

Unit K

ENERGY AND ENTROPY

Jon Ogborn

Department of Science Education, University of London, Institute
of Education

PLAN OF THE UNIT *page 266*

INTRODUCTION *268*

THE PLACE OF THE UNIT IN THE COURSE *269*

LIST OF SUGGESTED EXPERIMENTS AND DEMONSTRATIONS *270*

Section K1 **WHAT IS ENTROPY?** *271*

Section K2 **ENTROPY, ENERGY, AND TEMPERATURE** *278*

Section K3 **THE BOLTZMANN FACTOR** *289*

Section K4 **EXPLOITING ENTROPY CHANGES** *302*

Suggested time allocation: three weeks

PLAN OF THE UNIT

Section K1

What is entropy?

Qualitative idea of counting number of ways W

$$S = k \ln W$$

Shuffling particles between two halves of a box

$$W''/W = 2^N$$

kinetic theory; the mole;

Boltzmann constant

(Unit A)

Expansion of a gas:

$$\Delta S = kN\Delta(\ln V)$$

$$pdV = TdS \text{ (reversible isothermal)}$$

Section K2

Entropy, energy, and temperature

exponential function
(Units B and F)

Shuffling particles on energy levels

Exponential distribution with constant fraction f of particles in level one higher

$$W''/W = f \text{ for removing one quantum}$$

$$\ln f = -\epsilon/kT$$

$$f = e^{-\epsilon/kT}$$

energy levels: Unit L,
'Waves, particles, and
atoms'

Thermal transfer of energy

thermal conduction
(Unit G)

▶ entropy change = energy change/temperature

Thermodynamic temperature

Section K3
The Boltzmann factor

Activation processes, $e^{-E/kT}$:
vapour pressure of water
current in a thermistor
viscous flow
creep; rate of reaction

▶ charge carriers (Unit B)

▶ materials (Unit A)

▶ chemistry, biology

Importance of Boltzmann factor

Section K4

Exploiting entropy changes

▶ energy sources (Unit G)

The Second Law
inefficiency of engines

Further applications:

reversible compression of gas
equilibrium
concentration cell

▶ chemistry

INTRODUCTION

Few would deny the importance of the Second Law of Thermodynamics and the concept of entropy, but few would also deny their difficulty. We feel that experience since such work was first introduced into the Nuffield Advanced Physics course, even though the approach is still controversial, makes a sufficient case for a carefully limited and compact treatment.

Time is short, and the Unit aims to do no more than introduce the ideas and to offer examples of their use. The treatment is from a statistical point of view, but deliberately minimizes the amount of statistical or combinatorial mathematics. Instead, statistical 'games' are used, played with dice and counters, and on a microcomputer.

The Unit makes links with work in chemistry, and some with biology and engineering. The Revised Nuffield Advanced Chemistry course adopts rather similar arguments and ideas, but introduces them little by little in various topics, starting quite early in the course. For students taking both courses, this Unit in the physics course may serve to bring together ideas met before in chemistry.

The strategy suggested is to define entropy at the outset, with simple, mainly qualitative, examples. Section K2 makes the link with thermal energy transfer and Section K3 uses the results in the form of the Boltzmann factor. Finally, Section K4 states the Second Law and gives examples of its use and importance.

Section K1 uses qualitative examples to introduce the idea that a macroscopic object can have its particles arranged geometrically or amongst energy levels in many different ways, whilst having the same large-scale properties, regardless of these arrangements. Entropy is defined without explanation as the scaled-down logarithm of the number of arrangements. The section concludes with a dice game in which counters are shuffled between halves of a box, and applies the results to the expansion of gas, to give a first calculation of an entropy change.

Section K2 deals with entropy changes associated with exchanges of energy. Another dice game, shuffling particles on different energy levels, shows that the distribution of particles on levels is exponential, and allows one to look at a simple model of the approach to thermal equilibrium. The constant fraction of particles on any level as compared with a lower level (which is the Boltzmann factor) is interpreted as the ratio of the numbers of arrangements when one quantum is removed. Temperature is seen as deciding the change in the number of arrangements (and so the change in entropy) when energy is added or removed.

Thus the same argument gives
entropy change = energy change/temperature, and
the Boltzmann factor $e^{-\epsilon/kT}$.

Sections K3 and K4 show several applications and extensions of the basic ideas, with a choice of experiments.

Section K3 introduces the idea of an activation process and the meaning and general importance of the Boltzmann factor. Applications offered are the variations with temperature of vapour pressure, of current in a thermistor, of viscous flow or creep, and of rate of reaction.

Section K4 introduces the Second Law as following from a microscopic, statistical interpretation of entropy changes, and applies it to the inevitable inefficiency of heat engines. Further applications, again with experiments to be shared around the class, are about gases, a discussion of equilibrium, and work obtained by exploiting an entropy increase. Teachers may wish to add an account of the specific heat capacity of an Einstein solid.

The work has been pared to a minimum. These difficult ideas need all the time they can get, and it would be easy to make a case for doing more; but to do more would cut too much into time needed elsewhere in the course. Thus this Unit is no more than a very brief introduction, intended to help those who will study the ideas again later to have a first picture of what they are about, and to give those who will not a sight of an unusual, but deep and fundamental, aspect of scientific thinking.

THE PLACE OF THE UNIT IN THE COURSE

This Unit is less dependent on previous knowledge than most, requiring ideas about matter being made of particles, a willingness to accept that energy is quantized, and enough knowledge about gases to recognize $pV = NkT$, work = pdV in expansion, and the Boltzmann constant. Mathematically, logarithms and the exponential function are needed: both can be introduced using the functions on a scientific calculator. As a Unit reaching one main goal or end-point of the course, there is a case for having it rather late. The ideas in the Unit are subtle and sophisticated, and many teachers will want to wait until students are sufficiently able to think in a rather generalized and abstract fashion. Certainly it would be natural for it to come later than Unit G, 'Energy sources'. All this said, the timing of the Unit could be fairly flexible. It has less experimental work than many other Units, and it may be best to place it close to other more empirical work (perhaps alongside the Investigation).

LIST OF SUGGESTED EXPERIMENTS AND DEMONSTRATIONS

K1	Game	Particle shuffling <i>page 273</i>
K2	Game	Energy shuffling <i>280</i>
K3	Computer simulation	Equilibrium distribution of energy <i>281</i>
K4	Computer simulation	Thermal equilibrium <i>285</i>
K5	Experiment	Vapour pressure of water <i>291</i>
K6	Experiment	Current in a thermistor <i>294</i>
K7	Experiment	Viscous flow <i>296</i>
K8	Experiment	Creep <i>298</i>
K9	Experiment	Rate of reaction <i>300</i>
K10	Experiment	Entropy changes in a gas <i>306</i>
K11	Experiment	'Hot ice' <i>308</i>
K12	Experiment	Concentration cell <i>310</i>

SECTION K1

WHAT IS ENTROPY?

Reading for teachers

Teachers may not all feel secure about the material in this Unit themselves, and the treatment in this *Teachers' guide* and in the *Students' guide* is necessarily concise. The following suggestions for reading may therefore be helpful:

ATKINS, *The Second Law*. (A 'popular' but excellent introduction and overview. It presents a very useful elementary computational approach to statistical mechanics.)

GASSER and RICHARDS, *Entropy and energy levels*.

KITTEL and KROEMER, *Thermal physics*.

PSSC, *College physics*.

REIF, *Statistical physics*.

REVISED NUFFIELD ADVANCED CHEMISTRY (Topics 12, 15, and 16).

WYATT, *The molecular basis of entropy and chemical equilibrium*.

Qualitative introductory discussion

The Unit can begin with a qualitative discussion of the general ideas on which the statistical picture of processes of change is built. They are introduced briefly in the *Students' guide*, which may serve as a basis for class discussion.

The central ideas to bring out are:

Molecules don't care All processes happen ultimately by molecules, atoms, electrons, or other particles joining together, coming apart, hitting one another, changing energy, and so on. On the large scale, processes like ice melting or crystals forming look deceptively as if the molecules 'know what they ought to do'. But they don't: all such processes *must* happen purely by the blind operation of chance encounters.

People do care If all the events of the world happen, at the molecular level, purely at random, how can human beings make things happen the way they want them to? How can one chemist make, and another tear apart, the same useful molecule, when these processes are in opposite directions? Why does water sometimes evaporate and sometimes condense? The answer must be that to control events we have to arrange that what we want to happen is what, at random, is going to happen anyway.

Randomness means that molecules try everything The particles of a system can be arranged in many ways, having various positions, energies, and so on, without the system's overall properties being different. A gas with a steady pressure produced by random collisions is an example. A way to think about the molecular level is to imagine counting all the ways the 'insides' of the system can be arranged, and supposing that over time, the particles try them all without preferring one to another.

$S = k \ln W$ If the number of possible arrangements of a system in a given macroscopic state is W , then we define the entropy S to be $k \ln W$. Entropy S is just another way of talking about W .

It is not easy to convey a clear idea of what counting the number of ways W is supposed to mean. The *Students' guide* gives a number of examples, each intended to show a situation where one can imagine counting all the ways particles can be arranged. For concreteness, 'configurational' aspects are emphasized, despite the greater ultimate importance of distribution of particles amongst energy states. Examples suggested are:

- numbers of ways of arranging polymer chains in rubber, taking various paths between the same points;
- number of ways of exchanging water molecules and particles of another substance dissolved in the water;
- numbers of ways of arranging particles amongst different possible definite energies any one particle can have, for the same total energy.

Students may be asked to draw different pictures of such a system. Then W is simply the number of *different* pictures they could possibly draw. Models may also help: chains of paper clips randomly kinked to the same net length for chain molecules in rubber; counters of different colours spread out for molecules in a mixture or solution.

Other useful examples might be the arrangement of molecular magnets for a given degree of magnetization, or the arrangement of defects in a crystal lattice (see questions 6 and 7).

The teacher needs to bear in mind that these examples focus on just one kind of variation at a time. In reality there will be more than one: particles may vary in energy *and* in position or orientation. W is then the number of pictures combining all kinds of rearrangement. A correct general formulation is that W is the number of accessible distinct quantum states of the system: it is quantum mechanics that makes it possible to present finding W as simply a matter of *counting*.

Mathematically inclined students may like to see the binomial expression $\frac{N!}{n!(N-n)!}$ for the number of ways of arranging N objects, n in one condition or state and $(N-n)$ in another. It applies to defects in a lattice or to molecular magnets pointing 'up' or 'down'. No use will be made of this expression in the Unit, however.

Questions

Questions 1 to 7 give introductory practice in thinking about and working out numbers of possible arrangements. Questions 2 and 3 are simple ones about cards and coins; questions 4, 6, and 7 suggest applications to biology, physics, and chemistry.

CALCULATION OF AN ENTROPY CHANGE: PARTICLES SPREADING OUT

This subsection has three aims:

- to use dice to embody the idea of 'randomness';
- to make an actual calculation of a change in W ;
- to help students deal with expressions like $2^{6 \times 10^{23}}$.

GAME

K1 Particle shuffling

ITEM NO.	ITEM
1153	6 counters
1155	die
	graph paper

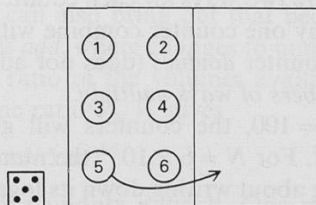


Figure K1

Shuffling counters between two halves of a box.

Students number six counters 1 to 6, and put them all in one half of a box drawn on graph paper. They throw a die, and move the counter whose number comes up into the opposite half of the box. A counter moves to the half it does not occupy, whenever its number is thrown.

Students keep a running tally of how many counters are in each half, and find the average number in each half. An average over the whole class will be close to three in each half.

The first point to bring out is that sooner or later the counters will go through every possible arrangement between the two halves. Secondly, no arrangement is preferred to any other. Thirdly, *because* of the second point, an average over all possibilities is close to an equal division of counters between halves. Pure chance produces a definite average equilibrium behaviour.

It is tempting to dwell on the different numbers of arrangements having various numbers of counters in each half. As all the arguments below and in the rest of the Unit will concern the *total number* of arrangements, not the pattern having the *largest* number, this is unnecessary and takes time (especially working out all the possibilities). The simple view taken will be that overall behaviour is determined by an average over *all* arrangements.

It is quite easy for a student to program a microcomputer to play the game, with variations (especially varying the number of particles, and perhaps the relative 'size' of each part of the box). It is also easy for such activities to take up too much time.

In a large class, or in a long enough run, someone will have all six counters go back into the half of the box where they started. This may be expected 1 in 64 times, because only 1 out of the 64 possible arrangements has all in a given half. Rare events do happen, as rarely as expected.

The main result is that there are 2^N possible different ways for N counters to be arranged between the two halves of the box. Doubling the space available increases the number of ways by the factor $W'/W = 2^N$. In the present case,

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64.$$

There are two ways for each counter to be placed, and since the two ways for any one counter combine with *all* possibilities for the others, adding a counter *doubles* (does not add 2 to) the previous number of ways. *Numbers of ways multiply*.

For $N = 100$, the counters will go back in one half 1 time in $2^{100} \approx 10^{30}$. For $N = 6 \times 10^{23}$ the number $2^{6 \times 10^{23}}$ can be approached by thinking about writing down its logarithm. Regarding the logarithm to base 10 as roughly the number of digits in a number written out in full, it can be shown that this number would stretch beyond the Solar System even if written in atom-sized digits.

Questions

Questions 3, 7, and 10 give practice in the use of the factor 2^N . Questions 8 and 9 are about numbers of ways multiplying. Questions 11 and 12 deal with the value of 2^N for $N = 100$ and for N equal to the number of molecules in a sample of gas.

Expansion of a gas

In the definition of entropy already given, the presence of the Boltzmann constant k , with units of energy divided by temperature, must seem arbitrary. Simple arguments about a gas may make it seem less so. The idea is to show that if k is included in the definition of entropy, the work pdV in a reversible isothermal expansion is equal to TdS , where dS is the entropy change of the expanding gas.

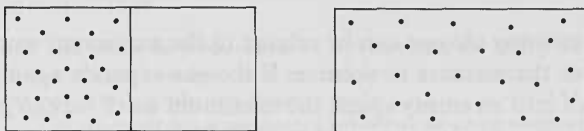


Figure K2

Doubling the volume of a gas.

One can calculate the entropy change for a doubling of volume. If there is one mole of molecules, $N = 6 \times 10^{23}$, then from

$$W'/W = 2^{6 \times 10^{23}}$$

and the definition $S = k \ln W$, one gets

$$\Delta S \approx 6 \text{ J K}^{-1}$$

(since $\Delta S = k \Delta \ln W = k \ln W'/W$, and using $\ln 2^{6 \times 10^{23}} = 6 \times 10^{23} \ln 2$).

It is worth emphasizing how taking the logarithm of W'/W and then scaling down by $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$ produces the much more comfortable value 6 J K^{-1} for ΔS in place of the disagreeably large number W'/W . One can also bring out that because the logarithm is used, entropy changes *add*, where changes to numbers of ways *multiply*.

Generalizing the ratio of the volumes available to the gas, previously 2, to the volume ratio V_2/V_1 gives

$$\Delta S = kN \ln(V_2/V_1) = kN \Delta \ln V$$

This step can be made unduly difficult. One way of presenting it is suggested in question 13, but it may be no worse simply to make the generalization directly without much comment.

To reach $pdV = TdS$, it may be best to work, as below, entirely in infinitesimal changes, and thus to avoid integrating pdV over varying pressures. For a small change, $d(\ln V) = dV/V$ (best regarded as the defining property of a logarithm, not as an accidental approximation), so

$$dS = kNdV/V$$

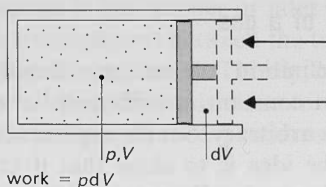


Figure K3
Expansion of a gas.

The entropy change can be related to the maximum work we could get out of the increase in volume. If the gas expands against a piston, instead of into an empty space, the maximum work we can get from it if the temperature stays the same is pdV . But

$$pV = kNT$$

so

$$pdV = kNTdV/V$$

which is just the expression for the entropy multiplied by the temperature, giving

$$pdV = TdS$$

Questions

Questions 13 and 14 deal with $dS = kNdV/V$ and $d(\ln x) = dx/x$. Questions 15 and 16 give practice in calculating entropy changes of gases. Question 17 is a derivation of $pdV = TdS$, but uses a graphical method.

Reading

A rather extensive elementary discussion of entropy changes in an expanding gas is to be found in *PSSC College physics*, Chapters 21 and 22, and this could provide teachers with a secure background from which to present the much more compact treatment suggested here. Valuable more advanced background is REIF, *Berkeley Physics Course Volume 5, Statistical physics*, Chapter 1.

The point of this work is to show that in one particular case, an entropy change is equal to an energy change divided by a temperature. In the present case, the temperature is clearly measured on the ideal gas scale: in the next Section temperature will be approached from a more fundamental point of view. From that point of view, we can ask *why* an entropy change should be an energy change divided by a temperature; here we see only that it is so in one special case.

The fact that the Boltzmann constant k cancels out is no accident. Since

$$pV = NkT$$

if the scale of T were changed, the value of k would change. Of course the scale of T is arbitrary, and therefore so is the value of k . If we had

$$S = k' \ln W$$

using some other constant k' in the definition of entropy, then for the gas we would get

$$pdV = (k/k')TdS$$

and TdS would be proportional to, not equal to, the energy change pdV . Thus the choice of k in the definition of entropy leads to the simplest result relating energy and entropy, which is why the choice is made.

The argument has some further subtleties, of which teachers need to be aware. The entropy change of the gas is $kN\Delta V/V$, and this is true whether the expansion is into a vacuum or not. If the expansion is a reversible one against a piston, the entropy change of gas *plus surroundings* is zero. The thermal transfer of energy Q out of the surroundings reduces their entropy by Q/T , exactly equal to the increase in entropy of the gas. If the gas is compressed, the two entropy changes both change sign but are still equal in magnitude. Thus although a gas will not spontaneously compress itself, thereby lowering its entropy, it can be compressed with an equivalent entropy increase in the surroundings.

The expression 'thermal transfer of energy' is used to avoid any identification of 'heat' with the energy of motion of the gas molecules, that is, with their internal energy. In the present case there is a thermal transfer of energy into or out of the gas but it does *not* get hotter or cooler, nor does its internal energy change.

Revised Nuffield Advanced Chemistry

The chance behaviour of molecules is dealt with in Topic 3.

SECTION K2

ENTROPY, ENERGY, AND TEMPERATURE

Overview

One aim of this Section is to show why there is a relation between entropy changes and energy changes which involves the temperature. But the more important aim is to create an image of particles sharing energy in many ways, always in a flux from one possible state to another. This entropy is a crucially important part of the total entropy change in most processes, and so is a vital aspect of what drives events in the direction in which they actually occur. Rigour of abstract argument ought if necessary to be sacrificed or reduced in emphasis for the sake of this more qualitative goal.

The whole argument can be summarized as follows. A warm object has entropy because its particles can share the total available energy in many ways. Adding energy to be shared increases the number of ways; removing energy decreases it. A statistical game (or of course a suitable theoretical analysis) shows that for a simple system with equally spaced energy levels, removing one quantum of energy multiplies the numbers of ways of sharing out the energy by a constant fraction f . The value of f depends on how much energy the system has already: removing a quantum from a system with little energy has a larger effect on the number of ways than removing a quantum from a system with more energy. This is why energy goes from hot to cold: removing energy from a hot body reduces W , but by a smaller factor than giving this energy to a cold body, so producing a net increase in W and in entropy. The fraction f is thus related to the quantity we call temperature. Since f is the ratio of numbers of ways when one quantum is removed, the corresponding entropy change $\Delta S = k \ln(W'/W)$ is $k \ln f$. T is thus defined such that

$$k \ln f = -\varepsilon/T \quad \text{or} \quad f = e^{-\varepsilon/kT}$$

where ε is the energy level spacing. Temperature can be regarded as the rate of change of energy with entropy; more fundamentally, $1/T$ can be seen as the rate of change of entropy with energy.

Reading

Teachers who need to review the arguments at greater length than is possible here, or who want alternative viewpoints, could consult:

POWLES, *Particles and their interactions*. Chapters 11 and 12.

GURNEY, *Introduction to statistical mechanics*. Chapter 1.

OPEN UNIVERSITY Course ST285, *Solids, liquids and gases*.

SHERWIN, *Basic concepts of physics*. Chapter 7.

BENT, *The Second Law*. Chapters 20–22.

WALTON, *Three phases of matter*. Chapters 6 and 8.

TABOR, *Gases, liquids, and solids*. Chapters 3 and 5.

FLOWERS and MENDOZA, *Properties of matter*. Chapter 4.

FEYNMAN *et al.*, *The Feynman Lectures on physics Volume 1*. Chapter 40.

The last two argue for the Boltzmann factor in terms of the distribution of molecules in an ‘atmosphere’ at constant temperature in a gravitational field. This simple argument makes it less clear that the Boltzmann factor represents the chance of extracting energy from a heat bath, and is thus related to an entropy change.

Teaching suggestions

The following Sections take the argument relating energy, entropy, and temperature step by step. Appendix II (page 423) discusses difficulties and background issues. Important steps in the arguments are italicized.

PARTICLES AND ENERGY LEVELS

Energies of bound particles are quantized, so one can imagine counting the numbers of particles in each possible state. Purely for simplicity, we consider equally spaced energy levels.

Quantization of energy belongs to Unit L, ‘Waves, particles, and atoms’: if this is yet to come one will have to refer forward to it. The choice of equally spaced levels can be presented as merely the simplest possible choice, or students can be told that it is a fairly good model for the oscillating atoms in a monatomic solid (Einstein solid). The important thing is to see what it might *mean* to count arrangements of particles on levels; however, the main result from this simple system does in fact generalize to more complicated ones.

Questions

Questions 18 to 20 are about the idea of arranging particles on levels, for very simple systems.

More energy to share means more ways of sharing energy. Cases studied should be kept simple. The main point is that the number of ways rises rapidly as either the number of particles or the total energy increase (figure K4). A useful device may be to pass pictures of simple systems round the class, asking each student to try to add to the set another arrangement with the same energy which has not yet been drawn. It may also be helpful to show the computer simulation K3, with just a few particles sharing a little energy, just to watch the particles hopping between levels and moving from arrangement to arrangement.

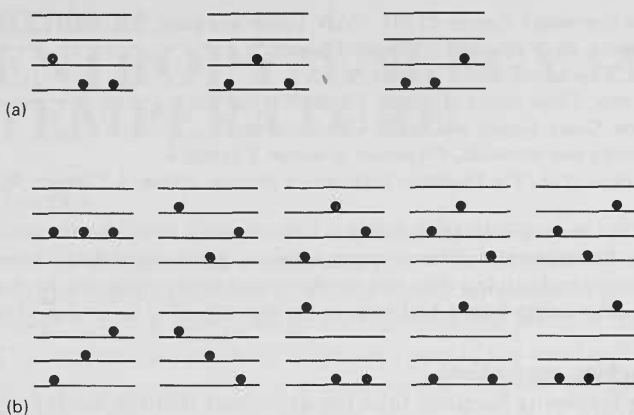


Figure K4

Particles on energy levels.

(a) The three ways for three particles to share one quantum.

(b) The ten ways for three particles to share three quanta.

ENERGY SHUFFLING

The distribution of particles on levels, if they share energy at random, is exponential in form (when there are many particles sharing appreciable energy).

GAME

K2 Energy shuffling

ITEM NO.	ITEM
1153	20–50 counters
	<i>either</i>
1155	2 dice
	<i>or</i>
	20-sided die
	<i>or</i>
	means of drawing numbers
	<i>or</i>
	many-sided spinner
	<i>or</i>
	computer programmed to pick counters at random
	graph paper

Students should rule about 10 or 15 ‘energy levels’ on a sheet of graph paper, and arrange a number of counters on them. Putting all counters on the first level above the lowest is a simple way to begin. For a first attempt, six counters representing particles is enough.

The shuffling rule is:

Throw a die, or in some other way pick one counter at random, as a candidate for demotion by one level. If this counter is on the lowest level, try again. If this counter is above the lowest level, demote it by one level. Again pick a counter at random, but this time promote it one level. Repeat the whole process.

To shuffle more than six counters, there are various options. 36 counters can be labelled in groups of six, using two six-sided dice, one to pick the group and one to pick a counter within a group. Better is to use a die with as many sides as there are counters (20-sided dice are obtainable from some games shops). With more risk of bias, simple many-sided spinners, or numbers drawn from a box, can be used.

Lastly, especially if a student will write *and test* the program, a computer could display pairs of numbers to select counters for demotion and promotion. The following will work for the BBC microcomputer. Each pair of numbers appears on the screen for a time P centiseconds.

```
100 P = 200
110 INPUT 'NUMBER OF PARTICLES', N
120 REPEAT
130 D = RND(N): U = RND(N)
140 PRINT D; 'DOWN': PRINT U; 'UP'
150 T = TIME: REPEAT: UNTIL TIME > T+P: CLS
160 UNTIL FALSE
```

The class will find that the higher the level, the less often a counter reaches that level. If the whole class starts from the same total energy, using 20 or more counters, the ratios of average numbers on pairs of levels should be roughly constant.

A computer version of the game is needed, to see the difference made by different starting points with the same or different total energy, the behaviour of a time-averaged distribution, and the flow of energy from one such shuffling system to another.

COMPUTER SIMULATION

K3 Equilibrium distribution of energy

microcomputer
program 'Quantum shuffling'

A video monitor, rather than a television set, is highly desirable for clarity of the display. Switch on (load BASIC if necessary), and LOAD and RUN the program 'Quantum shuffling' (Software for Nuffield Advanced Physics).

The program presents a set of energy levels, and a choice of the number of particles to be used. Particles are put on levels by pressing number keys. (The particles can be split into two groups, but here use one.) The display shows the particles on the levels, an instantaneous (and as an option a time-averaged distribution), and a measure of the mean energy per particle.

A simple start, close to the paper and pencil game, is to put 50 particles all on level 1. Observe the roughly exponential form of the instantaneous distribution after a number of moves, though with considerable fluctuations. Then try with 100, 200, or 255 (the largest number allowed), again starting with all on level 1. The exponential form becomes clearer. The fluctuations are not an undesirable or unimportant phenomenon: they demonstrate visually that the system is dynamic, and that the general form of the distribution arises by chance, not by design.

Note that it is the number of particles on a particular energy level (plotted as abscissa, i.e. 'horizontally') which varies exponentially with energy (plotted as ordinate, i.e. 'vertically'). In the exponential curves seen earlier in the course, the exponentially varying quantity (e.g. charge on a capacitor, number of undecayed nuclei) has been plotted as ordinate.

<input checked="" type="checkbox"/> S Shuffle 993	<input checked="" type="checkbox"/> A Average OFF ON
<input checked="" type="checkbox"/> Q Quit	<input checked="" type="checkbox"/> P Probability OFF ON

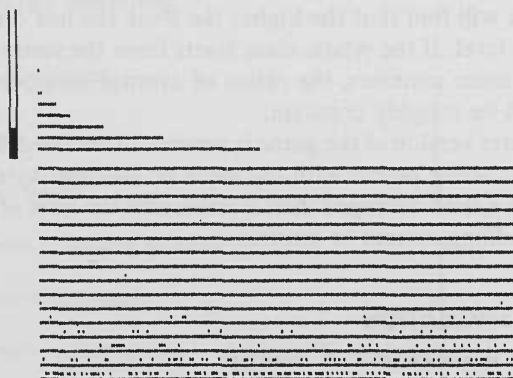


Figure K5 (part)

Screen display from the computer program 'Quantum shuffling', showing a simulation of energy shuffling between 200 particles on energy levels; above that is total number on each level and average energy per particle.

(a) Instantaneous distribution after nearly 1000 moves for 200 particles all initially on level 1.

<input checked="" type="checkbox"/> Shuffle	4006	<input checked="" type="checkbox"/> Average	OFF ON
<input checked="" type="checkbox"/> Quit		<input checked="" type="checkbox"/> Probability	OFF ON

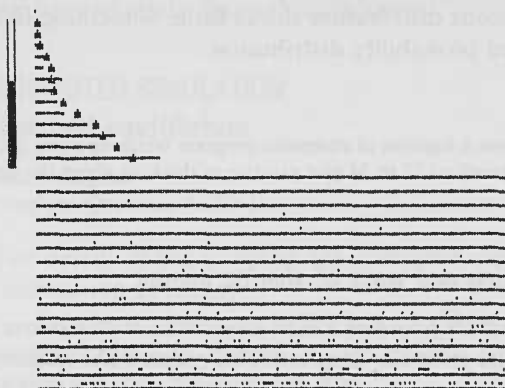


Figure K5 (part)

(b) 200 particles initially all on level 3. The instantaneous distribution shows random fluctuation about the average distribution over the last 2000 moves, shown by ▲

With all particles starting at level 1 (one quantum per particle) the ratio of numbers in adjacent levels is around 2 to 1. To compare different total energies it is convenient to use a display of two systems side by side. Both can have (say) 100 particles, but different initial total energies chosen when the particles are put on the levels. Good examples to look at include:

all on level 1 (as in figure K5(a));

all on level 2 (twice as much energy);

about half on level 1 and half on level zero (half as much energy).

Note that large initial energies (e.g. all on level 5) will take some time to reach the exponential form, and will transiently give the impression that the outcome is a peaked distribution. It may also be useful (perhaps by putting particles on levels at random), to start with an initially uniform distribution over levels.

The larger the initial energy per particle, the steeper the final distribution, with a greater chance of finding a particle at a high level.

That it is energy *per particle* which matters can be brought out by comparing systems with different numbers of particles, say 100 and 200, both from similar initial states. The ratios of numbers of particles in adjacent levels are similar.

The distribution-averaging facility can be used to show how the long-run average of numbers of particles in levels is rather accurately exponential in form, with fluctuation effects slowly averaging out. The display of probability of finding particles at various levels is useful to show that there is pattern in the behaviour of even a few particles, where the instantaneous distribution shows little. Something like 10 particles gives a graded probability distribution.

Questions

Question 21 shows a segment of computer program which operates the energy shuffling rule. Questions 22 to 24 give practice in thinking about the exponential distribution.

Energy shuffling and ways of sharing energy

A particle is rarely found at a high level because to rise one level, it must take a quantum from the rest, and this reduces the number of arrangements of the rest since they share less energy. Conversely, a particle falling one level gives a quantum to the rest, and increases the number of ways they share energy.

This is the crucial move in the argument. The distribution is exponential because each removal of a quantum reduces the number of ways of sharing energy by a constant factor (if there are many particles and many quanta). Thus the chance of finding a particle in a particular level is a constant fraction f of the chance of finding it in the next level down. The fraction f is better known as the Boltzmann factor.

The fraction f has a direct interpretation in terms of a ratio of numbers of ways. *If taking one quantum from a set of particles reduces the number of ways from W to W' , then*

$$f = W'/W$$

where f is also the fraction

$$f = \frac{\text{chance of a particle on a given level}}{\text{chance of a particle one level lower}}$$

Since the entropy change is $k \ln(W'/W)$ the entropy change when one quantum is removed is just $k \ln f$.

Note that since f is less than unity, $\ln f$ is negative and the entropy falls. Adding one quantum multiplies W by $1/f$ (more than unity) and the positive entropy increase is $k \ln(1/f) = -k \ln f$.

ENTROPY CHANGE AND TEMPERATURE

We now know the entropy change for adding or removing one quantum of energy ϵ ; we go on to find the entropy change for arbitrary energy changes, and to relate it to the temperature. To introduce temperature, we have to study thermal equilibrium.

COMPUTER SIMULATION

K4 Thermal equilibrium

microcomputer
program 'Quantum shuffling'

For details of the program see computer simulation K3, Equilibrium distribution of energy.

Two systems coming to thermal equilibrium can be simulated by the program shuffling energy in two systems isolated from one another, which are then merged into one.

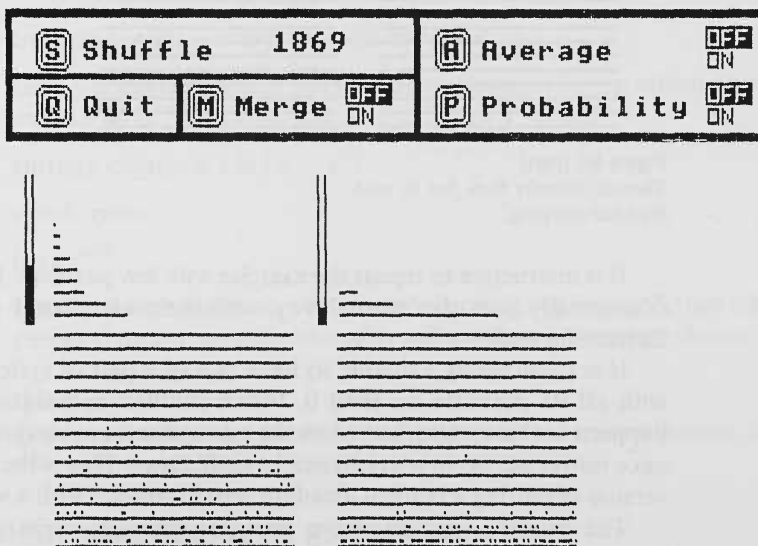


Figure K6 (part)
Thermal transfer from hot to cold.
(a) Isolated.

A suitable pair of systems to look at is one with all particles starting on level 2 in one system, and with half on level 1 and half on level 0 in the other. Run the program shuffling energy within each system

independently until the two distributions are roughly exponential, then 'merge' them and shuffle again. The distributions slowly equalize, and the average energies per particle also become equal. Other pairs of initially 'hot' and 'cold' systems may be tried.

<input checked="" type="checkbox"/> Shuffle 3904		<input checked="" type="checkbox"/> Average OFF ON	
<input checked="" type="checkbox"/> Quit	<input checked="" type="checkbox"/> Merge OFF ON	<input checked="" type="checkbox"/> Probability OFF ON	

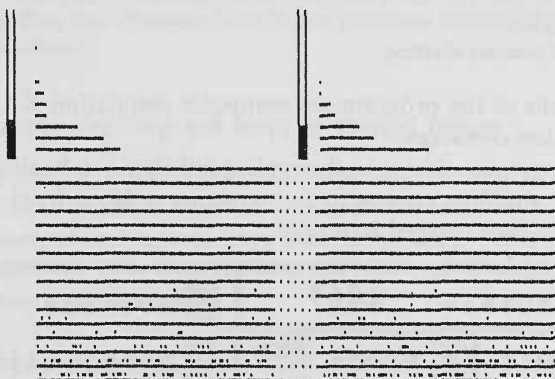


Figure K6 (part)
Thermal transfer from hot to cold.
(b) After merging.

It is instructive to repeat the exercise with few particles. Energy then occasionally goes the 'wrong' way, with the mean energy per particle fluctuating more.

It is particularly valuable to have one of a pair of systems starting with all its particles on level 0. When shuffled in isolation, nothing happens to this system, but when the two systems are merged, energy at once moves across in a dramatically visible way. This is the computer's version of putting a body at absolute zero in contact with a warmer one!

The crucial argument above, about one particle exchanging energy with a set of others, can be illustrated by having two systems, one with just one particle and the other with (say) 20. With the two 'merged', the probabilities for the lone particle come to be the same as those of the larger set.

When energy goes spontaneously, by chance, from one system to another, we say that the first was hotter than the second. When they reach equilibrium, we say that they are at the same temperature.

This statement is, of course, the essence of the 'Zeroth' Law of Thermodynamics, seen from a statistical point of view.

Consider systems A and B, with one quantum passing from A to B. We know that the entropy of A will be reduced by the magnitude of

$$k \ln f_A$$

and the entropy of B will be increased by the magnitude of

$$k \ln f_B$$

Only if $f_A = f_B$ will the entropy changes be equal and the total entropy be unchanged. Then the exchange will happen equally often in either direction and there will be equilibrium. Thus f and the temperature are related.

For energy to go from hot to cold, the reduction in entropy when energy is removed from a high-temperature system must be less than the increase in entropy when energy is added to the low-temperature system. Thus $\ln f$ must be inversely related to temperature.

In the case of a gas, we had

entropy change = energy change/temperature

Here the energy change (remove one quantum) is $-\varepsilon$, which suggests writing

$$\text{entropy change} = k \ln f = -\varepsilon/T$$

which gives

$$f = e^{-\varepsilon/kT}$$

Finally, if each quantum ε produces an entropy change ε/T , then adding energy Q (many quanta) reversibly will produce an entropy change Q/T .

Questions

Question 25 considers one quantum exchanged between two systems. Question 26 asks about entropy changes produced by adding many quanta.

THERMAL TRANSFER OF ENERGY

If energy Q is transferred thermally from A to B then the associated entropy changes are:

$$\Delta S_A = -Q/T_A \quad \Delta S_B = Q/T_B$$

Since $T_A > T_B$ (otherwise the thermal transfer would not occur):

$$|\Delta S_A| < |\Delta S_B|$$

so the entropy gain of B is greater than the entropy loss of A, and:

$$\Delta S_{\text{total}} = \Delta S_A + \Delta S_B > 0$$

In approaching equilibrium the total entropy increases because the total number of ways of sharing the energy increases. At equilibrium of course $T_A = T_B$ and $\Delta S_A = -\Delta S_B$, giving $\Delta S_{\text{total}} = 0$.

Energy transfer still occurs, but as the computer simulation shows, it is just as likely in one direction as the other. This dynamic equilibrium is reached when the net effect of transferring energy from one part of the system to the other leaves the total number of ways unchanged.

Questions

Questions 27 and 28 relate the fraction f to the temperature. Questions 29 and 30 give further practice.

THERMODYNAMIC TEMPERATURE

The temperature is the ratio of energy added to entropy produced. The choice of the Boltzmann constant k in the definition of entropy makes temperature described in this way the same as temperature on the ideal gas scale.

This introduction of a statistical definition of temperature must necessarily seem strange to students. Indeed, it is more than a definition, being rather a new and deeper way of looking at the concept. Previously, temperature meant somewhat vaguely ‘how hot’, and definitions laid down arbitrarily how to measure it precisely. The present idea is to think of temperature as the ratio of energy added to the extra thermal chaos this energy increase produces. Adding energy to something hot adds little to the chaos already there: adding the same amount of energy to something cold makes a much greater difference.

The upshot, which needs emphasizing, is that we can now deal with that part of the change in numbers of ways, and so of entropy, associated with flow of energy. Before, we could only deal with rearrangements of particles. To get there, we needed to recognize temperature as the quantity that helps translate energy change into entropy change.

SECTION K3

THE BOLTZMANN FACTOR

This Section consists of a general introduction to activation processes and to the importance of the Boltzmann factor in understanding them. Then there is an experimental study of five applications, each discussed only briefly in the *Students' guide*.

It is suggested that the experimental work, looking at the applications in greater detail, be divided amongst the class so that each student might tackle only one or two. A reporting-back session could be valuable. Teachers should feel free to develop other examples (such as diffusion in liquids or solids), particularly where they lead to experimental work. In essence, any process dominated by an activation energy of some kind will serve.

ACTIVATION PROCESSES

Many physical processes have their rates determined by the chance that a particle can acquire, by random interactions with others at a temperature T , an untypically large energy. They are called activation processes. The Boltzmann factor plays an important part in determining the chance that a particle will have a given energy.

In Section K2 we obtained the Boltzmann factor f in a special case, with

$$\ln f = -\varepsilon/kT \quad f = e^{-\varepsilon/kT}$$

for particles on equally spaced energy levels. More generally, the factor $e^{-E/kT}$ is often a good guide to the fraction of the total number of particles with energy E or greater, if E is considerably larger than kT .

Section K2 gives an essentially correct picture of the origin of the Boltzmann factor, as the chance of borrowing energy from a set of particles, in equilibrium at temperature T , by random exchanges of energy between particles. The larger the energy to be borrowed, the rarer such an event will be, just because it depends on chance. The value of f is determined simply by the reduction in W which is caused by removing that energy: f is the factor (less than unity) which multiplies W when the energy is removed. Practical examples are many, including:

- the number of ions in a flame
- the number of molecules which evaporate from water
- the number of fast-moving molecules in a gas

the number of defects in a solid
viscosity of liquids
conduction in a thermistor

The Reading 'Life in the Universe' in the *Students' guide* mentions the example of the neutron-proton ratio in the early Universe, when thermal processes could supply the energy corresponding to their difference in mass.

The Boltzmann factor is not restricted to equally spaced levels. In general, the levels may be unequally spaced, and there may be more than one quantum state at a given energy. It is always true (in Maxwell-Boltzmann statistics) that the ratio of numbers of particles in quantum states differing by energy E is given by $e^{-E/kT}$. The number of particles at or close to a given energy then depends on the Boltzmann factor and on the number of states in that range of energy. When one adds up the numbers of particles at or above some energy E substantially greater than kT , it is often the case that the dominant term is $e^{-E/kT}$, if only because of its strong dependence on temperature.

The rate of an activation process will depend on factors other than the Boltzmann factor, such as the rate of collision of particles – so that rates of reaction depend on concentration. But at given concentration, or its equivalent, the temperature dependence of the rate is often given rather accurately by the Boltzmann factor, even though other factors intervene.

Questions

Question 31 illustrates the rapid variation of the Boltzmann factor with temperature. Questions 32 and 33 give further practice in its use, question 33 showing how it gives an approximately exponential variation with temperature.

EXPERIMENTAL STUDY OF ACTIVATION PROCESS

A selection of activation processes should be studied in outline, so as to grasp their essential similarity, and each student should study at least one in experimental detail. An example not discussed in the course material, but which would be equally suitable, would be thermionic emission, studied with a hot-wire diode. Some of the experiments (creep, for example) offer useful possibilities for experimental design and instrumentation, or may be good sources of Investigations.

Common to all the examples is that each is a process in which particles have to acquire energy from others at temperature T . The Boltzmann factor gives the chance that this will happen. Thus in each case we have something like:

$\ln(\text{fraction of particles with extra energy } E) = -E/kT$

and some measurable quantity which is proportional to the fraction of particles with extra energy E . The logarithm of this quantity should thus

vary linearly with E/kT , and each experiment results in a plot of the logarithm of some quantity against $1/T$.

Questions

Questions 34 and 35 concern the energy needed to evaporate water. Question 36 is more general, and will be needed for other experimental work. It shows how to calculate an activation energy from the dependence of rate on temperature.

Question 37 is about current in a semiconductor device, with data to analyse.

Question 38 is about nuclear reactions in the Sun. Question 39 is about thermionic emission.

VAPOUR PRESSURE OF WATER

To escape from water, a molecule needs to break on average about two hydrogen bonds, and it is this which gives water its relatively large latent heat of vaporization. We may expect the number of molecules in the vapour to be determined in part by the Boltzmann factor, and since the vapour pressure p is proportional to the particle density, it is no surprise that to a rather good approximation

$$p \propto e^{-\Delta H/LkT}$$

where ΔH is the enthalpy of vaporization (heat of vaporization at constant pressure) per mole, and $\Delta H/L$ is thus the corresponding energy per molecule. This is not the whole story, but it is an important part of it.

This is a case where both the energy involved, and the variation of a property with temperature, can be measured, so it seems worth making both measurements unless the first is totally familiar. A suitable plot is of $\ln p$ against $1/T$, which should be a straight line. The slope, $-\Delta H/Lk$, can be predicted, and compared with the slope obtained.

EXPERIMENT

K5 Vapour pressure of water

K5a Energy needed to evaporate water

ITEM NO.	ITEM
1153	vacuum flask
1504	balance (resolution 0.1 g)
75	immersion heater
59	l.t. variable voltage supply
	<i>either</i>
1518	joulemeter
	<i>or</i>
1507	ammeter, 10 A a.c. and voltmeter, 25 V a.c.
1000	leads

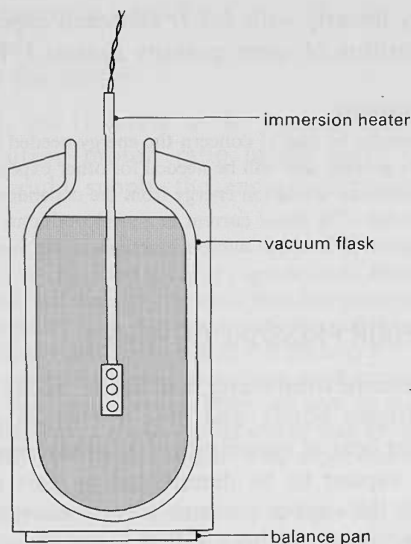


Figure K7
Vacuum flask with boiling water.

Safety note: Do not allow students to use any immersion heater which has a cracked seal, or to let the heater cool while immersed in water.

Any simple reliable method to measure the energy needed to evaporate water will serve. A suitable method is to boil water in a vacuum flask on a top-pan balance, recording the loss of mass by evaporation with time, and the energy delivered electrically. (See JARVIS, W. H. 'Specific latent heat of steam'. *School Science Review*, Vol. 53, No. 184, March 1972, page 596.) Note that with the apparatus suggested it is best to over-run the immersion heater at 20 V, and to pre-heat the water. Sufficient water to be measured accurately should boil away in a quarter of an hour.

K5b Variation of vapour pressure with temperature

ITEM NO.	ITEM
548	round-bottomed flask
542	thermometer, -10 to 110°C
1155	T-tube
67	Bourdon gauge
1155	pressure tubing
522	Hoffman clip
511	tripod
510	gauze
508	Bunsen burner
1502	safety spectacles
503-6	retort stand base, rod, boss, and clamp

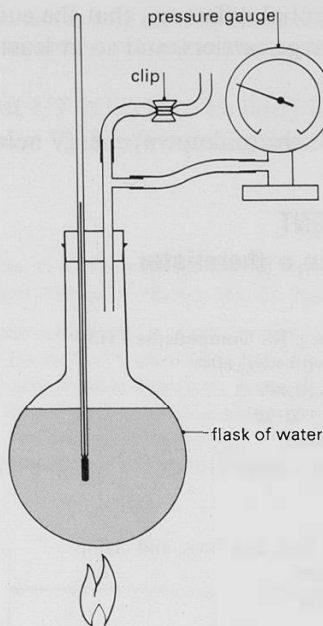


Figure K8
Pressure gauge measuring vapour pressure.

The method suggested is to boil the water, *with the clip open*, until the flask is full of vapour; then remove the flame, connect the pressure gauge, and close the clip. The temperature is then allowed to fall, and at intervals the water is made to boil by *gently* applying the flame. Readings of pressure and temperature can be obtained over a fair range. It is important to avoid water distilling over into the pressure gauge. *Safety spectacles should be worn.*

Other more elaborate methods can be devised, or students might be asked to find ways of measuring the much lower vapour pressure nearer to room temperature.

Current in a thermistor

An electrical example of a use of the Boltzmann factor can add variety and lead to simple experiments. There are difficulties: conduction mechanisms are complex and little has been said about them in the course. An example worth discussing, however, is a pure semiconductor—a material which, but for the Boltzmann factor, would be an insulator. That is, one may regard most electrons as bound to atoms, with a few free to move because they have acquired the necessary energy to escape from an atom. Ignoring any temperature-dependent effects on their mobility, one might expect, at a

given potential difference, that the current I would vary as the density of free charge carriers, and so at least approximately as

$$I \propto e^{-E/kT}$$

where E is the (unknown) energy needed to liberate a charge carrier.

EXPERIMENT

K6 Current in a thermistor

ITEM NO.	ITEM
1151	thermistor (e.g. RS Components TH3)
1033	cell holder with two cells
1507	milliammeter, 10 mA
1507	milliammeter, 100 mA
52K	2 crocodile clips
542	thermometer, -10 to 110°C
511	tripod
510	gauze
503-6	retort stand base, rod, boss, and clamp
508	Bunsen burner
512/1	beaker, 250 cm^3
1000	leads

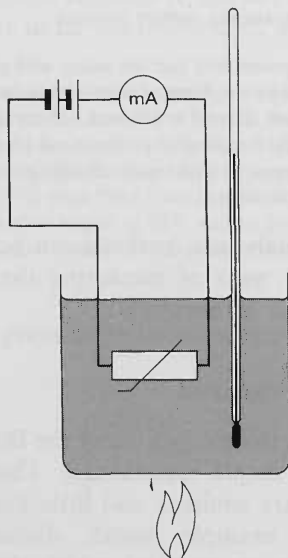


Figure K9
Current in a thermistor.

The current in the thermistor, for a fixed p.d., is simply measured at a number of different temperatures, with the thermistor heated by being immersed in water in a

beaker. It is of course important to keep the temperature of the water constant for a time before taking a reading, stirring well, and heating gently.

A graph of $\ln I$ against $1/T$ is likely to be fairly linear, with a gradient $-E/k$ of the order $-4 \times 10^3 \text{ K}$, giving an estimate of E of a fraction of an electronvolt.

