma rays. These extensions are pure assertions so far as pupils at this stage are concerned.

Even if special apparatus is available to demonstrate centimetre waves, or to show electrical oscillations in frequencies ranging from a few cycles a second to many millions, we should only be unrolling new mysteries and making further assertions by showing those experiments at this point. They belong to A-level or to special teaching for an unusually able group.

Nevertheless, pupils should have, even though it is given by assertion, a picture of something we call the spectrum, spreading far beyond the limited range which our eyes can see.

A permanent chart of this on the wall would be very helpful. It is part of a modern man's general knowledge of nature.

Calling this spectrum 'The Electromagnetic Spectrum' is probably necessary, since that has long been what we know about these waves, but the word 'electromagnetic' represents one more assertion that we can justify convincingly, at least for light.

Appendix C

BOHR'S ATOM MODEL AND A MODERN VIEW

When physicists were forced by experimental evidence to adopt a wave : particle view, Bohr's model, which had already run into some difficulties, was modified so greatly that we no longer speak of it as a good picture. Yet we still discuss atoms with many a phrase from Bohr's picture. And that picture was indeed the parent of modern atom models. Here is a brief description of Bohr's move in making his model.

In devising his model, Bohr did not simply discover and announce new facts, but rather announced, without much explanation, those rules which he could see were necessary to effect a working compromise between classical theory and the facts of experiment. He said, in effect, 'the Rutherford atom model is true, and the classical laws of motion and electrical forces apply to it'. But then he added some new rules which expressed clearly the conflict that physicists had been facing uneasily. (Remember the waiter who said apologetically to the customer, 'the Chef says he is very sorry; your soup *was* dishwater'.) Physicists knew clearly that, on the basis of reliable classical laws, a static version of the Rutherford atom with electrons at rest *could not be in stable equilibrium*;‡ and, equally clearly, a dynamic or planetary model *could not last* because it would radiate electromagnetic waves and collapse in a very short time. Bohr simply stated: 'Atoms *do* have electrons moving in orbits which are stable and do not radiate.' He gave no reason for that – it was simply an honest statement that the emperor has no clothes.

Then Bohr gave a rule which fitted the experimental facts of line spectra: 'Although the atom has stable states, an electron can change from a stable state of higher energy to one of lower energy, emitting the difference as a single quantum of radiation.' That meant that, if we know the energies of the atom in those two states, we can predict the frequency of the spectral line emitted; or, in reverse, we can obtain information concerning energy levels from spectral lines.

Bohr added a rule that specified the stable orbits. It was a rule concerning quantization of angular momentum that is still useful in modified form; but it came at the time as an unexpected empirical rule. Bohr gave a hint of justification by linking it to classical predictions for extreme cases (very large orbits) but it remained essentially an *ad hoc* rule.

‡ No group of particles exerting *only* inverse-square forces can remain at rest in *stable* equilibrium. This is *Earnshaw's Theorem*, worth remembering in our teaching; but not something to teach at this level.

Physicists were glad to take and use Bohr's rules, which proved very fruitful in predicting patterns of spectra, in giving estimates of the size of atoms, and in affording interpretation of X-ray spectra, ionization potentials, etc. But to some younger scientists, the need for the rules did not seem so pressing, and so became something to be learned by rote.

STATES AND ENERGY LEVELS

In our present view, the idea of stable states, each a definite energy level, remains; and so does the *general* Rutherford atom picture. A photon of each spectral line is emitted when the atom changes from one state to another, the electron switching from one energy level to another. But we now have a good, simple 'reason' for the stationary states; they are defined by stationary wave patterns of the electron's wave system.

For a circular orbit a stationary wave is embroidered on the circumference. A whole number of wavelengths must fit into the circumference. We feel it is reasonable to expect such a stationary wave to remain stable without growing or shrinking or running away. (And yet the electron wave patterns in our speculations are *not* mechanical waves, so perhaps we are unwise to think of them as things that could run away in any case.)

We can combine that condition for stationary waves with the expression that connects a moving electron's wavelength with its momentum – (wavelength) = h/(momentum). Then we obtain the very condition defining stable states that Bohr expressed in his surprising rule for orbits; but now the condition seems plausible. We should still have to apply classical expressions for potential energy and kinetic energy before we can calculate the energy of the electron at each level; but if we do that we can predict the lines of the simple hydrogen spectrum.‡

However, quantum mechanics describes the structure and behaviour of atoms with a much greater variety of wave patterns, in which any close correspondence with the old Bohr model disappears. Even for hydrogen, the picture of waves on circular 'orbits' proves incomplete and partially incorrect.

‡ We can calculate the radius of a hydrogen atom on this modified view, without using an expression for energy. We combine the wavelength : momentum rule with the requirement that the circumference of the normal stable orbit of the electron is one wavelength. And we add the orbit rule: mv^2/r = electrostatic attraction between nucleus and electron.

We are left with the view that the atom is hollow with a massive nucleus carrying a positive charge (equal to the atomic number * times one electron charge); that the atom has electrons that 'reside' in regions outside the nucleus (the atomic number gives the number of those electrons); that the whole system has many energy levels, the lower ones (with greatest negative potential energy) occupied by electrons, but an unending series of higher ones unoccupied. And when the atom changes from one state to another, an electron switches from one energy level to another; and the difference of energy is emitted as a photon of frequency given by (energy-difference)/*h*.

This fits with ('accounts for' ?) our knowledge of stability and elasticity of atoms. At low energies (a few hundredths of an electron·volt at room temperature) gas molecules make perfectly elastic collisions. Electrons with energies up to a few e.v bounce off gas molecules elastically. Only at higher energies are there inelastic collisions; and then the energy taken for excitation (or for ionization) is a *definite* amount – the difference between two definite energy-levels of the atom.

We still classify the levels, both occupied and unoccupied, by number systems which were developed for the Bohr orbit model and are now related to the quantum rules which guide our picture-making for atoms today. Those number systems are of vital importance to spectroscopists and are of value in some parts of chemistry – at least so far as the more loosely bound electrons are concerned – but learning them or even their classification does not seem to us to make valuable progress in a pupil's understanding of modern physics.

SUBJECT INDEX

NUFFIELD FOUNDATION SCIENCE TEACHING PROJECT PHYSICS SECTION

The physics programme was inaugurated in May 1962 under the leadership of Donald McGill. It suffered a severe setback with his tragic death on 22 March 1963, but those who were appointed to continue the work have done so in the spirit in which he initiated it, and in the direction he foreshadowed.

Consultative committee

Chairman Professor Sir Nevill Mott, F.R.S.
Professor C. C. Butler, F.R.S.
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Professor J. C. Gunn
Sister Saint Joan of Arc
Professor R. V. Jones, F.R.S.
W. K. Mace
J. M. Osborne
Dr C. W. W. Read
The Rev. R. G. Wickham

Organizer

Professor E. M. Rogers

Associate organizers

J. L. Lewis
E. J. Wenham

Assistant organizer

D. W. Harding

Area co-ordinators

Sister Saint Joan of Arc
Miss D. J. Alexander
Miss A. Lipson
Dr H. F. Boulind
B. R. Chapman
D. C. F. Chaundy
M. J. Elwell
L. Ennever
R. C. Hardwick
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C. L. Williams

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Associate organizers J L Lewis E J Wenham
Assistant organizer D W Harding

Other Nuffield Physics publications

Teachers' guide I
Teachers' guide II
Teachers' guide III
Teachers' guide IV

Guide to experiments I
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