Alternative

(See INMAN, F. W. and MILLER, C. E. 'The measurement of e/k in the introductory physics laboratory'. *American Journal of Physics*, Vol. 41, March 1973, pages 349–351.)

A transistor connected in the common base mode has a short circuit collector current given rather accurately by $I = I_0 e^{eV_{EB}/kT}$, since the collector current is mainly due to charge carriers that diffuse across the emitter–base junction. In this case, one has control over the height of the energy barrier eV_{EB} across which charge carriers have to acquire energy to pass, by varying V_{EB} . Over a range of current from 10^{-9} A or less to a few mA (6 orders of magnitude), one can get a linear dependence of $\ln I$ against V_{EB} .

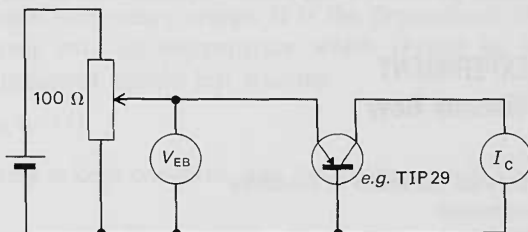


Figure K10
Alternative circuit.

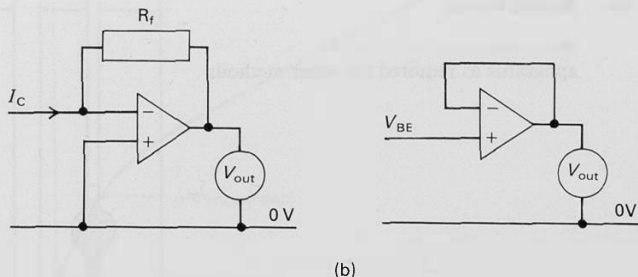


Figure K11

The current-measuring instrument needs to be able to measure down to 10^{-9} A , and the voltage-measuring instrument to have very high impedance. Instrumentation based on operational amplifiers is very suitable, and provides another use for ideas from Unit 1, 'Linear electronics, feedback and control' (see figure K11 for suggested circuits). A power transistor is a convenient device to study; its temperature can be controlled by immersing the heat sink fins in an oil bath.

Viscosity

One reasonable picture of many liquids is to think of each particle as free to move within a small 'cage' formed by its neighbours. The average kinetic energy of a particle is much less than that needed to thrust aside a neighbour or two and 'escape' into a nearby 'cage'. However, a particle will occasionally get the necessary energy, by chance, and the Boltzmann factor influences how often that will be. It will also matter how fast the particles are moving, since one can think of them making 'breakout attempts' about as often as they cross a 'cage' of the order of one particle diameter across. But the Boltzmann factor predominates in the temperature variation.

One application is to diffusion. Another is to viscosity, where the effect of shear forces in the liquid is to reduce the activation energy to escape from a 'cage' in one direction and increase it in another. Thus more particles escape one way than the other, and flow occurs.

EXPERIMENT

K7 Viscous flow

ITEM NO.	ITEM
	Ostwald viscometer (if available)
507	stopwatch
513	tall beaker
542	thermometer, -10 to 110°C
511	tripod
510	gauze
508	Bunsen burner
	or
	apparatus as required for other methods

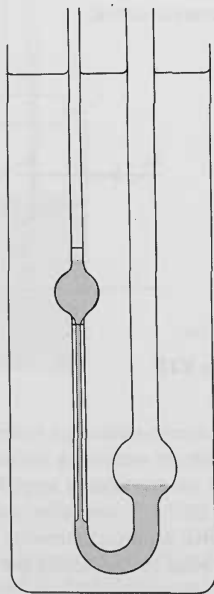


Figure K12
Ostwald viscometer.

The activation model works for a wide range of liquids, so a variety of experiments are possible, whether with water or with syrup. Note that only a rate of flow under standard conditions is needed, not a value of the viscosity. Students plot $\ln(\text{flow rate})$ against $1/T$.

The Ostwald viscometer is one possibility, but any method chosen for convenience or interest could be used, and exploited as a vehicle for teaching experimental design. Since a uniform temperature is required, methods using tall columns of liquid are least suitable. A simple device letting liquid run out of a hole may be as good as any: other methods might include measuring the rate of rotation of a disc or drum immersed in the liquid under a constant torque.

Creep

Creep is the slow extension of a material under stress. Typically it begins rapidly (primary creep) and then settles down to a steady rate of change of strain (secondary creep). It is the dependence of the linear secondary creep rate on temperature which should be investigated here. The experiment should test whether

$$d(\text{strain})/dt \propto e^{-E/kT}$$

where the stress is kept constant, and the energy E will depend on the stress.

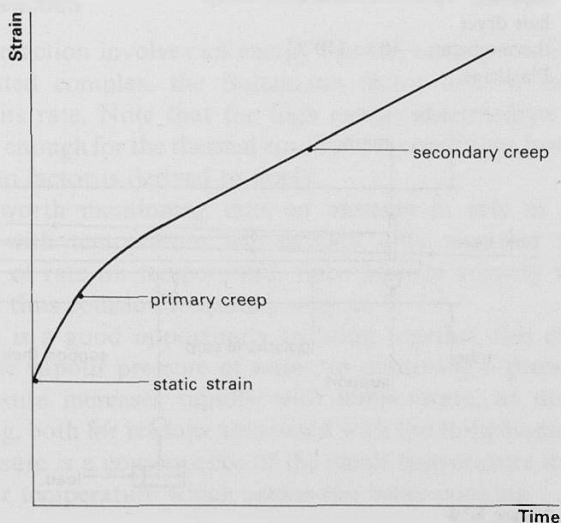


Figure K13
Creep.

At low temperatures, creep in metals is mainly by the movement of dislocations (Unit A, 'Materials and mechanics'). The movement of a dislocation can be impeded by meeting other dislocations, and also by impurity atoms in the lattice, and to move past these extra energy will be needed. That energy can come from the thermal energy, with probability given by the Boltzmann factor. The energy E involved is not one definite quantity, since there are different kinds of obstacle each with its own energy barrier, but it is possible to think of E as an average of such energies.

Reading

WALTON, *Three phases of matter*.

EXPERIMENT

K8 Creep

ITEM NO.	ITEM
51G	polythene strip
31/1	hanger and slotted masses, 10 g
116	2 plane mirrors
1505	laser (or compact light source and lens)
	screen (e.g. white paper on wall)
1153	75 mm diameter tube (e.g. plastic drain pipe)
504	supports (e.g. retort stand rods, short)
1515	hair dryer
542	thermometer, -10 to 110°C
1153	Plasticine

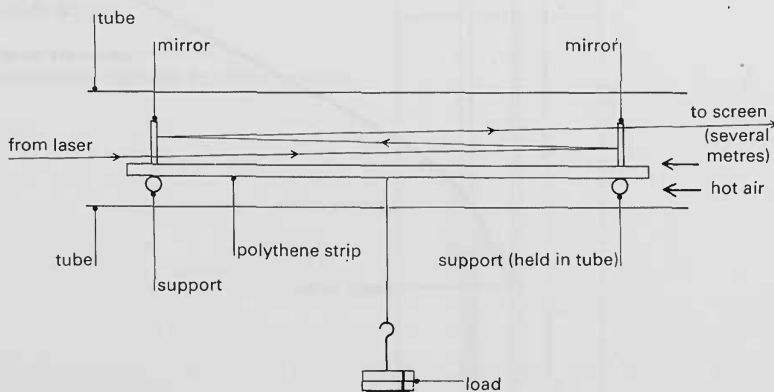


Figure K14

Apparatus to study creep in polythene.

Use of laser: See the 'Safety note' on page 207.

Creep in polythene is sufficiently rapid to be suitable for this investigation: even so it may be impractical to take measurements at more than two temperatures.

One method is to bend a polythene strip, supported at both ends, by hanging loads at the middle. The bending can be followed by mounting small plane mirrors on the ends of the strip, which deflect a beam of light reflected once from each mirror on to a screen several metres away (see figure K14). Such an optical lever can produce a rate of motion of the spot on the screen of several centimetres per minute.

When the beam is first loaded, there is an immediate static deflection. The deflection increases, at first rapidly but at a varying rate by primary creep, and then slowly and linearly by secondary creep. Recording positions of the spot on the screen allows students to plot a graph of the creep against time, and so to find the rate of creep.

The temperature can be varied by mounting the strip inside a tube (which carries its supports, and has a hole for the load to be attached to the strip), and blowing hot air from a hair dryer through the tube.

An alternative, more compact arrangement could be to cut a ring of polythene as the specimen, deform it into an ellipse by a load, and measure the deformation by mounting mirrors at a pair of '45-degree' positions on the ring.

Rate of reaction

Where a reaction involves an energy barrier, usually the formation of an activated complex, the Boltzmann factor will be important in deciding its rate. Note that the high energy intermediate state must exist long enough for the thermal equilibrium conditions from which the Boltzmann factor is derived to apply.

It is worth mentioning that an increase in rate of collision of particles with temperature will explain only a rather modest dependence of rate on temperature, since particle velocity will vary as $T^{1/2}$, and thus collision frequency only as $T^{-1/2}$.

There is a good opportunity to bring together this example and that of the vapour pressure of water, in discussing a pressure cooker. The pressure increases rapidly with temperature, as does the rate of cooking, both for reasons associated with the Boltzmann factor. The high pressure is a consequence of the small temperature increase. It is the higher temperature which causes the faster cooking.

EXPERIMENT

K9 Rate of reaction

ITEM NO.	ITEM
1156	sodium disulphate(iv), 5 g
1156	sodium sulphite (anhydrous), 1 g
1156	methanal, 10 cm ³
1156	phenolphthalein, 20 cm ³
1155	2 measuring cylinders, 100 cm ³
512/1	2 beakers, 250 cm ³
517	2 volumetric flasks, 1 dm ³ (1 litre)
542	thermometer, -10 to 110 °C
1502	safety spectacles
507	stopwatch or clock
533	plastic bucket or basin
	supply of hot water

One possible experiment involves the reaction of disulphate(iv) ions and methanal (formaldehyde). Two colourless solutions are mixed, and remain colourless until all the disulphate(iv) ions are used up, when the mixture rapidly turns pink. The solutions can be warmed before mixing, and the variation with temperature of the time t from mixing to the appearance of the pink colour found. The rate being proportional to $1/t$, we expect

$$\ln(1/t) \propto -E_A/T$$

where E_A is the activation energy.

Safety spectacles should be worn by the person making up the solutions (preferably someone with experience), especially since a drop of methanal in the eye can cause damage.

The two solutions should be made up just before they are required, and kept stoppered until used. One is 5 g sodium disulphate(iv) and 1 g anhydrous sodium sulphite, the other 10 cm³ methanal and 20 cm³ phenolphthalein, both made up to 1 dm³ water in volumetric flasks.

100 cm³ of each solution are mixed (quickly), and the clock is started. Before mixing, samples can be warmed by standing in hot water: temperatures in the range 20 °C to 50 °C are suitable.

The main reaction – the one which is timed – is one in which the methanal reacts with disulphate(iv) ions. At the same time, the sulphite ions present react with methanal to produce hydroxide ions, which are however immediately mopped up by disulphate(iv) ions. When all the disulphate(iv) ions are used up in the main reaction, the phenolphthalein responds to the hydroxide ions and the mixture turns pink.

IMPORTANCE OF THE BOLZMANN FACTOR

The Section concludes with a brief look at the importance and generality of the Boltzmann factor. It can be reinforced by the Reading 'Life in the Universe' on page 267 of the *Students' guide* (see question d on page 269) and by questions about various applications, such as questions 34 to 39.

Some discussion of the biological importance of the Boltzmann factor may interest students. The hydrogen bond is of great importance to life, in part because its strength is such that reactions proceed gently at 300 K, the temperature for which we have evolved. It is hydrogen bonds that hold together the base pairs in DNA which encode our heredity, for example.

No less important is the role of enzymes, which reduce the activation energies of specific reactions by providing suitable sites where the reactions can occur. Reducing the activation energy speeds up the reaction, but also makes it less temperature dependent. But despite our enzyme systems, we do in fact have to have rather precise temperature control, which must serve to control reaction rates in our bodies.

Life also depends critically on the Sun. In particular, the way in which the Sun burns slowly and steadily for billions of years gives time for evolution to work. To react, the hydrogen nuclei must be able to get close enough to each other for quantum mechanical tunnelling to be effective. The reaction is slow because even at the temperature of the centre of the Sun most particles do not have enough energy to overcome the Coulomb repulsion between them.

Revised Nuffield Advanced Chemistry

The Boltzmann factor is dealt with in Topic 6.

SECTION K4

EXPLOITING ENTROPY CHANGES

The Section begins with the Second Law of Thermodynamics, seen from a statistical point of view as a natural consequence of previous ideas. It is emphasized that the total entropy never decreases, entropy changes being zero only at equilibrium. But it is also emphasized that an entropy increase in one part of a system can be an opportunity for a process which would decrease the entropy to occur in another part of the system, by exploiting the increase to offset the decrease.

It is common in teaching about the Second Law to stress the inevitable running-down it implies. This, while correct, ignores the fact that the world contains structures (from stars to people) which arise spontaneously. Modern non-equilibrium thermodynamics pays attention to the way entropy-producing processes can 'drive' entropy-decreasing processes. Thus plants exploit the entropy increase as photons from the Sun are absorbed and re-radiated, to run the reactions of photosynthesis, making sugars or cellulose from carbon dioxide and water. See Appendix II (page 423).

A discussion of the limitations of heat engines is followed by a number of further applications:

energy and entropy changes in a gas
equilibrium
a concentration cell

These should be treated as selected examples, studied more to practise the ideas than for themselves. Thus a given student might not need to look at more than one of the experiments, and teachers should feel free to substitute other examples of their own, so long as the examples involve calculating a total entropy change from entropy changes of parts of a system. More mathematically inclined students may like to know how the ideas of Section K2 can be extended to the heat capacity of an 'Einstein solid'. Material for this is in Appendix III (page 428).

THE SECOND LAW OF THERMODYNAMICS

The Law is presented in the form:

All changes occur in the direction for which the total entropy increases. The total entropy never decreases. At equilibrium, the total entropy change is zero.

From the statistical view, the Law becomes in essence a restatement of the qualitative starting point of the Unit, namely that molecules ‘don’t care’. That is, a system simply tries out all the states available to it, without favouring one over another. What states *are* available depends on the constraints present: on where the piston in a cylinder is held, for example, or whether two substances are kept from mixing.

If the constraints are changed, other states become available, and certain processes (e.g. diffusion, mixing, ...) may occur, or they may not. The only processes that do occur are those for which the molecules are not somehow required to ‘know’ what to do, that is, to avoid getting into some of the available states. For the entropy to decrease, the system would have to steer itself away from some possibilities. This will never happen if the underlying behaviour is random (except for small fluctuations), and entropy does not decrease.

A difficulty in presenting the idea is that while the underlying fact (that at the molecular level all change is necessarily spontaneous random change) is in a sense rather obvious, the detailed consequences are not. Also difficult is the fact that the Law says nothing about how things come about, but only what things cannot come about.

It will be important to distinguish the whole system from parts of it. The entropy of part can decrease, if the entropy of the rest increases by a larger amount. Thus when steam condenses on the kitchen walls, the entropy of the water falls, but since the walls get warmer, the total entropy rises. If the walls are above the boiling-point, the total entropy increases by taking energy from the walls (decreasing their entropy) and evaporating the water (an increase in its entropy). Both condensation and evaporation – opposite processes – occur with an increase in entropy of the whole system. It follows that no change can take place in both directions under the same conditions.

Questions

Questions 40 to 42 ask for qualitative accounts of entropy changes.

THE INEFFICIENCY OF ENGINES

Energy goes spontaneously from hot to cold, because taking energy Q from something hot reduces the entropy less than giving energy Q to something cold increases the entropy. The net entropy change is

$$Q/T_{\text{cold}} - Q/T_{\text{hot}}$$

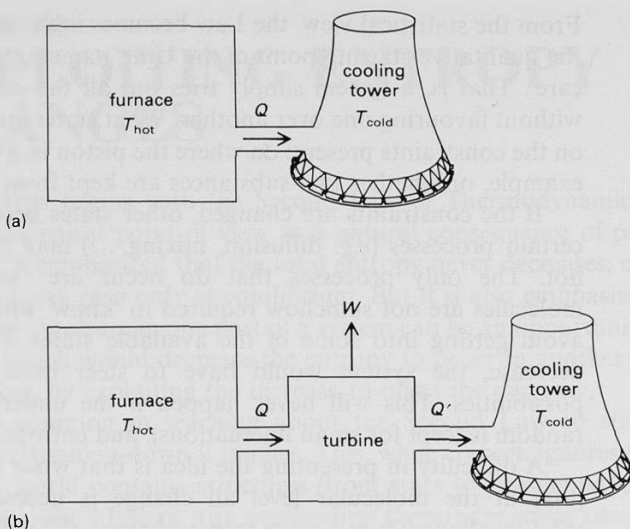


Figure K15

(a) Increasing entropy by heating the surroundings.

(b) Obtaining work, and increasing the entropy by less.

But entropy does not have to increase by any particular amount, only not to decrease. So we could afford to deliver less energy Q' to the cold thing, the smallest possible value of Q' being given by

$$Q'/T_{\text{cold}} - Q/T_{\text{hot}} = 0$$

or

$$Q' = Q(T_{\text{cold}}/T_{\text{hot}})$$

The difference, $Q - Q'$, can be delivered as work, $W = Q - Q'$, if someone is smart enough to engineer a device to do it. The best that can be done is therefore

$$W = Q(1 - T_{\text{cold}}/T_{\text{hot}})$$

and this is just the maximum thermal efficiency of such an engine.

It is useful to look at data for the efficiencies of heat engines, and plot the efficiency against the steam temperature as suggested in question 45.

Notice how this argument inverts the traditional sequence. Traditionally, the inefficiency of a given cycle is shown to be generalizable to any cycle which is reversible, and from that the concepts of thermodynamic temperature and entropy are constructed. Later, entropy may be given a microscopic interpretation. We have come at it all the other way around.

Note also that work, in these arguments, has to be understood as *energy exchange with zero entropy change*. This is just what distinguishes 'working' from other modes of energy transfer. It may also be useful to point out that the above argument just calculates the 'free energy' of a thermal transfer from hot to cold.

It is also important *not* to speak of 'the impossibility of turning all the heat in a body into work'. If in such a sentence, 'heat' means 'internal energy' (i.e. energy of motion, etc., of particles) then it is *wrong*. Nothing prevents us expanding a gas adiabatically to extract more of its internal energy (if we can let the volume get big enough), for example. If 'heat', correctly, means energy flow under a temperature difference, then there is no such thing as 'the heat in a body' (only heat flow plus work exchanged is constant, not either alone) so the sentence is incoherent. What is impossible is to build a device that exploits the entropy increase in a thermal transfer of energy from a hot body to a cooler one (without the device itself contributing to the entropy increase), which ends up with a negative entropy change, and so delivers more work than $Q(1 - T_{\text{cold}}/T_{\text{hot}})$.

Questions

Questions 43 to 45 concern thermal efficiencies.

FURTHER APPLICATIONS

Each of the following applications involves calculating a total entropy change, involving thermal transfer of energy to or from the surroundings of a system. Perhaps the most important example is the second, on chemical equilibrium, though it may be that the last, making a cell from solutions of different concentration, is the most interesting. The brief experimental work suggested can clearly be extended, for example to investigating the temperature dependence of a cell e.m.f., should time allow.

Energy and entropy changes in a gas

This example is intended to tie together the earlier work on gases with the later results about entropy and energy changes. One can now emphasize the thermal transfer of energy Q to the surroundings when the volume of a gas is altered reversibly at constant temperature, and the corresponding entropy change Q/T .

Suppose a gas is compressed reversibly with a piston, by an infinitesimal amount dV . Its entropy must decrease, by

$$dS_{\text{gas}} = kN dV/V$$

If the temperature is constant, the internal energy of the gas does not change, so energy dQ is transferred thermally to the surroundings. (See Appendix II, page 423, for a discussion of the notation used here.)

The entropy increase of the surroundings is

$$dS_{\text{surr}} = dQ/T$$

But if there is zero friction, and no large-scale motion in the gas to be damped out, the energy flow dQ is equal in magnitude to the work $p dV$:

$$dQ = -p dV = -NkT dV$$

Thus the entropy change of the surroundings balances the entropy change of the gas:

$$dS_{\text{surr}} = dQ/T = -Nk dV/V = -dS_{\text{gas}}$$

and the total entropy change is zero. The total entropy change is greater than zero if there is friction on the piston, turbulence in the gas, or if the compression is rapid, thus raising the gas temperature more than infinitesimally above that of the surroundings.

Questions

Question 42 uses rubber as an example, similar to the case of a gas, to rehearse the arguments again. Question 46 goes through the above argument with a numerical example. Questions 47 and 48 show how the ideas apply to other systems.

EXPERIMENT

K10 Entropy changes in a gas

ITEM NO.	ITEM
1070	gas energy transfer apparatus
77	aluminium block
75	immersion heater
59	l.t. variable voltage supply
1064	low-voltage smoothing unit
1507	ammeter, 0–5 A d.c.
1507	voltmeter, 0–15 V d.c.
1101	sensitive galvanometer
1501	copper wire, 0.28 mm diameter (bare)
1501	constantan wire, 0.28 mm diameter (covered)
44/1	G-clamp
1153	slab of expanded polystyrene
1153	transparent adhesive tape
513	measuring cylinder, 100 cm ³
1504	balance, resolution 0.1 g
507	stopwatch or clock
1000	leads

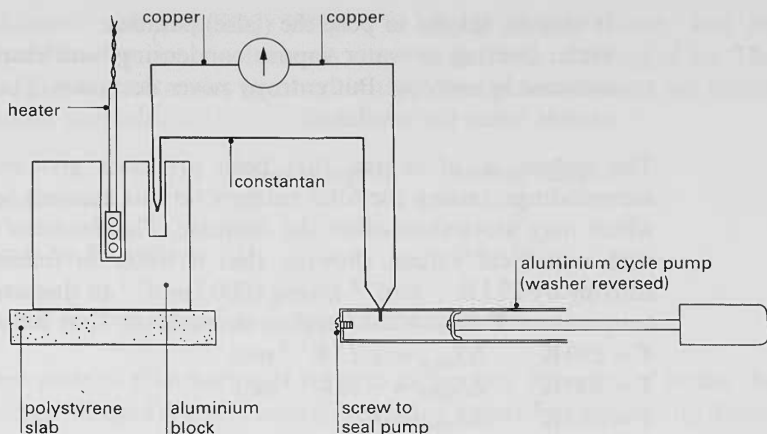


Figure K16
Gas energy transfer.

The idea of the experiment is to expand gas in a cycle pump, measuring the energy transferred thermally from the casing (by comparing the energy needed to warm a block of the same material). Entropy changes of gas and surroundings (casing) are compared. If the gas is expanded from V_1 to V_2 , and contains N molecules, its entropy increase is $kN \ln(V_2/V_1)$. If the casing gives up energy Q at temperature T , its entropy decrease is $-Q/T$.

The pump should be a lightweight aluminium cycle pump. The volume of the casing can be found by filling it with water. The end of the pump is sealed with a screw.

It is convenient to expand rather than compress the gas, using a pump with reversed washer, so that most of the aluminium pump casing is cooled slightly. A copper-constantan thermocouple with one junction on the casing and the other in an aluminium block records the cooling of the casing. Touching the junction with a finger shows that the temperature drop is very small, so that the expansion is essentially isothermal.

A second experiment delivers energy electrically to the aluminium block, and the energy taken from the casing is found by scaling the measured energy given to the block in proportion to the masses of casing and block, allowing for differences in the galvanometer readings obtained.

Equilibrium

A gas expanding reversibly, and so kept close to equilibrium, is a rather special case. The idea of the present example is to broaden this idea to include chemical and other physical equilibria.

It may be helpful to pose the (false) paradox:

Water freezing or water vapour condensing both clearly involve a decrease in entropy. But entropy never decreases. Therefore water cannot freeze (or condense).

The answer is, of course, that both processes give energy to the surroundings, raising the total entropy on this account by $(\text{energy})/T$, which may more than offset the decrease. The *Students' guide* works with numerical values, showing that if water in freezing drops in entropy by $22 \text{ J K}^{-1} \text{ mol}^{-1}$ giving 6000 J mol^{-1} to the surroundings at temperature T , then what happens depends on T , as follows:

$$T = 250 \text{ K} \quad \Delta S_{\text{total}} = +2 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$T = 300 \text{ K} \quad \Delta S_{\text{total}} = -2 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$T = 273 \text{ K} \quad \Delta S_{\text{total}} = 0$$

Similar arguments could be set out for water condensing from the vapour (see question 47).

Note that the same reasoning explains why muddy water can separate into clear water and silt. The mud particles, if more dense than water, lose gravitational potential energy if they fall, and deliver this energy to warm up the water or surroundings. The resulting entropy increase can more than compensate the fall in entropy of arrangement of particles. Note that the finer the particles, the larger the entropy drop, which is why very finely divided mud particles can stay suspended. This idea is also the basis of Perrin's method for measuring the Avogadro constant.

Because the entropy change when energy is given to or taken from the surroundings depends on the temperature, we can say qualitatively: if a process gives energy *to* the surroundings, the net entropy change can be increased (made more nearly positive) by lowering the temperature. This, however, may decrease the rate (Section K3). For this reason, explosives can be stored at low temperatures and then triggered off. If a process takes energy *from* the surroundings, the entropy loss can be reduced by making the surroundings *hotter*.

EXPERIMENT

K11 'Hot ice'

ITEM NO.	ITEM
1156	sodium sulphate decahydrate, 20 g
1155	boiling tube
542	thermometer, -10 to 110°C
508	Bunsen burner
1504	balance, resolution 0.1 g
1153	plastic cup
1155	measuring cylinder, 100 cm^3

Glauber's salt ($\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$) is clearly mostly water, and its crystallization out of solution is very similar to the freezing of ice. The energies and entropies involved in these two phase changes are rather similar (see table K1).

	Entropy change/ J K^{-1}	Enthalpy change/J
1 mole water	22	6×10^3
10 moles water	220	60×10^3
1 mole $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$	257	78.4×10^3

Table K1

Entropy and enthalpy changes on freezing or crystallization.

One mole of Glauber's salt behaves rather like 10 moles of water, the energy released when it crystallizes being somewhat greater, no doubt due to the extra bonding produced by the presence of the charged ions. The equilibrium temperature is a little higher:

$$T_{\text{eq}} = \Delta H / \Delta S = 305 \text{ K} = 32^\circ\text{C}$$

This is why the salt has been proposed as a means of energy storage for solar-heated homes. If a tank of the salt (possibly in the roof) is heated by the Sun on a hot day, more salt will dissolve, but on a cold day more will crystallize at 32°C , warming the house. The principle is just the same as that of an igloo whose interior is kept at 0°C by melting and freezing.

Students measure the enthalpy change by dissolving, say, 5 or 10 g of the salt in 50 cm^3 of water in a plastic cup, recording the temperature drop (several degrees).

To observe the equilibrium, and the energy released on crystallization, they warm about 10 g of the salt in a boiling tube (to about 50°C) until it dissolves in the water of crystallization. (The solution is clouded by some anhydrous salt.) They then cool the solution to about 25°C , and drop in a seed crystal of the hydrated salt. The solution at once crystallizes, and can be felt to be warm. The temperature will be found to be 32°C .

Questions

Question 47 is about the equilibrium of liquid and vapour, with Trouton's rule (that the entropies of evaporation of many liquids are very similar) as a by-product.

Question 48 asks about equilibrium more qualitatively.

A cell which runs on entropy

Adding salt to a weak solution increases the entropy more than adding the same amount to a stronger solution. Thus if we pass some salt

from a concentrated solution to a weaker one, there is an increase in entropy.

This can be compared with diffusion, and with adding energy to cold and to hot surroundings.

The entropy involved might be called 'interchange of particles' entropy. The argument in the *Students' guide* deliberately parallels the previous argument for gases (Section K1), with

$$W' = W \times 2^N$$

and

$$\Delta S = kN \ln 2$$

for doubling the dilution of N solute particles; and in general

$$\Delta S = -kN \ln(\text{concentration ratio})$$

In teaching, one might either begin with the calculation, or by showing a cell whose e.m.f. derives from a concentration difference, as in experiment K12.

EXPERIMENT

K12 Concentration cell

ITEM NO.	ITEM
1156	copper(II) sulphate solution, 1 M
512/1	2 beakers
1155	filter paper
1156	potassium nitrate solution
1509	high-impedance voltmeter, 100 mV d.c.
	distilled water

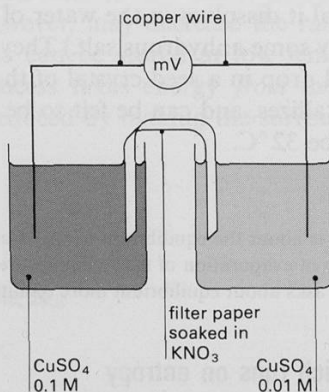


Figure K17
Concentration cell.

1/10 and 1/100 molar solutions of copper sulphate are made up with distilled water and are put in two beakers standing side by side. A salt bridge can be improvised from a strip of filter paper dipped in potassium nitrate solution, joining the two beakers. The high-impedance voltmeter is connected to copper wires dipping in the two solutions. An e.m.f. of the order 20 mV can be expected.

Note that the e.m.f. from pairs of solutions of the same concentration ratio is *not* in fact the same. This is because the energy of solution depends on concentration, and because the entropy change on dilution is affected by clustering of molecules in the solution, thus making 'hidden' contributions to the entropy change.

The argument for the cell e.m.f. depends on the principle, mentioned before, that where there is an entropy increase, it may be possible to extract energy from the surroundings (which decrease in entropy) since entropy does not have to increase by any particular amount, but only to change by zero or by a positive amount. Passing copper ions from the concentrated solution to the weaker one is just such a case, which can be exploited.

In the cell, copper ions are removed on the more concentrated side, electrons being taken from the copper, and replaced by electrons coming from the copper on the more dilute side, where copper ions are formed in solution. Since two electrons pass for each ion removed, the maximum work obtained is

$$2eN\mathcal{E}$$

where \mathcal{E} is the cell e.m.f.

The maximum work is obtained when the total entropy change is zero, with $\Delta S_{\text{surr}} + \Delta S_{\text{solutions}} = 0$

$$\text{work} = T\Delta S_{\text{solutions}} \quad (\text{See Appendix II.})$$

Thus

$$2eN\mathcal{E} = -kTN \ln(\text{concentration ratio})$$

$$\mathcal{E} = -\frac{1}{2}T \left(\frac{k}{e} \right) \ln(\text{concentration ratio})$$

For concentrations in the ratio 10 to 1 this gives $\mathcal{E} \approx 30 \text{ mV}$.

It may help to put the matter in a more general light by pointing out that osmosis is the same idea, but now getting *mechanical* work from a concentration difference, as liquid rises up one tube, for example. Many biological processes depend on osmotic pumps. Electrical nerve impulses also depend on concentration differences (as well as on the different ions on either side of cell walls). We drink water to help keep up these differences.

Questions

Questions 49 and 50 concern entropy changes and concentration differences.

Further reading for students

WEINBERG, *The first three minutes*, and DAVIES, *The runaway Universe*, both present accessible accounts of modern views of cosmic thermodynamics.

DAVIES, *The accidental Universe*, is a good source.

ATKINS, *The creation*, presents a highly partial challenge to philosophical ideas about the place of human beings in the Universe.

ATKINS, *The Second Law*, gives an excellent overview of the nature of the Second Law and its importance.

Revised Nuffield Advanced Chemistry

Cells are dealt with in Topic 15.

Unit L

WAVES, PARTICLES, AND ATOMS

John Harris

Centre for Science and Mathematics Education, Chelsea College,
University of London

PLAN OF THE UNIT *page 314*

INTRODUCTION *316*

THE PLACE OF THE UNIT IN THE COURSE *316*

LIST OF SUGGESTED EXPERIMENTS AND DEMONSTRATIONS *317*

Section L1 PHOTONS 319

Section L2 ELECTRONS 335

Section L3 ELECTRON WAVES IN ATOMS 342

Suggested time allocation: three weeks

PLAN OF THE UNIT

Section L1 Photons

radio waves (Unit J);
gamma rays (Unit F)

Electromagnetic radiation:
a review of similarities and differences

The photoelectric effect: theory and
evidence

diffraction grating (Unit J)

Light quanta and energy levels

Spectrum and energy levels of hydrogen

potential (Unit E)

Why are energy levels negative?

Light – waves or particles – or both?

energy of wave (Unit D);
probability (Unit K)

Looking ahead: chance and (amplitude)²

Section L2 Electrons

Electron waves

diffraction grating (Unit J)

Electron diffraction pattern of graphite

Section L3

Section L3

momentum, kinetic energy
(Units A and B)

▶ Further use of experimental results:
momentum \times wavelength is constant

Section L1

▶ Electron waves: waves of chance

Section L3

Section L3

Electron waves in atoms

size of atoms (Unit A)

▶ A reminder about atoms

standing waves (Unit D)

▶ A reminder about standing waves

A crude theory of hydrogen: electron
waves in an atom-sized box

The size of a hydrogen atom

strength of materials (Unit A)

▶ Why solids are hard to compress

protons, neutrons (Unit F)

▶ Wave-in-a-box idea applied to nucleus

A better model for the hydrogen atom

Standing waves in spherical atoms

A possible shape for the electron standing
wave in a hydrogen atom

Algebraic version of the $1/n^2$ rule

Other uses of wave mechanics

INTRODUCTION

This Unit is concerned with what is arguably the major innovation of twentieth century physics – quantum theory. Its aims are necessarily modest, but we hope that students will at least come away with an understanding of how quantum ideas can explain some key facts about atoms: their size, their stability, and the existence of energy levels. The example chosen is of course the hydrogen atom.

The Unit starts by re-examining some apparently familiar concepts. Section L1 introduces the notion that light – until now seen to behave as a form of wave motion – has particle-like properties. Major topics of the Section are the photoelectric effect, Planck's constant, spectra and how energy levels may be inferred from them, and the $1/n^2$ rule for the energy levels of a hydrogen atom.

Section L2 deals with the wave-like behaviour of electrons, introduced by electron diffraction, which can be demonstrated in the school laboratory. Planck's h appears again in $mv = h/\lambda$.

In Section L3 these ideas are used in building a theory of the hydrogen atom. The first, crudest attempt treats the atom as a box. It can be shown that if the size of the box is about 10^{-10} m, then a standing wave representing a bound electron can be fitted into the box. But much smaller or much bigger atoms are not possible.

A more sophisticated treatment takes note of the $1/r$ variation in potential, with its consequence that the electron's kinetic energy, and hence wavelength, must vary. A computer program based on a simplified form of Schrödinger's equation can be used to emphasize that the only possible energy levels are those that give rise to standing electron waves in the atom, that these levels are very sharply defined, and that they follow the $1/n^2$ rule.

It would be a great pity if students finished this Unit – and the course – thinking that wave mechanics can do no more than explain some facts about the hydrogen atom! The Unit ends by pointing to the relevance of wave mechanics in a wide variety of topics in chemistry and physics.

THE PLACE OF THE UNIT IN THE COURSE

This Unit is planned to be the last one in the course, and as such is intended to form an end-point that draws together many earlier ideas. These include:

the size of atoms
momentum, kinetic energy

Unit A, 'Materials and mechanics'

charge on an electron	Unit B, 'Currents, circuits, and charge'
standing waves	Unit D, 'Oscillations and waves'
inverse-square law, electric potential	Unit E, 'Field and potential'
nuclear atom	Unit F, 'Radioactivity and the nuclear atom'
diffraction	Unit J, 'Electromagnetic waves'

It would certainly be very difficult to teach this Unit in advance of any of those listed. The notion that an atom has a series of sharply defined energy levels, and that atoms exchange energy in quanta, is used in Unit K, 'Energy and entropy'.

LIST OF SUGGESTED EXPERIMENTS AND DEMONSTRATIONS

L1	Demonstration	Detection of electromagnetic radiation <i>page 319</i>
L1a		Radio waves <i>319</i>
L1b		Gamma rays <i>320</i>
L2	Demonstration	Photoelectric effect <i>320</i>
L2a		Demonstration of photoelectric effect with zinc plate and gold-leaf electroscope <i>320</i>
L2b		Simple photoelectric cell using magnesium ribbon <i>322</i>
L3	Demonstration	Colour of light and energy of photoelectrons <i>324</i>
L4	Demonstration	The spectrum of mercury vapour <i>327</i>
L5	Optional experiment	The hydrogen spectrum <i>329</i>
L6	Experiment	Interference of single photons <i>332</i>
L7	Demonstration	Electron diffraction by graphite <i>335</i>
L8	Optional experiment	Optical analogue of electron diffraction <i>337</i>

L9	Demonstration	Standing waves 343
L9a		Standing waves on a rubber cord 343
L9b		Vibrations in a rubber sheet 344
L10	Demonstration	Standing waves with variable wavelength 350
L10a		Rubber cords of different thickness 350
L10b		A hanging chain 351
L10c		Rubber strip of varying width 351

SECTION L1

PHOTONS

ELECTROMAGNETIC RADIATION: A REVIEW OF SIMILARITIES AND DIFFERENCES

One might begin by reviewing what has already been learned in the course (in Units D, 'Oscillations and waves', and J, 'Electromagnetic waves') about waves and about electromagnetic radiation. The continuity of the spectrum from radio waves through visible light to X-rays and gamma rays was stressed. All travel at the same speed in vacuum; all behave like waves; but the range of wavelengths is vast (from kilometres down to picometres, 10^3 m to 10^{-12} m).

Different regions of the spectrum have different properties. Microwaves (of about 12.5 cm wavelength) are useful for cooking because radiation of that wavelength is strongly absorbed by the water molecule; but water does not absorb visible light, so to our eyes it looks transparent. X-rays pass through soft tissue but not bone or metal; light doesn't pass through either. Radio waves can pass through brick and concrete walls into our homes, but we need an outside aerial on a metal motor car.

The different ways in which different regions of the electromagnetic spectrum are detected suggest other differences: aerials and tuned circuits respond to continuous variations in an electric or magnetic field, but a Geiger counter detects gamma rays as a series of discrete events. A simple demonstration shows that radiation at different ends of the electromagnetic spectrum is detected differently.

DEMONSTRATION

L1 Detection of electromagnetic radiation

L1a Radio waves

ITEM
portable radio

A simple demonstration with the radio tuned in to a broadcast will serve to remind students that the radio's aerial, or ferrite rod, is responding to a continuously changing electric or magnetic wave. If possible, emphasize this by showing how the strength of signal depends on the orientation of the aerial or ferrite rod.

One can also show that the radiation does not pass through even a thin sheet of metal, or metal gauze.

L1b Gamma rays

ITEM NO.	ITEM
130/6	gamma GM tube
130/3	GM tube holder
130/1	scaler
195/1	pure gamma source
196	source holder

Safety note: See the notes about the use of radioactive sources in *Teachers' guide 1* (page 352) and the students' laboratory notes (*Students' guide 1*, page 377) for Unit F, 'Radioactivity and the nuclear atom'.

Simply put the gamma source close enough to the GM tube so that individual counts can be detected (preferably by a click of the loud-speaker). The count rate should be low enough for it to be obvious that the clicks are irregular.

The behaviour of the GM tube may suggest – though it certainly does not prove – that it is responding to individual 'particles' in the gamma rays – not to a continuously changing field, as the radio seemed to be. Similarly, photographic film which responds to light because electrons are excited to higher energy levels, *might* suggest that visible light consists of particles which can excite or ionize electrons in atoms.

More conclusive evidence about the particulate nature of light depends on more subtle evidence from the photoelectric effect. Students familiar with the effect already may not need to see the next demonstration, and could go straight on to the quantitative version, demonstration L3.

DEMONSTRATION

L2 Photoelectric effect

These two simple alternative demonstrations are intended just to show that light of a short enough wavelength can eject electrons from a clean metal surface.

L2a Demonstration of photoelectric effect with zinc plate and gold-leaf electroscope

(REVISED NUFFIELD PHYSICS *Teachers' guide Year 5*, demonstration 92)

Safety note: Make sure that the students do *not* look directly at the ultra-violet lamp.

ITEM NO.	ITEM
51A	gold-leaf electroscope
190	zinc plate attachment
1153	wire mesh, about 5 cm diameter
14	e.h.t. power supply
1153	piece of fine emery cloth
1153	glass plate, about 25 cm square
189	ultra-violet lamp
503-6	retort stand base, rod, boss, and clamp
1000	leads

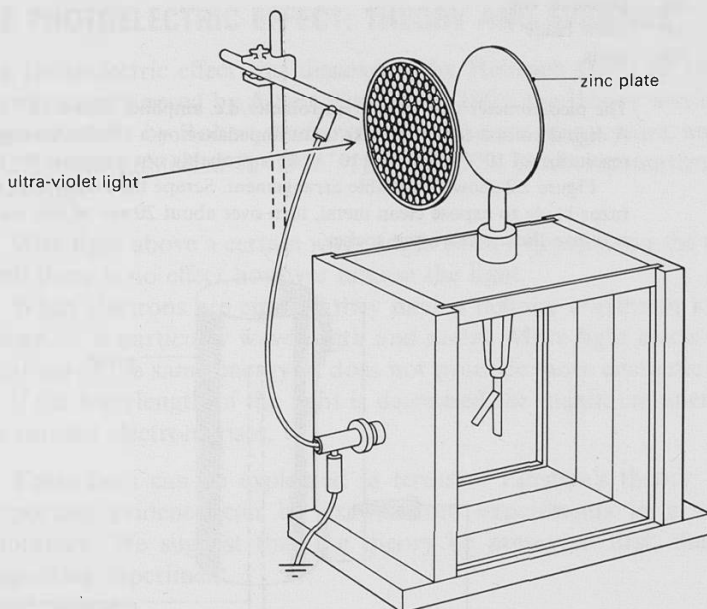


Figure L1

Demonstration of photoelectric effect with zinc plate and electroscope.

Thoroughly clean the zinc plate with emery cloth and attach it to the electroscope. Support the wire mesh a few centimetres away from the zinc plate. This mesh is connected to the case of the electroscope which is earthed.

Use the e.h.t. supply to charge the plate on the electroscope negatively. Then illuminate the plate with the ultra-violet lamp. Observe the effect on the gold leaf.

Repeat the experiment with the plate and electroscope charged positively.

Finally, repeat the experiment with the plate negatively charged again, but when the charge is clearly seen to be leaking away, interpose a sheet of glass between the light source and the charged plate.

L2b Simple photoelectric cell using magnesium ribbon

ITEM NO.	ITEM
1516	picoammeter (see note below)
1033	cell holder with four cells
189	ultra-violet lamp
1156	magnesium ribbon, 100 mm long
1153	glass plate, about 25 cm square
1155	wire gauze, 70 mm \times 60 mm, e.g. 20 mesh copper
503-6	retort stand base, rod, boss, and clamp
52K	crocodile clip
1153	razor blade
1000	leads

The picoammeter may be an electrometer/d.c. amplifier with a $10^{11} \Omega$ input resistor. A digital voltmeter (e.g. 10 M Ω input impedance on a 199.9 mV range, giving a resolution of $10^{-4} \text{ V}/10^7 \Omega = 10^{-11} \text{ A}$) is probably not adequate for this experiment.

Figure L2 shows a suitable arrangement. Scrape the magnesium ribbon with the razor blade to expose clean metal, turn over about 20 mm at one end, and push this end into the meter's input socket.

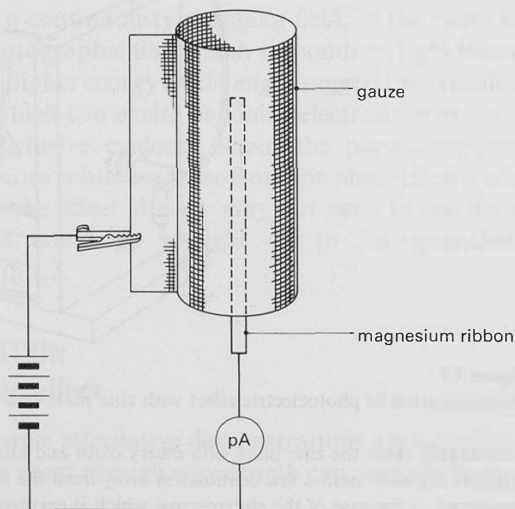


Figure L2

Demonstration of photoelectric effect with magnesium ribbon.

Make a gauze cylinder 60 mm tall by wrapping the gauze round a former 15 to 20 mm in diameter. Turn out the last few millimetres, as shown in figure L2, to give a means of clamping the cylinder over the ribbon, and of making connection to the cylinder. The gauze cylinder does not touch the magnesium ribbon.

There should be a current of about 5 pA when the ultra-violet lamp is 20–30 cm from the magnesium ribbon.

A glass plate absorbs the ultra-violet, and the current falls to zero. A copper or iron rod in place of the magnesium gives no photoelectric effect. Replacing the copper gauze with iron gauze makes no difference.

Some types of electrometer may be provided with a zinc plate and a gauze collector, with which the effect can be obtained using visible light.

THE PHOTOELECTRIC EFFECT: THEORY AND EVIDENCE

The photoelectric effect was discovered by Heinrich Hertz in 1888; a theory was proposed by Albert Einstein in 1905; the theory was tested experimentally by Robert Andrews Millikan over many years, and his results were published in 1916. The *Students' guide* summarizes the essentials:

- a With light above a certain wavelength (which depends on the metal used) there is no effect however intense the light.
- b When electrons are emitted they have a definite maximum kinetic energy for a particular wavelength and metal. More light ejects more electrons of the same energy, it does not produce more energetic ones.
- c If the wavelength of the light is decreased the maximum energy of the emitted electrons rises.

These facts can be explained in terms of Einstein's theory; some supporting evidence can be drawn from experiments in a school laboratory. We suggest that the theory be presented first, then the supporting experiment.

Theory: Einstein's equation

The maximum kinetic energy of an electron ejected from a metal surface by light is

$$K.E._{max} = hf - \phi$$

Essential to the theory is the idea that light energy is quantized in photons of energy $E = hf$. Energy ϕ is lost by an electron when it leaves the metal surface. ϕ depends on the metal and the state of its surface.

A more careful experiment using a specially constructed photocell allows one to test Einstein's relationship and to estimate a value for the constant h .

DEMONSTRATION

L3 Colour of light and energy of photoelectrons

ITEM NO.	ITEM
1068	parallel beam projector
59	l.t. variable voltage supply
69	high-dispersion prism
1074	photoelectric cell
1509	high-impedance voltmeter
1151	capacitor, 10^{-9} F
1033	cell holder with one cell
1153	card with slit
1167/1E	set of stops
1000	leads

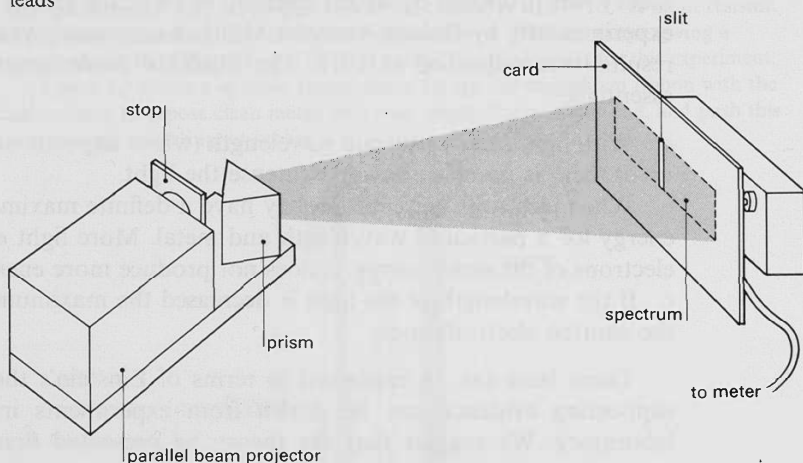


Figure L3
Projection of a spectrum on a photocell.

Figure L3 shows an arrangement for projecting a spectrum of white light on the photocell. A slit in a card held over the photocell allows only a narrow range of wavelengths to enter the cell. Figure L4 shows a suitable circuit.

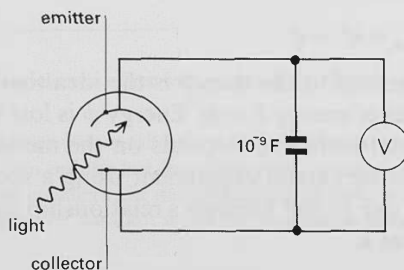


Figure L4
Circuit for a test of the photoelectric effect.

The slit should be about 2 mm wide, and should be centred on the aperture of the photocell. The parallel beam projector may be overrun by up to 30 per cent to obtain a bright spectrum. The spectrum should be formed at minimum deviation. Room lighting should be dim, and the photocell shielded from stray light.

The photocell circuit may contain a battery and a potentiometer for reducing the cell current to zero by applying an external p.d. to it. In the method suggested here, these are not used; the cell develops its own p.d. across itself and the capacitor.

When connecting the meter to the photocell, join the positive going input to the potassium emitter. (If using an electrometer, remove any input resistor so that the electrometer functions as a voltmeter of the highest possible input resistance, and adjust the sensitivity so that an input of 1.5 V gives a full scale deflection on the output meter.)

Variation with colour Start with the dark region of the spectrum beyond the red over the slit, and with the meter short-circuited. Then switch the meter to read the p.d. across its input terminals, when no appreciable reading should appear. Sweep the spectrum slowly across the slit, allowing time for the p.d. to rise at each step. The reading rises through the blue and into the ultra-violet. Beyond the ultra-violet it should stay nearly constant, even if the light is cut off, because the only way for the capacitor to discharge is through the high-impedance meter. If the meter is momentarily short-circuited, the reading falls to zero and stays there, if no radiation is entering the cell.

Warning A seemingly paradoxical result can be obtained by sweeping the spectrum back from blue to red, when the high reading falls steadily, though one would expect it not to, since low energy electrons produced by red light ought not to be able to reach the collector. The reason is that some potassium gets on to the collector, and it emits some electrons which flow the 'wrong' way. It is probably best to avoid the point by not sweeping the spectrum across the slit from blue to red.

To estimate the Planck constant, h , note the meter readings V_1 and V_2 first when red light (f_1) and then when violet light (f_2) fall on the cell. Estimates of the wavelengths give an acceptable value of h , using $h = e(V_2 - V_1)/(f_2 - f_1)$.

Variation with intensity Shine the blue part of the spectrum on the slit. Reduce the intensity by placing stops over the lens of the projector, with the stops exactly over the centre of the lens (since an off-centre stop produces a spectrum in a different place and alters the colour of the light falling on the slit). The p.d. should change by less than 10 per cent, though the intensity has changed by a much larger factor. If the intensity is reduced very much, the p.d. will fall much more, as the meter resistance becomes comparable to that of the photocell. The point needs to be admitted openly.

Planck's constant

When Einstein proposed his theory of the photoelectric effect in 1905 the idea that energy might be quantized had already been proposed five years earlier by Planck in an explanation of black-body radiation. That story is not part of our course, but students may wonder why the constant in Einstein's equation is named after someone else.

Time effects

The fact that there is no detectable delay in the emission of photoelectrons, even in dim light, helps to confirm the story. In one experiment performed in 1912, light of intensity less than $10^{-10} \text{ W m}^{-2}$ produced electrons with energy of the order of 10^{-19} J , with no detectable delay.

On a wave model, a wave will deliver energy into an area of the order of the square of its wavelength; certainly less than 10^{-12} m^2 in this instance (wavelength $\approx 5 \times 10^{-7} \text{ m}$). The wave delivers energy to this area at a rate of about $10^{-10} \text{ W m}^{-2} \times 10^{-12} \text{ m}^2 = 10^{-22} \text{ W}$. Using the wave model an electron could not obtain energy from the light more rapidly than this. To obtain 10^{-19} J it would be necessary to wait 1000 seconds: over a quarter of an hour. Any one such area, on a photon picture, must wait an average of a quarter of an hour between photons, but in a surface of several square centimetres some such areas will receive photons with as little delay as one pleases, and emission starts at once. On a wave model, all areas must wait together and no electron can emerge for a quarter of an hour.

Questions and reading

Questions 1–9 are on the photoelectric effect and $E = hf$. Before answering questions 2–4 students should read Einstein's own explanation of the photoelectric effect, on page 308 of the *Students' guide*. MILLIKAN, in *The electron*, Chapter 10, describes his test of Einstein's theory.

LIGHT QUANTA AND ENERGY LEVELS

We now need to develop the idea (already used in Unit K, 'Energy and entropy') that atoms gain or lose energy only in discrete lumps and link it with the new idea that light energy consists of parcels of size hf . The link between these two ideas is via spectra. Although the hydrogen spectrum is the one which will be treated in detail later in this Unit, we start by looking at the mercury spectrum, which is easier to demonstrate. Measurements of wavelength and use of $E = hf$ show that mercury emits a light quantum of energy $7.84 \times 10^{-19} \text{ J}$, showing that

among its energy levels mercury has two that differ by $7.84 \times 10^{-19} \text{ J}$ (4.90 eV).

DEMONSTRATION

L4 The spectrum of mercury vapour

ITEM NO.	ITEM
1071	mercury discharge lamp
1073	concave reflection grating
1153	screen with slit
1153	strip of green fluorescent paper, 20 mm wide, 0.5 m long
503-6	retort stand base, rod, boss, and clamp
1156	a little mercury in a polythene bottle
1155	microscope slide
191/2	fine diffraction grating

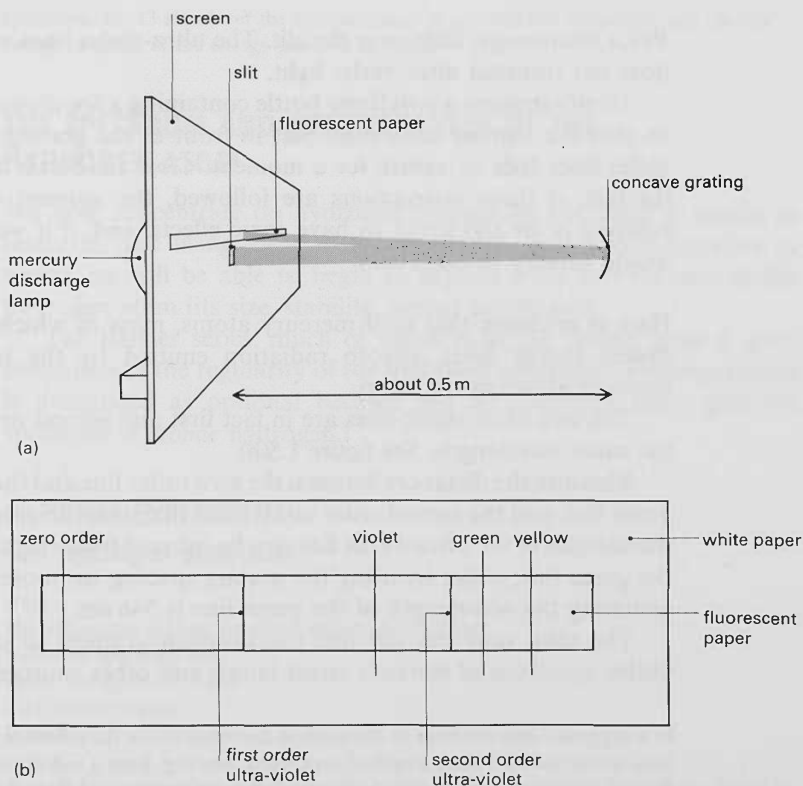


Figure L5

(a) Production of mercury spectrum using a reflection grating.

(b) Spectrum of mercury using fluorescent paper to detect ultra-violet light.

Safety note: This experiment involves ultra-violet light and mercury vapour: both are potentially dangerous. Please observe the safety precautions in the following instructions.

Figure L5(a) shows a suitable arrangement. A screen, 0.3 m high and 0.6 m wide, is placed in front of the mercury discharge lamp, with a slit 1 mm wide opposite the aperture in the lamp casing. A strip of green fluorescent paper is pinned across the screen above the slit, on top of white, non-fluorescent paper. *The screen must be large enough to protect the class from stray ultra-violet light, and no student should look directly at the lamp.*

Fix the grating at a distance from the screen equal to its radius of curvature (about 0.5 m), so as to cast a spectrum back on the screen, with the zero order image of the slit in focus and just above the slit. Tilt the grating so that the diffracted orders lie over the horizontal strip of fluorescent paper, but fall partly on the white paper, as in figure L5(b). Any ultra-violet lines appear only on the fluorescent paper, visible lines being seen on both. Run the lamp at a low level at this stage.

Put a microscope slide over the slit. The ultra-violet lines vanish: glass does not transmit ultra-violet light.

Gently squeeze a polythene bottle containing a few drops of mercury so that the vapour comes out just in front of the grating. The ultra-violet lines fade or vanish for a moment. Close the bottle immediately. (In fact, if these instructions are followed, the amount of mercury released is far too small to have toxic effects, and, if it were inhaled, would rapidly be excreted.)

Here is evidence that cold mercury atoms, most of which are in the lowest energy level, absorb radiation emitted by the hot, excited mercury atoms in the lamp.

The two ultra-violet lines are in fact first and second order lines of the same wavelength. See figure L5(b).

Measure the distances between the zero order line and the first order green line, and the *second order* ultra-violet line near the green line. The wavelength of the ultra-violet line can be inferred from its position near the green line, either by using the grating spacing, or, more simply, by assuming the wavelength of the green line is 546 nm.

The class may use the fine transmission gratings to observe the visible spectrum of mercury street lamps and other sources.

In a suggested improvement to the method described above the reflection grating is fixed to the end of a short length of cardboard tube (e.g. from a roll of paper towel). Rubber bellows and the polythene bottle containing mercury are arranged as in figure L6. Mercury vapour stays trapped in the space in front of the grating, giving plenty of time to observe its effect on the spectrum.

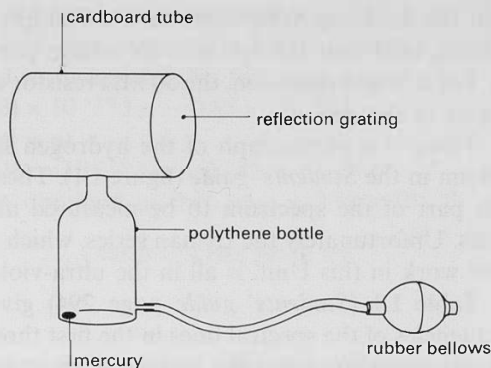


Figure L6

Questions

Questions 10–13 are about the interpretation of spectral line frequency, and photon energy in terms of the energy levels of mercury.

THE SPECTRUM AND ENERGY LEVELS OF THE HYDROGEN ATOM

We now concentrate on hydrogen because its spectrum is simple to interpret, and using ideas to be introduced later about electrons as waves, we will be able to begin to explain some key features of the hydrogen atom (its size, stability, energy levels, etc.).

The Balmer series, much of which is in the visible, gives a good indication of the regularity of the hydrogen spectrum. (The experiment is designated as optional because not all hydrogen tubes give the spectrum of *atomic* hydrogen.)

OPTIONAL EXPERIMENT

L5 The hydrogen spectrum

ITEM NO.	ITEM
191/2	fine diffraction grating (or direct vision spectroscope)
193/2	hydrogen spectrum tube
194	holder for spectrum tubes
14	e.h.t. power supply
1000	leads

Not all hydrogen tubes give the atomic hydrogen line spectrum. The trouble seems to be due to age. The demountable discharge tube (item 144) can be filled with hydrogen from a balloon, which has previously been filled from a hydrogen cylinder. The tube is then pumped out (vacuum pump, item 13) until the spectrum appears (with e.h.t. supply connected).

Run the discharge tube from the e.h.t. supply. Look at it through the grating, held near the eye with its rulings parallel to the tube.

For a bright spectrum, the $50\text{ M}\Omega$ resistor in the e.h.t. supply should not be in the circuit.

There is a photograph of the hydrogen spectrum from 656 nm to 389 nm in the *Students' guide* (figure L1). These lines are all part of the first part of the spectrum to be measured and analysed, the Balmer series. Unfortunately the Lyman series, which is the key to much of the later work in this Unit, is all in the ultra-violet.

Table L1 (*Students' guide* page 294) gives the wavelengths and frequencies of the spectral lines in the first three of the families which it is convenient to group the hydrogen spectrum into. By a few simple questions in class establish the fact that each frequency in the Balmer series is equal to the *difference* between two frequencies in the Lyman series (and the frequency of a Paschen line is equal to the difference between two Balmer frequencies). A few simple sums, such as those given below, should suffice to make the point.

a What is the connection between the frequencies of the first two Lyman lines (24.659 and $29.226 \times 10^{14}\text{ Hz}$) and the first Balmer line at $4.5665 \times 10^{14}\text{ Hz}$?

b The second Balmer line is at $6.1649 \times 10^{14}\text{ Hz}$. Which two Lyman lines can this be related to?

This line of thought justifies a scheme in which the Lyman series corresponds to transitions to the lowest possible energy level, the Balmer to the first level above, and the Paschen to the one above that. A diagram of the energy levels of a hydrogen atom showing the transitions which give rise to the Lyman, Balmer, and Paschen series is given as figure L2 in the *Students' guide*.

Questions

Questions 14–17 are on the hydrogen spectrum, and the energy levels of the hydrogen atom.

Why are energy levels negative?

Remind students that as an electron approaches a proton it loses potential energy. And since we have chosen to call the potential energy zero when the two are very far apart, the potential energy will be *negative* when the electron is bound to the proton. The highest frequency in the hydrogen spectrum is $32.881 \times 10^{14}\text{ Hz}$. The corresponding energy is $2.18 \times 10^{-18}\text{ J}$, which is the same as the ionization energy, 13.6 electronvolts. This is the energy which must be *added* to the atom to free the electron. When the electron is free the total energy is

zero: when it is bound the energy will be $-2.18 \times 10^{-18} \text{ J}$. An electron falling from one level above the lowest causes a photon of energy $1.63 \times 10^{-18} \text{ J}$ to be emitted. So this level is at $(-2.18 + 1.63) \times 10^{-18} \text{ J} = -0.55 \times 10^{-18} \text{ J}$

The next level is at

$$(-2.18 + 1.938) \times 10^{-18} \text{ J} = -0.242 \times 10^{-18} \text{ J}$$

The numbers obtained in this way, -2.18×10^{-18} , -0.55×10^{-18} , -0.242×10^{-18} , ..., form a series given by $\frac{-2.18 \times 10^{-18}}{n^2}$

Questions

Question 18 is about the total energy of the Earth orbiting the Sun. Questions 19 and 20 are about energy levels of hydrogen and the $1/n^2$ rule.

Interlude: a little philosophy

The $1/n^2$ rule is a simplified form of the rule found in 1885 by Johann Balmer, a Swiss physics teacher. Balmer was one of those who believed that numbers hold the key to understanding the Universe, and as soon as accurate measurements of the hydrogen spectrum were available he found a number pattern to explain the wavelengths. Pythagoras also believed in the power of numbers. One of his discoveries had to do with the musical notes of stretched strings of different lengths: for example, halving the length produces a note an octave higher. We now know that the simple numbers are important in explaining the energy levels of hydrogen atoms and the notes of stretched strings because both arise from standing wave patterns.

The class may be able to suggest other instances where the identification of a number pattern has later been shown to have a physical interpretation – and others where it has not.

Discussion: light – waves or particles – or both?

Diffraction is an example of the wave-like behaviour of light. On a particle model, although each particle can pass through only one slit, one might suppose that many particles passing through a grating simultaneously could somehow affect each other so that many leave at some angles, few at others, and so the usual pattern of varying intensity is built up. But the same pattern is formed even at very low light levels, where calculations show that only one photon can be in the apparatus at any one time. Many experiments of this kind have been made in an attempt to see if the interference effects disappear when there is no possibility of any co-operative effect between photons. Students may be

amused to hear of such an attempt made by G. I. Taylor in 1909. He took diffraction photographs with light so feeble that his exposure time reached 3 months. Rumour has it that he went on holiday during the experiment.

The next suggested experiment is a simplified – and much quicker – version of this type of experiment in which the intensity of light is reduced progressively until it is so low that calculations show that only one photon is in the system at any instant.

EXPERIMENT

L6 Interference of single photons

ITEM NO	ITEM
52A	m.e.s. lamp, 1.25 V, 0.25 A
52C	Worcester circuit board
52D	spring connector with lampholder
52B	cell
1153	fogged photographic film
1153	35 mm slide mounts
191/1	coarse grating
1155	photographic exposure meter
	slide projector

To make the fogged film, pull the film out of its cassette and expose it to daylight for a minute or so. The film can be developed in a tank or dish. Fixing and washing are advisable if the film is to be used in later years.

To calibrate the filters, put an exposure meter in the direct beam of a slide projector, and move it until it reads near maximum. Put a fogged film slide into the projector and, without moving the meter, take the new reading.

When calibrating the filters it will probably be necessary to change the meter range (often this is done by opening a louvre). A reduction of six or seven stops (f -numbers) is typical, giving an intensity reduction of 2^6 or 2^7 (64 and 128 respectively). The meter may read ‘light values’. A change of unity in light value is also a factor of two in intensity. Ideally, filters giving an intensity reduction of about 100 should be prepared, making pairs which on top of one another are calculated to give a reduction of up to 10^4 . (The reductions multiply, of course.) Students can be given ready-calibrated filters.

Students look at the lamp through a diffraction grating and observe the spectrum. Then slides containing fogged film are propped against the lamp, so as to reduce the intensity by a large, known factor (see below). If this factor is large enough, students can calculate that the photons in flight to their eyes are an arm’s length or more behind one another, so that they cross the grating and eye one at a time. At this intensity, in a well blacked out room, the lamp filament is still just visible, and the diffraction spectrum can still be seen. Students need to stand about

half a metre from a lamp so one lamp and one set of filters can serve about four observers.

Question

Question 21 relates to experiment L6.

Films

The PSSC films 'Photons' and, better, 'Interference of photons' are worth showing at this stage, if they can be obtained.

Looking ahead: chance and (amplitude)²

By the end of this Unit students should be able to understand something of how stable states of an atom can be understood in terms of standing waves, where the (amplitude)² of the wave represents the probability of finding an electron. The idea that chance \propto (amplitude)² can be introduced at this stage by applying it to photons.

The argument may be summarized. On the *photon* model, the intensity of light is proportional to the number of photons arriving per unit time. But photons do not arrive regularly: the random rate of arrival is obvious at low light levels and is well demonstrated in figure L3 of the *Students' guide* (and in the film 'Interference of photons'). The *chance* of a photon arriving in a given short time interval depends on the average rate of arrival. So intensity depends on the chance of a photon arriving.

On the *wave* model intensity depends on the square of the wave amplitude. Students have met a similar relationship with respect to springs ($E \propto x^2$), capacitors ($E \propto V^2$), and sound waves (intensity at a maximum in a superposition pattern is *four* times – not double – the intensity due to one source alone).

If the two models are to agree then it must be that the chance of a photon arriving \propto (amplitude)²

SUMMARY OF SECTION L1

'If one does not feel a little dizzy when discussing the implications of Planck's constant, h , it means that one does not know what one is talking about.'

Niels Bohr

It is likely that students, like many physicists, will be puzzled by the apparent need for two seemingly contradictory models of light. The Section could be summarized by a discussion bringing out that while *neither* model is satisfactory on its own, *both* are needed.

Some points that could be made in summary about light (or any electromagnetic radiation):

Light delivers its energy in lumps, or photons, of size $E = hf$, where f is the frequency of the electromagnetic wave.

The energy per photon $E = hf$ explains why different parts of the electromagnetic spectrum have different effects on molecules, atoms, and nuclei.

From the frequency of light emitted or absorbed by atoms we can learn about the energy levels of the atoms. Similarly, gamma ray spectroscopy gives information about nuclear energy levels and infra-red spectroscopy about molecular energy levels.

Photons travel at $3 \times 10^8 \text{ m s}^{-1}$ in vacuum.

A bright beam delivers many photons in a given time; a weak beam delivers few. At very low intensities the irregular rate of arrival of photons is noticeable.

Because the energy per photon depends on the frequency of the radiation there will be fewer photons in a high frequency beam than in a low frequency beam of the same power.

Although emission and absorption is often best explained in terms of photons, the wave model is needed to explain how the light is affected by slits and gratings. There is no satisfactory particle theory that explains this behaviour.

The puzzle of two models for the same phenomenon has given rise to many comments. Perhaps Feynman (see *Students' guide* page 308) best explains why we need both.

Question

Question 22 (for discussion or essay) is about the behaviour of light and models of light.

SECTION L2

ELECTRONS

ELECTRON WAVES

Section L1 showed that light has particle-like as well as wave-like properties. Now we need to explore in what sense electrons, until this stage in the course regarded as particles, exhibit wave-like behaviour. This wave-like aspect of the electron's behaviour is the key to the simplified wave mechanical theory of the atom presented in the next Section.

The *Students' guide* has photographs showing diffraction effects due to X-rays and due to electrons (figures L4, L5, and L6), chosen to allow direct comparison and to emphasize the electron's wave-like behaviour.

More direct evidence can come from a demonstration which shows that an electron beam is diffracted by a layer of graphite, and demonstrates the effect of varying the energy (momentum) of the electrons.

DEMONSTRATION

L7 Electron diffraction by graphite

ITEM NO.	ITEM
197	electron diffraction tube
14	e.h.t. power supply
27	transformer
50/1	cylindrical magnet
	ruler
1000	leads

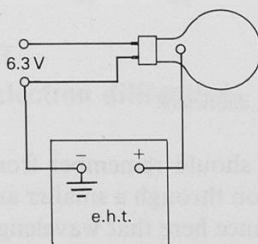


Figure L7

Circuit for electron diffraction tube.

The anode at the side of the tube is connected to the e.h.t. positive terminal, not using the 50 M Ω resistor. The negative terminal is earthed and connected to one of the filament sockets in the tube base. The filament requires 6.3 V.

Two rings should be visible at voltages between 3 kV and 5 kV. As the voltage is raised the rings shrink in size, and become brighter. The ring diameter may be measured with a rule marked in millimetres.

A magnet placed near the neck of the tube will bend the electron beam and shift the whole pattern sideways, reassuring evidence that we are still dealing with electrons previously familiar as charged particles.

Measure the ring diameters at several voltages; the ratio should be close to $1.73 (\sqrt{3})$ in each case. From the geometry of the tube (figure L8) the angle of diffraction is proportional to the diameter of the ring.

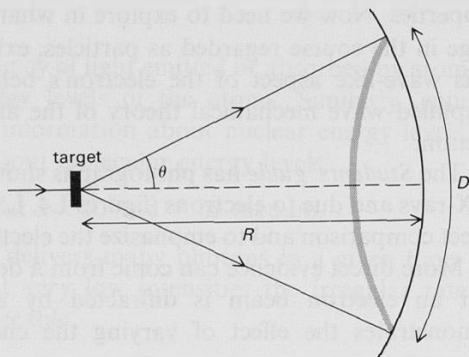


Figure L8

Geometry of electron diffraction tube ($\theta \approx \sin \theta \approx D/2R$).

Table L1 gives sample results.

Electron gun p.d./V	D_1/mm	D_2/mm	D_2/D_1
5000	26	45	1.73
4000	28.5	49.5	1.74
3000	34	56	1.65
Average			1.71

Table L1

Diffraction ring diameters.

Students should remember from Unit J, 'Electromagnetic waves', that diffraction through a smaller angle means a shorter wavelength. So there is evidence here that wavelength *decreases* as accelerating voltage *increases*.

Television or video

'Electron diffraction' (programme 5 in the Granada Television series *Experiment: physics*) is a filmed version of the experiment described here, and involves students in making measurements from the screen.

EXPLAINING THE ELECTRON DIFFRACTION PATTERN OF GRAPHITE

To account for the formation of the rings by a layer of graphite – which is necessary if de Broglie's $mv = h/\lambda$ is to be tested – needs a little care.

From X-ray diffraction a layer of graphite is known to consist of carbon atoms hexagonally arranged as in figure L8 of the *Students' guide*, page 299. The hexagons have sides 1.42×10^{-10} m long.

The target in the electron diffraction tube is a thin graphite film perpendicular to the electron beam. Each layer consists of many small areas in which the carbon atoms are regularly arranged, as in *Students' guide* figure L8. These many small areas are randomly oriented relative to each other. And the lines of carbon atoms in one layer are not necessarily parallel to those in the next layer, and so on.

Each small ordered layer acts like a two-dimensional diffraction grating, the net effect being the combined effect of many such gratings all in one plane but oriented at every possible angle in that plane.

A full treatment of two-dimensional diffraction is not simple. In what follows, we have treated a two-dimensional array of atoms as if parallel rows of atoms behave like parallel slits in an ordinary grating. Such a treatment is only approximate, but it happens that the system under discussion has too poor a resolution for any difference to be detected.

Some feeling for how the rings in the electron diffraction tube are produced can come from looking at a point source of light through a fine diffraction grating, while the grating is rotated.

OPTIONAL EXPERIMENT

L8 Optical analogue of electron diffraction

ITEM NO.	ITEM
	<i>either</i>
21	compact light source
27	transformer
	<i>or</i>
92R, 92T	m.e.s. lamp, 2.5 V, 0.3 A, in holder
1033	cell holder with 2 cells
191/2	fine diffraction gratings
1167/3R	colour filter (e.g. green)
1000	leads

The colour filter is placed in front of the light source which is viewed from several metres away with the diffraction grating held in front of the eye. Two spots are seen, one on either side of the central maximum. The

orientation of the spots depends on the direction of the grating's rulings, and as the grating is rotated the spots move round in a circle. If all orientations of the grating were present simultaneously, the spots would join together and a circle of diffracted light would be seen.

Two rings, as seen in the electron diffraction experiment, might be due to first and second order diffraction. But the measurements of the ring diameters which showed the ratio of the two angles of diffraction to be about 1.7 (and not 2) rule this out. In fact the graphite layer acts as a grating with two spacings, each giving rise to diffraction at a characteristic angle.

Questions

Questions 23 to 27 are about diffraction by gratings and the electron diffraction experiment.

Sets of parallel rows of atoms are clearly seen if figure L8 (*Students' guide*) is held at eye level and rotated. The spacings of the rows responsible for the rings seen in the diffraction tube are derived below.

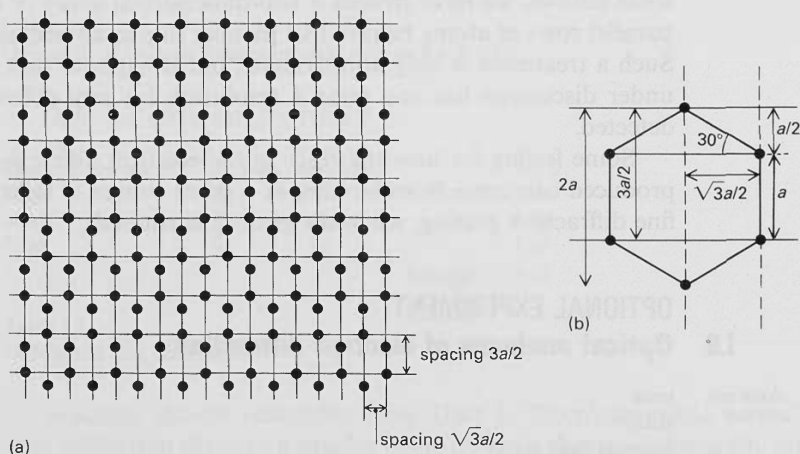


Figure L9

- (a) Two row spacings in a graphite layer.
(b) Geometry of row spacings.

Since a , the side of the hexagon, is $1.42 \times 10^{-10} \text{ m}$ the two sets of rows shown in figure L9 are $\frac{\sqrt{3}}{2} (1.42 \times 10^{-10}) = 1.23 \times 10^{-10} \text{ m}$ and $\frac{3}{2} (1.42 \times 10^{-10}) = 2.13 \times 10^{-10} \text{ m}$ apart.

The ratio of these two spacings is $1:\sqrt{3}$ or 1:1.73.

There are other rows of atoms which might be expected to give rise to diffraction rings: one can easily pick out a set of rows with spacing $a/2$ and another set with spacing a . The rows with spacing $a/2$ would be expected to give a diffraction ring at a larger angle than the $3a/2$ rows. (At 5000 V the $3a/2$ rows give a ring at angle θ , where $\theta = \sin^{-1} 0.167$. So there should be a ring at $\sin^{-1} (3 \times 0.167) \approx 0.5$; $\theta \approx 30^\circ$.) However, the intensity of electron scattering decreases with angle; also, for scattering through larger angles the imperfections in the structure have more effect. These two factors may explain why the wider ring is not seen – or at the most there may be a faint hint of it only. The rows with spacing a will not give rise to their own diffraction pattern: at angles where contributions from these rows would combine to form a maximum (phase difference 2π) there is, for each of the a rows, an $a/2$ row whose contributions will be exactly out of phase (phase difference π).

Questions

Question 28 deals with the geometry of the row spacings. Question 29 is about the wave properties of electrons.

Students should be able to answer confidently (from the grating formula $\lambda = s \sin \theta$) that the ring size ($\propto \sin \theta$) will increase with increasing wavelength, and that the closer spaced rows give the larger (bigger angle) diffraction ring.

Further use of experimental results: momentum \times wavelength is constant

The higher the accelerating potential difference in the electron diffraction tube the *smaller* the rings, hence the shorter the wavelength. The data should allow a test of de Broglie's suggestion that momentum $\propto 1/(\text{wavelength})$. Since it is the *kinetic energy* of the electrons which varies directly with accelerating p.d. it needs to be emphasized that it is momentum \times wavelength which is constant, not energy \times wavelength. Data from the experiment may be good enough to show that the constant in $mv \propto 1/\lambda$ is $6.6 \times 10^{-34} \text{ J s}$, previously met as the constant relating energy and frequency in the photoelectric effect. It is good practice for students to show that momentum \times wavelength and energy/frequency both have the same dimensions, ML^2T^{-1} .

Questions

Questions 30 to 33 are about wavelength, energy, and momentum of electrons.

Historical order and teaching order

We have suggested doing the experiment first, and then showing that the results support de Broglie's suggestion that $mv = h/\lambda$ (for any particle – not just for electrons). In fact de Broglie proposed the idea in his Ph.D. thesis of 1923; experiments by Davission and Germer (U.S.A.) and G. P. Thomson (U.K.) followed within a few years.

Energy and momentum at relativistic velocities

It is strictly true (on existing evidence) that $p = h/\lambda$ for electrons at all velocities. p is the momentum; if necessary in relativistic form. There is a corresponding rule for photons. If, using $E = mc^2$ naively, one says that, for photons, $p = mc = E/c$, then one gets $p = hf/c = h/\lambda$, which is the same rule as before. A more rigorous way is to use the exact relativistic rule connecting energy, momentum, and rest mass m_0 :

$$E^2 = p^2 c^2 + m_0^2 c^4$$

For a photon, the rest mass m_0 is zero, so the rule simplifies to $E = pc$. When the relativistic rule is used for electrons, putting

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

and expanding the square root as a series in v^2/c^2 gives

$$E \approx m_0 c^2 + \frac{1}{2} m_0 v^2 \quad \text{for } v \ll c$$

Non-relativistic formulae will cope with slow electrons, but never with photons, whose velocity is always c .

Practice with energy, momentum, and wavelength

In Section L3 students will need to move confidently from kinetic energy, via momentum, to wavelength, and vice versa. Some practice now should help to give that confidence.

They should know that K.E. = $\frac{1}{2}mv^2$; that momentum = mv ; and that $\lambda = h/mv$. And they should, for example, be able to work out the approximate energy of a neutron ($m = 1.7 \times 10^{-27}$ kg) which has a wavelength of 10^{-10} m (about 10^{-20} J). They should be able to convert this to electron volts ($10^{-20}/1.6 \times 10^{-19} = 1/16$ eV) and know that this is about the energy of a molecule at room temperature.

Questions

Questions 34, 35, and 36 are about energy, momentum, and wavelength. Question 37 is about the use of electron scattering to estimate nuclear sizes.

Film

The PSSC film 'Matter waves' is worth showing, if it can be obtained.

INTERPRETATION OF ELECTRON WAVES: WAVES OF CHANCE

Students may wonder ‘What are the electron waves waves of?’. For electromagnetic radiation we have been able to identify the waves with varying electric and magnetic fields. No answer like this can be given for electrons. Electrons are not waves, they are particles, but waves are needed to explain how they travel. Some writers speak of ‘pilot waves’ or ‘ghost waves’ – waves which tell the electron how to travel.

The interpretation of wave amplitude is exactly as it was for light. There intensity \propto chance of photon arriving \propto (amplitude)². Here chance of electron arriving \propto (amplitude)².

So we can work out the chance of finding an electron at a certain place by calculating the wave amplitude there – paying attention to phase if the contributions of several waves have to be added – and then squaring. The idea of chance \propto (amplitude)² is needed for the last Section of the Unit.

Reading: waves and particles

As at the end of the last Section students may be left with the uncomfortable feeling that physics can no longer give the kind of definite answer they have come to expect. Much has been written about ‘wave–particle duality’, and the work of this Section might well end with some guided reading. Suggested passages – and questions for students to keep in mind while reading – are given in the *Students’ guide* (pages 309 to 311). Such reading could well lead to writing an essay.

Questions

Questions 38, 39, and 40 also require rather longer written answers.

SECTION L3

ELECTRON WAVES IN ATOMS

A REMINDER ABOUT ATOMS

In this Section the ideas introduced in the first two Sections of the Unit, together with other ideas from earlier in the course, will be used to explain some of the properties of atoms. So, a reminder of what is to be explained is appropriate.

1 The stability of atoms Substances may change – but atoms do not. When water freezes or iron rusts it is only the arrangement of the atoms that changes – not the atoms themselves. New atoms of an element can be made (in nuclear reactors or by radioactive decay) and they behave just like any other atoms of that element. Originally, atoms were thought of as ‘uncuttable’, they could not be further subdivided. But now we know that atoms do have parts. There are electrons and there is a nucleus composed of neutrons and protons. And yet each atom of a particular kind is put together in exactly the same way as all the other atoms of that kind. Even different isotopes of an element behave in the same way chemically. Evidently the number of neutrons in the nucleus does not affect the behaviour of the outer electrons which determine the chemistry of an element.

2 The size of atoms We know that each atom has a characteristic size. For hydrogen it is of the order of 1.0×10^{-10} m. What prevents the electrical attraction between proton and electron making it collapse?

3 Energy levels Every atom has characteristic energy levels, revealed accurately by spectroscopy. For hydrogen these levels are at $-2.18 \times 10^{-18}/n^2$ J, and no hydrogen atom needs more than 2.18×10^{-18} J to be ionized.

Quantum theory – especially the idea that particles have wave-like properties – is the clue to some of these puzzles.

Question

Question 41 asks students to review the evidence for the stability, size, and energy levels of atoms.

The Bohr theory: not part of the course

Whether they have been taught it formally or not, students are likely to have a picture of the atom as a miniature Solar System with electrons orbiting the nucleus. There are, of course, difficulties with this model: why are the orbits stable and not perturbed by collisions between atoms? Why doesn't the atom lose energy by radiation, as it should because of the (centripetal) acceleration of charged particles? The Bohr theory attempted to deal with these difficulties in its own way. But that theory has now been superseded and is not intended to be part of this course.

A REMINDER ABOUT STANDING WAVES

A wave confined to a fixed space is a standing wave. So if electron waves are to be bound to atoms they must form standing waves. A reminder of familiar standing waves on strings, etc. is appropriate.

DEMONSTRATION

L9 Standing waves

L9a Standing waves on a rubber cord

(See also demonstration D16)

ITEM NO.	ITEM
1109	signal generator
1060	vibrator
134/2	xenon flasher
1153	rubber cord, 0.5 m long, 3 mm square cross-section
503-6	retort stand base, rod, boss, and clamp
1153	2 small wooden blocks (to clamp rubber)
44/2	2 small G-clamps
1000	leads

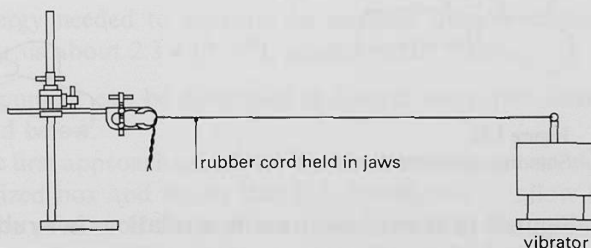


Figure L10

Safety note: When using the xenon flasher, teachers should be aware that frequencies around 7 Hz have been known to cause epileptic fits in certain people.

Figure L10 shows how the rubber cord is stretched between jaws held in a clamp and tied to the vibrator. The cord should not be so taut as to

prevent the vibrator from vibrating freely. White painted spots on a black rubber cord make for an effective demonstration, especially if the cord is viewed both in steady light and when illuminated by a stroboscope.

L9b Vibrations in a rubber sheet

ITEM NO.	ITEM
1109	signal generator
1044	large loudspeaker
134/2	xenon flasher
1153	sheet of rubber
503, 504	3 retort stand bases and rods
505, 506	boss and clamp
1076	large aluminium ring
	rubber band(s)
1000	leads

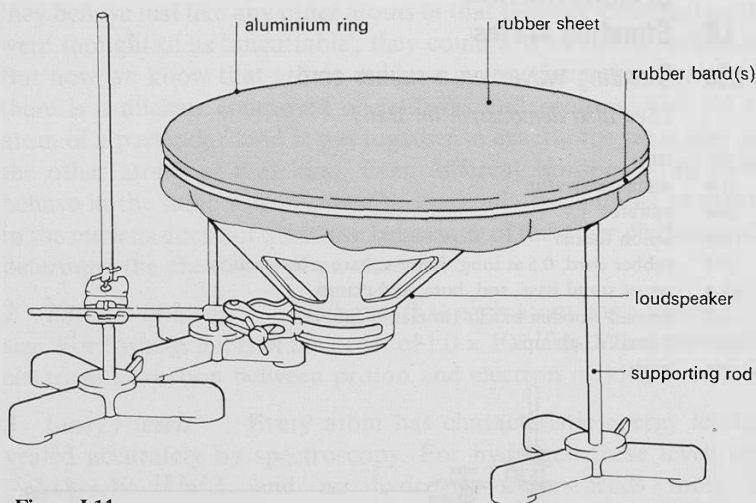


Figure L11
Standing waves on a rubber sheet.

Figure L11 shows how to excite oscillations in a rubber sheet. The sheet is stretched over the large metal ring, so as to be as evenly stretched as possible. Holding the sheet to the ring by a large rubber band makes it easy to make small adjustments to the evenness of the sheet's tension. The large loudspeaker is placed below the ring and rubber sheet. Frequencies in the range 10 to 100 Hz are required, and the larger the power delivered by the loudspeaker, the better. Try a central position of the speaker first, and a low frequency. Raise the frequency gradually, looking and listening for the lowest mode, in which the centre of the

rubber sheet rises and falls. Radial lines drawn on the rubber make the oscillations easier to see. The amplitude may be 10 to 20 mm at the centre.

At a higher frequency, with the speaker off centre, a mode of oscillation can be found in which the rubber surface tilts, one side rising as the other falls, the rim staying fixed of course.

Question

Question 42 is an introductory question about standing waves.

Film loops

'Vibrations of a drum' shows the rubber diaphragm. 'Soap film oscillations' shows standing waves on soap film. Neither is essential, but may be helpful.

A CRUDE THEORY OF HYDROGEN: ELECTRON WAVES IN AN ATOM-SIZED BOX

We now follow up the idea that the electron in a hydrogen atom is associated with a confined standing wave. The argument makes use of the following facts which could be brought out through questioning in class:

- a** an order of magnitude estimate for the size of a hydrogen atom is 10^{-10} m;
- b** wavelength of standing wave with one loop in distance l is $2l$;
- c** an electron of wavelength λ has momentum $mv = h/\lambda$;
- d** if momentum is mv , then kinetic energy $= \frac{1}{2}mv^2$ is $\frac{(\text{momentum})^2}{2m}$;
- e** the energy needed to separate an electron from a proton, given by $e^2/4\pi\epsilon_0 r$, is about 2.3×10^{-18} J, when r is 10^{-10} m.

The argument can be developed in several ways: two alternatives are outlined below.

The first approach calculates the kinetic energy for an electron in an atom-sized box and shows that it is insufficient to allow it to escape from the proton. If the box were much smaller, the potential energy would not be sufficient to stop the electron with its greater kinetic energy flying off. But a much larger atom in which the electron would have much smaller kinetic energy would shrink.

The second approach calculates the total energy, E , of the atom in terms of the radius of the atom, r . By differentiating E with respect to r the value of r for which E is a minimum can be found.

Structured questions in the *Students' guide* deal with both alternative approaches, and each is summarized below.

Alternative A: calculation of kinetic energy

Refer back to the facts listed on page 345.

From **a** and **b** it follows that the standing wave of longest wavelength which can fit into a hydrogen atom has a wavelength which is twice the diameter of the atom, say $\lambda = 4 \times 10^{-10} \text{ m}$.

From **c** we find that

$$mv = h/\lambda = 6.6 \times 10^{-34} \text{ J s} / 4 \times 10^{-10} \text{ m} \approx 1.6 \times 10^{-24} \text{ kg m s}^{-1};$$

and so from **d** we find that kinetic energy is

$$(1.6 \times 10^{-24})^2 \text{ kg}^2 \text{ m}^2 \text{ s}^{-2} / 2 \times 9.1 \times 10^{-31} \text{ kg} \approx 1.4 \times 10^{-18} \text{ J}.$$

From **e** we know that

$2.3 \times 10^{-18} \text{ J}$ is needed to remove the electron from the hydrogen atom.

So an electron with kinetic energy $= 2.3 \times 10^{-18} \text{ J}$ or more could fly away from the proton, but because it has less kinetic energy it is bound.

The next step might be to consider other standing waves in a box of the same size. A wave with two loops would have half the wavelength; twice the momentum; four times the kinetic energy. So the electrical potential energy is no longer enough to bind the electron to the proton. Similar argument shows that an electron couldn't be confined in a much smaller atom.

If the atom 'box' were much bigger – say by a factor of ten – then the electron's kinetic energy would be less by a factor of 100, while the electrical potential energy would change by a factor of 10. Electrical potential energy is a *negative* quantity (and is less negative for a larger atom), and so the total energy for the larger atom turns out greater than for the $1.0 \times 10^{-10} \text{ m}$ atom. By shrinking, the atom loses energy.

Questions

Question 43 presents the structured argument about a $1.0 \times 10^{-10} \text{ m}$ atom box.

Questions 44 and 45 are about the (im)possibility of much smaller or larger atoms.

Alternative B: use of calculus to find radius for minimum energy

Some teachers, and students, may prefer the following approach, which is derived from that given in section 38–4 of FEYNMAN, *The Feynman lectures on physics Volume I*. Whereas Feynman uses the uncertainty principle, we start from the assumption that a one-loop standing wave must fit into the atom.

The approach is based on the reasonable enough assertion that the atom will arrange itself so that its total energy, E , is a minimum; and it uses calculus to find the radius at which it is a minimum.

If the radius of the atom is r , then half a wavelength fits into $2r$; and so $\lambda = 4r$. Hence

$$mv = h/4r, \text{ and}$$

$$\text{kinetic energy } E_k = (\text{momentum})^2/2m = h^2/32mr^2.$$

Potential energy when the electron is at distance r from the proton is given by

$$E_p = -\frac{1}{4\pi\epsilon_0} \times \frac{e^2}{r}$$

$$\text{Total energy, } E = E_k + E_p = \frac{h^2}{32mr^2} - \frac{1}{4\pi\epsilon_0} \times \frac{e^2}{r}$$

The problem is to find the value of r for which the total energy is a minimum

$$\frac{dE}{dr} = -\frac{h^2}{16mr^3} + \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

When $dE/dr = 0$ we find that

$$\frac{h^2}{16mr^3} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

$$r = \frac{4\pi\epsilon_0 h^2}{16me^2}$$

whence $r \approx 1.3 \times 10^{-10} \text{ m}$

Question

Question 46 presents this argument in a series of short questions.

The size of a hydrogen atom

The arguments suggested are crude. One shows that $1.0 \times 10^{-10} \text{ m}$ is about the right size for the hydrogen atom. The second gives a value of $1.3 \times 10^{-10} \text{ m}$. Some students may object that $0.5 \times 10^{-10} \text{ m}$ is closer to the value usually quoted for the radius of the hydrogen atom. While more careful choice of constants might have given a 'better' result by the second method (Feynman seems to use $h/2\pi$) it is probably better not to try to claim too much: the arguments are crude and based on a very simple model and yet give reasonable results.

If pressed one could perhaps argue that because the amplitude of wave we have fitted into the hydrogen atom drops to zero towards the edge, the chance of finding the electron there, which is proportional to (amplitude)², also drops towards zero. The electron is much more likely to be nearer the centre of the atom where the amplitude is greater. A more exact argument, which can be used as the basis for a computer

program shows that the amplitude of a one-loop electron wave does reach a maximum at about 0.5×10^{-10} m.

On to a better model?

Students may rightly point out that the energy needed to remove an electron from a proton, which we have taken to be about 2.3×10^{-18} J depends on how far the electron is from the proton. As the electron moves about in its box the potential energy will vary, and our wave-in-a-box argument has taken no account of this. A more refined model which takes account of the $1/r$ variation of potential can be attempted later (see page 349). That model also explains the existence of other energy levels and correctly predicts their values. But before attempting that more difficult stage it is worth pointing out some other successes for the crude wave-in-a-box argument.

Another effect explained: why solids are hard to compress

Students can be told that the 'wave-in-a-box' argument, though admittedly crude, also explains why, if atoms are really mostly empty space, we don't fall through the chair when we sit down. Why are solids hard to compress?

When we try to compress a solid we are trying to force the atoms closer together: that means shorter wavelengths for the electrons, more momentum, and more kinetic energy. It is only by supplying that extra energy that we can compress the solid. A lot of energy is needed for a small compression.

The wave-in-a-box idea applied to the nucleus

By considering a much smaller box, about 10^{-14} m across, *i.e.* the size of a nucleus, it is easy to show that electrons cannot exist within the nucleus: there is no known force which acts on electrons which could confine them in such a small space against the very large kinetic energy they would necessarily have (about 2×10^{-9} J, or 10^{10} eV). On the other hand, we know that protons do exist in the nucleus. Calculations of their kinetic energy (≈ 10 MeV) give an indication of the strength of the binding force between nuclear particles: this is the strong nuclear force. When a proton captures a neutron to form a deuteron (deuterium nucleus) the energy released in the form of a gamma ray photon is 2.2 MeV. Contrast this with the 13.6 eV photon emitted when a proton captures an electron to form a hydrogen atom.

Knowledge of nuclear energy levels is obtained experimentally by firing particles of known energy at nuclei, in much the same way that knowledge of atomic energy levels can come from experiments in which electrons are fired at atoms, as well as from spectroscopy.

Questions

Questions 47 and 48 are about particles in the nucleus.

DISCUSSION: TOWARDS A BETTER MODEL FOR THE HYDROGEN ATOM

Students should remember from Unit E, 'Field and potential', that the potential energy of an electron near a proton varies as $-1/r$. This is the first step in the argument towards a better model of the hydrogen atom.

The key point to be grasped here is that when we say that the energy is $-2.18 \times 10^{-18} \text{ J}$ (or $-2.18 \times 10^{-18}/n^2 \text{ J}$ for some higher energy level), we mean the *total* energy. We do *not* have a model in which electrons orbit like *planets*, always at – or very nearly at – the same distance from the nucleus. By discussion establish that for the total energy to remain constant, the kinetic energy must decrease as the electron moves away from the nucleus. Rather crudely one might imagine a ball rolling around inside a $1/r$ shaped well, slowing down as it climbs up and gains potential energy. More formally, a diagram such as figure L12 shows how kinetic energy varies with r : kinetic energy is (total energy – potential energy).

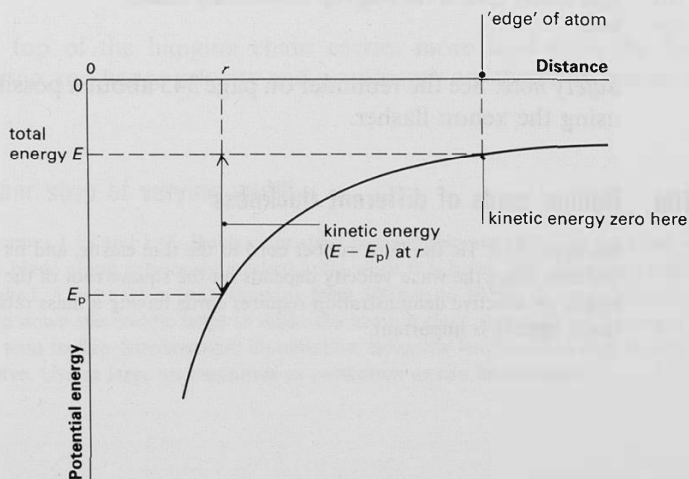


Figure L12

Finding the kinetic energy at distance r .

The edge of the atom is the place where the kinetic energy is zero. We would not expect to find the electron further from the centre than this. (But, as we shall see later, wave mechanics shows that there is a chance of finding the electron at greater values of r .)

Varying the kinetic energy means varying the wavelength (since $E_k = \frac{(mv)^2}{2m}$ and $\lambda = \frac{h}{mv}$). The greater the kinetic energy, the *less* the wavelength of the electron wave will be: according to this argument the electron's wavelength will be low near the centre of the atom, and higher near the edge. But before taking the idea of the standing waves with variable wavelength too seriously in this theoretical situation, students should be given a chance to see some – on cords, chains, and sheets. Since frequency is the same at all parts of the oscillating system, the speed at which a wave travels must be made to vary by making the tension or mass per unit length change.

DEMONSTRATION

L10 Standing waves with variable wavelength

ITEM NO.	ITEM
1109	signal generator
1060	vibrator
134/2	xenon flasher
1153	length of light chain
1153	V-shaped strip of rubber, about 0.5 m long, maximum width about 0.1 m
1153	rubber cord, 0.5 m long, 3 mm square cross-section
1153	light rubber cord, 0.5 m long (e.g. dressmaking elastic)
1000	leads

Safety note: See the reminder on page 343 about a possible hazard when using the xenon flasher.

L10a Rubber cords of different thickness

See figure L13. Tie the thick rubber cord to the thin elastic, and fix one of them to the vibrator. Since the wave velocity depends on the square root of the mass per unit length, an effective demonstration requires cords having a mass ratio of at least four. Good lighting is important.

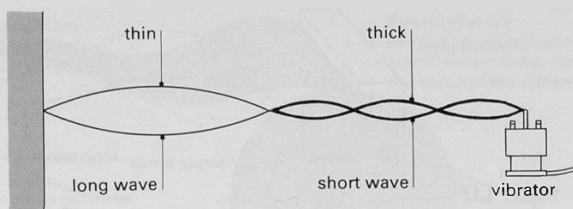


Figure L13
Wavelength depends on thickness of cord.

L10b A hanging chain

See figure L14. Chain sold for securing bath plugs is suitable. It is easiest to swing the top round in a small circle, but it will be clearer that a standing wave is involved if the top is oscillated sideways.

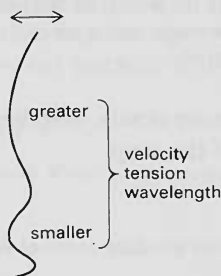


Figure L14
Wavelength depends on tension.

The top of the hanging chain carries more load than the bottom. Tension and hence velocity and wavelength are therefore greater at the top.

L10c Rubber strip of varying width

See figures L15 and L16. Rubber cot sheet is a suitable material. Cut the sheet with a razor blade along previously marked lines, while it is being held down and lightly stretched. A piece 0.5 m long, tapering from 100 mm to 10 mm, is about right. A line drawn down the middle helps to make the motion clear, especially as the edges of the strip tend to flap. Stroboscopic illumination along the length of the strip is very effective. Use as large an amplitude of oscillation as can be managed.

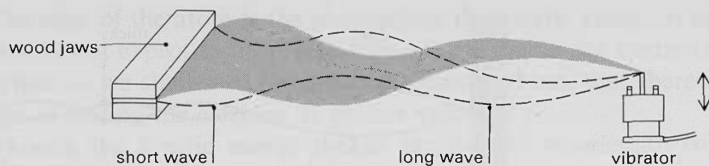


Figure L15

Wavelength depends on mass per unit length of the strip.

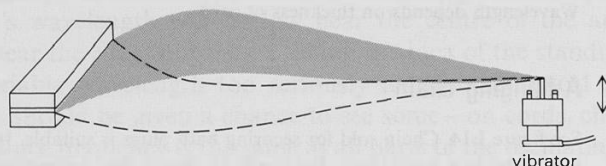


Figure L16

In the lowest mode the maximum displacement is not at the middle, but near the wide end. The curvature varies and is greatest at the wide end of the strip – where the wavelength is shortest.

Waves travel more slowly, and therefore wavelength is least, at the more massive end of the strip.

Question

Question 49 is about standing waves of varying wavelength.

Standing waves in spherical atoms – a piece of geometry

Students know (Section L2) that the chance of finding an electron at a particular place is proportional to the square of the wave amplitude at that place. But, unlike the standing waves seen in the previous demonstration, atoms are not one-dimensional – they are spherical. Two-dimensional standing waves have been seen (demonstration L9), and three-dimensional standing waves can exist. But how is a standing wave in a *spherical* atom to be interpreted? The three-dimensional equation is a rather difficult looking one, but it turns out that it can be very much simplified by a ‘change of variable’. By making the appropriate change of variable we can turn it into a simple one-dimensional equation, whose interpretation turns out to be simple enough.

Imagine a *one-dimensional* standing wave from the centre out along a radius, with amplitude A where the radius is r . Then $A^2 dr$ gives the chance of finding an electron in a spherical shell of thickness dr at radius r (figure L17).

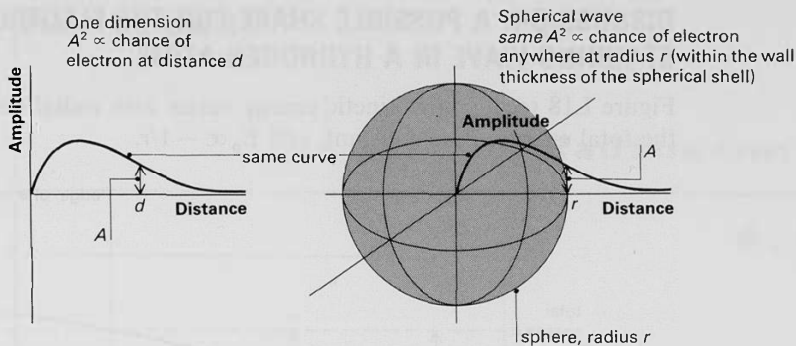


Figure L17

When $r=0$ the volume of this spherical shell ($4\pi r^2 dr$) becomes vanishingly small: so when $r=0$, $A=0$.

Students can be told this; any attempt to justify it would probably confuse. Some justification for teachers is offered below.

Spherical symmetry, A and Ψ

In one dimension, a standing wave Ψ can be represented by

$$\frac{d^2\Psi}{dx^2} = -k^2\Psi$$

In three dimensions, this becomes:

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} = -k^2\Psi$$

Transforming to polar co-ordinates, if there is no variation with angle (spherical symmetry), geometry transforms the above into

$$\frac{1}{r} \frac{\partial^2(r\Psi)}{\partial r^2} = -k^2\Psi$$

Or, writing A for $r\Psi$,

$$\frac{1}{r} \frac{\partial^2 A}{\partial r^2} = -k^2 \frac{A}{r}$$

$$\text{and } \frac{\partial^2 A}{\partial r^2} = -k^2 A$$

Now $A^2 = r^2\Psi^2$, and $\Psi^2 dV$ represents the chance of having an electron in a small volume dV at one place. So $r^2\Psi^2 dr = A^2 dr$ represents (apart from a constant 4π) the chance of having the electron in the volume $4\pi r^2 dr$, that is, at any point in a shell between radii r and $(r + dr)$. A^2 will be zero when $r=0$. (A note in the *Students' guide* points out that many treatments use Ψ rather than the A used in this course. And Ψ , unlike A , is not 0 at $r=0$.)

DISCUSSION: A POSSIBLE SHAPE FOR THE ELECTRON STANDING WAVE IN A HYDROGEN ATOM

Figure L18 recalls how kinetic energy varies with radial distance, r , if the total energy, E , is constant, and $E_p \propto -1/r$.

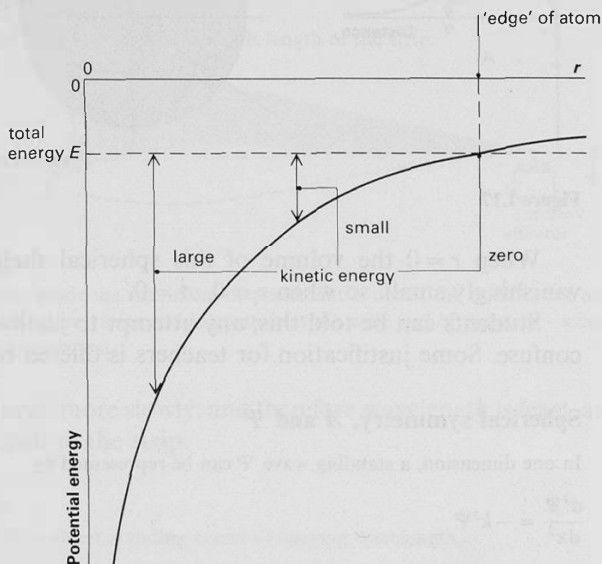


Figure L18

Students also know:

- that a bound electron is represented by a standing wave;
- that the amplitude of the wave, A , is zero at the centre of the atom;
- that the lower the kinetic energy the longer the wavelength (and the smaller the curvature) of the wave.

Questions

Question 50 is about the relevance of standing waves on a spring to electron waves in an atom. Question 51 is based on the points listed above.

Figure L19 shows how the wave amplitude varies with distance when the potential energy varies as $-1/r$.

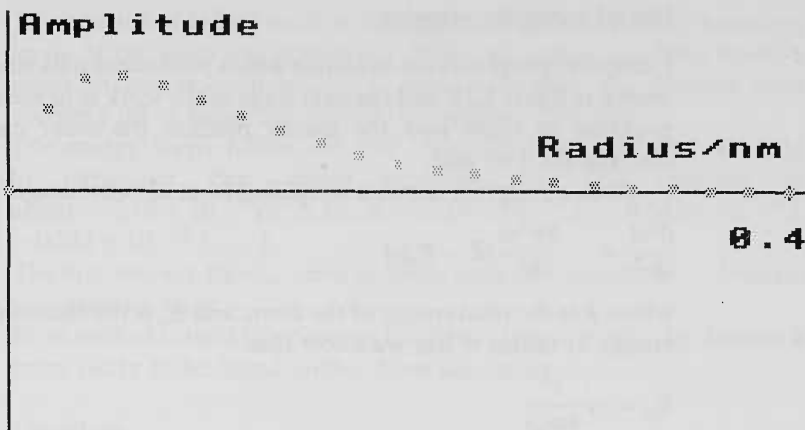


Figure L19

Computer-drawn wave function for hydrogen, at the energy of the lowest level
 $E = -2.18 \times 10^{-18} \text{ J}$.

Some points to emphasize about this wave function:

The energy must be very precisely specified: a very small change in total energy makes the wave amplitude shoot off to very high or very low values, which means that there is no standing wave at these energies: the electron will not be bound.

The wave amplitude reaches a maximum value at just over $0.5 \times 10^{-10} \text{ m}$. This is where the electron is most likely to be found, and corresponds well with quoted values for the radius of a hydrogen atom.

The wave amplitude is not zero at $1.0 \times 10^{-10} \text{ m}$, where the kinetic energy is zero, but continues to drop down slowly and smoothly as r increases. (This apparently absurd idea, that there is a chance – albeit a small one – of a particle being in a region where the kinetic energy is negative explains the phenomenon known as ‘tunnelling’ or barrier penetration. Examples include the escape of an alpha particle from an atomic nucleus, the ‘inversion’ of the ammonia molecule between

$\begin{array}{c} \text{H} \\ | \\ \text{H} - \text{N} - \text{H} \end{array}$ and $\begin{array}{c} \text{H} \\ | \\ \text{H} - \text{N} - \text{H} \\ | \\ \text{H} \end{array}$, and the operation of semiconductor devices.) Although there is only one loop, which makes it difficult to talk about wavelength, the wave function is more sharply curved at small values of r , and gets less and less curved as r increases. We should expect this since curvature depends on kinetic energy, and we know that kinetic energy is greatest near the centre of the atom.

Use of computer program

Computer programs are available which produce results similar to that shown in figure L19, and the next stage of the work is to use a computer program to show how the theory predicts the other energy levels obeying the $1/n^2$ rule.

Such a program solves a simplified form of Schrödinger's equation:

$$\frac{d^2 A}{dr^2} = -\frac{8\pi^2 m}{h^2} (E - E_p) A$$

where E is the total energy of the atom, and E_p is the electron's potential energy at radius r . But we know that

$$E_p = -\frac{e^2}{4\pi\epsilon_0 r}$$

And so

$$\frac{d^2 A}{dr^2} = -\frac{8\pi^2 m}{h^2} \left(E - \frac{e^2}{4\pi\epsilon_0 r}\right) A = -0.165 \times 10^{39} \left(E - \frac{2.30 \times 10^{27}}{r}\right) A$$

The computer works out the amplitude, A , at different values of radius, r , for a given total energy, E .

Teachers will decide how much detail their classes will benefit from. But as a minimum we suggest that before the program is used, the essentials should be rehearsed. These are:

- a** The electron is associated with a wave.
- b** If the amplitude of the wave is A at distance r from the centre of the atom, then $A^2 dr$ represents the chance of finding an electron in a spherical shell of radius r and thickness dr .
- c** The wavelength is related to the electron's kinetic energy, through $\lambda = h/mv$ and kinetic energy $= (mv)^2/2m$.
- d** The potential energy, E_p , varies as $1/r$ and is a negative quantity.
- e** In any energy level the total energy E is constant, so kinetic energy $= E - E_p$.
- f** We are only interested in electron waves with a finite number of loops – standing electron waves – because these represent bound electrons. A wave that shoots off to very high amplitudes (positive or negative) means that the electron is not bound.

After these reminders the computer program can be used. It will work out the form of the electron wave for values of total energy chosen by the user. It shows very clearly that:

- a** Standing waves – bound electrons – are only formed for certain specific values of total energy.

- b** These values of energy must be very precisely defined, that is, the energy levels of the atom are very sharp. When an energy has been found a small change in total energy is enough to send the electron wave shooting off in either the positive or negative direction.
- c** The energy levels follow the $1/n^2$ rule. (Teachers will find it useful to remember that closed solutions may be expected at about $-2.18 \times 10^{-18}/n^2$ J, i.e. at -2.18×10^{-18} J, -0.545×10^{-18} J, -0.242×10^{-18} J, ...)
- d** The first, second, third, ... energy levels have one, two, three, ... loops in the standing wave.
- e** In an excited state (higher energy level) the atom is larger; the electron is more likely to be found farther from the centre.

Questions

Questions 52 and 53 are review and revision questions about the wave theory of atoms.

Computer programs

The program 'Schrödinger's equation' ('Software for Nuffield Advanced Physics') is one of several that could be used here.

The 'Dynamic modelling system' can, of course, be used to produce solutions to the simplified form of Schrödinger's equation.

Equations:

$$A = A + B \cdot DR$$

A is amplitude

$$R = R + DR$$

R is radius

$$PE = -K \cdot Q \cdot Q/R$$

$$KE = E - PE$$

$$C = -A \cdot KE \cdot 1.65E38$$

C is curvature

$$B = B + C \cdot DR$$

B is slope

Initial values:

$$Q = 1.6E-19$$

charge on electron

$$K = 9E9$$

$$1/4\pi\epsilon_0$$

$$R = 0$$

$$A = 0$$

$$DR = 5E-12$$

$$B = 0.2E12$$

chosen so that $B \cdot DR = 1$

Try $E = -2.18E-18, -0.546E-18$, etc.

Plot A against R. Suitable maximum values for A and R are 10 and 1E-9 respectively.

The usual form of Schrödinger's equation

The equation used here is not quite like the usual form of Schrödinger's equation.

It is the time-independent part of it, usually written as $\nabla^2\psi = -(2\pi)^2 \frac{2m}{h^2} (E - E_p)\psi$.

As shown on page 353, for spherical symmetry we can write $A = r\psi$ and

$$\nabla^2\psi = \frac{1}{r} \left(\frac{d^2 A}{dr^2} \right) \text{ from which it follows that } \frac{d^2 A}{dr^2} = \frac{-8\pi^2 m}{h^2} (E - E_p) A$$

Many treatments deal with Ψ rather than A . Teachers, and students who may go on to study wave mechanics later, should be aware of the different significance of A^2 and Ψ^2 . (See page 353.) Graphs of A against r and Ψ against r have different shapes. In particular Ψ is not zero when $r = 0$.

Algebraic derivation of the $1/n^2$ rule

The computer program gives a convincing demonstration that the wave model implies the existence of sharply defined energy levels, and gives values for these energy levels which satisfy the $1/n^2$ rule. Some students may be interested to see how the $1/n^2$ rule may be derived by an algebraic argument, though such an argument does not, of course, allow one to say anything about the amplitude of the electron wave (the shape of the orbital) in any energy level. But it does bring out clearly the fact that the $1/n^2$ rule is a consequence of the $1/r$ variation of potential, something which may not be sufficiently emphasized in the computer solution.

The argument depends on making an assumption about the average kinetic energy of the electron as it moves between 0 and r_0 , the edge of the atom. A simple assumption would be to say that the average kinetic energy is equal to the kinetic energy at $r_0/2$, half way out to the edge of the atom. For a total energy $-E$, the potential energy at r_0 (where $E_k = 0$) will be $-E$; and so the potential energy at $r_0/2$ will be $-2E$ (since potential varies as $-1/r$).

So the kinetic energy (= total energy $-$ potential energy) at $r_0/2$ is $-E - (-2E) = E$. A more precise argument suggests that $E/2$ is a better value for average kinetic energy.

It turns out that it doesn't matter whether we take E or $E/2$; we simply assume that in a level with total energy $-E$, the mean kinetic energy $\propto E$. It follows that the mean momentum of electron $\propto \sqrt{E}$, and, since $\lambda = h/mv$, that the mean wavelength $\propto 1/\sqrt{E}$.

Now the total energy ($-E$) is equal to the potential energy at r_0 , the edge of the atom. By the $1/r$ rule $r_0 \propto 1/E$. If the electron is described by a standing wave with n loops, each half a wavelength long, then (since n loops are to fit into a distance r_0 , $n\lambda/2 \propto r_0$, where λ is the mean wavelength):

$$n\lambda \propto r_0$$

$$\Rightarrow n \left(\frac{1}{\sqrt{E}} \right) \propto \frac{1}{E} \quad \text{since } \lambda \propto \frac{1}{\sqrt{E}} \text{ and } r_0 \propto \frac{1}{E}$$

$$\Rightarrow n \propto 1/\sqrt{E}$$

$$\text{or } E \propto 1/n^2$$

which is Balmer's rule.

Question

Question 54 presents this derivation in structured form.

Visual aids

Cassette/slide programme 'The basis of quantum mechanics' covers very much the same ground as this Unit. (It also introduces Heisenberg's uncertainty principle.) It could usefully be used as a summary or for revision.

The Open University's 'Physics beyond experience' (television programme 14 in course S271 *Discovering physics*) may be useful. It deals with wave-particle duality, electron diffraction, and the relationship of quantum to classical physics. It also introduces the idea of the wave-packet and the Heisenberg uncertainty principle – not parts of this course.

THE SCOPE OF WAVE MECHANICS

It would be a great pity if, by default, students left the course feeling that the wave aspect of electrons can explain the energy levels of the hydrogen atom but no more. The last part of the *Students' guide* summary of this Unit mentions some of the fields in which wave mechanics is successfully applied (chemical bonding, properties of solids, semiconductors, ...).

The first edition of the course (*Unit 10 Waves, particles, and atoms*, 1971) gives brief notes on several applications of the ideas of wave mechanics, including the size and energy levels of the helium ion He^+ , X-ray spectra, the Periodic Table of the elements, molecular energy levels, and alpha decay. Perhaps the simplest of these is the He^+ ion. Here a single electron is in the field due to a nucleus containing two protons. The potential at a given radius is twice as great in magnitude as for a hydrogen atom. Treating the electron as a standing wave in a potential well which is everywhere twice as deep as the potential well of the hydrogen nucleus leads to a set of energy levels which are just four times as deep as the energy levels for hydrogen, *i.e.* they are at $-4 \times 2.18 \times 10^{-18}/n^2 \text{ J} = -8.72 \times 10^{-18}/n^2 \text{ J}$. Spectroscopy shows that there are lines in the spectrum of He^+ with one-quarter of the wavelength of lines in the spectrum of atomic hydrogen; that the lowest energy level of He^+ is at $-8.72 \times 10^{-18} \text{ J}$; and that the other energy levels are given by $-8.72 \times 10^{-18}/n^2 \text{ J}$. The spectrum and energy levels of the neutral He atom are not of course so simple – each electron is affected by the presence of the other.

Teachers who have access to the first edition of *Waves, particles, and atoms* may like to discuss this and also some of the other examples outlined there. As an alternative we give below outline sketches of how the ideas of wave mechanics are used in two other areas: in explaining the electrical properties of solids and chemical bonding.

Whichever examples they choose teachers should note that the aim of this part of the course is *not* to give students extra information. The aim is to help them appreciate the scope, value, and power of wave mechanics by showing how it is used in a variety of contexts. Although much of the detail is beyond the scope of this course, we do think it worthwhile attempting to show the nature of the argument in a few cases.

Electrons in solids

The wave mechanics of electrons offers an understanding of the wide range of thermal, electrical, optical, and mechanical properties of solids. In all cases the key is knowledge of the energy levels of electrons. As an example we suggest concentrating on the electrical properties and showing how the different classes of conductors, insulators, and semiconductors arise from the fact that solids have energy bands, instead of discrete energy levels, as in an atom.

It is the number of electrons in the solid in relation to the number and distribution of energy levels in the bands and the size of the gaps between the bands which determines whether the material will be a conductor, a semiconductor, or an insulator (figure L20). In a metal there are plenty of unfilled levels below the energy gap; an electron can easily be promoted to the slightly higher energy of a vacant level, and conduction is good.

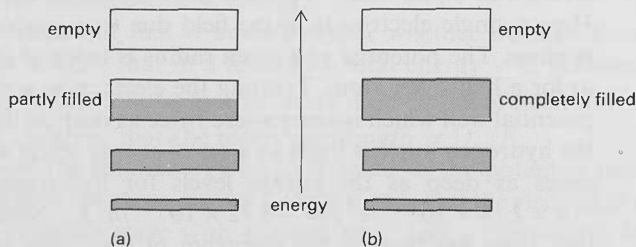


Figure L20

Simplified representation of energy bands in (a) conductor, (b) insulator.

In an insulator the energy gap is very wide (≈ 5.3 eV for diamond) and all levels below it are full. Electrons cannot gain energy and move in an electric field because they cannot get into higher, unfilled energy levels: the gap is too wide.

Semiconductors are an intermediate case: although all levels below the gap are full (and all above are empty) the gap is much smaller than in an insulator (1.1 eV for silicon, 0.7 eV for germanium). It is possible,

through random fluctuation of energy between particles, for an electron to gain enough energy to be promoted across the gap to an empty energy level. And the higher the temperature the more likely this becomes. So a semiconductor's resistivity decreases with temperature, according to the Boltzmann factor (Unit K, 'Energy and entropy').

Explaining how the bands and gaps arise is, of course, a job for quantum mechanics and really demands concepts beyond the level of this course. We suggest below approaches to the two methods usually used, which teachers may find worth passing on to interested students.

a The free electron approach

In Unit B, 'Currents, circuits, and charge', a model in which electrons (thought of as particles) are free to move in a metal was used. In the alkali metals, for example, each atom in the solid supplies one of these free electrons. The relationship $I = nQvA$ was developed.

How are we to understand electrical conduction if the wave behaviour of electrons is to be considered?

To simplify matters greatly we imagine the electrons to be in a one-dimensional box whose length is L . The potential is uniform within the box and rises abruptly at the ends – figure L21(a).

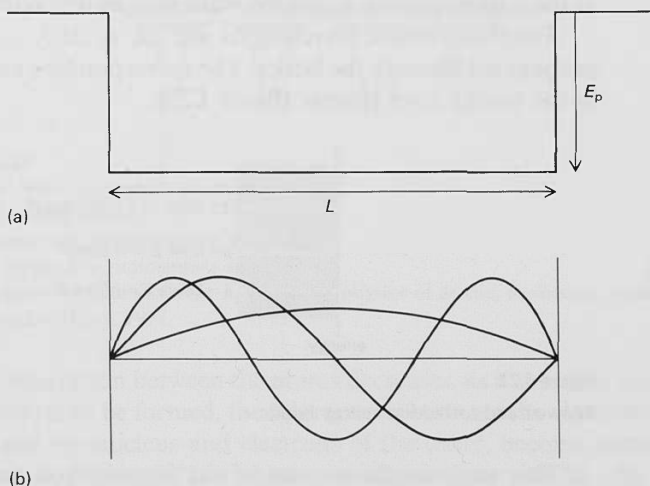


Figure L21

(a) Potential is uniform within the 'box', and rises abruptly at each end.

(b) The first few possible wave functions for an electron in a box length L .

Figure L21(b) shows some possible standing waves in the box. The possible wavelengths are $2L$, L , $2L/3$, $L/2$, $2L/5$, ..., $2L/n$. So, from $mv = h/\lambda$, the possible momenta for the electron are $h/2L$, h/L , $3h/2L$, ..., $nh/2L$, and the corresponding kinetic energies, $(mv)^2/2m$, are

$$\frac{n^2 h^2}{8mL^2}$$

Because L is very large (in atomic terms) for a reasonably sized piece of metal, the successive allowed energy levels are very closely spaced.

In an electric field the electron (viewed as a particle) accelerates – it gains kinetic energy. If the electron as a wave is to gain kinetic energy then its wavelength decreases. The electron is promoted to a higher energy level (greater n). Nothing that has been said so far rules this out.

But this treatment has ignored the presence of the positive ion cores – it has assumed their effect to be smoothed out throughout the crystal. A more realistic model allows interaction between the localized ion cores and the electron waves.

For low values of n (wavelength very long compared with distance between ions) the wave will pass through the lattice unhindered. At higher values of n the wavelength may be of the same order of magnitude as the lattice spacing, d , and the wave may be diffracted by the ion cores.

Electrons whose wavelengths are $2d$, d , $2d/3$, ..., $2d/n$, cannot be propagated through the lattice. The corresponding energies are missing in the energy level scheme (figure L22).

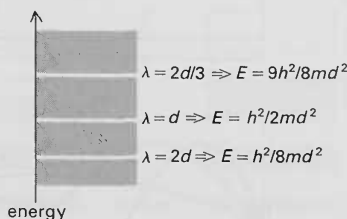


Figure L22

Allowed and forbidden energy bands.

This very simple account of the free electron model leaves much unexplained. Among other things it does not account for the *widths* of the energy gaps which are of crucial importance in determining whether a material will be an insulator or a semiconductor.

Teachers, and students, who want to see a more rigorous version of this argument will find that there are plenty to choose from in texts written for undergraduates. See, for example, DAVIES, *Waves, atoms, and solids*.

b The tight binding approach

In this approach we think about what happens when isolated atoms are brought together to form a solid.

At large separations, for instance in a gas, there is negligible interaction between any two atoms, and the energy levels of each are unaffected by the presence of the other – figure L23(a).

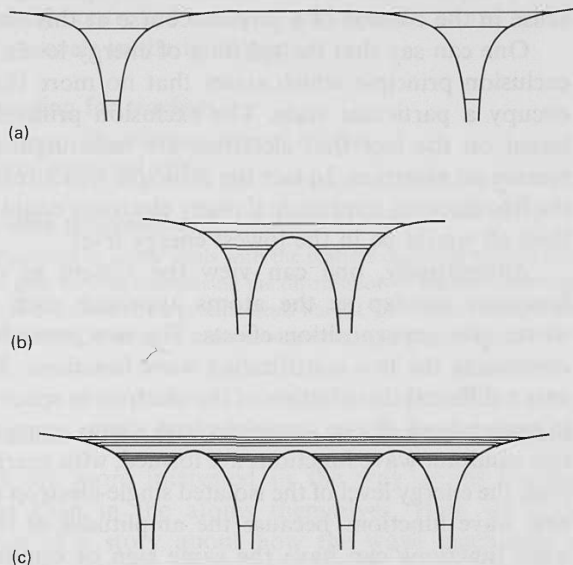


Figure L23

Potential wells and energy levels for:

(a) two isolated atoms;

(b) the same two atoms forming a molecule;

(c) four atoms in a rudimentary solid.

From EISEBERG, R. and RESNICK, R. *Quantum physics of atoms, molecules, solids, nuclei, and particles*. Wiley, 1974.

As the separation between the atoms decreases, as it must if a solid (or a molecule) is to be formed, the interactions between the electrons of one atom and the nucleus and electrons of the other, become increasingly important. Figure L23(b) shows two immediate results: the higher energy electrons become part of the whole system – the molecule; they are no longer bound to one nucleus but have become delocalized. If we imagine not a two-atom molecule but a crystal of, say, sodium, we can see how one electron per atom is now free to move through the lattice: the origin of electrical and thermal conductivity in metals – figure L23(c). Chemists will know that the concept of electrons being delocalized is also vital to much of our understanding of chemical

bonding: in conjugated $-\text{CH}=\text{CH}-\text{CH}=\text{CH}-$ chains and the benzene ring to take just two examples.

Another consequence of the interaction between two atoms is that the energy levels of each individual atom are split. In a two-atom molecule each level is split into 2; in an N -electron crystal each level is split into N levels.

It may be difficult to account for this splitting in terms which make sense in the context of a physics course at this level.

One can say that the splitting of energy levels follows from Pauli's exclusion principle which states that no more than one electron can occupy a particular state. The exclusion principle – which is in fact based on the fact that electrons are indistinguishable – will have to remain an assertion. In fact the principle was already used implicitly in the free electron approach: if many electrons could be in the same state then all would be in the lowest energy level.

Alternatively, one can view the system as one in which wave functions overlap as the atoms approach each other. Overlapping waves give superposition effects. The new wave function is found by combining the two contributing wave functions. The resultant represents a different distribution of the electron in space and hence a change in energy level. If two single-electron atoms come together in this way two resultant wave functions are formed, with energies slightly different from the energy level of the isolated single-electron atom. (There are *two* new wave functions because the amplitudes of the two contributing wave functions can have the same sign or opposite signs.) When N atoms come together N new energy levels are formed.

Clearly, the electrons in higher energy levels will be affected more because they tend to be further from their nucleus than those in the lower levels. Figure L23(c) shows that the higher levels are more split.

In a system containing many atoms the splitting of each atomic energy level gives rise to many very closely spaced levels. These levels are so close that they effectively constitute an energy band. These bands have finite width and there may remain gaps between them just as there are gaps between the energy levels of the single atom. The electrical (and other) properties of the solid then depend on whether or not a band is full, and the size of the gap between bands.

In an alkali metal there are only enough electrons to occupy half the levels in the highest band. In an insulator the highest band with electrons is completely full. To be promoted to a higher band where there are empty levels requires enough energy to cross the gap – in diamond it is 5.3 eV and so the material is an insulator. Semiconductors are those materials with a smaller band gap (≈ 1.1 eV in pure silicon,

≈ 0.7 eV in pure germanium). A few electrons will have enough energy to cross the gap where they are free to move (gain energy) under the influence of a potential difference. As the temperature is increased the equilibrium distribution of electrons between energy levels changes, proportionately more have higher energy, there are more in the upper, nearly empty band, and conductivity is greater. In practice the electrical properties of semiconductors are changed by 'doping' the pure material with carefully controlled amounts of impurity atoms which introduce new energy levels in the band gap.

Further reading for teachers

COLES and CAPLIN, *The electronic structure of solids*.

DAVIES, *Waves, atoms, and solids*.

Cassette/slide programme

Part I of 'Electrons in solids' deals with the matters discussed here in much the same way; it also goes further, introducing the distribution of electrons between states and the electronic contribution to specific heat. Part II deals with structure and bonding (ionic and covalent), resistance, superconductivity, and magnetism.

The relevance of the ideas of wave mechanics in chemistry

Chemists are more interested in the way atoms combine to form molecules than in the atoms themselves. Here we sketch out the beginnings of a story about how the wave mechanics of electrons explains some of the mysteries of chemistry. Some new ideas are needed and they are introduced as and when necessary. Certainly consultation with chemistry colleagues would be advisable: students doing an A-level chemistry course may not have heard an explanation of the kind outlined here.

Bonding

The simplest case to consider is of course hydrogen – why is the hydrogen molecule, H_2 , more stable than two separate hydrogen atoms?

Here wave mechanics scarcely seems necessary. If we agree that the four particles concerned (2 electrons plus 2 protons) will tend to be arranged in the state which has lowest energy it seems clear that both electrons ought to be near both protons. So $\bullet\bullet:\bullet\bullet$ (where \bullet is a proton and \cdot an electron) will be preferred to $\cdot\bullet\bullet\cdot$. And this is one way in which chemists represent the bond between two hydrogen atoms. The equivalent representation showing the electrons as distributed in space is shown in figure L24.

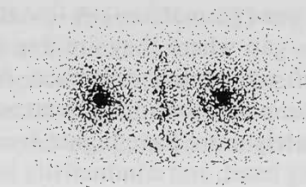


Figure L24

Electron cloud of H_2 molecule.

From ATKINS, P. W. *Quanta – a handbook of concepts*. Oxford University Press, 1974.

As the two atoms are brought closer together the wave functions overlap. Overlapping waves representing probabilities superpose in the same way as overlapping electromagnetic waves, for example. So the amplitude in the region between the two protons is the sum of the amplitudes of the two individual waves and there is a high probability of finding an electron there. What prevents the distance between the protons decreasing below about 0.07 nm? The potential energy due to the repulsion between the two protons increases as they get closer, and the kinetic energy of the electrons, squashed into a diminishing volume, increases too.

The shape of molecules

To begin to explain other more interesting molecules needs some new ideas. One of the most important features of a molecule in determining its behaviour is its shape. This applies equally well to simple compounds such as water or ammonia and to large and complex structures like enzymes and proteins, which are important in biology.

So far we have assumed that a wave function has spherical symmetry: the chance of finding an electron at a particular distance from the nucleus is the same in all directions. While this is true for the lowest energy level of hydrogen (and of other atoms), it is not necessarily true of higher energy levels. Not all standing wave patterns which can be excited on a circular flat plate are symmetrical about the centre, although the plate itself is. Non-symmetric patterns occur in higher modes of vibration – figure L25(b).

And so it is with atoms. For example, the second energy level of hydrogen can take several forms. The computer program used to produce the kind of results shown, for example, in figure L19 was restricted to spherically symmetrical solutions. Others are possible. All those shown in figure L26 are for $n = 2$ ($E = -0.545 \times 10^{-18} \text{ J}$).

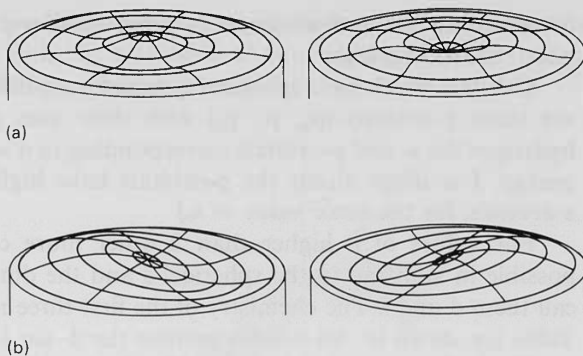


Figure L25

(a) Symmetric and (b) asymmetric vibrations of a circular membrane.

Drawing based on frames from the BFA Films/Ealing Scientific filmloop Vibrations of a drum.

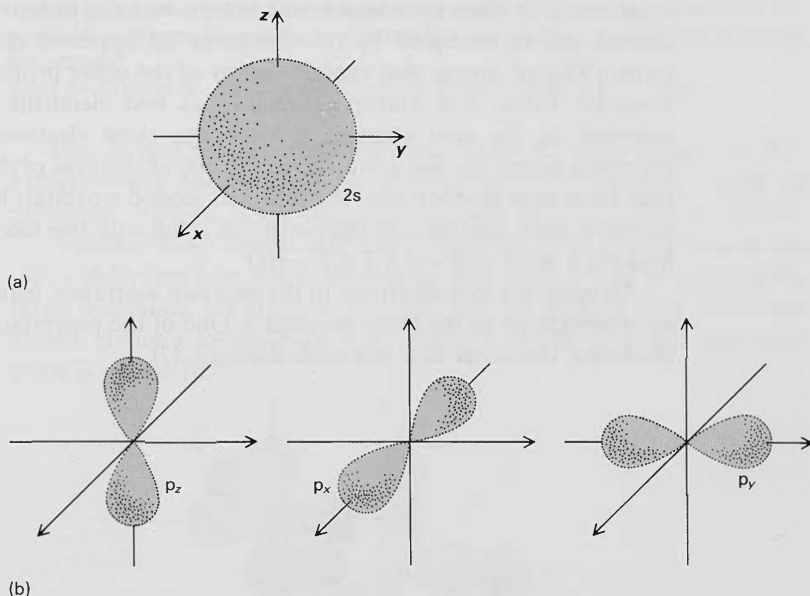


Figure L26

s- and p-orbitals of the hydrogen atom for $n=2$.

Based on COMPANION, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

The shaded regions in figure L26 which indicate where the wave amplitude is fairly large, that is, where the chance of finding the electron is fairly high, are called orbitals. Only figure L26(a) has spherical symmetry; this corresponds to the solution for $n=2$ given by the computer program. Other solutions predict that the electron is to be

found within one of three mutually perpendicular dumb-bell (or double pear-) shaped regions.

Orbitals which have spherical symmetry are called s-orbitals. There are three p-orbitals (p_x , p_y , p_z) with their axes perpendicular. (For hydrogen the s- and p-orbitals corresponding to $n = 2$ all have the same energy. For other atoms the p-orbitals have higher energy than the s-orbitals, for the same value of n .)

For values of n higher than 2 other more complex shapes are possible in addition to the spherical s and the dumb-bell p. Chemists call them d and f. The chemistry of the first three rows of the Periodic Table (i.e. down to Ar) doesn't involve the d- (or f-) orbitals.

The hydrogen molecule, formed by the overlap of two s-orbitals, has a simple linear shape. Clearly if a bond is formed which involves one or more p-orbitals more complex shapes are going to occur.

Oxygen has eight electrons. To understand how it and any atom more complex than hydrogen forms bonds, we have to know that each orbital can be occupied by two electrons (of opposite spin), and no more. (This of course also explains many of the other properties of the Periodic Table. For example, helium has two electrons in its first s-orbital; in the next element, lithium, the third electron goes into the much bigger second s-orbital. At the end of this row of the Table we find neon with 10 electrons. The first and second s-orbitals both have 2 electrons each, and each of the p-orbitals is full with two electrons each, making a total of $2 + 2 + 3 \times 2 = 10$.)

Oxygen has four electrons in the first two s-orbitals, leaving 4 to be accommodated in the three p-orbitals. One of the p-orbitals takes two electrons, the other two one each (figure L27).

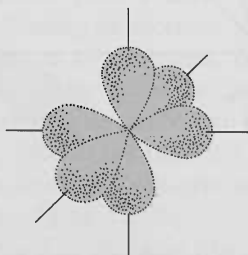


Figure L27

One of the p-orbitals of oxygen has two electrons; the others one each. (The s-orbitals are not shown.)

Based on COMPANION, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

A full orbital (for example the s-orbital of He, the p-orbitals of neon) cannot take part in bonding. A 'chemical bond' is formed if the overlap of two partly filled orbitals results in a *decrease in energy*. Thus, each of

the two p-orbitals of oxygen, which have only one electron each, can overlap with the s-orbitals of two hydrogen atoms to form a water molecule (figure L28).

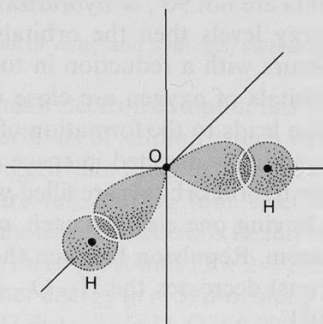


Figure L28

A water molecule formed by the overlap of the s-orbitals of two hydrogen atoms with two p-orbitals of oxygen. (For clarity the non-bonding p-orbital and the s-orbitals of oxygen are omitted.)

Based on *COMPANION*, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

This model predicts that the bond angle in water should be 90° . Measurement shows it to be about 104° . The increase could be explained in terms of the repulsion between the two hydrogen nuclei.

A similar story can be told about ammonia. The nitrogen atom has seven electrons: four are in s-orbitals, and there is one in each of the three p-orbitals. So three s-p bonds can be formed with hydrogen atoms, giving a molecule of NH₃ (figure L29). In ammonia the bond angle is about 107° .

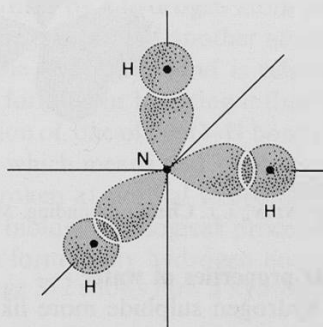


Figure L29

A molecule of ammonia formed by the overlap of the s-orbitals of three hydrogen atoms with the three p-orbitals of a nitrogen atom.

Based on *COMPANION*, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

Hybridization

An important concept in the theory of chemical bonding which offers, for example, an alternative explanation of why the bond angles in water and ammonia are not 90° , is 'hybridization'. If two atomic orbitals have similar energy levels then the orbitals may mix, or hybridize, when bonding occurs with a reduction in total energy. The one 2s- and the three 2p-orbitals of oxygen are close in energy level, and in bonding hybridization leads to the formation of four hybrid orbitals, called sp^3 . These are uniformly directed in space at the tetrahedral angle of 109° . Two of these hybrid orbitals are filled with a pair of electrons each. The other two, having one electron each, can take part in bonding with a hydrogen atom. Repulsion between the two filled p-orbitals (the 'lone pair' electrons) decreases the H—O—H bond angle from 109° to the observed 104° .

Now consider ammonia. Nitrogen has five electrons in the $n = 2$ level: two fill the 2s-orbital and one goes into each of the three 2p-orbitals. In bonding these 'mix' to form four symmetrically directed molecular orbitals. One of these is filled with a pair of electrons; the other three, with a single electron each, can overlap (bond) with the s-orbitals of three hydrogen atoms. The resulting ammonia molecule has a pyramidal shape, with the 'lone pair' projecting beyond the N apex.

Much of the chemistry of carbon derives from tetrahedrally directed hybrid orbitals, each of which has a single electron and so can take part in bonding.

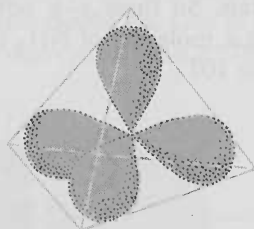


Figure L30

Hybrid sp^3 -orbitals.

Based on *COMPANION*, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

The peculiar properties of water

Why isn't hydrogen sulphide more like water? In H_2S there are two s-p bonds binding hydrogen to sulphur, and in water there are two s-p bonds binding hydrogen to oxygen. Oxygen and sulphur are in the same column of the Periodic Table. But a glance at table L2 reminds us of some of the important differences.

	m.p./°C	b.p./°C
H ₂ O	0	100
H ₂ S	-85.5	-60.3

Table L2

Melting- and boiling-points of water and hydrogen sulphide.

Whereas oxygen has 8 electrons, sulphur has 16. The sulphur atom is much bigger. The p-orbitals of sulphur which overlap with hydrogen's s-orbitals are much bigger than the p-orbitals of oxygen. In oxygen two spherical s-orbitals are full, leaving four electrons in the first set of p-orbitals. To accommodate 16 electrons in sulphur *three* s-orbitals are full ($3 \times 2 = 6$); so are *three* p-orbitals ($3 \times 2 = 6$); leaving four electrons in a second set of higher energy ($n = 3$) p-orbitals. Higher energy means larger orbits. The O–H distance in H₂O is 0.099 nm, the S–H distance in H₂S is 0.13 nm. The two hydrogen nuclei are further apart in the bigger molecule. The smaller repulsive force between them is reflected in the fact that the bond angle, measured to be 92°, is much closer to the 90° angle between the two original p-orbitals. (In terms of hybridization: the energy difference between the filled s- and partly filled p-orbitals ($n = 3$) is much greater for sulphur than between the corresponding $n = 2$ orbitals for oxygen, and mixing does not occur.)

Hydrogen bonding

Some of the most significant properties of water, including properties vital for life as it has evolved on Earth, are due to its ability to take part in the formation of hydrogen bonds, or hydrogen bridges. The hydrogen bond is a link formed by a hydrogen atom (in one molecule) and a pair of electrons in the p-orbital of another atom, either N, O, or F, in another molecule. The hydrogen bond is relatively weak; much less energy is involved in forming or breaking it than is needed to break, or is released on formation of, one of the O–H bonds in water, for example. It is this low energy, which means that the hydrogen bonds are rather easily formed and broken at normal temperatures, which gives them their significance in biology. Biological processes often involve large molecules which can form many hydrogen bonds.

Figure L31 on page 372 shows how a hydrogen bond may be simply visualized.

The fluorine atom has three p-orbitals, one of which overlaps with the hydrogen atom's s-orbital when an HF molecule is formed. The fluorine (which is very electronegative) attracts the two bonding electrons strongly, leaving the proton (the nucleus of the hydrogen atom) relatively 'bare'. The hydrogen bond is the result of the attraction

which exists between this only partly screened proton and one of the ‘lone pairs’ of electrons in a filled p-orbital of another HF molecule.

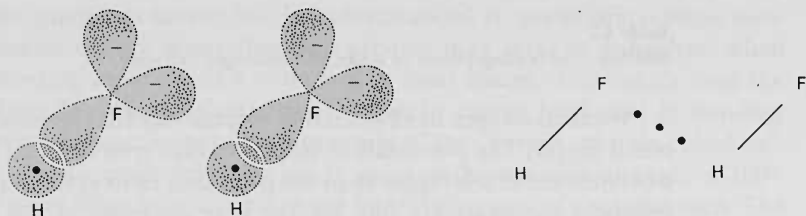


Figure L31

Hydrogen bond formation between two HF molecules. (a) Electron orbitals; (b) simplified representation.

Based on COMPANION, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

Hydrogen bond formation isn’t limited to a pair of molecules: a chain of HF can be formed.

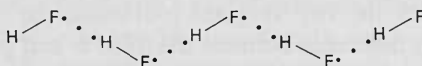


Figure L32

A chain of HF molecules formed by hydrogen bonds.

Inspection of the melting- and boiling-points of the hydrogen halides (table L3) shows that the temperatures increase from chloride to iodide. But hydrogen fluoride does not fit into this pattern: it has the highest boiling-point and the second highest melting-point. The energy needed to break the hydrogen bonds is the reason.

	m.p./K	b.p./K
HF	190	293
HCl	159	188
HBr	184	206
HI	222	238

Table L3

Melting- and boiling-points of the hydrogen halides.

But if the other halogens have the same electronic structure – at least as far as their outer electrons are concerned – one might ask (indeed *should* ask!) why isn’t hydrogen bond formation important in the rest of the family?

The answer lies in the size of the orbitals. The outer p-orbitals which take part in bonding with the hydrogen atom increase in size from fluorine to iodine. The bigger the orbital the deeper inside it the proton

can sit and the more effectively the electron cloud shields it. Charge is more uniformly distributed (and the dipole moment less) in HI than in HF. Because the proton is less exposed it is less likely to form a hydrogen bond with a non-bonding p-orbital. (When chemists speak of fluorine being the most electronegative member of the family this is what they mean; the electron pair which is the 'bond' between hydrogen and the halogen is more closely associated with fluorine in HF, than with chlorine in HCl, etc.)

Similarly, oxygen in water is more electronegative than sulphur in hydrogen sulphide: the negative charge tends to be more concentrated on the oxygen and the proton is more exposed. So hydrogen bonding is possible between water molecules but not between molecules of hydrogen sulphide.

The striking and vital differences between H_2O and H_2S stem from this fact.

The importance of the hydrogen bond

The energy needed to break a hydrogen bond between two water molecules is about $0.043 \times 10^{-18} \text{ J}$ ($\approx 20 \text{ kJ mol}^{-1}$). The data comes from the energy needed to convert ice to water vapour at 0°C . Contrast this with, for example, the $0.77 \times 10^{-18} \text{ J}$ (463 kJ mol^{-1}) needed to break the O-H bond in a water molecule.

The hydrogen bond between water molecules offers an explanation of:

- the unexpectedly high melting- and boiling-points of water;
- the large amount of energy needed to melt ice or boil water;
- the structure of ice and the fact that it has a lower density than water at 0°C ;
- the high specific heat capacity of water.

The low volatility of water compared with other substances of similar molecular mass helps to explain the relatively high abundance of water in the Earth's atmosphere ($\text{H}_2\text{O} \approx 1\%$; $\text{CH}_4 \approx 7.4 \times 10^{-4}\%$; $\text{Ne} \approx 2 \times 10^{-4}\%$).

The hybridization model in which a water molecule has four (nearly) symmetrically directed orbitals shows that water, uniquely among these molecules, can form hydrogen bonds. There are two 'lone pair' electrons and two 'exposed protons' in each molecule: each molecule can be bonded to four others (figure L33, page 374).

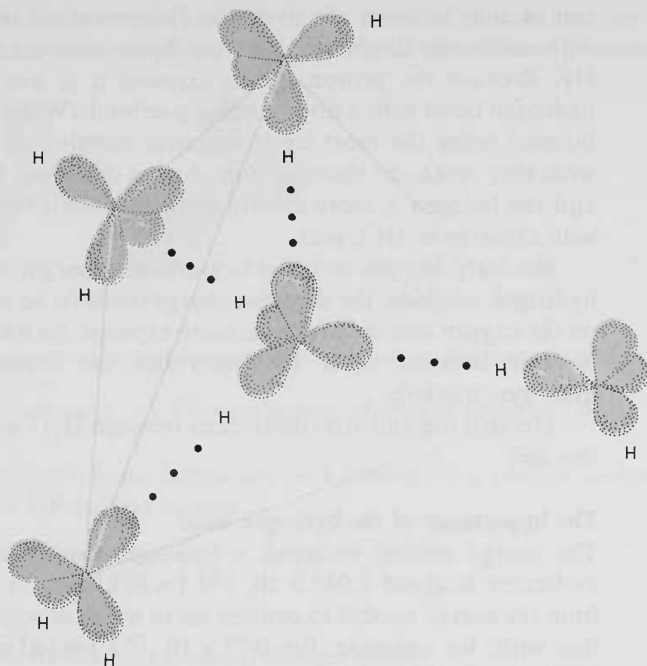


Figure L33

Each water molecule can form hydrogen bonds with four others. The bonds (including the hydrogen bond) are directed nearly towards the corners of a tetrahedron and a three-dimensional structure can be built up.

Based on *COMPANION*, A. L. Chemical bonding. McGraw-Hill Book Co., 1964.

The hydrogen bond in biology

The importance of hydrogen bonding in biology is not restricted to its consequences for the properties of water, nor to the effect of hydrogen bonding between water and other molecules. Hydrogen bonding can occur between any two atoms of highly electronegative elements (O, N, F) if one is bonded to H and the other has an unbonded lone pair of electrons. One example involving molecules of a much more complex type must serve as an illustration.

The DNA double helix consists of two long chains which spiral round each other (figure L34).

The backbone of each is a long chain of alternate sugar (deoxy-ribose) and phosphate groups. Attached to this backbone is a sequence of 'bases'. Only four different bases are involved (figure L35).

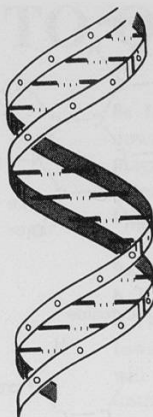
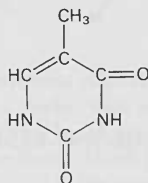
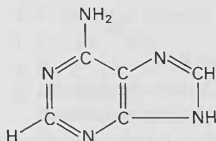


Figure L34

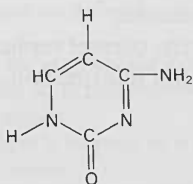
From WATSON, J. D. *Molecular biology of the gene*. 3rd edition. The Benjamin/Cummings Publishing Company, Inc., 1975.



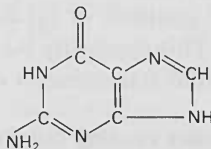
thymine



adenine



cytosine



guanine

Figure L35

The four bases present in DNA.

It is the ordering of these bases which constitutes the genetic code; they contain the information which determines the characteristics of the organism.

In the double helix these bases are on the inside, the backbones on the outside, and the two chains are held together by hydrogen bonding between base molecules. The way in which the hydrogen bonds form between the base pairs is shown in figure L36.

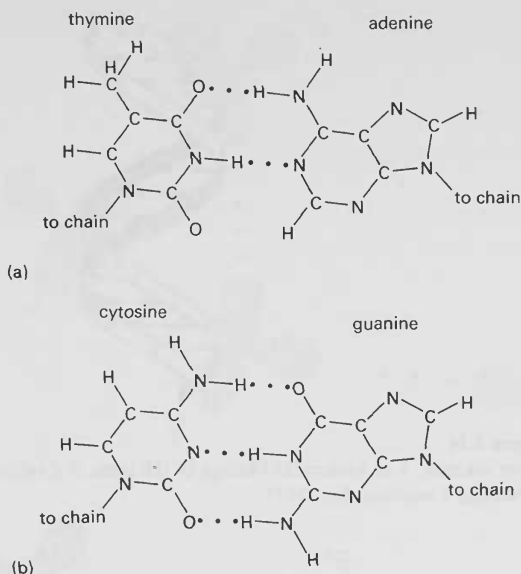


Figure L36

(a) Thymine to adenine.

(b) Cytosine to guanine.

Based on WATSON, J. D. *Molecular biology of the gene*. 3rd edition. The Benjamin/Cummings Publishing Company, Inc., 1975.

One characteristic is that the bases always form bonds between the same pairs – thymine and adenine, cytosine and guanine, never thymine and guanine, or cytosine and adenine.

This specificity is crucial for the correct replication of DNA and the faithful transmission of the genetic information.

Further reading for students

TAYLOR, *Water*.

References and further reading for teachers

COMPANION, *Chemical bonding*.

CBA *Chemical systems*.

NUFFIELD REVISED ADVANCED BIOLOGY *Students' book II* and *Teachers' guide II*.

WATSON, *Molecular biology of the gene*.

ANSWERS TO QUESTIONS

UNIT H

Magnetic fields and a.c.

- 1a Perpendicular to both x and y axes.
b A brick, a box, a die, a lump of sugar.

2 b and e only.

- 3a 0.04 T
b 0.002 N
c The force is T newtons per metre.

- 4a 0.004 N m
b 0.004 N m
c 0 (the forces are in the plane of the coil and tend to push the sides in or out, not to turn the coil).

- 5a 1300 A
b This is an enormous current. It would melt the wire. A thicker wire would need a greater current and would get hotter. An aluminium wire (lower density) would need less current. Note that the current would have to flow at all times, and because the Earth's magnetic field has a vertical component there would also be a horizontal force on the wire.

6ai Because the gravitational force of the Earth is acting on it, perpendicular to its direction of motion.

ii Because there is no force on it in its direction of motion.

b No; but even in the upper atmosphere there will be a force on a real satellite acting against its direction of motion, due to air resistance. The satellite will move into a lower orbit.

- 7a Ne
b Ne/t
c vt
d $F = BNe v$
e $Be v$
f BQv

8a Its weight (about 9.8 N) acting downwards; an upward force of equal magnitude due to the tension of the extended spring.

b There is no net force acting on it.

9a BQv

b Towards the front edge.

c The moving charge carriers are pushed towards the front edge, so the density there will build up. It will continue to increase until there is an equally strong electrical force in the opposite direction to the magnetic force, *i.e.* from front to back.

d The electric force (EQ) must be equal in magnitude to the magnetic force, BQv . So the electric field $E = Bv$.

e Front edge negative, rear edge positive.

f $V = Ed = Bvd$

g bd

h $n = BI/VbQ$

10 D

11a eV

b $v = \sqrt{2eV/m}$

c $F = Bev$; at right angles to v and B .

d $a = Bev/m$

e No

f $e/m = 2V/B^2 r^2$

12a $1.6 \times 10^6 \text{ m s}^{-1}$

b 0.34 T

c About 35 mm.

d Same time for both, $2 \times 10^{-7} \text{ s}$.

e The time it takes a particle to go round does not increase if it has more energy because it goes in a larger circle. Thus all the particles can be given a push at the same moment.

13a See figure Q1(a).

b See figure Q1(b).

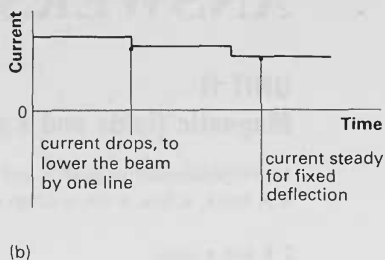
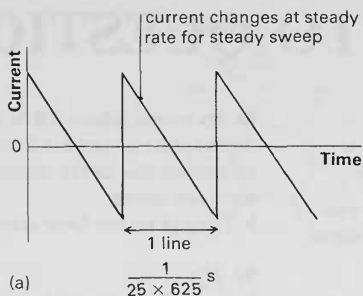


Figure Q1

(a) Current in coils for horizontal deflection.

(b) Current in coils for vertical deflection.

The field giving horizontal deflection must vary at a steady rate as the spot sweeps across the screen at a steady speed. If the middle of the screen corresponds to zero deflection and zero field, the current must vary in sign. The steps shown in the vertical deflection current are there to keep the spot at a fixed height for the whole of each line, dropping this height by one line spacing every time a line is completed. In practice, a very slowly changing current is used. Each line on the screen is tilted by an angle of about $1/625$ radian. Such a steadily changing current would be much easier to produce than a series of small sharp steps.

14a The path is a spiral because the momentum of the electrons is changing.

$$Bev = \frac{mv^2}{r}$$

$$mv = Ber$$

b About $2 \times 10^{-20} \text{ kg m s}^{-1}$.

15 C

16 B

17a The field is $k \text{ N A}^{-1} \text{ m}^{-1}$.

b z

c The force is k newtons.

d x

e $2 \times 10^{-7} \text{ N A}^{-2}$

f $4\pi \times 10^{-7} \text{ N A}^{-2}$

18ai $2 \times 10^{-4} \text{ T}$

ii $2 \times 10^{-5} \text{ T}$

b 10^{-4} N , 10^{-5} N

c Yes; the weight of 1 cm of the tickertape is about $6 \times 10^{-5} \text{ N}$.

19a $1.5 \times 10^{-3} \text{ T}$

b $0.75 \times 10^{-3} \text{ T}$

20a 5.3 A

b A larger current is needed (actually $5.3 \text{ A} / \cos 45^\circ = 7.5 \text{ A}$).

c The coil would have to be in a plane perpendicular to the Earth's B -field, that is it should be 24° above the horizontal and at an angle of 83° East of North.

21 Some of the points to be made in an answer include:

a Need to know or estimate the mass and charge of an electron; the p.d. of the electron gun; the gravitational field strength; the distance from gun to screen.

b The current in the wire; the formula for magnetic field strength (near a straight wire, a solenoid?); the value of μ_0 .

22a See figure Q2.

The field strength in the positions shown: 0.4×10^{-4} , 0.8×10^{-4} , 1.2×10^{-4} , 1.6×10^{-4} , $2.0 \times 10^{-4} \text{ T}$. It is difficult to show the magnitude of B on a drawing of this kind.

bi $4 \times 10^{-4} \text{ T}$

ii 2 A

c $6 \times 10^{-4} \text{ N}$

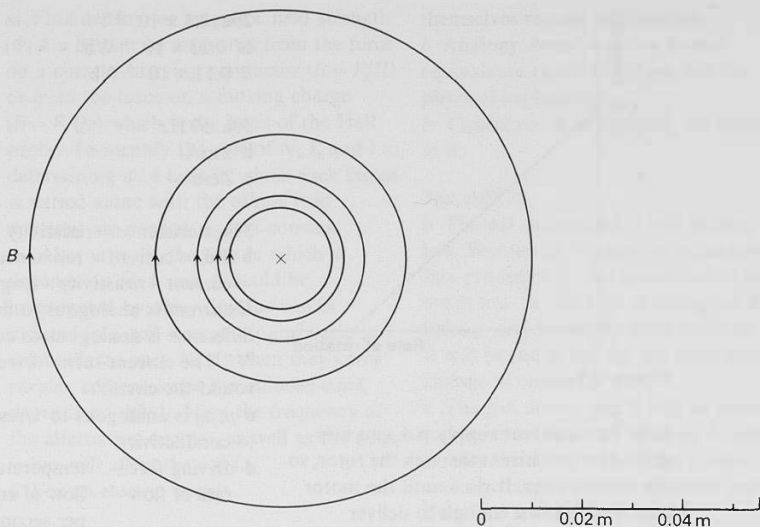


Figure Q2

23a A force BQv parallel to the length of the rod.

b A current will flow.

c $\mathcal{E} = \mathcal{E}/l$

d $Q\mathcal{E}/l$

e $\mathcal{E} = Blv$

f No; Q does not appear in the expression for the e.m.f.

g I^2Rt

h No. The force Bil must act against the motion of the rod. If it helped the motion one would only have to start the rod moving for it to go faster and faster, gaining energy without drawing on any supply.

i vt

j $Blvt$

k $IR = Blv$. Since IR is the p.d. across the resistor, which must be equal to the induced e.m.f., $\mathcal{E} = Blv$. Notice that the argument of steps **g** to **k**, which was about energy, has given the same result as the argument about force in steps **a** to **e**.

l No. Charges will be pushed to one end of both conductors as they move together through the B -field, but after this initial movement of charge, the rod and wire travel along with one end of the pair positively charged with respect to the other end, but with no current circulating.

24a In time t the rod travels distance vt , at right angles to its length l . The area over which it passes, shown shaded in figure H84 of the *Students' guide*, is lvt . Now the e.m.f. is

$\mathcal{E} = Blv$ (from question 23),

and this is B multiplied by the area worked out above, divided by t . That is, $\mathcal{E} = Blv$ is the rate at which the area swept out by the rod increases, multiplied by B .

b The induced e.m.f. would be just the same. Parts of the rod parallel to its velocity, v , contribute nothing to magnetic forces on charges within the rod. The area, perpendicular to B , swept out by the crooked rod is the same as the area swept out by the straight rod.

25 C

26a $I = V/R$

b See figure Q3 (page 380).

c The torque will fall as the rate of rotation increases.

d The rate of rotation must fall. If the motor is not producing enough torque to turn the larger load, it will slow down. As it does so, the induced e.m.f. in its rotor

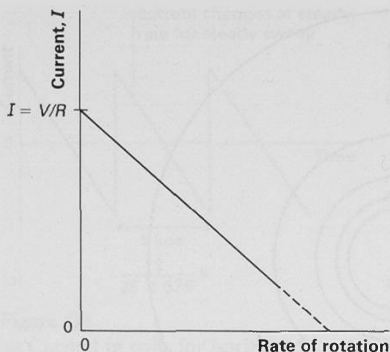


Figure Q3

falls. The constant supply p.d. can now drive a larger current through the rotor, so the current rises. It rises until the motor has slowed down enough to deliver sufficient torque to turn the load. In this way, an electric motor is a self-adjusting device.

- 27a** As long as the motor is turning, an e.m.f. is induced in the rotor and the current in it is given by $V - \mathcal{E} = IR$. The applied p.d., V , is constant at 5 V, so I is always less than $V/R = 5$ A, the value it would have if the rotor were stationary and $\mathcal{E} = 0$. The faster the rotor turns the larger \mathcal{E} and the smaller I become.
- b** The field, B , in which the rotor turns is constant, so the torque is proportional to the rotor current. The larger the load, the larger the current must be. The motor slows down as much as is necessary for this larger current to flow, if that is possible without it stalling.
- c** Raise the p.d. applied to the rotor, so that it exceeds the induced e.m.f. at 27 revolutions per second (about 4 V) by as much as is needed to drive 3.1 A through 1 Ω (3.1 V). This assumes that the torque required remains the same, and so, since the torque is proportional to the current, that the current will remain 3.1 A.

- 28a** $B = 0.75 \times 10^{-3}$ T;
 $\Phi = 3.7 \times 10^{-6}$ Wb.
b $B = 0.75 \times 10^{-3}$ T; $\Phi = 1.9 \times 10^{-6}$ Wb.
c $B = 0.75 \times 10^{-3}$ T; $\Phi = 6.3 \times 10^{-6}$ Wb.

- 29a** 2.3×10^{-3} T
bi 0.23×10^{-6} Wb
ii 0.11×10^{-6} Wb

- 30a** 50 Hz
b 2 mA
c Zero

- 31a** Resistance = resistivity \times length/area.
b P.d. = current \times resistance
 = current \times resistivity \times length/area.
 If current is analogous to flux, potential difference is analogous to current-turns, NI . The current-turns 'drive the flux round the circuit'.

c $\mu_r \mu_0$ is analogous to $1/\text{resistivity}$; that is, to conductivity.

- d**
- | | |
|---------------|-------------------------------------|
| driving force | temperature difference |
| rate of flow | flow of energy in joules per second |
| length | thickness of insulation |
| conductivity | thermal conductivity |
| area | surface area of hot water tank |

e Reluctance corresponds to resistance.

- 32a** About 5×10^7 A² m⁻¹ N⁻¹.
b If l is doubled (keeping the same number of turns per metre), then so is N , the total number of turns. But doubling l also doubles the reluctance. Since both current-turns and reluctance are doubled, the flux stays the same.
c About 6×10^{-6} N m A⁻¹ (or Wb).
d About 1.2×10^{-3} T (or Wb m⁻²).
e A steel retort stand rod might have a diameter of 10 mm, and a cross-sectional area of about 80 mm². The cross-section of the solenoid is about 5000 mm². If the steel 'conducts' flux 500 times better than the air, the ratio of the fluxes is about $500 \times 80/5000$, or about 8. (This estimate neglects the fact that the solenoid area not occupied by iron is only $(5000 - 80)$ mm², not 5000 mm².)

33 B

34 B

- 35** An answer should include at least the following points:

ai Flux density, or magnetic field strength ($\Phi/A = B$), can be measured from the force on a current-carrying conductor ($B = F/Il$) or from the force on a moving charge ($B = F/Qv$) which is the basis of the Hall probe. To identify the role of N , I , and l in determining Φ/A tests in which each factor is varied alone with the others held constant are needed. μ_0 is a constant.

Alternatively, the way in which Φ depends on N , I , and A could be investigated by observing the e.m.f.s induced in a coil wound around various solenoids (to vary N , A) when they carry *varying* current, since the induced e.m.f. depends on $d\Phi/dt$. Here the frequency of the alternating current, as well as the peak value, will affect the e.m.f.

ii For an electric circuit

$$V = I/R$$

If flux (Φ) is compared with current (I), we see that current-turns (NI) is analogous to p.d. (V), and the term $I/\mu_0 A$ to resistance (R). R can be written $\rho l/A$, or $l/\sigma A$ (ρ is resistivity, σ conductivity), and the term $l/\mu_0 A$, which plays the role of resistance in the 'magnetic circuit', is called reluctance. Like resistance it is proportional to length, inversely proportional to area, and depends on the properties of the material. So although we have no reason to suppose that anything does actually flow in a magnetic circuit the comparison is a useful one.

iii The constant μ_0 is a property of air (strictly vacuum, 'free space'). The presence of iron greatly increases the flux in a magnetic circuit, perhaps by a factor of 1000. The equation becomes

$$NI = \Phi l / \mu_r \mu_0 A.$$

iv It is sometimes helpful to think of the behaviour of a magnetic circuit in terms of an equivalent electric (or water) circuit. Thus increasing the driving force (potential difference, current-turns, pressure difference) increases the flow (current, flux, flow of water); adding a high resistance, an air gap, or a constriction decreases the flow.

bi Summary of observations; note that it involves ideas (current, flux) which

themselves require explanation.

ii Analogy; based only on formal equivalence of relationships, not on physical explanation.

iv Consequence of applying the analogy in **ii**.

36a 0.05 H

b The p.d. across coil B will be very much less. Because of the greater reluctance, the flux produced by the same current will be much less. So the rate of change of flux linkage and hence the e.m.f. induced in coil B will be much less for the same rate of change of current.

c The p.d. across coil B will be doubled: having twice as many turns in A will double the flux for the same current. The rate of change of flux is doubled and so therefore is the induced e.m.f.

d Doubling the number of turns in coil B doubles the flux linkage. The rate of change of flux linkage doubles, and so therefore does the induced e.m.f.

e Because the flux is decreasing, the p.d. across B will have the opposite sign. Because the rate of change of current, and therefore of flux and of flux linkage has half its previous value, the magnitude of the p.d. across coil B will be 0.75 V.

37a The p.d. across R is only 0.05 V when the current is 0.5 A, so the p.d. across L is very nearly 2 V. The rate of rise of current is 50 A s^{-1} , so the inductance is $2/50 \text{ H}$, or 0.04 H, approximately.

b By connecting an oscilloscope across R. **c** The p.d. across R when the current is 0.5 A is still only 0.1 V, so that the p.d. across L is still not much under 2 V. The rate of rise of current will be much the same.

d The *initial* rate of rise of current will be the same, 50 A s^{-1} . When the current is 0.5 A, the p.d. across R is 1 V, leaving only 1 V across L. The rate of rise of current must now be about half what it was when there was nearly 2 V across L, being now about 25 A s^{-1} .

38 C

39 A

40 B

41 E

42a 20:1 approximately, allowing nothing for losses.

b There must be enough turns in the 240 V coil for the current in it to be small when no current is drawn from the secondary. But if the number of turns is very large, the resistance of the coils begins to make a big difference.

c Not for the intended purpose.

43

Number of turns on primary P	Number of turns on secondary S	Brightness of lamp L	Alternating p.d. across S/volts
20	50	normal	2.5
50	20	dim	0.4
20	30	dim	1.5
40	100	normal	2.5
20	80	bright	4.0

Table Q1

Note that, if the coils are wound from fine wire, it may well be that the resistance of a coil with many turns is important, and the lamp may then not light as it would if the coil's resistance were negligible.

44a The flux through the secondary coil must be pretty much the same wherever the coil is on the core. The greater proportion of the flux must circle round within the core, with little flux passing through the air.

b The flux has to cross the air gaps. The system behaves for flux as an electric circuit behaves for current. The air gaps correspond to lengths of high-resistivity material inserted into a circuit of thick copper wire. The current then depends more on the high-resistance part of the circuit than on the low-resistance part. Similarly, a small air gap in an iron-filled 'magnetic circuit' reduces the flux in the 'circuit' by a large factor.

c The result b suggests that the flux is determined by the whole 'magnetic circuit'. If there are air gaps, the flux has a low value right round the circuit. (Similarly, one would have a low current in a thick copper bar if somewhere else in the electric circuit there were a high resistance.)

d The flux links each coil, and each has an e.m.f. induced in it. Current can be drawn from each, a corresponding current flowing in the single primary. That each coil can light a lamp is no stranger than that three times as much current could be drawn from one coil.

e The primary current should be about three times larger.

45 B

46 A

47 Suggested demonstrations using small laminated C-cores and 120 + 120 turn coils might include:

Connect primary to a d.c. source (e.g. 1.5 V cell), then to an a.c. source (2 V, 50 Hz) to show that transformers work only for a.c., not for d.c. (Take care that current in primary is not large enough to cause overheating on d.c.) Use 1.5 V lamp on secondary.

Show effect of turns ratio: for given a.c. input (say 2 V) and about 10 primary turns wound from insulated wire, increase secondary turns until lamp lights.

Show effect of using 2 V, 4 V, 6 V, ... a.c. input on e.m.f. in secondary with, say, 120-turn coils as primary and secondary, using a.c. voltmeters to measure V_p , \mathcal{E}_s ; hence establish that $\mathcal{E}_s/V_p \approx N_s/N_p$; show use as step-up and step-down transformer. Measure current in primary and secondary coils (a.c. meter 1 or 5 A) to show that although transformer can be used to step up voltage there is no gain in power, and in fact power out is slightly less than power in.

Increase load on secondary (perhaps by starting with no load on secondary, then connecting one lamp, then two or more lamps in parallel) to show that current in primary rises as current in secondary rises.

Show importance of magnetic circuit linking the two coils by making a small, then a large, air gap between two C-cores, and/or show effect of no iron core linking the two coils.

Point out that iron core is laminated – to reduce losses due to eddy currents.

Show that these losses become more important at high frequencies (signal generator, oscilloscope).

48a P/V

b $I^2 R = (P/V)^2 R$

c Reduce I (i.e. increase V); reduce R .

d a.c. can be stepped up and down using transformers; d.c. cannot. Although power should be transmitted at high voltage to reduce losses, generators do not produce such high-voltage a.c., nor do consumers use it.

e 3:1

f 1:9

g Nine times more for the 132 kV system.

h Insulators, e.g. between the conductor and the supporting tower, must be larger. Also clearances between conductors and surroundings must be greater.

49ai $I^2 R$ **ii** $I^2 R/2$ **iii** $I^2 R$

b $I/\sqrt{2}$

c Its square is the average, or mean, of the square of the varying current. So it is the root of the mean of the square of the varying current. The usual abbreviation is r.m.s.

d $1/2$

e $I/\sqrt{2}$

Note. It happens that $I_{\text{r.m.s.}} = I_{\text{maximum}}/\sqrt{2}$

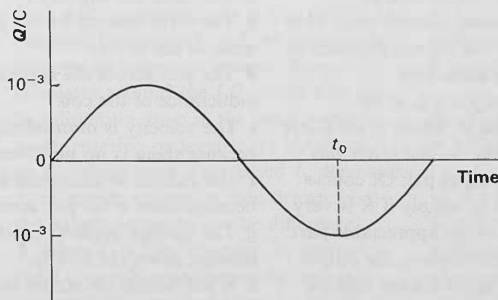


Figure Q4

in both **b** and **e**, though for different reasons. This result is *not* generally true for all varying wave forms. In **aiii**, for example, $I_{\text{r.m.s.}} = I_{\text{maximum}}$ for fairly obvious reasons, even though the current 'alternates'.

50a When $V = +10 \text{ V}$, $Q = 10^{-3} \text{ C}$.

When $V = 0$, $Q = 0$.

When $V = -10 \text{ V}$, $Q = -10^{-3} \text{ C}$ (this is at time t_0).

See figure Q4.

b The current I is a maximum when the rate of change of charge, dQ/dt , is greatest. The current is a minimum when the rate of change of charge is momentarily zero, which happens as the charge goes past its greatest value (of either sign). See figure Q5 (page 383).

At t_0 the current is zero.

c To estimate the magnitude of the current, the time scale would be needed. For example, if the complete oscillation took 0.04 s, the charge would rise from zero to its greatest magnitude in 0.01 s, requiring an *average* current of 0.1 A.

51a No, the current is zero.

b No, the p.d. is zero, so the coil is not in a magnetic field.

c The forces are equal in magnitude and opposite in direction as the current is the same size at both times but reverses its direction.

d Zero

e The wattmeter would give a reading. It must, as the lamp would light and would

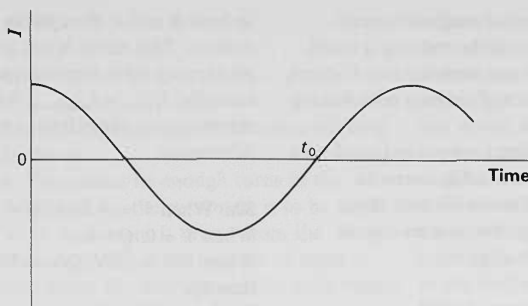


Figure Q5

be dissipating energy. If it does, the current and p.d. across the two together can no longer have the phase relationship shown in figure H103 of the *Students' guide*, for that phase relationship gives zero average force on the wattmeter coil. In fact, if the lamp is the only component in the circuit, the current and p.d. will be in phase.

52a 3 V

b 3 V

c Zero

d Zero

e Zero

f Zero

g Increasing

h Decreasing

i Zero

53a The same maximum current would have to be attained from zero in half the time, so the rate of change of current would have doubled.

b The time for the change of current has halved; to keep the rate of change constant, the maximum current must have halved, so that half the change happens in half the time, at the same rate.

c The maximum output p.d. is the maximum p.d. across R , which is IR if I is the maximum current. As the latter has halved, so has the output p.d. Of course this only works out so simply if R is very small, so that IR is not an appreciable part of the input p.d. Nevertheless, the output as a fraction of the input always falls as the frequency is raised, in this circuit.

d Less of a high frequency input appears across the output, than of a low frequency. The circuit acts as a 'filter' which only passes lower frequency signals to the next stage. The circuit is used to discourage unwanted mains frequency voltages from mixing with steady (zero frequency) voltages in devices which draw their power from the a.c. mains.

54 D

55a The p.d. across the coil is proportional to the rate of change of current, dI/dt . When I has its maximum value, dI/dt is zero.

bi OA, BC

ii AB, CD

When the product VI is positive the source is supplying energy; when this product is negative it is receiving energy. The energy comes from the magnetic field set up by the current in the coil.

56a Stored in the springs.

b Stored in the capacitor.

c The force exerted by the springs and the mass of the trolley.

d The p.d. across the capacitor and the inductance of the coil.

e The velocity is momentarily constant, because there is no net force on the trolley.

f The current is momentarily constant, because there is no p.d. across the coil.

g The springs begin to exert a force, slowing down the trolley.

h A p.d. builds up across the capacitor, and the current decreases.

i The system has as much energy as before, stored in the springs again. If energy is conserved, one could infer that the trolley had energy equal to the total energy, when it was in the centre position, and the undisplaced springs had no energy. This is, of course, the trolley's kinetic energy, $\frac{1}{2}mv^2$.

j The system has as much energy as before, stored in the capacitor again. If energy is conserved, one could infer that the inductor had energy equal to the total energy, when the capacitor was uncharged and had no energy. The energy of an inductor is the electrical analogue of kinetic energy, and is given by $\frac{1}{2}LI^2$.

57 E

58 D

59 An answer might include the following points, together with fuller explanations than are given here.

1 The current in the inductor I_L lags behind the p.d. between X and Y by a quarter of a cycle ($V \propto dI_L/dt$, so when $V=0$, $dI_L/dt=0$; when $V=V_0$, dI_L/dt has its maximum value, i.e. $I_L=0 \dots$). But current in the capacitor leads p.d. across it by quarter of a cycle (explanation in terms of Q , V , and $I_C = dQ/dt \dots$). Net effect is phase difference of half a cycle between the two currents. Thus one is positive when the other is negative, and vice versa: charge circulates around the LC circuit first in one direction, then in the other, then

2 Values of I_L and I_C depend on values of L , C , and frequency f . $I_C \propto f$; $I_L \propto 1/f$. At resonant frequency $I_L = I_C$ in magnitude. Because of difference in sign, charge circulates around the LC circuit first in one direction, then in the other, without much entering or leaving the circuit at X or Y. If frequency is increased I_L decreases, I_C increases: current is then drawn from the supply to 'make up the difference'.

3 When C is fully charged the current in the LC circuit is momentarily zero (at resonant frequency). Energy ($=\frac{1}{2}CV^2$) is stored in the electric field between

capacitor plates (cf. trolley at rest with spring fully compressed). The capacitor discharges through the inductor, current grows (cf. spring pushes trolley which accelerates). Current in the inductor causes a magnetic field around it which can store energy (cf. kinetic energy of moving trolley). Capacitor charges up in opposite sense

UNIT I

Linear electronics, feedback and control

(Note: answers to questions requiring graphs are given as lists of co-ordinates. This is done in the hope that it will encourage *thinking*, rather than a quick glance at the graph.)

1a $V \frac{R_2}{R_1 + R_2}$

b $6\frac{2}{3} V$

c $\frac{2}{3} \text{ mA}$

d $6\frac{2}{3} V$, 0.067 mA

2 $1.5 V$

3a $V_1 + V_2$

b $\frac{R_1}{R_2}$

c a doubled, b the same.

4a 0.2 mA

b 0.2 mA

c $20 V$

d X

e Negative.

f Equal resistances.

5a $20 \mu C$

b $100 \mu C$

6a 0.5 mC

b $0.67 \times 10^{-3} F$

c Very high resistance voltmeter (e.g. electrometer), or oscilloscope. (Charge would leak away through a normal voltmeter.)

7a Exponential decay curve.

b Current \propto slope of first graph.

c Linear rise.

8 Inverting: if the input is positive the output is negative, and vice versa.

Limiting value: over a certain range

$V_{\text{out}} \propto -V_{\text{in}}$, but V_{out} cannot exceed certain values (+ and -), which depend on the supply voltage.

9a Perfectly 'linear'; output \propto - input.

b At some value of V_{in} , V_{out} stops changing and remains steady for further change of V_{in} .

ci Gradient doubled, same maximum V_{out} .

ii Gradient divided by 3, same maximum V_{out} .

iii Same gradient, but maximum V_{out} is higher.

10a Pure sine.

b Sine, except peaks flattened.

11 In amplifier, output energy > input energy 'extra' energy being drawn from power supply. Transformer merely changes V and I values, contributing no extra energy. Many amplifiers can handle d.c.; transformers never can.

12 Both input current (to the amplifier itself) and p.d. between inputs are negligible so long as saturation not reached, or [input voltage] greater than V_{s} .

13a 20, 26, 32, 38

b 10^5

c 2×10^5

14a 5×10^{-5} V

b 5×10^{-11} A

15a 0 V

b 0.2 V

c 2×10^{-5} A

d 2×10^{-5} A

e -2 V

f Negative

g 2×10^{-6} A

h 2 V

16a V_{in}

b $V_{\text{in}}/R_{\text{in}}$

c $-V_{\text{out}}$

d $-V_{\text{out}}/R_{\text{f}}$

e $V_{\text{out}}/V_{\text{in}} = -R_{\text{f}}/R_{\text{in}}$

17a 3 V, 1 V, 0 V

b Because they both have the same current (V/R).

c In terms of co-ordinate axes, assuming a scale of ohms on the x-axis:

i (0 Ω , +2 V), A is (2 k Ω , 1 V), B is (4 k Ω , 0 V), (8 k Ω , -2 V);

ii (0 Ω , -5 V), (10 k Ω , 0 V), (110 k Ω , +50 V).

d (0 Ω , +3 V), (10 k Ω , 0 V), (110 k Ω , -30 V).

ei (0 Ω , +2 V), (10 k Ω , 0 V), (30 k Ω , -4 V) becoming (0 Ω , -1 V), (10 k Ω , 0 V), (30 k Ω , +2 V);

ii (0 Ω , +2 V), (10 k Ω , 0 V), (25 k Ω , -3 V);

iii (0 Ω , +2 V), (20 k Ω , 0 V), (50 k Ω , -3 V).

18a -1 mV

b 1001 mV

c 1001 mV

d -1002 mV

e +2000 mV

f 500 mV, +499 mV.

g Opposite direction.

h Much too large.

i 0.5 mV rise.

j -0.5 mV

k +1000 mV; nearly.

19a 0 V

b 3 V

c 3×10^{-5} A

d 5×10^{-5} A

e 8×10^{-5} A

f 8 V

g -8 V

h $V_{\text{out}} = -(V_{\text{A}} + V_{\text{B}})$

i $V_{\text{out}} = -(2V_{\text{A}} + V_{\text{B}})$

j -7 V

k $V_{\text{out}} = -2(V_{\text{A}} + V_{\text{B}})$; $V_{\text{out}} = -1.6$ V

l $V_{\text{out}} = -10(V_{\text{A}} + V_{\text{B}})$; $V_{\text{out}} = -8$ V

20a 0

b $V_{\text{A}}/R_{\text{A}}$, $V_{\text{B}}/R_{\text{B}}$, $V_{\text{C}}/R_{\text{C}}$

c $I_{\text{f}} = I_{\text{A}} + I_{\text{B}} + I_{\text{C}}$

- d $I_f = -V_{out}/R_f$,
 $V_{out} = -R_f(V_A/R_A + V_B/R_B + V_C/R_C)$
 e All must be the same.

- 21a $R_C = 5R_f$
 b $R_B = \frac{5}{2}R_f$
 c $R_A = \frac{5}{4}R_f$
 d e.g. $R_C = 40\text{ k}\Omega$, $R_B = 20\text{ k}\Omega$, $R_A = 10\text{ k}\Omega$, $R_f = 8\text{ k}\Omega$.
 e Assume 63 will be represented by 12.6 V, then resistors R_A, \dots, R_f should be in the ratios $1:\frac{1}{2}:\frac{1}{4}:\frac{1}{8}:\frac{1}{16}:\frac{1}{32}$ (inputs):2 (feedback). Assume 63 represented by 6.3 V and the ratios become: $1:\frac{1}{2}:\frac{1}{4}:\frac{1}{8}:\frac{1}{16}:\frac{1}{32}:1$.

22a Ramp and analogue negative terminal to comparator 1; ramp and analogue positive terminal to comparator 2; comparators to bistable inputs A and B respectively; bistable and clock into AND; AND to counter 1 Mz input; comparator 1 to counter input P.

b Display goes to zero at start of each count and begins to count up. If voltage was near level of top of ramp, the reading would be displayed for negligible time. Either add (unspecified) circuitry to 'hold' (latch) the reading until next reading is ready, or make the ramp generator have long rest spaces between ramps so that display is steady for a large proportion of the time.

- 23a Summer with $R_A:R_B:R_f = 1:\frac{1}{10}:1$, followed by a 1:1 inverter.
 b '10V_A' must first be inverted (1:1) to give $-10V_A$. This is then added to $3V_B$ using a summer with $R_A:R_B:R_f = 3:10:30$.

- 24a 1 V
 b 1 μA
 c The capacitor.
 d Yes
 e $10\text{ }\mu\text{C}$, $20\text{ }\mu\text{C}$
 f 0.1 V, 0.2 V
 g Lefthand positive.
 h Falling steadily.
 i -0.01 V s^{-1}
 j Fall at 0.02 V s^{-1} .

k V_{out} moves with uniform gradients from (0 s, 0 V) to (2 s, -0.02 V) to (7 s, -0.17 V) to (8 s, -0.15 V).

- l All rates of change multiplied by 10, V values at the times given in answer to k: 0 V, -0.2 V , -1.7 V , -1.5 V .

- 25ai $I = V_{in}/R$
 ii $dQ/dt = V_{in}/R$
 b $dQ/dt = -C dV_{out}/dt$

$$\text{c } dV_{out}/dt = -\frac{1}{CR} V_{in}, \text{ so}$$

$$V_{out} = -\frac{1}{CR} \int V_{in} dt + \text{constant.}$$

- 26a V_{out} falls at 0.5 V s^{-1} .
 b Triangular wave, amplitude 62.5 mV, starting at zero, negative half-cycle first.
 c Like a sine wave, but each half-cycle is a parabola. Starts at zero, first half-cycle negative. (Gradient at $V = 0$ is $\pm 1\text{ V s}^{-1}$; peak value $\pm 0.5\text{ V}$.)

- 27a Current through R = rate of change of charge on C, whence $-V_{out}/R = C \frac{dV_{in}}{dt}$, whence $V_{out} = -CR dV_{in}/dt$.
 b The outputs (a), (b), (c), and (d) in figure 170 in the *Students' guide* correspond to the inputs (ii), (iv), (i), and (iii) respectively in figure 171.
 c Differentiators do not always differentiate exactly.

- 28a 0.2 mA
 b 2 V
 c +2 V
 d +2 V
 e $V_{out} = 3V_{in}$
 f In place of the 1:2 resistance ratio of the diagram, it would need a 1:9 ratio.
 g Line runs from (0 Ω , +6 V) straight to (30 k Ω , 0 V). X is at (20 k Ω , 2 V).
 h All V values half those in g.

29ai $V_X = V_{out} \frac{R_1}{R_1 + R_2}$

ii $V_{in} = V_X$

b $\frac{R_1 + R_2}{R_1}$

- 30a 11, 6, 2, 1.5, 1.1, 1.01
 b $R_2 = 0$ or $R_1 \rightarrow \infty$ (=open circuit).

- 31a** 3 V
b $15\ \mu\text{A}$
c $1.5 \times 10^{-5}\ \text{V}$

- 32a** $V_{\text{in}} = V_{\text{out}}$
b $I_L = V_{\text{in}}/R_L$
c Because the p.d. between the + and - inputs is very small indeed, and the resistance between them is very large indeed.

33 Either inverting amplifier, e.g. amplification factor = 1, output current through a load of $1\ \text{k}\Omega$ is $1\ \text{mA}$, or amplification 10, load $10\ \text{k}\Omega$, output current $1\ \text{mA}$; or non-inverting amplifier of similar specification; or unity gain follower with $1\ \text{k}\Omega$ load.

34 A variety of possible solutions, e.g. current to be measured passes through a $10\ \Omega$ resistor. The p.d. across this ($0.1\ \text{V}$) is made the input to an inverting or non-inverting amplifier with a gain of 10.

- 35ai** $V_X = (V_{\text{out}} + 3)/2$,
ii $V_Y = \frac{5}{2}\ \text{V}$
b $V_{\text{out}} = 2\ \text{V}$
c $V_X = (V_{\text{out}} + V_A)/2$, $V_Y = V_B/2$, whence $V_{\text{out}} = V_B - V_A$
d Two intersecting straight lines AC and BD. A is ($0\ \Omega$, $3\ \text{V}$), B is ($0\ \Omega$, $5\ \text{V}$), C is ($2\ \text{M}\Omega$, $2\ \text{V}$), D is ($2\ \text{M}\Omega$, $0\ \text{V}$), lines intersect at $X \equiv Y$ ($1\ \text{M}\Omega$, $2.5\ \text{V}$).
e A is (0 , 1); B is (0 , 3), C is (1.5 , 4), D is (1.5 , 0), intersecting at $X \equiv Y$ (0.5 , 2); $V_{\text{out}} = 4\ \text{V}$.

36a Differential amplifiers: $R_f/R_{\text{in}} =$

- i* 1
ii 10
iii $1/5$
b Possibly bridge circuit null detector.

- 37a** Limiting value, positive.
b Limiting value, negative.
c Thermistor and fixed resistor in series between $+V_s$ and $-V_s$, thermistor at $-V_s$ end. Junction of these to V_{out} .
d Alter the temperature at which V_{out} changes sign.
e Lamp should switch off momentarily during changeover - otherwise it would stay lit.
f Diode in amplifier's output, to prevent current into amplifier when V_{out} negative.

- 38a** $+3\ \text{V}$
b $0\ \text{V}$
c Yes
d Rise (approaching $+6\ \text{V}$ 'exponentially').
e Change to $-6\ \text{V}$.
f Change to $-3\ \text{V}$.
g Fall (approaching $-6\ \text{V}$ 'exponentially').
h Change to $+6\ \text{V}$.
i Cycle repeats.
j See figure Q6(a).
k See figure Q6(b).

- 39a** They are equal.
b $dV/dt = -(1/CR) V$; $V = V' e^{-t/CR}$ ($V = V'$ when $t = 0$).
c $1.84\ \text{V}$
d $0.68\ \text{V}$
e $2.27 \times 10^{-4}\ \text{V}$

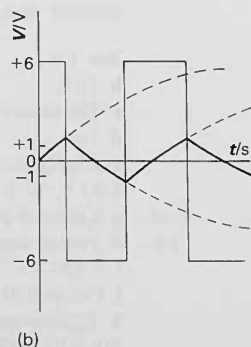
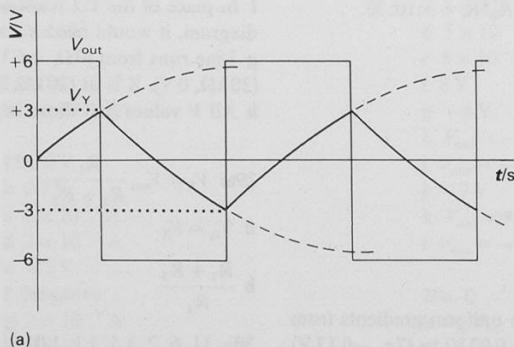


Figure Q6

40a -1 V

b -1 V

c 10^{-6} A towards A.

d Will change its charge; potential at B will rise at 1 V s^{-1} .

e -10 V

f $-dV/dt$

g $dV/dt = 10\text{ V}$, $V = 0.1e^{10t}$

h Exponential rise, becoming constant at limiting value.

i 0.107 s

41a $d^2V/dt^2 = -10^8\text{ V}$

b 1600 Hz (2 s.f.)

42 The values not given in table I2 are:

10 s , $\pi/2$

2 Hz , 0.125 s

10 ms , π

43a 2π or zero.

b π

c $\pi/2$

d 4π (or 2π or zero).

e a 2 V b 0 V

c In between ($\sqrt{2}\text{ V}$). d 2 V

44ai $Z/1000$

ii $S - Z/10$

b $Z/S = 9.9(0)$

c $Z/S = 9.99$

d $G = \frac{1}{\beta + 1/A}$ or $\frac{A}{A\beta + 1}$ or $\frac{A/\beta}{A + 1/\beta}$

e $G = 1/\beta$: depends only on β .

45 The answers to this question depend upon interpretation of what is being controlled: level, or inflow.

Feedback: position of ball controls inflow.

Reference signal: setting of ball relative to valve position, or intended water level.

Error signal: discrepancy between ball level required to stop flow, and present level.

Input: could mean reference signal, or water input.

Output: could mean rate of flow into tank, or water outflow.

Disturbance: change of water pressure and therefore flow rate, or drop in level when

water is being used.

46a Potential at A is $\frac{V-\mathcal{E}}{2R} \times R = \frac{V-\mathcal{E}}{2}$.

Potential at B = $V/2$, so p.d. between A and B is $\mathcal{E}/2$.

b A and B inputs to differential amplifier; its output $\propto \mathcal{E}/2$. This could be fed into a comparator, being compared with a second input from a potential divider fed from an independent source of p.d. The output will rise or fall as \mathcal{E} falls or rises (care needed over signs!), and so can control a power amplifier regulating V .

47a Lights control flow of cars. For example: car approaches red light, providing feedback. But this must not instantly change the lights – a waiting period is needed unless lights have not changed for, say, 15 seconds. Further complications need thinking out.

b A lift is a common example.

48a Body tends to move in a straight line (Newton's First Law).

b Body moves to 'outside' of bend.

i Will correct the course.

ii Will increase the veering.

c The same.

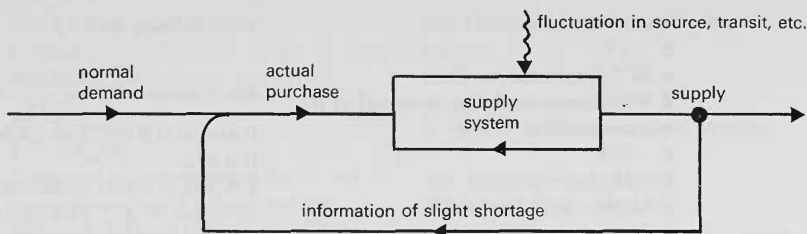
d Accelerator pressed, car accelerates backwards, driver's inertia makes him move forwards relative to the car, force on accelerator increased, car moves faster. Driver pulls foot back in alarm reaction, car slows, leg tends to leave accelerator, car slows more, driver reacts again, etc. Frequency depends on driver's reaction time.

49a

No.	3	4	5	6	7	8	9	10	11
m/g	90	100	110	110	100	90	90	100	110

Table Q2

b Reducing the correction by 50% will give damped oscillation, but not immediate correction. Moving the sensor one place to the left will give stable correction.



50 People buy in reserves of sugar, so shortage *does* happen, i.e.:
thought of shortage \rightarrow extra demand
 \rightarrow real shortage.

Possibly the fear of shortage arises from an actual temporary hold-up in supply, i.e.:
small actual shortage \rightarrow more purchase
 \rightarrow more shortage.

An example of positive feedback is shown above.

An example of the other kind: expectation of cold winter leads to stocking of fuel, which does *not* make any difference to the weather: there is no feedback.

Some people believe that spending on defence makes war more likely, because of its feedback effect on attitudes. There is room for argument here.

UNIT J

Electromagnetic waves

1 The ripples do not affect one another in the sense that the spreading of the ripple is

unaffected even though other ripples cross part of it. The ripples *do* affect one another in the sense that the motion of the water at a place where there are waves from two sources is the sum of the motion due to each wave alone. To a physicist, it is this simple adding together of two or more waves at the same place which is implied by the use of the term 'superposition'.

In shallow water, a wave must not have too great a height, or it will collapse into a 'breaker'. It can happen that two waves, each too low to break, add together to give a wave which will break. Then the combined effect of the waves is *not* the same as the sum of their separate effects.

2 Figure Q7 shows what the wave might look like. A similar effect can be seen in the aerial photograph, figure J96 in the *Students' guide*.

3a AC is the incident wave front; NB the reflected wavefront. Waves start out from the places such as K on the mirror (figure J97 in the *Students' guide*) when the

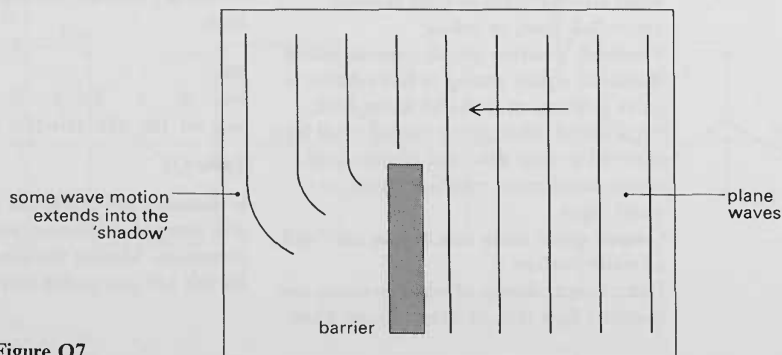


Figure Q7

incoming wave reaches those places. One starts from A and travels out in a circle, radius AN, in the time that the part of the wavefront at C takes to get to B, so that $CB = AN$. The reflected wavefront is the flat surface common to all the little waves from different places on the surface. Because $CB = AN$, the angle of reflection is equal to the angle the incoming wave makes with the surface (angle $ABN = \text{angle } BAC$).

Huygens introduced the idea of imagining a wave as the net result, or envelope, of many little waves from imaginary point sources along the wavefront, an idea which is also used in discussions of diffraction in this Unit. The little waves are often called Huygens's or secondary wavelets.

b AC is the incident wavefront, travelling in direction HK; NB is the refracted wavefront.

The explanation follows the same lines as a, except that in the water the waves go more slowly than in air. As the wavefront arrives along the boundary, wavelets start out one after the other into the water. The wavelet starting at A travels a distance equal to AN, in the time that the part of the wave at C takes to get to B, along the longer distance CB in air, where the speed is greater. Clearly $CB/AN = c/v$, where c and v are the two speeds. Since CB/AB is the sine of angle CAB, and AN/AB is the sine of angle NBA, the ratio c/v is also the ratio of the sines of these angles. The ratio of the sines is constant

for a given pair of materials (Snell's Law). **c** Since DAC is a right angle, the angle of incidence $i = \text{angle } CAB$. Similarly, the angle of refraction $r = \text{angle } NBA$. So $\sin CAB/\sin NBA = c/v = \sin i/\sin r$.

4 The new wavefront is formed by the envelope of the secondary wavelets. It shows the wave diffracting (spreading) around the obstacle. (See figure Q8.)

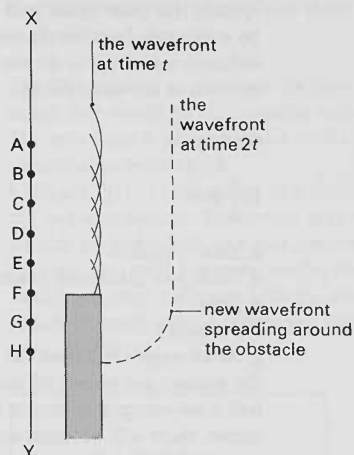


Figure Q8

5a Figure Q9 shows the regions in front of the loudspeakers where the sound heard will be loud or quiet. The sound will be quiet at places whose distance from one speaker is an odd number of half-wavelengths different from its distance from the other speaker.

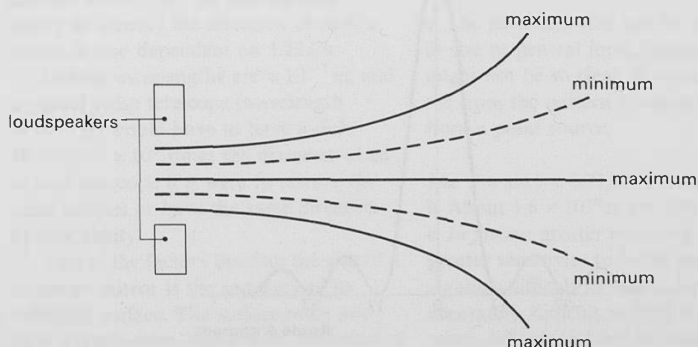


Figure Q9

b One loudspeaker in a room can give superposition effects, as there are quiet spots at which two waves, one reflected from walls, floor, or ceiling, arrive out of step. In general, there will be waves coming from several directions at once, but unless the room has a very irregular shape, there will still be quiet spots where a number of these waves superpose to give little net effect. In listening to music or speech, the 'quiet spots' will not normally be noticeable because the large number of different wavelengths do not produce these minima at the same place.

6a $b/2$

b $\frac{b}{2} \sin \theta$

c Zero resultant.

d Each pair gives zero resultant.

e Zero

f $\lambda = b \sin \theta_1$

g At all angles less than the angle θ_1 , if the sources are paired off into pairs with half a wavelength difference in path to the screen, there will be some sources left unpaired. At $\theta = 0$, no pairs can be found, since the light from all sources travels

equally far to the screen, and at this angle there will be most light.

h $\frac{\lambda}{2} = \frac{b}{4} \sin \theta_2$

$2\lambda = b \sin \theta_2$

Zero resultant on the screen.

i $n\lambda = b \sin \theta_n$

where $n = 1, 2, 3$, etc. gives the 1st, 2nd, 3rd, etc. minima.

7 See figure Q10.

8a In figure J103 in the *Students' guide* the angle to the central position is plotted on a linear scale along the x -axis, while in figure J102, the angle is plotted directly around the central point.

The 'polar plot' J102 gives a better picture of the pattern.

b $b = \frac{\lambda}{\sin \theta}$ for first minimum.

$b = \frac{330 \text{ m s}^{-1}}{3 \times 10^3 \text{ s}^{-1}} \times \frac{1}{\sin(0.5)} = 0.23 \text{ m}$

c Reflected and direct waves can superpose and confuse the diffraction pattern. A suitable environment could be produced by performing the experiment

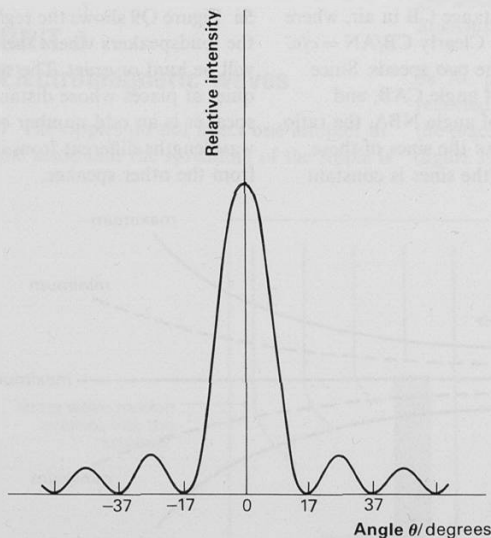


Figure Q10

either out of doors and well above the ground or in a large room with reflection-free inner surfaces.

d If b is reduced by a factor of ten, then $\lambda/b = 0.11/0.023 = \sin \theta > 1$, which implies that the central maximum extends over an angle of $\pi/2$ either side of the centre. The absence of minima is important in the faithful reproduction of recorded sound, for otherwise there will be directions with little or no signal for a given frequency. This is particularly undesirable in a stereo system, where the acoustic perspective of the sound stage is conveyed to the listener by the combination of sound from the two loudspeakers.

This explanation takes no account of unwanted reflections and the quite considerable variations of sound intensity with frequency that are quite common in most rooms.

9 Solving these should not be taken too seriously; it can be quite fun and quite a valuable exercise in detective work.

(a) O (b) Q (c) Y (d) I (e) V (f) A

10 E

11 C

12 An optical instrument can only resolve two objects of angular separation θ , according to the Rayleigh criterion, if the ratio of wavelength to aperture diameter $1.22\lambda/b < \theta$. Also, since the central maximum of a distant point source has angular width $= 2\theta$, the instrument's ability to identify the direction of such a source is also dependent on $1.22\lambda/b$.

Optical wavelengths are $\approx 10^{-7}$ m, and a typical radio telescope (wavelength $\approx 10^{-2}$ m) would have to have a dish $10^{-2}/10^{-7} \approx 10^5$ times the diameter of an optical telescope if it were to resolve the same sources or have the same direction-finding ability.

One of the factors limiting the size of a telescope mirror is the accuracy of its reflecting surface. The surface must not have irregularities bigger than a fraction of

a wavelength. This is a particular problem for optical telescopes, and to grind and polish an accurate, smooth mirror, 5 m across, is no mean feat. It is also a problem for designers of radio telescopes, especially if the telescope is to operate with wavelengths of a few centimetres or less.

Large diameters of telescope mirrors are also demanded by the very small intensity of the radiation being detected, so that it is important to collect as much of it as possible.

13a The pattern would have the same shape, but would be on a smaller scale. The reduction is in proportion to the reduction in wavelength.

b Figure Q11 is an attempt at a sketch of the two-star pattern. Remember that you wouldn't know which star was responsible for each part of the pattern, so the rings would not keep a distinct identity, and a sketch showing clear, overlapping rings must be wrong.

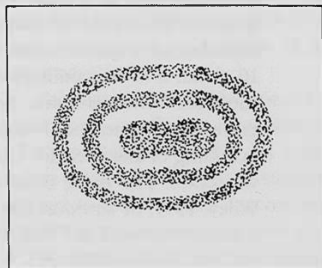


Figure Q11

c The pattern would not be very different in size or general form, though the 'rings' might not be so clear. It would be hard to tell from the pattern produced by light from a point source.

14a $\theta \approx \sin \theta \approx 0.21/25 \approx 0.008$ rad

b About 1.6×10^{18} m (or 160 light years).

c *In favour:* greater resolving power; greater sensitivity to feeble radiation.

Against: difficult to steer a large object accurately; difficult to hold it still in a wind; difficult to keep its shape accurate

under deforming forces of its own weight and of wind; expense.

$$15a \quad \theta \approx \lambda/b \approx \frac{5 \times 10^{-7}}{2 \times 10^{-3}} \text{ rad}$$

$$\approx 3 \times 10^{-4} \text{ rad.}$$

b Separation of images on retina

$$= 5 \times 10^{-4} \times 20 \times 10^{-3} \text{ m} = 10 \times 10^{-6} \text{ m.}$$

c The separation of the receptors is half this, which is adequate.

d The structure of the retina, in particular the separation of the receptors (cones) imposes a limit, but other factors, such as irregularities in the material and shape of the cornea and the lens, mean that this limit is not achieved. The sensation of vision is a product of the eye and brain together, so the limitations may be very complex.

16 When the eye pupil is fully open (as it may well be at night) its diameter is about 5 mm. So $\theta \approx 5 \times 10^{-7}/5 \times 10^{-3} \approx 10^{-4} \text{ rad}$.

In fact, the best the eye can do is about twice this, that is, $2 \times 10^{-4} \text{ rad}$ (see question 15). As the Morning Star Venus subtends an angle of rather less than 10^{-4} rad , it is very unlikely that the eye could see Venus as a disc. Some other explanation is needed. For more about the twinkling of stars see, for example, WALKER, J. *The flying circus of physics*. Wiley, 1977, or MINNAERT, M. *Light and colour in the open air*. Bell, 1959.

$$17a \quad 0.6 \times 10^{-3} \text{ rad}$$

b We estimate the mean diameter of the ellipse to be about $8 \times 10^{17} \text{ km}$, subtending an angle of about $1.6 \times 10^{-4} \text{ rad}$ at a distance of $5 \times 10^{21} \text{ km}$.

c The ratio of the wavelength to the diameter of the telescope must be of the order 10^{-4} , from **b**. At a wavelength of 0.15 m, the diameter comes to 1.5 km: not far short of a mile. In fact, the plot shown in figure J108 was made with the 'one-mile radio telescope' at Cambridge University, but this telescope is not a dish a mile across. It consists of smaller, separate dishes which are moved around over a region about a mile across, the signals from which are combined so as, in total, to simulate the presence of a much larger single telescope. A dish one mile in diameter would not be a practical proposition on mechanical grounds, leaving aside the question of its expense.

Cygnus A is the second 'brightest' radio object in the sky, despite its great distance. It was first thought that the optical source, midway between the two patches of radio intensity seen in figure J108, was a pair of colliding galaxies. When high resolution radio measurements showed that the source consisted of two separate regions, well away from the visual object, most people held that the galaxy in the middle of the picture had exploded, emitting a pair of rapidly moving clouds of material in opposite directions, and that these were the radio sources.

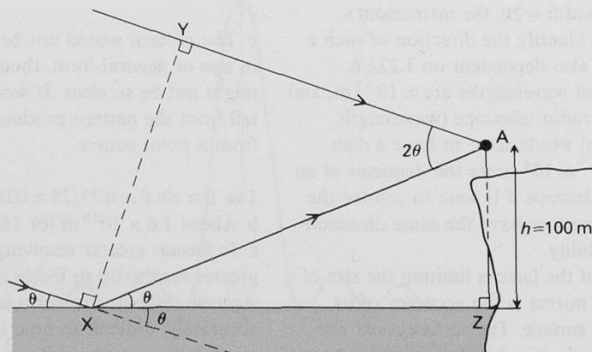


Figure Q12

18a Figure Q12 shows the point Y. Waves from the Sun just reaching Y are in phase with those at X. Since $AX - AY$ is an odd number of half-wavelengths, reception at A would be at a minimum if it were not for the reversal of the phase of the wave as it is reflected at X.

b As the Sun sets, θ decreases, and so AX increases, so that the angle at which the wave is reflected at X is the same as that at which the direct waves reach A. AY also increases, since the angle at Y is a right angle. As the Sun sets, the path difference $AX - AY$ increases. If it is, say, three half-wavelengths in figure Q12, it will soon be four half-wavelengths, and the signal at A will be a minimum. The signal at A will alternately be a maximum and a minimum as the path difference progressively becomes larger, according to whether it is an odd or even number of half-wave-lengths.

Algebraically:

$$\angle YXA = 90^\circ - \theta$$

$$\Rightarrow \angle YAX = 2\theta$$

from $\triangle AXZ$,

$$AX = \frac{h}{\sin \theta}$$

from $\triangle AYZ$,

$$AY = AX \cos 2\theta$$

$$= \frac{h \cos 2\theta}{\sin \theta}$$

Path difference = $AX - AY$

$$= \frac{h}{\sin \theta} (1 - \cos 2\theta)$$

But $(1 - \cos 2\theta) = 2 \sin^2 \theta$,

$$\begin{aligned} \text{so, path difference} &= \frac{h}{\sin \theta} (2 \sin^2 \theta) \\ &= 2h \sin \theta \end{aligned}$$

For maxima

$$\begin{aligned} 2h \sin \theta &= (n + \frac{1}{2})\lambda \\ \Rightarrow \sin \theta &= \frac{(2n + 1) \lambda}{4h} \end{aligned}$$

Since $h = 100 \text{ m}$ and $\lambda = 1.5 \text{ m}$

$$\sin \theta = \frac{(2n + 1)}{4} \times 0.015 \text{ for a maximum}$$

signal at A.

19i On a large scale, light does cast sharp shadows and the wave properties are relatively unimportant – the so-called rectilinear propagation of light.

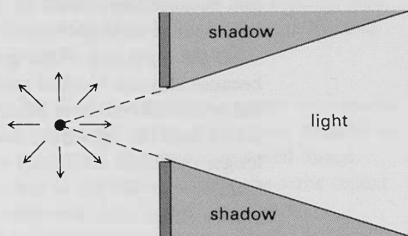


Figure Q13

However, if the border between light and shadow is inspected closely, or if the size of the aperture or obstacle is comparable to the wavelength, wave effects become noticeable in that light spreads into the shadow region non-uniformly, producing a pattern of light and dark regions. See experiments J1 or J2 or demonstration J5.

Calculations along the lines of questions 7 and 8 show the relative size of θ for a particular λ and b . An explanation comes from the principle of superposition applied to the Huygens's secondary wavelet construction (questions 4 and 6).

ii The resolution of any instrument is limited fundamentally by the diffraction effects of the smallest aperture which it presents to the incoming wavefront. If the aperture has diameter b , the minimum angular separation $\theta \approx 1.22\lambda/b$. Magnification cannot overcome this limitation of resolution, but increasing b , or effectively increasing it (see demonstration J8 and question 18), allows an improvement.

20a λ

b From the geometry of figure J110(d), the path difference is $s \sin \theta_1 \Rightarrow \lambda = s \sin \theta_1$.

c Next constructive superposition occurs when path difference = 2λ , $\Rightarrow 2\lambda = s \sin \theta_2$.

d $n\lambda = s \sin \theta_n$, where $n = 1, 2, 3$, etc.

21 When every other line is blocked out, the grating spacing, s , doubles, so the maxima appear at angles whose sines are half what they were previously. Half as much light goes through the grating, so the maxima will be less bright. (They may not be half as bright, because they may not have the same angular width as before, and because there could be more of them in the range of angle from 0 to 90° .)

If the regularity of the grating is lost, because slits are blocked out at random, the whole co-operative effect of the grating is lost, and the light from each slit no longer combines additively with the light from every other slit at certain sharp angles. In fact, the most likely thing to see would be the broad, hazy diffraction pattern of a single slit, brighter than that from one slit according to the number of slits left open. If there were some mean spacing not too large compared with a wavelength, the 'grating' could produce a diffuse halo at an angle corresponding to that spacing. Students should be content with having thought of any point like this; especially if they notice that losing the grating's regularity would be important.

22 If the slats repeat every 0.1 m, the wavelength should be of that order, so the frequency will be 3400 Hz or lower, a rather high pitched whistle.

23 Here are some points that might be mentioned. A full discussion would be a very long one.

In diffraction at a single slit the light does not 'just spread out', but has maxima and minima at angles where waves from different parts of the slit combine constructively or destructively. The pattern from two slits contains extra bright and dark bands which are the result of combining the light from the two slits, but it also involves diffraction, because the light from the two slits must spread out enough to overlap.

If one wished to distinguish between interference and diffraction, it might be best to reserve the term interference for

cases like a pair of small loudspeakers, whose effect is the sum of waves from point sources, the waves having a finite phase difference. Diffraction could be kept for the case where, as for a wide single slit, waves with a continuously varying phase difference are to be combined. Young's two-slit experiment involves both, and so does a diffraction grating. Both are special cases of the principle of superposition: the principle of calculating the combined effect of waves by adding their amplitudes, taking due account of their phases.

$$\mathbf{24} \quad n\lambda = s \sin \theta \Rightarrow s = \frac{n\lambda}{\sin \theta} \text{ and } \lambda = \frac{s \sin \theta}{n}$$

$$\mathbf{a} \quad s = \frac{1 \times 550 \times 10^{-9}}{\sin 20^\circ} \text{ m} \\ = 1.6 \times 10^{-6} \text{ m}$$

$$\mathbf{b} \quad \lambda = \frac{1.6 \times 10^{-6} \times \sin 32^\circ}{2} \text{ m} \\ = 420 \text{ nm}$$

$$\mathbf{25} \quad n\lambda = s \sin \theta \Rightarrow \sin \theta = \frac{n\lambda}{s}$$

$$\mathbf{a} \quad \sin \theta = \frac{5 \times 10^{-7} \text{ m}}{10^{-6} \text{ m}} = 0.5$$

$$\theta = 30^\circ$$

$$\mathbf{b} \quad \text{Path difference} = \lambda.$$

$$\mathbf{c} \quad \text{Path difference} = 5000\lambda.$$

$$\mathbf{d} \quad \text{Path differences are all a whole number of wavelengths.}$$

$$\mathbf{e} \quad (5000 + \frac{1}{2})\lambda$$

$$\mathbf{f} \quad \text{Zero}$$

$$\mathbf{g} \quad BC = 50\lambda, BC' = 50.5\lambda. \text{ A change in path difference of 1 in 100 must be about 100 times bigger than the 10000-slit case where it is 1 in 10000. Thus the new angle } \theta' \text{ would have to be much larger to create such a large path difference.}$$

$$\mathbf{h} \quad BC' = 5000\lambda'$$

$$\mathbf{i} \quad BC' = (5000 + \frac{1}{2})\lambda$$

$$BC' = 5000\lambda'$$

$$5000\lambda' = (5000 + \frac{1}{2})\lambda$$

$$10000(\lambda' - \lambda) = \lambda$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{1}{10000}$$

j It would be about 100 times worse, i.e.

$$\frac{\lambda' - \lambda}{\lambda} = \frac{1}{100}$$

26a In the earlier derivations of the diffraction pattern formula (for example, *Students' guide* figure J111) the rays from adjacent slits were drawn parallel, whereas they are actually shown coming together at the screen in figure J115. If the screen is a long way off, is this an acceptable approximation? For a typical case of $s = 0.5 \text{ mm}$ and $L = 2.5 \text{ m}$ then the angle between the rays at the screen is not zero but approximately $\frac{0.5 \times 10^{-3}}{2.5}$

radian or 0.01° which is very small.

b Path difference, $s \sin \theta_n = n\lambda$.

c For $\theta = 10^{-2} \text{ rad}$, $\sin \theta = 0.0099998 \approx 0.01000$ (4 significant figures), $\tan \theta = 0.0100003 \approx 0.01000$ (4 significant figures)

$$n\lambda = sx_n/L \text{ or } \lambda = sx_1/L.$$

$$\mathbf{d} \quad (n+1)\lambda = sx_{n+1}/L$$

e Subtracting the result of **c** from **d**

$$[(n+1) - n]\lambda = \frac{s}{L}(x_{n+1} - x)$$

$$\lambda/x = s/L$$

27a Nearly $5 \times 10^{-7} \text{ m}$.

b The first few fringes are alternately bright and dark. Because of the range of wavelengths present in the light transmitted by a colour filter, outer fringes look blurred. If the spacing of bright fringes is estimated, the wavelength obtained is a rough average over the range of wavelengths present.

28a The fringe spacing will be halved. The wavelength, now 250 nm ($2.5 \times 10^{-7} \text{ m}$), is in the ultra-violet part of the spectrum, so the fringes would be invisible to the eye, though a photographic film could record them.

b The fringe spacing now returns to its previous value. In interference or diffraction, the angle at which an effect is seen depends only on the ratio of the wavelength to the dimensions of the slit or aperture.

c The electron wavelength is 5000 times smaller than the wavelength of the visible light used, so to get the same fringe spacing, the slits would have to be 5000 times closer. But fringes 250 times closer together are said to be acceptable, so the slits need only be 20 times nearer together, with a spacing of about 0.005 mm .

Such an experiment has actually been done. See Unit L, 'Waves, particles, and atoms'.

29a X-rays cannot be focused by material or magnetic lenses, so it is not possible to produce a directly recombined image.

b The figures have both the same repeat spacing of dots and, as far as a photograph can show, similar intensity distributions across the dots. Thus the periodicities of the gratings are the same and the shapes of the scattering centres similar. Hence the 'ball-and-stick' model seems very good.

There are difficulties to overcome in the design of the X-ray camera and readers should consult one of the references to X-ray crystallography for a discussion of various solutions.

c The diffraction pattern shows no repeat spacing and hence implies no periodicity or long-range order within distilled water.

d The first is broadly similar to that for distilled water and implies no order, but the second has a definite repeated pattern, so stretched rubber seems to have a regular structure or order. Both figures J124 and J125 in the *Students' guide* are actually electron diffraction patterns.

30a Spherical waves. As drawn in *Students' guide* figure J126, the waves become circular in two dimensions.

b Plane waves.

c A, B, C, D, and E are points of constructive superposition since the two waves are in phase.

d There are minima at roughly the mid-points between adjacent maxima, and the resulting pattern of light and dark on the plate varies continuously between the two extremes. In fact the variation is nearly sinusoidal, as shown in figure Q14(a).

When the recombination takes place, the reconstructed image consists of a three-slit pattern (no higher orders beyond the first being formed), the central slit corresponding to the reference beam and the two outer slits giving a real and a virtual image of the original scattering point.

e The waves are always in phase so the conditions in **c** and **d** are always true.

f If the phase relationship varies in space or time, then the superposition pattern will cease to be stable and a jumble will result. It is the great coherence of laser light which made it possible to produce the first holograms in 1960.

g The hologram consists of a complex set of superposition patterns. As has been noted earlier, limiting the pattern to only a few orders removes only a little fine detail. If the piece of hologram is anything other than miniscule even this limitation will not

be reached and since all the patterns cover all the plate, complete reconstruction is possible. The only rider to this is that the field of view will become increasingly limited as the piece of hologram becomes smaller.

$$\begin{aligned} \text{31a } \lambda &= b \sin \theta \\ b &= 7 \times 10^{-7} \text{ m} / (7 \times 10^{-3}) \\ &= 10^{-4} \text{ m} \end{aligned}$$

$$\text{bi } n\lambda = s \sin \theta_n$$

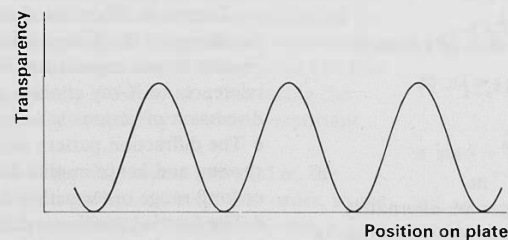
See figure Q15.

$$\text{Maxima at } \sin \theta = \frac{\lambda}{s}, \frac{2\lambda}{s}, \frac{3\lambda}{s}, \text{ etc.}$$

$$= 2 \times 10^{-3}, 4 \times 10^{-4}, \text{ etc.}$$

ii Intensity scale is $\times 4$ (since amplitude at central maximum is $\times 2$).

c The intensity scale would be $\times 4$. The full line envelope would be only half as wide, for example, 1st minimum at $\sin \theta = 3.5 \times 10^{-3}$.



(a)



Diffraction image of (a)

(b)

Figure Q14

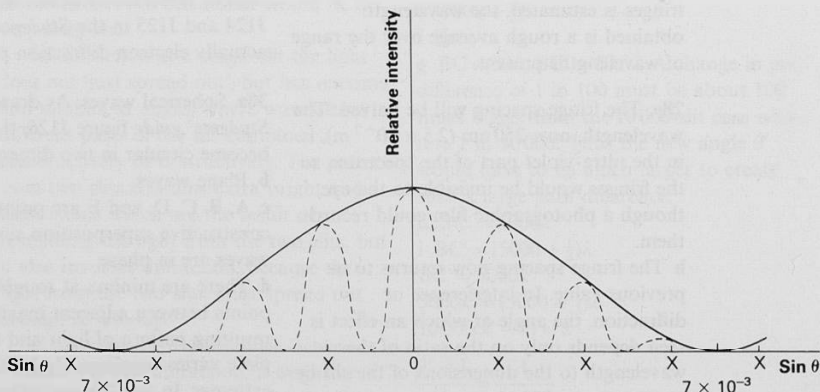


Figure Q15

32a $n\lambda = s \sin \theta_n, n = 1$

$\sin \theta_1 = 5 \times 10^{-7} / 12.5 \times 10^{-7} = 0.40$

$\theta_1 = 24^\circ$

b $\theta_2 = \sin^{-1} 0.80$

$\theta_2 = 53^\circ$

c For $n = 3$, $\sin \theta_3 > 1$, thus no such order exists.

d See figure Q16.

New path difference $= a + b$

$= s(\sin \theta + \sin \phi)$

for 3rd order, $3\lambda = s(\sin \theta + \sin \phi)$

$\Rightarrow (\sin \theta + \sin \phi) = \frac{3\lambda}{s} = 1.2$

This is possible as $(\sin \theta + \sin \phi) \leq 2$.

33 A

34 B

35 Microwaves are partially reflected from the end of the guide (it is not important to discuss the relative phase of the reflected wave). These waves superpose with the oncoming waves to give a standing wave pattern of nodes and antinodes. The distance between adjacent nodes or antinodes is equal to a half-wavelength.

$\lambda/2 = 15 \text{ mm}$

$\Rightarrow \lambda = 30 \text{ mm}$

$c = f\lambda$

$= 9.5 \times 10^9 \times 30 \times 10^{-3} \text{ m s}^{-1}$

$= 2.8(5) \times 10^8 \text{ m s}^{-1}$

a value close to that found in other electromagnetic wave experiments.

36a Drift speed is about 10^{-4} m s^{-1} for a typical conductor carrying 5 A through a cross-section of 1 mm^2 . In this case the cross-sectional area is about 100 times greater, but the current is at least this number of times smaller, so 10^{-4} m s^{-1} is a reasonable estimate.

b See figure Q17. (Note that for clarity many field lines have been omitted from this sketch.)

c In order for uncharged regions of the rods to become charged, the battery removes positive charge from the uncharged region of the lefthand rod and drives it into the uncharged region of the righthand rod, constituting a current which flows in opposite directions in the two rods.

d See figure Q18.

e A speed approaching $3 \times 10^8 \text{ m s}^{-1}$, the speed of the pulse.

37a The electrical signal wavefront moves at $3 \times 10^8 \text{ m s}^{-1}$ in a vacuum. It has not had time to reach the distant place E and thus this region has no net charge.

b $BC = 3 \times 10^8 \text{ m s}^{-1} \times 10^{-9} \text{ s}$
 $= 0.3 \text{ m}$

c See figure Q19.

d Capacitance of section BC
 $= (200 \times 10^{-12} \text{ F m}^{-1}) \times (0.3 \text{ m})$
 $= 6 \times 10^{-11} \text{ F}$

Since the p.d. between the conductors $= 1.5 \text{ V}$

then charge $Q = CV$

$= 6 \times 10^{-11} \text{ F} \times 1.5 \text{ V}$
 $= 9 \times 10^{-11} \text{ C}$

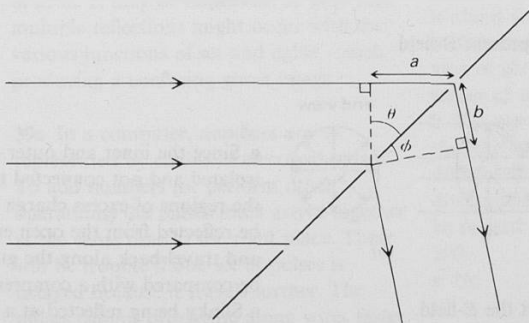


Figure Q16

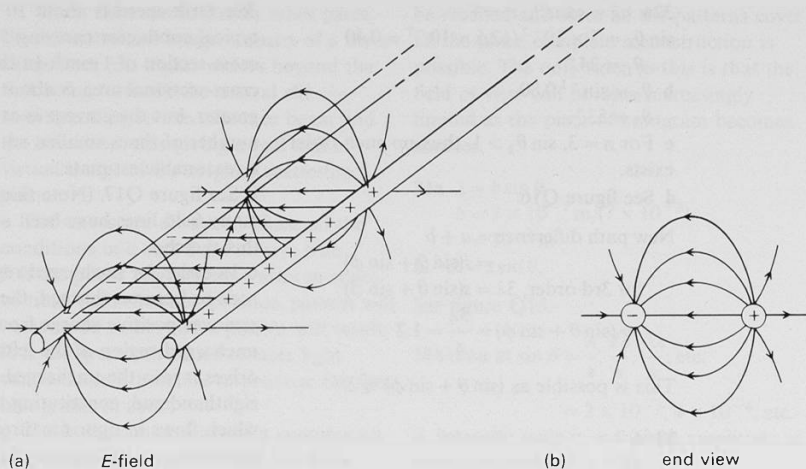


Figure Q17

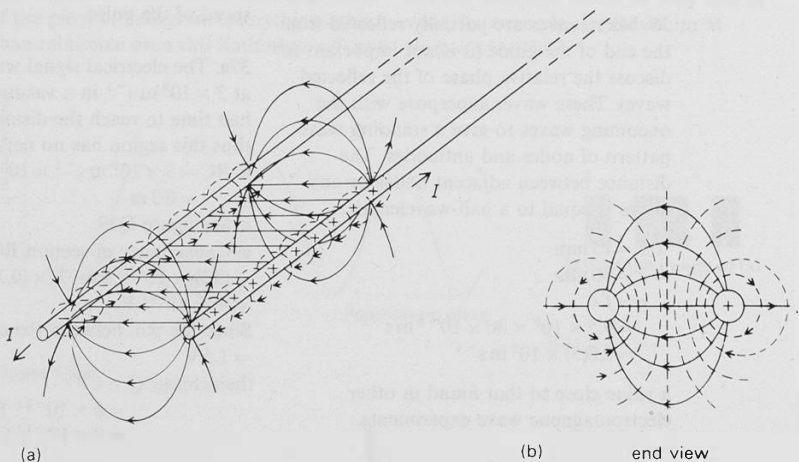


Figure Q18

Dashed lines represent B -field

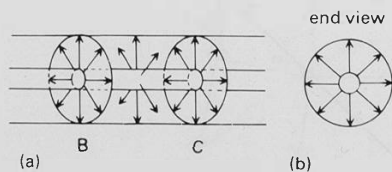


Figure Q19

Arrows represent the E -field.

e Since the inner and outer conductors are isolated and not connected to each other, the regions of excess charge are likely to be reflected from the open end of the cable and travel back along the guide. This may be compared with a compression pulse in a Slinky being reflected at a fixed end.

Note that after reflection the electric field is still directed outwards from the inner wire but the magnetic field will circulate around the wires in the opposite direction since the current associated with it is reversed. The E -field has experienced no phase change on reflection, while the B -field has changed phase by π .

38a Some of the received signal at the input to the television set travels out along the second length of coaxial cable, is partially reflected at the free end, and travels back down to the input of the television set. The journey time up and down this cable means that a second or 'ghost' image is delayed and thus appears a short time after the original. Since the electron beam scans from left to right the ghost picture appears to the right of the original.

b Time taken by electron beam to travel 10^{-2} m

$$= \frac{10^{-2} \text{ m}}{10^4 \text{ m s}^{-1}}$$

$$= 10^{-6} \text{ s}$$

hence speed of the signal

$$= \frac{2 \times 125 \text{ m}}{10^{-6} \text{ s}}$$

$$= 2.5 \times 10^8 \text{ m s}^{-1}$$

c The uncertainty in the measurement of the displacement of the ghost picture is likely to be about 2 mm. Thus, using the figures already given, the fractional uncertainty will be $2 \text{ mm}/10^{-2} \text{ m} = 20/100$, or 20 % corresponding to a length of cable of 25 m. It may be important to note that multiple reflections might occur with the various junctions of set and cable – each producing a confusing ghost image.

39a In a computer, numbers are represented by chains of electrical pulses. To add numbers (or perform other operations) the pulses must arrive together at the same time at the right place. There will be trouble if one set of pulses is delayed because it travels further. The pulses cannot propagate along wires faster than the speed of light, so if the electrical

connections in the computer are some metres in length, there will be delays of the order 10^{-8} s. This may not seem much, but it is important, because computer designers would like to be able to use pulses recurring as often as 10^9 every second if they could, so that complex sequences of operations would take as little time as possible.

b A long length of coaxial cable could be used, but it would need to be about 100 km long, so this is not a very practical suggestion. A row of capacitors and inductors along which the pulse would travel more slowly might serve, but would be likely to distort the pulse. To get a delay of 2 ms within a reasonably sized piece of apparatus, one needs a propagation speed of the order of 10^3 m s^{-1} , which suggests using sound waves in water or in a solid. This method has been used, employing a loudspeaker and a microphone in a tank of water or other liquid.

There is, of course, no need to use wave propagation to obtain the delay. If the recording and replay electromagnets of a tape recorder were 10 mm apart, the tape would need to travel at 5 m s^{-1} to give the desired delay. This is perfectly practical, especially if the tape is replaced by the surface of a rapidly rotating drum or disc. Students may well be able to think of other ways of doing the job.

40a Assuming the power to be 100 W, and assuming an inverse-square law, at the distance of the Earth from the Moon there is about 6×10^{-17} watt per square metre. A dish 10 m in diameter has a collecting area of about 80 m^2 , and so it collects a power of some $5 \times 10^{-15} \text{ W}$.

b The use of an inverse-square law implies that the space probe's aerial is not directional. Since it almost certainly is directional, the estimate above had better be revised upwards by perhaps a factor of 100.

c No

41 In both cases the signal is carried on a rapidly oscillating 'carrier' wave, the amplitude or frequency of which is modified to convey the programme information. In both cases the signal is conveyed by an electromagnetic wave; in 1 the wave goes through space from the transmitter to the receiver, in 2 it goes in the space between the conductors of the cable. The second signal travels a little slower than the first, but only because the cable has a solid insulating material in it.

Anybody in the radio beam can pick up the signals in 1, but only a set connected to the cable can obtain the signal in 2 (indeed, outside the cable there is no evidence that a wave is going along it). The final results are very similar, because the propagation of the signal is similar in the two cases. The sending of a signal along a wire is also the sending of an electromagnetic wave, though if one is thinking more in terms of switching on a light or of using the telephone, this aspect may escape attention. The currents in the wires can be thought of as running along at the edges of the electromagnetic wave. Indeed, for electromagnetic waves sent over the conducting surface of the Earth, there are currents in the ground below the wave.

42a, b As is usual when a battery charges a capacitor, charge is removed from one plate and delivered to the other.

In terms of conventional current, positive charge is removed from the 'extra area' of the lefthand plate, leaving it negative, flows through the charged area, through the battery and the charged area of the righthand plate and is delivered to the 'extra area' of the righthand plate, making it positive.

c It

d avt , since the leading edge travels a distance vt in time t .

e $E = I/\epsilon_0 av$

The time t cancels. Physically, t is not involved because in twice as long a time, twice as much charge covers twice the area, giving the same charge density and

the same field, E . It is this which makes it convenient to argue in terms of E .

f $B = \mu_0 I/a$

One could think of the one-turn solenoid in *Students' guide* figure J137(b) as part of a longer one with one turn in each distance a . Or one could argue that a current I/N in the N -turn solenoid ought to give the same field as a current I in the one-turn solenoid.

g $B = \epsilon_0 \mu_0 Ev$.

h For a given E -field, the charge density on the plates is fixed. But a faster speed v requires this same charge to be delivered more rapidly, so a bigger current is needed. A bigger current implies a bigger B -field.

i vt

j Bdv ; Bdv = rate of change of flux.

k The battery p.d. = $V = Bdv$.

l $V/d = E = Bv$

m $v^2 \epsilon_0 \mu_0 = 1$

$$v = 1/\sqrt{\epsilon_0 \mu_0}$$

n $v = 3 \times 10^8 \text{ m s}^{-1}$

o The units of $\mu_0 \epsilon_0$ are $\text{NA}^{-2} \text{Nm}^2 \text{C}^{-2}$. Cancelling the unit N, and writing Cs^{-1} for the unit of current A gives units $\text{m}^{-2} \text{s}^2$, the unit of charge C also cancelling. Thus $\mu_0 \epsilon_0$ has the units of the reciprocal of the square of a velocity, and $1/\sqrt{\epsilon_0 \mu_0}$ has the units of a velocity.

p The speed would be the same, whatever changes were made to the dimensions of the plates, or the battery voltage, because all these quantities cancel out in the final expression for the speed. It would be correct to take this as a hint that the particular shape of plates chosen is rather unimportant: indeed, so long as the fields in one part of the plates do not affect the state of affairs in other parts, which means using long straight conductors, the speed is the same for any such shape of conductors, whether they are pairs of wires, or coaxial cables, etc. The speed is changed if insulating material is introduced between the conductors, because the charge on the conductors is then different, for a given voltage across the conductors, or E -field between them. Polythene, for example, in

the space between the conductors in a coaxial cable, reduces the speed from $3 \times 10^8 \text{ m s}^{-1}$ to a somewhat smaller value, though still of the order 10^8 m s^{-1} .

43a Light takes a small but finite time to travel to the distant mirror and back. In this time the adjacent hole will have moved on a small distance so some of the light is prevented from reaching the detector.

b Eventually the wheel rotates fast enough for the hole *two* away from the direct beam hole to move into the position previously occupied by the adjacent hole when a large signal was detected.

The hole has thus moved $1/300$ of the circumference. Since the wheel rotates at 100 rev s^{-1} , this must have taken $1/(300 \times 100) \text{ s} = 3.3 \times 10^{-5} \text{ s}$.

$$\begin{aligned}\text{Hence speed of light} &= \frac{2 \times 5 \times 10^3 \text{ m}}{3.3 \times 10^{-5} \text{ s}} \\ &= 3 \times 10^8 \text{ m s}^{-1}.\end{aligned}$$

44a The electromagnetic wave consists of the pair of oscillating *E*- and *B*-fields as shown in figure Q20.

$$\text{As } c = f\lambda, \lambda = c/f = \frac{3 \times 10^8 \text{ m s}^{-1}}{600 \times 10^6 \text{ s}^{-1}} = 0.5 \text{ m}.$$

b The wave would be displaced $3 \times 10^8 \text{ m s}^{-1} \times 5 \times 10^{-10} \text{ s}^{-1} = 0.15 \text{ m}$ (about $\lambda/3$). On figure Q20 point *O'* becomes *O*.

ci The wire should be placed along the direction of the electric vector.

ii The loop should be placed with its axis parallel to the magnetic vector. (But note that the loop aerial used with a television

set may in fact be a 'folded dipole', and so should be in the plane of the *E*- and *B*-vectors.)

iii A large metal sheet should be parallel to the plane of the electric vector, for example plane *yz* and $\lambda/4$ behind the aerial. There is a phase change of π on reflection.

d The electric field vector exists in only one plane. If the receiving aerial is rotated through 90° in the plane at right angles to the direction of propagation and the signal virtually disappears, then the waves must be polarized.

45a The transmitted waves are electromagnetic waves consisting of moving *E*- and *B*-fields with the electric field vector vertical (vertically polarized). Consequently the electrons in the vertical aerial are accelerated by the action of these fields. The movements of these electrons constitute the currents in the aerial.

b The coil produces an induced e.m.f. whose value is largest when the rate of change of flux is at a maximum. For this to happen the axis of the coil must be parallel to the *B*-field. Thus reception is poor if the axis of the coil points to the transmitter because no flux links the coil.

c The inductor *L* and capacitor *C* form a resonant oscillator (or tuned circuit) whose natural frequency is given by $f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$

(see Section H3, 'Alternating current').

When driven by the alternating e.m.f. induced in the aerial by the carrier wave at this frequency, the amplitude of oscillation

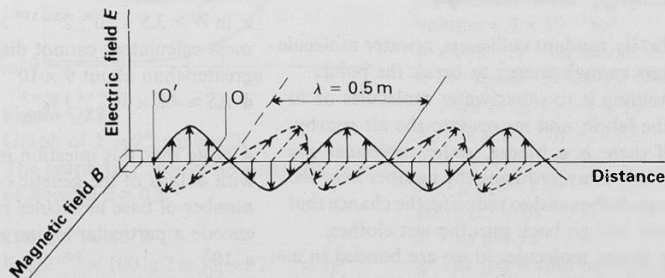


Figure Q20

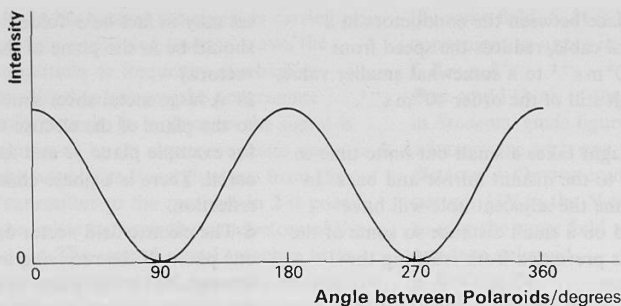


Figure Q21

of the current in the LC loop builds up to a large value.

Turning the knob which varies C changes the resonant frequency allowing the tuning circuit to produce large oscillating currents at a particular frequency.

46a Figure Q21 indicates how the intensity varies with angle. Like the energy dissipated in a resistor carrying alternating current, over one cycle, it is a graph like that of $\cos^2 \theta$. It has a similar shape to a cosine graph of twice the frequency, displaced upwards on the axes so that its least value is zero [$\cos^2 \theta = \frac{1}{2}(\cos 2\theta + 1)$].

b The average value of $\cos^2 \theta$ is $1/2$, so one Polaroid transmits half the light energy that falls on it, if it is 'ideal' (that is, non-absorbing apart from its effect on polarized light).

47 B

UNIT K

Energy and entropy

1a By random collisions, a water molecule gets enough energy to break the bonds holding it to other water molecules or to the fabric, and moves into the air nearby. If there is a breeze, water molecules are swept away, reducing the number near the wet clothes and so reducing the chance that any will go back into the wet clothes.

b Water molecules in ice are bonded in a regular lattice, with permanent neighbours. As the ice melts, a molecule is still closely

surrounded by others (the density actually increases) but has shifting neighbours, and itself moves about.

c The water already has tannin and other molecules from the tea dispersed rather uniformly through it. When milk is stirred in fat, casein, and other molecules, previously close together, become dispersed so that there is a roughly equal average distance between molecules of the same kind, and the chance of having a neighbour of a given kind is proportional to the total number of that kind of molecule in the mixture.

d Energy leaves the radiator in two ways, by emission of photons of infra-red radiation, and by giving extra energy to nearby air molecules.

$$2 \quad W \approx 8 \times 10^{67}; \ln W \approx 156;$$

$$k \ln W \approx 2 \times 10^{-21} \text{ J K}^{-1}.$$

$$\Delta S = k \ln W - k \ln 1 \approx 2 \times 10^{-21} \text{ J K}^{-1}.$$

3a Possibly 5×10^8 (10 coins per person).

b Each coin can be put down in 2 ways, independently of the others, and numbers of ways multiply.

c $\ln W \approx 3.5 \times 10^8$; $2^5 \times 10^8 > 10^{100}$, and most calculators cannot display numbers greater than about 9×10^{99} .

$$\mathbf{d} \quad \Delta S \approx 4.8 \times 10^{-15} \text{ J K}^{-1}$$

4 Note that this question is *not* concerned with details of the genetic code, such as the number of base molecules needed to encode a particular amino acid.

$$\mathbf{a} \quad 10^7$$

$$\mathbf{b} \quad 4^{10^7}$$

$$\mathbf{c} \quad 1.9 \times 10^{-16} \text{ J K}^{-1}$$

5 There are fewer ways to arrange the marbles as shown on the right of figure K40(a) than as on the left, whilst keeping the same numbers of the two kinds. In figure K40(b) there are too few marbles to be sure of the order.

6a Figure Q22 shows two examples.

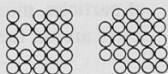


Figure Q22

b N

7i There are two ways for each of the N magnets to point, each independent of the others. (Note that this idea will be roughly correct if the molecular magnets are far apart, as in some paramagnetic materials. It is *not* correct for a ferromagnet.)
ii $\Delta S = Lk \ln 2 \approx 6 \text{ J K}^{-1}$

8 There are $4 \times 6 \times 5 \times 2 = 240$ possible selections. Possibilities multiply if any one choice goes with any other.

9 $2^{100} \approx 10^{30}$; 1 chance in 10^{30} .

10 See figure Q23.

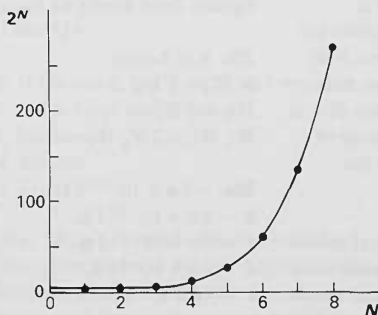


Figure Q23

Graph of $Y = 2^N$ against N .

The graph $\lg Y$ would be a straight line of slope $\lg 2 \approx 0.3$.

11a $2^{100} \approx 10^{30}$

b $\lg 2^{100} = 100 \lg 2 \approx 30$; so $2^{100} \approx 10^{30}$

c 10^{24} s

d 3×10^6

12a About $4 \times 10^{-4} \text{ m}^3$ (0.4 litre).

b 3×10^{21}

c 3×10^{21} digits; $3 \times 10^{18} \text{ m}$; 300 light years; Betelgeuse (alpha Orionis).

13ai To choose equally probably one of the cells, when there are V/dV cells.

ii The number of ways of placing N molecules in the cells.

bi The number of ways of placing N molecules in the larger volume $V + dV$.

ii The ratio of the numbers of ways of placing N molecules in the two volumes.

$$\text{iii } dS = k \ln [(V + dV)/V]^N$$

$$\approx Nk \ln (1 + dV/V)$$

$$\approx NkdV/V.$$

14 $\ln 10 = 2.302$

$\ln 10.5 = 2.351$

$\ln 10.5 - \ln 10 = 0.049$; compared with $0.5/10 = 0.05$.

$\ln(x + dx) - \ln x \approx dx/x$ if $dx \ll x$

15a $+5.76 \text{ J K}^{-1}$

b -5.76 J K^{-1}

c $+11.5 \text{ J K}^{-1}$

d $+23.0 \text{ J K}^{-1}$

e -11.5 J K^{-1}

f $+46.1 \text{ J K}^{-1}$

16 The volume of air inside a car tyre is about 10 litres, at an excess pressure of about 2 atmospheres (*i.e.* total pressure $\approx 3 \text{ atm}$). So the volume at atmospheric pressure would be about 30 litres (rather more than 1 mole).
 $\Delta S \approx 10 \text{ J K}^{-1}$.

17a Cycle pump: about 0.2 m long; cross-section about $2 \times 10^{-4} \text{ m}^2$; volume $\approx 4 \times 10^{-5} \text{ m}^3$.

b About 10^{21} molecules in $4 \times 10^{-5} \text{ m}^3$.

c See figure Q24, page 406.

d Area from $V_1 = 3 \times 10^{-5} \text{ m}^3$ to $V_2 = 9 \times 10^{-5} \text{ m}^3 \approx 4.5 \text{ J}$.

e $\ln V_1 = -10.41$; $\ln V_2 = -9.32$;

$\ln(V_2/V_1) = 1.09$

$\Delta S \approx 1.5 \times 10^{-2} \text{ J K}^{-1}$

f $T\Delta S \approx 4.5 \text{ J}$

g The gas would become hotter when compressed; its internal energy would increase. To keep the temperature

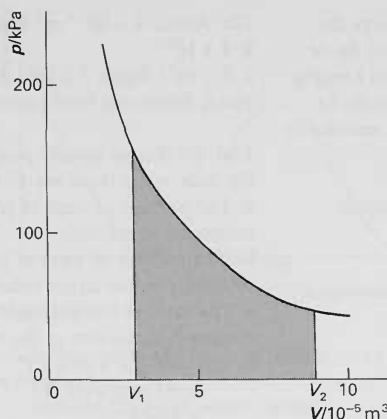


Figure Q24

constant, the gas must give energy to the surroundings, which it can do if it briefly becomes a little hotter than the surroundings.

18 See figure Q25. For two particles sharing 6 quanta there are seven ways.

19 See figure Q26. More energy means more ways to share.

20 *N*

21 Line 1000 picks one particle at random, and line 1010 picks another (or the same particle). Lines 1030 and 1040 lower the level of the first by one, and raise the second by one, unless the first is on the lowest level, when no change of level is made. Line 1050 repeats the sequence.

22 See figure Q27. Both are 'equal ratio' distributions, therefore in (b), where more energy is being shared by the same number

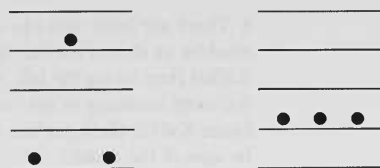


Figure Q26

of particles, more are on higher levels, fewer are on the very lowest level.

23 See table Q3.

Level	$f = 0.5$ Number	$f = 0.25$ Number	$f = 0.75$ Number
0	256	256	256
1	128	64	192
2	64	16	144
3	32	4	108
4	16	1	81
5	8		≈ 61
6	4	less than 1	≈ 45
7	2		≈ 34
8	1		≈ 26

Table Q3

24 For equal steps in level, the exponential function is the function which changes by a constant factor. A graph of the logarithm of the number of particles on each level against level would be linear.

25a X is hotter.

b $W'_X = \frac{2}{3} W_X$

$W'_Y = \frac{3}{1} W_Y$

$W'_X W'_Y = 2W_X W_Y$

26a $-9.6 \times 10^{-24} \text{ J K}^{-1}$

b $-9.6 \times 10^{-22} \text{ J K}^{-1}$

c $-9.6 \times 10^{-21} \text{ J K}^{-1}$

d $-9.6 \times 10^{-3} \text{ J K}^{-1}$

e -9.6 J K^{-1}

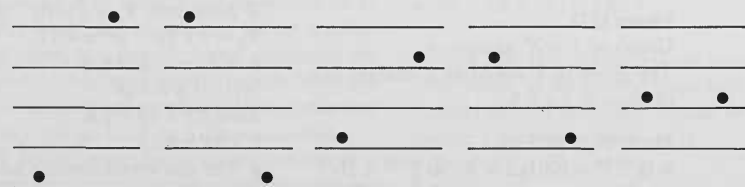


Figure Q25

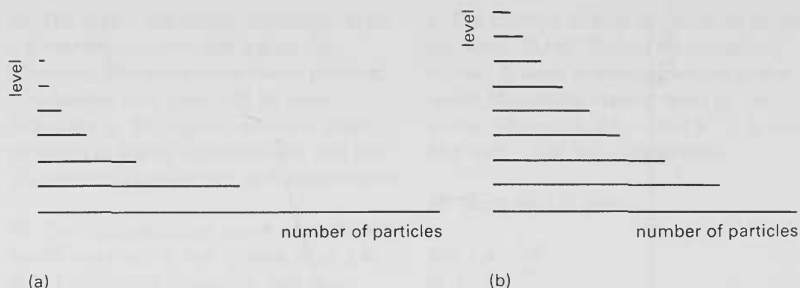


Figure Q27

27a 105 K

b 0.785

ci $n/N \approx 3.7$

ii About 1/10.

28a $-\ln x = \ln 1/x$, so if $x < 1$, $\ln x$ is negative and $-\ln x$ is positive.

b $-2k \ln f$

c Q/ε

d $n = Q/\varepsilon$, so $\Delta S = -(Q/\varepsilon)k \ln f$.

If $\ln f = -\varepsilon/kT$, $\Delta S = Q/T$.

29a	f	$-\ln f$
	1	0
	0.5	+0.69
	0.1	+2.3
	0.01	+4.6

Table Q4

b If f is small, $\ln f$ is large and negative, $-\ln f$ is large and positive.

c Increases

d Infinite

e Zero

30ai Mass of cupful of water ≈ 0.1 kg. Energy to warm it by one degree ≈ 420 J. Entropy change ≈ 1.5 J K $^{-1}$.

ii Normal body temperature $\approx 40^\circ\text{C}$, room temperature $\approx 20^\circ\text{C}$. Energy needed ≈ 8400 J. Mean temperature ≈ 300 K. Entropy change ≈ 28 J K $^{-1}$.

b An ice cube is about 1 mole of H₂O molecules (say 20 g). Energy required ≈ 1700 J. Mean temperature ≈ 280 K. Entropy change ≈ 6 J K $^{-1}$.

31a	E/kT	$e^{-E/kT}$
	1	0.37
	2	0.14
	5	0.0067
	10	4.5×10^{-5}
	20	2.1×10^{-9}
	50	1.9×10^{-22}
	100	3.7×10^{-44}

Table Q5

b 14

32a 5000 K

b 5×10^9 K

c 126 K

33 See figure Q28, page 408. The Boltzmann factor increases very rapidly with temperature.

34a 41 kJ mol $^{-1}$

b 6.8×10^{-20} J molecule $^{-1}$

c 3.4×10^{-20} J bond $^{-1}$

d 4.1×10^{-21} J; 8.3; 2.5×10^{-4}

35 $e^{-1.6} \approx 1.1 \times 10^{-7}$

$e^{-1.3} \approx 2.3 \times 10^{-6}$

$e^{-1.3}/e^{-1.6} = e^3 \approx 20$

Many more molecules acquire the energy needed to evaporate.

36a $\ln r_1 = \ln C - E/kT_1$

$\ln r_2 = \ln C - E/kT_2$

$$\ln \left(\frac{r_2}{r_1} \right) = \ln r_2 - \ln r_1 = \frac{E}{k} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

bi 1.4×10^{-19} J

ii 4.5

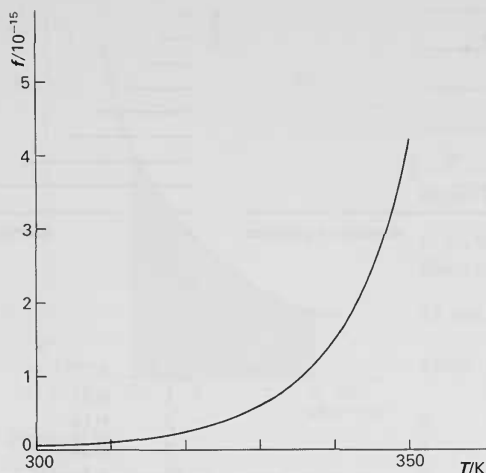


Figure Q28

37ai Slope = e/kT

ii T does not vary much on the Kelvin scale.

b About 40 C J^{-1} for 273 K; about 30 C J^{-1} for 366 K, giving $e/k \approx 1.1 \times 10^4 \text{ C K J}^{-1}$.

c, d Consult especially Unit I, 'Linear electronics, feedback and control'. In the circuit shown in the *Students' guide* figure K47 $V_{\text{out}} = -IR_r$.

38a $1.5 \times 10^{-13} \text{ J}$ (about 1 MeV).

b Extremely slow; in fact the reaction requires quantum mechanical 'tunnelling' through the 'potential barrier' for it to work.

c The rate of fusion could rise very rapidly.

39 The current increases by a factor of about 5 between $T = 900 \text{ K}$ and $T = 1000 \text{ K}$. $\Phi \approx 2 \times 10^{-19} \text{ J}$.

40 The bouncing ball passes kinetic energy to the molecules of the ball and floor, making ball and floor warmer, and sharing the energy out amongst many more particles than before.

Mixing a drink means that particles can exchange places with unlike particles in many more ways than before.

Dissolving sugar in water is in essence the same as mixing a drink.

Air escaping from a tyre occupies a larger volume, so that the molecules can be arranged in many more ways than before.

41 Friction, arising from local adhesion between two surfaces which is then stressed and broken, repeatedly, transfers energy to the sliding objects which warms them. Energy becomes spread out amongst many molecules, and so able to be shared in many ways.

42 Stretched chains can follow few paths between fixed points, so that the number of chain arrangements falls on stretching. The work of stretching, if the rubber becomes hot, has been shared out amongst the motions of its molecules, so that the number of ways of arranging the energies of molecules increases.

43a 85 %

b 5.2 %. The temperature difference is maintained by the World climate, and affects large masses of water, so that a substantial amount of renewable energy might be extracted, provided that doing so did not materially affect the temperature gradient.

44 The higher the source temperature, for a given environment, the higher the efficiency. The Boltzmann factor (Section K3) ensures that there will be more molecules in the vapour, and so a higher pressure, at higher temperatures, and that the pressure rises rapidly with temperature.

45 The relationship of steam temperature to efficiency exists, but is weak (a plot is rather scattered), indicating that many other factors (turbine design, friction) affect efficiency. Over time there has been a steady though small improvement.

46a $5.1 \times 10^{-3} \text{ m}^3$

bi 51 J

ii -0.17 J K^{-1}

c $p\Delta V$ and $T\Delta S$ are equal in magnitude.

d 51 J

e $+0.17 \text{ J K}^{-1}$

47a See table Q6.

b About 60 J K^{-1} .

c About 70 %. If there is structure in the liquid and not in the vapour, the entropy increase on evaporation must be larger than that due to the volume change above.

d The entropy change for water is larger (by some 20 J K^{-1}) than for the other liquids. If there is more structure in the water, this might explain some of the excess. Methanol, $\Delta S = 104 \text{ J K}^{-1}$, is more like water than the other liquids.

48 See table Q7 below.

49a 1.2×10^{23}

bi 1.1 J K^{-1}

ii 3.8 J K^{-1}

iii 7.6 J K^{-1}

50a More energy can be taken from the surroundings for a given entropy decrease.

b The entropy increase for the same number of ions reacting is the same.

c The energy available per ion is fixed by the entropy increase in the cell and the temperature; an ion with less charge delivers fewer electrons so there is a higher p.d. for the same energy change.

Liquid	$\Delta H_{\text{evap}}/\text{kJ mol}^{-1}$	T/K	$\Delta S_{\text{evap}}/\text{J K}^{-1} \text{ mol}^{-1}$
carbon disulphide	27.2	319	85.3
trichloromethane	29.3	335	87.5
hexane	28.8	342	84.2
octane	34.9	399	87.5
sulphuric acid	50.2	617	81.4
water	40.6	373	108.8

Table Q6

	Steam condensing	Water freezing	Material more magnetized
a energy of surroundings increases because:	bonds form	bonds form	molecular magnet aligned along field has less energy
b entropy of surroundings:	increases	increases	increases
c entropy of substance:	decreases	decreases	decreases
d effect of raising temperature:	more steam	more liquid water	less magnetized

Table Q7

UNIT L

Waves, particles, and atoms

1 $1.6 \times 10^{-19} \text{ J}$

2 $5.8 \times 10^{15} \text{ Hz}$

3a 375 nm

b Ultra-violet

c Shorter wavelength light will eject more energetic electrons.

4 $6.4 \times 10^{-34} \text{ Js}$

5 D

6 See table Q8.

7 $5.6 \times 10^{-13} \text{ m}$. This is the longest wavelength radiation that could break apart a deuteron. The radiation is in the gamma ray region of the electromagnetic spectrum.

8 About 1.5×10^{20} photons per second;
 1.5×10^{10} photons per cycle of oscillation
 ($f \approx 10^{10} \text{ Hz}$).

9 About $2 \times 10^7 \text{ W}$.

The gamma ray source emits far fewer than 10^{20} photons in a second. (The strength of radioactive sources is given in disintegrations per second or becquerel (Bq). A school gamma ray source may have an activity of around 0.2 MBq. This means that of the order of 10^5 gamma ray photons are emitted per second.)

10 $7.83 \times 10^{-19} \text{ J}$; 4.89 eV

11a Energy levels A and B.

b 582 nm

c Emitted.

12 D

13 A

14a $2.18 \times 10^{-18} \text{ J} = 13.6 \text{ eV}$

b Ionization energy of hydrogen is 13.6 eV.

15 Longest wavelength.

16a $4.567 \times 10^{14} \text{ Hz}$

b $0.458 \times 10^{-18} \text{ J}$

17 $1.63 \times 10^{-18} \text{ J}$; $1.94 \times 10^{-18} \text{ J}$;
 $2.04 \times 10^{-18} \text{ J}$; $2.09 \times 10^{-18} \text{ J}$. (Note that since h is given to 3 significant figures, the energies cannot be given more precisely, even though frequencies are given to 5 significant figures.)

18a $-Gm/r$

b $\sqrt{Gm/r}$

c $Gm/2r$

d $-Gm/2r$

19a $-2.18 \times 10^{-18} \text{ J}$

b $-0.55 \times 10^{-18} \text{ J}$; $-0.24 \times 10^{-18} \text{ J}$;
 $-0.14 \times 10^{-18} \text{ J}$; $-0.09 \times 10^{-18} \text{ J}$.

20b $-0.034 \times 10^{-18} \text{ J}$

21a 0.3 W

b About 2.3×10^{16} photons per second.

c About one in 50 000 (the area of your eye pupil divided by total area of sphere 0.3 m radius).

d About 5×10^{11} photons enter your eye per second; time between photons is about 2×10^{-12} seconds.

e About $6 \times 10^{-4} \text{ m}$.

f About 6 m.

22 Behaviour: interference, diffraction, photoelectric effect, reflection, refraction, polarization.

BBC Radio 4

λ 1500 m

f $2.0 \times 10^5 \text{ Hz}$

E $\begin{cases} 1.3 \times 10^{-28} \text{ J} \\ 8.2 \times 10^{-10} \text{ eV} \end{cases}$

Microwaves

$3 \times 10^{-2} \text{ m}$

$1 \times 10^{10} \text{ Hz}$

$7 \times 10^{-24} \text{ J}$

$4 \times 10^{-5} \text{ eV}$

Visible light

$6 \times 10^{-7} \text{ m}$

$5 \times 10^{14} \text{ Hz}$

$3 \times 10^{-19} \text{ J}$

2 eV

Gamma rays

$1 \times 10^{-12} \text{ m}$

$3 \times 10^{20} \text{ Hz}$

$2 \times 10^{-13} \text{ J}$

$1 \times 10^6 \text{ eV}$

Table Q8

Models: wave model (superposition effects: diffraction, interference; polarization); particle model (photoelectric effect). Both models can explain reflection and refraction.

Models which refer to objects and phenomena we can see aren't adequate to explain matter on a very small scale.

23 Red light is diffracted through a larger angle than blue light.

24 Shorter wavelength.

25 $\sin \theta_1 / \sin \theta_2 = s_2 / s_1$; does not depend on λ .

26a 0.167

b 0.167

27a 1.73; 1.74; 1.65. Average = 1.71.

b 1:1.71

c Smaller spacing.

d Decrease; wavelength decreases as energy is increased.

28a 1:1.73

b 1:1.73 agrees well enough with **27b**.

c Horizontal rows (wider spacing) give smaller rings (D_1).

29 B

30a \sqrt{V}

b $D \propto \sin \theta \propto \lambda$

31 $D\sqrt{V}$ should be constant. Values are 410; 427; 413; 424; 436; 419; 435; 429.

(Since values of V were estimated from the size of a spark gap and so are not accurately known, the agreement is quite good.)

32a $0.205 \times 10^{-10} \text{ m}$

b $\frac{1}{2}mv^2 = Ve$

c $mv = \sqrt{2mVe}$

d $h = 7.78 \times 10^{-34} \text{ Js}$ (the accepted value is $6.6 \times 10^{-34} \text{ Js}$).

e Yes; both are Js.

$$\begin{aligned} \text{33a Slope is } \frac{\lambda}{1/\sqrt{V}} &= \lambda\sqrt{V} = \frac{h}{mv} \times \sqrt{V} \\ &= \frac{h}{\sqrt{2mVe}} \times \sqrt{V} = \frac{h}{\sqrt{2me}} \end{aligned}$$

b From the graph the slope is approximately $2 \times 10^{-10}/0.15 = 13 \times 10^{-10}$. So $h \approx 13 \times 10^{-10} \times \sqrt{2me} \approx 7.1 \times 10^{-34} \text{ Js}$.

34 About 2000 times more slowly.

35 $\lambda \approx 10^{-34} \text{ m}$

36 Smaller wavelength – smaller angles; wavelength of X-rays is about the same as interatomic spacing in crystals. Wavelength of light is much larger, and no diffraction pattern is produced. Wavelength associated with tennis balls is tiny: a structure with spacing of the same order of magnitude as wavelength ($\approx 10^{-34} \text{ m}$) would be needed to give any diffraction effect.

37a Wavelengths of 420 MeV and 183 MeV electrons are $2.96 \times 10^{-15} \text{ m}$ and $6.79 \times 10^{-15} \text{ m}$ respectively.

b Estimates of nuclear diameters:

C $4.7 \times 10^{-15} \text{ m}$

O $5.2 \times 10^{-15} \text{ m}$

V $9.1 \times 10^{-15} \text{ m}$

38 Points to make include:

a Spectral lines (e.g. hydrogen) and photon model ($E = hf$) suggest atom emits or absorbs light in definite packets of energy corresponding to transition between discrete energy levels.

b Evidence: electron diffraction $\lambda \approx 10^{-10} \text{ m}$. Theory (de Broglie) $\lambda = h/mv$: confirmed by experiment, $\lambda \propto 1/\sqrt{V}$.

Usefulness of electron-as-wave idea; stability, energy levels of atoms (e.g. hydrogen). Other applications (chemical bonding, electrical properties of solids, e.g. semiconductors).

39 Points to make include:

Mistake or ambiguity: they still behave as 'very tiny billiard balls', and as waves *as well*.

UNIT L

Waves, particles, and atoms

1 $1.6 \times 10^{-19} \text{ J}$

2 $5.8 \times 10^{15} \text{ Hz}$

3a 375 nm

b Ultra-violet

c Shorter wavelength light will eject more energetic electrons.

4 $6.4 \times 10^{-34} \text{ Js}$

5 D

6 See table Q8.

7 $5.6 \times 10^{-13} \text{ m}$. This is the longest wavelength radiation that could break apart a deuteron. The radiation is in the gamma ray region of the electromagnetic spectrum.

8 About 1.5×10^{20} photons per second; 1.5×10^{10} photons per cycle of oscillation ($f \approx 10^{10} \text{ Hz}$).

9 About $2 \times 10^7 \text{ W}$.

The gamma ray source emits far fewer than 10^{20} photons in a second. (The strength of radioactive sources is given in disintegrations per second or becquerel (Bq). A school gamma ray source may have an activity of around 0.2 MBq. This means that of the order of 10^5 gamma ray photons are emitted per second.)

10 $7.83 \times 10^{-19} \text{ J}$; 4.89 eV

11a Energy levels A and B.

b 582 nm

c Emitted.

12 D

13 A

14a $2.18 \times 10^{-18} \text{ J} = 13.6 \text{ eV}$

b Ionization energy of hydrogen is 13.6 eV.

15 Longest wavelength.

16a $4.567 \times 10^{14} \text{ Hz}$

b $0.458 \times 10^{-18} \text{ J}$

17 $1.63 \times 10^{-18} \text{ J}$; $1.94 \times 10^{-18} \text{ J}$; $2.04 \times 10^{-18} \text{ J}$; $2.09 \times 10^{-18} \text{ J}$. (Note that since h is given to 3 significant figures, the energies cannot be given more precisely, even though frequencies are given to 5 significant figures.)

18a $-Gm/r$

b $\sqrt{Gm/r}$

c $Gm/2r$

d $-Gm/2r$

19a $-2.18 \times 10^{-18} \text{ J}$

b $-0.55 \times 10^{-18} \text{ J}$; $-0.24 \times 10^{-18} \text{ J}$; $-0.14 \times 10^{-18} \text{ J}$; $-0.09 \times 10^{-18} \text{ J}$.

20b $-0.034 \times 10^{-18} \text{ J}$

21a 0.3 W

b About 2.3×10^{16} photons per second.

c About one in 50000 (the area of your eye pupil divided by total area of sphere 0.3 m radius).

d About 5×10^{11} photons enter your eye per second; time between photons is about 2×10^{-12} seconds.

e About $6 \times 10^{-4} \text{ m}$.

f About 6 m.

22 Behaviour: interference, diffraction, photoelectric effect, reflection, refraction, polarization.

BBC Radio 4

λ 1500 m

f $2.0 \times 10^5 \text{ Hz}$

E $\begin{cases} 1.3 \times 10^{-28} \text{ J} \\ 8.2 \times 10^{-10} \text{ eV} \end{cases}$

Microwaves

$3 \times 10^{-2} \text{ m}$

$1 \times 10^{10} \text{ Hz}$

$7 \times 10^{-24} \text{ J}$

$4 \times 10^{-5} \text{ eV}$

Visible light

$6 \times 10^{-7} \text{ m}$

$5 \times 10^{14} \text{ Hz}$

$3 \times 10^{-19} \text{ J}$

2 eV

Gamma rays

$1 \times 10^{-12} \text{ m}$

$3 \times 10^{20} \text{ Hz}$

$2 \times 10^{-13} \text{ J}$

$1 \times 10^6 \text{ eV}$

Table Q8

Models: wave model (superposition effects: diffraction, interference; polarization); particle model (photoelectric effect). Both models can explain reflection and refraction.

Models which refer to objects and phenomena we can see aren't adequate to explain matter on a very small scale.

23 Red light is diffracted through a larger angle than blue light.

24 Shorter wavelength.

25 $\sin \theta_1 / \sin \theta_2 = s_2 / s_1$; does not depend on λ .

26a 0.167

b 0.167

27a 1.73; 1.74; 1.65. Average = 1.71.

b 1:1.71

c Smaller spacing.

d Decrease; wavelength decreases as energy is increased.

28a 1:1.73

b 1:1.73 agrees well enough with **27b**.

c Horizontal rows (wider spacing) give smaller rings (D_1).

29 B

30a \sqrt{V}

b $D \propto \sin \theta \propto \lambda$

31 $D\sqrt{V}$ should be constant. Values are 410; 427; 413; 424; 436; 419; 435; 429. (Since values of V were estimated from the size of a spark gap and so are not accurately known, the agreement is quite good.)

32a 0.205×10^{-10} m

b $\frac{1}{2}mv^2 = Ve$

c $mv = \sqrt{2mVe}$

d $h = 7.78 \times 10^{-34}$ J s (the accepted value is 6.6×10^{-34} J s).

e Yes; both are J s.

$$\begin{aligned} \text{33a Slope is } \frac{\lambda}{1/\sqrt{V}} &= \lambda\sqrt{V} = \frac{h}{mv} \times \sqrt{V} \\ &= \frac{h}{\sqrt{2mVe}} \times \sqrt{V} = \frac{h}{\sqrt{2me}} \end{aligned}$$

b From the graph the slope is approximately $2 \times 10^{-10}/0.15 = 13 \times 10^{-10}$. So $h \approx 13 \times 10^{-10} \times \sqrt{2me} \approx 7.1 \times 10^{-34}$ J s.

34 About 2000 times more slowly.

35 $\lambda \approx 10^{-34}$ m

36 Smaller wavelength – smaller angles; wavelength of X-rays is about the same as interatomic spacing in crystals. Wavelength of light is much larger, and no diffraction pattern is produced. Wavelength associated with tennis balls is tiny: a structure with spacing of the same order of magnitude as wavelength ($\approx 10^{-34}$ m) would be needed to give any diffraction effect.

37a Wavelengths of 420 MeV and 183 MeV electrons are 2.96×10^{-15} m and 6.79×10^{-15} m respectively.

b Estimates of nuclear diameters:

C 4.7×10^{-15} m

O 5.2×10^{-15} m

V 9.1×10^{-15} m

38 Points to make include:

a Spectral lines (e.g. hydrogen) and photon model ($E = hf$) suggest atom emits or absorbs light in definite packets of energy corresponding to transition between discrete energy levels.

b Evidence: electron diffraction $\lambda \approx 10^{-10}$ m. Theory (de Broglie) $\lambda = h/mv$: confirmed by experiment, $\lambda \propto 1/\sqrt{V}$.

Usefulness of electron-as-wave idea; stability, energy levels of atoms (e.g. hydrogen). Other applications (chemical bonding, electrical properties of solids, e.g. semiconductors).

39 Points to make include:

Mistake or ambiguity: they still behave as 'very tiny billiard balls', and as waves *as well*.

Explanation: neither model complete; need both ideas to understand behaviour of matter on a very small scale. Most scientists accept the need for two models, but some are unhappy with this and search for 'a better idea'.

40 Points to make include:

Light: some behaviour (superposition) explained on wave model; other behaviour (photoelectric effect) needs particle model.

Particles: small particles (e.g. electrons, but also neutrons, protons, etc.) show wave-like behaviour (electron, neutron diffraction). Wavelength \ll wavelength of light.

Duality: neither model complete – need both, depending on behaviour to be 'explained'.

41a Atoms do not change in chemical reactions, or even when subjected to extreme temperatures and pressures.

b X-ray diffraction; electron micrographs; Avogadro constant; oil film experiment; etc.

c Spectra

42a 2l

b Longest

43a 4×10^{-10} m

b Largest

c 1.65×10^{-24} kg m s $^{-1}$

d No

e 1.5×10^{-18} J

f Minimum

g Electrical attraction between proton and electron.

h 2.3×10^{-18} J

i No

44a Wavelength: ten times smaller; momentum: ten times larger; kinetic energy: hundred times larger.

b 23×10^{-18} J

c No, because kinetic energy would be 150×10^{-18} J.

45a Electron's kinetic energy will decrease by factor of 100; electrical potential energy

will decrease in magnitude by factor of 10.

Total energy

$$= 0.015 \times 10^{-18} \text{ J} - 0.23 \times 10^{-18} \text{ J} \\ = -0.215 \times 10^{-18} \text{ J}.$$

Total energy is more (i.e. a smaller negative quantity) than for the atom with radius 1×10^{-10} m.

b Total energy (for atom with radius 0.1×10^{-10} m) is

$$150 \times 10^{-18} \text{ J} - 23 \times 10^{-18} \text{ J} \\ = 127 \times 10^{-18} \text{ J}.$$

$$46a \quad \frac{h}{4r}$$

$$b \quad \frac{h^2}{32\pi r^2}$$

$$c \quad -\frac{e^2}{4\pi\epsilon_0 r}$$

$$d \quad E = \frac{h^2}{32\pi r^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$e \quad \frac{dE}{dr} = -\frac{h^2}{16\pi r^3} + \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$f \quad r = \frac{4\pi\epsilon_0 h^2}{16\pi e^2} = 1.3 \times 10^{-10} \text{ m}$$

47a About 2.4×10^{-9} J.

b About -23×10^{-15} J.

c No

48a 1.3×10^{-12} J

b Potential energy must be less (i.e. more negative) than -1.3×10^{-12} J.

49a Yes

b Waves travel more slowly on the heavier cord.

c Wave speed $\propto 1/\sqrt{\text{mass per unit length}}$.

50a Tension in the spring; mass per unit length of the spring.

b Speed decreases because tension is less near the lower end (less weight to support).

c Similarity: electron standing waves in atoms have variable wavelength.

Difference: electron standing wave in atom shows chance of finding electron, it does not represent displacement of a material object like a spring.

51 (c)

53i Explanation: some properties of electrons are only understood by thinking of them as waves, but this does not mean they *are* waves.

Support: electron diffraction (experimental evidence); success of wave mechanics in explaining, for example, energy levels of hydrogen atom (theory, evidence).

ii Explanation: electron confined to a box must be a standing wave (model). If size of box is about 10^{-10} m, then, because at least one half-wavelength loop must fit in the box, wavelength can't be more than 2×10^{-10} m. This puts a lower limit to momentum (since momentum $\propto 1/\text{wavelength}$) and to kinetic energy ($[\text{momentum}]^2/2m$). Smaller box means shorter wavelength, so higher momentum and kinetic energy (theory, model, calculations).

Support: atoms have finite size (evidence, calculations); nuclear forces which hold neutrons, protons in the nucleus, are much stronger as they must be able to confine a particle to a much smaller box (evidence, model, calculations).

iii Explanation: if the total energy is not negative the electron will fly away from the nucleus (*cf.* satellite orbiting a planet, etc.). Wave mechanics shows that kinetic energy of electron exceeds 10 eV if it is confined within a space much smaller than 10^{-10} m (theory, model, calculation).

Support: calculations of this kind correctly predict the size of atoms (calculations, evidence).

iv Explanation: atoms cannot have any arbitrary amount of energy: only specific, well-defined amounts called energy levels are allowed. The values of energy which the atom can have are $c/1$, $c/4$, $c/9$, ..., c/n^2 .

Value of c : -2.18×10^{-18} J (evidence, theory, calculations).

Support: measurement of frequencies of spectral lines; (evidence, calculations, theory).

v Explanation: electron is standing wave because it is confined. Standing wave must have a whole number of half-wavelength loops, so only certain wavelengths are possible. Since kinetic energy depends on wavelength, only discrete values of kinetic energy are possible. This model predicts energy $\propto n^2$; in fact energy $\propto 1/n^2$.

Support: momentum $\propto 1/\text{wavelength}$, and kinetic energy $\propto (\text{momentum})^2$. Wave with 2 loops has half the wavelength, so four times the kinetic energy of one-loop standing wave. Three-loop standing wave has 9 times, etc. The $1/n^2$ rule arises because potential as well as kinetic energies contribute to total energy of atom; also because the standing waves with more loops occupy more space (model, theory and evidence).

54a Average momentum $= \sqrt{2mE}$

b Average wavelength $= h/\sqrt{2mE}$

c $E \propto -1/r_0$

d $n \propto r_0/\lambda$

e $E \propto -1/n^2$

REFERENCE MATERIAL

TEXTBOOKS AND FURTHER READING

The students have a list of textbooks and other references on pages 348 and 349 of *Students' guide 2*.

Here we give a list of books and references for teachers and the library.

Books useful throughout the course

AKRILL, T. B., BENNET, G. A. G., and MILLAR, C. J. *Physics*. Arnold, 1979.

BOLTON, W. *Patterns in physics*. McGraw-Hill, 1974.

DUNCAN, T. *Physics: a textbook for advanced level students*. Murray, 1982.

or

DUNCAN, T. *Advanced physics: materials and mechanics*. 2nd edn. Murray, 1981.

DUNCAN, T. *Advanced physics: fields, waves, and atoms*. 2nd edn. Murray, 1981.

FEYNMAN, R. P., LEIGHTON, R. B., and SANDS, M. *The Feynman lectures on physics Volume 1: Mainly mechanics, radiation, and heat*. Addison-Wesley, 1963.

FEYNMAN, R. P., LEIGHTON, R. B., and SANDS, M. *The Feynman lectures on physics Volume 2: The electromagnetic field*. Addison-Wesley, 1964.

NUFFIELD REVISED PHYSICS *Years 1 and 2, Year 3, Year 4, and Year 5*. Longman, 1978-80.

ROGERS, E. M. *Physics for the inquiring mind*. Oxford University Press, 1960.

WENHAM, E. J., DORLING, G. W., SNELL, J. A. N., and TAYLOR, B. *Physics: concepts and models*. 2nd edn. Addison-Wesley, 1984.

Unit H Magnetic fields and a.c.

Applications of electromagnetism

BARCLAY, A. W. and GIBBON, J. A. *Physics principles at work*. BP Educational Service, 1983.

Oscillations in electrical and mechanical systems

SHIVE, J. N. and WEBER, R. L. *Similarities in physics*. Hilger, 1982.

Further reading for teachers and students

BENNET, G. A. G. *Electricity and modern physics*. 2nd edn. Edward Arnold, 1974.

CARO, D. E., McDONNELL, J. A., and SPICER, B. M. *Modern physics*. 3rd edn. Edward Arnold, 1978.

MORGAN, D. V. and HOWES, M. J. Wykeham Science Series, No. 20. *Solid state electronic devices*. Wykeham, 1972.

STAFFORD, G. H. Rutherford Appleton Laboratory Monograph. *The use of high energy machines in particle physics*. Rutherford Appleton Laboratories, 1980. (Out of print.)

WILSON, R. R. and LITTAUER, R. Science Study Series No. 15. *Accelerators: machines of nuclear physics*. Heinemann, 1962.

WRIGHT, J. P. *The vital spark*. Heinemann, 1974. (Out of print.)

Data book

NUFFIELD REVISED ADVANCED SCIENCE *Book of data*. Longman, 1984.

Unit I Linear electronics, feedback and control

Teaching suggestions

BEVIS, G. and TROTTER, M. (eds). *Microelectronics: practical approaches for schools and colleges*. BP Educational Service, 1981.

Not much on operational amplifiers, but gives a bibliography, and suppliers' addresses.

BARCLAY, A. W. and GIBBON, J. A. *Physics principles at work*. BP Educational Service, 1983.

A variety of accounts by teachers of industrial processes, including principles of transducers. Some 'school laboratory' suggestions.

GOUGH, C. E. *et al.* *Notes for guidance on the electronics option.* 2nd edn. Joint Matriculation Board, 1981.

Electronics and operational amplifiers

- CLAYTON, G. B. *Experiments with operational amplifiers.* Macmillan, 1975.
- CLAYTON, G. B. *Operational amplifiers.* 2nd edn. Butterworth, 1979.
- CLOSE, K. J., and YARWOOD, J. *Electronics.* University Tutorial Press, 1976.
- ELECTRONIC SYSTEMS TEACHING PROGRAMME ESP700 Book 4 *Feedback systems.* Feedback Instruments Ltd.
- FOXCROFT, G. E. *Operational amplifier.* Griffin and George, 1975.
- HOROWITZ, P. and HILL, W. *The art of electronics.* Cambridge University Press, 1980.
- NUFFIELD ADVANCED PHYSICS Students' book *Unit 6 Electronics and reactive circuits.* Longman, 1971.
- PLANT, M. *Operational amplifier applications.* NCST Trent Polytechnic, 1974.

Instrumentation and control

- BOLTON, W. *Engineering instrumentation and control.* Butterworth, 1980.
- ENGINEERING CONCEPTS CURRICULUM PROJECT *The man-made world.* McGraw-Hill, 1971.
- ENGINEERING SCIENCE PROJECT *Electronics, systems, and analogues.* Macmillan, 1975. (Out of print.)
- HASLAM, J. A., SUMMERS, G. R., and WILLIAMS, D. *Engineering instrumentation and control.* Edward Arnold, 1981.
- JONES, B. E. *Instrumentation, measurement and feedback.* McGraw-Hill, 1977.
- MARSTON, R. M. *110 Operational amplifier projects.* Hayden, 1975.
- PROJECT TECHNOLOGY Handbook 14, *Simple computer and control logic.* Heinemann/Schools Council, 1972.
- RAMSEY, D. C. *Engineering instrumentation and control.* Thornes, 1981.

Feedback in biological systems

- HARDY, R. N. *Homeostasis.* 2nd edn. Edward Arnold, 1983.

NUFFIELD REVISED ADVANCED BIOLOGY *Study guide I.* Longman, 1985.

ROBERTS, M. B. V. *Biology, a functional approach.* 2nd edn. Nelson, 1976.

Unit J Electromagnetic waves

Optics and electromagnetic waves

- BENNET, G. A. G. *Electricity and modern physics.* 2nd edn. Edward Arnold, 1975.
- BRADDICK, H. J. J. *Vibrations, waves, and diffraction.* McGraw-Hill, 1965.
- DAMPIER, W. C. *A history of science.* Cambridge University Press, 1971.
- FRENCH, A. P. MIT Introductory Physics Series. *Electromagnetic waves and optics.* Nelson, 1968.
- FRENCH, A. P. MIT Introductory Science Series. *Vibrations and waves.* Van Nostrand Reinhold (U.K.) Co. Ltd., 1982.
- HECHT, E. and ZAJAC, A. *Optics.* Addison-Wesley, 1974.
- LONGHURST, R. S. *Geometrical and physical optics.* 3rd edn. Longman, 1973.
- MASON, P. *The light fantastic.* Penguin, 1981.
- RUTHERFORD, S. J. *Optical fibres in school physics* (Experimenting with industry, No. 2). SCSST and ASE, 1985.
- SHIVE, J. N. and WEBER, R. L. *Similarities in physics.* Hilger, 1982.
- WALKER, J. *The flying circus of physics.* Wiley, 1977.
- WALKER, J. *Light and its uses: Readings from Scientific American.* W. H. Freeman and Co., 1980.
- WHELAN, P. M. and HODGSON, M. J. *Essential principles of physics.* Murray, 1978.

Special relativity theory

- BONDI, H. *Relativity and common sense: a new approach to Einstein.* Heinemann, 1974.
- EPSTEIN, L. C. *Relativity revisualized.* Adam Hilger, 1985.
- FRENCH, A. P. MIT Introductory Physics Series. *Special relativity.* Nelson, 1968.
- HOFFMAN, B. *Relativity and its roots.* A Scientific American Book.
- W. H. Freeman and Co., 1983.

KITTEL, C., KNIGHT, W. D., and RUDERMAN, M. A. *Berkeley Physics Course Volume 1 Mechanics and relativity*. McGraw-Hill, 1964.

LANDAU, L. D. and RUMER, G. B. *What is relativity?* Oliver and Boyd, 1980.

LILLEY, S. *Discovering relativity for yourself*. Cambridge University Press, 1981.

PURCELL, E. M. *Berkeley Physics Course Volume 2 Electricity and magnetism*. McGraw-Hill, 1964.

SANDAGE, A. R. 'The red shift'. *Scientific American* Volume 195 (3), Sept. 1956. (Offprint No. 240.)

SHANKLAND, R. S. 'The Michelson-Morley experiment'. *Scientific American* Volume 211 (5), Nov. 1964. (Offprint No. 321.)

SHERWIN, C. W. *Basic concepts in physics*. Holt, Rinehart, and Winston, 1961. (Out of print.)

Radioastronomy

HENBEST, N. 'Jodrell under Merlin's spell'. *New Scientist*, 96, 1332, Nov. 1982.

KELLERMAN, K. I. 'Intercontinental radio astronomy'. *Scientific American* Volume 226 (27) Feb. 1972.

READHEAD, A. S. C. 'Radio astronomy by very long baseline interferometry'. *Scientific American* Volume 246 (6) June, 1982.

Images

CANNON, T. M. and HUNT, B. R. 'Image processing by computer'. *Scientific American* Volume 245 (4), Oct. 1981.

HARBURN, G., TAYLOR, C. A., and WELBERRY, T. R. *An atlas of optical transforms*. Bell and Hyman, 1983.

HOLLIGAN, P. J. (ed.) *Open University second level course ST 291, 'Images and information', Units 9 and 10. An introduction to holography*. The Open University Press, 2nd edition, 1978.

TAYLOR, C. A. Wykeham Science Series. *Images*. Wykeham, 1978.

Unit K Energy and entropy

ATKINS, P. W. *The creation*. W. H. Freeman, 1981.

ATKINS, P. W. *The Second Law. A Scientific American Book*. W. H. Freeman and Co., 1984.

BENT, H. A. *The second law: an introduction to classical and statistical thermodynamics*. Oxford University Press, 1965. (Out of print.)

DAVIES, P. C. W. *The accidental Universe*. Cambridge University Press, 1982.

DAVIES, P. C. W. *The runaway Universe*. Cambridge University Press, 1982.

FLOWERS, B. H. and MENDOZA, E. *Properties of matter*. Wiley, 1970.

GASSER, R. P. H. and RICHARDS, W. G. *Oxford Chemistry Series No. 19 Entropy and energy levels*. Oxford University Press, 1974.

GURNEY, R. W. *Introduction to statistical mechanics*. Dover, 1966.

KITTEL, C. and KROEMER, H. *Thermal physics*. 2nd edn. W. H. Freeman and Co., 1980.

OGBORN, J. M. 'Introductory statistical physics', in E. J. Wenham (ed.) *New trends in physics teaching* Volume 4 UNESCO, 1984.

OPEN UNIVERSITY Course ST285, *Solids, liquids, and gases*. Open University, 1973.

POWLES, J. G. *Particles and their interactions*. Addison-Wesley, 1968.

PSSC *College physics*. 5th edn. Raytheon, 1981.

REIF, F. *Berkeley Physics Course Volume 5, Statistical physics*. McGraw-Hill, 1965.

SHERWIN, C. H. *Basic concepts of physics*. Holt, Rinehart, and Winston, 1961. (Out of print.)

TABOR, D. *Gases, liquids, and solids*. Penguin, 1970.

WALTON, A. J. *Three phases of matter*. McGraw-Hill, 1976.

WEINBERG, S. *The first three minutes*. Fontana, 1983.

WYATT, P. A. H. *Royal Institute of Chemistry Monographs for Teachers*, No. 19. *The molecular basis of entropy and chemical equilibrium*. 1971.

WYATT, P. A. H. *Royal Institute of Chemistry Monographs for Teachers*, No. 35. *A thermodynamic bypass GOTO Log K*. 1982.

Unit L Waves, particles, and atoms

- ARONS, A. B. *Development of concepts of physics*. Addison-Wesley, 1965.
- ATKINS, P. W. Oxford Chemistry Series, No. 21. *Quanta: a handbook of concepts*. Oxford University Press, 1974.
- BOORSE, H. A. and MOTZ, L. *The world of the atom*. Volume 1. Basic books, 1966.
- CROPPER, W. H. *The quantum physicists*. Oxford University Press, 1970.
- MILLIKAN, R. A. Phoenix Science Series. *The electron*. University of Chicago Press, 1963.
- MOTT, N. F. Wykeham Science Series, No. 22. *Elementary quantum mechanics*. Wykeham, 1972.
- NUFFIELD ADVANCED PHYSICS Unit 10 *Waves, particles, and atoms*. Longman, 1971.
- SHERWIN, C. H. *Basic concepts of physics*. Holt, Rinehart, and Winston, 1961. (Out of print.)
- WICHMANN, E. H. Berkeley Physics Course Volume 4 *Quantum physics*. McGraw-Hill, 1971.

Electrons in solids

- COLES, B. R. and CAPLIN, A. D. *The electronic structure of solids*. Edward Arnold, 1976.
- DAVIES, D. A. *Waves, atoms, and solids*. Longman, 1978.

Chemical and biological applications

- COMPANION, A. L. *Chemical bonding*. McGraw-Hill, 1964.
- CBA (Chemical bond approach) *Chemical systems*. Webster/McGraw-Hill, 1964.
- NUFFIELD REVISED ADVANCED BIOLOGY *Students' book II and Teachers' guide II*. Longman, 1986.
- TAYLOR, R. J. Unilever Educational Booklets Advanced Series No. 5. *Water*. Unilever, 1966.
- WATSON, J. D. Biology Teaching Monograph Series. *Molecular biology of the gene*. 2nd edn. Benjamin, 1970.

Further reading for students

In the 'Readings' section of the *Students' guide* the following sources are referred to:

- AKRILL, T. B., BENNET, G. A. G., and MILLAR, C. J. *Physics*. Edward Arnold, 1979.
- BENNET, G. A. G. *Electricity and modern physics*. 2nd edn. Edward Arnold, 1974.
- BOLTON, W. *Patterns in physics*. McGraw-Hill, 1974.
- BORN, M. *The restless Universe*. Dover, 1951.
- CARO, D. E., MCDONELL, J. A., and SPICER, B. M. *Modern physics*. Edward Arnold, 1966.
- DUNCAN, T. *Advanced physics: fields, waves, and atoms*. 2nd edn. Murray, 1981.
- DUNCAN, T. *Physics: a textbook for advanced level students*. Murray, 1982.
- FEYNMAN, R. P., LEIGHTON, R. B., and SANDS, M. *The Feynman lectures on physics, Volume 1: Mainly mechanics, radiation, and heat*. Addison-Wesley, 1963.
- HOFFMAN, B. *The strange story of the quantum*. Penguin, 1970.
- OPEN UNIVERSITY Science Foundation Course S101 Unit 9 'Energy: light, waves or particles', Units 10 and 11 'Atomic structure'. Open University Press, 1979.
- PROJECT PHYSICS Reader, Unit 5 *Models of the atom*. Holt, Rinehart, and Winston, 1971.
- PROJECT PHYSICS Text, Unit 5 *Models of the atom*. Holt, Rinehart, and Winston, 1971.
- PSSC *College physics*. 5th edn. Raytheon, 1981.
- PSSC *Physics*. 3rd edn. Heath, 1971.
- ROGERS, E. M. *Physics for the inquiring mind*. Oxford University Press, 1960.
- ROTHMAN, M. A. *The laws of physics*. Penguin, 1966.
- TOLANSKY, S. *Revolution in optics*. Penguin, 1968.
- TOULMIN, S. E. and GOODFIELD, J. *The architecture of matter*. Penguin, 1965.
- WENHAM, E. J., DORLING, G. W., SNELL, J. A. N., and TAYLOR, B. *Physics: concepts and models*. 2nd edn. Addison-Wesley, 1984.

Computer programs and visual aids

Unit H Magnetic fields and a.c.

Computer program

'Mass spectrometer' in the Computers in the Curriculum Project. Longman Micro Software, 1982.

Unit I Linear electronics, feedback and control

Television or video

'Control systems' – television programme 1 of Open University course T391, *Control engineering*.

Open University programmes may be recorded only by licence holders. A school licence costs £40 and a college licence costs £80. Both may be obtained from: Guild Organization Ltd., Guild House, Oundle Road, Peterborough PE2 9PZ.

Unit J Electromagnetic waves

Television or video

'The determination of the velocity of light', and 'The determination of the velocity of radio waves' are programmes 3 and 4 respectively in the Granada Television series *Experiment: physics*. The former uses a rotating mirror method and not the pulsed light beam described in this Unit; in the latter standing waves are set up on a transmission line. In these programmes students take measurements from the screen, and notes obtainable from Granada Television Ltd., Manchester M60 9EA, or from the Education Office (Schools Information) of regional ITV companies are essential.

Computer programs

'1 SLIT', '2 SLITS', and 'SLITS' are physics models for use with the 'Dynamic modelling system'. Longman Micro Software, 1985.

'FRAUN' (Fraunhofer diffraction) is one of the *Physics Demonstration Programs* in the Computers in the Curriculum Project. Longman Micro Software, 1982.

'Interference and diffraction of waves'. Chelsea Science Simulations. Edward Arnold.

'Multiple slit interference'. Physics Pack, G. S. N. Educational Software, Manchester University Press, Oxford Road, Manchester M13 9PL.

'Optical diffraction' by John Campbell is part of the *Physics Software Pack 2* from Hutchinson Software, 17–21 Conway Street, London W1P 6JD.

'RCL' (Resistor, capacitor, inductor) illustrates the operation of RCL circuits, and includes phasor diagrams. It is available from MUSE (Microcomputer Users in Secondary Education), P.O. Box 43, Hull HU1 2HD.

'SLITS' is part of the *A-level physics suite* from Software Production Associates, P.O. Box 59, Royal Leamington Spa, Warwickshire CV31 3QA.

'Young's slits'. Five Ways Software. Heinemann Computers in Education.

Unit K Energy and entropy

Computer program

'Quantum shuffling' is part of Software for Nuffield Advanced Physics. Longman Micro Software, 1985.

Unit L Waves, particles, and atoms

Film loops

'Soap film oscillations' Ealing Scientific No. 80–2660/1.

'Vibrations of a drum'. Ealing Scientific No. 80–3924/1.

These film loops (super 8) are available from BFA Educational Films, 56 Copthorne Road, Leatherhead, Surrey. In due course they will also be available on videotape.

16 mm films

'Interference of photons', 14 minutes, black and white, sound.

'Matter waves', 28 minutes, black and white, sound.

'Photons', 19 minutes, black and white, sound.

These three films were originally produced for the PSSC course in the USA. They are now difficult to obtain, but are worth showing if copies can be found.

Television or video

'Electron diffraction' – programme 5 in the Granada Television series *Experiment: physics*. Notes on the broadcast, which can be obtained from Granada Television Ltd., Manchester M60 9EA, or from the Education Officer (Schools Information) of regional ITV companies, are essential.

'Physics beyond experience' – television programme 14 in the Open University course S271 *Discovering physics*.

Open University programmes may be

recorded only by licence holders. A school licence costs £40 and a college licence costs £80. Both may be obtained from Guild Organization Ltd., Guild House, Oundle Road, Peterborough PE2 9PZ.

Computer programs

'Dynamic Modelling System'. Longman Micro Software, 1985.

'Solving the Schrödinger equation' is part of Software for Nuffield Advanced Physics. Longman Micro Software, 1985.

Cassette/slide programmes

'The basis of quantum mechanics'. Two tracks of recorded discussion (each about 25 minutes) with 15 slides and supplementary booklet, *PHA002*. 'Electrons in 'Electrons in solids'. Two tracks of recorded discussion (each about 25 minutes), with 17 slides and supplementary booklet, *PHA009*. Both from Audio Learning Ltd., 105a Toriano Avenue, London NW5 2RA.

APPENDICES

APPENDIX I

PROBLEMS OF TEACHING ELECTROMAGNETISM

This appendix concerns some of the problems, many of them problems of logic, of thinking through a sequence of teaching in electromagnetism. There are several difficulties in the way of such teaching. The logic of electromagnetism happens to be particularly complicated; there are very many relationships which interlock with one another, but which have to be put into some pattern.

As elsewhere in science, there is no one 'correct' pattern; no one 'true' view. There is a complex network of relationships, and one may start where one likes, and go which way round the network one pleases. What is a definition on one view may be an empirical relationship on another. What is a measured quantity on one view may be a defined quantity on another.

Another difficulty is that logic in this sense has very little to do with teaching, which has to do with the gradual growth of concepts through the experience of trying to use them; not with precise definitions which are clear to the expert, but impenetrable to the student. A scientific definition, such as that of the unit of current, is not an attempt to *explain* what one means. The definition of the ampere is a *prescription* for measuring current, chosen by professional physicists, not with the convenience of students in mind, but with a view to stability and accuracy of measurement. Nor is science so closed and certain an enterprise that a physicist knows clearly from the start what he or she means. A definition is an end to be striven towards, not a starting point. At the least, there is no sense in 'saying what one means', until one has found out by experiment what is worth talking about. These notes are intended to clear the ground, so that in teaching, as little time as possible may be spent on logical considerations, which students are not very likely to appreciate. At the same time these notes may help to avoid the taking up of positions which would later on have to be abandoned.

Defining current

The SI definition of the ampere involves the force between long straight parallel wires. It is a convenient definition for professional physicists, but not a thing which means much at first to the student at school. At an early stage, it is better to use simple ammeters (and maybe current

balances), trying them in circuits to see what they do. In particular, ammeter readings are the same all round a series circuit, and add up at junctions.

Because the unit of current will in the end be the fourth primary unit added to the units of mass, length, and time in SI, we feel free to use an ammeter to measure the strength of a B -field, via $F = BIl$, at the start of Unit H, 'Magnetic fields and a.c.'. But because current is measured by a magnetic effect, whether in an ammeter, or in a current balance based on the definition, it is not as simple as it might be to say why one supposes the force to be proportional to Il . The essence of the matter is that magnetic forces vary linearly with whatever it is (current) that adds up at circuit junctions.

An experiment to test the point would be to place two current balances in two arms of a circuit, with a third in a wire which both arms join. The experiment will show that the force on the third balance is always equal to the sum of the forces on the other two, if all the balances are identical. Important as the experiment is in principle, it and others like it often make less than the expected impact on students, who care less for logic than do some physicists. Moreover, it needs some skill to make it work convincingly, and if it 'doesn't work', the point is lost.

Defining B

As indicated above, B is introduced in the suggested sequence by way of the force on a current, and not by way of flux density and flux. We regard this as a free choice of tactics, and we regard as irrelevant the fact that another route, taking flux and flux density from induction experiments before using $F = BIl$, is likely to appear in schemes of logical chains of definitions in SI. For the learner, induction has the disadvantage of being a good deal more complex than the force on a current, though even the latter is not so simple as it might be, especially having regard to the directions of current, field, and force. Those who judge otherwise will do it the other way, no doubt. We only urge them to be swayed by clarity and ease of teaching, rather than by 'logic'.

A note on μ_0

In SI, the ampere has been chosen as a primary unit. To settle upon a value for an ampere means choosing what force will exist between, say, two wires when one ampere is said to flow in each. Since μ_0 expresses the magnitude of such forces, an arbitrary choice of the size of an ampere is also an arbitrary choice of μ_0 , which is given the value $4\pi \times 10^{-7} \text{ N A}^{-2}$. Clearly no experiment can *measure* μ_0 in this system.

Such an experiment would discover what current was flowing.

Note that this is not 'the truth about the nature of μ_0 ', but is an artefact of a particular choice of starting point in the network of definitions and relationships. It has nothing to do with the physical realities expressed by the whole pattern of relationships.

The constant ε_0 was introduced in Unit E, 'Field and potential', to express the magnitude of the forces between charges. But ε_0 and μ_0 are related, through the result

$$c^2 = 1/\varepsilon_0\mu_0$$

for the velocity c of electromagnetic waves.

On a relativistic view, when charges move with velocity v , the electric force between them is modified by terms of order v^2/c^2 . This modification (when both move) is the 'magnetic' force between them. The one physical measurement of a constant that it is possible to make here, is the *ratio* of the 'electric' to the 'magnetic' forces between charges at 'rest' and in 'motion'. The constant quantity c^2 expresses the magnitude of this ratio (which also depends on velocity v). This statement still holds good if one has not seen a relativistic interpretation, nor come to suppose with Maxwell that $1/\varepsilon_0\mu_0$ is the square of a wave velocity. c^2 is then a constant, dimensionally the square of a velocity, whose deeper 'meaning' is unknown.

There is much to be said for the view that one of the two quantities ε_0 or μ_0 need never be introduced. ε_0 could be replaced by $1/c^2\mu_0$, or μ_0 by $1/c^2\varepsilon_0$, throughout. Formulae for electric and magnetic forces then fall into a form in which the ratio of forces in comparable situations has the value v^2/c^2 (or v_1v_2/c^2). We have not, however, found any way of making this move which seems to us to be intelligible to a student at the time when μ_0 first appears, before any discussion based upon Maxwell's theory or on relativity suggests that the product $\varepsilon_0\mu_0$ is equal to $1/c^2$.

It is sometimes claimed that ε_0 is not a measurable constant, but is to be calculated from c^2 (measured) and μ_0 (chosen by definition). It may be so regarded, but it need not be. It was, after all, the electrical calculation of c^2 via (in effect) a *measurement* of ε_0 , that gave Maxwell hopes of his electromagnetic theory of light.

The vectors B and H

Although only B is mentioned in the course, H is present, in a sense, as ampere turns per metre. Just as current turns produce flux in iron, so, for the purpose of dealing with flux in materials at a later stage, H may in some ways be regarded as the 'cause' of B . We think, then, that the pair of quantities, B and current turns, is an adequate representation of

the structure of ideas which will come later, and simple enough for a student to grasp the ideas without the confusion of introducing too many quantities, too soon.

Relativity and electromagnetism

One difficulty of teaching about electromagnetism is that one tends to treat magnetic effects as divorced from the rest of electricity. From a relativistic point of view, magnetic effects *are* electric effects, seen from a moving reference frame. More exactly, different observers assign to ‘electric’ and ‘magnetic’ effects different proportions of one multi-component electromagnetic field.

In this sense, relativity ‘explains’ magnetic effects; that is, it relates them to other ideas.

Relativity is involved in some of the simplest experiments in electromagnetism. It is soon found that the induced e.m.f. in a wire moving relative to a magnet depends only on the relative motion of the two. Yet when the wire moves, the force on electrons in the wire is written as BQv and is called ‘magnetic’, whilst when the magnet moves, there is no help for it but to call the *same* force on electrons in the wire ‘electric’, for they are at rest, and no magnetic force exists on charges at rest. One’s prejudice that the name of the effect is decided by the nature of the source (a magnet) has here to be set aside. An observer travelling along at the appropriate velocity will say just the same, but will assign ‘electric’ and ‘magnetic’ in the opposite way. It is as well, in teaching the subject, to have this larger perspective at the back of one’s mind.

APPENDIX II DIFFICULTIES AND BACKGROUND ISSUES IN TEACHING THERMODYNAMICS

Indistinguishability of molecules

Teachers need to be aware that the model, used in Section K1, of marked counters moving in various regions of space cannot strictly be applied to a gas, simply because gas molecules are in principle indistinguishable. A correct calculation of entropy change with volume gives the same answer, however. Its form is as follows. A molecule is thought of as a particle confined in a box, so that it is described by a three-dimensional de Broglie standing wave. Possible energies are then determined by the wavelength associated with the numbers of nodes in each direction. It is easily calculated that energy kT for all but very low temperatures corresponds to very high-order modes, so that gaps

between energies of quantum states are negligible and the energy is to a good approximation continuous. For just this reason the simple kinetic theory works well. Even so, possible numbers of ways of distributing molecules amongst the various states available can be written down, and changes of volume accounted for by imagining that the confining box is expanded, so altering possible wavelengths of de Broglie waves. The argument is complicated, but the result is simple: the number of possible arrangements is proportional to V^N (recall that the simple dice game in Section K1 gave $(V_2/V_1)^N$ as the factor by which W increases). The different arrangements are best thought of as arising from different *motions* the molecules can have rather than from different *positions* as in the dice game.

It may well be worth telling students that the entropy change calculated for a gas is called a *translational entropy*, because it arises from molecules moving from place to place.

Free energy

In Section K1 the quantity TdS , where dS is the total entropy change, is calculated, and shown to be equal to the work that could be obtained. The quantity TdS is a free energy: that is, the maximum work available from a process. In the case of doubling the volume of a mole of gas at (say) 300 K, the free energy change is

$$T\Delta S = 300 \text{ K} \times 6 \text{ J K}^{-1} = 1800 \text{ J}$$

One may get less work out of doubling the volume of a mole of ideal gas, but not more. The reason is that free energy is just another way of talking about entropy. The *net* entropy *cannot decrease*: if the entropy of the gas increases, a balancing decrease in the entropy of the surroundings can be achieved by taking energy $T\Delta S$ from surroundings at temperature T , with the gas doing this amount of work. For this purpose, 'work' means energy change with zero entropy change.

Free energy is generally regarded as an advanced and difficult concept, and it is rarely made clear that it is no more than a way of considering the total entropy change in a process, expressed in terms of energy. It is *not* suggested that it be discussed in the physics course, unless to do so would be helpful, particularly to chemists who find considerable use for the idea. It may help teachers if they realize that the quantity called 'energy' or 'fuel' in very elementary beginnings of teaching about energy is in fact much closer to the concept of free energy than to that of energy itself.

The idea has further use in Section K4. Thus the entropy increase in a concentration cell can be exploited to get work, and the maximum

electrical work, from which the e.m.f. is found, is again a free energy.

The argument for the inefficiency of a heat engine exploits the same idea once again. Thermal transfer of energy from hot to cold increases entropy, and $T\Delta S$ is the maximum work obtainable.

Quantum mechanics and counting

In Section K2, energies are treated as discrete and energy states as countable. Thus quantum mechanics has the conceptual advantage of allowing us to think simply in terms of *counting*. Boltzmann and others had to find artificial devices, such as splitting up the 'space' defined by the momenta of particles into 'cells'. Where energy and momentum were continuous, progress could only be made in this way: the quantum view is simpler and more fundamental.

In the argument of Section K2, the distinction between energy level and quantum state is blurred. What is the case is that any closed assembly of particles has a definite number of possible quantum states, and W is just the total number of such states. All are taken to be equally likely. There is no reason for all such states to have different energies, and it will often be the case that many share the same energy (degeneracy).

In the argument, we have supposed something less general, namely a set of energies for the whole system, with one state for each particle associated with one of these energies. The next step to greater generality would be to think of the levels as unequally spaced, then to allow several states with the same energy. Finally, one would admit that the quantum states belong to the system as a whole, not individually to particles one at a time. (All these steps are needed to deal with gases, for example.)

The value of the simple, equally spaced level argument is that it throws into prominence the part of the answer which is common to more complex systems. Thus the exponential distribution reappears in any system (if it has Boltzmann statistics to a good approximation) but modified by other terms which reflect the varying intervals between levels and the numbers of states with the same energies. In general, one gets

energy distribution = density of states factor \times Boltzmann factor

For equally spaced levels, with one state of a particle for each energy, the density of states factor vanishes, leaving only the central result obtained in the argument. Since it is the Boltzmann factor that comes from the statistics, whilst the density of states comes from the quantum mechanics of the system in question, the simplification loses less generality than might appear.

The argument presented also avoids the calculation of W for a given number of particles with given total energy. Appendix III shows how to calculate W .

Energy, entropy, and temperature

Section K2, building on the work on gases in Section K1, introduces temperature and the entropy change associated with energy transfer in a sequence whose logic may be unfamiliar.

First, entropy is taken to be defined by

$$S = k \ln W$$

In the arguments about a gas we find that, given this choice,

$$pdV = TdS$$

in which T is necessarily on the ideal gas scale of temperature. However, consideration of entropy changes associated with particles distributed on energy levels leads us to identify

$$k \ln f$$

as the entropy change for removing one quantum. We argue that $\ln f$ is $d(\ln W)$ so that for a change of internal energy dU , with dU/ε quanta added, the entropy change is

$$dS = dU k d(\ln W)/\varepsilon$$

so that if we choose to write

$$kT = \varepsilon/d(\ln W)$$

we have

$$T = dU/dS$$

so that T is a rate of change of energy with entropy both for the gas and for the particles distributed on energy levels.

In all these expressions, the presence of the Boltzmann constant k is arbitrary. It might perhaps be clearer to employ some arbitrary constant k' , and identify it with k when showing that if we do so, temperature defined statistically is identical with the ideal gas scale.

Thermodynamic temperature

The equation

$$T = dU/dS$$

is not of course generally true. T can always be regarded as the rate of some energy change with entropy, but what the energy change has to be depends on the circumstances. In the present case, it is the energy change when the energy levels do not shift, but only the population of a fixed set of levels varies. This has a simple interpretation: it is what the quantum mechanics says will happen if the volume is constant. Thus one can correctly write

$$T = (dU/dS) \quad (\text{constant volume})$$

If, instead, the *pressure* is constant, then the energy change includes work pdV , with enthalpy change $dH = dU + pdV$, when

$$T = (dH/dS) \quad (\text{constant pressure})$$

Quite generally, if a process is carried out *reversibly*, so that the total entropy change is zero, then the entropy change of the material in which we are interested must exactly balance that of the surroundings, which will be dQ_{surr}/T , where dQ_{surr} is the energy transferred thermally between the material and the surroundings, that is, an energy exchange taking place under a temperature difference (albeit infinitesimal). Thus we also have,

$$dS = dQ_{\text{surr}}/T \quad (\text{reversible})$$

Notation for energy changes

We write dQ , not Q , since 'heat' is *not* a property of the material in question, unlike, say, the internal energy dU . The magnitude of dQ depends on the path from initial to final state, as well as on the two states, where dU , dH , etc. depend only on initial and final states. This is simply because a process can occur with any non-negative entropy change overall, so that the entropy change of the surroundings, dQ/T , has whatever value is required to add to or subtract from the entropy change dS of the material in question to give the actual total entropy change.

The essential point the teaching is intended to bring out is that temperature has the form

$$T = (\text{energy change})/(\text{change in chaos resulting from energy change})$$

Note that from this point of view, if T turns out to be related to mean energy per particle, we should regard that as a fact about the system in question, not as a defining property of T . Thus which of two systems is hotter depends not on which has most energy per particle, but on the direction of spontaneous energy flow between them. One should

therefore avoid saying which of two systems in the computer simulation is hotter until the direction of energy flow is known.

Entropy and life

Thermodynamics raises a question which ought to be unavoidable, but is often avoided. It might be put: 'If entropy always increases, how does life evolve?'. A common answer is that living beings are not closed systems, so that equilibrium thermodynamics does not apply. This, while correct, avoids the issue, since the Second Law *does* still apply.

Since the development of non-equilibrium thermodynamics, a better answer can be given. Firstly, entropy can decrease locally if paid for by a bigger increase elsewhere, and this is as true of a body making tissue from food and 'excreting heat', as it is of freezing ice cubes in a refrigerator. More deeply, it has recently been recognized that in systems far from equilibrium, large-scale structures can arise. A familiar example is convection currents in a heated fluid: a large-scale structure of motion driven by a large rate of production of entropy. Further, if a sufficient entropy increase is available, structured systems can stay structured and far from equilibrium. Thus the Sun, flooding the Earth with high-energy photons which increase entropy by being re-radiated as low-energy photons in greater numbers, provides a continual entropy increase which permits entropy-decreasing reactions to rebuild our bodies. It also allows grass to grow, through the entropy-reducing reaction of carbon dioxide and water to make sugars and cellulose. Another way of putting it is that the Sun provides a large supply of *negative* entropy (the high-energy photons), or better, a large supply of free energy. It is not the *energy* from the Sun that keeps us alive, but the *free energy* we get from it just because it is much hotter than the Earth.

It is perhaps too easy to talk about the Second Law in a pessimistic way, as stating a continual progress towards decay. Correct though this is, it remains true that new beings arise, and we can put off decay, not in conflict with the law but actually because of it. Newness comes from decay, and as the structures around us in the Universe show, newness and structure are far from exceptional.

APPENDIX III THE EINSTEIN SOLID

It was suggested that the energy shuffling game, Section K2, be presented just as shuffling particles on levels, without any specific interpretation. This appendix interprets the game as a model of internal energy in a monatomic solid, and uses it to get the specific heat capacity.

The essential steps in the argument are:

- 1 The energy levels of a harmonic oscillator are equally spaced, $\varepsilon = h\nu$. (ν is the classical vibration frequency.)
- 2 If atoms in a solid all oscillated independently then each atom would have energy levels the same as every other, but there would be no exchange of energy. If atoms oscillating affect their neighbours a lot, the levels of neighbours will depend on each other's motion. The *Einstein model* is a clever compromise: atoms oscillating with just a little interaction, so that there is energy exchange, but levels are not significantly affected. One may imagine this as each atom in a prison cell with slightly flexible prison walls.
- 3 We need to find how $\ln f$ depends on the number N of oscillators and the number n of quanta they share, where $n = U/\varepsilon$, U being the total internal energy. But $1/f = W'/W$, for one quantum added. The result of doing the statistics (see below) is:

$$W'/W = 1/f = (N + n)/(n + 1)$$

or

$$1/f \approx 1 + N/n = 1 + N\varepsilon/U$$

But

$$\ln(1/f) = \varepsilon/kT$$

so

$$kT = \varepsilon/\ln(1 + N\varepsilon/U)$$

which for large energy (high temperatures, $n \gg N$) is approximately

$$kT = U/N$$

- 4 Finally, since each atom oscillates independently in three directions, $N = 3L$ for one mole of atoms, and we have

$$U = 3kLT$$

so that the specific heat at constant volume is $3kL$. This is just the Dulong and Petit Law.

Experimental results confirm this result for monatomic solids. Discrepancies occur where they should: carbon has light, tightly bound atoms, so ν is large, ε is large, and n is small for a given U . (In fact if we do not make the approximation, $kT = \varepsilon/\ln(1 + N\varepsilon/U)$ gives a rather good prediction of its specific heat.) The form

$$kT = \varepsilon/\ln(1 + N\varepsilon/U)$$

can be rewritten

$$U = N\varepsilon/(e^{\varepsilon/kT} - 1)$$

which, with $\varepsilon = h\nu$, is how it generally appears in textbooks. The heat capacity dU/dT can then be obtained by differentiating.

It would be appropriate here to measure the specific heat capacity of, say, aluminium. A student who wanted a bigger challenge could try measuring the heat capacity of a block of carbon, which would present significant problems of experimental design and instrumentation.

The statistical part of the argument leading to the result $W'/W = (N + n)/(n + 1)$ can begin by noting that the state of a system of oscillators can be written as a list

o...o...o...oo.o...o...o.o.ooo.o.o..

where each o represents one of the N oscillators, and each dot following an o represents one quantum of the energy it has. The total energy is represented by all n dots.

Any permutation of such a list of $N + n$ symbols is a possible state of the same system, if the first symbol is an o. There are $(N + n - 1)!$ such permutations. But interchanging a pair of oscillator symbols or a pair of quantum symbols refers to the same state, and there are respectively $(N - 1)!$ and $n!$ ways of doing this, so we get

$$W = (N + n - 1)!/(N - 1)! n!$$

If one more quantum is added, we get

$$W' = (N + n)!/(N - 1)!(n + 1)!$$

and so

$$W'/W = (N + n)/(n + 1)$$

A perhaps simpler argument attempts to obtain directly the effect of adding a quantum. One more dot (quantum) can be put into the list anywhere, except at the front, so there are $N + n$ places for it to go. Whatever W (the number of lists) may be, adding one quantum multiplies it by $N + n$. But the $n + 1$ quanta are all alike, so we must divide by $n + 1$ to get the correct factor multiplying W , giving $W'/W = (N + n)/(n + 1)$ directly.

INDEX

A

accelerator (economics), 166
 accelerators, particle, 26, 27
 acoustic feedback, 147–8
 acoustical holography, 243
 activation energy, 300, 301
 activation processes, 289–90
 experimental studies, 290–300
 adenine, 375, 376
 adrenalin, 165
 aerials, 244, 257–9, 319
 alkali metals, electrical conduction in,
 361, 363, 364
 alpha particle, escape from nucleus, 355
 alternating current, 8, 49–60, 71–96
 in circuits containing capacitors, 80–83
 in inductors, 83–5
 in *LC* circuits, 85–8, mechanical
 analogue, 89–90
 in resistive circuit, 77–9
 induction of, 50–51
 induction using, 51–6
 measurement, 79
 operational amplifier response, 117
 signal mixer, with fade-in, 174–5
 smoothing rectified, 91
 three-phase, 71
 aluminium, dopant, 20
 Hall effect in, 21
 specific heat capacity, 430
 ammeters, 32
 moving-coil, 10
 response to a.c., 50
 ammonia molecule, inversion, 355
 shape, 369, 370
 ampere, defined, 32–4, 420–21
 amplitude, related to probability,
 204, 333, 341, 352, 353
 related to wave energy, 203, 204, 333
 analogue computers, 142, 178–81
 analogues, 93–4
 mechanical, in *LC* circuits, 89–90
 anemometer, 177
 arsenic, dopant, 20
 astable multivibrator, 185
 atomic nucleus, energy levels, 334, 349
 wave-in-a-box model, 348–9
 atoms, electron waves in, 204, 342–76
 energy levels, 222, 279, 334, 342;
 computer prediction, 356–7, 358; of
 helium ion, 359; of hydrogen atom,
 330–31; splitting, 364

 size, 342
 stability, 342
 total energy, 345, 346–7
 Avogadro constant, 274, 275
 Perrin's method of measurement, 308

B

bacterial population growth, 165
 Balmer spectrum, 329, 330
 Balmer's rule, 331, 358
 band brake, 44, 45
 benzene ring, 364
 beta gauge, use in rolling mill, 159–60
 biological organisms, evolution, 428
 feedback in, 164–6
 hydrogen bonding in, 374–6
 importance of Boltzmann factor in, 301
 osmotic pumps in, 311
 temperature control in, 301
 blazing (grating), 233
 blood, circulation, 165
 sugar level, 165
 body temperature control, 164
 Bohr, N., quoted, 333
 Bohr's theory of atom, 343
 Boltzmann, L., 425
 Boltzmann constant (*k*), 275, 277, 288,
 426
 Boltzmann factor (*f*), 279, 284, 289–301,
 361, 425
 bond angles, 369
 bonding, 365–76
 brain, 164, 165, 243
 eye an extension of, 218
 breathing, 165
 bridge circuit, balance detector for, 139,
 176–7
 buffer circuits, 135

C

capacitance meter, 182–3
 capacitors, in a.c. circuits, 80–83, 85–8;
 mechanical analogue, 89–90
 to improvise, 91
 voltage across, measurement, 137–8
 carbon, conjugated chains, 364
 heat capacity, 430
 orbitals, 370
 specific heat capacity, 429
 see also diamond; graphite
 carbon dioxide, effect of blood level, 165

cassette/slide programmes, electrons in solids, 365
 quantum mechanics, 359
 'catapult field', 8, 10–11
 chance, 271
 related to amplitude, 333, 341
 see also randomness
 chemical plant control, 105, 157
 chemistry, wave mechanics in, 365–76
 chiffon, diffraction due to, 222, 238
 closed-loop control systems, 153–5
 coaxial cable, electromagnetic wave pulse in, 248–52
 cobweb theory, 166
 coils, fields near, 27, 30, 31–2
 mutual inductance, 61–3
 see also solenoids
 common-mode effects, 129, 135, 139
 communications satellites, orientation, 106
 comparator, 138, 154, 177
 as switch, 173–4
 complex gratings, 237–40
 compressibility of solids, 348
 computer programs, amplitude of
 electron waves, 348
 diffraction and interference patterns, 207, 210, 233
 dynamic modelling system, 96, 210, 233, 357
 energy level prediction, 356–7, 358
 energy shuffling, 281
 equilibrium distribution of energy, 279, 281–4
 mass spectrometer simulation, 26
 particle shuffling, 274
 phasor diagrams, 210
 thermal equilibrium, 285–6
 computers, in control systems, 157
 see also analogue computers; digital computers
 concentration cell, 302, 309–11, 424–5
 condensation, 303, 308
 conservation of energy, 42
 control systems, 105–6, 152–8
 closed-loop, 153–5
 continuous, 107–10, 155, 157, 158
 feedback in, 153–4
 illumination, 106–10, 155–7
 on-off, 155–6
 open-loop, 152–3
 oscillations in, 158–63
 temperature, 153–4, 158, 183; biological, 164, 301; with thermal inertia, 161–3
 'copper loss', 76
 copper sulphate concentration cell, 309–11

creep, 297–9
 crossover filters, 92, 93
 crystallization, 308–9
 crystals, 240, 260
 current balances, 13–15, 16–17, 23, 34–5
 currents, absolute measurement, 34–5
 fields near, 27–36
 forces between, 9–10
 forces on, 9–10, 12–16, 18
 forces on induced, 11–12
 in operational amplifier, 125–6
 in thermistor, 293–5
 unit, 32–4, 420–21
 cybernetics, 163
 see also control systems
 cytosine, 375, 376

D

damped oscillations, 88–9, 94, 95–6, 151–2, 159, 160
 de Broglie relation, 337, 339, 340
 de Broglie wave, 423–4
 delayed feedback, 148–9, 151–2, 158–9, 160, 166
 delocalized electrons, 363–4
 demonstrations, 5
 deuteron formation, 348
 diabetes mellitus, 165
 diamond, insulating properties, 360, 364
 differential amplifier, 129, 138–9, 177
 differentiating circuit, 127, 131–2
 diffraction, 331
 at grating, 221–43; effect of number of slits, 225–8; effect of slit width, 230, theory, 225
 at single aperture, 194–202, 205–15
 effect on resolution, 216–19
 Fraunhofer and Fresnel, 205
 see also electron diffraction; X-ray diffraction
 digital computers, 142, 157
 digital control systems, 163
 digital-to-analogue conversion, 128, 171
 diode, hot-wire, 290
 dipole aerials, 244
 direct current, measurement of
 unsmoothed, 79
 dislocations, movement, 298
 disulphate(iv) ion, reaction with methanal, 300
 DNA, 301, 374–6
 doped semiconductors, 20, 365
 Dulong and Petit Law, 429
 dynamic modelling system, 96, 210, 233, 357
 dynamo, induced e.m.f.s in, 41–3

E

ear, separation of frequencies by (home experiment), 222

economics, feedback in, 166–7

eddy currents, 45, 74, 75–6

efficiency, of d.c. motor, 45–6
of engines, 303–5, 425

Einstein, A., and photoelectric effect, 323
and relativity theory, 8

Einstein solid, oscillating atoms in, 279,
428–30

electric charge, force on moving, 18–27

electric fields, 244
associated with ‘tied’ wave, 247; in
coaxial cable, 250–52
related to magnetic field, 40

electrical conduction, 360–65

electromagnetic fields, 8

electromagnetic induction, 5, 6, 37–70

electromagnetic waves, 194–264
detection, 319–20; *see also* aerials
free, 244, 247–8, 255
nature, 334
similarities and differences, 319
speed, 245, 422; constancy of, 255;
measurement, 248–52
‘tied’, 244, 245–7, 255
see also light; and headings such as
gamma rays; microwaves; etc.

electromagnetism, applications, 12
teaching of, 4–6, 244, 420–23

electron diffraction, 240, 335
by graphite, 335–9
optical analogue, 337–9

electronics, linear, 100–186
applications, 169–86

electron–positron storage ring, 26

electrons, 335–41
in semiconductors, 293–4
in solids, 360–65
mass, 25
specific charge, to measure, 24–5
waves, 240, 335–41; in atoms, 204,
342–76
see also orbitals; photoelectric effect

energy, changes, notation, 427–8
conservation, 42
equilibrium distribution, 281–4
of waves, 203–4
quantization, 279, 323, 326, 334
related to entropy and temperature,
278–88, 426
sharing, 278, 279, 280, 284
storage for solar-heated homes, 309
thermal transfer, 277, 285–8
transfer in vibrating systems, 94

energy bands, 360–65

energy levels, 278, 334
molecular, 334
nuclear, 334, 349
of particles, 279–80, 281–4, 285–6, 290,
425
see also under atoms

energy shuffling (game), 280–84, 428

engines, inefficiency, 303–5, 425

enthalpy change of crystallization, 309
of vaporization, 291–2

entropy, 268–9, 271–7, 304
and free energy, 424–5
and life, 428
change of, to calculate, 273–7; to
exploit, 302–12
defined, 272
related to energy and temperature,
278–88, 426

enzymes, 165, 301

equilibrium, 302, 307–9
distribution of energy, 281–4
in particle shuffling game, 274
thermal, 285–6
zero total entropy change, 302

error detector, 154

etalons, 233

evaporation, 291–2, 303

exclusion principle, 364

explosives, 308

exponential change, 140, 141, 144, 165

exponential distribution (energy), 282–4,
425

eye, 194
resolving power, 217, 218
unable to separate frequencies (home
experiment), 222

F

fan control, 183

Faraday, Michael, 8

Faraday’s Law, 40, 47, 51

feedback, 140–68
examples in biology, 164–6
examples in economics, 166–7
in control systems, 153–4
use of term, 106n.
see also negative feedback; positive
feedback

feedback amplifier circuit, 119–20
applications, 126–32
input–output characteristic, 120–21

ferrite rod aerials, 244, 258, 319

ferrites, 76

Feynman, R. P., 334, 346, 347

film loops on standing waves, 345

films, matter waves, 340
photons, 333

filter circuits, 92–3
 flasher, xenon (safety note), 46, 343
 flow, viscous, 296–7
 flux, *see* magnetic flux
 fluxmeter, 176
 follower circuits, 133–5
 forced oscillations, 149, 150
 formaldehyde, *see* methanal
 Fraunhofer diffraction, 205
 free electromagnetic waves, 244, 247–8, 255
 free energy, 424–5, 428
 freezing, 308, 309
 frequency generators, 151
 Fresnel, A. J., 202
 Fresnel diffraction, 205

G

gamma ray spectroscopy, 334
 gamma rays, detection, 320
 polarization by scattering, 262
 gases, energy/entropy changes in
 compressed, 302, 305–7
 entropy change on expansion, 275–7, 306, 423–4
 free energy change on expansion, 424
 spectra, *see under* hydrogen; mercury
 Geiger–Müller (GM) tube, 320
 generators, 71
 genetic code, 375
 germanium, semiconducting properties, 360, 365
 ‘ghost waves’, 341
 gigahertz (30 cm) waves, polarization, 256–7
 Glauber’s salt, *see* sodium sulphate
 decahydrate
 glucagon, 165
 glucose, level in blood, 165
 gramophone record, as reflection grating, 235–6
 graphite, electron diffraction by, 335–9
 gratings, complex, 237–40
 reflection, 233–6, 327
 see also under diffraction
 guanine, 375, 376
 ‘guided’ electromagnetic waves, *see* ‘tied’
 electromagnetic waves

H

Hall effect, in aluminium, 21
 in semiconductors, 18–20, 21–2
 Hall probe, 27–8
 to calibrate, 22–3
 Hall voltage, 19–20
 ‘heat’, 305, 427
 heat engines, 303–5, 425

heater control, 183
 helium ion, 359
 henry (unit), 63
 Hertz, Heinrich, 323
 hi-fi systems, filter circuits, 92–3
 ‘holes’ (in semiconductors), 20
 holography, 240–43
 homeostasis, 164
 hormones, 165
 ‘hot ice’, 308–9
 hot-wire anemometer, 177
 hot-wire diode, 290
 Huygens’s construction, 201–2
 hydrogen atom, formation, 348
 electron waves in, 345–50, 352–9;
 models: $1/r$ potential well, 348,
 349–50, 352–9, wave-in-a-box, 345–8
 energy levels, 330–31, 342, 366
 nuclear reaction in Sun, 301
 orbitals, 368
 size, 347–8, 355
 spectrum, 329–30
 wave function, 355
 hydrogen bonds, 371–6
 in biological organisms, 301
 hydrogen fluoride, hydrogen bonding in, 371–3
 hydrogen–hydrogen bond, 365–6
 hydrogen sulphide, compared with water, 370–71, 373
 hypothalamus, 164
 hysteresis, magnetic, 45, 74, 76

I

illumination control, 106–10, 155–7, 158
 image recombination, 210, 212–15
 immersion heaters, safety note, 292
 income, circular flow, 166–7
 induced e.m.f., 37–42
 and magnetic flux, 46–9
 Faraday’s Law, 40, 47, 51
 in d.c. motor, 44
 in motor/dynamo, 41–3
 Lenz’s rule, 41
 inductance, 61–70
 induction, *see* electromagnetic induction
 inductors, a.c. in, 83–8
 mechanical analogue, 89–90
 inertia, 161–3
 effect on oscillations, 158
 information storage by holography, 243
 infra-red radiation, identification in
 spectrum, 233, 235
 infra-red spectroscopy, 334
 insulators, 360, 364
 insulin, 165
 integrating circuit, 127, 129–31

- with feedback, 140–44
- intensity, wave, 203, 204, 333
- intercom, 174
- interference, single photons, 331–3
 - see also* superposition of waves
- interferometers, radio, 219
 - model, 219–20
- inverter circuit, 143
- ionization energy, 330
- iron, effect on flux in solenoid, 56–7
 - in magnetic circuits, 57–60
 - in transformer, 72
- 'iron losses', 45, 74, 76

J

- Jodrell Bank radio telescope, 219
- 'jumping ring' demonstration, 11–12, 69–70

K

- kinetic energy, related to momentum and wavelength, 340
- kinetic theory of gases, 424
- Kirchhoff, G. R., 202
- Kirchhoff's Second Law, 42, 44, 65

L

- laminated transformer core, 76
- lasers, diffraction patterns using, 207–8, 211–15, 226–7, 237–40
 - safety note, 207
 - see also* holography
- left hand (force-direction) rule, 15–16, 20
- Lenz's rule, 41, 64
- letters, diffraction patterns produced by, 211–12, 214
- life, entropy and, 428
- light, diffraction, 194–5, 198–9, 205–15, 222–4, 226–40; effect on resolution, 216–18; home experiment, 200
 - eye unable to separate frequencies, (home experiment), 222
 - intensity, 333, 334
 - nature, 201, 320, 326, 331–2, 333–4
 - polarization: by Polaroid, 259–60; by reflection and scattering, 261–2; home experiment, 263
 - spectrum, 234–5
 - speed, measurement, 253–4
 - wavelength, measurement: using diffraction grating, 228, 230–31; using Young's fringes, 230, 232–3
 - see also* holography; lasers;
 - photoelectric effect; photons
- light follower, 110–13, 163
- light meter, 171

- linear electronics, 100–186
- linear variable differential transformer, 184
- linear variable transformer, 184
- 'lone pair' electrons, 370
- Lyman spectrum, 330

M

- magnetic field constant, 33
- magnetic fields, 6, 8–36, 244
 - associated with 'tied' wave, 247; in coaxial cable, 250–52
- catapult, 8, 10–11
 - continually changing, 50–51
 - direction of forces in, 13–16
 - measurement, 5, 12–18, 27–36; using Hall effect, 19, 22
 - moving wire in, 37–40
 - near currents, 27–36
 - strength (H), 422–3
- magnetic flux, 4, 47
 - and induced e.m.f., 46–9
 - continuous nature, 57
 - effect of iron, 56–8
 - linkage, 48
- magnetic flux density, 17, 421, 422–3
 - measurement, fluxmeter for, 176
 - unit, 18
- magnetic hysteresis, 45, 74, 76
- magnetic induction, *see* magnetic flux density
- magnetization cycle, 176
- mass-and-spring system, 89–90
- mass spectrometers, 26, 27
- Maxwell, J. Clerk, 8, 163
 - theory of light, 422
- mercury, safety note, 328
 - spectrum of vapour, 221–2, 326–9
- mercury discharge lamp, safety note, 221
- metals, creep in, 298
 - electrical conduction in, 360, 361, 362, 363
- methanal, reaction with disulphate(IV) ion, 300
- microwaves, 319
 - diffraction, 197–8
 - model of radio interferometer, 219–20
 - polarization, 255–6
- Millikan, R. A., 323
- mixer amplifier, 174–5
- molecules, energy, 340
 - energy levels, 334
 - indistinguishability, 423–4
 - random behaviour, 271, 272, 303
 - shape, 366–9
- momentum, related to kinetic energy and wavelength, 340

monatomic solid, *see* Einstein solid
motors, d.c., 42–6
 as dynamo, induced e.m.f.s in, 41–3
 efficiency, 45–6
 model, 10
motor rule, 15, 20
moving-coil ammeter, 10
muddy water, separation, 308
multiplication with operational amplifier,
 124
musical notes on strings, 331
mutual inductance, 61–3

N

National Grid, 71
negative feedback, 106, 123, 124–5, 141,
 142–3, 159, 160
 in biological organisms, 164, 165
nerve impulses, 311
neutron, energy, 340
neutron:proton ratio (in early Universe),
 290
nitrogen, 369
non-inverting amplifier circuits, 133, 135
NOT gate, operational amplifier as, 157
n-type semiconductors, 20–21, 22
nuclear binding energy, 348
nucleus, *see* atomic nucleus
number patterns, 331
numerical control systems, 163

O

‘obliquity factor’, 202
ohmmeter, linear, 172
on–off control systems, 155–6
 illumination, 156–7, 158
open-loop control systems, 152–3
operational amplifiers, 100, 105–39, 295
 closed-loop gain, 125
 currents and voltages in, 125–6
 equipment, 101
 input bias current, 123
 internal circuit, 114
 introductory experiments, 115–21
 open-loop voltage gain, 122, 123
 theory, 121–3, 124–5
 use of term, 113
 using non-inverting input, 132–9
 virtual earth, 123
orbitals, 367–74
 hybridized, 370, 373
 overlapping, 368–9
oscillations, atoms in Einstein solid, 279,
 428–30
 in control systems, 158–63
 in feedback system, 149–52
 in light follower, 112–13

 in public address system, 147–9
 of charge, origin of waves, 244
oscillator circuits, 5, 87–8, 93–4, 144–7,
 181
 model, 96
osmosis, 311
Ostwald viscometer, 296
oxygen, effect of blood level, 165
 electronic structure, 368
 in water molecule, 369, 370

P

particle accelerators, 26, 27
particle shuffling (game), 273–4, 423, 424
particles, energy levels, 279–80, 281–4,
 285–6, 290, 425
Paschen spectrum, 330
Pauli exclusion principle, 364
permeability of free space, 33, 421–2
permittivity of free space, 422
Perrin’s method for Avogadro constant,
 308
phase change on reflection, 197
phase-sensitive detector, 184
phase shift in oscillating system, 148–9,
 151
phase shift oscillators, 151
phasor diagrams, 207
photoelectric cell (simple), 322–3
photoelectric effect, 320–26, 339
photographic enlarger, timer for, 181–2
photons, 204, 222, 319–34
 energy, 334, 348
 energy and momentum, 340
photosynthesis, 302, 428
‘pilot waves’, 341
Planck’s constant, 323, 325, 326, 333
polarization of waves, 255–7, 258–93
 home experiment, 263
Polaroid, 259–60
polythene, creep in, 298–9
 stretched, between Polaroids, 260
population growth and control, 165
positive feedback, 142–4, 147–8, 150–51,
 160
 in biological organisms, 164, 165
potassium manganate(vii), absorption
 band, 231
potential difference across inductor, 65
power, in a.c. circuits, 5
 in capacitor, 82–3
 in inductor, 84
 in resistive circuit, 77–9
 input and output, of transformer, 73–4
 output of d.c. motor, 45
power stations, 304
power transistor, 295

power transmission, 49, 71
 predator-prey interaction, 165
 pressure cooker, 299
 proton synchrotron, 26
 protons, in nucleus, 348
 see also neutron:proton ratio
 proximity meter, 60
 p-type semiconductors, 20–21, 22
 public address system, feedback in, 147–8
 Pythagoras, 331

Q

Q-factor, 94–5
 quadrature oscillator, 151
 quantization, of energy, 279, 326
 of light energy, 323, 326, 334
 quantum mechanics, 272, 342, 361, 425–6
 'Quantum shuffling' (computer simulation), equilibrium distribution of energy, 279, 281–4
 thermal equilibrium, 285–6

R

radar, *Q*-factors of receivers, 95
 radio, 90–91, 92, 319
 Q-factors of receivers, 95
 radio interferometer, 219
 model, 219–20
 radio telescopes, 219
 radio waves, 319
 detection, 319
 polarized, 258
 predicted by Maxwell, 8
 reception, 257–9
 radioactive decay, computing circuit for, 178, 179–81
 randomness, 271, 303
 Rayleigh's criterion, 217–18, 229
 reaction rates, 290
 biological, 301
 disulphate(iv)-methanal reaction, 299–300
 reflection, phase change on, 197
 polarization by, 262
 reflection gratings, 233–6, 327
 relative permeability, air, 33
 iron, 59; measurement, 58
 variation with flux, 60
 relativity theory, 8, 18, 263, 340
 relay, protective diode for, 174
 reluctance, 4, 59–60
 repeaters (telecommunications), 163
 resistance, measurement, ohmmeter for, 172
 of strained wire, measurement, 177
 resistance thermometer, 177
 resolution, 216–18, 229

resonance, 94–6, 149
 in *LC* circuits, 86–7; applications, 90–93
 right hand fingers and thumb rule, 15–16
 ripple tank, 196–7, 201–2, 224–5
 rolling mill, 159–60
 root mean square values, 78–9
 rubber, electron diffraction by, 240
 standing waves in sheet, 344–5; on cords, 343–4, 350; on V-shaped strip, 351–2
 rule, metal, as reflection grating, 236

S

satellites, communications, 106
 scattering, polarization by, 261–3
 Schrödinger's equation, 357–8
 simplified form, 356
 search coils, 27–8, 50
 Second Law of Thermodynamics, 268, 269, 302–3, 428
 self inductance, 63–9
 semiconductors, 293, 355, 360–61, 364–5
 Hall effect in, 18–20, 21–2
 types of, 20–21
 shape of molecules, 366–9
 shipbuilding, 166
 silicon, doped, 20
 semiconducting properties, 360, 364
 similarities in physics, 93–4
 sink, battery or inductor as, 65
 slide projector, 215
 diffraction pattern using, 209–10
 Slinky solenoid, field in, 29, 30, 31
 sodium disulphate(iv), reaction with methanal, 300
 sodium sulphate decahydrate ('hot ice'), crystallization, 308–9
 solar heating, energy storage, 309
 solenoids, fields around, 31
 fields in, 17, 23, 29, 30, 31, 34, 35–6
 flux in, effect of iron, 56–7
 induced e.m.f. in, using a.c., 51–6; effect of iron, 56–7
 sound waves, amplitude and energy of superposed, 203–4
 separation of frequencies by ear (home experiment), 222
 see also ultrasound
 specific heat capacity of Einstein solid, 428–30
 spectra, 222
 helium ion, 359
 hydrogen, 329–30
 mercury, 326–9
 using reflection grating, 234–5
 spectroscopy, 228
 energy levels, 334

- simple spectroscope, 222
- standing waves, 331, 343
 - electrons as, 343, 345–50, 352–9
 - formed by reflection, 246, 248
 - in rubber sheet, 344–5
 - in spherical atoms, 352–3
 - molecule described by, 423–4
 - on hanging chain, 351
 - on rubber cords, 343–4, 350–51
 - on V-shaped rubber strip, 351–2
 - with variable wavelength, 350
- steam, detection, 174
- steam engine governors, 163
- steel rolling mill, 159–60
- strained wire, to measure resistance, 177
- strong nuclear force, 348
- subtractor circuit, 127, 128–9, 139
- sugar level in blood, 165
- summing amplifier, 127–8
- Sun, 301, 309, 428
- sunlight, spectrum (home experiment), 222
- superposition of waves, 197, 198, 203–4, 240
 - see also* holography
- supply cycles, 158, 166
- swing, oscillations of, 159
- switch, comparator as, 173–4

T

- Taylor, G. I., 332
- telecommunications, repeaters for, 163
- television, *Q*-factors of receivers, 95
- television and video programmes, control systems, 167
 - determination of wave velocities, 254
 - electron diffraction, 336
 - physics beyond experience, 359
- temperature, 276, 427–8
 - related to entropy change, 285–7, 426
 - thermodynamic, 288, 304, 426–7
- temperature control, 153–4, 158, 183
 - biological, 164, 301
 - with thermal inertia, 161–3
- tesla (unit), 18
- thermal equilibrium (computer simulation), 285–6
- thermal transfer of energy, 277, 285–8
- thermionic emission, 290
- thermistors, 102
 - current in, 293–5
 - use in temperature control, 161–3, 183
- thermodynamic temperature, 288, 304, 426–7
- Thermodynamics, Second Law, 268, 269, 302–3, 428
 - teaching, 423–8

- 'Zeroth' Law, 287
- thermometer, resistance, 177
- thymine, 375, 376
- 'tied' electromagnetic waves, 244, 245–7, 255
- time lag in oscillating system, 148–9, 151
- timer, for photographic enlarger, 181–2
 - short-interval, 175
- torque produced by motor, 44
- transducer, 109
 - variable-reluctance, 184–5
- transformers, 5, 50, 51, 71–6
 - 'jumping ring' system, 69
 - linear variable, 184; linear variable differential, 184
- transistor, 295
- translational entropy, 424
- 'tunnelling', 355

U

- ultrasound, diffraction, 199–200
- ultra-violet light, identification in spectrum, 233, 234–5
 - photoelectric effect, 320–23
 - safety notes, 221, 320, 328
- umbrella material, diffraction due to, 222
- units, current, 32–4, 420–21
 - inductance, 63
 - magnetic flux, 47
 - magnetic flux density, 18

V

- vapour pressure, water, 291–3, 299
- variable-reluctance transducer, 184–5
- vector notation, 18
- velocity damping, 151
- vertical motion, computing circuit for, 178
- vibration monitor, 60
- video tapes, *see under* television and video programmes
- viscous flow, 296–7
- voltage, in operational amplifier, 126
- voltage follower, 135–8
- voltmeter, high impedance, 172

W

- water, compared with hydrogen sulphide, 370–71, 373
 - condensation, 303, 308
 - detection, 174
 - evaporation, 303
 - freezing, 308, 309
 - hydrogen bonding in, 371, 373–4
 - muddy, separation, 308
 - shape of molecule, 369, 370

vapour pressure, 291–3, 299
water waves, diffraction, 196–7, 201–2,
224–5
Watt, James, 163
wave detectors, 204
wave functions, 354–5
 overlapping, 364, 366
 without spherical symmetry, 366–7
wave mechanics, relevance in chemistry,
365–76
 scope, 359–65
waveguide, 245, 246
wavelength, of light, measurement: using
 diffraction grating, 228, 230–31;
 using Young's fringes, 230, 232–3
of X-rays, measurement, 235, 236
related to kinetic energy and
momentum, 340
weber (unit), 47
Wheatstone bridge, *see* bridge circuit
white-light holograms, 243
wires, field of carpet of, 35–6

fields near, 27, 29, 30, 32, 33
induced e.m.f. in moving, 37–40, 48
measurement of resistance of strained,
177
work, 424–5
 done by heat engine, 304, 305

X

xenon flasher, safety notes, 46, 343
X-ray diffraction, 237, 240
X-rays, 319
 measurement of wavelength, 235, 236
 polarization by scattering, 262

Y

Young's double-slit experiment, 230,
232–3

Z

'Zeroth' Law of Thermodynamics, 287

**General editor,
Revised Nuffield
Advanced Physics**
John Harris

Consultant editor
E. J. Wenham

**Editors of Units in
this Guide**

Stephen Borthwick
Peter Bullett
David Chaundy
John Harris
Wilf Mace
Jon Ogborn

**This Teachers' guide supports the last five
of the twelve Units in the Revised
Nuffield Advanced Physics course.**

**The Units are: Unit H, 'Magnetic fields
and a.c.'; Unit I, 'Linear electronics,
feedback and control'; Unit J,
'Electromagnetic waves'; Unit K, 'Energy
and entropy'; and Unit L, 'Waves,
particles, and atoms'.**

**For each Unit there is a plan
summarizing the relationship of the
work in the Unit to topics covered
elsewhere in the course. There are very
many teaching suggestions, including
details of the suggested experiments
and demonstrations; and there are
references to other resources, for
example, books, videos, and computer
programs. Full answers to the questions
in the Students' guide are provided.**

ISBN 0 582 35418 8